Math 7A
Midterm 1

You must show all work to receive full credit. Answers without any work shown will not receive any points, even if the answer is correct. Please upload your answers to the exam on Gradescope **before the Gradescope time is up**. Please be sure to assign the pages that contain your answers to corresponding questions. If you have trouble uploading to Gradescope, please email the instructor (gtsai001@ucr.edu) **before the Gradescope time is up**.

Calculators are **not** allowed. Collaboration is **not** allowed. Electronics are **not** allowed unless it is to access the exam and/or Zoom. You may only use your cheat sheet.

Please do your best to write your answers to parts of the same question on the same page. (For example, keep 1a, 1b, and 1c together on the same page.)

1. Let c be some constant. Consider the function

$$g(x) = \begin{cases} -x^2 + c & \text{if} \quad x \le 1 \\ x + 3 & \text{if} \quad x > 1 \end{cases}$$

   a) (1 pt) What is $\lim_{x \to 1^-} g(x)$?

   b) (1 pt) What is $\lim_{x \to 1^+} g(x)$?

   c) (4 pts) Suppose $c = 0$. Use the definition of continuity to show that $g(x)$ would not be continuous at $x = 1$.

   d) (4 pts) Use the definition of continuity at a point to solve for c so that $g(x)$ will be continuous at $x = 1$.

2. Evaluate the following limits using methods learned thus far in the course. Use of L'Hopital will not receive the full credit.

   a) (3 pts) $\lim_{x \to \infty} \frac{12x^8 + \pi x^5 - \sqrt{3}}{2x^3 - x^4 - 17x^6}$

   b) (2 pts) $\lim_{x \to -2} \frac{x^3 - x^2 - 6x}{x^2 - 4}$

   c) (4 pts) $\lim_{x \to 0} \frac{1 - \cos^2(3x)}{9x^2}$

3. (7 pts) Use The Sandwich Theorem, also known as The Squeeze Theorem, to show

$$\lim_{x \to \infty} \frac{\sin^2 x + 1}{7e^x} = 0$$

Continue to the next page for the rest of the exam.

4.   a) (6 pts) Use the **limit definition** to find the derivative of $f(x) = \sqrt{3x - 2}$.

     b) (4 pts) Write the equation of the **normal** line of $f(x) = \sqrt{3x - 2}$ at $x = 2$.

5. (Microtutorials) A tank contains 5000L of pure water. Brine that contains 30g of salt per liter is pumped into the tank at a rate of 25L/min. The concentration of salt after t minutes (in grams per liter) is

$$C(t) = \frac{30t}{200 + t}$$

     a) (3 pts) What happens to the concentration as $t \to \infty$? You must show or explain how you got your answer.

     b) (1 pt) Interpret what part (a) means.

6. Note that if you are unable to do part (b), you will not receive credit for part (c).

     a) (0.25 pt) Take a picture or scan your cheat sheet with the rest of your exam (assign the cheat sheet to this question).

     b) (0.25 pt) Upload your exam answers to Gradescope before time is up.

     c) (0.25 pt) Assign the pages that contain your answers to corresponding exam questions.