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Measurement of thermal fields in transparent fluids by optical techniques

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Summary

- **Physical basis**
  - Electromagnetic wave's theory
  - Optics
- **Principles of operation**
  - Refraction → Schlieren visualization technique
  - Holography and interferometry → Holographic interferometry
  - Speckle effect → Speckle photography
- **Relevant applications**
  - Forced convection heat transfer
  - Free convection heat transfer

Electromagnetic waves - 1

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Monocromatic electromagnetic wave
$$\begin{cases} \vec{E}(\vec{r},t) = \vec{E}_0(\vec{r})\, e^{-i\omega t} \\ \vec{H}(\vec{r},t) = \vec{H}_0(\vec{r})\, e^{-i\omega t} \end{cases}$$

where $\omega = 2\pi f$ Angular frequency

$$\boxed{\begin{aligned} &\vec{\nabla} \wedge \vec{H}_0 + ik_0\varepsilon\vec{E}_0 = 0 \\ &\vec{\nabla} \wedge \vec{E}_0 - ik_0\mu\vec{H}_0 = 0 \\ &\vec{\nabla} \cdot \varepsilon\vec{E}_0 = 0 \\ &\vec{\nabla} \cdot \mu\vec{H}_0 = 0 \end{aligned}}$$

Maxwell equations

$$k_0 = \frac{\omega}{c_0} = \frac{2\pi}{\lambda_0}$$

Wave number

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Electromagnetic waves - 2

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Simplifications

Far field $r >> \lambda_0$

$$\begin{cases} \vec{E}_0(\vec{r}) = \vec{e}(\vec{r})\, e^{ik_0 S(r)} \\ \vec{H}_0(\vec{r}) = \vec{h}(\vec{r})\, e^{ik_0 S(r)} \end{cases}$$

$k_0 S(r)$ phase $S(r)$ Eikonal

$S(r) = \text{constant}$ Iso-phase surface or **wavefront**

Very short wavelengths (as for visible light) $\lambda_0 \to 0 \quad k_0 \to \infty$

$$\left(\vec{\nabla} S\right)^2 = n^2$$ Eikonal equation

$$n = \sqrt{\varepsilon\mu} = c_0/c$$ Refractive index

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Electromagnetic waves - 3

Wave propagation

$$\frac{\vec{\nabla} S}{n} = \frac{\vec{\nabla} S}{\left|\vec{\nabla} S\right|} = \vec{s}$$

Wavefront normal versor

⇒ Direction of propagation (geometric ray)

Poynting vector (power density) $\quad \vec{P} = \frac{c}{4\pi} \vec{E} \wedge \vec{H}$

w Sum of the electric and magnetic energy density

$$\left\langle \vec{P} \right\rangle = \frac{c_0 \langle w \rangle}{n^2} \vec{\nabla} S = c \langle w \rangle \vec{s}$$

$\langle \bullet \rangle$ Time average

$I = \left| \left\langle \vec{P} \right\rangle \right|$ Radiation intensity (detected quantity)

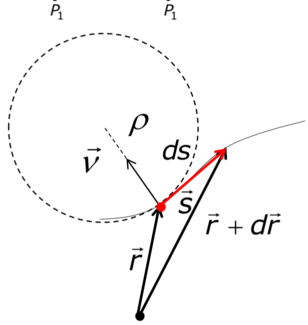
Wave propagation - 1

$$\vec{\nabla} \wedge \vec{\nabla} S = \vec{\nabla} \wedge n\vec{s} = 0$$

$$\int_F \left(\vec{\nabla} \wedge \vec{\nabla} S\right) dF = \int_F \left(\vec{\nabla} \wedge n\vec{s}\right) dF = \int_C n\vec{s}\, d\vec{r} = \int_C \vec{\nabla} S\, d\vec{r} = 0$$

$$\int_{P_1}^{P_2} n\vec{s}\, d\vec{r} = \int_{P_1}^{P_2} n\, ds = S(P_2) - S(P_1)$$

optical path length = **phase shift**



$$\vec{s} = \frac{d\vec{r}}{ds} \qquad \frac{\vec{\nabla} S}{n} = \frac{\vec{\nabla} S}{\left|\vec{\nabla} S\right|} = \vec{s} \qquad n\frac{d\vec{r}}{ds} = \vec{\nabla} S$$

$$\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right) = \vec{\nabla} n$$

Ray equation

$$\vec{K} = \frac{d\vec{s}}{ds} = \frac{\vec{\nu}}{\rho} \qquad n\vec{K} = \vec{\nabla} n - \frac{dn}{ds}\vec{s}$$

$$\left|\vec{K}\right| = \frac{1}{\rho} = \frac{\left|\vec{\nabla} n\right|}{n} \sin\alpha$$

α angle between $\vec{\nabla} n$ and $\vec{s}$

Wave propagation - 2

Summary of the main concepts

Wave propagation

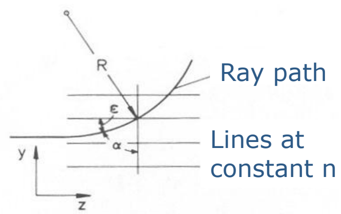
⇩

Phase shift

Constant refractive index	Variable refractive index
⇩	⇩
Rays are straight lines	Rays bend toward the direction of increasing n
	⇩
	Refraction

Wave propagation - 3

One-dimensional distribution of the refractive index

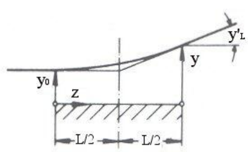


$$n = n(y)$$

Ray equation (Snell's law)

$$\frac{1+y'^2}{1+y_0'^2} = \left(\frac{n}{n_0}\right)^2 = \frac{\sin^2\alpha_0}{\sin^2\alpha}$$

If y' is small (~0.01, in typical applications) $\frac{1}{R} = \frac{y''}{(1-y'^2)^{3/2}} \approx y''$



$y'' = \text{const}$ Parabolic approximation

$$y'(L) = \frac{y(L)-y(0)}{\frac{L}{2}}$$

Effect of thermal fields -1

Lorentz-Lorenz equation
(transparent media)

$$\frac{n^2-1}{n^2+2}\frac{1}{\rho}=\frac{\bar{r}}{M_m}$$

$\bar{r}$ Molar refractivity [m³/mol]

M_m Molar mass [kg/mol]

ρ Density [kg/m³]

Gladstone-Dale equation
(gases)

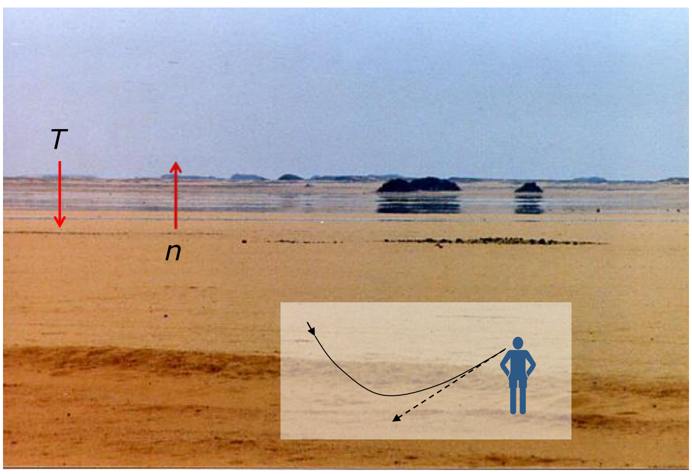
$$\frac{n-1}{3}\frac{2}{\rho}=\frac{\bar{r}}{M_m}$$

Ideal gas

$$n(T,p)=1+\frac{3}{2}\frac{p\bar{r}}{RT}$$

Effect of thermal fields -2

The mirage



The Schlieren method - 1

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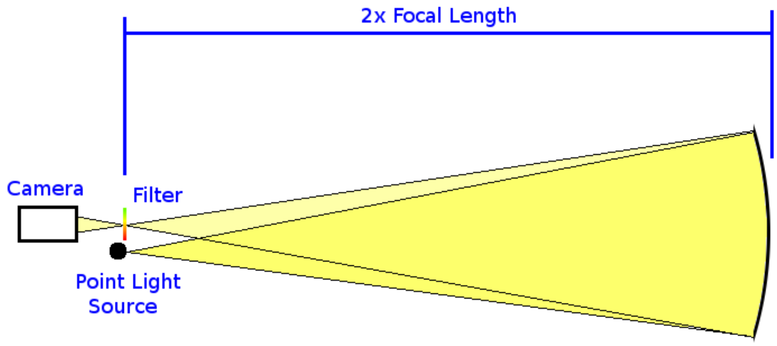

A Schlieren Object, i.e. a variable refractive index field is placed between the source and the mirror

The Schlieren method - 2

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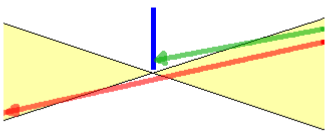
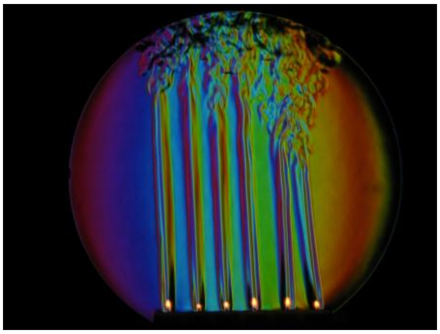
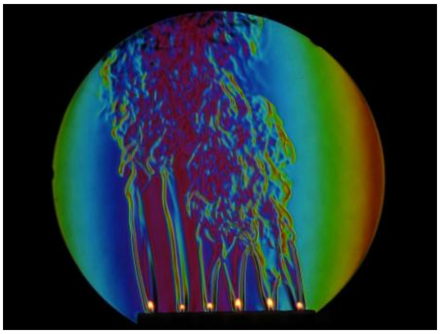


Reflected rays after being refracted by the Schlieren Object do not converge to the focus

- By using a “knife edge” part of the refracted light can be cut off.
- The resulting image is characterized by an alternating bright and dark pattern.
- A colored filter works as well, coloring the intercepted light.

The Schlieren method - 3





Interference - 1

Superposition of monochromatic electromagnetic waves

$$I = I_1 + I_2 + J_{12}$$

$J_{12} = 2\sqrt{I_1 I_2}\cos\delta$ Interference term

$\delta = k_0 (S_2 - S_1)$ (Initial) phase difference

Iso-δ lines = **interference fringes**

Constructive interference

$$\begin{cases} I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \\ |\delta| = 0, 2\pi, 4\pi, \ldots \end{cases}$$

Destructive interference

$$\begin{cases} I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \\ |\delta| = \pi, 3\pi, \ldots \end{cases}$$

Fringe visibility (contrast) $V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

Interference - 2

$\Delta\nu << \bar{\nu}$ **Almost** monochromatic electromagnetic waves

Interference pattern $$I = I_1 + I_2 + |\gamma_{12}| \cdot 2\sqrt{I_1 I_2}\cos\delta$$

(Initial) phase difference $$\delta = \bar{k}_{02}S_2 - \bar{k}_{01}S_1$$

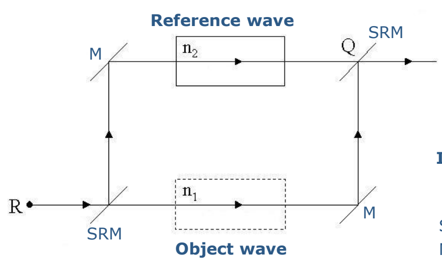
Complex degree of coherence γ_{12}

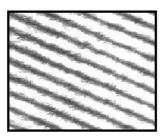
It depends on the degree of correlation between the arrival times of the wave packets and on the polarization state

Fringe visibility (contrast) $$V = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} \cdot |\gamma_{12}|$$

Interferometry - 1

Mach-Zender interferometer





Interference pattern

SRM – Semi-reflecting mirror
M - Mirror

$$\delta = \bar{k}_0[S_2(Q) - S_2(R) - S_1(Q) + S_1(R)] = \bar{k}_0[S_2(Q) - S_1(Q)] = \bar{k}_0\Delta\left(\overline{RQ}\right)$$

$$\Delta\left(\overline{RQ}\right) = \int_R^Q n_2 ds - \int_R^Q n_1 ds$$

The fringe modulation is related to the variation of the optical path length due to refraction

Interferometry - 2

Two-dimensional distribution of the refractive index

z direction of propagation $\quad n = n(x,y)$

$$\Delta\left(\overline{RQ}\right) = \int_{z_1}^{z_2} \left[n(x,y) - n_0\right] dz = \frac{\delta(x,y)}{\bar{k}_0} = \left[n(x,y) - n_0\right] L \qquad L = z_2 - z_1$$

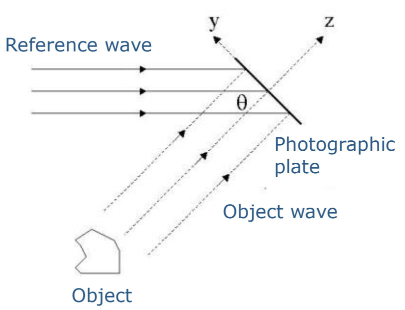
$$n(x,y) = \frac{\delta(x,y)}{\bar{k}_0 L} + n_0$$

$$T(x,y) = \left[\frac{R\bar{\lambda}_0}{3\pi M_m p\bar{r} L}\,\delta(x,y) + \frac{1}{T_0}\right]^{-1}$$

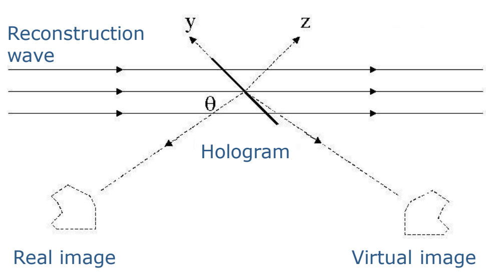
Corrections for the ray deflection are needed

Holography - 1

Holography is a **complete recording** of a wave, that is of both its **amplitude** and **phase**



Hologram recording



Object Reconstruction

Holography - 2

Object wave $\underline{a}(x,y) = a(x,y)\exp[-j\varphi(x,y)]$

Reference wave $\underline{A}(x,y) = A(x,y)\exp[-j\psi(x,y)]$

Radiation intensity on the photographic plate

$$I(x,y) \propto |\underline{A}(x,y)|^2 + |\underline{a}(x,y)|^2 + 2\gamma_{aA}(x,y)A(x,y)a(x,y)\cos[\psi(x,y) - \varphi(x,y)]$$

Exposure of the photographic plate

$$H(x,y,\bar{t},\Delta t_e) = \int_{\bar{t}}^{\bar{t}+\Delta t_e} I(x,y,\tau)d\tau$$

Δt_e Exposure time

Amplitude transmittance of the plate is a function of exposure

$T(H) = T_b + k(H_b - H)$ k plate sensitivity $H_b = |\underline{A}|^2 \Delta t_b$

$T = T_b + k\Delta t_b \left(|\underline{a}|^2 + \gamma_{aA}\underline{A}^*\underline{a} + \gamma_{aA}\underline{A}\underline{a}^* \right)$ **Diffraction grating**

Holography - 3

Reconstruction

Reconstruction wave = Reference wave

$$\underline{A}(x,y)T(x,y) = \left(T_b + k\Delta t_b |\underline{a}|^2\right)\underline{A} + k\Delta t_b \gamma_{aA} |\underline{A}|^2 \underline{a} + k\Delta t_b \gamma_{aA} \underline{A}\underline{A}\underline{a}^*$$

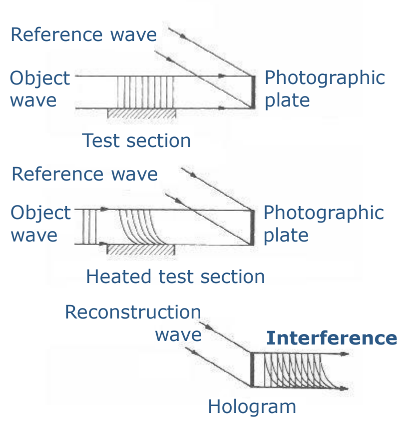
Transmitted wave

Copy of the **object wave** (virtual image)

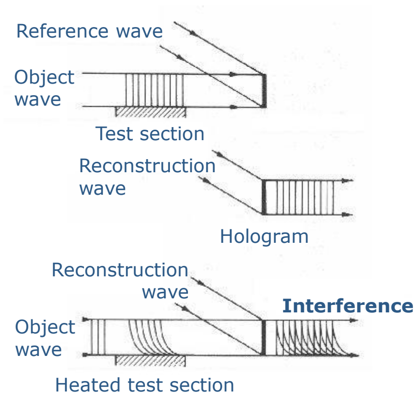
Distorted object wave

Holographic interferometry

Double exposure



Real time



Interference pattern $I(x,y) = a(x,y) + b(x,y)\cos\left[\delta(x,y)\right]$

Interference pattern analysis - 1

Maxima and minima recognition

$$\delta(x,y) = \begin{cases} 2k\pi & \text{maxima} \\ 2(k+1/2)\pi & \text{minima} \end{cases} \qquad (k = 0, \pm 1, \pm 2, \ldots)$$

The phase difference is calculated only for maxima and minima

Fourier Transform method

$$I(x,y) = a(x,y) + c(x,y) + c^*(x,y) \qquad c(x,y) = \frac{1}{2} b(x,y) \exp\left[j\,\delta(x,y)\right]$$

⇩ Fourier Transform

$$I(u,v) = A(u,v) + C(u,v) + C^*(u,v)$$ If spectral contents do not overlap

$$\delta(x,y) = \arctan \frac{\mathrm{Im}\left[c(x,y)\right]}{\mathrm{Re}\left[c(x,y)\right]}$$

Interference pattern analysis - 2

Phase shifting

$$I(x,y,t) = a(x,y) + b(x,y)\cos\left[\delta(x,y) + \delta_R(x,y,t)\right]$$

δ_R is introduced by passing the reconstruction wave through a moving wedge between the source and the hologram

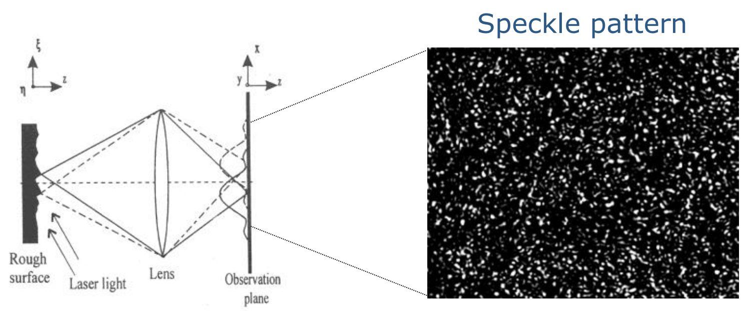
Carré method

Four shifted interferograms are required

$$\delta_R(x,y) = \arccos\frac{I_1(x,y) - I_2(x,y) + I_3(x,y) - I_4(x,y)}{2\left[I_2(x,y) - I_3(x,y)\right]}$$

$$\delta(x,y) = \arctan\frac{(I_3 - I_2) + (I_1 - I_3)\cos\delta_R + (I_2 - I_1)\cos 2\delta_R}{(I_1 - I_3)\sin\delta_R + (I_2 - I_1)\sin 2\delta_R}$$

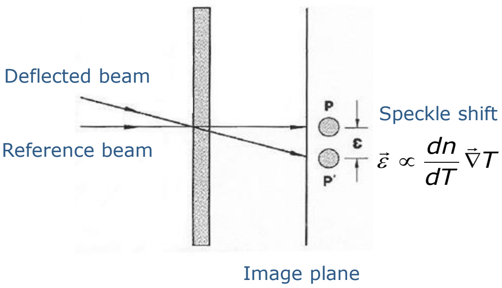
Speckle effect - 1

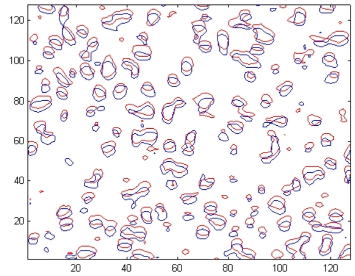


Objective speckle $\quad \sigma_{SP} = 1.22\lambda\dfrac{z}{D}$

Subjective speckle $\quad \sigma_{SP} = 1.22\lambda(1+M)\dfrac{f}{D}$

Speckle effect - 2

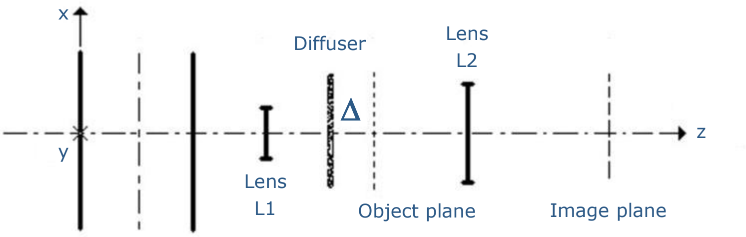


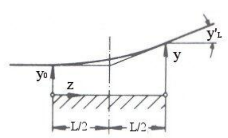


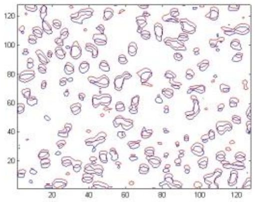
Conditions for a "rigid" shift

- perfectly coherent source and plane wave incident on the diffuser
- the phase shift arising from diffusion is randomly distributed with uniform density of probability
- unchanged wave polarization
- a large number of point sources on the diffuser contributes to the luminous intensity in one point on the image plane
- little deflection angle (~ 1 mrad)

Speckle metrology - 1







Δ: defocusing

$$\vec{\varepsilon}_i = \frac{L}{n_0}\frac{M_2}{M_1}\Delta\frac{dn}{dT}\vec{\nabla}T$$

Speckle metrology - 2

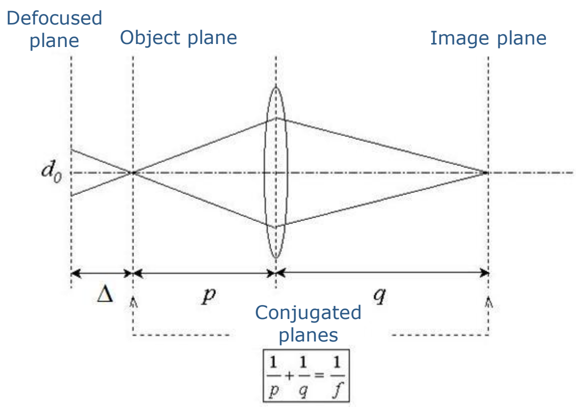
Operating parameters

Sensitivity

$$K = \frac{M_2}{M_1}\Delta$$

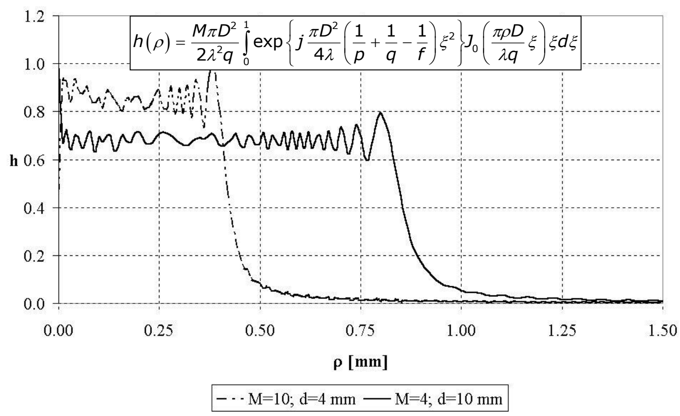
Spatial resolution (geometrical optics)

$$d_o = \frac{D\Delta}{f}\frac{M_2}{M_2+1}$$



Speckle metrology - 3

Spatial resolution (impulse response)



Δ [mm]	M_2	f [mm]	D [mm]	$d_{o,geom}$ [mm]	d_o [mm]
4	10	50,8	12,7	0,91	0,98
10	4	50,8	12,7	2,00	1,88
4	1	50,8	12,7	0,50	0,60

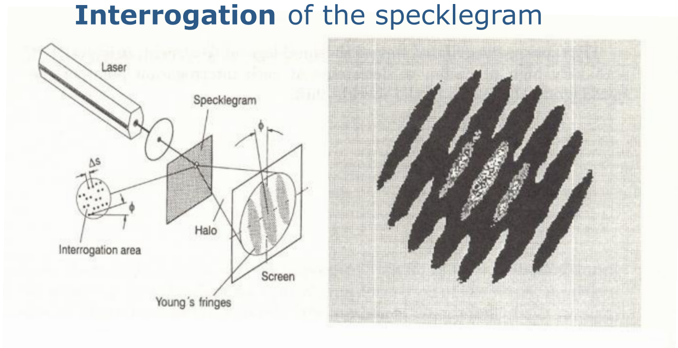
Speckle photography - 1

Double exposure technique

Recording of the "specklegram" (photographic plate)

1. First exposition without interposing the object
2. Second exposition by interposing the object

Interrogation of the specklegram



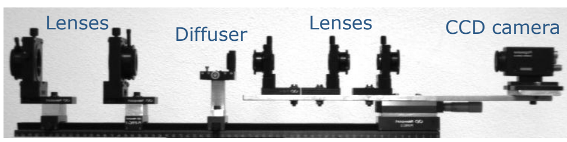
$$|\vec{\varepsilon}| = \frac{\lambda L}{d_f}$$

d_f: fringe distance
L: heated length

Fringe inclination is normal to the direction of the speckle shift

Speckle photography - 2

Digital speckle photography

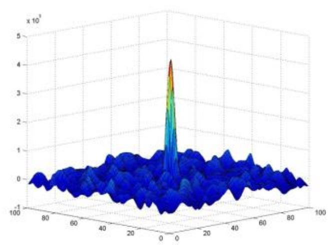


$I_1(\underline{x};t_1)$ Speckle pattern relative to a reference condition

$I_2(\underline{x};t_2)$ Speckle pattern relative to the measurement condition

Comparison by **cross-correlation**

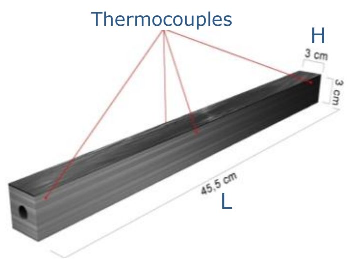
$$\hat{I}_1(f_x,f_y) = FFT_2\left[I_1(\underline{x};t_1)\right]$$
$$\hat{I}_2(f_x,f_y) = FFT_2\left[I_2(\underline{x};t_2)\right]$$
$$\hat{C}(f_x,f_y) = \hat{I}_1 \cdot \hat{I}_2^*$$
$$C(\xi,\eta) = IFFT_2\left[\hat{C}(f_x,f_y)\right]$$

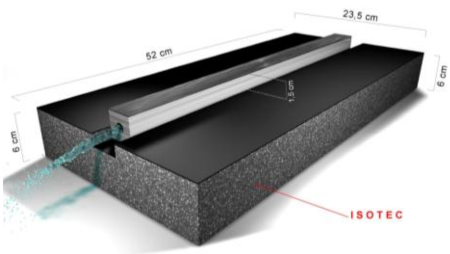


Parabolic interpolation for sub-pixel resolution

Applications: Interferometry – 1.1

Free convection past a protruding object at constant temperature





Aspect ratio $AR = \frac{L}{H} > 15$

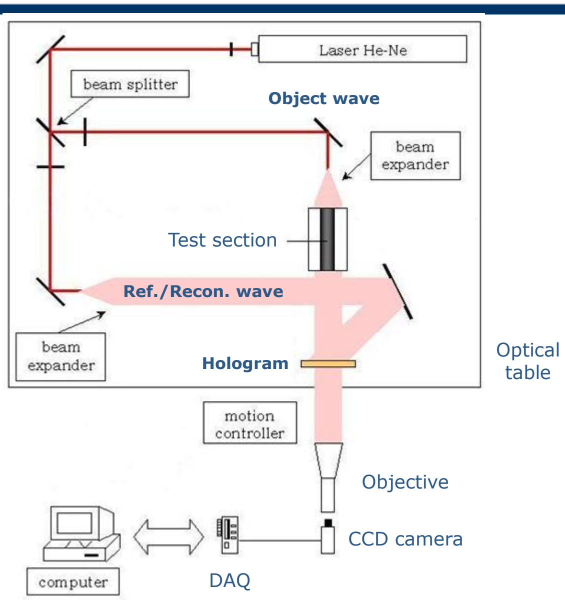
2D boundary layer

$$L_c = \frac{H \cdot L}{2(L+H)}$$

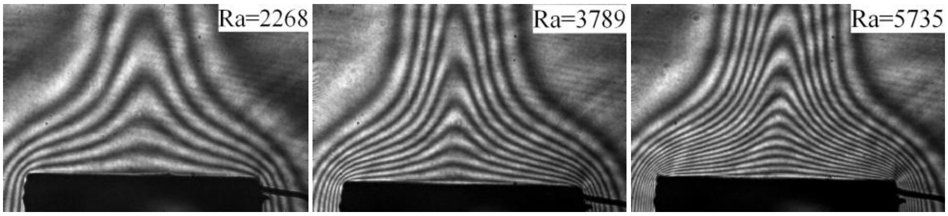
$$Ra = Gr \cdot \Pr = \frac{g \cdot \beta \cdot \Delta T \cdot L_c^3}{\nu \cdot \alpha}$$

T_p [°C]	T_{amb} [°C]	Ra
35.1	26.1	2268
40.0	24.7	3789
49.8	25.2	5735
59.6	25.6	7466
69.7	26.2	8969

Applications: Interferometry – 1.2

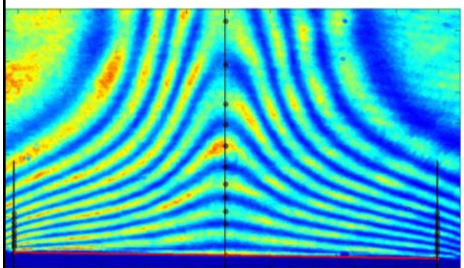


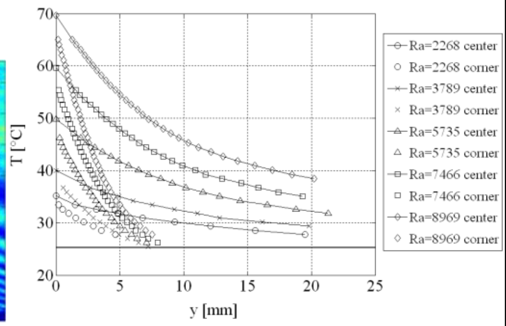
Applications: Interferometry – 1.3



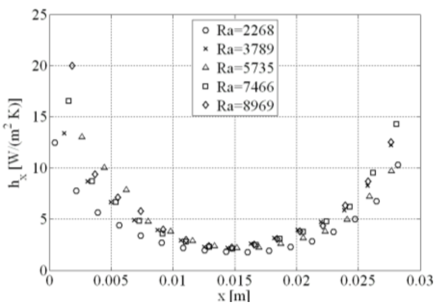
Temperature distribution within the boundary layer (maxima and minima algorithm)

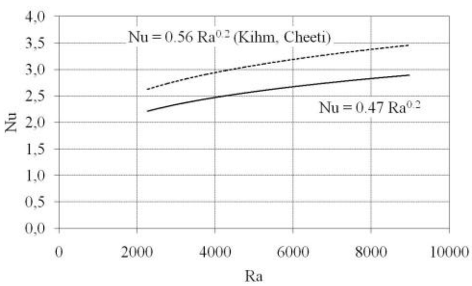
$$T(x,y)=\left[\frac{R\bar{\lambda}_0}{3\pi M_m p\bar{r}L}\delta(x,y)+\frac{1}{T_0}\right]^{-1}$$





Applications: Interferometry – 1.4





Local heat transfer coefficient

$$h(x)=\frac{-k_f\left.\dfrac{\partial T}{\partial y}\right|_{y=y_w}}{T_w(x)-T_{amb}}$$

$$h(x)\cong\frac{k_f\dfrac{T_w-T_{\text{1st fringe}}}{y_{\text{1st fringe}}(x)-y_w(x)}}{T_w(x)-T_{amb}}$$

Average heat transfer coefficient

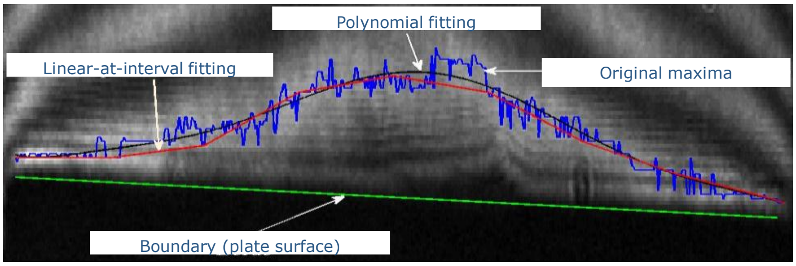
$$\bar{h}=\frac{1}{L}\int_0^L h_x dx$$

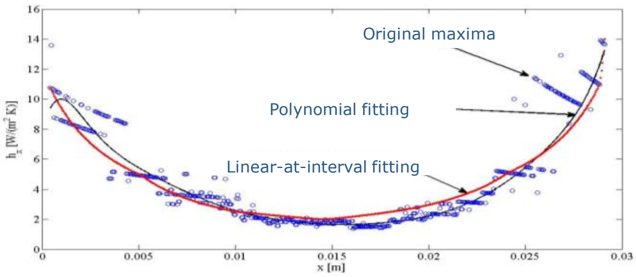
Average Nusselt number

$$\overline{Nu}=\frac{\bar{h}L_c}{k}$$

Applications: Interferometry – 1.5

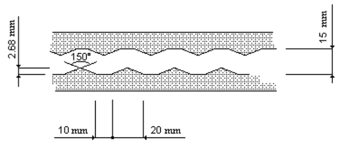
Errors due to diffraction

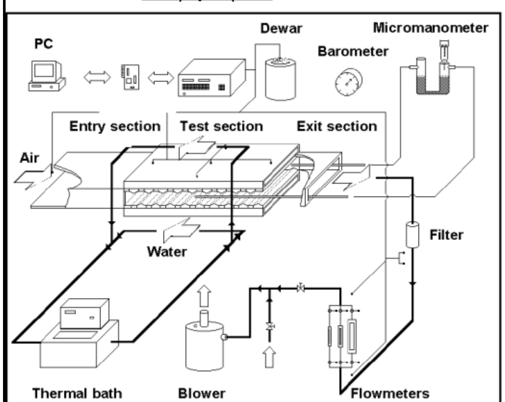




Applications: Interferometry – 2.1

Convective heat transfer of an air-flow through a wavy channel





Objective:
study of the heat transfer enhancement provided by corrugations

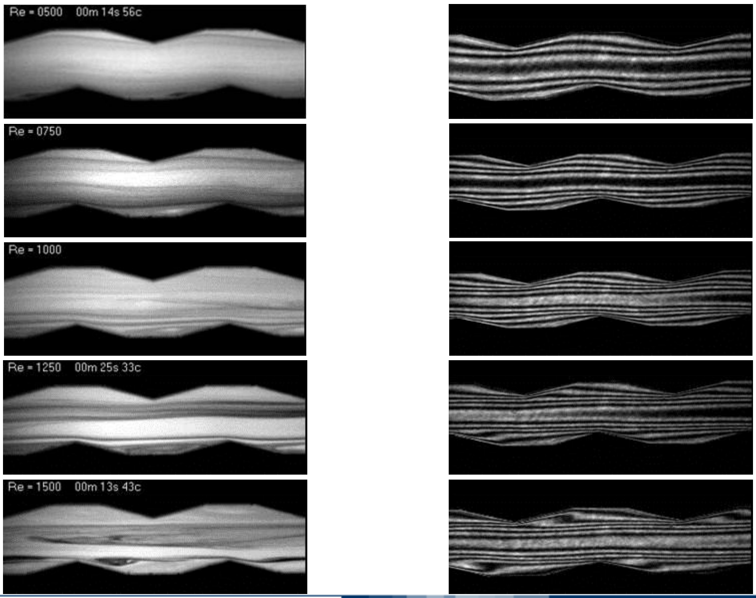
Flow visualization:
fast cinematography

Velocity measurement:
laser doppler velocimetry (LDV)

Temperature measurement:
holographic interferometry

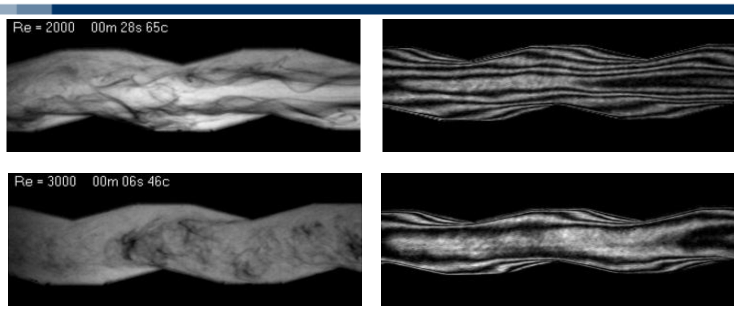
Applications: Interferometry – 2.2

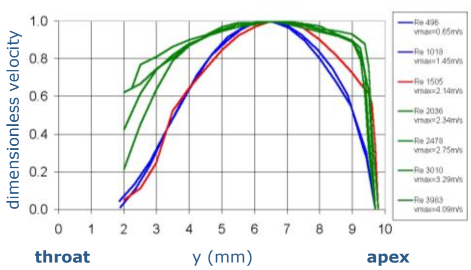
Flow

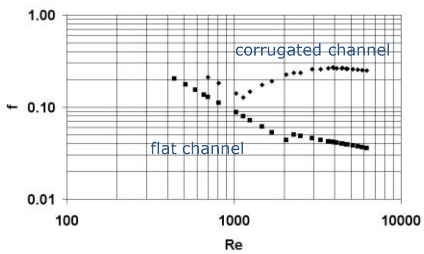


Applications: Interferometry – 2.3

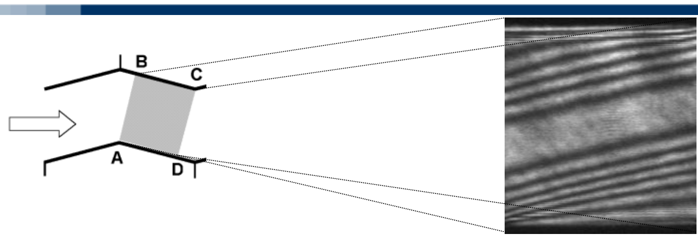
Flow

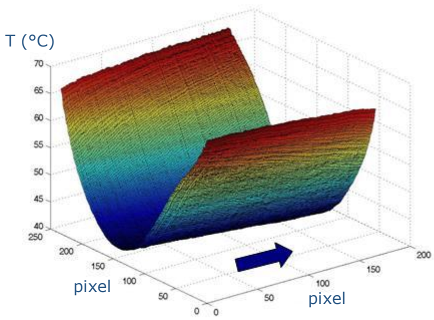


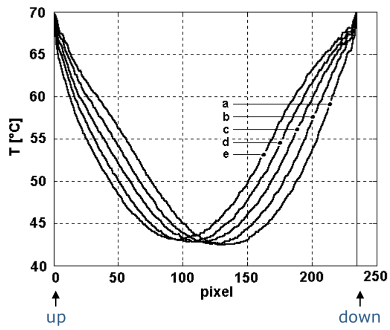




Applications: Interferometry – 2.4







Applications: Interferometry – 2.5

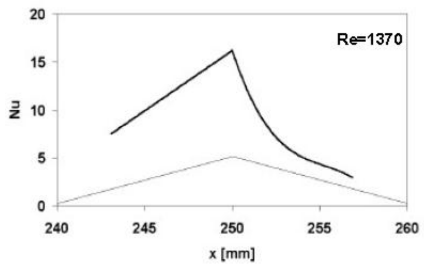
Local Nusselt number

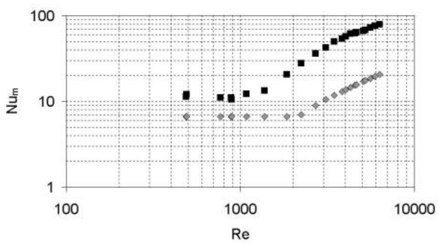
$$h_x = -\frac{k \left.\dfrac{\partial T}{\partial n}\right|_{w,x}}{T_p - T_m}$$

$$Nu_x = \frac{h_x D_H}{k}$$

$$Nu_m = \frac{1}{L}\int_0^L Nu_x dx$$

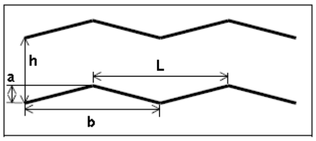
Average Nusselt number



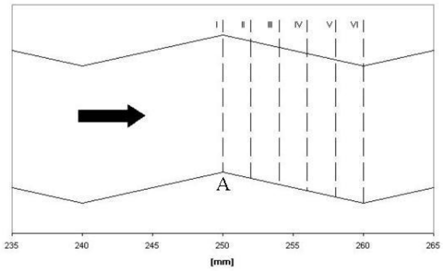


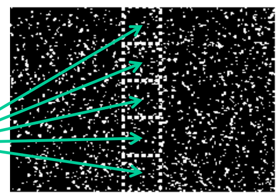
Applications: Speckle Photography – 1.1

Convective heat transfer of an air-flow through a wavy channel



h	[mm]	10.00
a	[mm]	2.68
b	[mm]	20.00
L	[mm]	20.00
Vertex angle		150°
Channel width	[mm]	200.00
Hydraulic diameter	[mm]	18.18
Heigth to width ratio		0.05
Ribs number		30

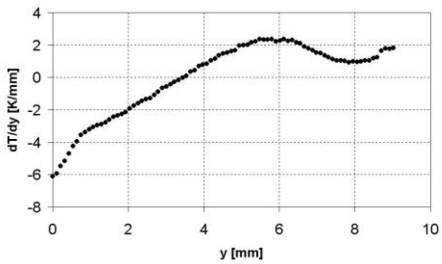


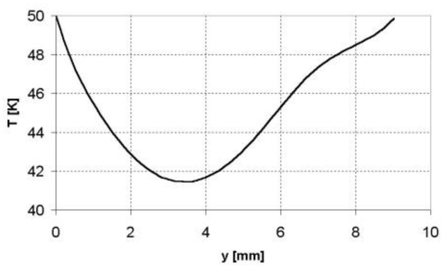


Subdomains for cross-correlation: $90 \times 90\ \mu m^2$

Field of view: $640 \times 295\ \mu m^2$

Applications: Speckle Photography – 1.2





Temperature gradient

Numerical integration

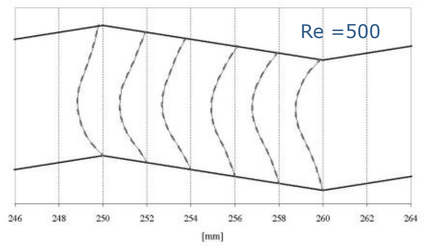
Temperature

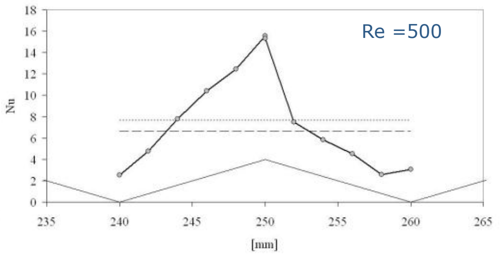
$$T(x,y) - T_p = \int_0^L \frac{\partial T(x,y)}{\partial y} dy$$

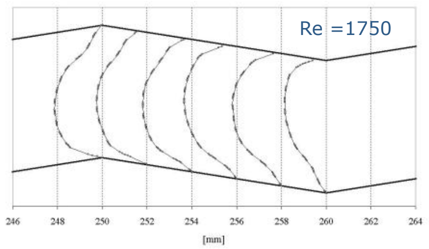
Applications: Speckle Photography – 1.3

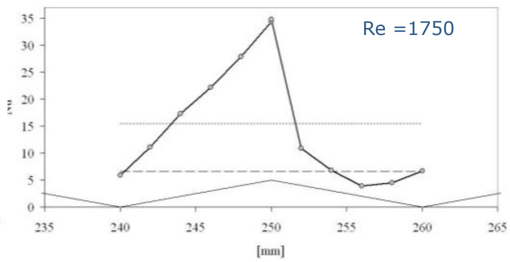
Temperature profiles

Local Nusselt number









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