

COMP3141

Software System Design and Implementation

More on the Curry Howard Isomorphism

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What is Intuitionistic Logic?

- Classical logic is the logic that most people know about.
- Intuitionistic logic does not contain the axiom of excluded middle $p \vee \neg p$ or equivalently $\neg\neg p \rightarrow p$.
- In classical logic more can be proven but less can be expressed.
- Intuitionistic proof of an existence statement gives a witness for the statement.

Example of Existence in the Classical Sense

- Let \mathbb{Q} be the set of rational numbers and \mathbb{I} be the set of irrational numbers.
- Consider the statement $\exists x, y. (x \in \mathbb{I}) \wedge (y \in \mathbb{I}) \wedge (x^y \in \mathbb{Q})$.
- Proof:
 - Consider the number $\sqrt{2}^{\sqrt{2}}$.
 - ① If $\sqrt{2}^{\sqrt{2}} \in \mathbb{Q}$
 -
 -
 - ② Otherwise if $\sqrt{2}^{\sqrt{2}} \in \mathbb{I}$
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 - Pick $x = \sqrt{2}$ and $y = \sqrt{2}$
 - Then $x^y = \sqrt{2}^{\sqrt{2}}$ so $x^y \in \mathbb{Q}$
 - ② Otherwise if $\sqrt{2}^{\sqrt{2}} \in \mathbb{I}$
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 - ② Otherwise if $\sqrt{2}^{\sqrt{2}} \in \mathbb{I}$
 - Pick $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$
 - Then $x^y = (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = 2$ so $x^y \in \mathbb{Q}$

Recall: The Curry-Howard Isomorphism

This correspondence goes by many names, but is usually attributed to **Haskell Curry** and **William Howard**.

It is a *very deep* result:

Logic	Programming
Propositions	Types
Proofs	Programs
Proof Simplification	Evaluation

It turns out, no matter what logic you want to define, there is always a corresponding λ -calculus, and vice versa.

Constructive Logic	Typed λ -Calculus
Classical Logic	Continuations
Modal Logic	Monads
Linear Logic	Linear Types, Session Types
Separation Logic	Region Types

Translating

We can translate logical connectives to types and back:

Conjunction (\wedge)	Tuples
Disjunction (\vee)	Either
Implication	Functions
True	()
False	Void

We can also translate our *equational reasoning* on programs into *proof simplification* on proofs!

Examples

```
prop_or_false :: a -> (Either a void)
prop_or_false a = Left a
```

```
prop_or_true :: a -> (Either a ())
prop_or_true a = Right ()
```

```
prop_and_true :: a -> (a, ())
prop_and_true a = (a, ())
```

```
prop_double_neg_intro :: a -> (a -> void) -> void
prop_double_neg_intro a f = f a
```

```
prop_triple_neg_elim ::
  (((a -> void) -> void) -> void) -> a -> void
prop_triple_neg_elim f a = f (\g -> g a)
```


Wrap-up

- ➊ Assignment 2 is due in just over a week.
- ➋ There is a quiz for this week, but no exercise.
- ➌ Next week's lectures consist of a **guest lecture** on Tuesday, from **Dr. Hira Taqdees Syeda** of the Trustworthy Systems group at data61 (CSIRO), and a **revision lecture** on Wednesday.
- ➍ Come up with **questions** to ask us for the revision lecture! It will be over very quickly otherwise.
- ➎ If you enjoyed the course and want to do more in this direction, ask us for thesis topics, taste of research projects, and consider attending COMP3161 and COMP4161.
- ➏ Fill in the myExperience reports, it is important for us to receive your feedback.