Randomness and Computation: Written Problem Set 1

John Marangola

February 26, 2021

1.

(a)

$$b(n, p, k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{1}$$

Under the assumption that Latouch has the ability that she claims, the sample space Ω is given by the number of times Latouch guesses correctly:

$$\Omega = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, \text{ such that } m(\omega) = b(10, 0.75, \omega) \ \forall \omega \in \Omega$$

Let E_1 be the event that Latouch wins the bet such that:

$$E_1 = \{7, 8, 9, 10\}$$

$$\begin{aligned} P(E_1) &= \sum_{k=7}^{10} b(10, 0.75, k) = \sum_{k=7}^{10} \binom{10}{k} (0.75)^k (0.25)^{10-k} \\ &= \binom{10}{7}(0.75)^7(0.25)^3 + \ \dots \ + \binom{10}{10}(0.75)^{10} \end{aligned}$$

$$P(E_1) = 0.7758750916 \approx 0.776$$

(b) Now, assume that Latouch is bluffing. If Latouch has no special ability, the probability that she makes a single guess correctly is 0.5. Likewise, the mean of the binomial distribution corresponding to the probabilities of X is given by $np = (10)(0.5) = 5$. Letting E_2 be the event that Latouch loses the bet given that she is lying about her ability:

$$E_2 = \{0, 1, 2, 3, 4, 5, 6\}$$

$$P(E_2) = \sum_{i=0}^{6} b(10, 0.5, i) = 0.828125 \approx 0.828$$

2.

We define the null hypothesis H_0 to be that Dr. Venkman is guessing. It follows naturally that α, β are the probabilities of Type I and Type II errors, respectively. More precisely, α is the probability that H_0 is rejected when Dr. Venkman is simply guessing and β is the probability that we fail to reject the null hypothesis when Dr. Venkman does indeed possess ultrasensory powers.

From eqn (1) above,

$$\alpha(m) = \sum_{k=m}^{N} b(N, p, k)$$