Determining the Best Overtime System for the NFL Through the Use of Markov Chains

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1 Introduction

For many people in the United States, 14.9 million people to be exact (Statista), Sundays in the fall consist of gathering around the television, and watching their favorite National Football League team play. Even though football is one of, if not the most popular sport in the United States today, it is interesting to see that there are still many fundamental rules in the game that are subject to heavy criticism.

Overtime is one of these basic parts of the game has always been at issue. In the NFL, many have called for changes to the overtime system, as people have noticed that the winner of the coin toss at the start of overtime seems to win the game much more often than the other team. This is due to the rule which states that first touchdown scored in overtime wins the game (which gives one team the chance to win the game before the other team gets to posses the ball). NCAA college football, on the other hand, runs an overtime system that has been praised much more than the NFL's overtime system. It is believed to be much more fair to both teams, as each team gets a chance to possess the ball.

Since there is so much debate about which overtime system is better, this paper will determine if the NCAA overtime system is in fact a more fair way to determine the winner of a football game than the NFL overtime system. To do this, a period of each overtime system will be modeled through the use of Markov chains, a modeling technique which combines linear algebra and probability to determine the probability that any given sequence of events occurs.

2 Markov Chains

2.1 General Overview

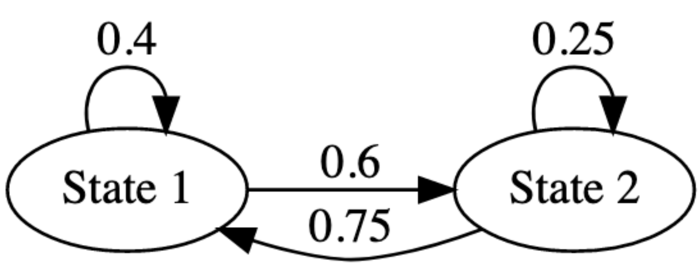

Figure 1: A sample Markov chain

A Markov chain is a random process that has a discrete amount of time, a discrete state space, and maps out all of the possible transition between states in this state space. Observe Figure 1, which is a sample Markov chain. The state space of Figure 1 consists of State 1 and State 2. Note that each arrow stemming from each state represents the probability of the current state transitioning into that state. For example, if the current state (X_n) is State 1, then the probability of (X_n) progressing to State 2 from State 1 is 0.6, and the probability of State 1 remaining at State 1 is 0.4

What makes Markov chains so unique is that they follow the Markov Property. The Markov property implies that $P(X_{n+1} = S_{n+1}|X_n = S_n, X_{n-1} = S_{n-1}, X_{n-2} = S_{n-2}, ...) = P(X_{n+1} = Sn + 1|X_n = S_n)$ (with S_n being a state in the state space). Essentially, this means that the probability distribution for each state does not depend on any past states, but only on current states.

The Markov property is what allows Markov chains to be considered memoryless, meaning that past events have no impact on any future events. The combination of this fact, and the fact that Markov chain calculate the probability of any state in the state space occurring is what makes it the best model to use for football. Even though it may not be a completely perfect model (as it is very hard to account for factors such as weather, player caliber, and injuries), it still provides a way to take all possible scenarios of a football game, and then calculate the probability any of these events occurs through the use of a transition matrix.

2.2 Transition Matrices

A transition matrix is what allows for the probability of any event, given that another event happened before it, to be calculated. Simply put, a transition matrix is a matrix that gives the probabilities that any state in the state space succeeds to another state in the state space. Finding the probability that a state e_j, succeeds another state, e_i, in m steps can be found by finding the entry in row i and column j of transition matrix p^m. Mathematically this can be written as: $(p^m)_{i,j} = P(X_{n+m} = e_j|X_n = e_i)$ (Rocca).

To illustrate the transition matrix concept, observe Figure 1. To create the sample transition matrix S from Figure 1, let each row in S represent an a state X_i, and each column in S also represent a state X_i (for i in the cardinality of the state space). Then fill in the body of the matrix with the probability of each state transitioning from the row state to the column state. It follows that S equals:

$$\begin{pmatrix} 0.4 & 0.6 \\ 0.75 & 0.25 \end{pmatrix}$$

Note that the sum of each row is equal to one, satisfying the law of total probability. In order to find the probability of State 1 transitioning to State 2 ($P(X_2|X_1)$) in one step, we would find $(S^1)_{1,2} = 0.2$.

3 NFL Overtime System Model

3.1 Rules

When an NFL game ends regulation in a tie, the game will head to a single fifteen minute overtime period, and will be declared a tie if no winner is found at the end of the fifteen minutes (unless the game is a playoff game, in which overtime periods will continue to be played until there is a winner). The overtime period will begin with a kickoff, with the receiving team determined by a coin toss. The first touchdown scored will end the game, even if only one team has possessed the ball (which gives the receiving team a chance to win on their opening dive by scoring a touchdown). If the first score is a field goal, then the other team will have a chance to possess the ball and tie the game (or win with a touchdown).

3.2 Assumptions, Given Probabilities, and State Space

Two teams will be featured in this model, team A and team B. It is going to be assumed that both teams are evenly matched, and that team A begins the overtime period with the ball. In addition, it will also be assumed that all extra points scored will be matched. Finally, since the average NFL drive lasts 2 minutes and 34 seconds (Zauzmer), it will be assumed that each team will possess the ball 3 times in the fifteen minute overtime period (for a total of 6 drives, and 7 minutes and 42 seconds of possession for each team).

In addition to these assumptions, the probabilities of certain events happening in the game are needed to create the model. Specifically, the probability of a team scoring a touchdown (α), kicking a field goal (β), or turning the ball over (λ) are needed. In 2020, for drives starting from a team's 25 yard line (the yard line at which a the ball is placed for a touch back) $\alpha = 0.2420$, $\beta = 0.1280$, and $\lambda = 0.630$ (Stathead Football).

Now that all assumptions have been made, and given probabilities have been listed, the state space of the NFL overtime period is can now be defined. Note that the states listed on the Markov chain are listed in the format: X_i:[Number of points team A leads by, Number of points team B leads by, Team with Possession of the Ball]. The state space for this model is: [1:[0, 0, A], 2: [A wins], 3:[3, 0, B], 4:[0, 0, B], 5:[B wins], 6:[0, 3, A]].

3.3 Model

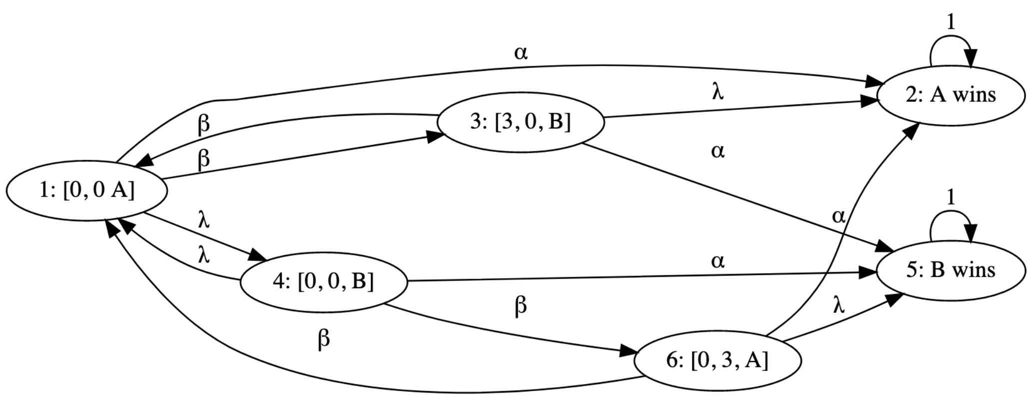

Figure 2: A Markov chain mapping the relations between states in the NFL overtime system

Observe that Figure 2 is a Markov chain which shows the possible paths between states in the NFL overtime period. From Figure 2, transition matrix N can be created:

$$\begin{pmatrix} 0 & \alpha & \beta & \lambda & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \beta & \lambda & 0 & 0 & \alpha & 0 \\ \lambda & 0 & 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \beta & \alpha & 0 & 0 & \lambda & 0 \end{pmatrix}$$

which is equal to:

$$\begin{pmatrix} 0 & 0.2420 & 0.1280 & 0.630 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0.1280 & 0.630 & 0 & 0 & 0.2420 & 0 \\ 0.630 & 0 & 0 & 0 & 0.2420 & 0.1280 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0.1280 & 0.2420 & 0 & 0 & 0.630 & 0 \end{pmatrix}$$

Note that the probability that team A wins on the first drive of the game is equivalent to $N_{1,2} = 0.242$, while the probability that team B wins is equivalent to $N_{1,5} = 0$.

Since it is assumed that each team will be possessing the ball three times, there will be six total possessions in this overtime period. This means that there will be six steps, so the probability of team A winning will be $N^6_{1,2}$, and the probability of team B winning will be $N^6_{1,5}$. Multiplying N by itself 6 times yields N^6:

$$\begin{pmatrix} 0.0708 & 0.5443 & 0.0011 & 0.0054 & 0.3648 & 0.0138 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0.0011 & 0.6976 & 0.0028 & 0.0138 & 0.2846 & 0.0001 \\ 0.0082 & 0.3716 & 0.0138 & 0.0679 & 0.5375 & 0.0012 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0.0011 & 0.3096 & 0.0028 & 0.0138 & 0.6728 & 0.0001 \end{pmatrix}$$

It follows that $N^6_{(1,2)} = 0.5443$, and $N^6_{(1,5)} = 0.3648$. Playing out the full overtime period allows it to be concluded that the probability that team A wins is 0.5443, the probability that team B wins is 0.3648, and the probability of a tie (or continued overtime periods in a playoff game) is 0.0909.

4 NCAA Overtime System Model

4.1 Rules

When an NCAA football game ends regulation in a tie, the game will head to a single overtime period. In this overtime period, each team will be given one possession, with each drive beginning at the opposing team's 25 yard line. Whoever leads after these two possessions is declared the winner. If the game remains tied, it will continue to another overtime period, until a winner is found. A coin toss determines who starts with the ball in the first overtime, but possession alternates as more overtime periods are played (for example, if team A began overtime with the ball, team B would begin the second overtime with the ball, then team A would begin with the ball again in the third overtime, and so on).

4.2 Assumptions, Given Probabilities, and State Space

Team A and team B will be used again in this model. It is going to be assumed that both teams are evenly matched, and that team A starts the overtime period with the ball. Since most college football overtimes end after one period, it will also be assumed that there will only be one period, with 2 possessions (one per team).

In addition to these assumptions, we will again need to know the probabilities of a team scoring a touchdown (α), a team kicking a field goal (β), and a team turning the ball over (λ). For a drive starting at the opposing team's 25 yard line, we now have $\alpha = 0.5694$, $\beta = 0.3125$, $\lambda = 0.1181$ (Stathead Football).

Now that all assumptions have been made, and all given probabilities have been listed, the state space of the NCAA overtime period can now be defined. Once again, the states on the Markov chain are listed in the format X_i:[Number of points team A leads by, Number of points team B leads by, Team with Possession of the Ball]. The state space for this model is: [1:[0, 0, A], 2:[6, 0, B], 3:[3, 0, B], 4: [0, 0, B], 5:[A wins], 6:[B wins]].

4.3 Model

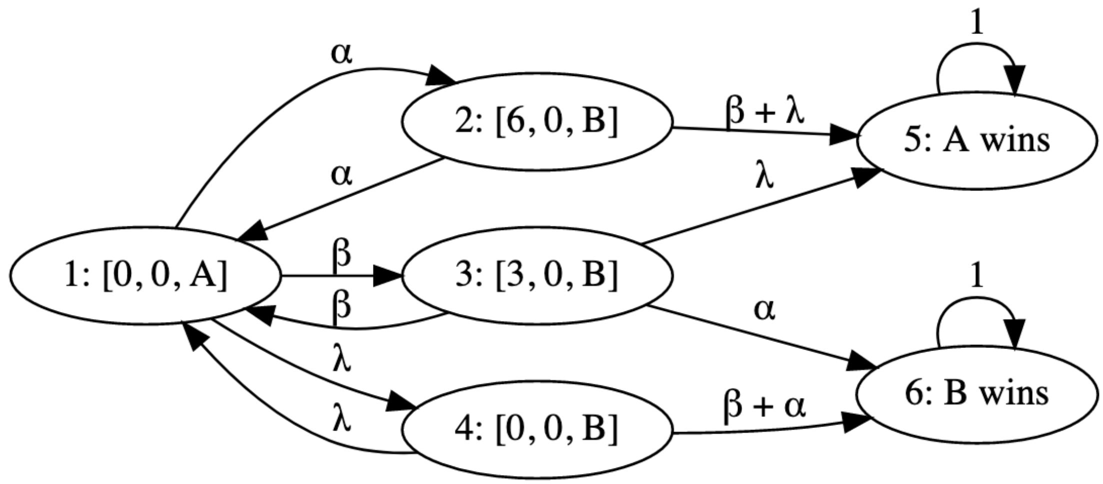

Figure 3: A Markov chain mapping the relations between states in the NCAA overtime system

Observe that Figure 3 is a Markov chain which shows the possible paths between different states in the NCAA overtime period. From Figure 3, transition matrix C can be created:

$$\begin{pmatrix} 0 & \alpha & \beta & \lambda & 0 & 0 \\ \alpha & 0 & 0 & 0 & \beta+\lambda & 0 \\ \beta & 0 & 0 & 0 & \lambda & \alpha \\ \lambda & 0 & 0 & 0 & 0 & \alpha+\beta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

which is equal to:

$$\begin{pmatrix} 0 & 0.5694 & 0.3125 & 0.1181 & 0 & 0 \\ 0.5694 & 0 & 0 & 0 & 0.4306 & 0 \\ 0.3125 & 0 & 0 & 0 & 0.1181 & 0.5694 \\ 0.1181 & 0 & 0 & 0 & 0 & 0.8819 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Since it is assumed that each team will be possessing the ball once, there will be two total possessions. This means that there will be two steps, so the probability of team A winning will be $C^2_{(1,5)}$, and the probability of team B winning will be $C^2_{(1,6)}$. Multiplying C by itself once yields C^2:

$$\begin{pmatrix} 0.4358 & 0 & 0 & 0 & 0.2821 & 0.2821 \\ 0 & 0.3242 & 0.1779 & 0.0672 & 0.4306 & 0 \\ 0 & 0.1779 & 0.0977 & 0.0369 & 0.1181 & 0.5694 \\ 0 & 0.0672 & 0.0369 & 0.0139 & 0 & 0.8819 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

It follows that $C^2_{(1,5)} = 0.2821$, and $C^2_{(1,4)} = 0.2821$. Playing out one overtime period shows that the probability that team A wins in the first overtime is equivalent to the probability that team B wins in the first overtime, which is 0.2821. It shows that the probability of the game continuing to another overtime period is 0.4358.

Note that if the game continues to a second overtime, Team B starts with the ball, so they now take on team A's probabilities in C^3, and team A takes on team B's probabilities. Multiplying C^2 by C yields C^3:

$$\begin{pmatrix} 0 & 0.2482 & 0.1362 & 0.0515 & 0.2821 & 0.2821 \\ 0.2482 & 0 & 0 & 0 & 0.5912 & 0.1606 \\ 0.1362 & 0 & 0 & 0 & 0.2063 & 0.6576 \\ 0.0515 & 0 & 0 & 0 & 0.0333 & 0.9152 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Note that if the game makes it to a second overtime, the probability that team B wins is now $C^3_{1,5} = 0.2821$, and is still equivalent to the probability that team A wins ($C^3_{1,6} = 0.2821$). The probability that the game continues on to a third overtime is still 0.4358.

5 Conclusion

In the NFL overtime system, the probability that team A wins is 0.1795 higher than the probability that team B wins. In addition to this, the probability that team A wins the game on their opening drive, before team B even has a chance to possess the ball, is 0.2420. Since the probability that team A wins (0.5443) is the most likely outcome of the overtime period, it is fair to conclude that winning the coin toss, and receiving the ball, provides a significant advantage.

On the other hand, in the NCAA overtime system, both teams have an equal probability of winning the game in all overtime periods, no matter who starts with the ball. In addition, the probability of one team winning is not the most likely outcome (greater than 0.5), as in each overtime period, the probability of the game continuing to another overtime period is 0.4358, and the probability of the game ending (either team scoring) is 0.5642.

When comparing the two systems, it is clear that the current NFL overtime system provides a clear advantage to the team that starts with the ball. It would benefit the league to consider transitioning to NCAA system, or at least a system similar to it, as the model shows that this system gives both teams a fair chance of winning the game.

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