Midterm for MATH 7A, Winter 2022

Midterm contains 6 problems on 6 pages. Number of each problem is circled. **In order to obtain full credit, all work must be shown.** The exam is Open Book which means that you can make use of the textbook or class notes. Using calculators, software or Internet sources is not permitted. The points attached to each problem are indicated beside the problem.

1 (*20 points*) Find limits

$(x-1)(x-3)$ $\quad x^2-3x-x+3$

x^2-4x+3

(a) $\lim\limits_{x\to 1} x^2 - 4x + 3$

$\lim\limits_{x\to 1} (x-1)(x-3)$

$\lim\limits_{x\to 1} \underset{0}{(1-1)}\underset{-2}{(1-3)}$

$\boxed{\lim\limits_{x\to 1} x^2-4x+3 = 0}$

(b) $\lim\limits_{x\to 1} \dfrac{x^2 - 4x + 3}{x - 1}$

$\lim\limits_{x\to 1} \dfrac{(x-3)\cancel{(x-1)}}{\cancel{x-1}}$

$\lim\limits_{x\to 1} (x-3) \to \lim\limits_{x\to 1} (1-3) \to \boxed{\lim\limits_{x\to 1} \dfrac{x^2-4x+3}{x-1} = -2}$

(c) $\lim\limits_{x\to 1+} \dfrac{|x^2 - 1|}{x - 1}$

$\lim\limits_{x\to 1+} \dfrac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$

$\lim\limits_{x\to 1+} (1+1) \to \boxed{\lim\limits_{x\to 1+} \dfrac{|x^2-1|}{x-1} = 2}$

(d) $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{2x}$

$\lim\limits_{x\to 0} \dfrac{\sin(x)}{x} = 1$

$\lim\limits_{x\to 0} \dfrac{\sin(5x)}{2x} \to \lim\limits_{x\to 0} \dfrac{5\cancel{x}}{2\cancel{x}} \to \boxed{\lim\limits_{x\to 0} \dfrac{\sin(5x)}{2x} = \dfrac{5}{2}}$

2 (*15 points*) Find limits

(a) $\lim_{x\to\infty} \frac{x^2-4x+3}{x-1}$

$1-2=-1$

$$\lim_{x\to\infty} \frac{\frac{x^2}{x}-\frac{4x}{x}+\frac{3}{x}}{\frac{x}{x}-\frac{1}{x}} \to \lim_{x\to\infty} \frac{x-4+\cancel{\frac{3}{x}}}{1-\cancel{\frac{1}{x}}} = \lim_{x\to\infty} \frac{x-4}{1} = \boxed{\lim_{x\to\infty} \frac{x^2-4x+3}{x-1} = \infty}$$

leading term ∞

(b) $\lim_{x\to-\infty} \frac{x^2-4x+3}{x-1}$

$$\lim_{x\to-\infty} \frac{\frac{x^2}{x}-\frac{4x}{x}+\cancel{\frac{3}{x}}}{\frac{x}{x}-\cancel{\frac{1}{x}}} \to \frac{x-4}{1} \to \lim_{x\to-\infty} \frac{x-4}{1} \to x^1 \to \text{odd} \xrightarrow{\text{limit of}} -\infty \longrightarrow -\infty$$

leading term $-$

$$\boxed{\lim_{x\to-\infty} \frac{x^2-4x+3}{x-1} = -\infty}$$

(c) $\lim_{x\to\infty} \frac{x^2-4x+3}{x^2-1}$

$$\lim_{x\to\infty} \frac{\frac{x^2}{x^2}-\frac{4x}{x^2}+\cancel{\frac{3}{x^2}}}{\frac{x^2}{x^2}-\cancel{\frac{1}{x^2}}} \to \frac{1-\cancel{\frac{4}{x}}}{1} \to \lim_{x\to\infty} \frac{1}{1} = 1$$

$$\boxed{\lim_{x\to\infty} \frac{x^2-4x+3}{x^2-1} = 1}$$

③ (*15 points*) Find derivatives

(a) $(x^{9-1} - 3x^{2-1} + 1)'$

$(x^9)' - (3x^2)' + (1)'$

$9x^8 - 6x + 0$

$\boxed{9x^8 - 6x}$

(b) $((x^9 - 3x^2 + 1)(\sqrt{x} + 2x))'$

$$\boxed{(9x^8 - 6x)(\sqrt{x} + 2x) + (x^9 - 3x^2 + 1)\left(\frac{1}{2\sqrt{x}} + 2\right)}$$

(c) $\left(\dfrac{x^9 - 3x^2 + 1}{\sqrt{x} + 2x}\right)'$ $\rightarrow$ $(x^9 - 3x^2 + 1)'\sqrt{x} + 2x - (3x - 1)(\sqrt{x} + 2x)$

$9x^8 - 6x$

(4) (*20 points*) Find derivatives

(*a*) $((2x-1)^2)'$

$$2\cdot(2x-1)\cdot\underbrace{(2x-1)}_{}^{2} = \boxed{4(2x-1)}$$

(*b*) $\left(\sqrt{x^2-5x}\right)'$

$$\overbrace{(\sqrt{x^2-5x})}^{2x-5}$$

$$\frac{1}{2\sqrt{x^2-5x}}$$

$$\boxed{\frac{2x-5}{(2\sqrt{x^2-5x})}}$$

5 (*20 points*) Consider function $y(x)$ given by the implicit relation $2x^3 + 3y^2 = 5$.

(*a*) Find $y'(x)$ using implicit differentiation.

$(2x^3+3y^2=5)'$ $\qquad y=f(x)$

$(2x^3+3y^2)' = 0$

$\Rightarrow (2x^3)' + (3y^2)' = 0$

$2x \cdot x \cdot x' + 3y \cdot y = 0$

$(2x+y)x$

$y' = \dfrac{-x-2y}{2x+y}$

(*b*) Find an equation for the tangent line at point $(1, -1)$.

$(1,-1)$

$y' = \dfrac{-1-2(-1)}{2(1)+(-1)}$

$y = 1 + \frac{1}{2}($ $\qquad y' = \dfrac{-1+2}{2} \rightarrow y' = \dfrac{1}{2}$

(6) (*10 points*) Let N denotes the population of species and F is the amount of nutrients. It was experimentally observed that $N = 0.8F^3$. Let F evolves in time t. Find growth rate of $N(t)$ if $F(t) = \dfrac{1+t}{2+t}$ as $t \to \infty$. In other words, find $\lim_{t\to\infty} N'(t)$.

$N = 0.8F^3$ $(N\ 0.8F^3 = t)$

$f \to t$

$0.8F^3 = 0$

$N' 0.8F^3 + 0.8F^{3)} = 0$

$N 0.8F^3 + 2.4F^2 \cdot F$

0.8
3.0
00
4