tbd

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Outline

1. Implementation of Grover's algorithm
   - Implementation of 2-Qubit States

Definiciones básicas

[

Producto tensorial] Dado $A \in \mathbb{C}^{m\times n}$, $B \in \mathbb{C}^{p\times q}$, el producto tensorial $A \otimes B$ es la matriz $D \in \mathbb{C}^{pm\times nq}$ tal que:

$$D := A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ a_{21}B & \cdots & a_{2n}B \\ \vdots & & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}$$

$$x \otimes y = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Inicialización

Buscamos una forma (operador unitario) de que todos los estados sean equiprobables. Se explota el principio de superposición

Inicialización

Buscamos una forma (operador unitario) de que todos los estados sean equiprobables. Se explota el principio de superposición

$$
\begin{aligned}
(I^{\otimes n} \otimes X) \left|0\right\rangle_{n+1} &= \left|0\right\rangle_n \otimes \left|1\right\rangle \\
H^{\otimes (n+1)} \left[(I^{\otimes n} \otimes X) \left|0\right\rangle_{n+1}\right] &= H^{\otimes n} \left|0\right\rangle_n \otimes H \left|1\right\rangle \\
&= \sum_{j \in \{0,1\}^n} \frac{1}{\sqrt{2^n}} \left|j\right\rangle_n \otimes \frac{1}{\sqrt{2}} \left(\left|0\right\rangle - \left|1\right\rangle\right) \\
&= \sum_{j \in \{0,1\}^n} \alpha_j \left|j\right\rangle_n \otimes \frac{1}{\sqrt{2}} \left(\left|0\right\rangle - \left|1\right\rangle\right) \\
&= \psi_n \times \psi_1 = \psi
\end{aligned}
$$

Definiciones básicas

Inicialización

$$
\begin{aligned}
(I^{\otimes 3} \otimes X)\,|0\rangle_{3+1} &= I^{\otimes 3}\,|0\rangle_3 \otimes X\,|0\rangle && (1)\\
&= \begin{pmatrix} 1 &&&&&&& \\ & 1 &&&&&& \\ && 1 &&&&& \\ &&& 1 &&&& \\ &&&& 1 &&& \\ &&&&& 1 && \\ &&&&&& 1 & \\ &&&&&&& 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} && (2)\\
&= |0\rangle \otimes |1\rangle && (3)
\end{aligned}
$$

$$
\begin{aligned}
& H^{\otimes(3+1)}\left[\left(I^{\otimes 3}\otimes X\right)|0\rangle_{n+1}\right] && (4)\\
= \; & H^{\otimes 3}|0\rangle \otimes H|1\rangle && (5)\\
= \; & \frac{1}{\sqrt{2^3}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} && (6)\\
= \; & \frac{1}{\sqrt{2^3}}\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1
\end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} && (7)\\
& && (8)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{2^3}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} && (9) \\
&= [\frac{1}{2\sqrt{2}} \left|000\right\rangle + \frac{1}{2\sqrt{2}} \left|001\right\rangle + \frac{1}{2\sqrt{2}} \left|010\right\rangle + \frac{1}{2\sqrt{2}} \left|011\right\rangle && (10) \\
&+ \frac{1}{2\sqrt{2}} \left|100\right\rangle + \frac{1}{2\sqrt{2}} \left|101\right\rangle + \frac{1}{2\sqrt{2}} \left|110\right\rangle + \frac{1}{2\sqrt{2}} \left|111\right\rangle] && (11) \\
&\otimes \frac{1}{\sqrt{2}} (\left|0\right\rangle - \left|1\right\rangle) && (12)
\end{aligned}
$$

$$
\begin{aligned}
H^{\otimes(3+1)}\left[(I^{\otimes 3}\otimes X)\,|0\rangle_{n+1}\right] &= [\frac{1}{2\sqrt{2}}\,|000\rangle + \frac{1}{2\sqrt{2}}\,|001\rangle + \frac{1}{2\sqrt{2}}\,|010\rangle + \frac{1}{2\sqrt{2}}\,|011\rangle \\
&+ \frac{1}{2\sqrt{2}}\,|100\rangle + \frac{1}{2\sqrt{2}}\,|101\rangle + \frac{1}{2\sqrt{2}}\,|110\rangle + \frac{1}{2\sqrt{2}}\,|111\rangle] \\
&\otimes \frac{1}{\sqrt{2}}(\,|0\rangle - |1\rangle) \\
&= \sum_{j\in\{0,1\}^n} \frac{1}{\sqrt{2^n}}\,|j\rangle_n \otimes \frac{1}{\sqrt{2}}(\,|0\rangle - |1\rangle) = \psi_n \times \psi_1 \qquad (13)
\end{aligned}
$$

Cambio de signo

Buscamos una forma (operador unitario), que nos permita cambiar el signo de las amplitud de interés. OJO: AÑADIR GRAFICO QUE pase de inicialización a cambio de signo gracias a U_f

Cambio de signo

Quantum Oracle

Se define el operador U_f tal que

$$U_f : |j\rangle_n \otimes |y\rangle_1 \rightarrow |j\rangle_n \otimes |y \oplus f(j)\rangle_1$$

, donde $\oplus$ es el operador suma en modulo 2, y $f(j) = \left\{ \begin{array}{ll} 1 & j = l \\ 0 & j \neq l \end{array} \right\}$

A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

Cambio de signo

Aplicamos el operador U_f (*Quantum Oracle*) a nuestro estado ψ

$$\begin{aligned} U_f\psi &= U_f\left(\sum_{j\in\{0,1\}^n} \alpha_j \left|j\right\rangle_n \otimes \frac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right)\right) \\ &= U_f\left(\alpha_l \left|l\right\rangle_n \otimes \frac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right) + \sum_{j\in\{0,1\}^n;\, j\neq l} \alpha_j \left|j\right\rangle_n \otimes \frac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right)\right) \\ &= \left(-\alpha_l \left|l\right\rangle_n + \sum_{j\in\{0,1\}^n;\, j\neq l} \alpha_j \left|j\right\rangle_n\right) \otimes \tfrac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right) \end{aligned} \tag{14}$$

Cambio de signo

Aplicamos el operador U_f (*Quantum Oracle*) a nuestro estado ψ

$$U_f\psi = U_f\left(\sum_{j\in\{0,1\}^n} \alpha_j |j\rangle_n \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right)$$

Amplitud cambiada de signo!

$$= U_f\left(\alpha_l |l\rangle_n \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) + \sum_{j\in\{0,1\}^n;\, j\neq l} \alpha_j |j\rangle_n \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right)$$

$$= \left(-\alpha_l |l\rangle_n + \sum_{j\in\{0,1\}^n;\, j\neq l} \alpha_j |j\rangle_n\right) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \tag{14}$$

Cambio de signo

Aplicamos el operador U_f (*Quantum Oracle*) a nuestro estado ψ

Amplitud cambiada de signo!

Extra qubit!

$$
\begin{aligned}
U_f\psi &= U_f\left(\sum_{j\in\{0,1\}^n} \alpha_j \left|j\right\rangle_n \otimes \frac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right)\right) \\
&= U_f\left(\alpha_l \left|l\right\rangle_n \otimes \frac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right) + \sum_{j\in\{0,1\}^n;\ j\neq l} \alpha_j \left|j\right\rangle_n \otimes \frac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right)\right) \\
&= \left(\boxed{-\alpha_l \left|l\right\rangle_n} + \sum_{j\in\{0,1\}^n;\ j\neq l} \alpha_j \left|j\right\rangle_n\right) \otimes \boxed{\frac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right)}
\end{aligned}
\tag{14}
$$

Quantum Oracle

$$
\begin{aligned}
U_f \psi &= [\frac{1}{2\sqrt{2}}\,|000\rangle + \frac{1}{2\sqrt{2}}\,|001\rangle + \frac{1}{2\sqrt{2}}\,|010\rangle - \frac{1}{2\sqrt{2}}\,|011\rangle \\
&+ \frac{1}{2\sqrt{2}}\,|100\rangle + \frac{1}{2\sqrt{2}}\,|101\rangle + \frac{1}{2\sqrt{2}}\,|110\rangle + \frac{1}{2\sqrt{2}}\,|111\rangle] \\
&\otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \\
&= \sum_{j\in\{0,1\}^n} \frac{1}{\sqrt{2^n}}\,|j\rangle_n \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \psi_n \times \psi_1
\end{aligned} \tag{15}
$$

Inversión con con respecto la media

Buscamos una forma (operador unitario) de invertir el valor de la amplitud con respecto la media.

$$\sum_{j\in\{0,1\}^n} \alpha_j \left|j\right\rangle_n \xrightarrow{U_d} \sum_{j\in\{0,1\}^n} \left(2\left(\sum_{k\in\{0,1\}^n} \frac{\alpha_k}{2^n}\right) - \alpha_j\right) \left|j\right\rangle_n ,$$

donde $\sum_{k\in\{0,1\}^n} \frac{\alpha_k}{2^n}$ es el valor medio.

$$\sum_{k\in\{0,1\}^n} \frac{\alpha_k}{2^n} = \frac{1}{2^3}\frac{6}{2\sqrt{2}}$$

Inversión con respecto la media

Difussion operator

$$U_d = \begin{pmatrix} \frac{2}{2^n} & \frac{2}{2^n} & \cdots & \frac{2}{2^n} \\ \frac{2}{2^n} & \frac{2}{2^n} & \cdots & \frac{2}{2^n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{2}{2^n} & \frac{2}{2^n} & \cdots & \frac{2}{2^n} \end{pmatrix} - I^{\otimes n} = \cdots = -H^{\otimes n} D H^{\otimes n}, \tag{16}$$

donde $D = diag\left(-1, 1, 1, \cdots, 1\right)$

Inversion con respecto la media

Difussion operator in python

```
>>> import numpy as np
>>> H1 = 1/np.sqrt(2)*np.array([[1,1],[1, -1]]) # Hadamard operator 1 qubit
>>> H2 = np.kron(H1,H1) # Hadamard operator 2 qubit
>>> H3 = np.kron(H2,H1) # Hadamard operator 3 qubit

>>> D = np.eye(8) # Diagonal operator
>>> D[0,0] = -1

>>> Ud = -np.dot(np.dot(H3,D),H3) # Difussion operator
>>> psi = 1/(2*np.sqrt(2))*np.array([1,1,1,-1,1,1,1,1]) # Initial state

>>> psi_2 = np.dot(Uw,psi)
>>> print(psi_2)
array([0.176 , 0.176 , 0.176 , 0.883, 0.176, 0.176, 0.176, 0.176])
```

Inversión con respecto la media

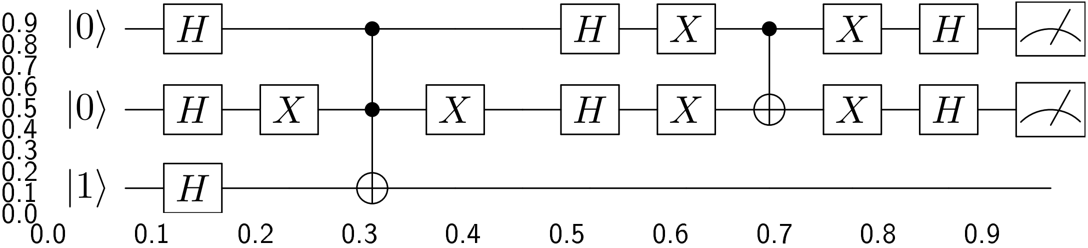
Difussion operator

$$U_d = -\frac{1}{8}\frac{1}{2\sqrt{2}}\begin{pmatrix} 6 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & 6 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & 6 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & 6 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & 6 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & 6 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & 6 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & 6 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{4\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 5 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

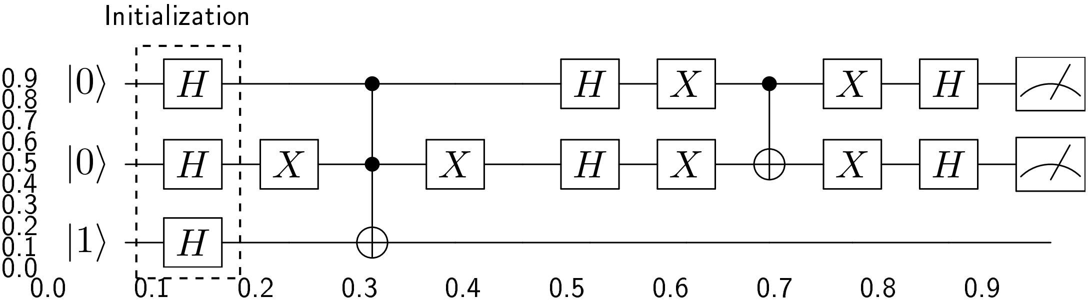
Outline

1. Implementation of Grover's algorithm
   - Implementation of 2-Qubit States

2-qubit


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$|0\rangle$
$|0\rangle$
$|1\rangle$
H
H
H
X
X
H
X
X
H
X
X
H
H
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

2-qubit


Initialization
|0⟩
|0⟩
|1⟩
H
H
H
X
X
H
H
X
X
X
X
H
H
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9