

CSCE 531 Homework 1

1. Let $C(x)$ be the statement “ x has a cat,” let $D(x)$ be the statement “ x has a dog,” and let $F(x)$ be the statement “ x has a ferret.” Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

- a. A student in your class has a cat, a dog, and a ferret.

$$\exists x(C(x) \wedge D(x) \wedge F(x))$$

- b. For each of the three types of animals, there is a student in your class who has one of these animals as a pet.

$$\exists xC(x) \wedge \exists yD(y) \wedge \exists zF(z)$$

2. (Problem 36) Find counterexamples, if possible, to these universally quantified statements, where the domain for all statements consists of all real numbers.

- a. $\forall x(x^2 \neq x)$

$$x = 0, x = 1$$

- b. $\forall x(x^2 \neq 2)$

$$x = \sqrt{2}$$

- c. $\forall x(|x| > 0)$

$$x = 0$$

3. (Inspired by Problem 28) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers. You may need to refer to Appendix 1.

- a. $\forall x \exists y(x^2 = y)$

T

- b. $\forall x \exists y(x = y^2)$

F, fails for negative numbers.

- c. $\exists x \forall y(xy = 0)$

T ($x=0$)

- d. $\exists x \exists y(x + y \neq y + x)$

F

e. $\forall x[x \neq 0 \rightarrow \exists y(xy = 1)]$

T ($y=1/x$)

f. $\exists x\forall y(y \neq 0 \rightarrow xy = 1)$

F, pick an x and two different y values.

g. $\forall x\exists y(x + y = 1)$

T ($y=-x+1$)

h. $\exists x\exists y(x + 2y = 2 \wedge 2x + 4y = 5)$

F (parallel lines)

i. $\forall x\exists y(x + y = 2 \wedge 2x - y = 1)$

F ($x=5$)

j. $\forall x\forall y\exists z(z = \frac{x+y}{2})$

T