Scattering n-p interaction

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The nucleon-nucleon potential is assume as:

$$V_{ij} = (V_R + \frac{1}{2}(1+P^{\sigma}_{ij})V_t + \frac{1}{2}(1-P^{\sigma}_{ij})V_s)(\frac{1}{2}u + \frac{1}{2}(2-u)P^{\tau}_{ij}) + \frac{1}{4}(1+\tau_{iz})(1+\tau_{jz})\frac{e^2}{r_{ij}} \tag{1}$$

take $u \approx 1$

$$V_{ij} = \frac{1}{2}\left[V_R + \frac{1}{2}(1+P^{\sigma}_{ij})V_t + \frac{1}{2}(1-P^{\sigma}_{ij})V_s\right](1+P^{r}_{ij}) + \frac{1}{4}(1+\tau_{iz})(1+\tau_{jz})\frac{e^2}{r_{ij}} \tag{2}$$

in which the operator come defined as $P^{\sigma}_{ij} = (-1)^{s+1}$ and $P^{r}_{ij} = (-1)^{l}$. For $n-p$ interaction we have $\tau_{iz} = 1$ and $\tau_{jz} = -1$. We take account all possible cases to s and l. In each case, the quantum number is in 0,1 set hence there are 4 possible combination:

Case I: $l = 0$ y $s = 0$

$$V_{ij} = \frac{1}{2}(V_R + \frac{1}{2}(1+(-1))V_t + \frac{1}{2}(1-(-1))V_s)(1+1) + \frac{1}{4}(1+1)(1+(-1))\frac{e^2}{r_{ij}} \tag{3}$$

$$V_{ij} = V_R + V_s \tag{4}$$

Case II: $l = 0$ y $s = 1$

$$V_{ij} = \frac{1}{2}(V_R + \frac{1}{2}(1+1)V_t + \frac{1}{2}(1-1)V_s)(1+1) + \frac{1}{4}(1+1)(1+(-1))\frac{e^2}{r_{ij}} \tag{5}$$

$$V_{ij} = V_R + V_t \tag{6}$$

Case III: $l = 1$ y $s = 0$

$$V_{ij} = \frac{1}{2}(V_R + \frac{1}{2}(1+(-1))V_t + \frac{1}{2}(1-(-1))V_s)(1+(-1)) + \frac{1}{4}(1+1)(1+(-1))\frac{e^2}{r_{ij}} \tag{7}$$

$$V_{ij} = 0 \tag{8}$$

Case IV: $l = 0$ y $s = 1$

$$V_{ij} = \frac{1}{2}(V_R + \frac{1}{2}(1+1)V_t + \frac{1}{2}(1-1)V_s)(1+(-1)) + \frac{1}{4}(1+1)(1+(-1))\frac{e^2}{r_{ij}} \tag{9}$$

$$V_{ij} = 0 \tag{10}$$

In this way is possible take account only two firsts cases

$$V(r) = \begin{cases} V_R + V_s & \text{if} \quad l = 0 \quad \text{and} \quad s = 0 \\ V_R + V_t & \text{if} \quad l = 0 \quad \text{and} \quad s = 1 \\ 0 & \text{other case.} \end{cases} \tag{11}$$

with

$$V_R = V_{0R} \exp(-\kappa_R r^2),$$
$$V_t = -V_{0t} \exp(-\kappa_t r^2),$$
$$V_s = -V_{0s} \exp(-\kappa_s r^2),$$
with
$$V_{0R} = 200.0 \text{ MeV}, \quad \kappa_R = 1.487 \text{ fm}^{-2},$$
$$V_{0t} = 178.0 \text{ MeV}, \quad \kappa_t = 0.639 \text{ fm}^{-2},$$
$$V_{0s} = 91.85 \text{ MeV}, \quad \kappa_s = 0.465 \text{ fm}^{-2}.$$

to find $f(\theta)$ put potential in the next integral

$$f(\theta) = \int_0^\infty r \sin(qr) V(r) dr \tag{12}$$

for all non-vanish cases

Case I:

$$f(\theta) = \int_0^\infty r \sin(qr)(V_R(r) + V_s(r)) dr \tag{13}$$

$$f(\theta) = \int_0^\infty r \sin(qr) \left(V_{0R} \exp(-\kappa_R r^2) - V_{0s} \exp(-\kappa_s r^2) \right) dr \tag{14}$$

Case II:

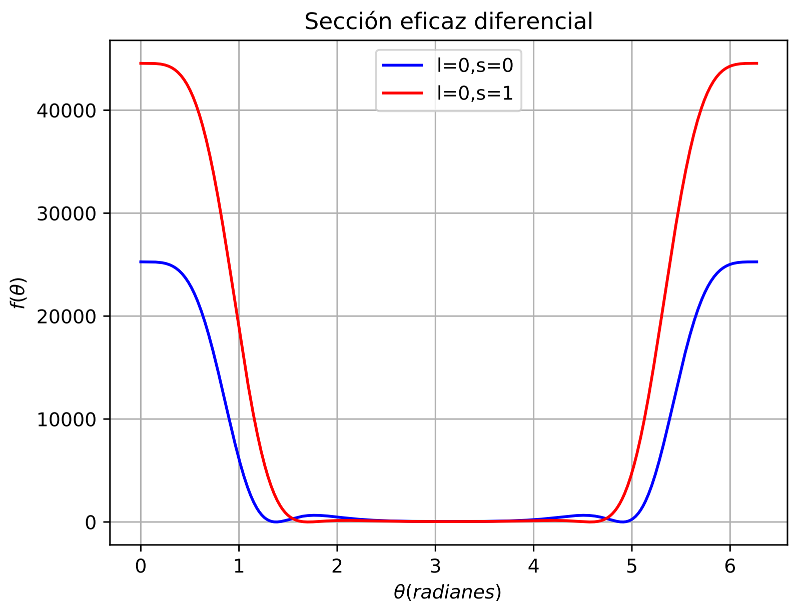
$$f(\theta) = \int_0^\infty r \sin(qr)(V_R(r) + V_t(r)) dr \tag{15}$$

$$f(\theta) = \int_0^\infty r \sin(qr) \left(V_{0R} \exp(-\kappa_R r^2) - V_{0t} \exp(-\kappa_t r^2) \right) dr \tag{16}$$

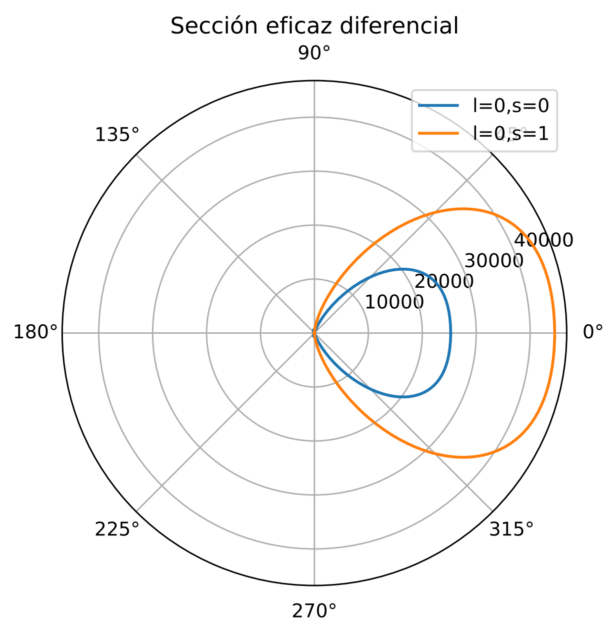
solving integral numericaly we get $f(\theta)$, with this

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \tag{17}$$

the results are shown in Figure 1



(a) Plot in cartesian coordinates.



(b) Plot in polar coordinates.

Figure 1: Plot of numerical results for differential cross-section in cartesian and polar coordinates.