# Gausian Elimination Notes

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## Gausian Elimination

Gausian elimination is a method for solving systems of linear equations. It involves transforming the system of equations into an upper triangular form, from which the solutions can be easily obtained through back substitution.

### Steps of Gausian Elimination

1. Write the augmented matrix of the system of equations.
2. Use row operations to transform the matrix into upper triangular form.
3. Perform back substitution to find the values of the variables.

### Row Operations

The following row operations can be performed on the augmented matrix:

- Swap two rows.
- Multiply a row by a non-zero scalar.
- Add or subtract a multiple of one row to another row.

### Example

Consider the system of equations:

$$\begin{aligned} 2x + 3y &= 5 \\ 4x + y &= 11 \end{aligned}$$

The augmented matrix is:

$$\begin{bmatrix} 2 & 3 & | & 5 \\ 4 & 1 & | & 11 \end{bmatrix}$$

We can perform the following row operations to transform it into upper triangular form: 1. Multiply the first row by 2 and subtract it from the second row:

$$\begin{bmatrix} 2 & 3 & | & 5 \\ 0 & -5 & | & 1 \end{bmatrix}$$

2. Divide the second row by -5:

$$\begin{bmatrix} 2 & 3 & | & 5 \\ 0 & 1 & | & -\frac{1}{5} \end{bmatrix}$$

Now we can perform back substitution: From the second row, we have:

$$y = -\frac{1}{5}$$

Substituting $y$ into the first row:

$$2x + 3\left(-\frac{1}{5}\right) = 5$$

$$2x - \frac{3}{5} = 5$$

$$2x = 5 + \frac{3}{5} = \frac{28}{5}$$

$$x = \frac{14}{5}$$

Thus, the solution to the system is:

$$x = \frac{14}{5}, \quad y = -\frac{1}{5}$$

# Reduced Row Echelon Form

The reduced row echelon form (RREF) of a matrix is a form where:

- Each leading entry in a row is 1.
- Each leading 1 is the only non-zero entry in its column.
- The leading 1 of a row is to the right of the leading 1 of the previous row.
- Any rows consisting entirely of zeros are at the bottom of the matrix.

## Finding RREF

To find the RREF of a matrix, you can use the following steps:

1. Start with the original matrix.
2. Use row operations to create leading 1s in each row.
3. Use row operations to ensure that each leading 1 is the only non-zero entry in its column.
4. Ensure that the leading 1s are in a stair-step pattern from left to right.
5. Move any rows of zeros to the bottom of the matrix.

### Example of RREF

Consider the matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

This matrix is already in RREF because:

- Each leading entry is 1.
- Each leading 1 is the only non-zero entry in its column.
- The leading 1s are in a stair-step pattern.
- There are no rows of zeros.

## Example of RREF Calculation

Let's find the RREF of the following matrix:

$$\begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

1. Start with the original matrix:

$$\begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

2. Divide the first row by 2 to create a leading 1:

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

3. Subtract the first row from the second row:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

4. Now, we can swap the second and third rows to move the non-zero row up:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

5. Finally, we can subtract 2 times the second row from the first row to eliminate the second column in the first row:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

This matrix is now in RREF.