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Outline

1. Introduction to Quantum Computing
2. Basic definitions
3. Grover's algorithm
   - Implementation of Grover's algorithm: 2-Qubit States
   - Implementation of Grover's algorithm: 3-Qubit States
4. Conclusion

Outline

1. Introduction to Quantum Computing
   - Implementation of Grover's algorithm: 2-Qubit States
   - Implementation of Grover's algorithm: 3-Qubit States

Outline

Definiciones básicas

[

Producto tensorial] Dado $A \in \mathbb{C}^{m\times n}$, $B \in \mathbb{C}^{p\times q}$, el producto tensorial $A \otimes B$ es la matriz $D \in \mathbb{C}^{pm\times nq}$ tal que:

$$D := A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ a_{21}B & \cdots & a_{2n}B \\ \vdots & & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}$$

$$x \otimes y = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Outline

3. Grover's algorithm
   - Implementation of Grover's algorithm: 2-Qubit States
   - Implementation of Grover's algorithm: 3-Qubit States

Initialization

We find a method (unitary operator) to have all the states with the same probability (*principle of superposition*).

Initialization

We find a method (unitary operator) to have all the states with the same probability (*principle of superposition*).

$$
\begin{aligned}
\left(I^{\otimes n} \otimes X\right)|0\rangle_{n+1} &= |0\rangle_n \otimes |1\rangle \\
H^{\otimes(n+1)}\left[\left(I^{\otimes n} \otimes X\right)|0\rangle_{n+1}\right] &= H^{\otimes n}|0\rangle_n \otimes H|1\rangle \\
&= \sum_{j\in\{0,1\}^n} \frac{1}{\sqrt{2^n}}|j\rangle_n \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \\
&= \sum_{j\in\{0,1\}^n} \alpha_j |j\rangle_n \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \\
&= \psi^{[1]}
\end{aligned}
$$

Initialization

3-qubit example

$$
\begin{aligned}
(I^{\otimes 3} \otimes X)\,|0\rangle_{3+1} &= I^{\otimes 3}\,|0\rangle_3 \otimes X\,|0\rangle \\
&= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
&= |0\rangle_3 \otimes |1\rangle
\end{aligned}
$$

Initialization

$$\begin{aligned}
\psi^{[1]} &= H^{\otimes(3+1)}\left[\left(I^{\otimes 3}\otimes X\right)|0\rangle_{n+1}\right] \\
&= H^{\otimes 3}|0\rangle \otimes H|1\rangle = \frac{1}{\sqrt{2^3}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
&= \frac{1}{\sqrt{2^3}}\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1
\end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}
\end{aligned}$$

Initialization

3-qubit example

$$
\begin{aligned}
&= \frac{1}{\sqrt{2^3}}\left(11111111\right)^{\dagger} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \\
&= [\frac{1}{2\sqrt{2}}\,|000\rangle + \frac{1}{2\sqrt{2}}\,|001\rangle + \frac{1}{2\sqrt{2}}\,|010\rangle + \frac{1}{2\sqrt{2}}\,|011\rangle \\
&+ \frac{1}{2\sqrt{2}}\,|100\rangle + \frac{1}{2\sqrt{2}}\,|101\rangle + \frac{1}{2\sqrt{2}}\,|110\rangle + \frac{1}{2\sqrt{2}}\,|111\rangle] \\
&\otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)
\end{aligned}
$$

Initialization

3-qubit example

$$
\begin{aligned}
\psi^{[1]} &= H^{\otimes(3+1)}\left[\left(I^{\otimes 3}\otimes X\right)|0\rangle_{n+1}\right] \\
&= [\frac{1}{2\sqrt{2}}\,|000\rangle + \frac{1}{2\sqrt{2}}\,|001\rangle + \frac{1}{2\sqrt{2}}\,|010\rangle + \frac{1}{2\sqrt{2}}\,|011\rangle \\
&+ \frac{1}{2\sqrt{2}}\,|100\rangle + \frac{1}{2\sqrt{2}}\,|101\rangle + \frac{1}{2\sqrt{2}}\,|110\rangle + \frac{1}{2\sqrt{2}}\,|111\rangle] \\
&\otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \\
&= \sum_{j\in\{0,1\}^n} \frac{1}{\sqrt{2^n}}\,|j\rangle_n \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = \psi^{[1]}
\end{aligned}
$$

Sign flip

We find a method (unitary operator) which flip the sign of the state of interest. OJO: AÑADIR GRAFICO QUE pase de inicialización a cambio de signo gracias a U_f

Sign flip

Quantum Oracle

It is defined the operator U_f:

$$U_f : |j\rangle_n \otimes |y\rangle_1 \rightarrow |j\rangle_n \otimes |y \oplus f(j)\rangle_1 ,$$

where $\oplus$ is the sum operator in mod 2, and $f(j) = \left\{ \begin{array}{cc} 1 & j = l \\ 0 & j \neq l \end{array} \right\}$

A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

Sign flip

We apply U_f (*Quantum Oracle*) to the previous state $\psi^{[1]}$

$$\begin{aligned}
\psi^{[2]} &= U_f \psi^{[1]} \\
&= U_f \left(\sum_{j \in \{0,1\}^n} \alpha_j \, |j\rangle_n \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) \right) \\
&= U_f \left(\alpha_l \, |l\rangle_n \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) + \sum_{j \in \{0,1\}^n;\, j \neq l} \alpha_j \, |j\rangle_n \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) \right) \\
&= \left(-\alpha_l \, |l\rangle_n + \sum_{j \in \{0,1\}^n;\, j \neq l} \alpha_j \, |j\rangle_n \right) \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) \qquad (1)
\end{aligned}$$

Sign flip

We apply U_f (*Quantum Oracle*) to the previous state $\psi^{[1]}$

$$\begin{aligned}
\psi^{[2]} &= U_f \psi^{[1]} \\
&= U_f \left(\sum \alpha_j \, |j\rangle_n \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) \right) \\
&= U_f \left(\alpha_l \, |l\rangle_n \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) + \sum_{j \in \{0,1\}^n;\, j \neq l} \alpha_j \, |j\rangle_n \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) \right) \\
&= \left(-\alpha_l \, |l\rangle_n + \sum_{j \in \{0,1\}^n;\, j \neq l} \alpha_j \, |j\rangle_n \right) \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) \qquad (1)
\end{aligned}$$

Amplitud cambiada de signo!

Sign flip

We apply U_f (*Quantum Oracle*) to the previous state $\psi^{[1]}$

$$
\begin{aligned}
\psi^{[2]} &= U_f \psi^{[1]} \\
&= U_f \left(\sum \alpha_j |j\rangle_n \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) \right) \\
&= U_f \left(\alpha_l |l\rangle_n \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) + \sum_{j \in \{0,1\}^n;\, j \neq l} \alpha_j |j\rangle_n \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) \right) \\
&= \left(\boxed{-\alpha_l |l\rangle_n} + \sum_{j \in \{0,1\}^n;\, j \neq l} \alpha_j |j\rangle_n \right) \otimes \boxed{\frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right)}
\end{aligned}
\tag{1}
$$

Amplitud cambiada de signo!

Extra qubit!

Sign flip[1]

Quantum Oracle

$$
\begin{aligned}
\psi^{[2]} &= U_f \psi^{[1]} \\
&= [\frac{1}{2\sqrt{2}}\,|000\rangle + \frac{1}{2\sqrt{2}}\,|001\rangle + \frac{1}{2\sqrt{2}}\,|010\rangle - \frac{1}{2\sqrt{2}}\,|011\rangle \\
&+ \frac{1}{2\sqrt{2}}\,|100\rangle + \frac{1}{2\sqrt{2}}\,|101\rangle + \frac{1}{2\sqrt{2}}\,|110\rangle + \frac{1}{2\sqrt{2}}\,|111\rangle] \\
&\otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \\
&= \sum_{j\in\{0,1\}^n} \frac{1}{\sqrt{2^n}}\,|j\rangle_n \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)
\end{aligned}
$$

[1]This slide shows the result of applying U_f but not how is applied. This is because this step of the algorithm depends on the specific problem.

Inversion about the average

Buscamos una forma (operador unitario) de invertir el valor de la amplitud con respecto la media.

$$\sum_{j\in\{0,1\}^n} \alpha_j \left|j\right\rangle_n \xrightarrow{U_d} \sum_{j\in\{0,1\}^n} \left(2\left(\sum_{k\in\{0,1\}^n} \frac{\alpha_k}{2^n}\right) - \alpha_j\right) \left|j\right\rangle_n ,$$

donde $\sum_{k\in\{0,1\}^n} \frac{\alpha_k}{2^n}$ es el valor medio.

$$\sum_{k\in\{0,1\}^n} \frac{\alpha_k}{2^n} = \frac{1}{2^3}\frac{6}{2\sqrt{2}}$$

Inversion about the average

Difussion operator

$$U_d = \begin{pmatrix} \frac{2}{2^n} & \frac{2}{2^n} & \cdots & \frac{2}{2^n} \\ \frac{2}{2^n} & \frac{2}{2^n} & \cdots & \frac{2}{2^n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{2}{2^n} & \frac{2}{2^n} & \cdots & \frac{2}{2^n} \end{pmatrix} - I^{\otimes n} = \cdots = -H^{\otimes n} D H^{\otimes n}, \qquad (2)$$

donde $D = diag\left(-1, 1, 1, \cdots, 1\right)$

Inversion about the average

Difussion operator in python

```
>>> import numpy as np
>>> H1 = 1/np.sqrt(2)*np.array([[1,1],[1, -1]]) # Hadamard operator 1 qubit
>>> H2 = np.kron(H1,H1) # Hadamard operator 2 qubit
>>> H3 = np.kron(H2,H1) # Hadamard operator 3 qubit

>>> D = np.eye(8) # Diagonal operator
>>> D[0,0] = -1

>>> Ud = -np.dot(np.dot(H3,D),H3) # Difussion operator
>>> psi_2 = 1/(2*np.sqrt(2))*np.array([1,1,1,-1,1,1,1,1]) # Initial state

>>> psi_3 = np.dot(Uw,psi)
>>> print(psi_3)
array([0.176 , 0.176 , 0.176 , 0.883, 0.176, 0.176, 0.176, 0.176])
```

Inversion about the average

Difussion operator

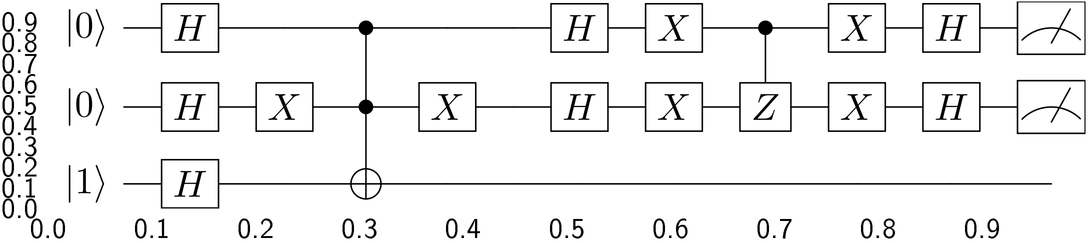
$$U_d = -\frac{1}{8}\frac{1}{2\sqrt{2}}\begin{pmatrix} 6 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & 6 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & 6 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & 6 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & 6 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & 6 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & 6 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & 6 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{4\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 5 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Outline

- Implementation of Grover's algorithm: 2-Qubit States
- Implementation of Grover's algorithm: 3-Qubit States

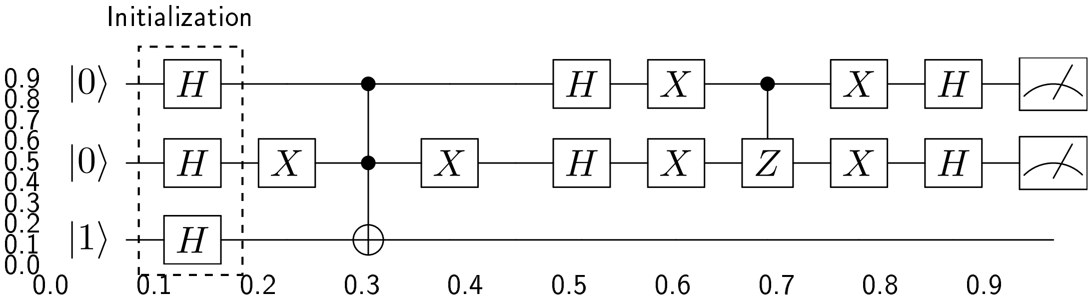
4. Conclusion

2-Qubit Quantum Circuit



2-Qubit Quantum Circuit

Initialization



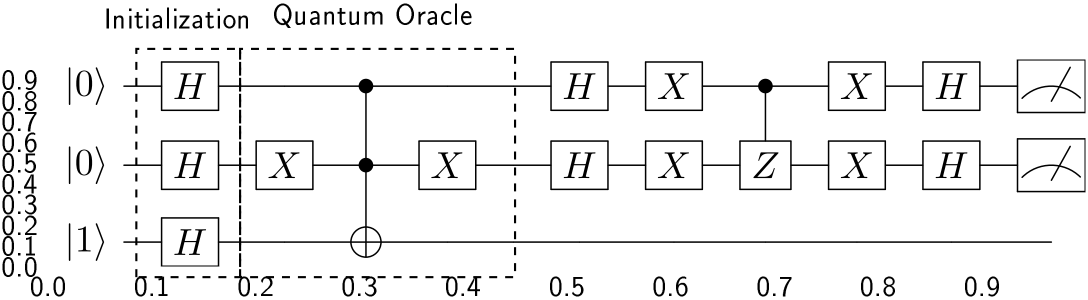
2-Qubit Quantum Circuit

Initialization

$$
\begin{aligned}
\psi^{[0]} &= |001\rangle \\
\psi^{[1]} &= H^{\otimes 3}\psi^{[0]} \\
&= \frac{1}{\sqrt{8}}\left(|000\rangle + |010\rangle + |100\rangle + |110\rangle - |001\rangle - |011\rangle - |101\rangle - |111\rangle\right)
\end{aligned}
$$

2-Qubit Quantum Circuit

Quantum Oracle



2-Qubit Quantum Circuit

Quantum Oracle

$$U_f = (I \otimes X \otimes I) \cdot T \cdot (I \otimes X \otimes I) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

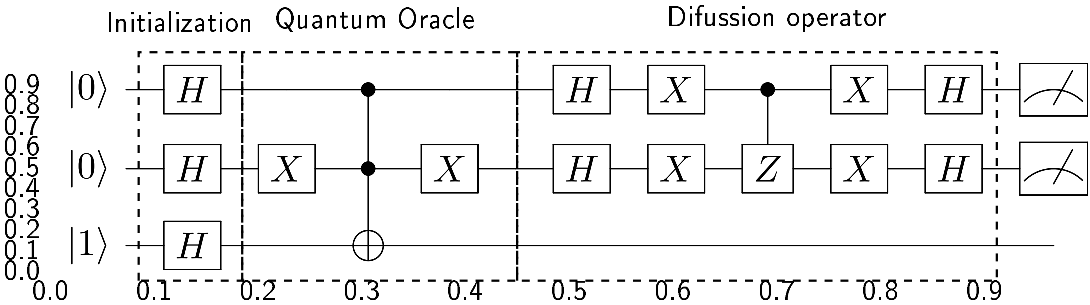
2-Qubit Quantum Circuit

Quantum Oracle

$$
\begin{aligned}
\psi^{[1]} &= \frac{1}{\sqrt{8}}\left(|000\rangle + |010\rangle + |100\rangle + |110\rangle - |001\rangle - |011\rangle - |101\rangle - |111\rangle\right) \\
\psi^{[2]} &= U_f \, |\psi^{[1]}\rangle \\
&= \frac{1}{\sqrt{8}}\left(|000\rangle + |010\rangle + |101\rangle + |110\rangle - |001\rangle - |011\rangle - |100\rangle - |111\rangle\right) \\
&= \frac{1}{\sqrt{8}}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right) \otimes \left(|0\rangle - |1\rangle\right) \\
&= \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle + |11\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)
\end{aligned}
$$

2-Qubit Quantum Circuit

Difussion Operator



2-Qubit Quantum Circuit

Diffusion Operator

$$U_d \quad = \quad (H \otimes H) \cdot (X \otimes X) \cdot (CZ)\,(X \otimes X) \cdot (H \otimes H) = \frac{1}{4}\begin{pmatrix} 2 & -2 & -2 & -2 \\ -2 & 2 & -2 & -2 \\ -2 & -2 & 2 & -2 \\ -2 & -2 & -2 & 2 \end{pmatrix}$$

$$\psi^{[3]} = U_d \psi^{[2]}$$

$$\frac{1}{8}\begin{pmatrix} 2 & -2 & -2 & -2 \\ -2 & 2 & -2 & -2 \\ -2 & -2 & 2 & -2 \\ -2 & -2 & -2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 2 & -2 & 2 & -2 \\ -2 & 2 & 2 & -2 \\ -2 & -2 & -2 & -2 \\ -2 & -2 & 2 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} = -\,|10\rangle$$