POLITECNICO DI MILANO

Temperature Measurements

A. Coghe

Department of Energy – Politecnico di Milano, Milano – ITALY

Temperature Measurement

Aldo Coghe

Dep. of Energy
Politecnico di Milano

March 2015

Temperature Measurement

➢ **Introduction**

➢ **Temperature scales**

➢ **Constant temperature gas thermometer**

➢ **The ITS-90**

➢ **Thermal radiation**

➢ **Platinum resistance thermometers / Thermistores**

➢ **Thermocouples / Thermopiles**

➢ **Temperature measurement / Data acquisition**

➢ **Bibliography**

POLITECNICO DI MILANO

Temperature is an ***intensive*** quantity whereas length, time and mass are ***extensive***.

The idea of a standard unit of mass or length that can be divided or multiplied to generate any arbitrary magnitude cannot be applied to the concept of temperature.

Statistical mechanics relates temperature to the mean kinetic energies of molecules (which depend on mass, length and time), but those energies are not measurable.

Thus an ***independent temperature standard*** is required!

Thermodynamics considers finite systems and *macroscopic* quantities, such as temperature and pressure (thermodynamic coordinates), which have a direct relation to our sense perceptions and are measurable.

The temperature characterizes the thermodynamic state of a systems.

A system at equilibrium exhibits *fixed* properties, independent of time, which may be measured with precision.

The zero-law of thermodynamics gives a useful concept:

*When two bodies (A and B) are each in **thermal equilibrium** with a third body (T), they are in thermal equilibrium with each other.*

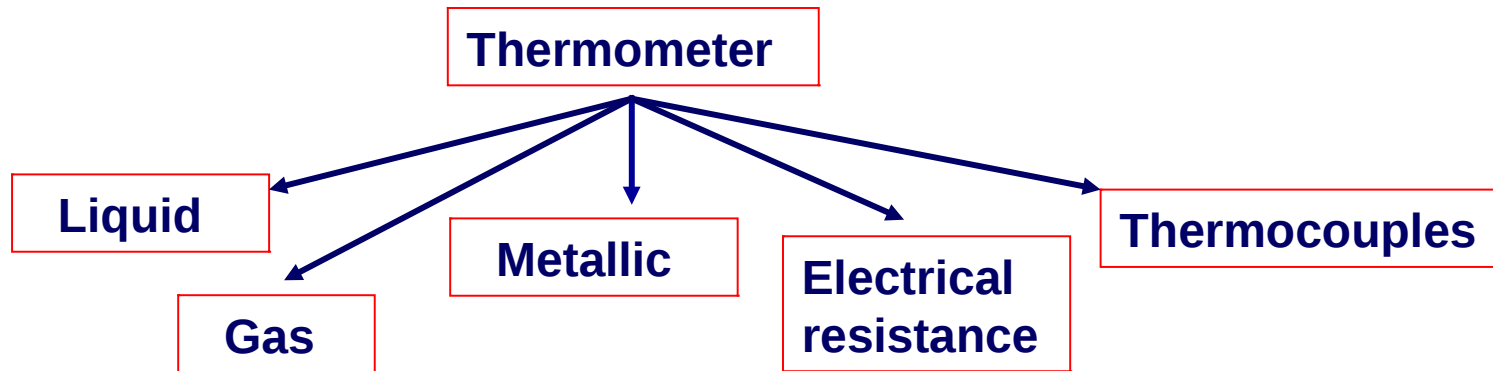*By definition two bodies have the same temperature if they are in thermal equilibrium.*
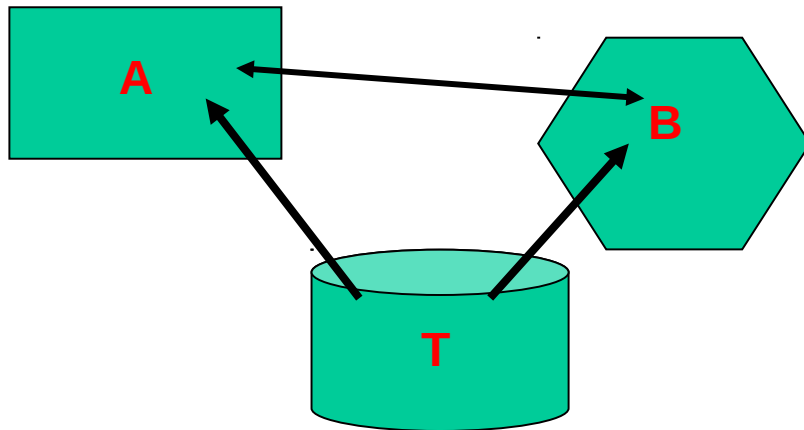
If we can establish a reproducible "temperature standard", unknown temperatures of any body may be compared with the standard, by using a **Thermometer** and successively allowing equilibrium to occur when T is posed in contact with A or B.

Alternatively, the "Thermometer" can be calibrated against the standard and then used to read unknown temperatures.

Thermometer

The first thermometer seems to have been introduced by Galileo before 1597.

Since pressure, volume, electrical resistance, etc…., are all related to temperature, we could employ any of the physical properties of materials that vary reproducibly with T to realize a thermometer.

To define a temperature scale, we must choose a *reference temperature* and state a rule for defining the ratio between the reference and other temperatures.

```
                  Thermometer


  Liquid                                      Thermocouples
              Metallic
                          Electrical
       Gas                resistance
```

Temperature Scales

Two examples of temperature scales are the Celsius and Fahrenheit scales, both based on the specification of the number of increments between the freezing and boiling points of water at standard atmospheric pressure.

The **Celsius scale** has 100 units between the two points (centigrade).

The freezing point of water at 1 atm is arbitrary taken as 0° on the Celsius scale.

The boiling point of water at 1 atm is arbitrarily taken as 100° on the Celsius scale and 212° on the Fahrenheit scale.

The relationship between the two scales is:

°F = 32.0 + (9/5) °C

Mercury Thermometer

POLITECNICO DI MILANO

Absolute Temperature Scale   (1/2)
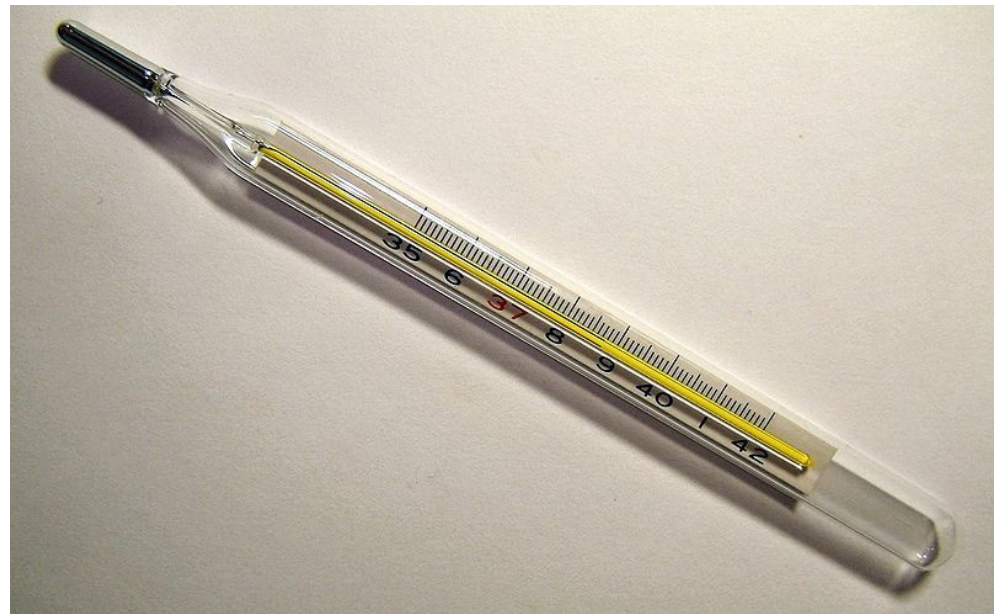
Any arbitrarily defined temperature scale is, in principle, as good as the other scales based on some material property, but a way of defining a temperature scale independent of any substance is desirable.

The thermodynamic temperature scale proposed by **Kelvin** in 1848 provides the theoretical base for such independent definition and is based on the *Carnot cycle* of a perfectly reversible heat engine working between two infinite capacity reservoirs at temperatures T_1 and T_2

For a Carnot cycle

$$Q_2 / Q_1 = T_2 / T_1$$

if the reference temperature value is selected for a fixed point, T_1, then the temperature scale is completely defined.

The fixed point is taken as *the triple point* of water (*the point or temperature at which ice, water and water vapors coexist*) because this is the most reproducible state known. The number assigned is **273.16 K.**

Assuming the temperature interval from the ice point to the steam point = 100 K it would coincide with the previously established Celsius scale.

Absolute Temperature Scale (2/2)

While the Kelvin absolute scale is ideal, it is not physically realizable since it depends on an ideal Carnot cycle.

However, a temperature scale defined by a ***constant-volume gas thermometer*** using an ideal gas is identical to the thermodynamic scale.

The constant-volume gas thermometer keeps a fixed mass of gas at constant volume and measures the pressure changes (easily measurable) caused by temperature changes.

The ideal-gas equation is: pV = mRT
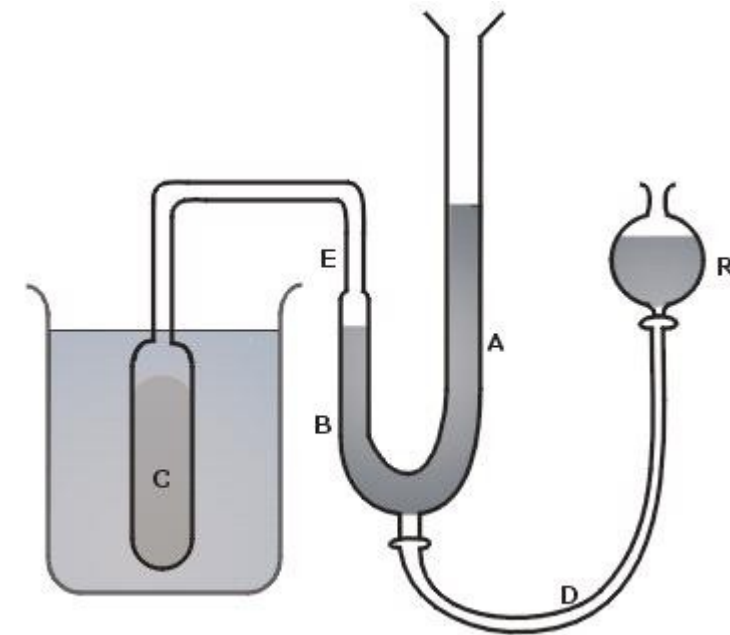
p = pressure of the gas
V = volume
m = mass of the gas
R = R/M = gas constant
R= universal gas constant (= 8314.5 J/kg)
M = molecular weight of the gas

At constant volume $$T = 273.16 \left(p / p_{ref} \right)$$

Constant volume gas thermometer

Thermodynamic Scale

According to the perfect-gas law, temperature ratios are identical to pressure ratios

$$T / T_{ref} = p / p_{ref}$$

These ratios are identical to those of the thermodynamic scale.

Thus, if the same fixed point (triple point of water) is selected as the reference point, the two scales are numerically identical.

However, **the ideal gas is a mathematical model, not a real substance!**

Real gases approach ideal-gas behavior if their pressure reduces to zero.

Since we cannot use zero mass of gas, the zero-pressure point must be obtained by extrapolation.

Temperature measurements are repeated with progressively smaller mass of gas generating a slight difference in the pressure ratio as the quantity of gas is varied.

Constant-volume gas thermometer

If the experiment is repeated with different gases, all the curves intersect at the same point, showing that the procedure is independent of the type of gas used.

Such a scale is not practical for day-to-day temperature measurements since the procedures involved are extremely tedious and time-consuming.

This situation led to the acceptance in 1927 of the ***International Practical Temperature Scale* (IPTS)** which is the temperature standard today.

The last revision was in 1990 **(ITS-90).**

POLITECNICO DI MILANO

The International Temperature Scale
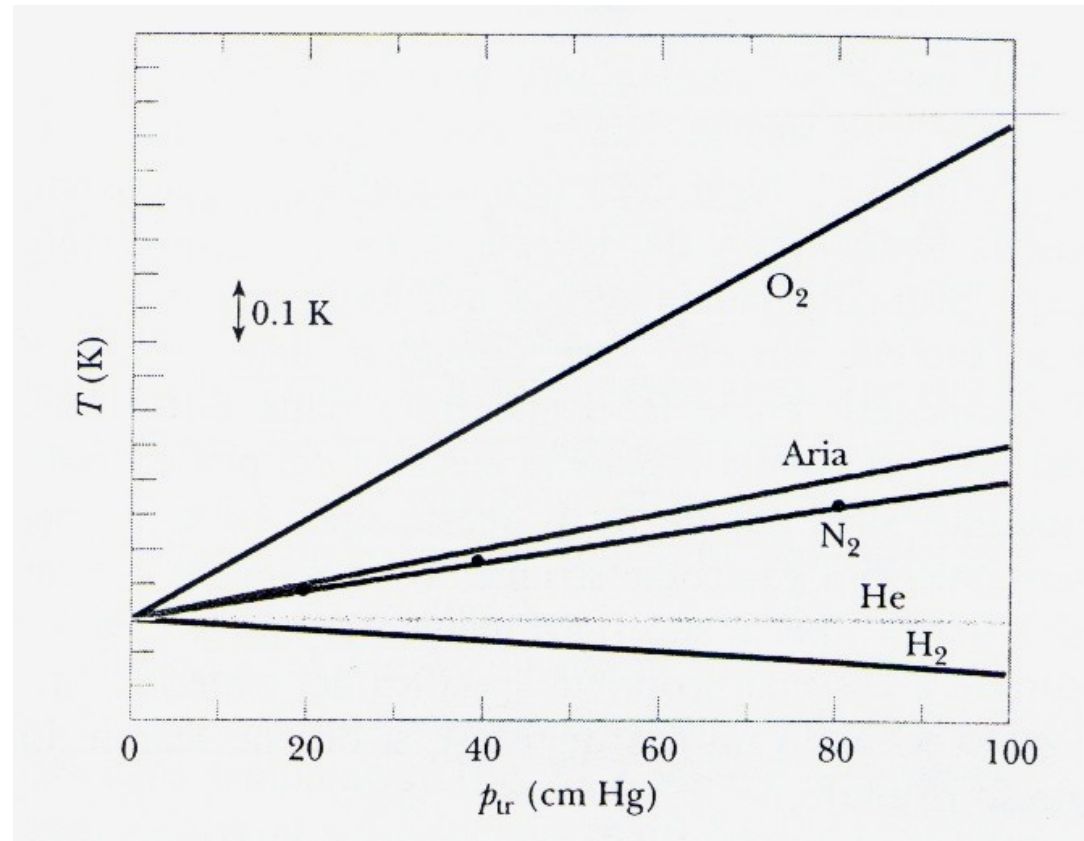
The ITS is set up to conform as closely as practical with the thermodynamic scale.

At the triple point of water, the two scales are in exact agreement, by definition.

A number of **fixed points** are used, selected between the state transitions of particular substances in order to cover the widest temperature range.

Values assigned to fixed points are determined by gas thermometry and are subject to refinement over the years as improved techniques are developed.

The ITS-90 also specifies instruments, equations and procedures to interpolate between the fixed points.

From -260 to 630 °C a platinum resistance thermometer is the suggested interpolating instrument.

Between 630 and 1084 °C the interpolating instrument is a platinum/10% rhodium – platinum (type S) thermocouple.

Above the Copper point temperature, the ITS is defined using the **Planck equation** of the spectral radiance of a blackbody (thermal radiation).

The ITS-90

	Temperature (°C)	Temperature (K)
Triple Point of Hydrogen	**- 259.3467**	**13.8033**
Equilibrium liquid/vapour of Hydrogen at 25/76 atm	**- 256.15**	**≈ 17**
Equilibrium liquid/vapour of Hydrogen at 1 atm	**- 252.87**	**≈ 20.3**
Triple Point of Neon	**- 248.5939**	**24.5561**
Triple Point of Oxygen	**- 218.7916**	**54.3584**
Triple Point of Argon	**- 189.3442**	**83.8058**
Triple Point of Water	**0.01**	**273.16**
Equilibrium solid/liquid of Gallium at 1 atm	**29.7646**	**302.9146**
Equilibrium solid/liquid of Tin at 1 atm	**231.928**	**505.078**
Equilibrium solid/liquid of Zinc at 1 atm	**419.527**	**692.677**
Equilibrium solid/liquid of Silver at 1 atm	**961.78**	**1234.93**
Equilibrium solid/liquid of Gold at 1 atm	**1064.18**	**1337.33**
Equilibrium solid/liquid of Copper at 1 atm	**1084.62**	**1357.77**

Thermal radiation

Thermal radiation is the electromagnetic radiation emitted by a body as a result of its temperature.

Thermal radiation lies in the wavelength region from about 0.1 to 100 μm.

The emissive power of a *blackbody* has a continuous spectral distribution according to the **Planck equation**

$$E_b(\lambda, T) = \frac{c_1}{\lambda^5 \left[e^{\frac{C_2}{\lambda T}} - 1 \right]}$$

The total thermal radiation emitted by a **blackbody** is given by the **Stefan-Boltzmann** formula, which may be obtained by integration of the Planck law over the entire wavelength interval.

$$\sigma = 5.669 \, 10^{-8} \quad \left[W \, m^{-2} \, K^{-4} \right]$$

The emissive power varies with temperature and the Planck or Stefan-Boltzmann laws can be used as an alternative of the ideal gas law to establish a thermometer.

Visible light

(0.4 to 0.7 μm) covers a small fraction of the entire thermal wavelength range.

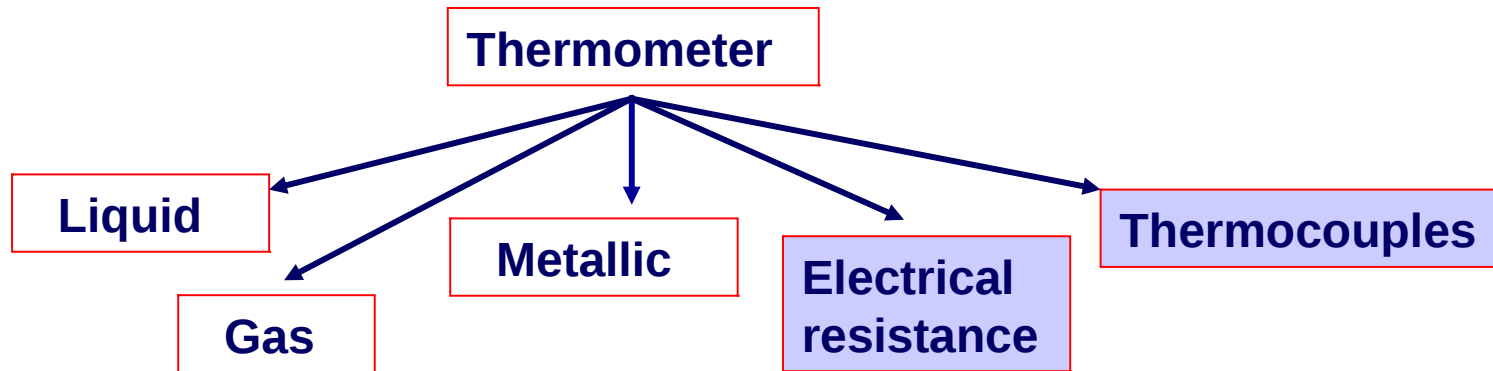# *Spectrum of the electromagnetic radiation*
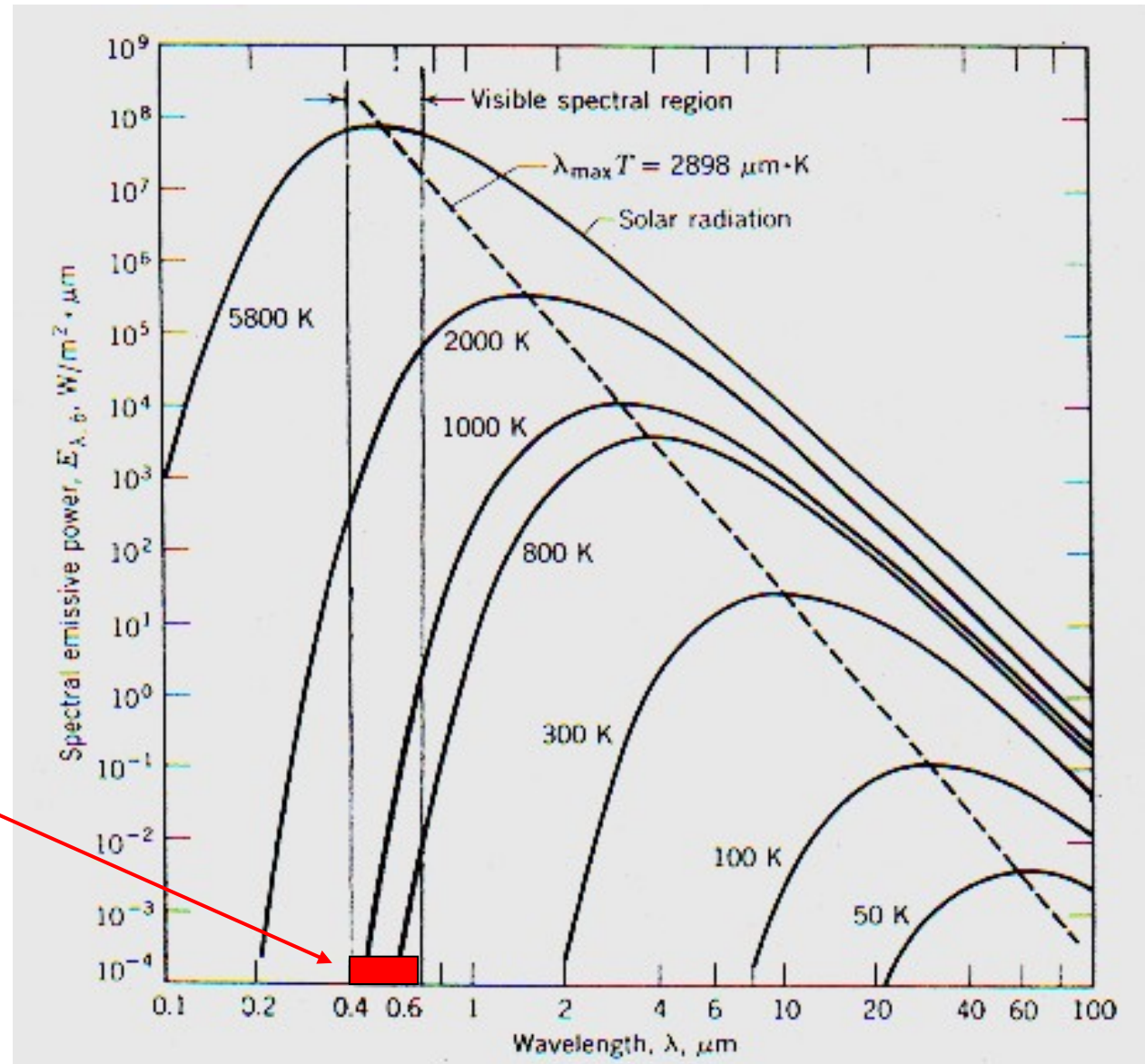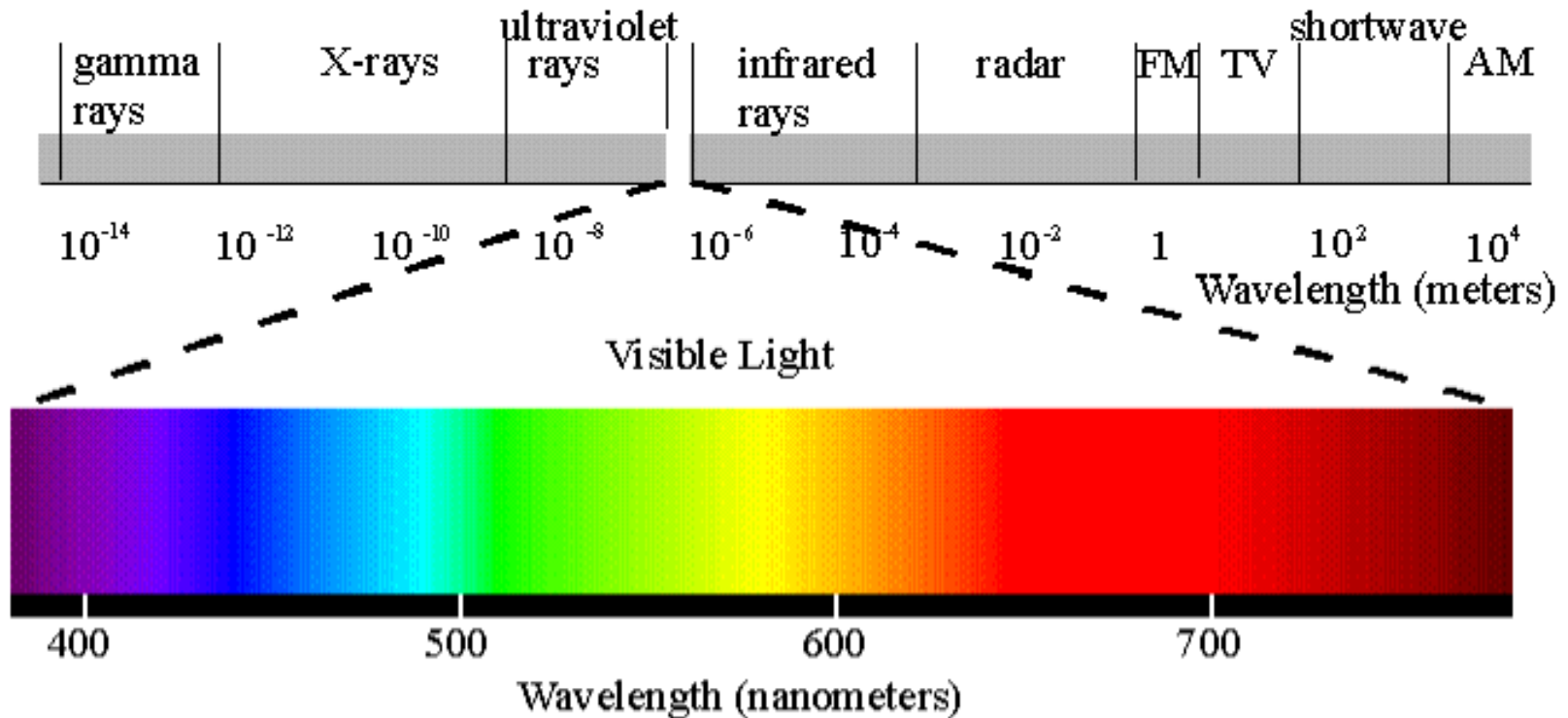
The energy distribution of the electromagnetic spectrum is mainly determined by the temperature of the emitting surface (radiator).

Thermometers

Since pressure, volume, electrical resistance, etc…., are all related to temperature, we could employ any of the physical properties of materials that vary reproducibly with T to realize a thermometer.

The most used thermometers are:

✓ Electricl resistence thermometers

✓ Thermocouples.

| Thermometer |
| Liquid |
| Gas |
| Metallic |
| Electrical resistance |
| Thermocouples |

Resistance Temperature Detectors (RTD)

Electrical methods are very convenient because they furnish a signal that can be easily detected, amplified or used for control purposes.

The electrical resistance of various materials changes in a reproducible manner with temperature and they may be used as resistive Thermometers.

Conducting materials (metals) ⟶ Resistance Temperature Detectors

Semiconductors ⟶ Thermistors
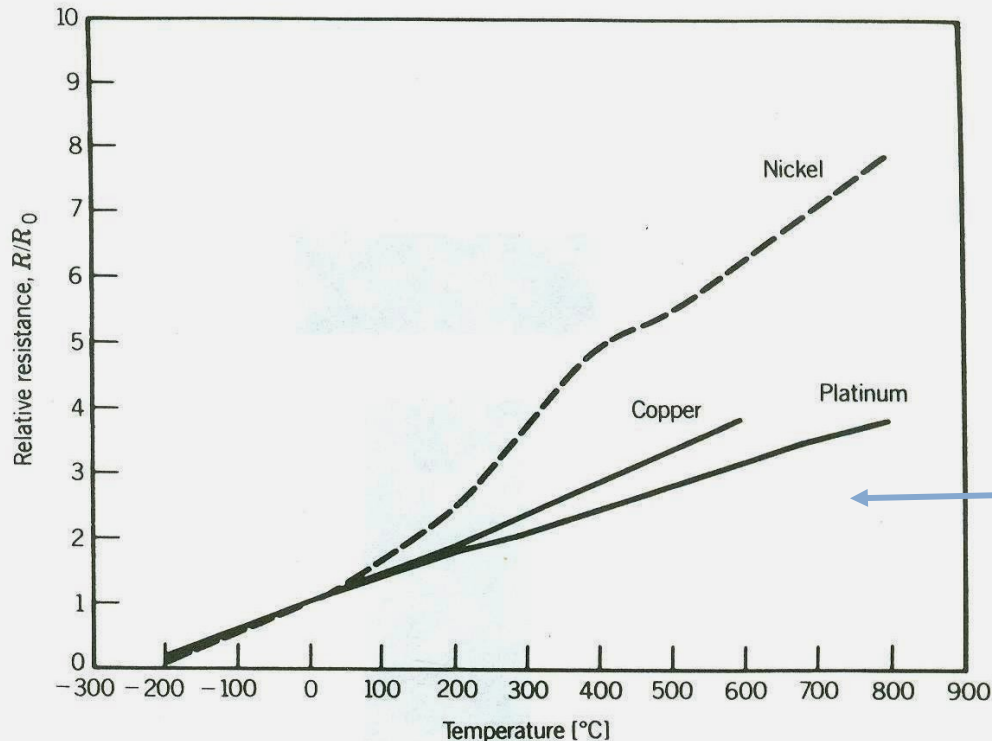
For most metallic materials the reletionship between R and T has the form:

$$R = R_0\left[1 + \alpha\left(T - T_0\right) + \beta\left(T - T_0\right)^2 + ...\right]$$

Platinum is linear within $\pm0.3\%$ between 0 – 200 °C and within $\pm1.2\%$ in the range 200-800 °C.

Platinum Resistance Thermometer (PRT)

The linear temperature coefficient of resistance is defined by between two temperatures over a narrow temperature range, typically $(T_1 - T_0) = 100$ °C. For platinum $\alpha = 0.003927$ (°C^{-1})

$$\alpha = \frac{R_1 - R_0}{R_0 (T_1 - T_0)}$$

Sensing elements are made in different forms, depending on the application.

In all cases care must be taken to ensure that the resistance wire is free of mechanical stresses. It is normally encased in a stainless-steel bulb to protect from moisture or corrosive liquid and gases.

Thin deposited films of platinum on a ceramic substrate are also used.

The resistance measurement may be performed with some type of bridge circuit.

For steady-state measurements a null condition will suffice, while transient cases will usually require the use of a deflection bridge.

POLITECNICO DI MILANO
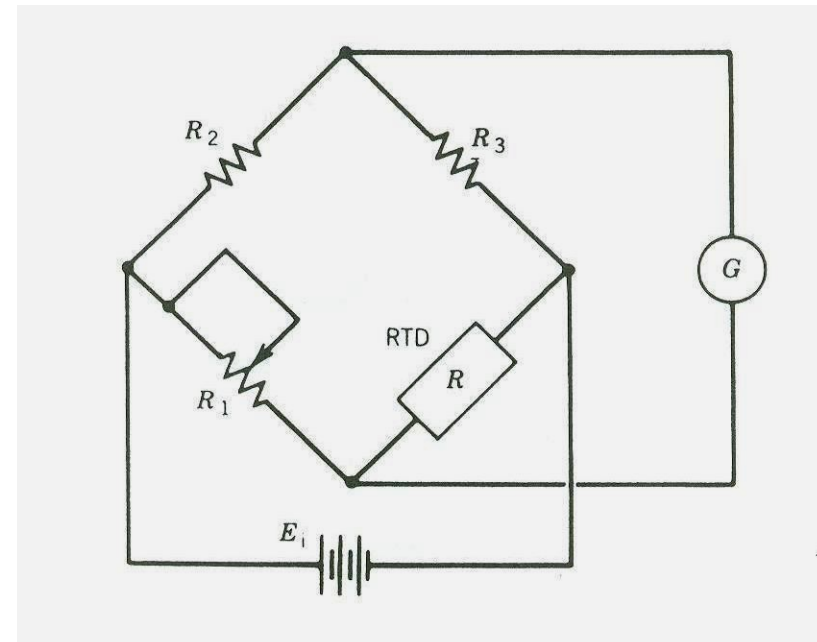
# *Platinum Resistance Thermometer (PRT)*

The four-wire "ohmmeter" technique is widely employed in digital thermometers and data acquisition systems, instead of the bridge techniques.

Since a precise current source (a few mA) is utilized, resistance changes in the two lead wires have no effect on sensor current, while high voltmeter input impedance (~200MΩ) makes current in its two lead wires negligible.

Resistance Thermometer (RTD)
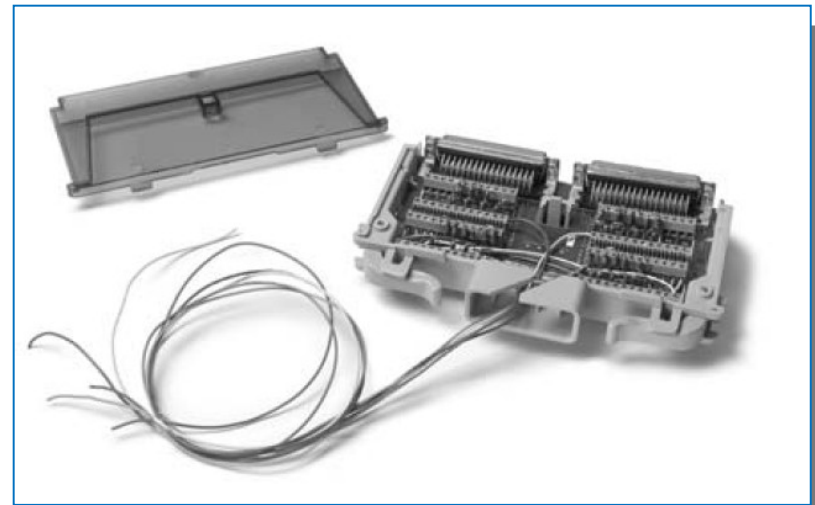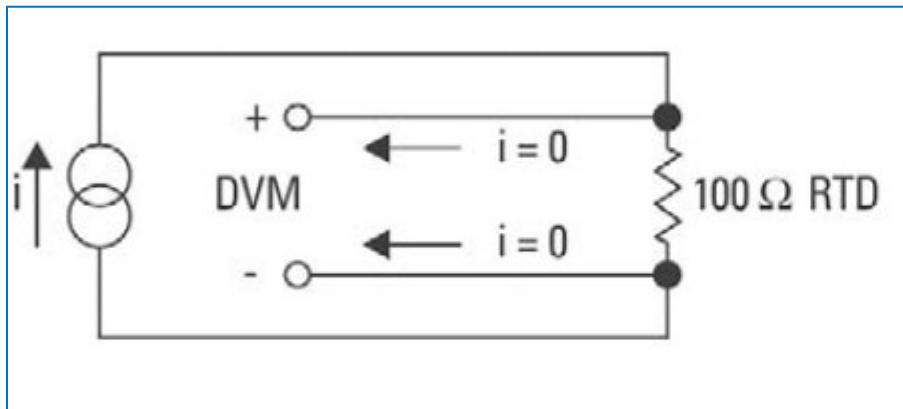
One source of error is the effect of the resistance of the leads which connect the element to the bridge circuit, in case of significant temperature variations.

Methods of correcting for lead resistance include the three-lead or the four-lead arrangements.
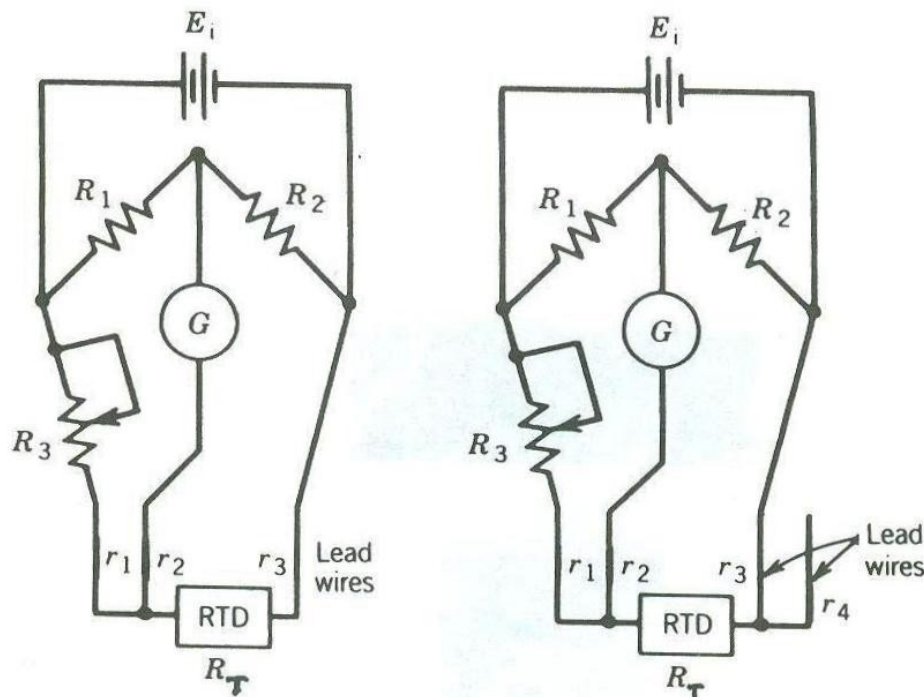
Three-lead: at balance conditions the center lead carries no current and the effect of the resistance of the other two leads is canceled out.

Four-lead: solves the problem by inserting one additional lead wire so that the effect of the lead wires on the resistance thermometer is canceled out. The extra lead may be used to check the equality of lead resistance.

POLITECNICO DI MILANO

Typical linear calibration for a resistance thermometer

$$T = 0.001022*R^2 + 2.349420*R - 245.111497$$
$$R^2 = 1.000000$$
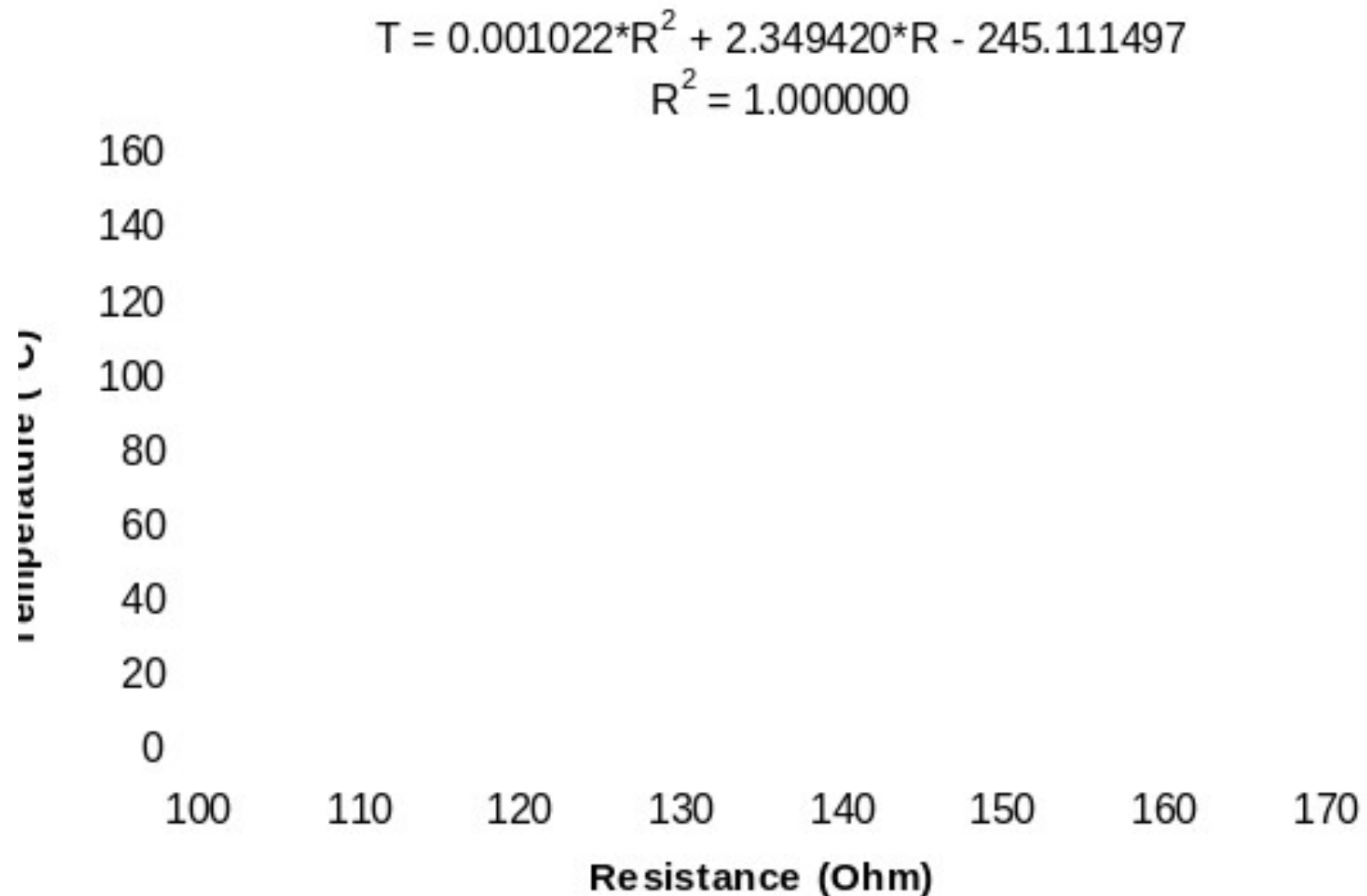
They have a very large negative coefficient, while conductors have a small positive temperature coefficient.

Their resistance/temperature relation is of the form:

$$R = R_0 \, e^{\,\beta\left(1/T - 1/T_0\right)}$$

The reference temperature T_0 is generally taken as 298 K (25 °C) while the constant β is ~4000.

Sensitivity:

$$\frac{dR}{dT} = R_0 \left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]\left(-\frac{\beta}{T^2}\right) \qquad\longrightarrow\qquad \frac{dR/dT}{R} = -\frac{\beta}{T^2}$$

At T=300 K

$$\frac{dR/dT}{R} = -0.045 \; \left[K^{-1}\right]$$

thus larger than the value of α for platinum: $\alpha = 0.003927$ [K^{-1}] .

Thermistors are extremely sensitive devices and performances within 0.01 °C may be obtained with proper calibration.

Thermistors are available commercially in form of beads, rods and disks.

The usable temperature range is limited to temperature below 300 °C, due to deterioration at high temperatures.

Resistivity of three thermistor materials compared
with platinum
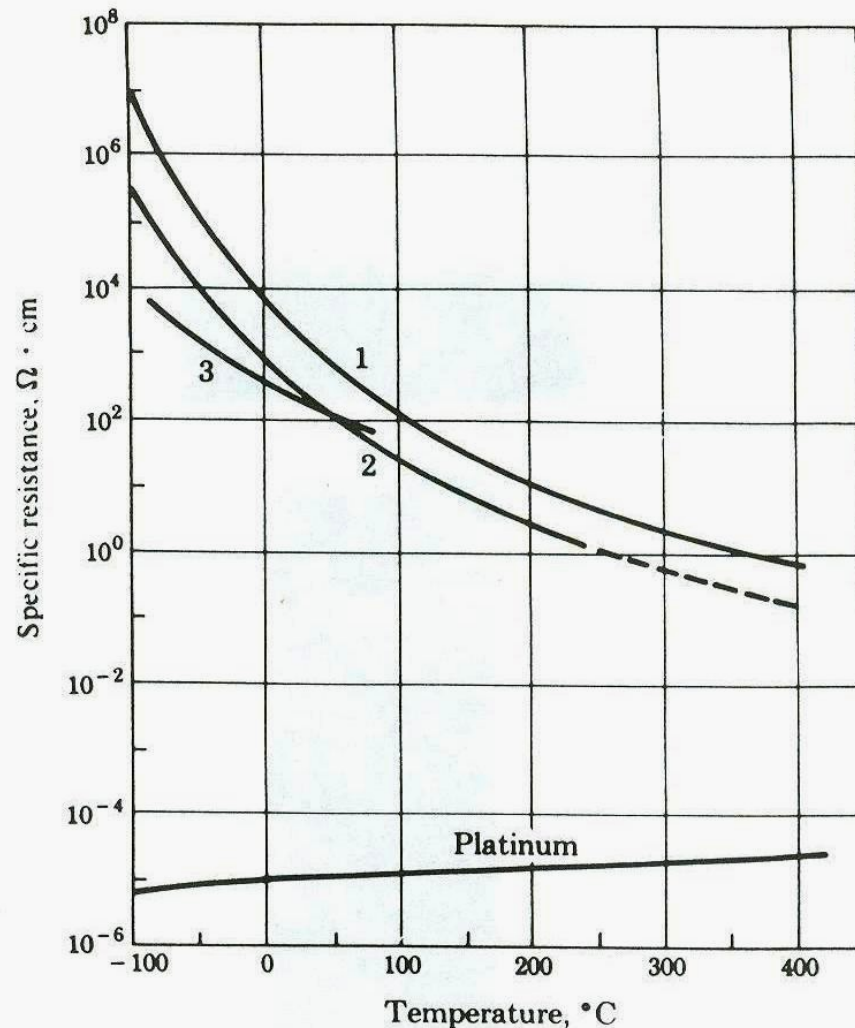
Because the resistance of the
thermistor is so high, the error due
to lead resistance is small
compared to that for RTD, and the
four-wire leads are not required.

The high resistance of the
thermistors means that lower
currents are required for the
measurement and thus errors due
to self-heating are very small.

They are the most common electrical method of temperature measurement.

When two dissimilar metals are joined together as in figure, an emf will exist between the two junctions 3 and 4, which is primarily a function of the temperature difference ($T_2 - T_1$).
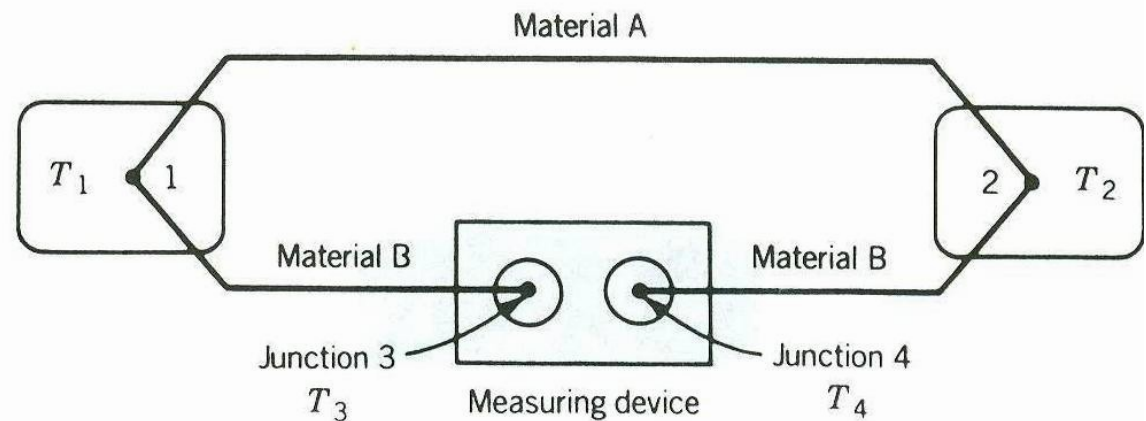
This phenomenon is called the **Seebeck effect**.

Peltier effect: *when* a current is drawn, a temperature difference is generated

Thomson effect: it results from a temperature gradient in the materials

Joule effect: $I^2 R$

When two *dissimilar* materials are connected to a measuring instrument, there will be another thermal emf generated at the junction.

The Thermoelectric laws describe the thermocouple behavior.

1. Law of homogeneous materials: the thermal emf is totally unaffected by temperature elsewhere in the circuit if the two metals are each homogeneous

- *two different metals are needed;*

- *the emf is only function of the two junction temperatures.*
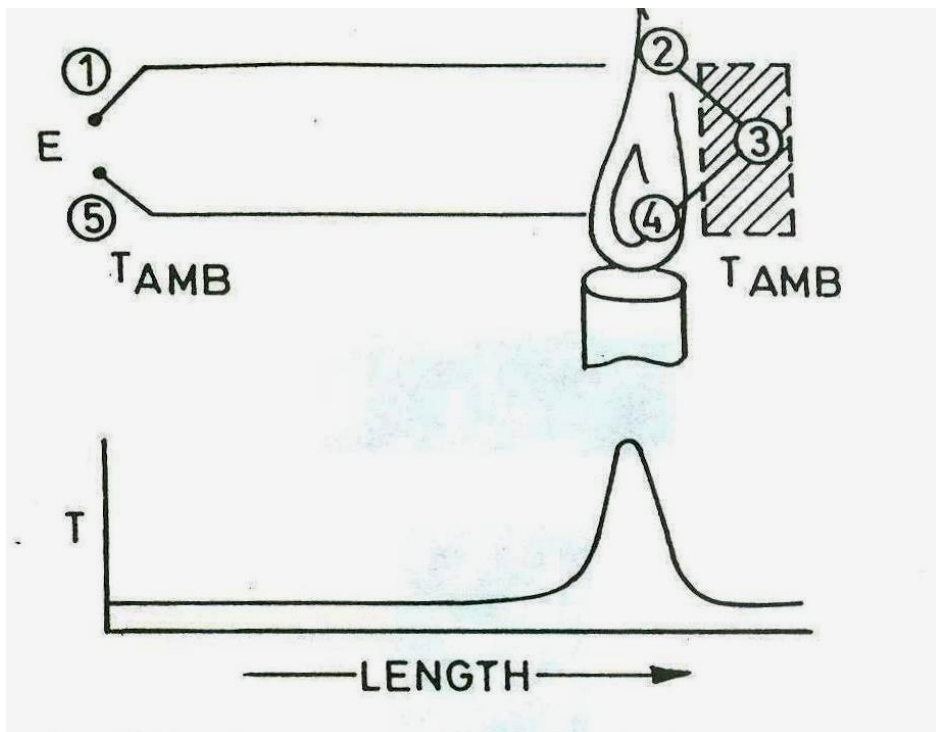
Note: the presence of a local temperature gradient allows to evidence any lack of homogeneity in the metal wires.

2. Law of intermediate metals: if a third homogeneous metal (*copper*) is inserted into the thermocouple two wires, the net emf is unchanged as long as the new connections are at the same temperature.

It may be proved with the aid of the second law of thermodynamics.

(b)

POLITECNICO DI MILANO
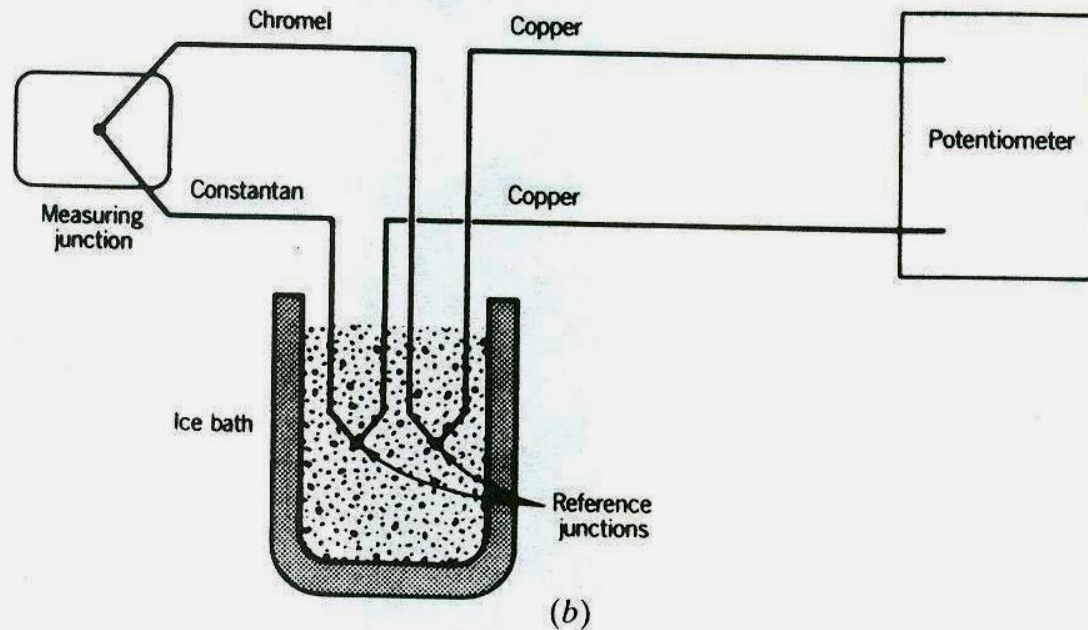
2. Law of intermediate temperatures: if a thermocouple produces emf = V_{AB} when its junctions are at T_A and T_B, and emf = V_{BC} when at T_B and T_C, then it will produce emf = V_{AC} = V_{AB} + V_{BC} when the junctions are at T_A and T_C.
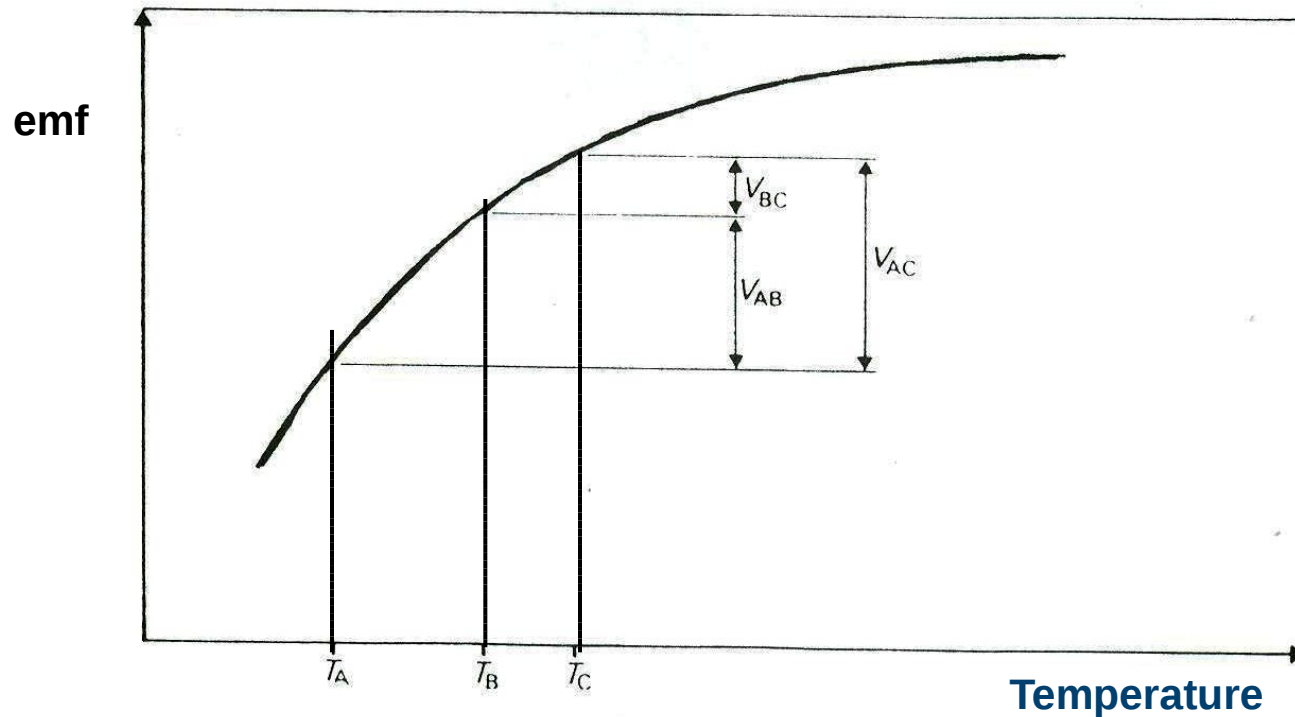
In order to provide a more sensitive output, thermocouples can be connected in a **series arrangement** and the output would be n times that of a single device, provided the temperatures of the hot and cold junctions are uniform.

The *thermopile* is useful for obtaining a higher emf for measurement of a small temperature difference between two junctions. The circuit could just as well be employed for the measurement of a differential temperature.

Voltage-
measurement
device
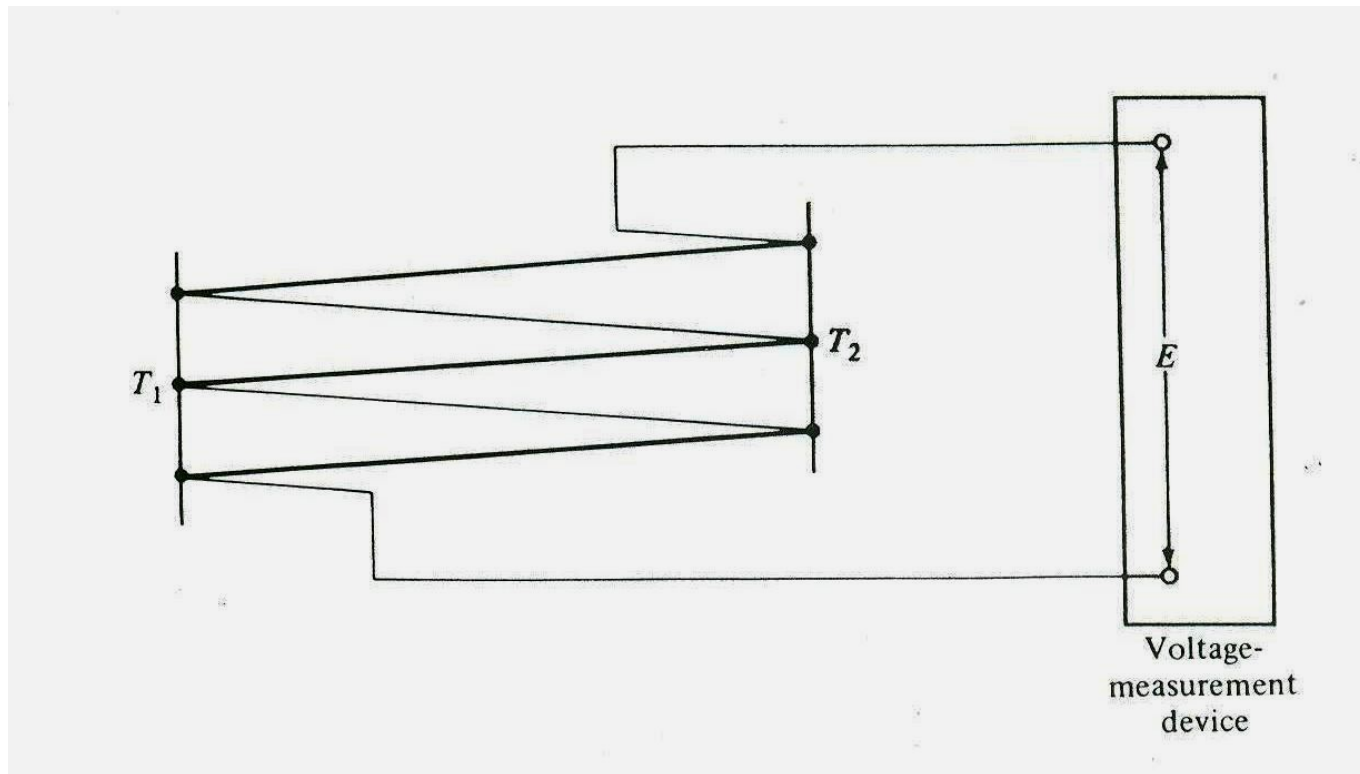
A **parallele connection** may be used for obtaining the average temperature of a number of points. The potential furnished by the potentiometer will be the average of the n junction potentials.

There can be a small error in the reading because there is a small current flow in the lead wires as a result of the difference in potential between the junctions.

Voltage-measurement device
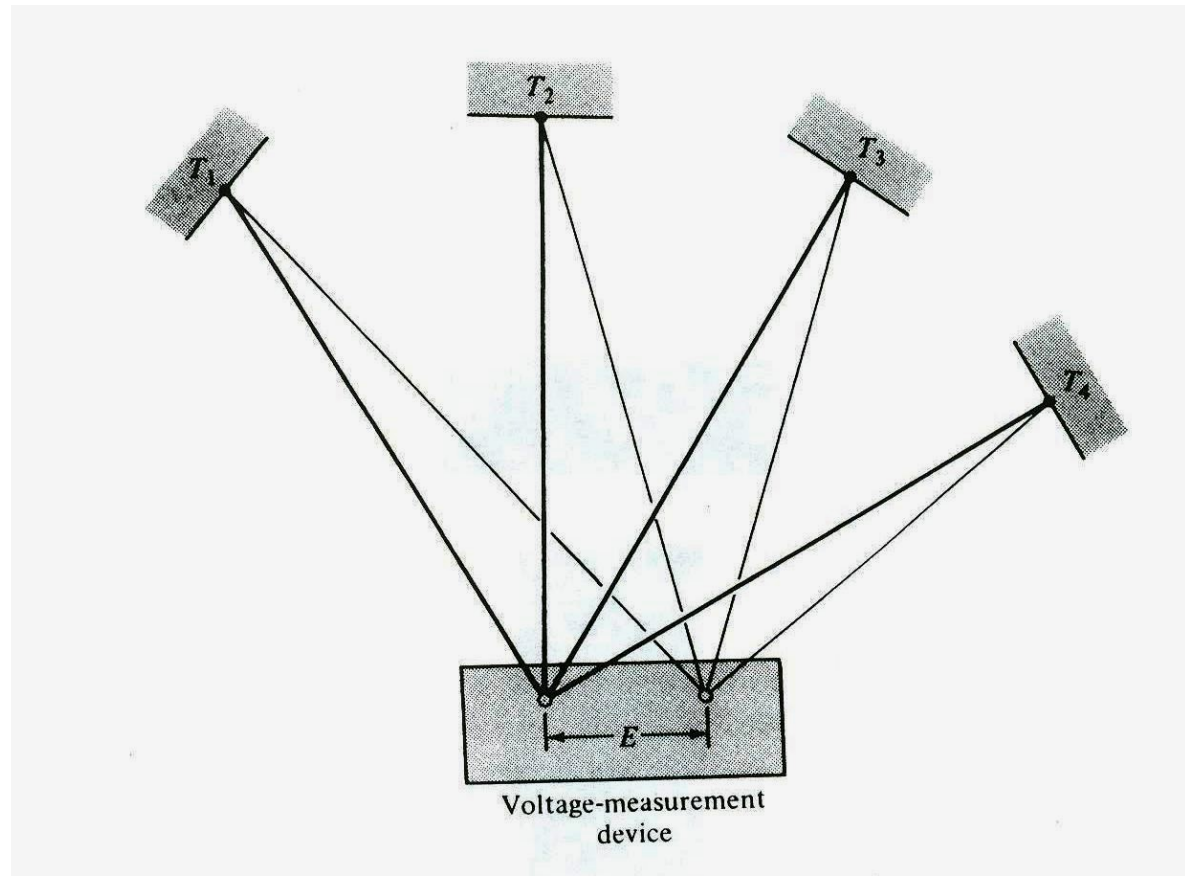
The most common thermocouple combinations and their standard limits of error

TABLE 4.2 Standard Thermocouple Compositions[a]

| | Wire | | Standard Limits |
Type	Positive	Negative	of Error[b]
S	Platinum	Platinum/ 10% rhodium	+1.5 °C or 0.25%
R	Platinum	Platinum/ 13% rhodium	±1.5 °C
B	Platinum/ 30% rhodium	Platinum/ 6% rhodium	±0.5%
T	Copper	Constantan	±1.0 °C or 0.75%
J	Iron	Constantan	±2.2 °C or 0.75%
K	Chromel	Alumel	±2.2 °C or 0.75%
E	Chromel	Constantan	±1.7 °C or 0.5%

Alloy Designations

Constantan: 55% copper with 45% nickel
Chromel: 90% nickel with 10% chromium
Alumel: 94% nickel with 3% manganese, 2% aluminum, and 1% silicon
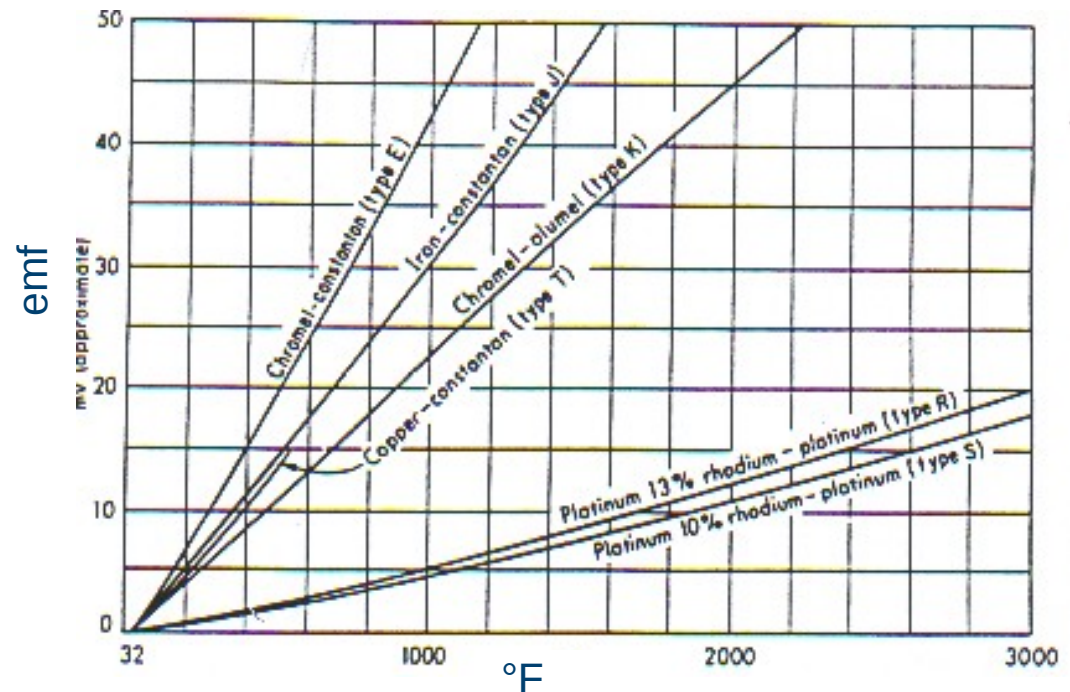
[a] From Temperature Measurements ANSI PTC 193-1974.
[b] Use greater value; these limits of error do not include installation errors.

It is common to express the emf in terms of the potential generated with the refernce junction at 0 °C (32 °F)

The output voltage E of a thermocouple circuit is usually written in the form:

$$E = AT + \frac{1}{2}BT^2 + \frac{1}{3}CT^3$$

where the constants A, B and C are dependent on the thermocouple material.

The need for precise calibration is particularly acute when the thermocouples are used for a small-defferential temperature measurement.

The accuracy of the thermocouples for a direct temperature difference measurement will be much better than for an absolute temperature measurement.

Using software to calculate temperatures from thermocouple voltages, a high-order polynomial can be used in the form

$$T = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ... + a_n x^n$$

In the next Table is indicated the accuracy with which each polynomial fits the NBS tables.

Up to a ninth-order polynomial can be used.

	Type E	Type J	Type K	Type R	Type S	Type T
	Chromel(+) vs. constantan(−)	Iron(+) vs. constantan(−)	Chromel(+) vs. nickel-5%(−) (aluminum silicon)	Platinum-13% rhodium(+) vs. platinum(−)	Platinum-10% rhodium(+) vs. platinum(−)	Copper(+) vs. constantan(−)
	−100°C to 1000°C* ±0.5°C 9th order	0°C to 760°C* ±0.1°C 5th order	0°C to 1370°C* ±0.7°C 8th order	0°C to 1000°C* ±0.5°C 8th order	0°C to 1750°C* ±1°C 9th order	−160°C to 400°C* ±0.5°C 7th order
a_0	0.104967248	−0.048868252	0.226584602	0.263632917	0.927763167	0.100860910
a_1	17189.45282	19873.14503	24152.10900	179075.491	169526.5150	25727.94369
a_2	−282639.0850	−218614.5353	67233.4248	−48840341.37	−31568363.94	−767345.8295
a_3	12695339.5	11569199.78	2210340.682	1.90002E + 10	8990730663	78025595.81
a_4	−448703084.6	−264917531.4	−860963914.9	−4.82704E + 12	−1.63565E + 12	−9247486589
a_5	1.10866E + 10	2018441314	4.83506E + 10	7.62091E + 14	1.88027E + 14	6.97688E + 11
a_6	−1.76807E + 11		−1.18452E + 12	−7.20026E + 16	−1.37241E + 1	−2.66192E + 13
a_7	1.71842E + 12		1.38690E + 13	3.71496E + 18	6.17501E + 17	3.94078E + 14
a_8	−9.19278E + 12		−6.33708E + 13	−8.03104E + 19	−1.56105E + 19	
a_9	2.06132E + 13				1.69535E + 20	

J.P. Holman, Experimental Methods for Engineers, McGraw-Hill, Inc., 1994

Thermocouples

The output of thermocouples is in the mV range and may be measured by a digital millivolmeter.

The problem of the reference junction may be alleviated with the *hardware* compensation: a thermistor is placed in thermal contact with the terminal strip to which the thermocouple wires are attached. The circuit must be adjusted so that V_c will match the thermocouple temperature coefficient in mV/°C.

Direct digital readout of temperature is then provided.

An alternate technique is to provide *software* compensation in the data acquisition system.
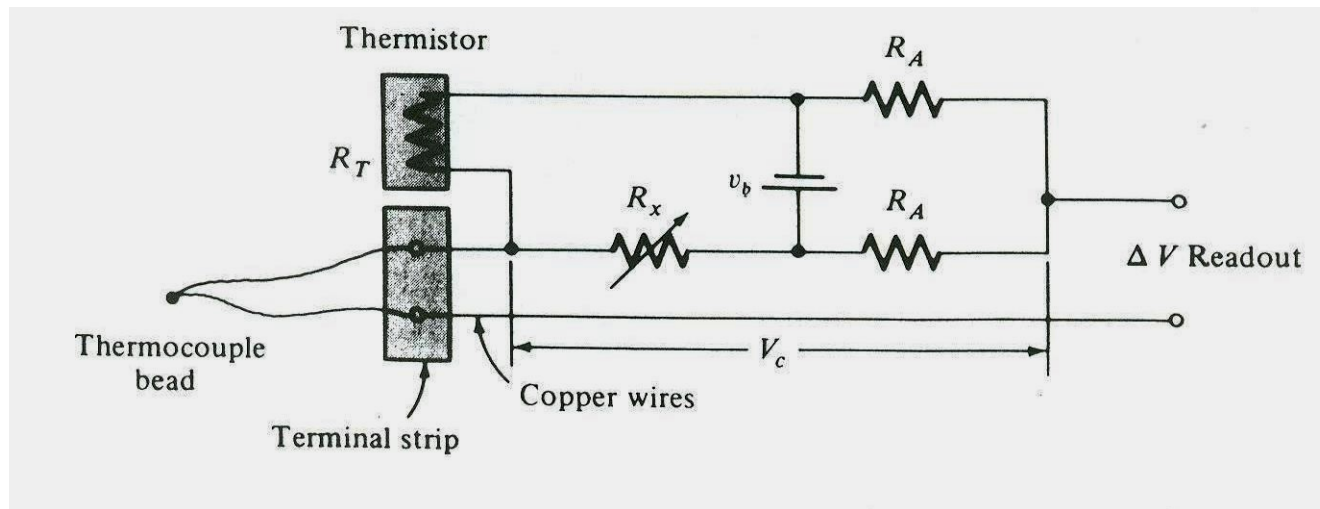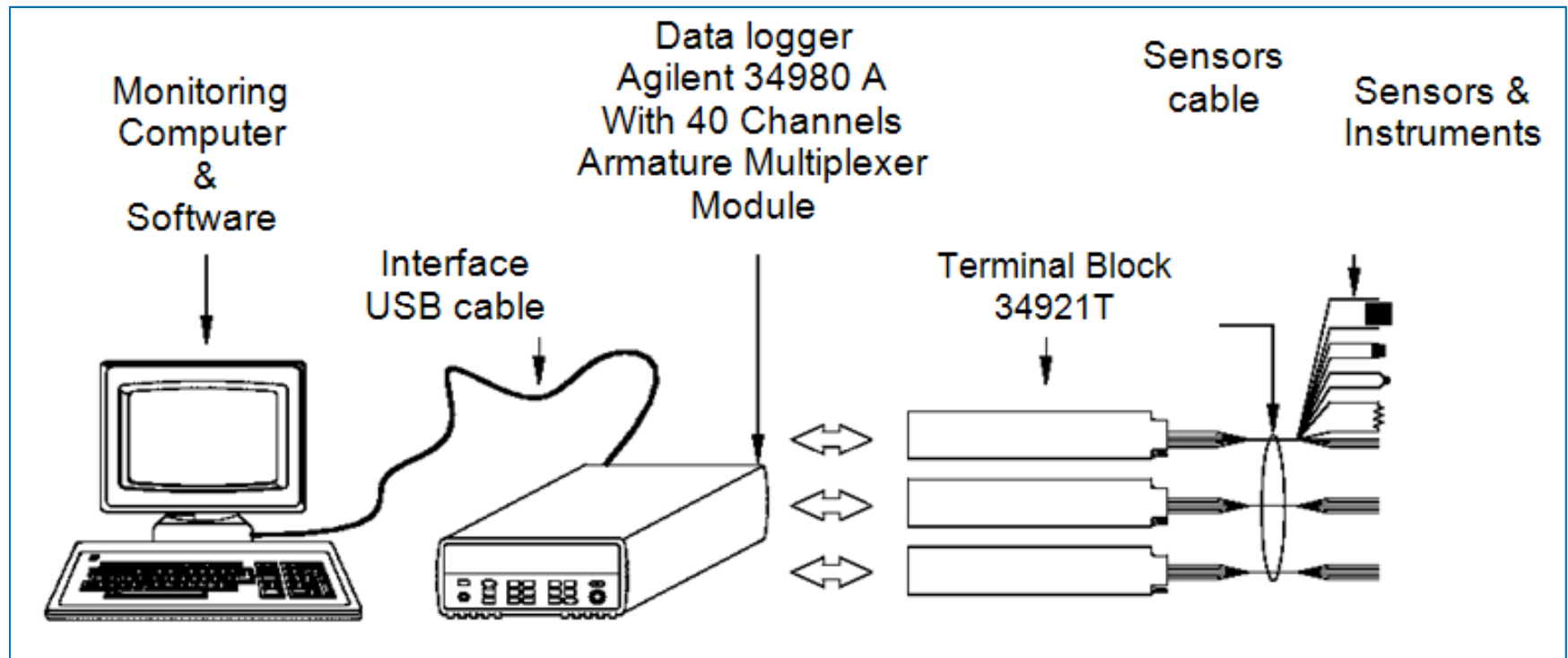

Accuracy ~ $\pm$ 0.1% that means an error of $\pm$0.7 °C

Display resolution = 0.1°C

Data acquisition system

A typical data acquisition system comprises:

➢ Terminal block with cold junction,

➢ Data logger with A/D converter and digital millivolmeter,

➢ linearization circuit and filters,

➢ PC for storage and data processing.

The complete system may be calibrated directly to furnish a reasonably accurate temperature determination.

Calibration oil-bath

Typical oil-bath used for the static calibration of thermocouples or resistance thermometers

Temperature range: -30 to 150°C
Accuracy: ±0.25°C
Stability: ±0.03°C at -25°C (oil, 5010)
                   ±0.05°C at 125°C (oil, 5010)
Uniformity   ±0.02°C
Resolution   0.01°C/F
Heating Time   25°C to 100°C: 35 minutes
Cooling Time   25°C to –25°C: 45 minutes

Cooling Fan and Thermal Electric Devices (TED)

A heat-transfer process is associated with all temperature measurements.

When a thermometer is exposed to an environment, the temperature is determined in accordance with the total *heat-energy* exchange with the temperature sensing element.

In some instances the temperature of the thermometer can be substantially different from the temperature which is to be measured.

The differences are also depending on the type of substance: gas, liquid or solid.

With gases, heat transfer may take place as a result of one or more of the three modes:

- ✓ **Conduction**
- ✓ **Convection**
- ✓ **Radiation**

The Thermodinamic Temperature is only defined at equilibrium states, which rarely are encountered in the practical applications.

In the ideal case in which both the probe and the gas are fixed with respect to the walls of a container, the temperature indicated by a probe will represent a measure of the average kinetic energy of the thermal motion of the molecules, assuming the medium as a continuum.

If the gas and the probe move at the same velocity, the probe output will give a gas temperature lower than in the previous case, the reduction being equal to that fraction of the thermal kinetic energy associated with the mean gas motion.

If only the gas is moving, the probe indicates a higher temperature due to the partial conversion in thermal energy of the kinetic energy of the gas.

If the gas is brought to rest adiabatically, the stagnation temperature is

$$T_0 = T_s + \frac{V^2}{2c_p}$$

T_0 = stagnation temperature of the gas
T_s = static temperature of the gas
V = gas mean velocity
c_p = constant-pressure specific heat of the gas

Temperature measurement in high-speed flow

The stagnation temperature of the gas may also be expressed in terms of the Mach number

$$\frac{T_0}{T_s} = 1 + \frac{\gamma - 1}{2} M^2$$

Where: M = Mach number of the gas

         $\gamma = c_p/c_v$ (=1.4 for air)

The conversion kinetic-to-thermal energy may result incomplete due to convection losses and viscous dissipation, depending on probe geometry and Prandtl number of the fluid

$$Pr = \frac{c_p \mu}{k_g}$$

where μ = dynamic viscosity of the gas

        k_g = thermal conductivity of the gas

Prandtl number relates friction heating in the boundary layer with dissipation due to conduction through the fluid;

Pr < 1; in air (0.65 < Pr < 0.7), and thus thermal diffusion effects are larger than viscous effects.

Temperature measurement in high-speed flow

For the actual case of a probe inserted in a high-speed gas stream, the indicated temperature will **not** in general be equal to the stagnation temperature.

It is introduced a recovery factor

$$\alpha = \frac{T_r - T_s}{T_0 - T_s}$$

where T_r is the recovery temperature and is strongly dependent on the probe configuration.

For a probe oriented normally to the flow direction, it is usually considered acceptable a value $\alpha = 0.68 \pm 0.07$, while for a probe parallel to the flow stream $\alpha = 0.86 \pm 0.09$.

In general, we can express the error due to the gas velocity as

$$\varepsilon_V = T_0 - T_r = (1 - \alpha)\frac{V^2}{2c_p}$$

Temperature measurement in high-speed flow

One objective of a probe design is to achieve a configuration that produces a near constant **recovery factor** over a wide range of flow velocities.

A typical high-speed temperature probe is shown in figure: the gas stream enters the front of the probe and is diffused to a lower velocity in the enclosed shield, which also serves to reduce radiation losses from the thermocouple hot junction.

The flow leaves the probe through the side vent holes.

$$\alpha = \frac{T_t - T_s}{T_0 - T_s} \approx 1$$
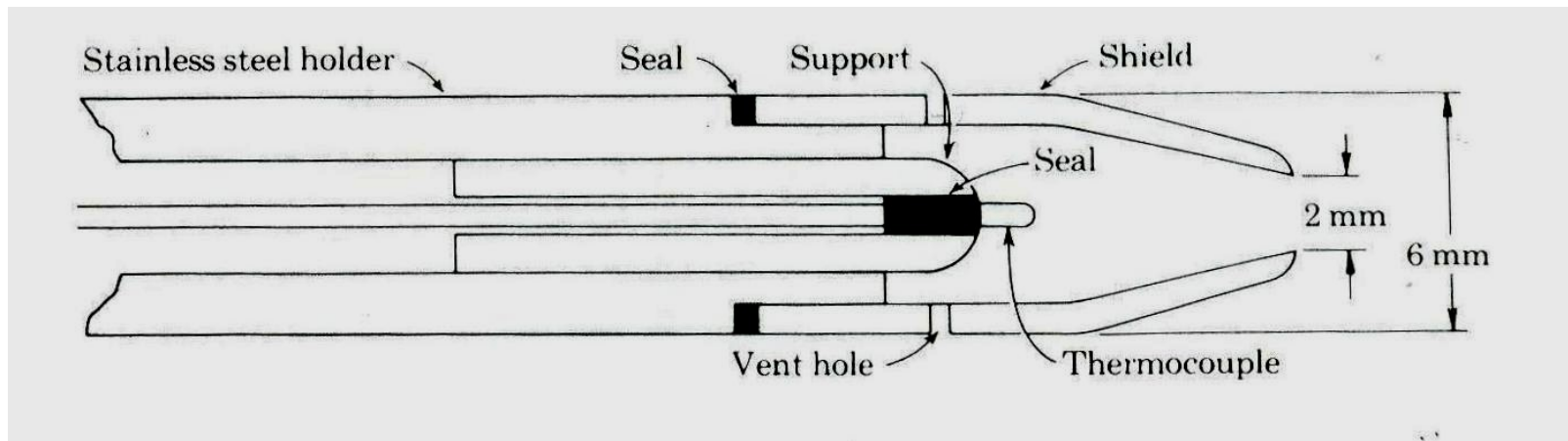
Conduction error: if a temperature gradient exists along a thermometer, heat may be conducted out along the probe support and the temperature of the sensing element will be lower.

The measurement error can be expressed by this relation

$$\varepsilon_C = T_t - T_g = \frac{T_w - T_g}{\cosh(mL)}$$

$$m = \sqrt{\frac{h\pi D}{k_s \pi \frac{D^2}{4}}} = \sqrt{\frac{4h}{k_s D}} = \frac{2}{D}\sqrt{\frac{k_g Nu}{k_s}} \qquad Nu = \frac{hD}{k_g}$$

h = convection heat transfer coefficient
D = probe diameter
L = length of probe support
k_s = thermal conductivity of the theromocouple
T_w = wall temperature
T_g = fluid temeprature
k_g = thermal conductivity of the fluid

If the probe is inserted in a wall whose temperature is different from that of the fluid, and the term cosh(mL) has a finite value, then the error is ≠ 0.

Note: cosh(mL) increases with the argument mL; thus a sufficiently large aspect ratio L/D will reduce the error : the condition L/D > 50 assures that the error will be negligible.
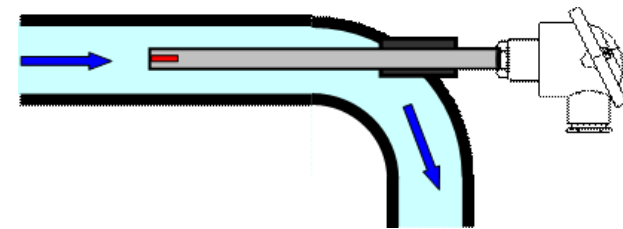
Errors due to radiation losses: heat transfer by radiation from a probe immersed in a hot gas stream can reduce the sensing element temperature, if the surroundings are at lower temperature.

The possible error can be expressed by the relation

$$\varepsilon_R = T_t - T_g = \frac{K_R A_R \varepsilon \sigma}{h A_c}(T_t^4 - T_w^4)$$

A_c = surface area of the sensing element for convective heat exchange

A_R = surface area of the sensing element for radiative heat exchange

T_w = temperature of the surroundings (wall or other)

K_R = radiation-shape factor

σ = Stefan-Boltzmann universal constant (= 5.669 x 10^{-8} W m^{-2} K^{-4})

ε= surface emissivity of the thermometer

h = convection heat transfer coefficient

The error is deduced through the energy balance between heat transfer from the gas to the probe by convection

$$q_c = h A_c (T_g - T_t)$$

and heat transfer from the probe to the sussoundings by radiation

$$q_R = \varepsilon \sigma A_R (T_t^4 - T_w^4)$$

Correction of radiation error:

In the previus relation used to evaluate the radiation error there are two terms which are difficult to determine:

➢ **ε** surface emissivity of the thermometer, which depends on the material
➢ **h** convection heat transfer coefficient from the gas to the thermometer

Uncertainties in the values of **h** of $\pm$ 25% are not uncommon.

The coefficient h can be deduced from the many convection heat transfer formulas relating the Nusselt and Reynolds numbers (see Holman, p.368).

$$Nu = \frac{hD}{k} \qquad \mathrm{Re} = \frac{VD}{\nu}$$

k_g = thermal conductivity of fluid
D = diameter of probe
V = mean fluid velocity
ν = kinematic viscosity of fluid
h = convection heat transfer coefficient

Typical correlation:

$$\mathrm{Nu} = (0.44 \pm 0.06)\,\mathrm{Re}^{0.50} \qquad \text{(probe oriented normally to the gas stream)}$$

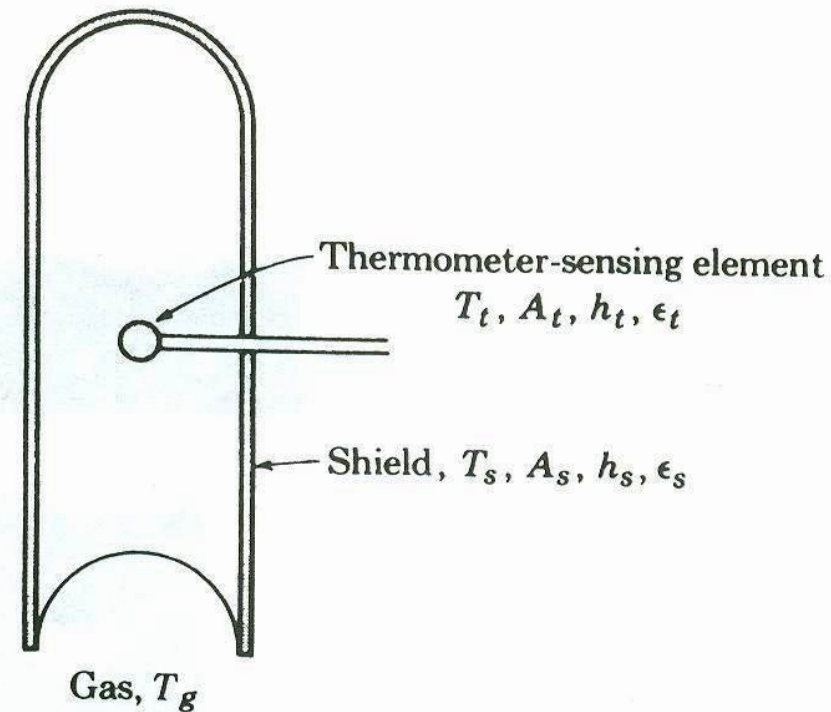$$\mathrm{Nu} = (0.85 \pm 0.009)\,\mathrm{Re}^{0.674} \qquad \text{(probe oriented parallel to the gas stream)}$$

Mitigation of radiation effects:

The radiation error in a temperature measurement is reduced by placing a radiation shield around the thermometer, which reflects most of the radiant energy back to the thermometer.

The radiation shield should be as reflective as possible (ϵ very small)
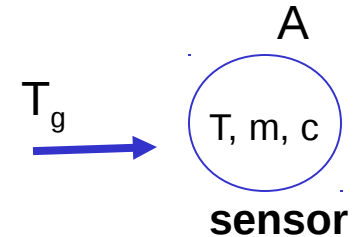

Environment, T_e

Thermometer-sensing element T_t, A_t, h_t, ϵ_t

Shield, T_s, A_s, h_s, ϵ_s

Gas, T_g
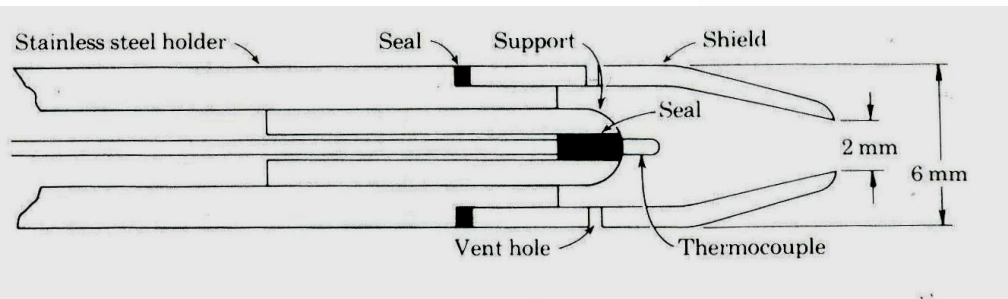
Stainless steel holder — Seal — Support — Shield — Seal — 2 mm — 6 mm — Vent hole — Thermocouple

Transient response

Consider only convection heat transfer between the gas and the probe.

Energy balance for the transient process

$$mc\frac{dT_t}{dt} = hA(T_g - T_t)$$

m = mass of the sensor
c = specific heat of the sensor material
A = surface area of the sensor
h = convection heat transfer coefficient
k_g = thermal conductivity of the fluid

The solution gives the temperature of the thermometer as function of time:

$$\frac{T_t - T_g}{T_r - T_g} = e^{-t/\tau}$$

$$\tau = \frac{mc}{hA} = \frac{\rho c V}{hA} = \frac{1}{6}\frac{\rho c D}{h} = \frac{\rho c}{6}\frac{D^2}{Nu\,k_g}$$

$$Nu = \frac{hD}{k_g}$$

τ = *time constant* of the temperature probe

Dynamic error

The dynamic response of a temperature sensor will depend on its design, material properties, and the nature of the heat transfer process during the measurements.

Any temperature probe is characterized by the familiar expomemtial behavior of *first-order systems* subjected to a step input.
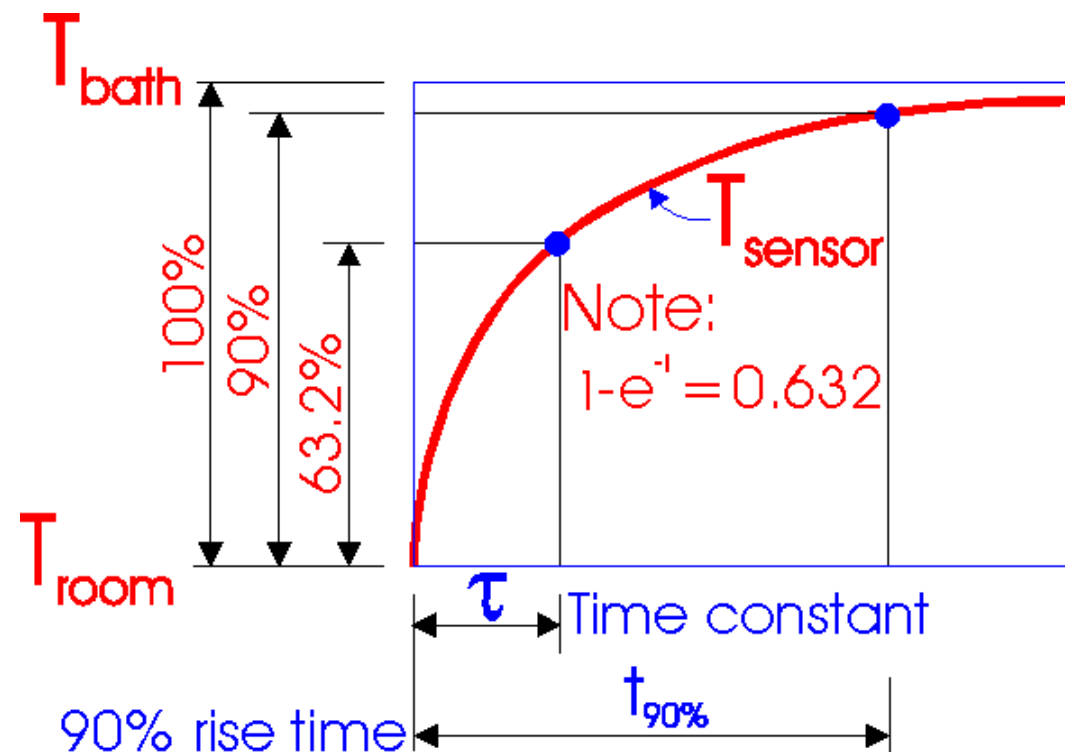
$$\tau = \frac{mc}{hA} = \frac{1}{6}\frac{\rho cD}{h}$$

Dynamic response of a sensor for a step-change (increase) of the input temperature from T_{room} to T_{bath}.
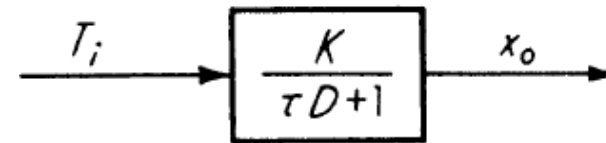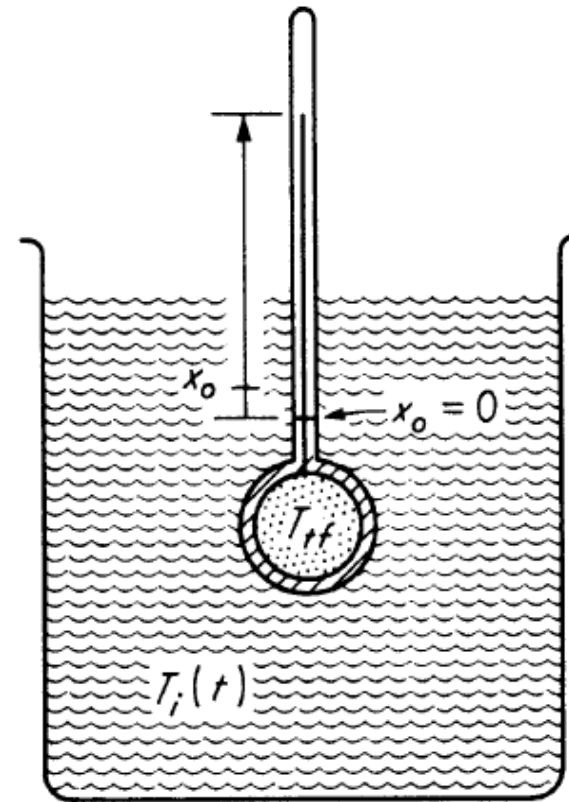
First order systems

$$a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i$$

$$\frac{a_1}{a_0} \frac{dq_o}{dt} + q_o = \frac{b_0}{a_0} q_i$$

$$(\tau D + 1) q_o = K q_i$$

Transfer function

$$\frac{q_o}{q_i}(D) = \frac{K}{\tau D + 1}$$

POLITECNICO DI MILANO

First order systems

Step-function response of first order instrument

$$(\tau D + 1)q_o = Kq_{is}$$

$$q_o(t) = Ce^{-t/\tau} + Kq_{is}$$
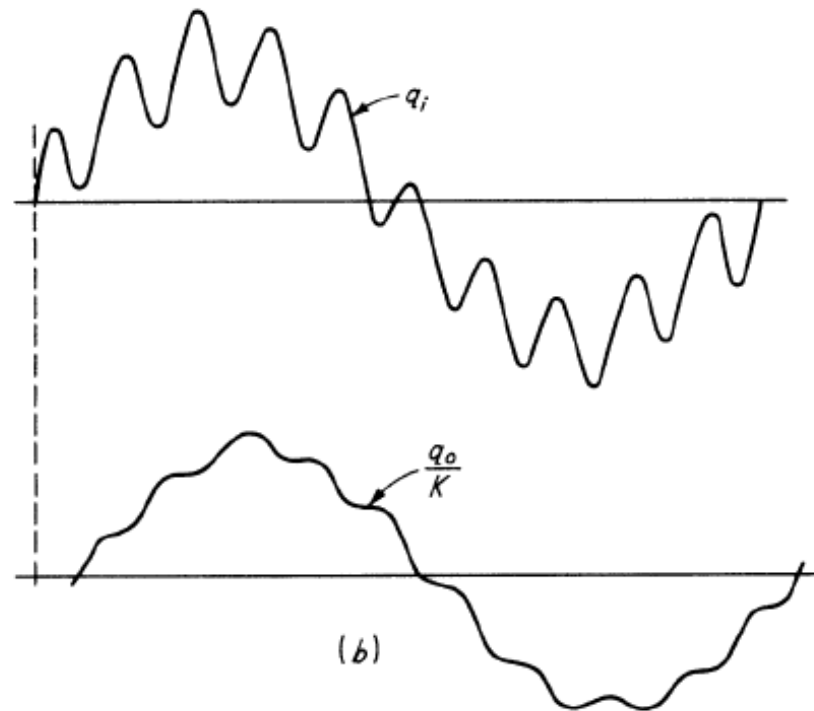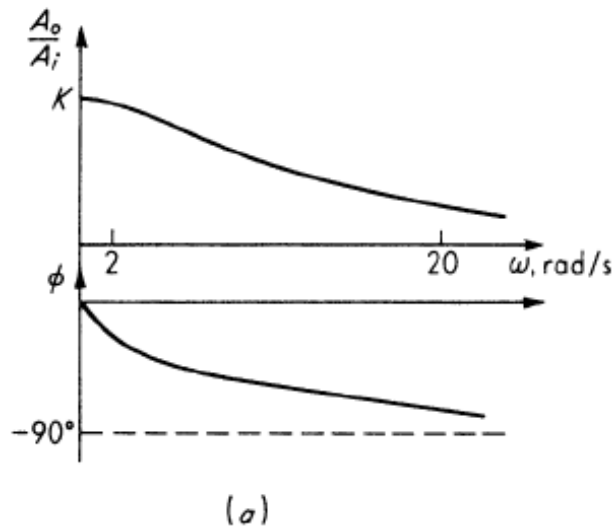
Initial conditions:

$0 = C + Kq_{is}$

$C = -Kq_{is}$

$$\frac{q_o(t)}{Kq_{is}} = 1 - e^{-t/\tau}$$

POLITECNICO DI MILANO
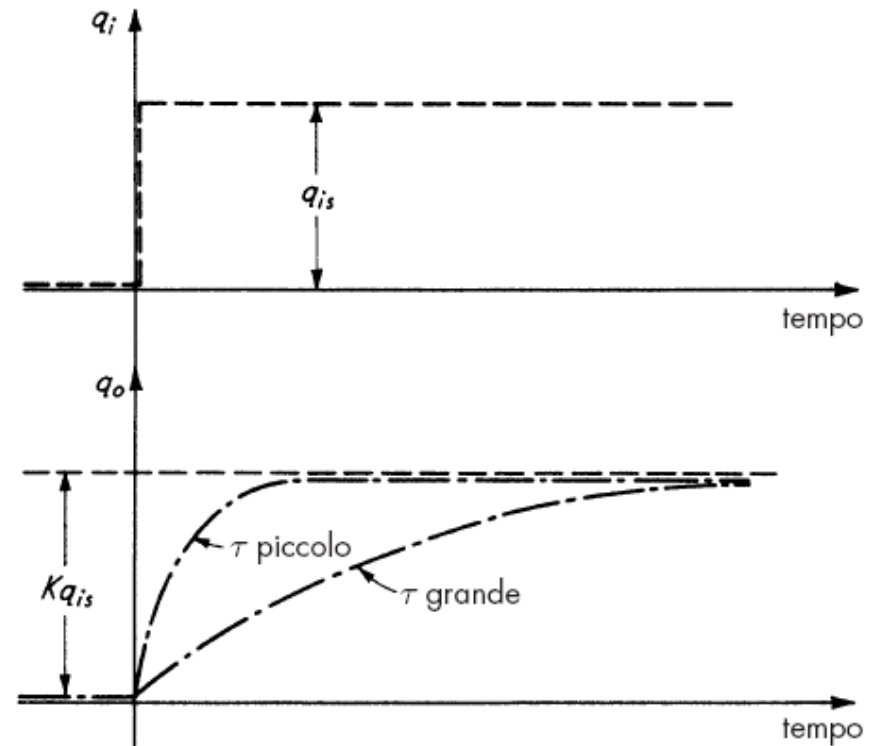
First order systems

Frequency response of first order instruments

$$\frac{q_o}{q_i}(i\omega) = \frac{K}{i\omega\tau + 1}$$
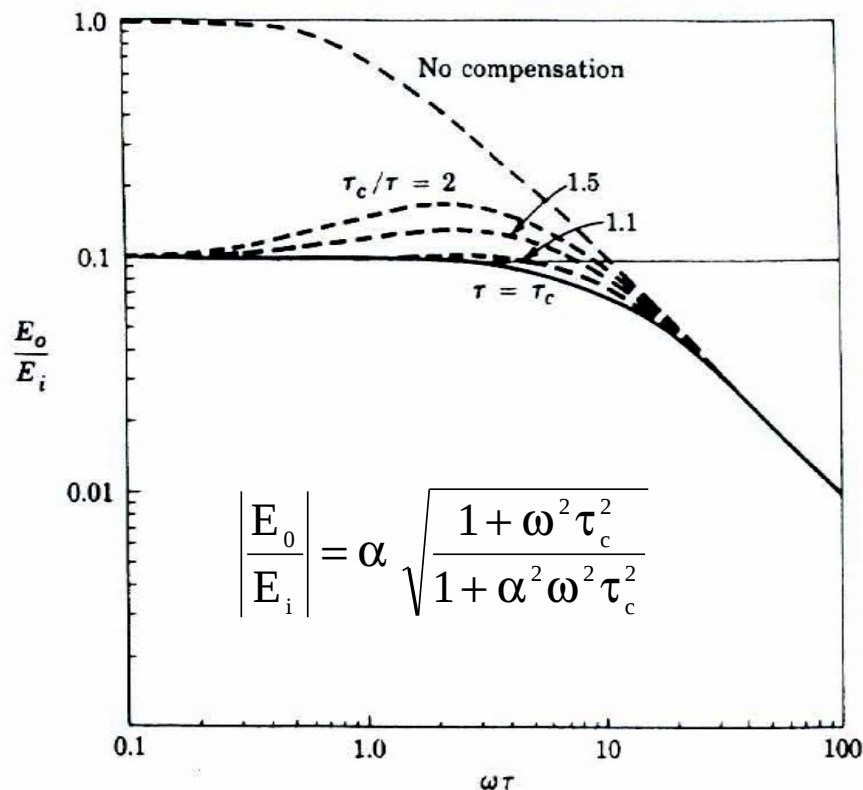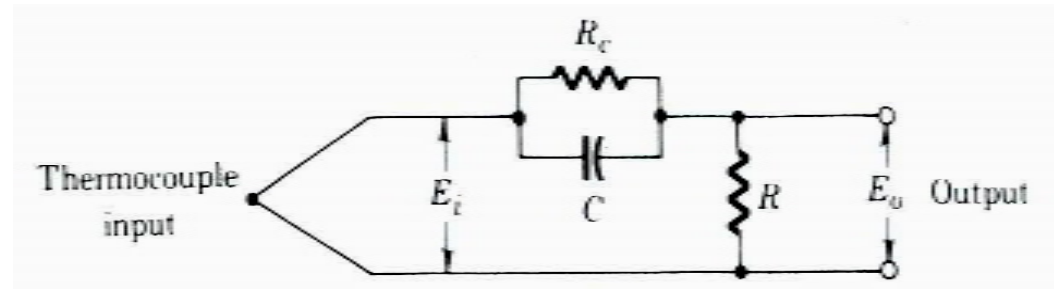

(a)

(b)

$$\frac{A_o}{A_i} = \left| \frac{q_o}{q_i}(i\omega) \right| = \frac{K}{\sqrt{\omega^2\tau^2 + 1}}$$

$$\phi = \tan^{-1}(-\omega\tau)$$

Thermocouple compensation

If a compensating electric
network is applied, it is possible
to increase the frequency
response of the thermocouple

The output of the network is given by

$$\frac{E_0}{E_i} = \alpha \; \frac{1 + j\omega\tau_c}{1 + \alpha j\omega\tau_c}$$

$$\alpha = \frac{R}{R + R_c}$$ static gain

$$\tau_c = R_c C$$ time constant of the network

$$\omega = \quad \text{frequency of the input signal}$$

$$\left|\frac{E_0}{E_i}\right| = \alpha \sqrt{\frac{1 + \omega^2 \tau_c^2}{1 + \alpha^2 \omega^2 \tau_c^2}}$$

Bibliography

Books

1. Doebelin E.O., *Strumenti e metodi di misura*, 2° edizione, McGraw-Hill, 2008

2. Holman J. P., *Experimental Methods for Engineers*, McGraw-Hill Inc., 1994

3. Figliola R.S. and Beasley D.E., *Theory and Design for Mechanical Measurements*, J. Wiley, 1991

Papers

Shaddix C.R., Correcting thermocouples measurements for radiation loss: a critical review, ASME – NHTC99 – 282, 1999.

Heitor M. V. and Moreira A. L. N., Thermocouples and Sample Probes for Combustion Studies, Prog. Energy Combust. Sci. 1993, Vol. 19, pp. 25~278.