Modeling NFL Overtime with Markov Chains
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1 Introduction

In recent years, the National Football League has come under fire for the way in which it conducts overtime, as, in the current NFL overtime system, it seems as if the team who possesses the ball first wins more often than not. This is due to a rule which declares the team that scores the first touchdown is the winner of the game (even if only one team has had possession). NCAA football, on the other hand, has been praised for its overtime system, as it is believed to give both teams a fair chance of winning. To see if this is actually true, each overtime system will system will be explored more in depth through the use of Markov Chains.

2 Mathematics

To create the models in this project, two mathematical structures will be used. The first structure that will be used is a Markov Chain:

▶ A Markov chain is a random process that has a discrete amount of time, a discrete state space, and maps out all of the possible transitions between states in this state space.

▶ What makes Markov chains so unique is that they follow the Markov Property. The Markov property is what allows Markov chains to be considered memoryless, meaning that past events have no impact on any future events.

From the Markov Chain, a transition matrix can be created.

▶ A transition matrix is a matrix that gives the probabilities that any state in the state space succeeds to another state in the state space.

▶ Finding the probability that a state e_j, succeeds another state, e_i, in m steps can be found by finding the entry in row i and column j of transition matrix p^m.

3 NFL Model

When a regular season NFL game ends regulation in a tie, the game heads to a single 15 minute overtime period. In this overtime period:

▶ Play is begun with a kickoff, with the receiving team determined by a coin toss. The first team to score a touchdown wins the game.

▶ If the first score is a field goal, the other team will then get the ball, and be given a chance to tie/win the game. If the game is still tied at the end of this period, it is declared a tie.

To create the model, certain facts are going to be assumed:

▶ In the model, two evenly matched teams will be featured: Team A, and Team B. Team A will start with the ball.

▶ On each drive, the probability of either team scoring a touchdown (α) is 0.2420, kicking a field goal (β) is 0.1280, and turning the ball over (λ) is 0.630 [1].
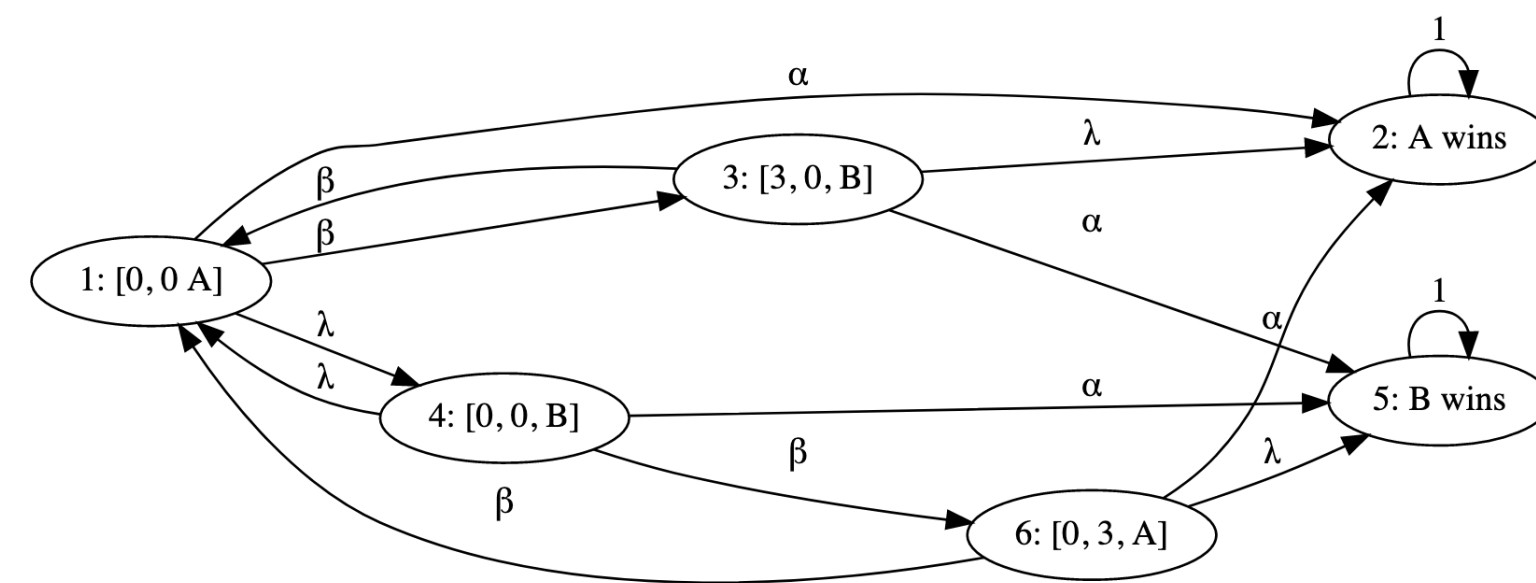
Each state is formatted as: X_i:[Number of points team A leads by, Number of points team B leads by, Team with Possession of the Ball]

Above is a Markov Chain that maps out all of the possible states and transitions in the NFL system. It follows that transition matrix N can be created:

$$\begin{pmatrix} 0 & 0.2420 & 0.1280 & 0.630 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0.1280 & 0.630 & 0 & 0 & 0.2420 & 0 \\ 0.630 & 0 & 0 & 0 & 0.2420 & 0.1280 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0.1280 & 0.2420 & 0 & 0 & 0.630 & 0 \end{pmatrix}$$

In this overtime, it will be assumed that each team possess the ball 3 times (6 total possessions). It follows that N^6 is:

$$\begin{pmatrix} 0.0708 & 0.5443 & 0.0011 & 0.0054 & 0.3648 & 0.0138 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0.0011 & 0.6976 & 0.0028 & 0.0138 & 0.2846 & 0.0001 \\ 0.0082 & 0.3716 & 0.0138 & 0.0679 & 0.5375 & 0.0012 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0.0011 & 0.3096 & 0.0028 & 0.0138 & 0.6728 & 0.0001 \end{pmatrix}$$

From these two matrices, observe that:

▶ The probability that Team A wins on its opening drive is $N_{1,2} = 0.242$, while the probability that Team B wins is $N_{1,5} = 0$.

▶ At the end of the overtime period, the probability Team A wins is $N^6_{(1,2)} = 0.5443$, the probability Team B wins is $N^6_{(1,5)} = 0.3648$, and the probability of a tie is 0.0909.

4 NCAA Model

When an NCAA football game ends regulation in a tie, the game heads to a single overtime period in which:

▶ Both teams will possess the ball once, beginning their drives at the opposing team's 25 yard line. The team that starts with the ball is determined by a coin toss.

▶ Whoever leads after these two possessions is declared the winner. If the game is still tied, it heads to another overtime period, until a winner is found.

To create the model, certain facts are going to be assumed:

▶ The evenly matched Team A and Team B will be used in this model as well, with team A starting with the ball.

▶ On each drive, the probability of either team scoring a touchdown (α) is now 0.5694, kicking a field goal (β) is 0.3125, and turning the ball over (λ) is 0.1181 [1].
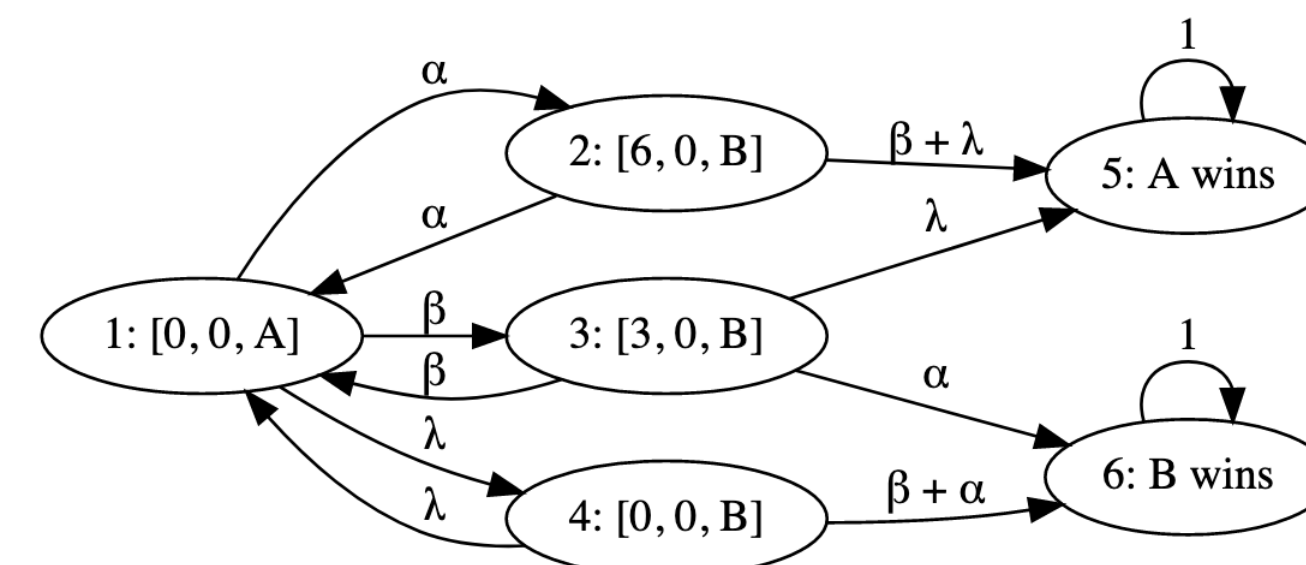
Above is a Markov Chain that maps out all of the possible states and possible transitions between states for this system. It follows that transition matrix C can be created from this:

$$\begin{pmatrix} 0 & 0.5694 & 0.3125 & 0.1181 & 0 & 0 \\ 0.5694 & 0 & 0 & 0 & 0.4306 & 0 \\ 0.3125 & 0 & 0 & 0 & 0.1181 & 0.5694 \\ 0.1181 & 0 & 0 & 0 & 0 & 0.8819 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

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Since each team will posses the ball once, there will be two total possessions in each overtime period. It follows that C^2 is:

$$\begin{pmatrix} 0.4358 & 0 & 0 & 0 & 0.2821 & 0.2821 \\ 0 & 0.3242 & 0.1779 & 0.0672 & 0.4306 & 0 \\ 0 & 0.1779 & 0.0977 & 0.0369 & 0.1181 & 0.5694 \\ 0 & 0.0672 & 0.0369 & 0.0139 & 0 & 0.8819 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

From these matrices, observe that:

▶ Playing out the overtime period shows that the probability Team A wins is $C^2_{(1,5)} = 0.2821$, which is equivalent to the probability Team B wins $C^2_{(1,6)} = 0.2821$.

▶ The probability of the game heading to another overtime period is 0.4358.

5 Conclusion

After examining each overtime system, it can be concluded that the NFL should switch to an overtime system similar to the NCAA model, as:

▶ The most likely outcome of the NFL system was the team that possesses the ball winning the game.

▶ The team that starts with the ball in the NFL system has a chance to win the game before the other team can possess it.

▶ Both teams in the NCAA system have an equal chance of winning in every overtime period.

6 References

[1] Stathead Footabll. Football — nfl drive finder. `https://stathead.com/tiny/7WIVt`, 2021. [Online; accessed 5-April-2021].