

# The Double Pendulum

## Creating the Perfect Baseball Bat

Jared Baur

Physics Department  
Occidental College

February 25, 2019

# Scope

1 Introduction

2 Equations of Motion

3 Experimental Results

# The Double Pendulum

# The Double Pendulum

Chaos increases exponentially

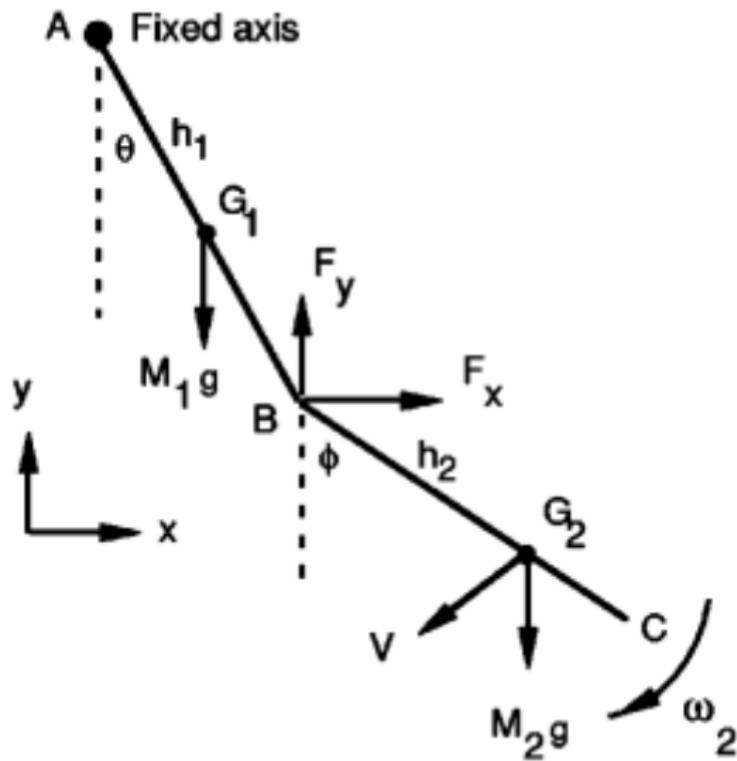
$$\Delta x(t) \sim \Delta x(t_0) e^{\lambda t} \quad (1)$$

# The Double Pendulum

Chaos is not as prevalent in a baseball swing, due to the swing only occurring in the first half cycle of a double pendulum

Can we design a “perfect” baseball bat?

# Equations of Motion



# Equations of Motion

Coordinates  $(x, y)$  of  $G_2$ , with respect to origin  $A$ :

$$L_1 \sin \theta + h_2 \sin \phi - L_1 \cos \theta - h_2 \sin \phi \quad (1)$$

With  $V$  as the velocity of  $G_2$ , components of  $V$  are given as:

$$V_x = \frac{dx}{dt} = -L_1 \omega_1 \cos \theta - h_2 \omega_2 \cos \phi \quad (2)$$

$$V_y = \frac{dy}{dt} = -L_1 \omega_1 \sin \theta - h_2 \omega_2 \sin \phi \quad (3)$$

# Equations of Motion

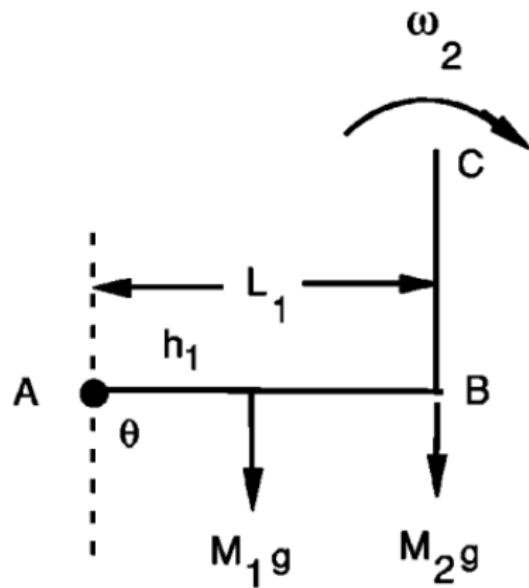
Forces arm exerts on rod:

$$\begin{aligned} F_x &= M_2 \frac{dV_x}{dt} \\ &= -M_2 \left[ L_1 \cos \theta \frac{d\omega_1}{dt} + L_1 \omega_1^2 \sin \theta + h_2 \cos \phi \frac{d\omega_2}{dt} + h_2 \omega_2^2 \sin \phi \right] \end{aligned}$$

$$\begin{aligned} F_y - M_2 g &= M_2 \frac{dV_y}{dt} \\ &= -M_2 \left[ L_1 \sin \theta \frac{d\omega_1}{dt} - L_1 \omega_1^2 \cos \theta + h_2 \sin \phi \frac{d\omega_2}{dt} - h_2 \omega_2^2 \cos \phi \right] \end{aligned}$$

## Equations of Motion

Torque from muscles,  $C_1$  is applied on arm and  $C_2$  is applied on the rod.  
The arm applies an equal and opposite torque  $-C_2$  to the rod.  
Initial conditions:  $\theta = 90^\circ$ ,  $\phi = 180^\circ$ , and  $\beta = \theta - \phi = 90^\circ$ .



# Equations of Motion

When  $C_1$  and  $C_2$  equal zero, initial angular accelerations are

$$\frac{d\omega_1}{dt} = (M_1 h_1 + M_2 L_1)g/A \quad (6)$$

$$\frac{d\omega_2}{dt} = 0 \quad (7)$$

Rod acts as point mass at the end of the arm since it has no initial angular acceleration, so the total moment of inertia of the arm and rod is

$$I_1 + M_2 L_2^2 = A \quad (1)$$

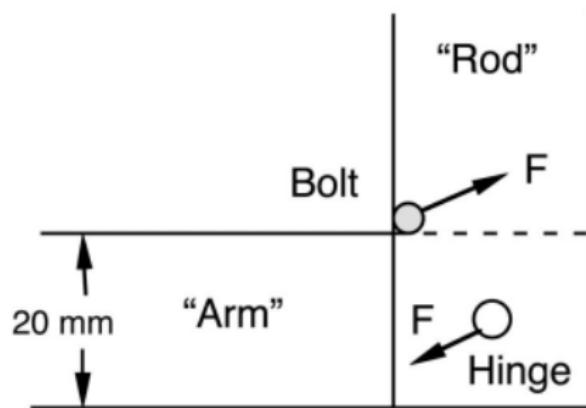
# Equations of Motion

In practice, initial stage of swing is in "wrist-cock" position. After arm has rotated to about  $\theta = 45^\circ$ , torque from centripetal force is large enough to swing the rod without assistance from the wrist.



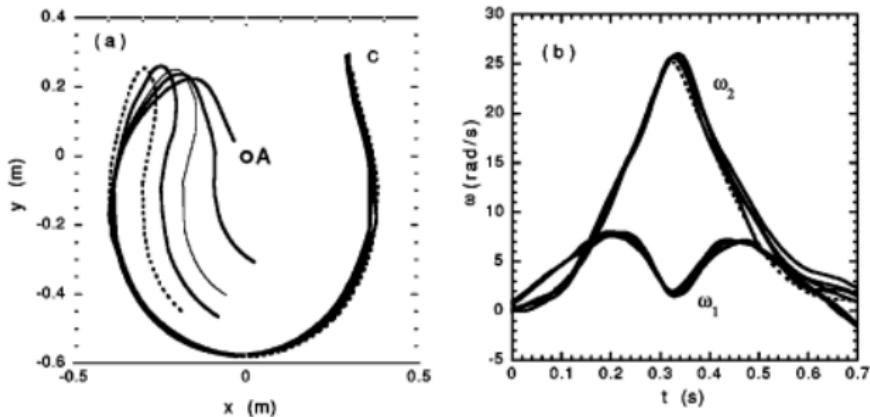
# Experimental Results

With the spot mechanism,  $\beta$  cannot exceed  $90^\circ$  and the pendulum initially rotates as a rigid body



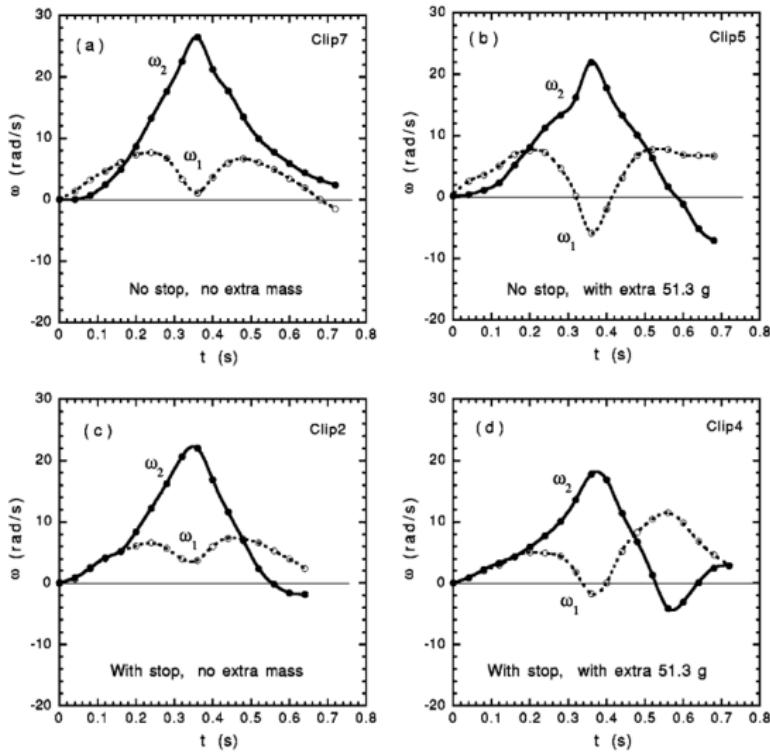
# Experimental Results

Chaotic motion is highly dependent on the initial conditions of the arm and rod, however this chaos develops mostly after the first half cycle of motion.



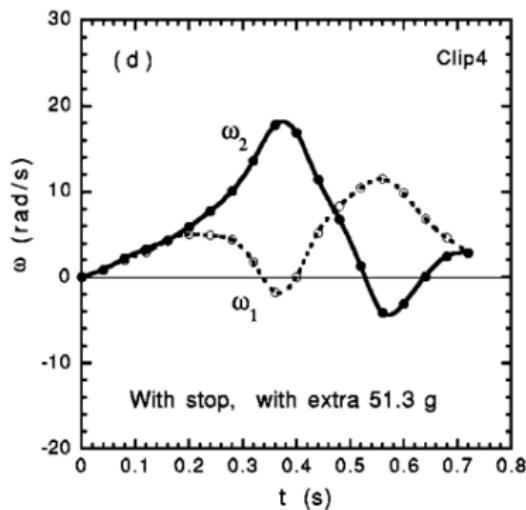
The area of interest with the double pendulum swing is the point where the lower segment of the rod reaches its maximum speed and shortly afterward.

# Experimental Results



# Experimental Results

- $\omega_1 = \omega_2$  for first 0.2 seconds with stop
- Angular speed of arm is at minimum when angular speed of rod is at maximum (angular momentum is transferred efficiently)



# Experimental Results

Experimental and analytical results are consistent

