



Software System Design and Implementation

Data Invariants, Abstraction and Refinement Practice

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Term 2 2019

Recall: Structure, Data Invariants and Refinement

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Refinement and Specifications

In general, all **functional correctness specifications** can be expressed as:

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- ② the implementation is a refinement of an abstract correctness model.

Editor Example

Consider this ADT interface for a text editor:

```
data Editor
einit :: String -> Editor
stringOf :: Editor -> String
moveLeft :: Editor -> Editor
moveRight :: Editor -> Editor
insertChar :: Char -> Editor -> Editor
deleteChar :: Editor -> Editor
```

Data Invariant Properties

```
prop_einit_ok          s = wellformed (einitA s)
prop_moveLeft_ok       a = wellformed (moveLeftA a)
prop_moveRight_ok      a = wellformed (moveRightA a)
prop_moveInsert_ok x a = wellformed (insertCharA x a)
prop_moveDelete_ok     a = wellformed (deleteCharA a)
```


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```
einitA s = A s 0
stringOfA (A s _) = s
moveLeftA (A t c) = A t (max 0 (c-1))
moveRightA (A t c) = A t (min (length t) (c+1))
insertCharA x (A t c) = let (t1, t2) = splitAt c t
                        in A (t1 ++ [x] ++ t2) (c+1)
deleteCharA (A t c) = let (t1, t2) = splitAt c t
                      in A (t1 ++ drop 1 t2) c
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Concrete Implementation

Our concrete version will just maintain two strings, the left part (in reverse) and the right part of the cursor:

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```
einit s = C [] s
stringOf (C ls rs) = reverse ls ++ rs
moveLeft (C (l:ls) rs) = C ls (l:rs)
moveLeft c = c
moveRight (C ls (r:rs)) = C (r:ls) rs
moveRight c = c
insertChar x (C ls rs) = C (x:ls) rs
deleteChar (C ls (_:rs)) = C ls rs
deleteChar c = c
```

Refinement Functions

Abstraction function to express our refinement relation in a QC-friendly way: such a function:

```
toAbstract :: Concrete -> Abstract
```

```
toAbstract (C ls rs) = A (reverse ls ++ rs) (length ls)
```

Properties with Abstraction Functions

```
prop_init_r s =
  toAbstract (einit s) == einitA s
prop_stringOf_r c =
  stringOf c == stringOfA (toAbstract c)
prop_moveLeft_r c =
  toAbstract (moveLeft c) == moveLeftA (toAbstract c)
prop_moveRight_r c =
  toAbstract (moveRight c) == moveRightA (toAbstract c)
prop_insChar_r x c =
  toAbstract (insertChar x c)
  == insertCharA x (toAbstract c)
prop_delChar_r c =
  toAbstract (deleteChar c) == deleteCharA (toAbstract c)
```