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Outline

Definiciones básicas

[

Producto tensorial] Dado $A \in \mathbb{C}^{m\times n}$, $B \in \mathbb{C}^{p\times q}$, el producto tensorial $A \otimes B$ es la matriz $D \in \mathbb{C}^{pm\times nq}$ tal que:

$$D := A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ a_{21}B & \cdots & a_{2n}B \\ \vdots & & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}$$

$$x \otimes y = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Inicialización

El primer paso es el de *inicializar* nuestro sistema. Para ello se hace uso del principio de superposicíon, es decir, en lugar de buscar en un único lugar, buscamos en varios al mismo tiempo.

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$$
\begin{aligned}
(I^{\otimes n} \otimes X) |0\rangle_{n+1} &= |0\rangle_n \otimes |1\rangle \\
H^{\otimes(n+1)} \left[(I^{\otimes n} \otimes X) |0\rangle_{n+1}\right] &= H^{\otimes n} |0\rangle_n \otimes H |1\rangle \\
&= \sum_{j\in\{0,1\}^n} \frac{1}{\sqrt{2^n}} |j\rangle_n \otimes \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \\
&= \sum_{j\in\{0,1\}^n} \alpha_j |j\rangle_n \otimes \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \\
&= \psi_n \times \psi_1 = \psi
\end{aligned}
$$

Definiciones básicas

Inicialización

$$
\begin{aligned}
(I^{\otimes 3} \otimes X)\,|0\rangle_{3+1} &= I^{\otimes 3}\,|0\rangle_3 \otimes X\,|0\rangle && (1)\\
&= \begin{pmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} && (2)\\
&= |0\rangle \otimes |1\rangle && (3)
\end{aligned}
$$

$$
\begin{aligned}
& H^{\otimes(3+1)}\left[\left(I^{\otimes 3} \otimes X\right)|0\rangle_{n+1}\right] && (4)\\
= \; & H^{\otimes 3}|0\rangle \otimes H|1\rangle && (5)\\
= \; & \frac{1}{\sqrt{2^3}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} && (6)\\
= \; & \frac{1}{\sqrt{2^3}}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} && (7)\\
& && (8)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{2^3}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} && (9) \\
&= [\frac{1}{2\sqrt{2}} |000\rangle + \frac{1}{2\sqrt{2}} |001\rangle + \frac{1}{2\sqrt{2}} |010\rangle + \frac{1}{2\sqrt{2}} |011\rangle && (10) \\
&+ \frac{1}{2\sqrt{2}} |100\rangle + \frac{1}{2\sqrt{2}} |101\rangle + \frac{1}{2\sqrt{2}} |110\rangle + \frac{1}{2\sqrt{2}} |111\rangle] && (11) \\
&\otimes \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) && (12)
\end{aligned}
$$

$$
\begin{aligned}
& H^{\otimes(3+1)}\left[\left(I^{\otimes 3} \otimes X\right) |0\rangle_{n+1}\right] \\
= & \left[\frac{1}{2\sqrt{2}}|000\rangle + \frac{1}{2\sqrt{2}}|001\rangle + \frac{1}{2\sqrt{2}}|010\rangle + \frac{1}{2\sqrt{2}}|011\rangle \frac{1}{2\sqrt{2}}|100\rangle + \frac{1}{2\sqrt{2}}|101\rangle + \frac{1}{2\sqrt{2}}|110\rangle\right. \\
\otimes & \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \\
= & \sum_{j\in\{0,1\}^n} \frac{1}{\sqrt{2^n}} |j\rangle_n \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \psi_n \times \psi_1
\end{aligned}
$$

Cambio de signo

Quantum Oracle

Se define el operador U_f tal que

$$U_f : |j\rangle_n \otimes |y\rangle_1 \to |j\rangle_n \otimes |y \oplus f(j)\rangle_1$$

, donde $\oplus$ es el operador suma en modulo 2, y $f(j) = \left\{ \begin{array}{ll} 1 & j = l \\ 0 & j \neq l \end{array} \right\}$

A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

$$
\begin{aligned}
U_f\psi &= U_f\left(\sum_{j\in\{0,1\}^n} \alpha_j \, |j\rangle_n \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\right) \\
&= U_f\left(\alpha_l \, |l\rangle_n \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) + \sum_{\substack{j\in\{0,1\}^n \\ j\neq l}} \alpha_j \, |j\rangle_n \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\right) \\
&= \left(-\alpha_l \, |l\rangle_n + \sum_{\substack{j\in\{0,1\}^n \\ j\neq l}} \alpha_j \, |j\rangle_n\right) \otimes -\tfrac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)
\end{aligned}
$$

$$
\begin{aligned}
U_f\psi &= U_f\left(\sum_{j\in\{0,1\}^n} \alpha_j \,|j\rangle_n \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\right) \\
&= U_f\left(\alpha_l \,|l\rangle_n \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) + \sum_{\substack{j\in\{0,1\}^n \\ j\neq l}} \alpha_j \,|j\rangle_n \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\right) \\
&= \left(-\alpha_l \,|l\rangle_n + \sum_{\substack{j\in\{0,1\}^n \\ j\neq l}} \alpha_j \,|j\rangle_n\right) \otimes -\tfrac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)
\end{aligned}
$$

Amplitud cambiada de signo!

$$
\begin{aligned}
U_f\psi &= U_f\left(\sum_{j\in\{0,1\}^n} \alpha_j \left|j\right\rangle_n \otimes \frac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right)\right) \\
&= U_f\left(\alpha_l \left|l\right\rangle_n \otimes \frac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right) + \sum_{\substack{j\in\{0,1\}^n \\ j\neq l}} \alpha_j \left|j\right\rangle_n \otimes \frac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right)\right) \\
&= \left(-\alpha_l \left|l\right\rangle_n + \sum_{\substack{j\in\{0,1\}^n \\ j\neq l}} \alpha_j \left|j\right\rangle_n\right) \otimes \boxed{-\tfrac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right)}
\end{aligned}
$$

Amplitud cambiada de signo!

Extra qubit intacto!