

$$\vec{S}'' = \begin{pmatrix} (S \sin^2 \theta \cos \omega t + S \cos^2 \theta) \cos \omega t + S \sin \theta \sin \omega t \sin \omega_1 t \\ -(S \sin^2 \theta \cos \omega t + S \cos^2 \theta) \sin \omega t + S \sin \theta \sin \omega t \cos \omega_1 t \\ S \sin \theta \cos \theta (\cos \omega t - 1) \end{pmatrix}$$

Let $\omega, t = \alpha$

Let $k = \frac{\gamma B_1}{\omega_1}$

Then

$$\sin \theta = \frac{\omega_1}{\gamma B_{\text{eff}}} = \frac{\omega_1}{\omega} = \frac{\omega_1}{\sqrt{(\gamma B_1)^2 + \omega_1^2}} = \frac{1}{\sqrt{k^2 + 1}}$$

$$\cos \theta = \frac{B_1}{B_{\text{eff}}} = \frac{\gamma B_1}{\sqrt{(\gamma B_1)^2 + \omega_1^2}} = \frac{k}{\sqrt{k^2 + 1}}$$

$$\omega t = (\omega_1 t) \frac{\omega}{\omega_1} = \alpha \cdot \sqrt{k^2 + 1}$$

Thus,

$$\vec{S}'' = \begin{pmatrix} \left(S \left(\frac{1}{\sqrt{k^2 + 1}} \right)^2 \cos(\alpha \sqrt{k^2 + 1}) + \frac{k^2}{k^2 + 1} \right) \cos \alpha + S \frac{1}{\sqrt{k^2 + 1}} \sin(\alpha \sqrt{k^2 + 1}) \sin \alpha \\ - \left(S \frac{1}{k^2 + 1} \cos(\alpha \sqrt{k^2 + 1}) + S \frac{k^2}{k^2 + 1} \right) \sin \alpha + S \frac{1}{\sqrt{k^2 + 1}} \sin(\alpha \sqrt{k^2 + 1}) \cos \alpha \\ S \frac{k}{k^2 + 1} (\cos(\alpha \sqrt{k^2 + 1}) - 1) \end{pmatrix}$$