POLITECNICO DI MILANO

Tecniche di misura PIV e LDV, principi ed applicazioni

Laser Doppler Velocimetry

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Aprile 2014

Why LDV- PIV ?

Conventional techniques (Pitot, HWA) have some limitations:

- ✓ perturb the medium
- ✓ have limited space or time resolution
- ✓ produce ambiguous results (→ NO directional sensitivity)
- ✓ not suitable in two-phase flows or combustion

LDV- PIV are optical techniques:

- ✓ no directional ambiguity (3 velocity components)
- ✓ no calibration required
- ✓ do not perturb the flow
- ✓ insensitive to ambient conditions
- ✓ have good space-time resolution (LDV better than PIV)
- ✓ work also in two-phase flows (sprays) and combustion environment

LDV allow direct measurement of local and instantaneous particle velocity, widely used in fluid mechanics and combustion.

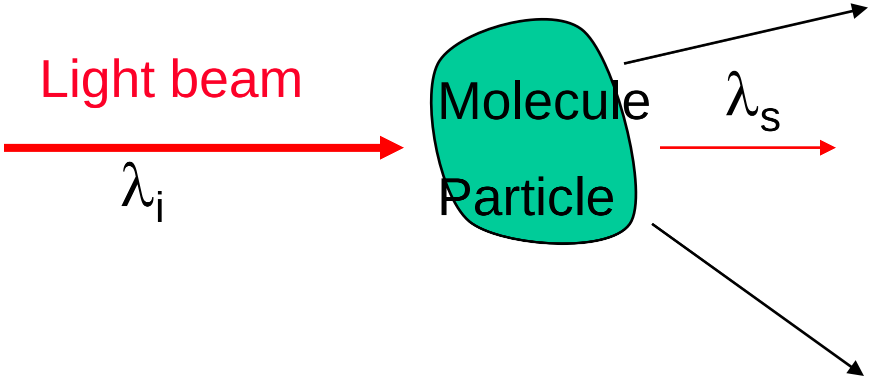
LDV has been demonstrated by Yeh and Cummins in 1964

Application of LDV (and PIV)

- Laminar and turbulent flows
- Two-phase flows (solid or liquid in gases)
- Investigations on aerodynamics
- Supersonic flows
- Turbines, automotive etc.
- Liquid flows
- Sprays (steady or unsteady)
- Surface velocity and vibration measurement
- Hot environments (flames, plasma, etc.)
- Velocity of particles (drops, bubbles or solid particles)
- ...etc., etc., etc.

Particle-Light Interaction

It is called Mie scattering the interaction between light and micrometric particles (D ~ λ)



Refraction

Phase shift
Interferometry
(phase modulation)

Doppler shift

Absorption

Diffraction

Rayleigh scattering

Mie scattering

Raman scattering

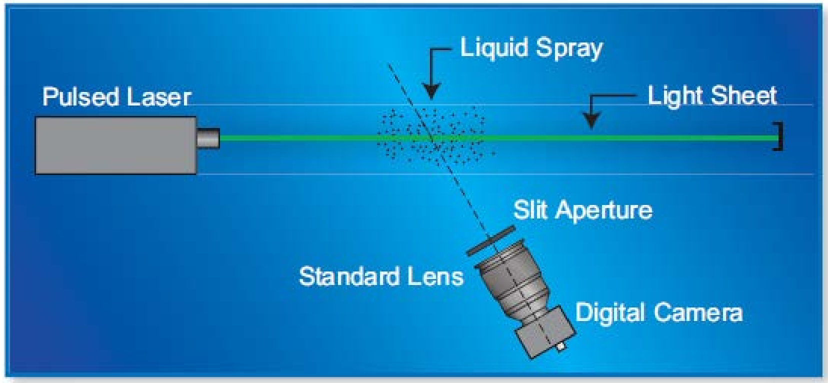
Fluorescence

Incandescence

Mie Scattering is an elastic scattering process $\lambda_i = \lambda_s$

Scattering techniques

- ➢ single-drop/particle
- ➢ ensemble averaging
- ➢ imaging techniques

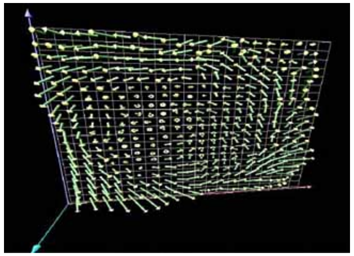


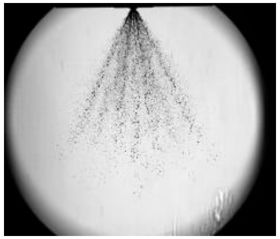
Laser-Doppler Velocimetry (LDV) is ***single-drop/particle techniques.***

Diffraction based measurements produce ensemble averaging over the cloud of drops in the control volume.

Imaging methods give instantaneous information on single- or two-phase flow structure. They can be used to visualize:

- 2-D, 3-D velocity pattern (PIV)
- drop/particle distribution in space
- vapor/liquid distribution in space

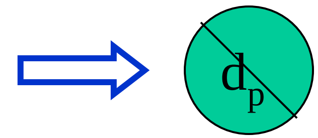


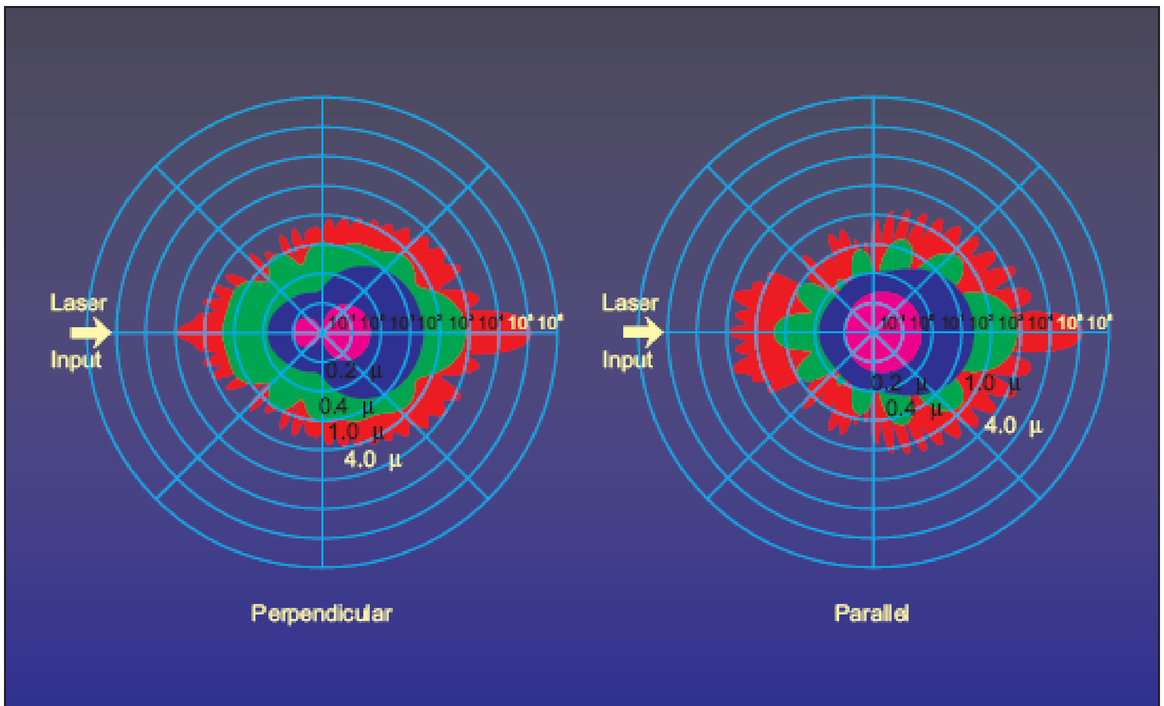


Scattering from small particles

The light scattering properties of a homogeneous and isotropic sphere illuminated by a plane wave can be described by geometrical optics or using the more general (Lorenz-Mie) theory which solves the wave equation with respect to the boundary conditions on the spherical particle.

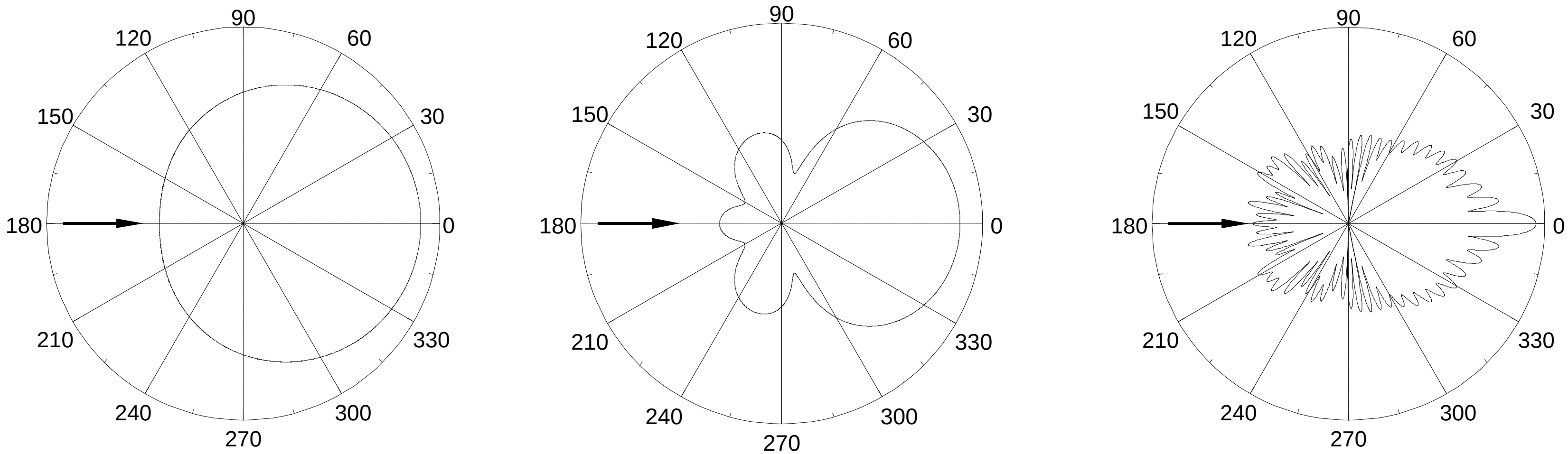
Geometrical optics is only applicable when $d_p >> \lambda$





Mie scattering patterns for different particle sizes, as a function of Perpendicular and Parallel laser Polarization.

Particles: scattered light distribution



$d_p \cong 0.2\lambda$ $d_p \cong 1.0\lambda$ $d_p \cong 10\lambda$

Polar plot of scattered light intensity versus scattering angle.

The intensity is shown on a logarithmic scale.

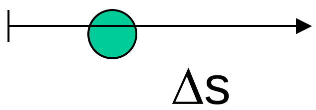
The scattered intensity distribution has a complicate shape.

It depends on particle size and is max. in the forward direction, for a given incident laser power.

Velocity measurement

*A **direct** velocity measurement requires the evaluation of the elapsed time Δt that an object requires to pass a known distance Δs.*

$$U = \frac{\Delta s}{\Delta t}$$



The measured velocity is a spatial and temporal mean value with respect to Δs and Δt.

Optical techniques allow to generate very small Δs ($\approx \lambda$ of light), i.e. two laser spots or an interference fringe pattern.

Modern electronic processors resolve very short Δt ($\approx$ 10 ns).

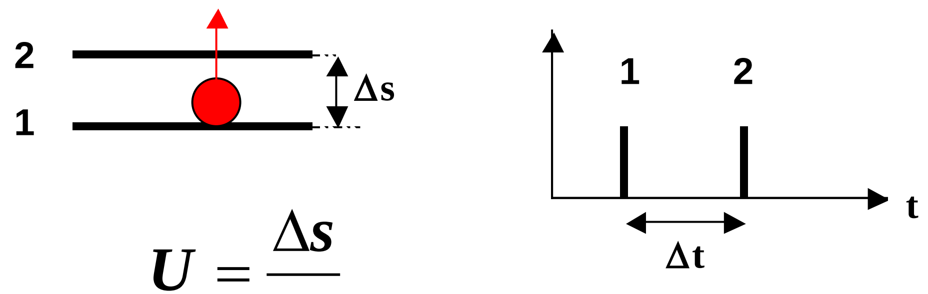
Micrometric tracer particles in a flow may constitute the sample objects. If properly chosen, the velocity of scattering particles corresponds to the local flow velocity, thus allowing a ***direct measurement***.

The scattered light from tracer particles brings the information to the observer (photoreceiver).

This illustrates the basic principle of any **optical technique** used to measure flow velocities.

Time-of-flight measurement

This technique allows direct velocity measurement through space and time

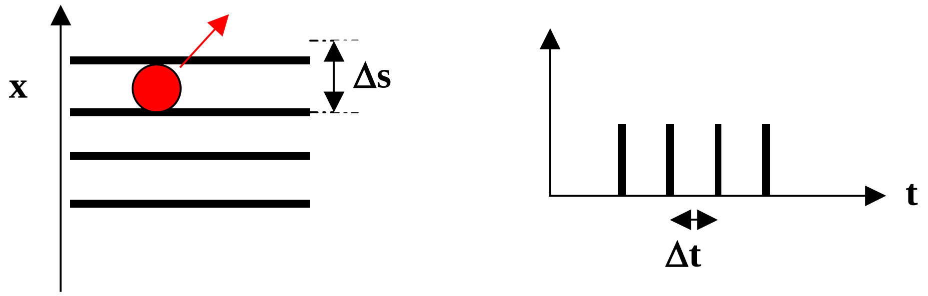


$$U = \frac{\Delta s}{\Delta t}$$

This approach is simple but disadvantageous at high flow turbulence levels, since the particle trajectory may cross only one of the two beams, resulting an a missed signal

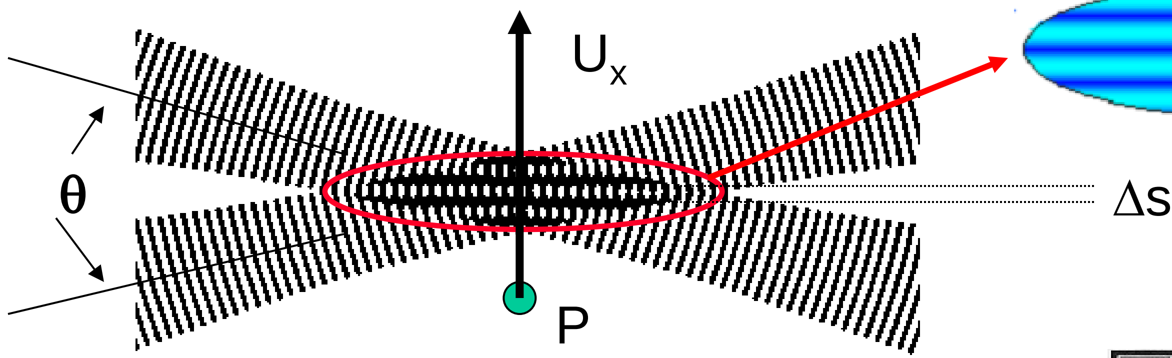
A multiple line grating, with uniformly spaced lines, resolves the velocity over the entire measurement volume, since the frequency of the signal pulses is directly proportional to the velocity normal to the grating lines

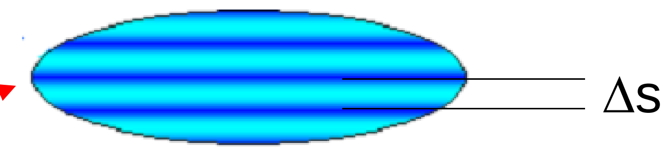
$$U_x = \frac{\Delta s}{\Delta t} = f\,\Delta s$$



Laser Doppler Velocimetry (Interferential Fringes Model)

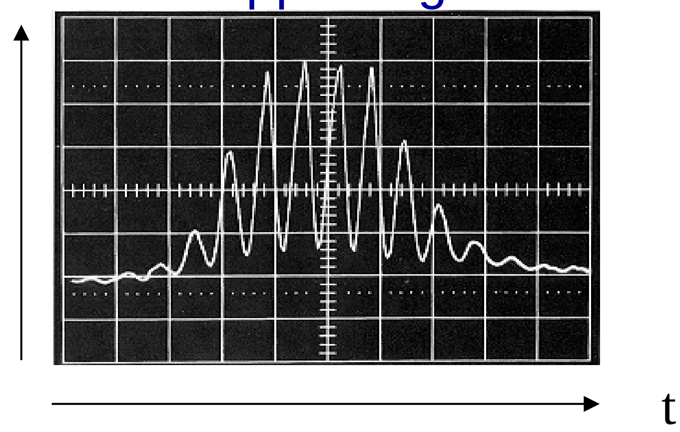
The LDV technique (in its differential mode) may be considered to use a multiple line grating (interferential fringes) to measure particle velocity.





$$f_D = \frac{U_x}{\Delta s}$$

Doppler signal



When the particle traverses the interference fringes (a pattern of light and dark surfaces) generated at the intersection of two focused laser beams, the scattered light fluctuates in intensity with a frequency equal to the velocity of the particle divided by the fringe spacing (fringe model)

$$\Delta s = \frac{\lambda}{2 sin(\theta/2)}$$

$$f_D = \frac{2U_x}{\lambda}\sin(\theta/2)$$

Laser Doppler Velocimetry (Doppler Effect)

The basic principle of the technique invokes the Doppler effect twice

1) the incident laser light, characterized by the wavelength λ_b and frequency f_b , impinges on a *moving* tracer particle.
2) when light with a frequency f_p is scattered from the *moving* tracer particle and received by a stationary detector with the frequency f_r

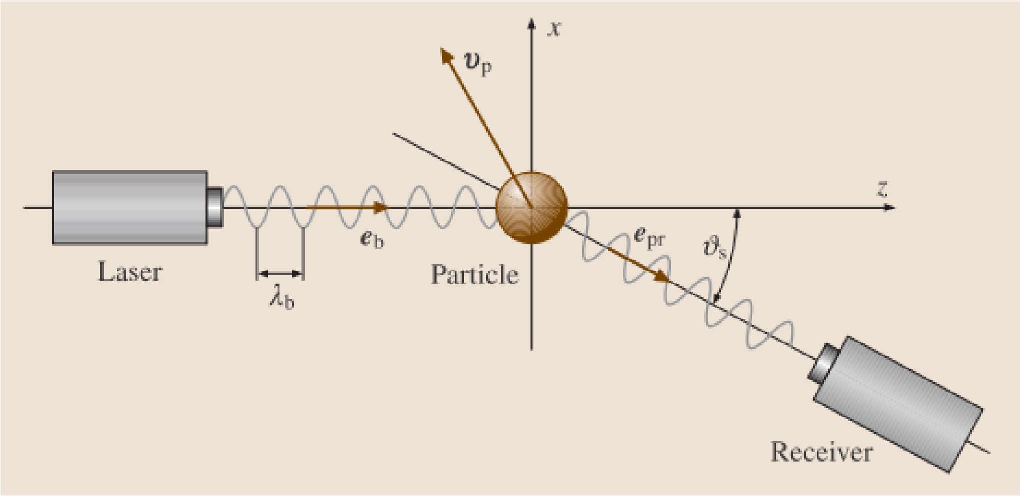


Fig. 5.69 Defining geometry for applying the Doppler effect in the laser Doppler technique

Scalar product!

$$f_r = f_b \frac{\left(1 - \frac{v_p e_b}{c}\right)}{\left(1 - \frac{v_p e_{pr}}{c}\right)} \approx f_b + \frac{v_p (e_{pr} - e_b)}{\lambda_b}$$

for $|v_p| \ll c, c = f_b \lambda_b$,

Laser Doppler Velocimetry (Doppler effect)

In the differential mode the frequency shift due to the double Doppler effect is extracted by the optical mixing of waves with frequencies f_1 and f_2 on the detector.

$$f_1 = f_b + \frac{\boldsymbol{v}_p(\boldsymbol{e}_{pr} - \boldsymbol{e}_1)}{\lambda_b}\,, \quad f_2 = f_b + \frac{\boldsymbol{v}_p(\boldsymbol{e}_{pr} - \boldsymbol{e}_2)}{\lambda_b}\,.$$

$$f_D = f_1 - f_2 = +\frac{\boldsymbol{v}_p(\boldsymbol{e}_1 - \boldsymbol{e}_2)}{\lambda_b} = \frac{2\sin(\Theta/2)}{\lambda_b}\underbrace{|\boldsymbol{v}_p|\cos\alpha}_{\text{Velocity component along versor } \boldsymbol{n}}$$

f_D indipendent from detector orientation

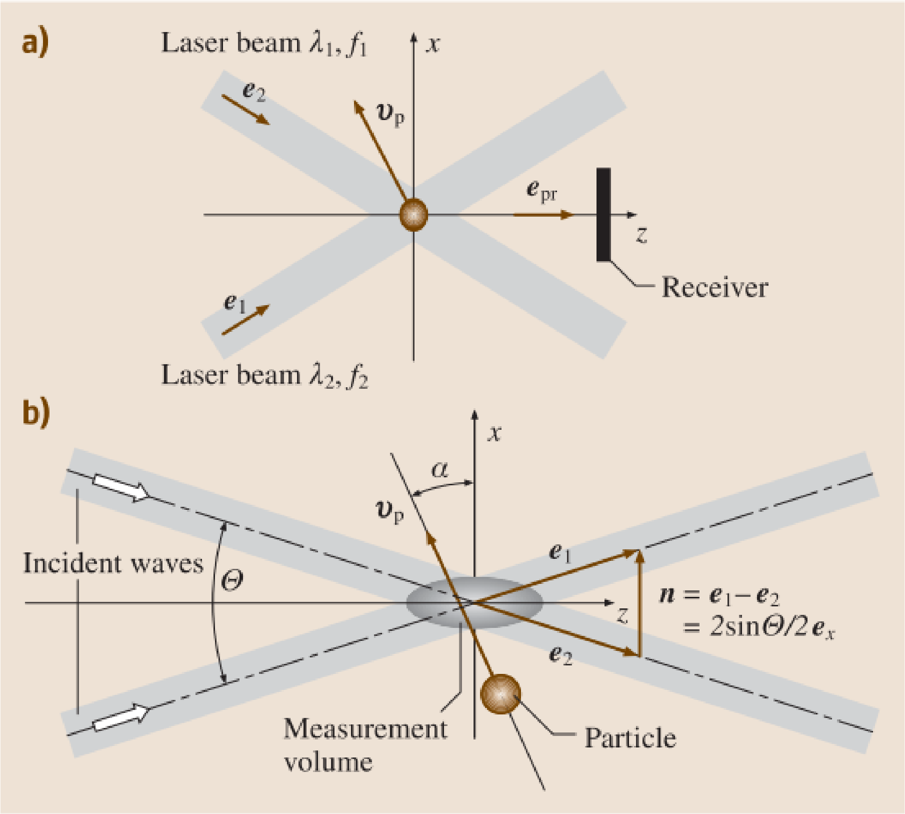


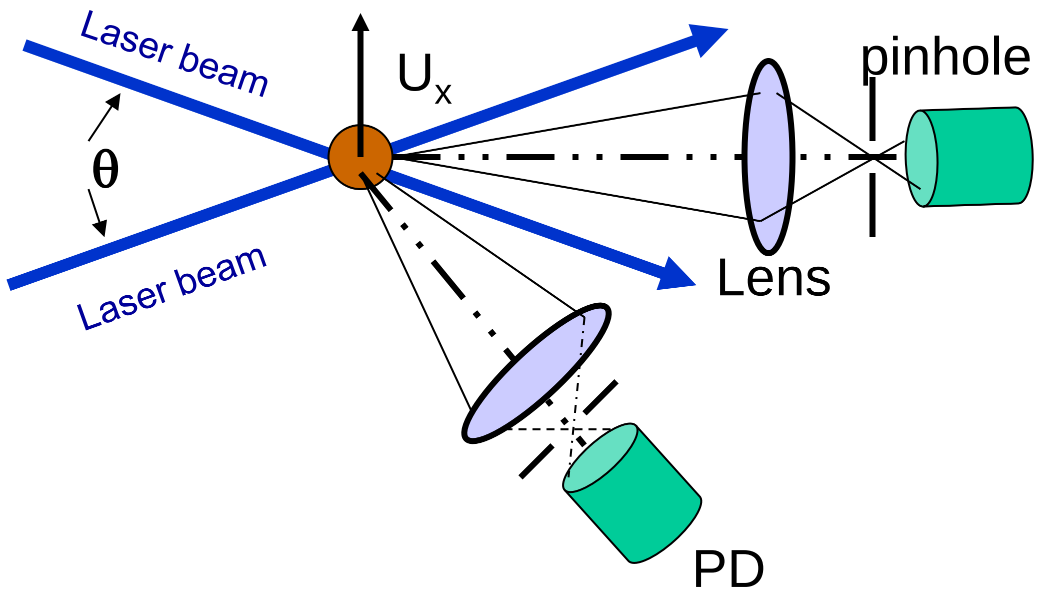
Fig. 5.70 (**a**) Dual-beam configuration. (**b**) Vector relations relevant to determining the Doppler frequency

LDV – Differential mode

Main advantages of the differential LDV optical scheme:

- Easy to align and almost insensitive to vibrations, resulting in a versatile and robust instrument.
- The Doppler frequency is independent of the detection angle, allowing off-axis and even back-scatter configurations.
- The differential mode assures very good spatial resolution.

Using a collecting lens and a pinhole (spatial filter), only the scattered light originating from a particle crossing the probe volume will be received by the photo-detector.



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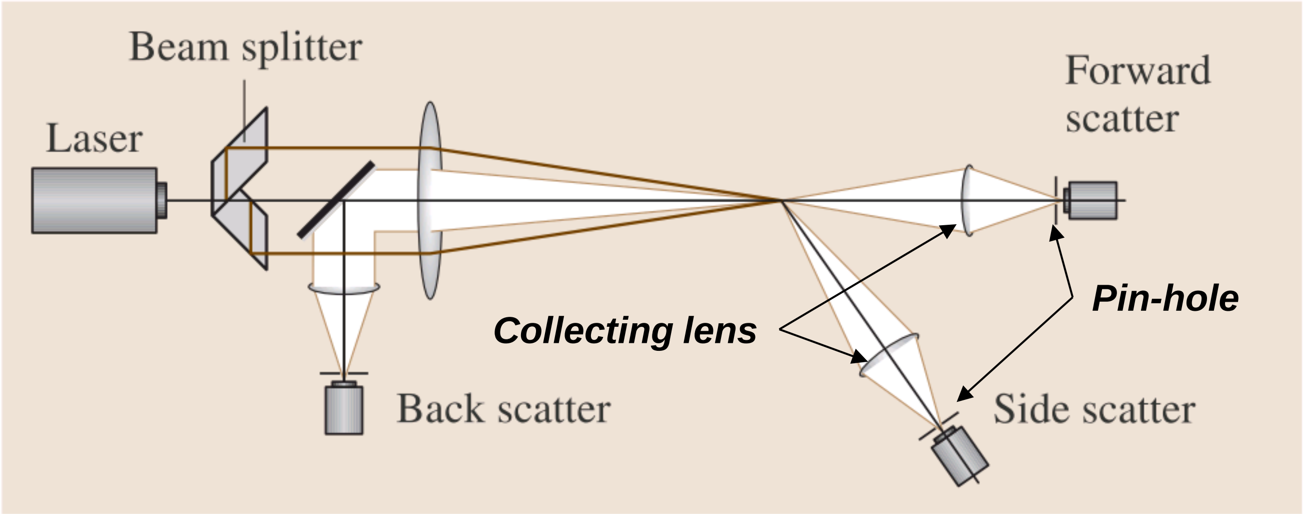


Fig. 5.73 Possible optical configurations of a laser Doppler system

LDV – Fringe model (1/3)

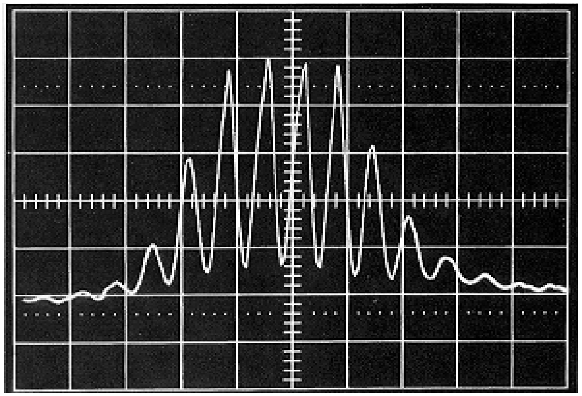
The differential LDV system can be interpreted by using the Fringe Model:

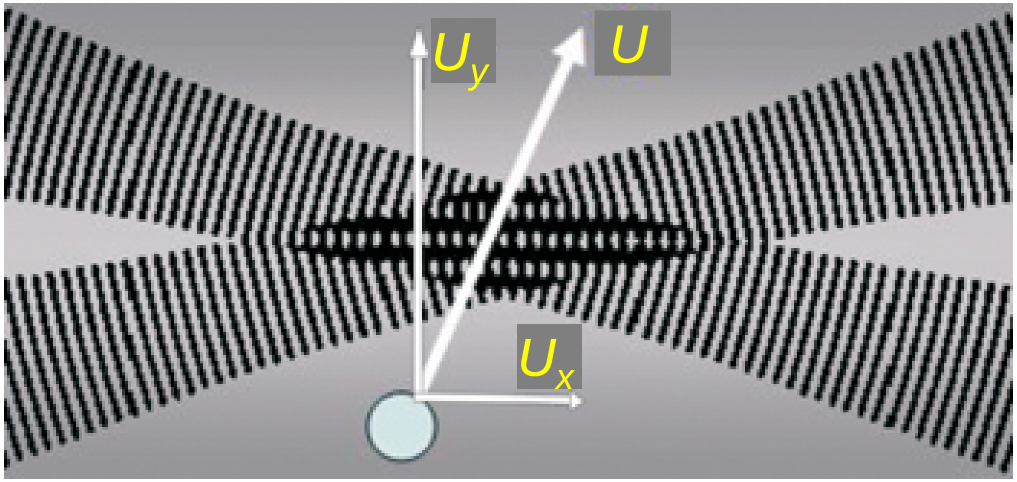
A particle that passes through the measurement volume crosses the fringe pattern and scatters a periodic light flux, which is detected as a frequency f_D in the receiving optics.

The frequency is independent of the direction of the scattered light beams and is proportional to the particle velocity perpendicular to the fringes.

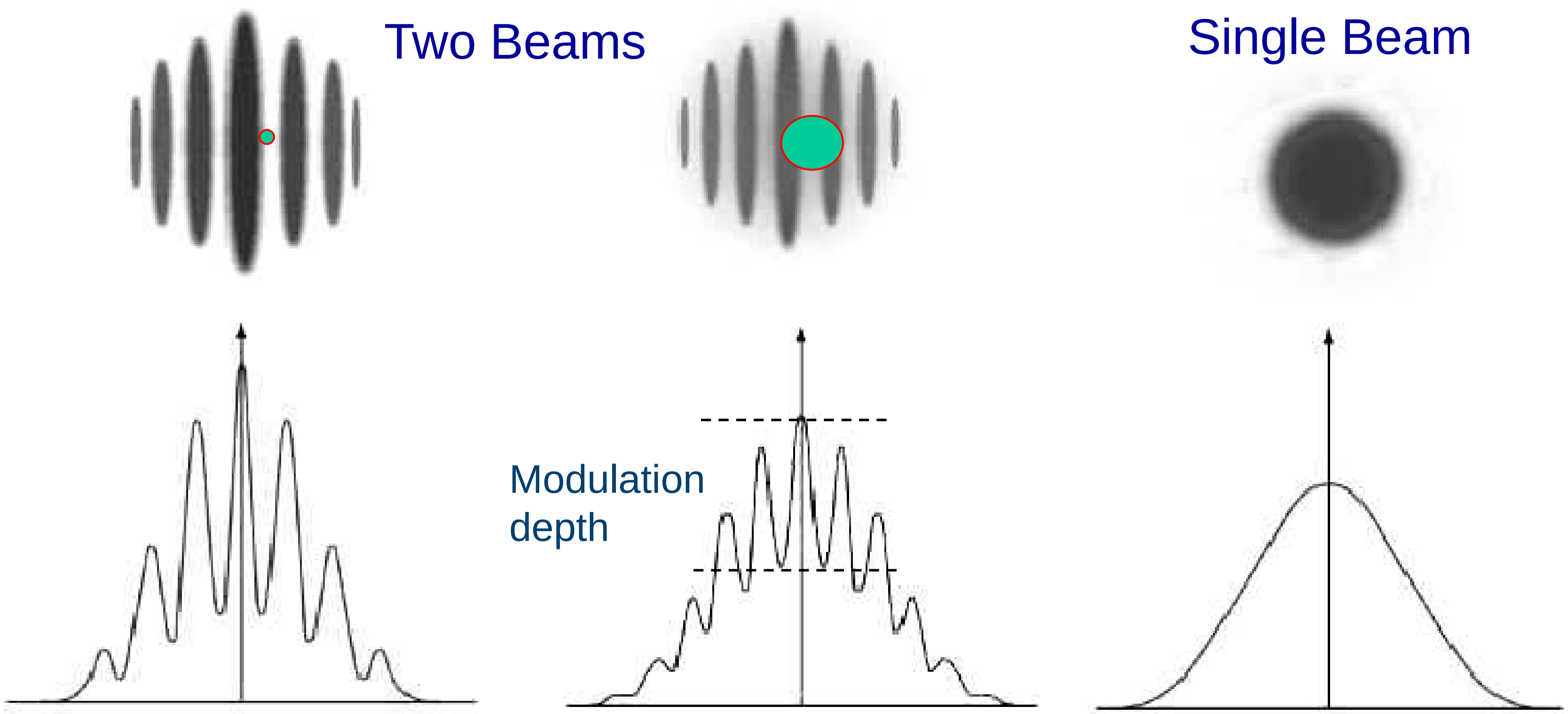
The LDV system thus measures the velocity component perpendicular to the fringe planes:

$$f_D = \frac{2U_y}{\lambda} sin(\ \theta / 2\ )$$





LDV – Fringe model (2/3)



Fringe quality (Visibility) defines the Doppler signal quality (Modulation depth).

Also the particle size influences the Doppler signal modulation, but a residual, measurable modulation is always present, except for a single beam.

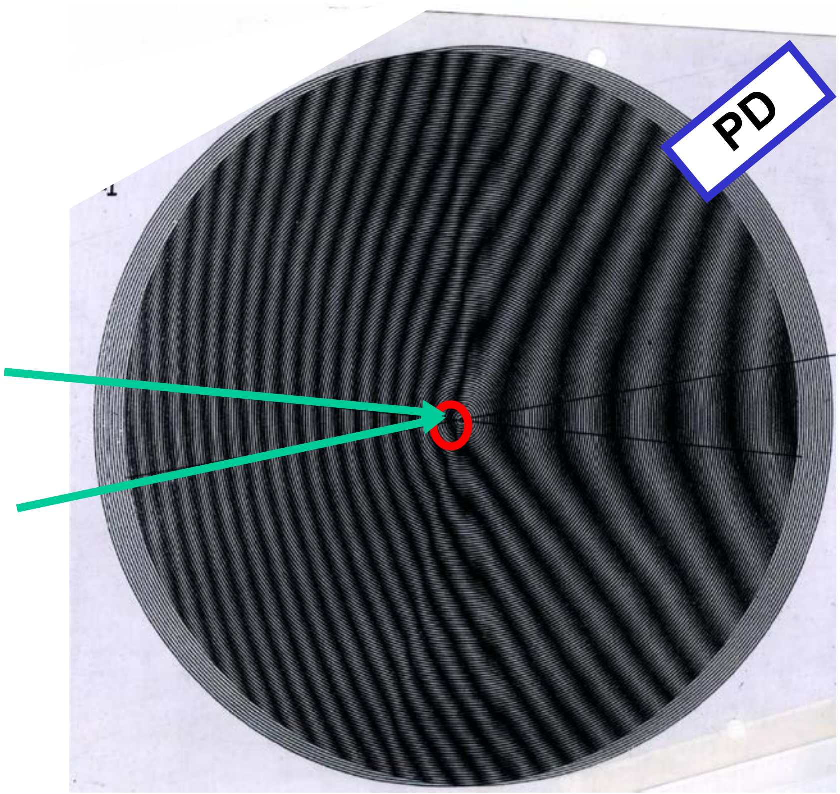
LDV – Fringe model (3/3)

Actually the fringes are formed by the superimposition of the two systems of wave front scattered by the particle from the two incident laser beams.

A photodetector (PD) located at any point in space will “see” these fringes.

When the particle moves the fringes will move and the detector will measure the Doppler frequency.

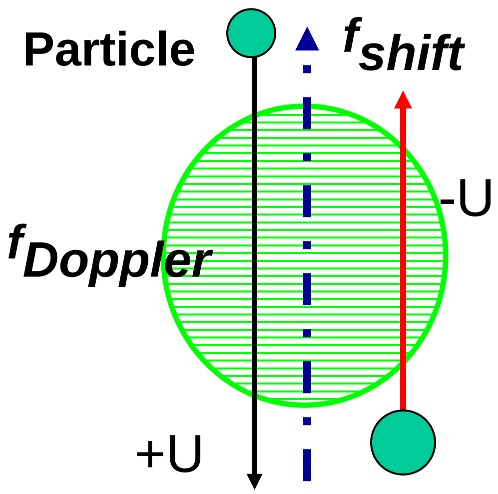
This explain why even very large particles generate a modulated Doppler signal.

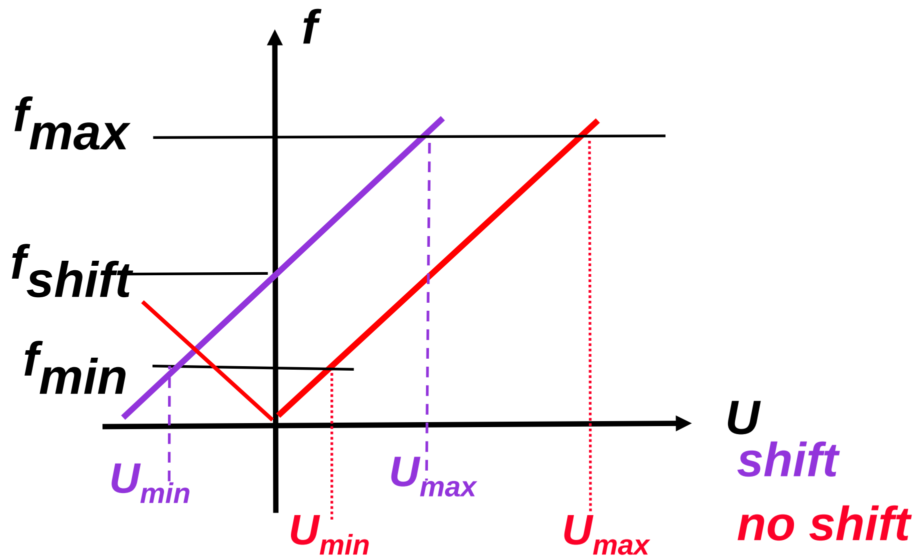


It also illustrate why, in the differential mode, the detector can be located everywhere in the space around the particle without altering the Doppler frequency.

Directional ambiguity

Particles moving in either the forward or reverse direction will produce identical Doppler signals and frequencies since a vector quantity (velocity) cannot be fully defined by a scalar quantity (frequency).





With frequency shift of one laser beam relative to the other, the interference fringes appear to move at constant velocity and the signal frequency exhibits an offset equal to the shift frequency.

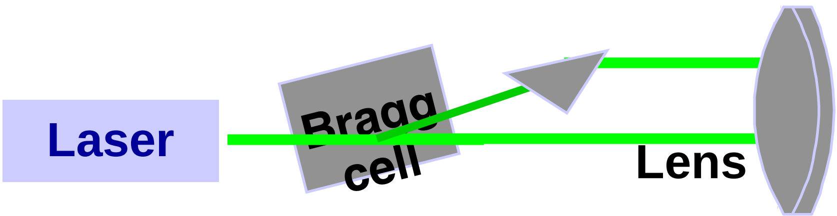
The particle velocity is now measured with a reference moving frame.

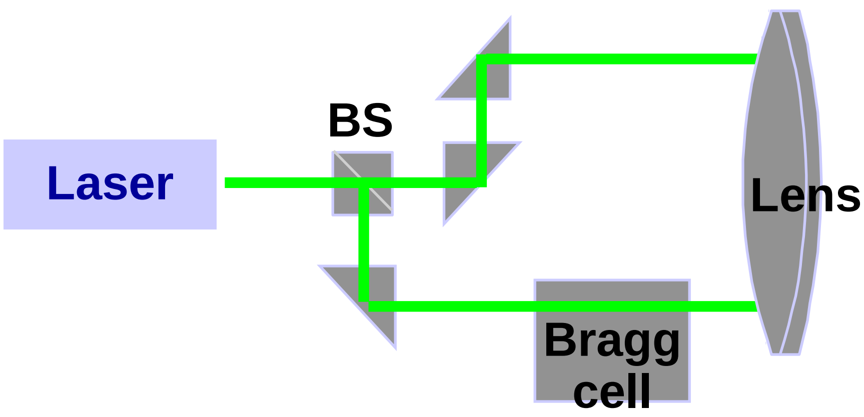
Positive velocities generate $f_D > f_s$, negative $f_D < f_s$

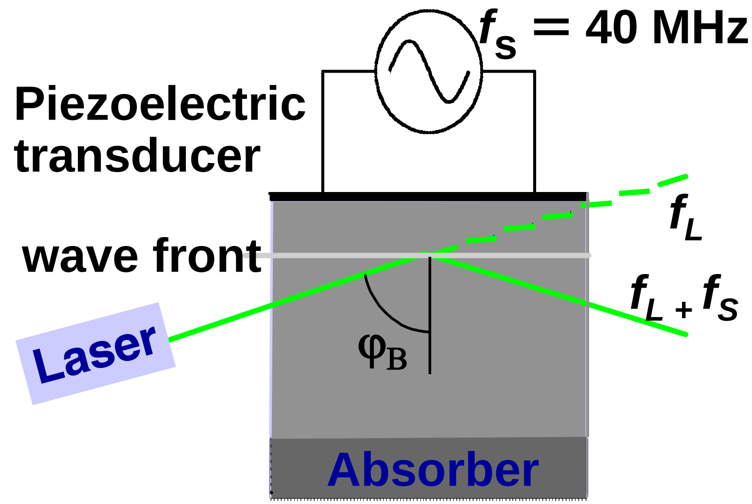
Frequency shift / Bragg cell

Frequency shifting can be obtained by using acousto-optical modulators (Bragg Cell)

The Bragg cell is inserted in the transmitting optics or used as a beam splitter (BS)







Bragg cells are powered at a fixed frequency (typically: 40 MHz)

Frequency of laser light is increased (reduced) by the same f_s, depending on the diffraction order (± 1)

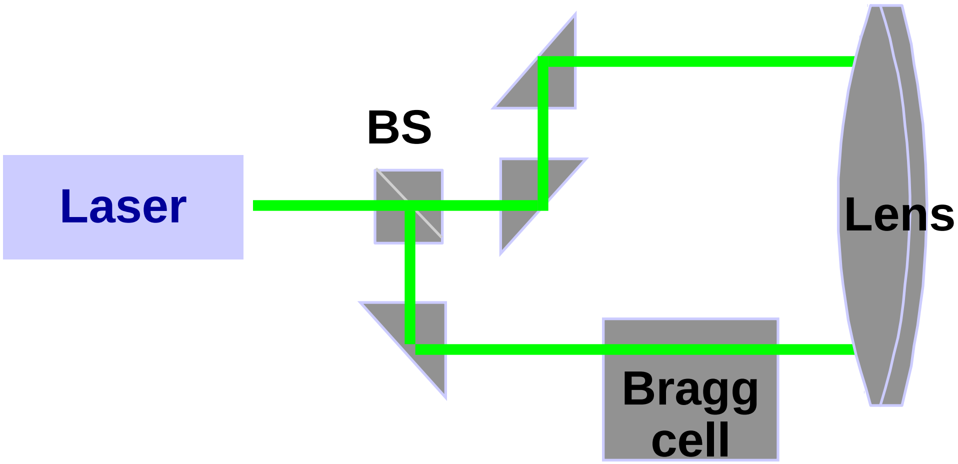
The frequency shift is equivalent to a change of phase with time which produces a fringe movement

$$\varphi = 2\pi f_s t$$

Transmitting optics

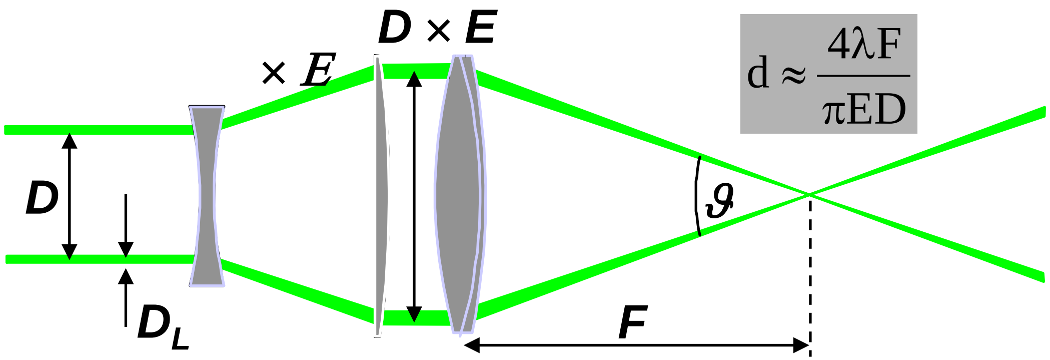
Basic modules:

- ✓ Beam splitter
- ✓ Mirrors
- ✓ Achromatic lens
- ✓ Bragg cell (frequency shift)



Options: Beam expanders instead of single lens

- reduce measurement volume dimensions
- increase power density in the probe volume
- allow measurements near a solid wall



A **beam waist** (minimum beam diameter, d) is generated in the plane of intersection (focal point) of the two laser beams.

Probe volume dimensions

The measurement volume results an ellipsoid and has a Gaussian intensity distribution in all 3 directions.

Dimensions are defined by the *1/e²* intensity points.

Length

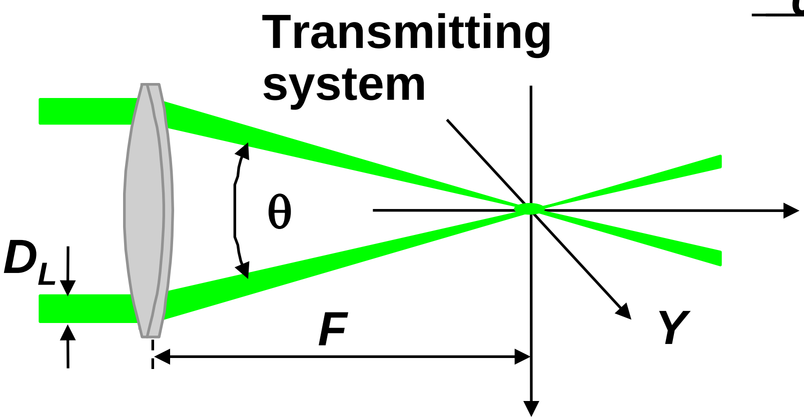
$$\delta_z = \frac{4F\lambda}{\pi E D_L \sin\left(\frac{\theta}{2}\right)}$$

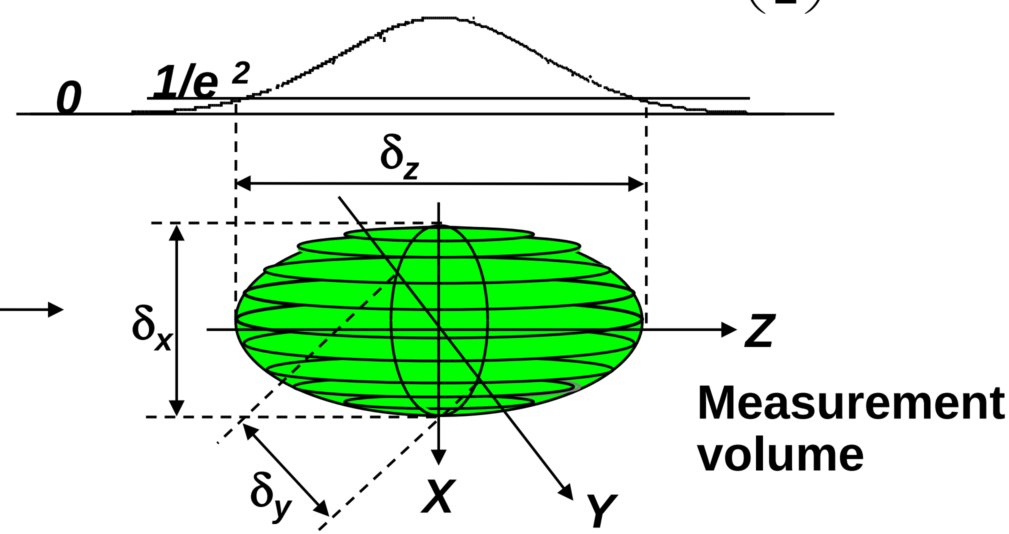
Width

$$\delta_y = \frac{4F\lambda}{\pi E D_L}$$

Height

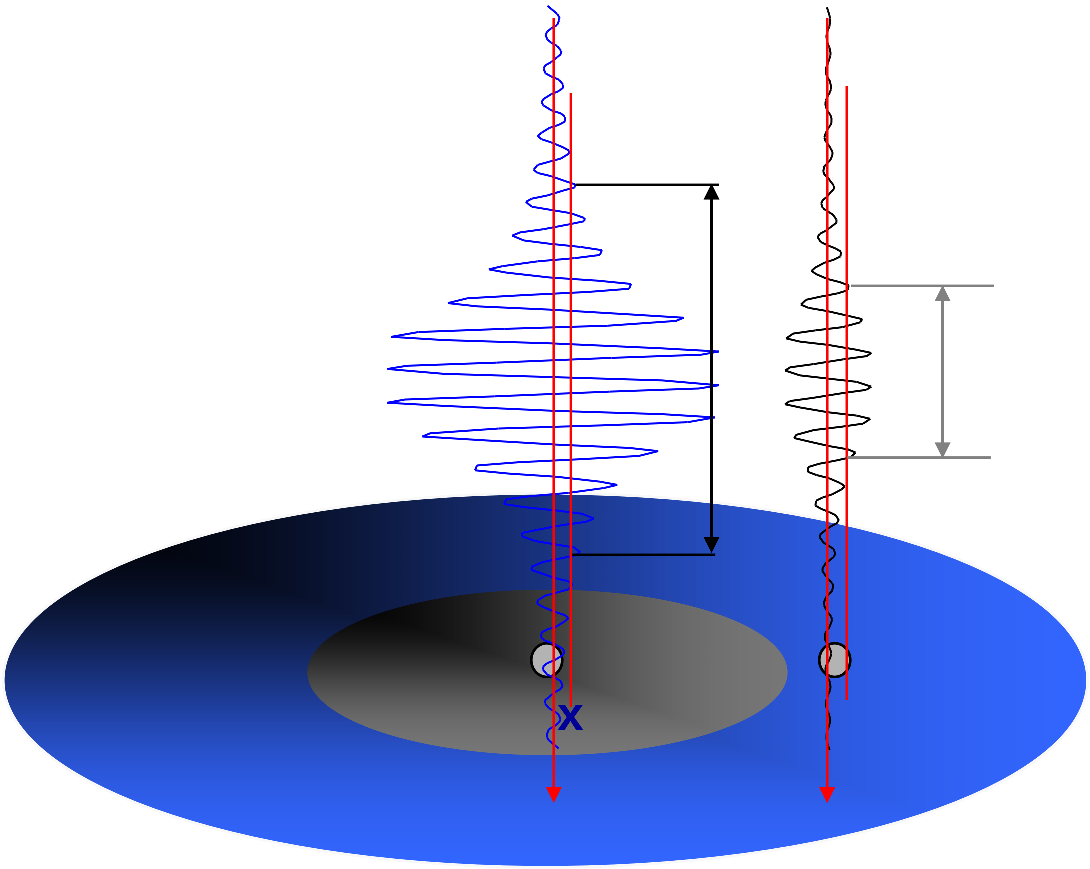
$$\delta_x = \frac{4F\lambda}{\pi E D_L \cos\left(\frac{\theta}{2}\right)}$$





Probe volume dimensions

The actual probe volume may be smaller than the geometric one, depending on laser power, signal amplification, system sensitivity, particle size and trajectory. Particles with same size but different trajectory, produce a different Doppler signal if the trigger level of the electronics is the same.

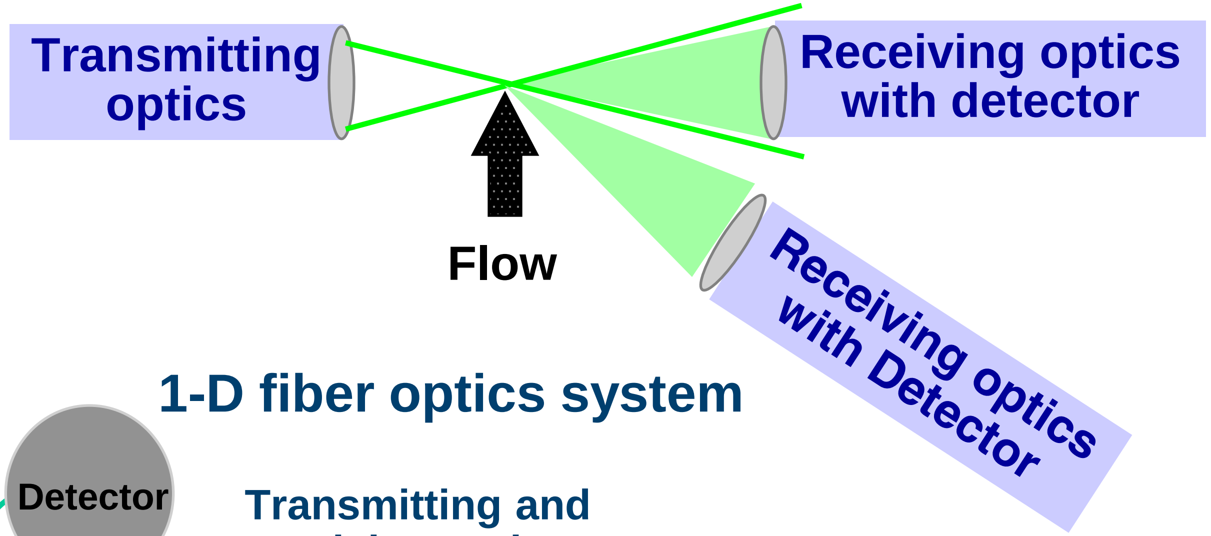


A particle passing through the center of the probe volume scatters light from the region with max. intensity, while a particle passing through the edge will produce a lower Doppler signal

LDV System configurations

Forward scatter
and side scatter
(off-axis)

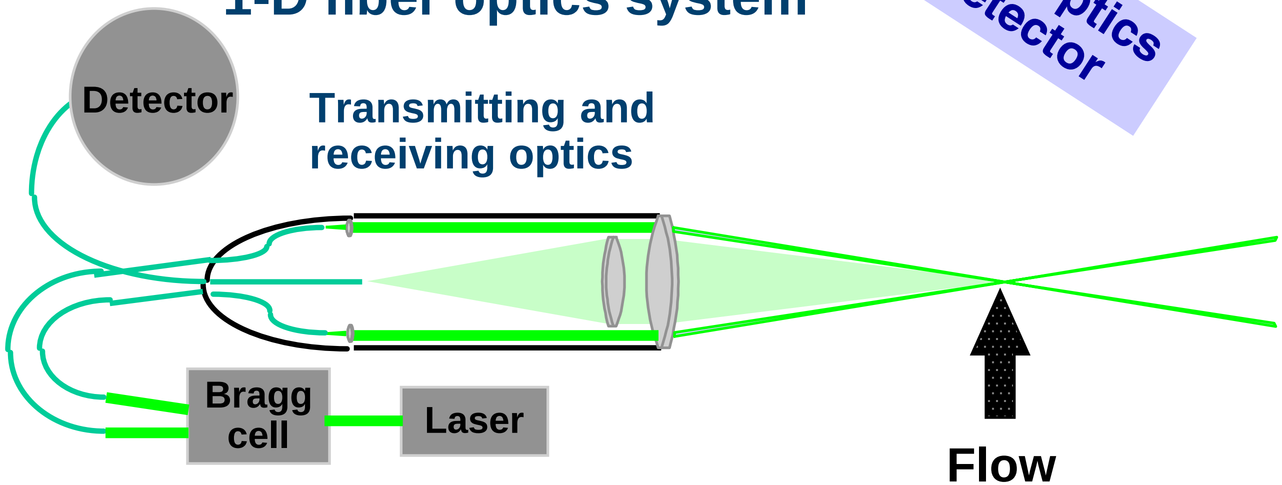
- Difficult to align
- Vibration sensitive

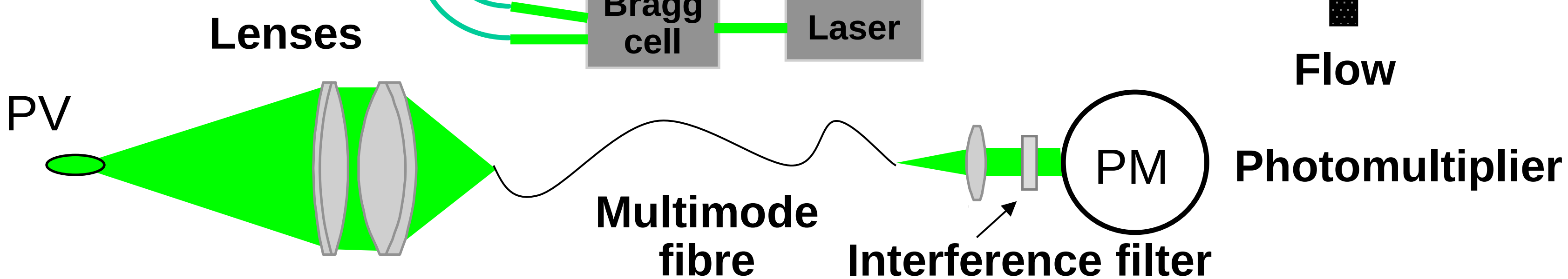


Backscatter

- Easy to align
- User friendly

1-D fiber optics system

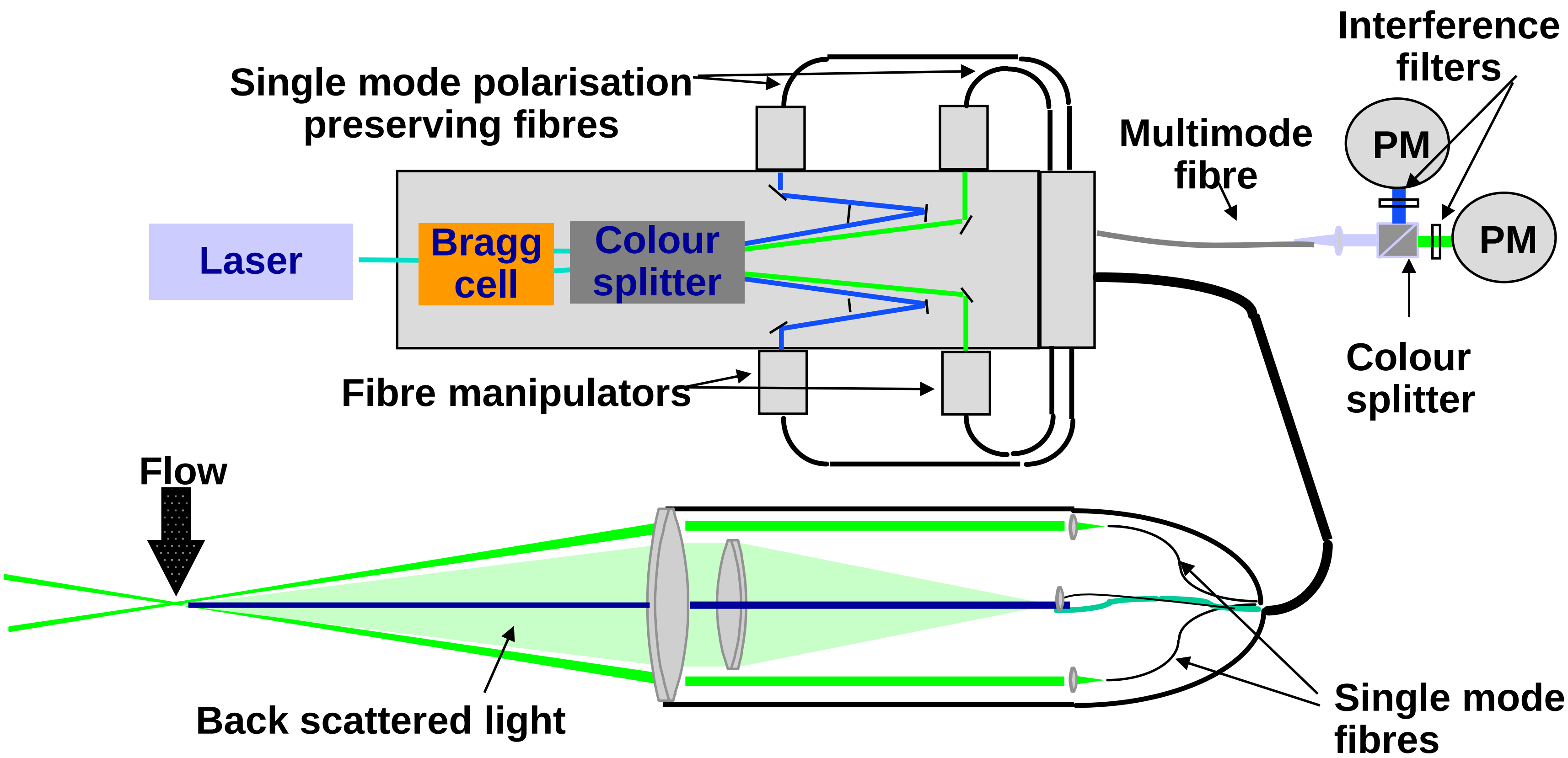




Backscatter 2-D configuration

Simultaneous measurements of two velocity components are possible using two color lasers (Argon-Ion) and a double detection system with proper filters.

The third velocity component can be also measured but requires an additional (separate) 1-D LDV system.



Dantec Dynamics documentation

LDV – Particle dynamics Introduction

The LDV instruments rely on laser light scattered from spherical particles. Only the particle velocity is measured (Velocimetry)

For applications to gaseous flows (Anemometry), fine particles (tracers) are suspended in the fluid and need to be sufficiently small to follow the velocity fluctuations at the highest frequencies; the lower limit to the acceptable size is given by the particle insensitivity to molecular thermal motion.

The particle capability to follow turbulent gas motion is measured by the ratio of the particle relaxation time to the time scale of the flow (Stokes number).

$$\tau_p = \frac{\rho_p d_p^2}{18\mu} \qquad T_f = \frac{L}{U} \qquad St = \frac{\tau_p}{T_f} << 1$$

Particle sizes from 0.1 to few microns are usually adequate in turbulent flow fields; 1 μm aerosol particles can be used up to 5 kHz fluctuations.

More precise estimation can be obtained through the analysis of particle dynamics.

LDV – Particle dynamics (1/5)

Under the conditions:

- Spherical particles
- No particle/particle interaction (particle separation > 1000 diameters)
- No lift force
- Low relative velocity between particle and medium
- High density ratios (particle/fluid)

the acceleration force acting on the particle can be equated to the Stoke's drag due to viscous forces

$$\frac{\pi}{6} d_p^3 \rho_p \frac{dU_p}{dt} = 3\pi\mu d_p \left(U_f - U_p\right)$$

with the assumption $Re_p = \frac{\rho_f \left|U_f - U_p\right| d_p}{\mu_f} < 1 \longrightarrow C_D = \frac{24}{Re_p}$

The equation of particle motion results: $\frac{d}{dt} U_p = 18 \frac{\mu}{d_p^2} \frac{U_f - U_p}{\rho_p}$

LDV – Particle dynamics (2/5)

$$\frac{\mathrm{d}}{\mathrm{dt}}U_p = \frac{18\mu}{d_p^2\rho_p}\left(U_f - U_p\right)$$

This is a *first order* differential equation and particle dinamics is characterised by a *characteristic time*

$$\tau_p = \frac{\rho_p\, d_p^2}{18\,\mu}$$

The particles manifest a first order low-pass filter response:

$$\frac{U_p}{U_f} = \frac{1}{\sqrt{1+\omega^2\tau_p^2}} \quad \text{valid for } \rho_p >> \rho_f$$

that can be re-arranged in the form: (ω= 2πf)

$$f = \frac{1}{2\pi\tau_p}\sqrt{\frac{1}{\left(U_p / U_f\right)^2} - 1}$$

The *cut-off frequency* results:
(for 1% slip (U_p/U_f) = 0,99)

$$f_c = \frac{0.0227}{\tau_p}$$

The *particle diameter* for a given cut-off frequency must be:

$$d_p < \left[\frac{18\mu}{2\pi\rho_p f_c}\sqrt{\frac{1}{\left(U_p / U_f\right)^2} - 1}\right]^{1/2}$$

LDV – Particle dynamics (3/5)

Particle response for 1% slip [$(U_p/U_f) = 0{,}99$] in Air ($\mu = 15\times10^{-6}$ Pa s)

Max. particle diameters for given cut-off frequency

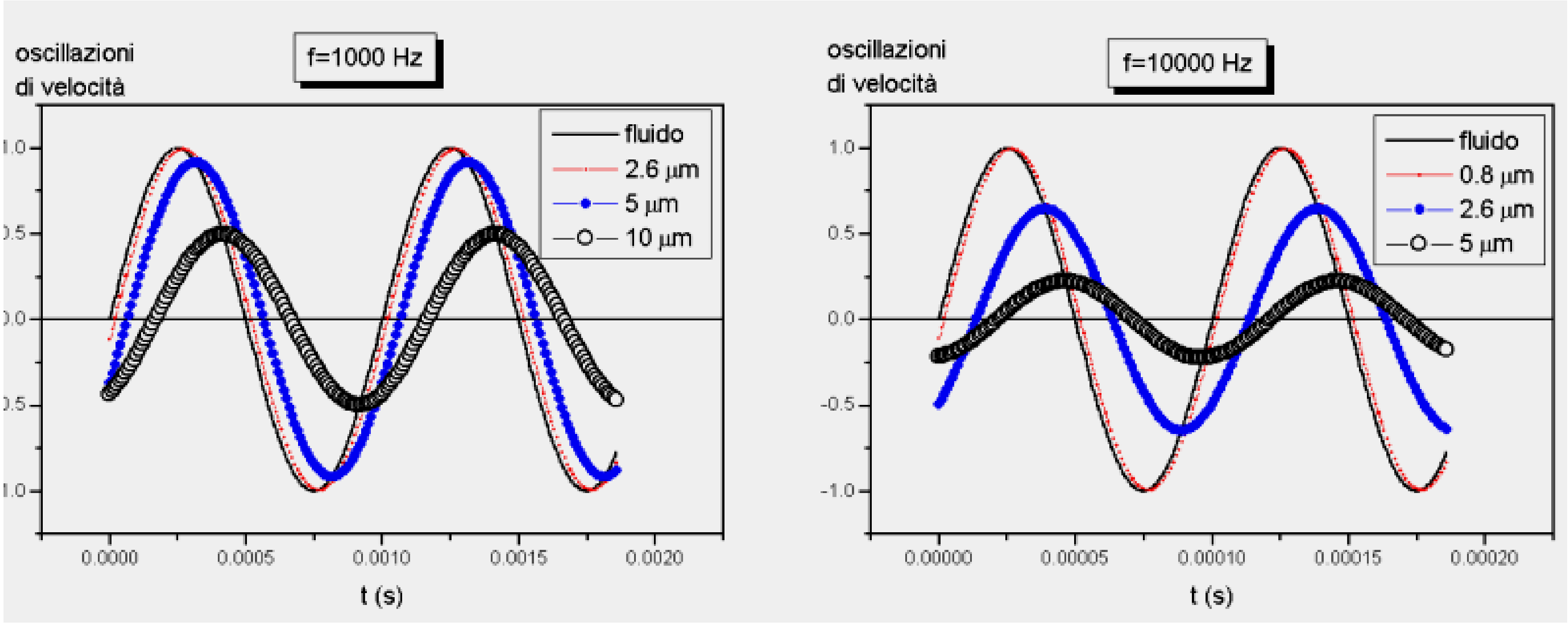
Fluid	Frequency	Particle [μm]			
	Fc [kHz]	TiO_2	Glass spheres	Water	Silicon Oil
Air	1	1.25	1.5	2.5	2.6
	2	0.9	1.1	1.8	1.8
	5	0.56	0.69	1.1	1.2
	10	0.4	0.5	0.78	0.82

Particle densities: TiO_2 $\rho = 3900$ [kg m^{-3}] Glass $\rho = 2600$ [kg m^{-3}]

Water $\rho = 1000$ [kg m^{-3}] Oil $\rho = 912$ [kg m^{-3}]

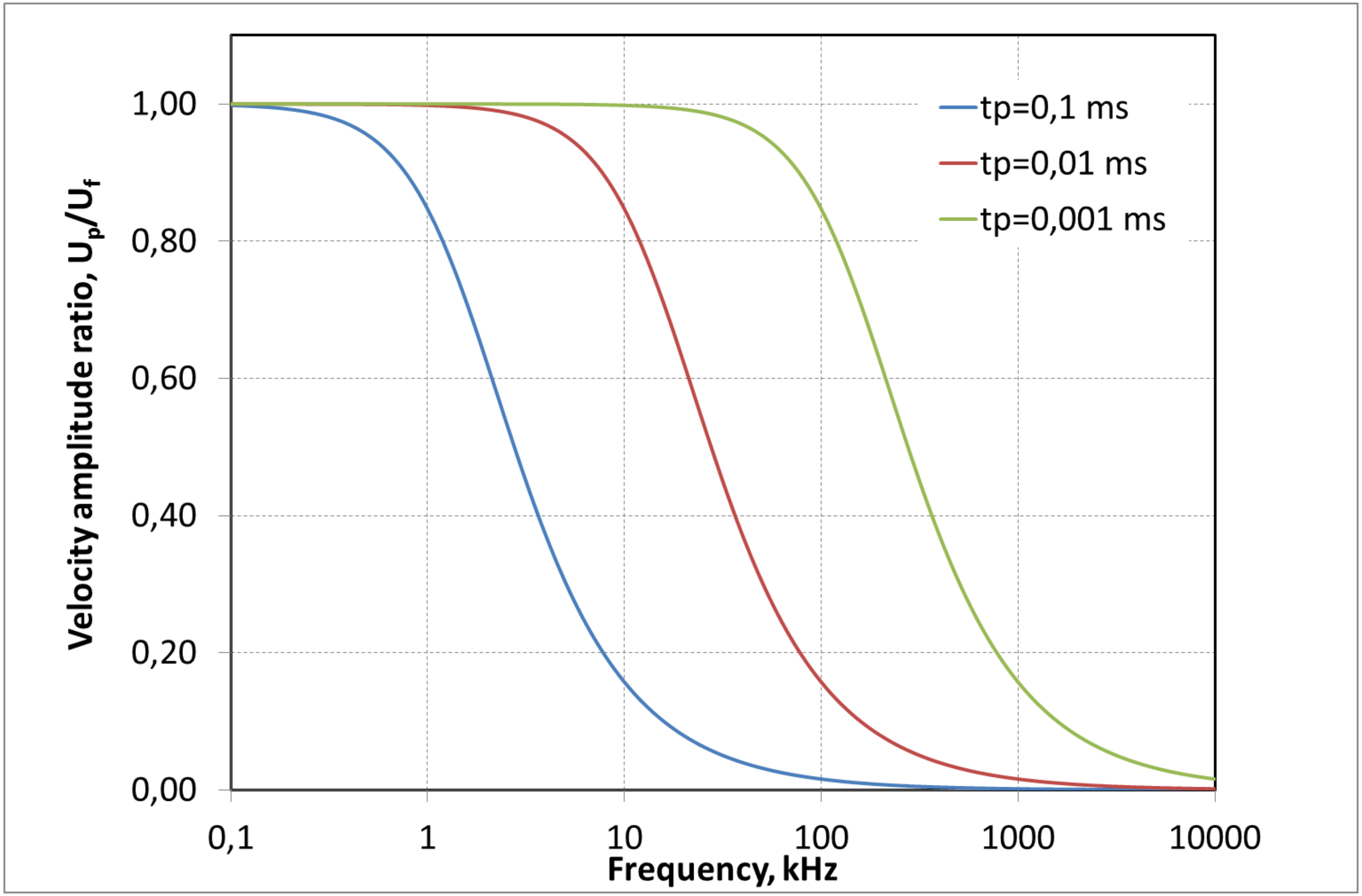
LDV – Particle dynamics (4/5)

Particle response to turbulent fluctuations (silicon drops in air)

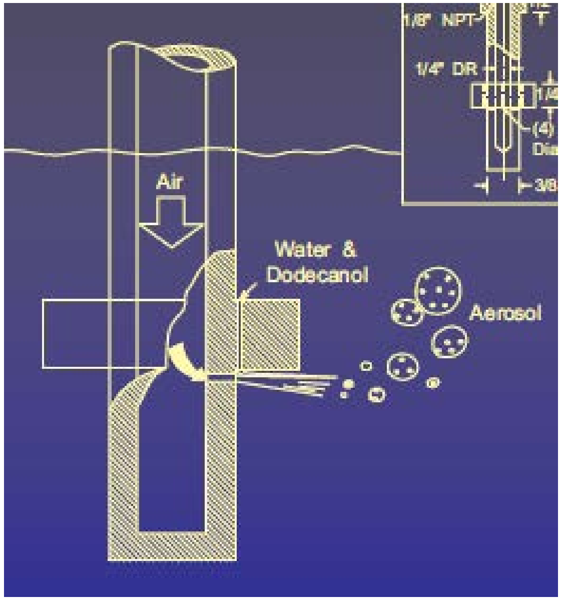


LDV – Particle dynamics (5/5)

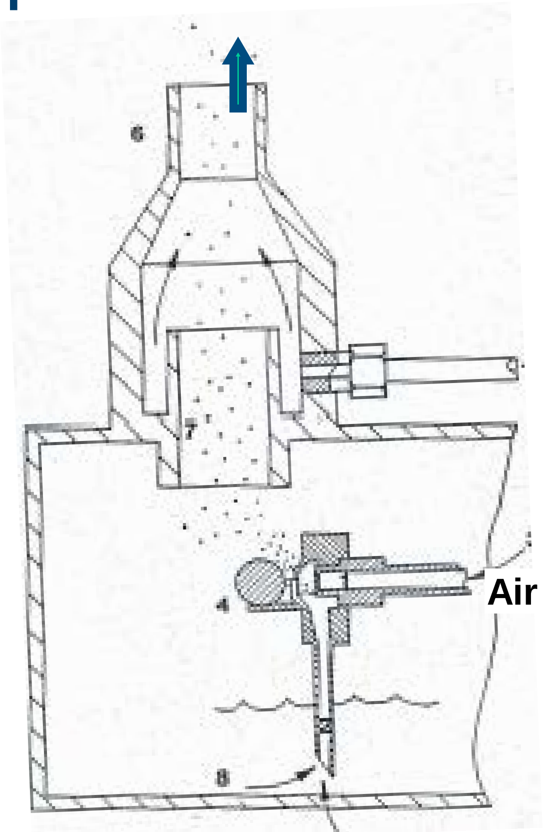
Particle response to turbulent fluctuations



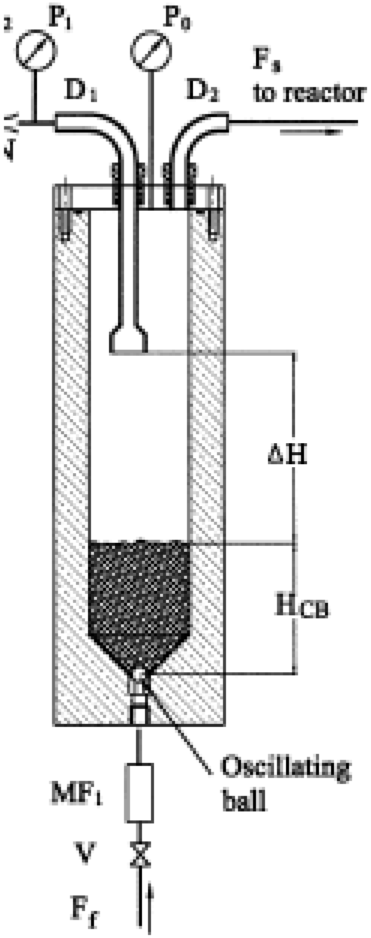
LDV – Particle generation



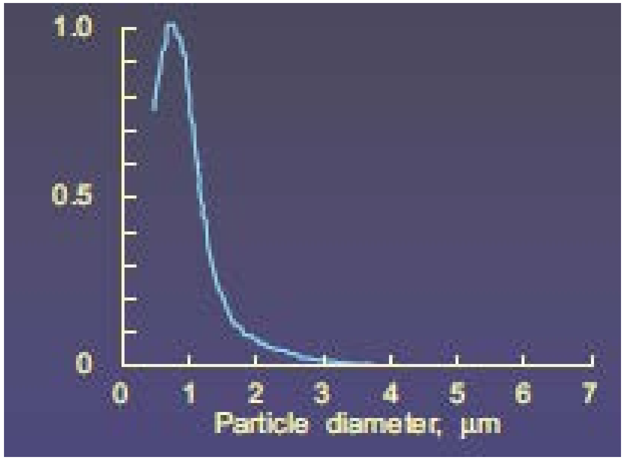
Laskin nozzle



Jet atomizer



Fluidized bed



Typical size distribution obtained with aerosol generators

Particle disturbance

For LDV applications in isothermal flows the best solution is to filter the air and introduce artificially generated aerosol particles.

The injection of seeding particles can perturb the reacting flow through:

- Catalitic effects
- Heat exchange (cooling of the gas)
- Absorption of light (obscuration)
- Particles/vortices interactions (typically particles are smaller then the smallest turbulence scales)

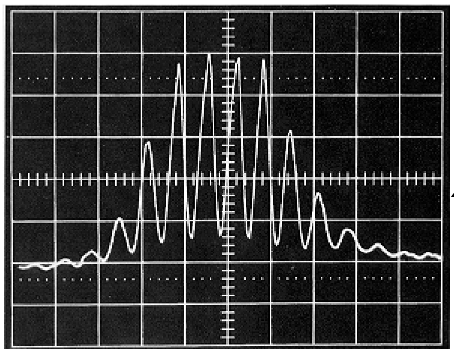
These effects depend also on the particle number concentration.

Typical probe volume dimension: 10^{-11} m^3

Max. $N_p \approx 10^{11}$ m^{-3} (single scattering, LDV)

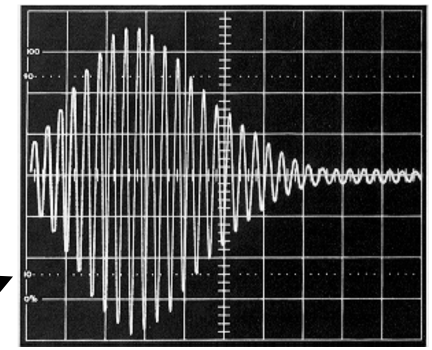
Particle concentration determines the average Data Rate and frequency resolution of the turbulent velocity spectra.

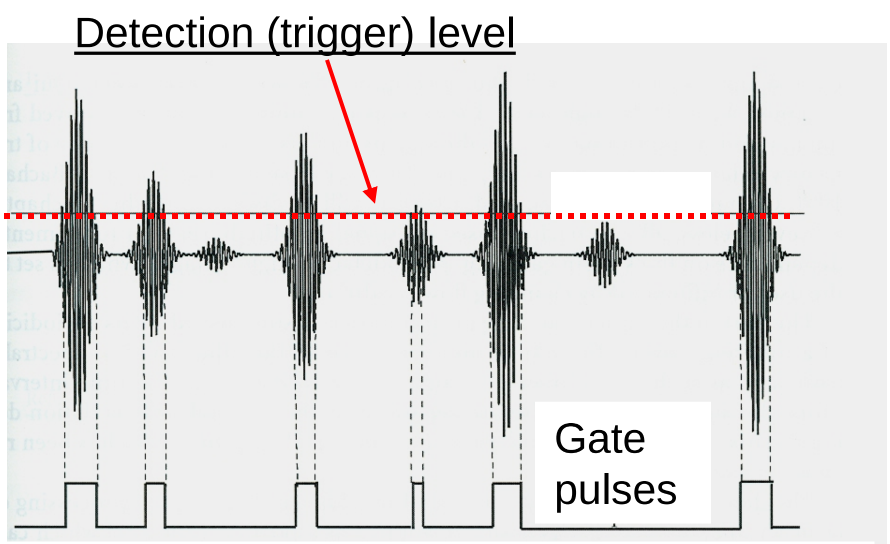
Signal characteristics



Single particle
Doppler signal

H-P filtered
Doppler signal





Doppler signals occur randomly and vary in amplitude with particle size and trajectory.

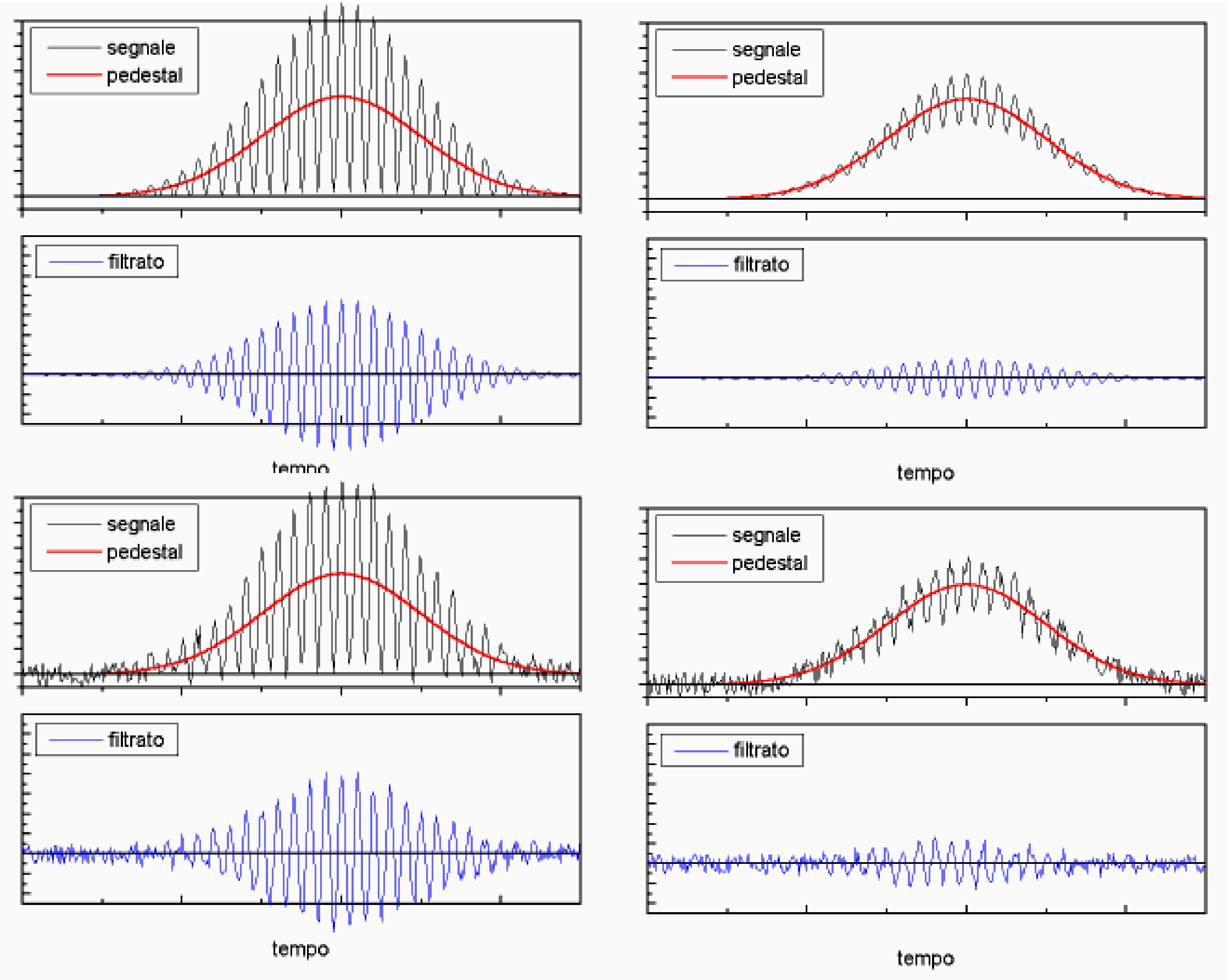
To analyse a time series of Doppler signals an amplitude level must be choosen to distinguish between signals and background noise.

It can also reject small particles.

A gate pulse indicates the beginning and the end of each particle signal.

It can be used to monitor data rate and particle arrival time or transit time.

Signal characteristics



LDV - Signal processing (1/2)

Signal processing estimates the primary measurement quantity (Doppler frequency) from the analog or digitized Doppler signal.

Classification of the signal processing methods

Technique/Device	Domain	Remarks
Tracker (phase or frequency locked loop)	Spectral	Operates at low SNR but requires high data rate (e.g. liquid flows)
Counter (period timing device)	Time	Operate at low data rates; high bandwidth but sensitive to SNR
Burst/Real time spectrum analyser	Spectral	Uses digital FFT and benefits from spectral domain analysis
Auto-correlation	Time	Exhibits similar benefits to burst spectrum analysers

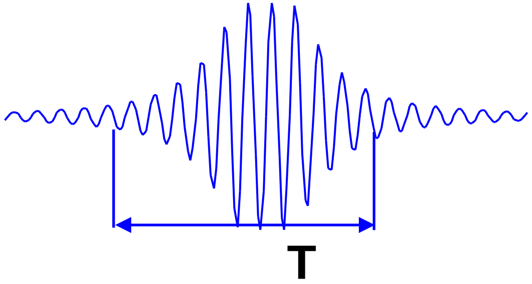
Chronological sequence of major signal processing developments for laser Doppler systems.

LDV - Signal processing (2/2)

Modern LDV processors are based on Spectral Analysis of Doppler signal performed by Discrete Fourier Transform (DFT)

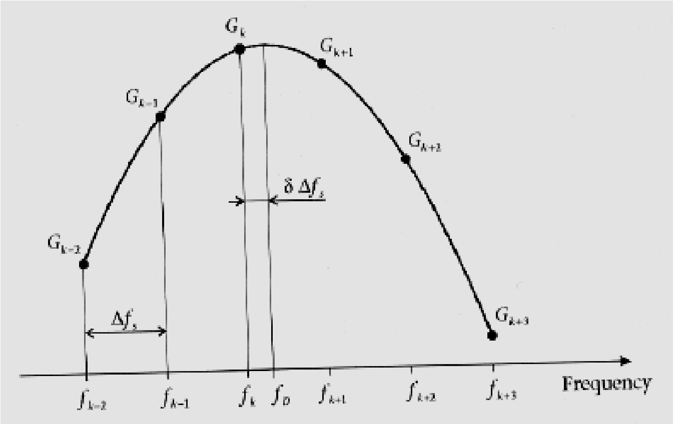
The max. resolvable frequency is half the sampling frequency (Nyquist frequency) and the resolution is determined by the Doppler burst duration.

$$\Delta f = \frac{1}{N\Delta t} = \frac{1}{T} = \frac{f_s}{N}$$



Transit time T of the particle in the probe volume is inversely proportional to flow velocity.

Finite transit times generate a spectral broadening which limits the resolution of the frequency estimation, since the product of observation time and frequency resolution is constant: ***T* Δ*f* = 1** *(Heisenberg's uncertainty principle).*

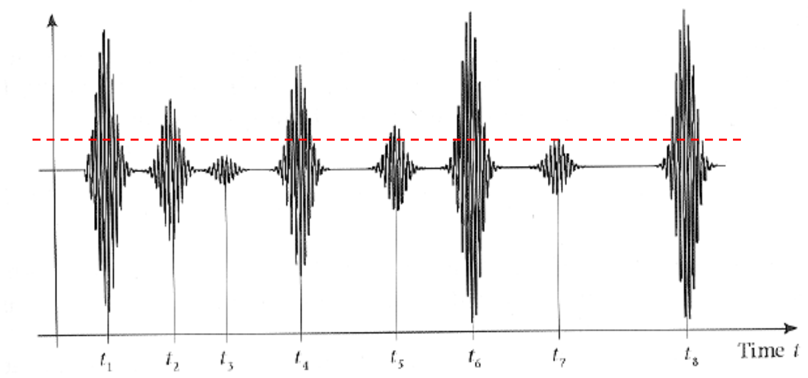


A significant resolution improvement ($\approx$10) is possible by interpolating the peak position around the maximum by a parabolic curve.

LDV – Data processing (1/3)

The LDV samples the flow velocity at discrete times dictated by the detection and validation of signals generated by scattering particles

Doppler signals occur randomly with *mean data rate* $\dot{N} = \eta n U A$



n volumetric particle concentration
A measurement cross-area
U mean particle velocity
η validation efficiency

A lower detection threshold increases data rates but can introduce noise.

It is desirable to operate in single realization mode to avoid signals overlap.

Peculiarities of laser Doppler data:

- ❑ The sample times of flow velocity, given by the particle arrival times in the probe volume, is irregular ($\Delta t \neq$ const.)
- ❑ The instantaneous rate of random particle arrival may be correlated with the flow velocity (*bias error*)

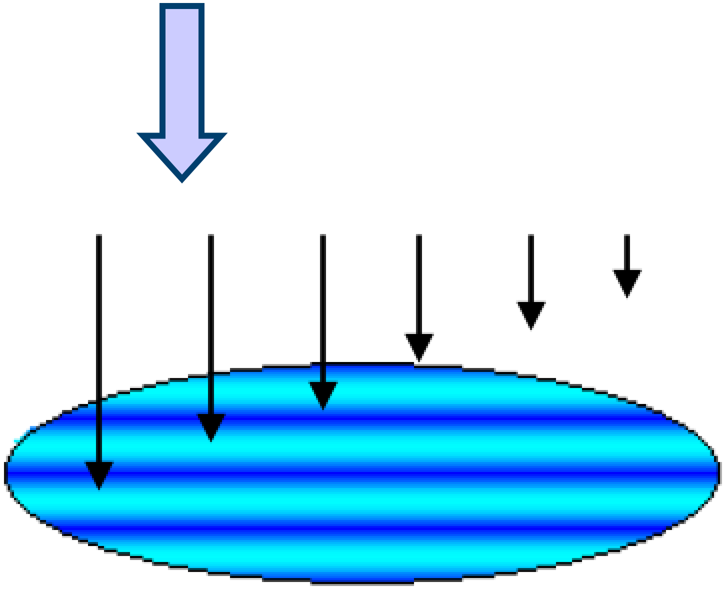
LDV – Data processing (2/3)

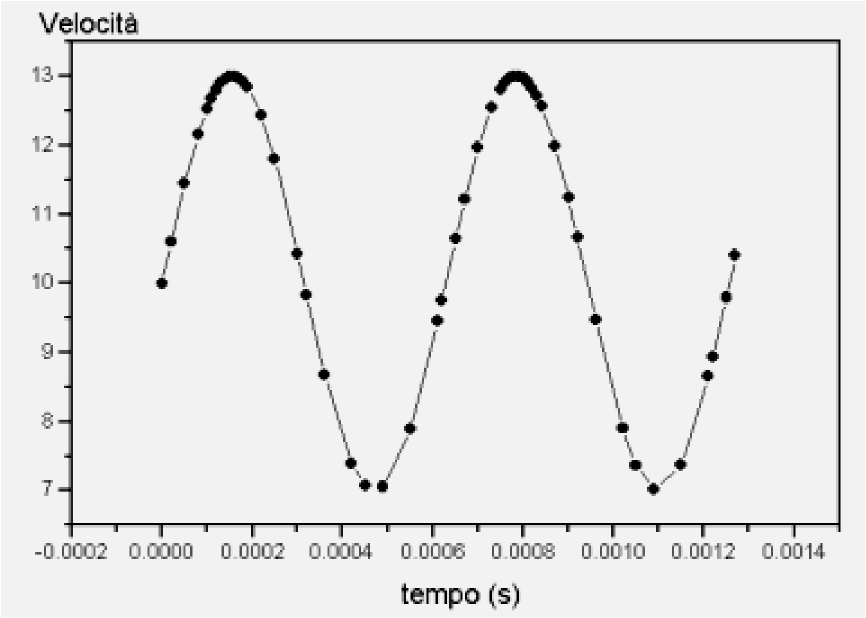
Sources of error in LDV signals

- ✓ *Low SNR*
- ✓ *Bias error*
- ✓ *Velocity gradients*

$$\overline{U} = \frac{\sum_{1}^{N} U_i \tau_i}{\sum_{1}^{N} \tau_i}$$

Transit time correction





LDV – Data processing (3/3)

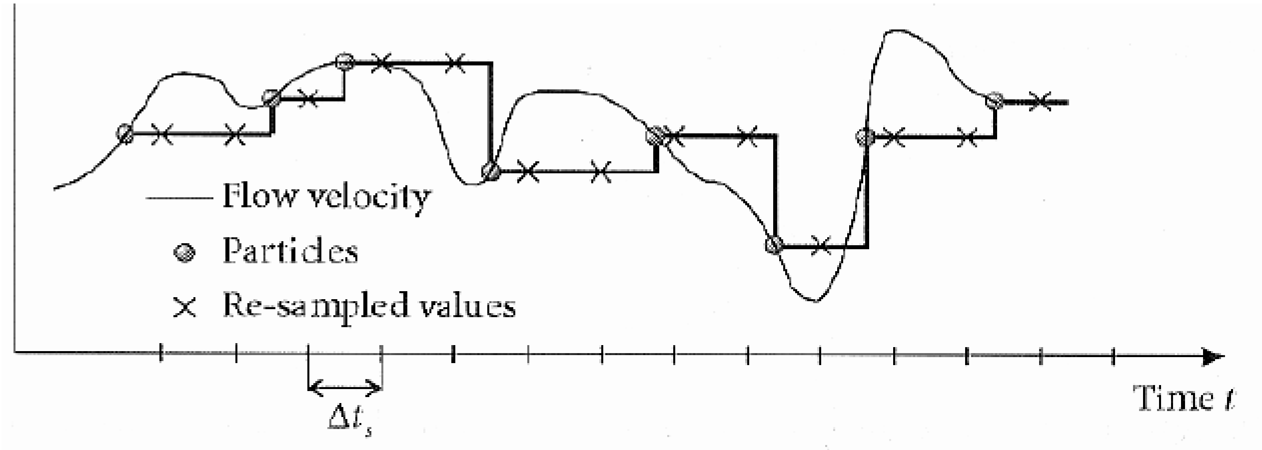
Estimation of turbulent velocity spectra

Due to the *randomly sampled* data, the FFT method is not applicable.

However, there is no equivalent of the Nyquist frequency and, with suitable estimators, it is possible to achieve alias-free and unbiased estimates of signal power at frequencies larger than the mean particle arrival rate.

The three principal approaches are:

- Direct transform (integral formula)
- Slot correlation followed by cosine transform
- Reconstruction with equidistant re-sampling and FFT



The Sample and Hold (S+H) is the simplest reconstruction algorithm.

It requires $\dot{N}T_u > 5$

Suggested Bibliography

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