MLHW1

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1 Problem 1 Part 1

The gradient of $f(x, y) = x^2 + \ln(y) + xy + y^3$.

First, calculate the partial derivative with respect to x:

$$\frac{\partial f}{\partial x} = 2x + y$$

Next, calculate the partial derivative with respect to y:

$$\frac{\partial f}{\partial y} = \frac{1}{y} + x + 3y^2$$

Now, substituting $(x, y) = (10, -10)$:

For $\frac{\partial f}{\partial x}$:

$$\frac{\partial f}{\partial x}(10, -10) = 2(10) + (-10) = 10$$

For $\frac{\partial f}{\partial y}$:

$$\frac{\partial f}{\partial y}(10, -10) = \frac{1}{-10} + 10 + 3(-10)^2 = -0.1 + 10 + 300 = 309.9$$

Therefore, the gradient at $(x, y) = (10, -10)$ is:

$$\nabla f(10, -10) = (10, 309.9)$$

2 Problem 1 Part 2

Now, for the function $f(x, y, z) = \tanh(x^3y^3) + \sin(z^2)$, the gradient is calculated as follows:

First, calculate the partial derivative with respect to x:

$$\frac{\partial f}{\partial x} = (1 - \tanh^2(x^3y^3)) \cdot 3x^2y^3$$

Next, calculate the partial derivative with respect to y:

$$\frac{\partial f}{\partial y} = (1 - \tanh^2(x^3y^3)) \cdot 3x^3y^2$$

Finally, calculate the partial derivative with respect to z:

$$\frac{\partial f}{\partial z} = 2z\cos(z^2)$$

Now, substituting $(x, y, z) = (-1, 0, \frac{\pi}{2})$:

For $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, since $y = 0$, both partial derivatives with respect to x and y are:

$$\frac{\partial f}{\partial x}(-1, 0, \frac{\pi}{2}) = 0, \quad \frac{\partial f}{\partial y}(-1, 0, \frac{\pi}{2}) = 0$$

For $\frac{\partial f}{\partial z}$:

$$\frac{\partial f}{\partial z}(-1, 0, \frac{\pi}{2}) = \pi \cdot \cos\left(\frac{\pi^2}{4}\right)$$

Therefore, the gradient at $(x, y, z) = (-1, 0, \frac{\pi}{2})$ is:

$$\nabla f(-1, 0, \frac{\pi}{2}) = (0, 0, \pi \cdot \cos\left(\frac{\pi^2}{4}\right))$$

Problem 2 part 1: Matrix multiplication

We are given the matrices:

$$\mathbf{A} = \begin{bmatrix} 10 \\ -5 \\ 2 \\ 8 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 & 3 & 0 & 1 \end{bmatrix}$$

The product $\mathbf{A} \times \mathbf{B}$ is:

$$\mathbf{A} \times \mathbf{B} = \begin{bmatrix} 10 \times 0 & 10 \times 3 & 10 \times 0 & 10 \times 1 \\ -5 \times 0 & -5 \times 3 & -5 \times 0 & -5 \times 1 \\ 2 \times 0 & 2 \times 3 & 2 \times 0 & 2 \times 1 \\ 8 \times 0 & 8 \times 3 & 8 \times 0 & 8 \times 1 \end{bmatrix}$$

Simplifying the multiplication, we get:

$$= \begin{bmatrix} 0 & 30 & 0 & 10 \\ 0 & -15 & 0 & -5 \\ 0 & 6 & 0 & 2 \\ 0 & 24 & 0 & 8 \end{bmatrix}$$

Problem 2 part 2: Matrix multiplication

We are given two matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 6 & 7 \\ 9 & 0 & 8 & 1 \\ -8 & 1 & 2 & 3 \\ 10 & 4 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 6 & 2 & 0 \\ 0 & -1 & 1 \\ -3 & 0 & 4 \\ 3 & 4 & 7 \end{bmatrix}$$

The product $\mathbf{A} \times \mathbf{B}$ is calculated by multiplying each row of $\mathbf{A}$ by each column of $\mathbf{B}$.

Step-by-step calculation:

Row 1 of $\mathbf{A}$ multiplied by each column of $\mathbf{B}$:

$$C_{11} = (1 \times 6) + (-1 \times 0) + (6 \times -3) + (7 \times 3) = 6 + 0 - 18 + 21 = 9$$

$$C_{12} = (1 \times 2) + (-1 \times -1) + (6 \times 0) + (7 \times 4) = 2 + 1 + 0 + 28 = 31$$

$$C_{13} = (1 \times 0) + (-1 \times 1) + (6 \times 4) + (7 \times 7) = 0 - 1 + 24 + 49 = 72$$

Row 2 of $\mathbf{A}$ multiplied by each column of $\mathbf{B}$:

$$C_{21} = (9 \times 6) + (0 \times 0) + (8 \times -3) + (1 \times 3) = 54 + 0 - 24 + 3 = 33$$

$$C_{22} = (9 \times 2) + (0 \times -1) + (8 \times 0) + (1 \times 4) = 18 + 0 + 0 + 4 = 22$$

$$C_{23} = (9 \times 0) + (0 \times 1) + (8 \times 4) + (1 \times 7) = 0 + 0 + 32 + 7 = 39$$

Row 3 of $\mathbf{A}$ multiplied by each column of $\mathbf{B}$:

$$C_{31} = (-8 \times 6) + (1 \times 0) + (2 \times -3) + (3 \times 3) = -48 + 0 - 6 + 9 = -45$$

$$C_{32} = (-8 \times 2) + (1 \times -1) + (2 \times 0) + (3 \times 4) = -16 - 1 + 0 + 12 = -5$$

$$C_{33} = (-8 \times 0) + (1 \times 1) + (2 \times 4) + (3 \times 7) = 0 + 1 + 8 + 21 = 30$$

Row 4 of $\mathbf{A}$ multiplied by each column of $\mathbf{B}$:

$$C_{41} = (10 \times 6) + (4 \times 0) + (0 \times -3) + (1 \times 3) = 60 + 0 + 0 + 3 = 63$$

$$C_{42} = (10 \times 2) + (4 \times -1) + (0 \times 0) + (1 \times 4) = 20 - 4 + 0 + 4 = 20$$

$$C_{43} = (10 \times 0) + (4 \times 1) + (0 \times 4) + (1 \times 7) = 0 + 4 + 0 + 7 = 11$$

Final Result:

$$\mathbf{A} \times \mathbf{B} = \begin{bmatrix} 9 & 31 & 72 \\ 33 & 22 & 39 \\ -45 & -5 & 30 \\ 63 & 20 & 11 \end{bmatrix}$$