Worksheet 5

MATH 006B - Schmidt

Winter 2021

Instructions:

- **Show ALL your work to receive credit!** Cross off anything you do not wish to be graded.
- Simplify your answers as much as possible. For instance, evaluate 2^2, but not $\sqrt{2}$.
- **Work with your group** on the following exercises. Each of you will turn in your own work via Gradescope.
- **Your group** may ask the TA questions, which the TA will answer with leading questions (not answers) to help guide you to the answer.

1. (4 points) Let $c(x) = \dfrac{4 - x^2}{x^2 - 2x}$. Show your work and/or justify your answers.

$c(x) = \dfrac{4 - x^2}{x^2 - 2x}$ $\quad (-x-2)(x-2)$ $\quad -x^2+2x-2x+4$ $\quad -x^2+4$

$x^2 - 2x$ $\quad x(x-2)$

$c(x) = \dfrac{(-x-2)\cancel{(x-2)}}{x\cancel{(x-2)}}$

$c(x) = \dfrac{(-x-2)}{x}$

$\dfrac{(+1-2)}{-1} \to \dfrac{-1}{-1} = 1$

$\dfrac{1-2}{1} \to \dfrac{-1}{1} \to -1$

$\dfrac{(-0-2)}{0} \to \dfrac{-2}{0} \to \text{DNE}$

$+ \quad -$

$-1 \quad 0 \quad 1$

(a) (2 points) Evaluate the following limits.

$\lim_{x \to 0^-} c(x) = \infty$

$\lim_{x \to 0^+} c(x) = -\infty$

$\lim_{x \to 0} c(x) = \text{DNE}$

(b) (2 points) Evaluate the following limits.

$c(x) = \dfrac{(-x-2)}{x}$

$\dfrac{(-1-2)}{1} \to \dfrac{-3}{1} \to -3$

$\dfrac{(-3-2)}{1} \to \dfrac{-5}{1} \to -5$

$\dfrac{(1-2)}{-1} \to \dfrac{-1}{-1} \to 1$

$\dfrac{(3-2)}{-3} \to \dfrac{1}{-3} \to$

$\ominus - \oplus \quad \ominus - \ominus$

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

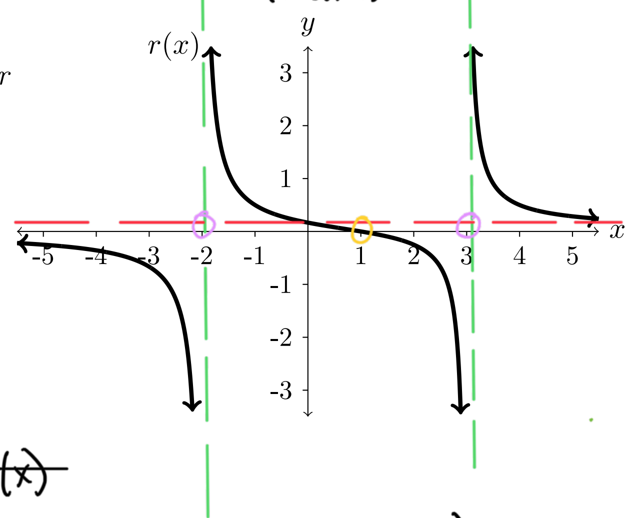
$\lim_{x \to 2^-} c(x) = -2 \quad -\infty$

$\lim_{x \to 2^+} c(x) = -2 \quad -\infty$

$\lim_{x \to 2} c(x) = -2 \quad -\infty$

2. (6 points) Suppose $r(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions such that $\frac{p(x)}{q(x)}$ cannot be simplified further. r is graphed at right.

$r(x) = \frac{(x-1)}{(x+2)(x-3)}$



(a) (1 point) What are the vertical asymptotes of r? *You may assume they happen at integer values.*

Vertical asymptotes

↳ -2 and 3

(b) (1 point) Find all roots of p.

$q(x) \cdot r(x) = \frac{p(x)}{q(x)} \cdot q(x) \rightarrow \cancel{q(x) \cdot r(x) = p(x)}$

$1 \cdot 1 = 1$

The root of p would 1

$\frac{p(x)}{q(x)} = \frac{(x-1)}{(x+2)(x-3)}$

(c) (1 point) Find all roots of q.

roots of q

↳ -2, 3

(d) (1 point) Does r have a horizontal asymptote? If so, what is it?

r does have a horizontal asmptote at 0

$y=0$

(e) (1 point) On what interval(s) is r continuous?

$(-\infty,-2) \cup (-2,3) \cup (3,\infty)$

$$\frac{3(x-a)^3(x-b)}{(x-c)^4}$$

3. (4 points) Let $h(x) = \frac{6\cancel{(x-a)}^{\cancel{4}\,3}(x-b)\cancel{(x-c)}}{2\cancel{(x-a)}\cancel{(x-c)^5}\,4}$, where a, b, and c are real numbers satisfying $a < b < c$. Do not choose values for a, b, and c.

(a) (2 points) What are the roots, vertical asymptotes, and holes of h? Clearly label your answers.

$2(x-a)(x-c)^5$

$\underbrace{(x-c)(x-c)}_{c}\cdot\underbrace{(x-c)(x-c)}_{c}\cdot -c \rightarrow -c$

$x = a, -c$

↳ roots

$(c-c)^5$

↳ 0

There are no vertical asymptotes.

c is the hole

(b) (1 point) Does h have a horizontal asymptote? If so, what is it?

The horizontal asymptote would be 3.

$\frac{6}{2} \rightarrow 3$

$2(x-a)(x-c)^5$

(c) (1 point) On what interval(s) is h continuous?

$(-\infty, -2a) \cup (-2a, c) \cup (c, \infty)$

NOTE: c is greater than a. $a<b<c$.

4. (1 point) Participation – no submission