A Model of the Effect of Vaccinations on the Spread of the COVID-19 Virus

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MATH 142

Introduction

- Effect of spreading of COVID-19 on US population
- A Mathematical model:
  - What: a description of a system using mathematical concepts and language
  - Why: Design effective strategies for controlling the contagion
- A SIRDVS (Susceptible-Infected-Recovered-Deceased-Vaccinated-Susceptible) model
- In 3 different scenarios
  - Neglecting the effect of natural birth rate and natural death rate (vital dynamics), and rate of vaccinated individuals on the US population (Case 1)
  - Neglecting the effect of the rate of vaccinated individuals on the US population (Case 2)
  - All model parameters take into account (Case 3)

Introduction

- Study the equilibrium and stability of the ODE system of the model
  - Whether the infection can be controlled
- Use official data about COVID-19
  - Simulate the model
    - effects of different factors on the US population.
  - create predictions by training the model
    - Predict how vaccine affects the US population related to COVID-19 (infected/reinfected/deceased)

Model Explanation

Components of our Model:

- S(t): the number of individuals susceptible to getting infected at time t;
- I(t): the number of infected individuals at time t;
- R(t): the number of individuals that have recovered from the disease at time t;
- D(t): the cumulative number of individuals that are deceased due to the disease at time t;
- V(t): the cumulative number of individuals that are fully vaccinated at time t.

Assumptions:

- The US population is isolated from other regions with a total population N=S+I+R.
- Recovered individuals can still be susceptible to getting reinfected.
- Vaccinated individuals can not be infected nor reinfected.
- The unit time t is expressed in days.
- Units S,I,R are expressed in persons.
- All rates are constant.
- The initial conditions are $S_0>0$, $I_0>0$, $R_0\geq0$, $D_0\geq0$, and $V_0\geq0$.

Case 1: Neglecting Natural Birth Rate, Death Rate, and Vaccination Rate

$$\begin{cases} \dfrac{dS}{dt} = -\dfrac{\beta SI}{N} + \xi R \\[2ex] \dfrac{dI}{dt} = \dfrac{\beta SI}{N} - \delta I - \nu' I \\[2ex] \dfrac{dR}{dt} = \delta I - \xi R \\[2ex] \dfrac{dD}{dt} = \nu' I \end{cases}$$

Notations:

- β is the infection rate
- δ is the recovery rate
- ν' is the mortality rate from the disease
- ξ is the reinfection rate

Equilibrium and Stability

Case 1 :

$$\xi R = \frac{\beta SI}{N}$$

No factors affect the population of susceptible people

$$\frac{\beta SI}{N} = \delta I + v'I$$

$$\delta I = \xi R$$

$$v' = 0 \ or \ I = 0$$

COVID data suggests v’ ≠ 0

$\Rightarrow I = 0$

$$\Rightarrow \quad \xi R = \frac{\beta SI}{N} = \delta I$$

then , the model is stable when there are no people being infected

Case 3: Accounting for Natural Birth Rate, Death Rate, and Vaccination Rate

$$
\begin{cases}
\frac{dS}{dt} = \mu N - \frac{\beta SI}{N} + \xi R - \nu S - \alpha S \\
\frac{dI}{dt} = \frac{\beta SI}{N} - \delta I - \nu' I - \nu I \\
\frac{dR}{dt} = \delta I - \xi R - \nu R \\
\frac{dD}{dt} = \nu' I \\
\frac{dV}{dt} = \alpha S
\end{cases}
$$

Notation:

- μ is the birth rate
- v is the general mortality rate
- α is the birth rate

Equilibrium and Stability

Case 3:

$$\mu N + \xi R = \frac{\beta S I}{N} + vS + \alpha S$$

$$\frac{\beta S I}{N} = \delta I + v' I + vI$$

$$\delta I = \xi R + vR$$

$$v' = 0$$

$$\Rightarrow \mu N = vR + vI + vS + v' I + \alpha S$$

$$or\ I = 0$$

$$\Rightarrow \mu N = vS + \alpha S$$

v’ = 0

$\Longrightarrow$ stable when birth rate = death rates

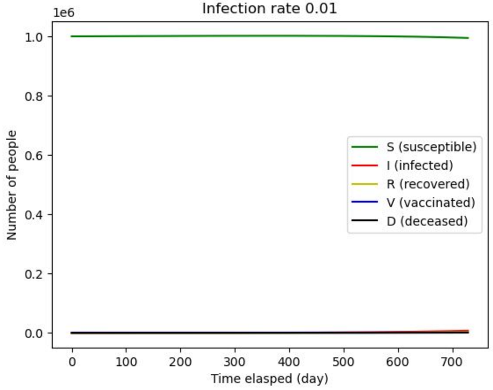
However, COVID data suggests v’ ≠ 0

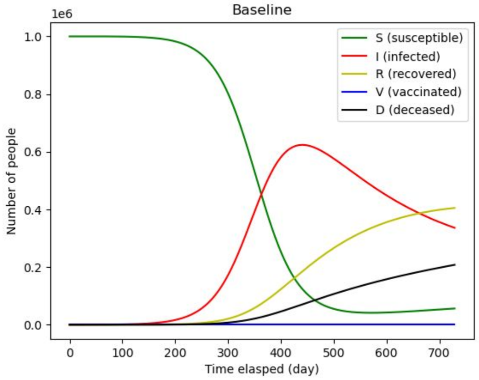
$\Longrightarrow$ *I* = 0

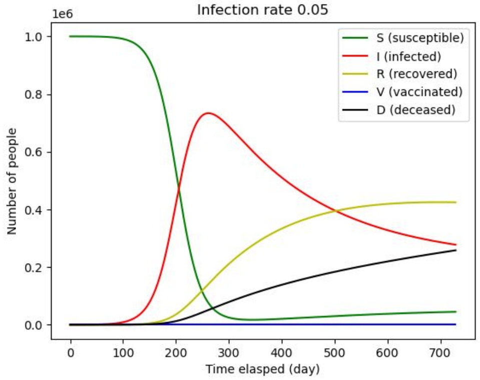
Then, the system is stable when birth rate = natural death in S + vaccination in S

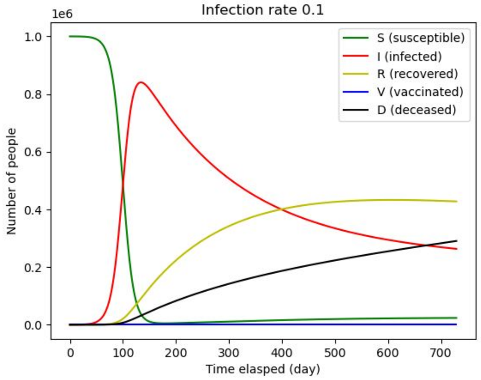
Reaches equilibrium most quickly, as demonstrated in our simulations

Simulation: Infection Rate (□)









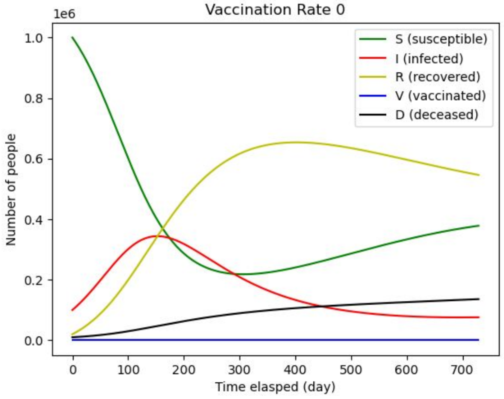
- infection rate increases

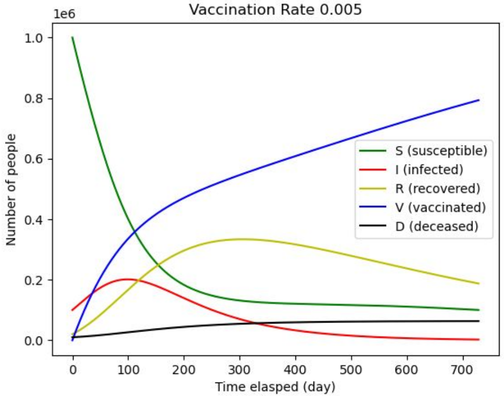
  => steeper curve for infected cases in the ascending phase

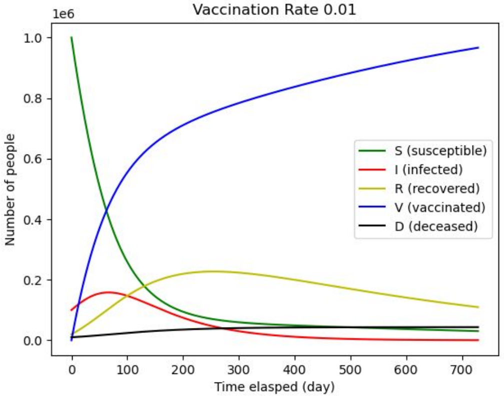
  => peak comes earlier

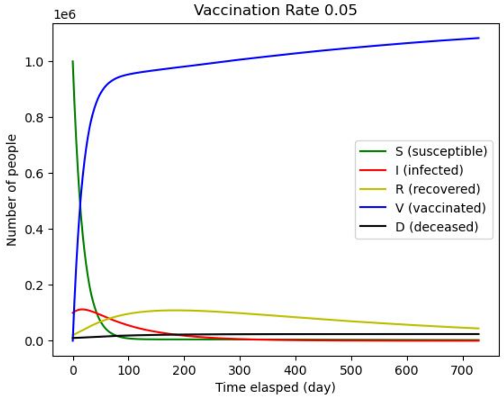
- Higher infection rate: higher and thinner peak
- Around peak of infection: Deceased population start to climb at a higher speed

Simulation: Vaccination Rate (α)



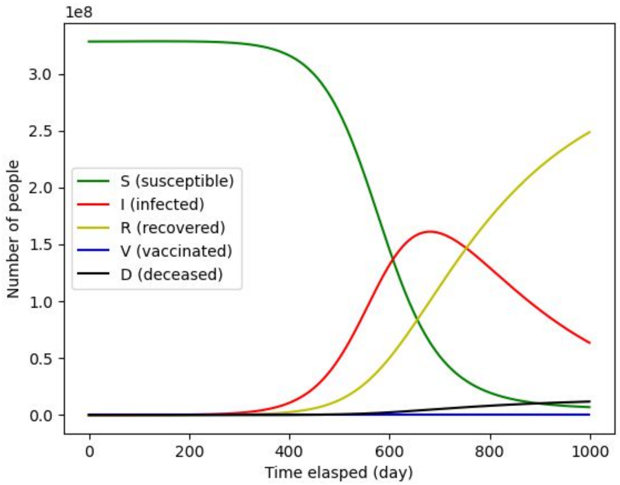






- Use of official data for initial conditions:
  - $S_0 = 10^6$, $I_0 = 10^5$, $R_0 = 2*10^4$, $D_0 = 1*10^4$, $V_0 = 0$
- Middle of pandemic: reduce the number of infections and deaths fairly effectively
- Faster the vaccination rate, lower the peak of infections and deaths
- peak of infection occurs around 40% to 60% vaccination rate
  - indicates that when roughly half of the population are vaccinated
  - => achieve herd immunity for the population

Simulation based on prediction



- Besides fixed values, we also did some model training on the ground truth of 10 months of data.
- Compared 2 models and chose gradient boosted linear regression model
- Curve without vaccination

Looking forward and additional considerations

Summary:

- In total we considered 3 scenarios for our modified SIR model
- We then simulated our model and highlighted the effects of each parameter

Future considerations:

- Vaccine and age groups
- Herd immunity
- Immigration

References

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