Transformer Design
60Hz & 60kHz

Joseph Arsenault

Abstract

The design and simulation of a non-ideal, step-down transformer are described. One design steps down a sinusoidal 60 Hz input voltage waveform, while the other steps down a sinusoidal 60 kHz input voltage waveform. Both were required to be rated for a $120V_{RMS}$, $100mA_{RMS}$ input. The standby current, the current when no load is connected, was required to be between 4mA and 5mA. The percentage of the cross sectional area of the core available to be filled with wire windings was to be 50%. Both transformer designs minimized dimensions and losses.

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1 Introduction

This report describes the design and simulation of two different transformers. Both are required to minimize physical dimensions as well as losses. The amount of cross sectional area occupied by the core windings is limited to 50% of the core. The approximations present for the ideal model form the foundation for the non ideal analysis. The ideal model fails to account for the presence of magnetizing inductances, linkages inductances, core losses, and resistance of the windings. In order to design a transformer, physical dimensions must be taken into account, the corresponding magnetic circuit must be analyzed, and finally the resulting electric model circuit must be analyzed. Section 2 of this report describes the design of both the 60 Hz and 60 kHz transformer. Section 3 covers the simulations. A discussion of the results and conclusion of the report are found in section 4.

2 Circuit development

This section covers the design choices associated with the two transformer designs. Both designs feature similar approaches; first applying physical constraints, analyzing the magnetic model, and then finally analyzing the resulting electric model. The 60 Hz transformer is described first followed by the 60kHz transformer.

2.1 Magnetic model

The design of the non ideal transformer proceeds from the application of physical constraints covered in lecture. The design specifications are as follows; the transformer must have a ratio of N2/N1 of 1/12, The input waveform is 120 V_{rms} and a current of 0.1 A_{rms}, the standby current is to be between 4 and 5 mA_{RMS}, and the percentage of the cross sectional area to be filled by insulated windings is 50%.

The physical dimensions are determined first as the other components are derived from these. The ratio of the horizontal length of the interior window to the width of the core was provided to be 0.45. Therefore that interior horizontal length, x, can be solved for which results in Equation 1,

$$x = 0.45w \tag{1}$$

where w is the width of the core. The ratio of the vertical length of the interior window to the width of the core was provided as 1.5. The vertical window length, y, can be found by Equation 2.

$$y = 1.5w \tag{2}$$

The corresponding height and total length of the core can be found from these window dimensions. A sketch of one half of the core can be seen in Figure 1

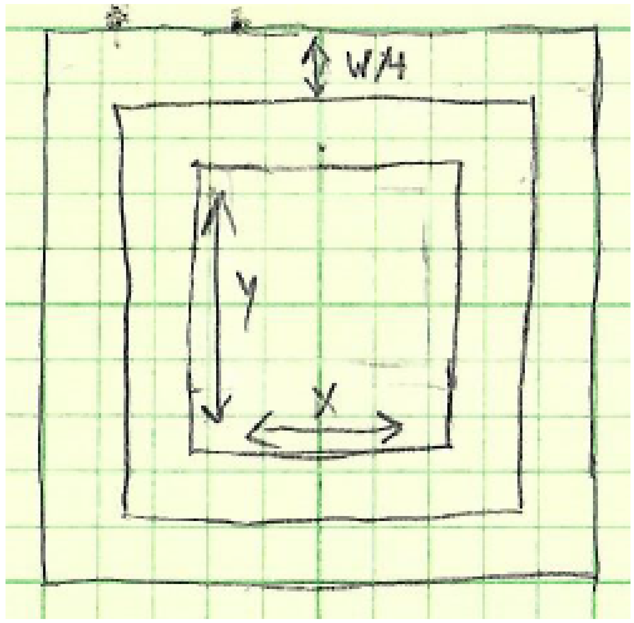

Figure 1: Sketch of 1/2 core.

The height of the core can be found to be sum of y and the width. The length of the core is then twice the sum of x and the width. The area of the inner windows is the product of x and y. In order to proceed with the magnetic model, the flux path length must be found. The horizontal path length can be found by inspection of Figure 1. The horizontal path length, H_{pl} is found in Equation 3,

$$H_{pl} = \frac{l}{2} - 2\frac{w}{4}, \tag{3}$$

where l is the total length of the core. The vertical path length, V_{pl} can be found similarly, which results in Equation 4,

$$V_{pl} = h - 2\frac{w}{4}, \tag{4}$$

where h is the height of the core. There are then two different path lengths for the transformer. This can be seen in Figure 2.

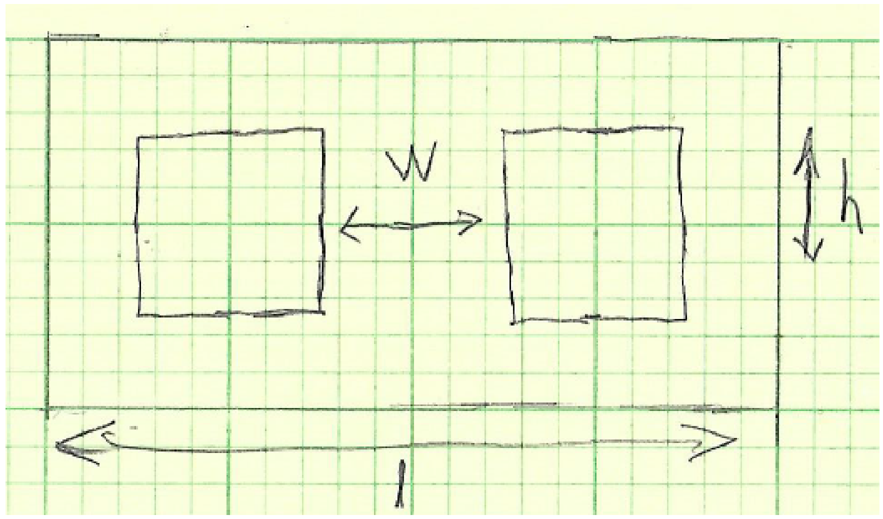

Figure 2: sketch of full core

There exists the path length through the center of the core and then mirror imaged paths through the exterior of the core. By inspection, the path length of the exterior is found by Equation 5

$$Flux_{pathexterior} = 2(H_{pl}) + (V_{pl}), \tag{5}$$

The majority of the flux flows through the center of the core with a negligible amount making it to the most outer extreme edges. The interior path length is the equivalent vertical path length from Equation 4. The number of turns that can fit is determined by the physical constraints as well. The amount of area that can be filled is 50% of the interior window area. From this, the wire area can be expressed as Equation 6

$$0.5(interiorwindowarea) = 12N(primarywirearea) + N(secondarywirearea) \tag{6}$$

The area of the wires can be found by applying the max current density constraint. The input current is 100 mA and them max current density is 5A/mm2 . From this the area of the wire can be found, as well as the resulting wire gauge. It is known by application of the ideal model that the current through the secondary is 12 times the input, or 1.2A. The wire area and wire gauge for the secondary can be found by the same process. The required AWG wires is 34 for the primary and 23 for the secondary, found by looking at the AWG table. From this the magnetic field strength, H, can be found by Equation 7

$$H = \frac{Ni}{l_c}, \tag{7}$$

where i is the input current, and lc is the path that the flux takes through the core, in this case the sum of two times the exterior flux path and the vertical flux path. The product Ni is known as the magnetomotive force (MMF). Since there are two sources in this model, the primary and the secondary windings of the transformer, the total MMF is actually the sum of the individual MMFs. Equation 7 can then be written as Equation 8.

$$H = \frac{MMF_{total}}{l_c} \tag{8}$$

H is now a function of MMF_{total} in the equivalent magnetic model circuit. From H the magnetic flux density, B, can be found from the provided MATLAB code. B is also a function of the core material. The magnetic permeability of the core can then be found from B. With magnetic permeability the permeances of the core can be found by Equation 9

$$P = \frac{\mu A}{L}, \tag{9}$$

where μ is the permeability of the core, A is the cross sectional area of the core, and l is the length of the core through which flux flows. There exists in the equivalent model two separate permeances, an interior and exterior path length. The equivalent magnetic model is shown in Figure 3

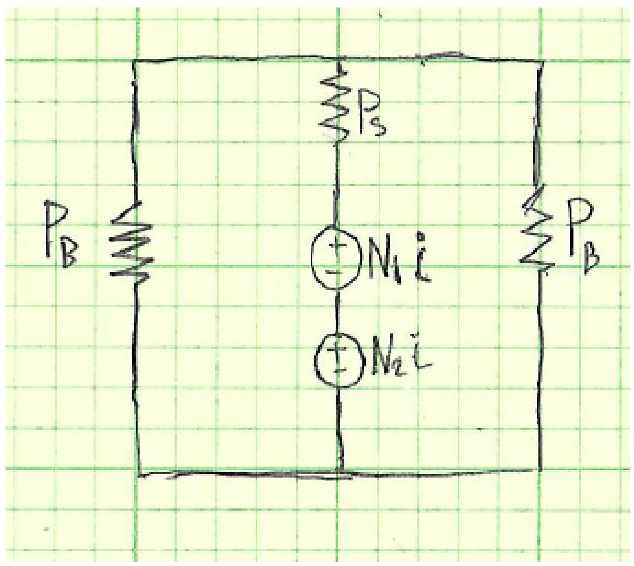

Figure 3: Equivalent Magnetic Model

The two exterior branches are in fact in parallel with each other. Permeances,being the magnetic equivalent of conductance, add as the sum of reciprocals, and are easier to discuss as reluctance, the magnetic equivalent of resistance. The total reluctance of the circuit can be expressed as Equation 10

$$R_{total} = \frac{1}{P_S} + \frac{1}{2P_B} \tag{10}$$

Now the total permeance can be found by Equation 11

$$P_{total} = \frac{1}{R_{total}} \tag{11}$$

The permeance can now be used to solve for the magnetizing inductance, L_m, through Equation 12

$$L_m = (N1)^2 P_{total}, (12) \tag{12}$$

N1 is the number of turns in the primary and this is now enough information to proceed with the describing the electric circuit model.

2.2 Electric model

The electric model analysis requires application of the standby current specification. The standby current is required to be between 4mA and 5mA. Phasor circuit analysis can be applied to the circuit seen in Figure 5.

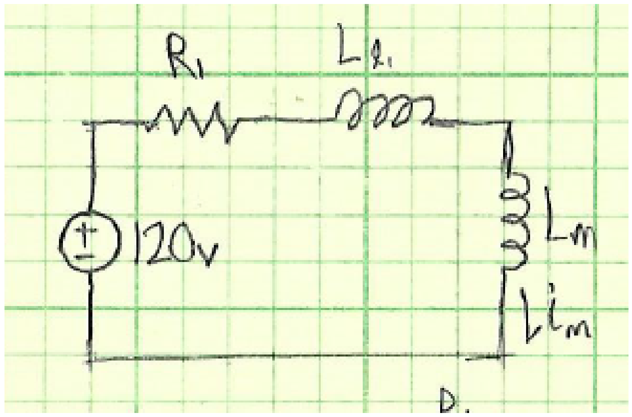

Figure 4: Modelling standby current

The current i_m can be found by Ohm's Law and is shown in Equation 13

$$i_m = \frac{V}{j2\pi f L_m}, \tag{13}$$

where f is the operating frequency and L_m is the magnetizing inductance. Here is where the frequency effects how the circuits behave. Equation 13 can then be solved for a range of L_m values based on the standby current restriction. L_m is then found, for 60 Hz, to be between 62 and 79 Henries. While at 60 kHz, L_m is found to be between 62 and 79 milli-Henries. The linkage inductance can be found by a similar process as seen in Equation 14

$$L_l = N^2 P \alpha, \tag{14}$$

The variable α is the linkage coefficient and is a function of the magnetizing flux, B_m. For the purposes at hand, α is approximately 1%

Through the use of MATLAB code, the width of the core can be varied until a sufficient magnetizing inductance can be found. The resistance from the wire, R, and linkage inductance L_l can be ignored for this calculation due to the magnetizing inductance having the most dominate impedance. Once L_m is found, the magnetizing flux, B_m can be found by Equation 15

$$B_m = \frac{L_m i_m}{N_1 A_c} \tag{15}$$

A_c is defined as the area of the core. The core losses are then be found from the magnetizing inductance which, in turn, can be represented as a resistance in the electric model. Equation 16 shows the relationship between the core losses and this resistance.

$$R_{ce} = \frac{V^2}{P_{cl}} \tag{16}$$

where P_{cl} is the core losses in Watts. The final electric model is presented in Figure 6

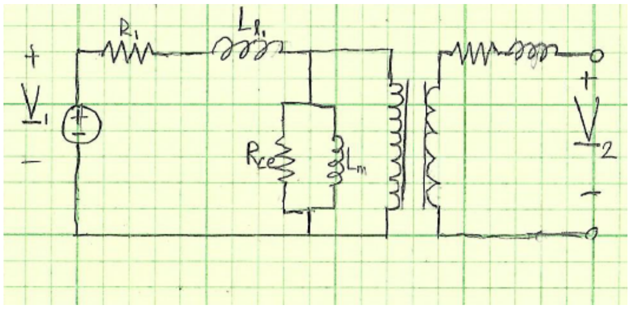

Figure 5: Final Electric Circuit

Attached at the end of this report is the MATLAB code mentioned above.

3 Simulations

The simulated circuit can be seen below.

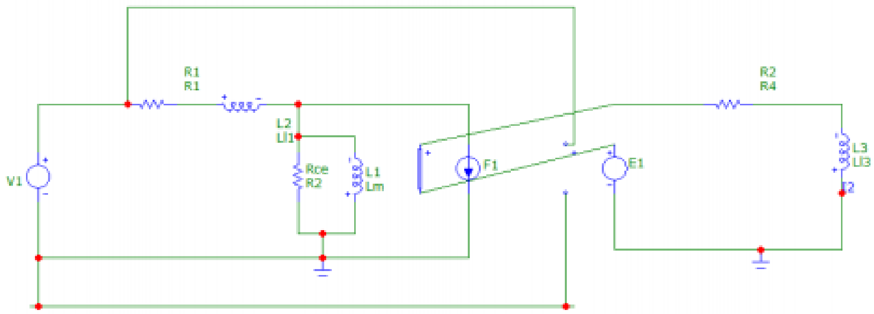

Figure 6: Microcap circuit used for simulations

The circuit above was used for both 60 Hz and 60 kHz versions of the transformer with changing component values. Figure 7 below shows the simulated output voltage of the 60 Hz transformer.

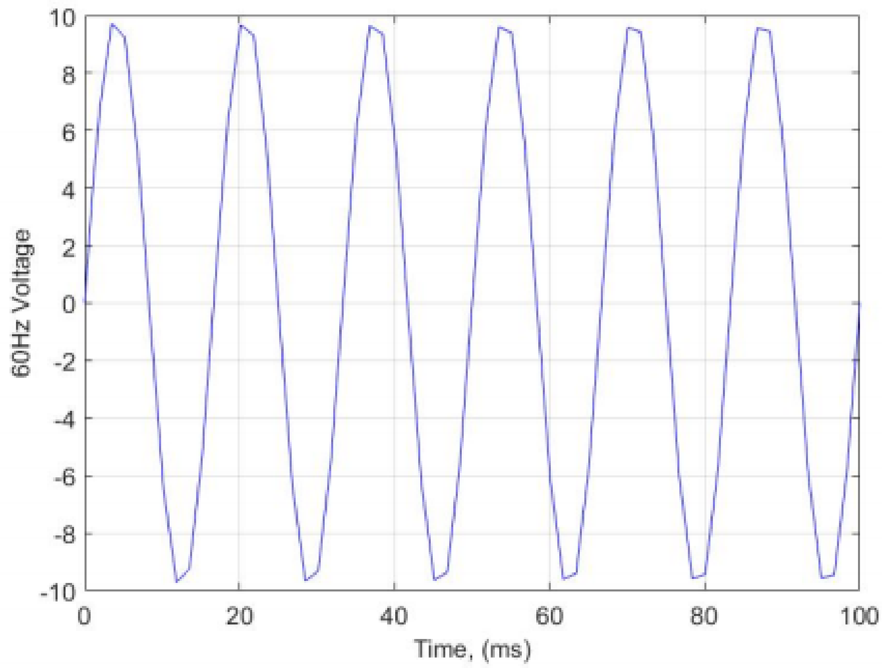

Figure 7: Simulated 60 Hz output voltage

As expected of a non ideal transformer, the output falls just shy of the 10v expected. Table 1 below summarizes the values provided by the MATLAB code.

Table 1: 60 Hz Values.

Frequency	60 Hz
Width	2.9 cm
R, Primary	570 Ω
Linkage Inductance, Primary	681 mH
Magnetizing Inductance	68.1 H
Core Loss Resistance	219 kΩ
R, Secondary	3.7 Ω
Linkage Inductance, Secondary	4.7 mH
B_m	114 mT
Mass, Total	1.45 kg
Turns, Primary	5736
Turns, Secondary	478

The 60 Hz comes out seeming practical in terms of size and mass, as well as ohmic losses. The 60kHz output is shown below and performs closer to the ideal transformer.

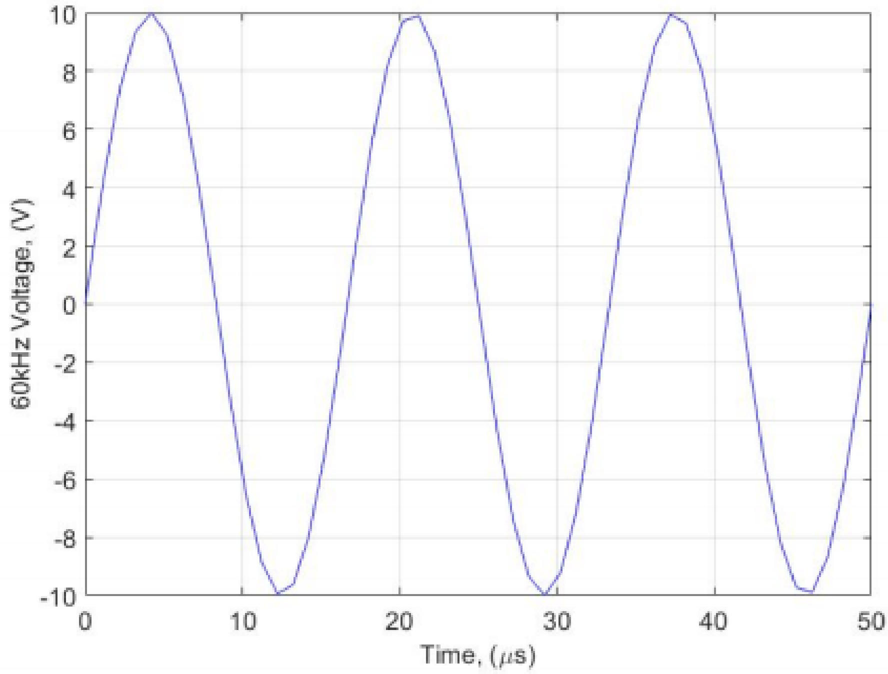

Figure 8: Simulated 60 kHz output voltage

The voltage output comes much closer to the desired 10v in the 60kHz transformer, a table of the values found using the MATLAB code are shown below.

Table 2: 60 Hz Values.

Frequency	60 kHz
Width	6.7 mm
R, Primary	7.15 Ω
Linkage Inductance, Primary	668 μH
Magnetizing Inductance	66.8 mH
Core Loss Resistance	13.4 MΩ
R, Secondary	46.5 mΩ
Linkage Inductance, Secondary	4.64 μH
B_m	39.4 mT
Mass, Total	131 g
Turns, Primary	312
Turns, Secondary	26

The 60 kHz featured a much smaller design, but with much higher core losses. It seems that some sacrifices must be made in making the transformer so small and decreasing the ohmic losses.

4 Discussion

The two transformers varied pretty significantly in regards to physical dimensions. The 60 Hz transformer ended up being quite a bit larger than the 60 kHz. The factor that ultimately determined the physical dimensions for this design is in fact the standby current. This effectively sets the possible range of values for the magnetizing inductance. With the 60 Hz, the transformer has to be quite large in order to accommodate much more turns in order to get the correct L_m value. This can be seen with Equation 13. As the operating frequency decreases, L_m must increase to ensure the standby current stays within specification. By extension, as the frequency increases, a smaller magnetizing inductance is needed, so at 60 kHz a much smaller magnetizing inductance is needed, which allows for far fewer turns and a smaller core overall. This ties back into the discussion of linear vs switch mode power supplies at the beginning of the semester. The trade off for the smaller design is the fact that core losses increase quite alarmingly. The core is switched to a Ferrite material for the 60 kHz . As the current through the transformer changes polarity, so do the magnetic fields of the core. The faster the polarity changes the more energy is going to be lost by the core as a result, so a core made of Ferrite is used. The 60 kHz transformer, however, results in a much smaller and lighter design while achieving comparable electrical results. It is among these reasons that consumer electronics made the switch to the smaller and more cost effective design.

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Transformer design m-file

```
% Joseph Arsenault
% Date  12/13/2017
% design two transformers:
% Don't change anything in this code cell

n = 12;             % required turns ratio
Vrms = 120;         % rms input voltage
Irms = 0.1;         % rated rms input current
Isb = 5e-3;         % rms standby current

Muo = pi*4e-7;      % permeability of free space
GammaC = [7650 4982];  % mass density of core materials in Kg/m^3
% GammaC(1) = steel mass density, Gammac(2) = ferrite mass density
GammaW = 8.95e3;    % mass density of wire material in Kg/m^3
RhoW = 1.7241e-8; % resistivity of wire at 20degC in Ohm*m

Jmax = 5;           % maximum wire currend density in Arms/mm^2

Kfill = 0.5;        % maximum normalized amount of winding space
 occupied by covered wire
```

Functions given by Duane Hanselman

```
% Functions you can call in your design

% Leakage flux fraction for computation of leakage inductances
% Bm = flux density in the core when the input current is at its peak
 RMS value
alpha = @(Bm) 0.01 + Bm/20;
% Example: alpha(1) = 0.06 = 6% Bm is in tesla
% As the steel saturates and becomes less permeable, the more flux
 leaks out

% COPPER WIRE Functions
 ---------------------------------------------------
% bare wire diameter in mm from Gauge G
DiamWb = @(G) 8.251463*(0.8905257)^G;
% Example: DiamW(20) returns diameter of 20 gage wire in mm

% covered wire diameter in mm from Gauge G
DiamWc = @(G) 1.15*DiamWb(G);

% bare wire cross-sectional area in m^2 from Gauge G
```

```
AreaWb = @(G) pi*(DiamWb(G)/2000)^2;

% covered wire cross-sectional area in m^2 from Gauge G
AreaWc = @(G) pi*(DiamWc(G)/2000)^2;

% wire Gauge G from bare wire diameter in mm
GaugeWd = @(d) log(d/8.251463)./log(0.8905257);

% wire Gauge G from bare wire cross-sectional area in (mm)^2
GaugeWA = @(A) log((2*sqrt(A/pi))/8.251463)./log(0.8905257);

% Wire Resistance for length z in meters and Wire Gauge G
Rwire = @(z,G) RhoW*z./(pi*(DiamWb(G)/2000)^2);


% Core Parameter Data---------------------------------------------
cp1 = [3.8 26];
cp2 = [2.17 12.2];
cp3 = [396.2 206.2];
cp4 = [9.55 1.06e-4];

% Magnetic CORE Functions
 -------------------------------------------------

% Two materials are available, as selected by variable k
% k = 1 is steel
% k = 2 is ferrite

% BH curve of core material, H as a function of B
Hcore = @(B,k) B.*(cp1(k)*exp(cp2(k)*B.^2) + cp3(k));
% Example: Hcore(0.2) returns H for B=0.2 Tesla

% BH curve of core material, B as a function of H
Bcore = @(H,k) fzero(@(B) Hcore(B,k)-H,0.15);
% Example: Bcore(450) returns B for H=450 A/m

% dB(H)/dH of core material as a function of B
dBdHb = @(B,k) 1./(cp1(k)*exp(cp2(k)*B.^2) .*(2*cp2(k)*B.^2 + 1) +
 cp3(k));

% Permeability of core material as a function of B
Muc = @(B,k) (B./Hcore(B,k));
% if you already have B and H, this is simply Muc = B/H

% Relative Permeability of the core material as a function of B
Mucr = @(B,k) Muc(B,k)/Muo;

% Core loss density in Watts/m^3, f in Hz, B in Tesla
Pcore = @(f,B,k) cp4(k)*(f.*B)^2;
```

Transformer Design

```
%Find area of wire necessary for given current density, A/(A/mm^2)
```

```
Primary_area = Irms/Jmax;
asd = GaugeWA (Primary_area);

%Floors value as decimals are nonsensical values
Primary_gauge =  floor(asd);
Area_secondary = (n*Irms)/Jmax;
Gauge_secondary = floor(GaugeWA(Area_secondary));

%Prompts user for frequency,Steel core for 60hz, Ferrite core for 60
 kHz,
%changes width accordingly, checks to make sure answer is valid.
checkprompt = 1;
while checkprompt == 1
prompt = 'What is the frequency? Choose 60 or 60,000. ';
if ~exist('f', 'var')
f = input(prompt);
end
Omega = 2*pi*f;
if f == 60
     k_picked = 1;
     width =  29e-3;
     checkprompt = 0;
elseif f == 6e4
    k_picked = 2;
    width = 6.7e-3;
    checkprompt = 0;
else
    checkprompt = 1;
end
end
y = 1.5 * width;
x = .5 * width;
height = y + width;
length = 2*x + 2* width;

%Area of inner window
Inner_area = x * y;

%Volume of inner window
Inner_volume = Inner_area * width;
i=0;

%%Magnetic model
Horizontal_core_length = length/2 - 2*(width/4);
Vertical_core_length = height - 2*(width/4);
Length_core_parallel = 2*Horizontal_core_length +
 Vertical_core_length;
Length_core_series = height - 2*(width/4);
Full_core_length = 2*Length_core_parallel + Length_core_series;

%Initiliaze wire area, loops to find optimal wire area
Wire_area = 0;
while Wire_area < (Kfill*Inner_area)
    i = i+1;
```

```
    Wire_area = 12*AreaWc(Primary_gauge)*i +
 AreaWc(Gauge_secondary)*i;
    Primary_turns = 12*i;
    secondary_turns = i;

%Turn length parameter, also known as bobbin parameter
Bobbin = 4*width;

%Primary Wire Length
Primary_wire_length = Bobbin * Primary_turns;
Wire_volume_primary = Primary_wire_length * AreaWc(Primary_gauge);
Primary_mass = GammaW * Wire_volume_primary;

%Secondary Wire Length
Wire_length_secondary = Bobbin * secondary_turns;
Wire_volume_secondary = Wire_length_secondary *
 AreaWc(Gauge_secondary);
Mass_secondary = GammaW * Wire_volume_secondary;

%Magneto Motor Force (MMF),  F = Ni
MMF = Primary_turns*Irms + secondary_turns*n*Irms;

%Finding H, the ratio of F to core length
H = MMF/Full_core_length;

%B from H and core material
B = Bcore(H,k_picked);

%Mu core from B
Mu_core = Muc(B,k_picked);

%Permeance of middle branch
Permeance_series = (Mu_core*width^2)/Length_core_series;

%Permeance of parallel branch
Permeance_branch = (Mu_core*width*(width/2))/Length_core_parallel;

%Reluctance of core is 1/Permeance
Total_reluctance = (1/Permeance_series) +(1/(2*Permeance_branch));
Total_permeance = 1/Total_reluctance;

%%Electric model
% Lm = N^2 P
Lm = Primary_turns^2 * Total_permeance;

%Alpha is linkage coeffecient, let alpha = 1%
alpha = .01;
%Linkage inductances of primary and secondary
Llink_1 = Primary_turns^2 * Total_permeance*alpha;
Llink_2 = secondary_turns^2 * Total_permeance*alpha;

%resistivity
R_wire_primary = Rwire(Primary_wire_length, Primary_gauge);
```

```
R_wire_secondary = Rwire(Wire_length_secondary, Gauge_secondary);

end

%Magnetizing current
Im = Vrms/(R_wire_primary+1i*Omega*(Llink_1+Lm));

%Magnetizing current magnitude
Im_final = abs(Im);

%Magnetizing flux with weighted areas, since area of core is not
 uniformed,
%center has larger area than branches
Bm = (Lm*Im_final)/
(Primary_turns*((1/7)*(width*width)+(6/7)*(width*(width/2))));

%Core loss density for material
Core_loss_density = Pcore(f,Bm,k_picked);

%Volume of core is volume of full core minus window volumes
Volume = length*height*width - 2*Inner_volume;
Mass_core = GammaC(k_picked) * Volume;

%Wire volumes
Volume_wire = Primary_wire_length * AreaWc(Primary_gauge) +
 Wire_length_secondary * AreaWc(Gauge_secondary);
Mass_wire = GammaW * Volume_wire;

%Total mass
Total_mass = Mass_core + Mass_wire;

%Core losses
Core_losses = Core_loss_density * Volume;

%Resistivity of the core
Rce = (Vrms^2)/Core_losses;
```

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