The Double Pendulum:
Creating a Better Baseball Bat

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To be written later.

I. INTRODUCTION

The double pendulum is a classic example of chaotic motion[1]. The trajectories of various trials will show that the motion of a double pendulum is highly unpredictable. The chaos in a classic double pendulum is described by Equation 1. The exponent λ is a positive constant, Δx is the trajectory of the bottom arm of the pendulum, and t is time. For small times t, the trajectories are relatively the same, however for increasing times, the trajectories exponentially increase in distance from trial to trial.

$$\Delta x(t) \sim \Delta x(t_0)e^{\lambda t} \tag{1}$$

The double pendulum is used in real life situations such as in double-trailer trucks (officially known as "long combination vehicles") where two large-wheeled vehicles are combined at a hinge point. These vehicles are essentially semi-trucks with two trailers attached instead of one. The swinging motion in sports is also a common instance of a double

FIG. 1. Chaos in a double pendulum. This figure depicts the trajectory of the end of the bottom arm in the double pendulum. The light is traced along the route of the bottom arm. As seen, there is no evident pattern in the bottom arm's trajectory.

pendulum. For this paper, the double pendulum will be compared to a baseball swing. In a baseball swing, a bat (striking implement) is used in conjunction with the arms to strike a baseball. In general, the arms are held up and back from the player's head, with the bat pointed upwards. The arms initiate the swing, moving towards the center of the player's body; the bat follows the arms, and then swings through as the transfer of energy from the arms to the bat occurs. This motion occurs mostly in the horizontal plane, but nonetheless acts as a double pendulum with the player's arms and the bat as the upper and lower "arms" of the pendulum, respectively.

Since the swing of a baseball bat only occurs once per trial (does not swing back and forth and time t is relatively low), this motion is equivalent to the first half cycle of a double pendulum swing. Thus, the chaotic motion prevalent in classic double pendulums will not be applicable for the purpose of the baseball swing.

Although the focus of this paper will be on the baseball swing, the double pendulum motion can be applied to the motion of a swing of a baseball bat, golf club, tennis racquet, or any other "swinging" motion that occurs with a striking implement. The goal of this paper will be to quantitatively describe the double pendulum in the context of a baseball swing and determine whether all of the energy in the striking implement can be transferred to the ball on impact. From this, we will decide whether we can design a baseball bat or ball that would allow for this perfect transfer of energy to occur.

II. EQUATIONS OF MOTION

Describing the motion of a double pendulum has been argued as highly complicated[2]. For this reason, Lagrangians have been used in past studies to represent this motion. However, when we look at the forces on each arm of the pendulum with respect to each arm's center of mass, Newtonian equations of motion can be used to describe the motion.

Figure 2 depicts the components in a simple double pendulum. The x, y axes for position and forces are shown in the figure. The upper arm of the pendulum is the first arm attached to a fixed axis A. This arm has length L_1, center of mass at point G_1, and distance to its center of mass h_1 (measured from point A). This arm makes an angle θ with the vertical axis and has a gravitational force acting on its center of mass equal to $M_1 g$, where M_1 is the mass of the arm and g is the acceleration due to gravity.

The second arm (lower arm) rotates on a "fixed" axis at point B. If we isolate the second arm from the system, it would rotate about point B as if it were fixed, however point B is rotating around point A since it is at the end of the first arm.
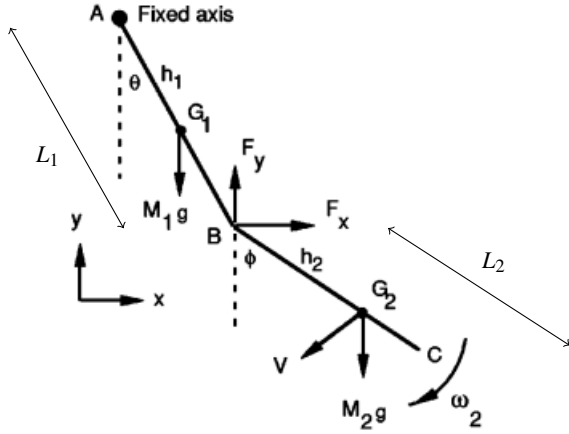
FIG. 2. The components in a double pendulum. Since the equations that describe a double pendulum can be rather confusing, we use the center of mass G_1 and G_2 to describe the forces acting on the pendula arms.

The second arm has a length L_2 and its center of mass at point G_2, a distance of h_2 away from point B. The arm makes an angle of ϕ with the vertical axis and has a gravitational force of M_2g acting on its mass M_2. Point C is the end of the second arm, and will act as a point of interest for our paper as we track the trajectory of this point. The velocity vector V is the direction of motion for the second arm's center of mass through the swinging of a double pendulum.

We start writing our equations of motion by describing the coordinates of our center of mass on the second arm. We want the coordinates of the center of mass since we will write our forces with respect to that (as opposed to point C). Point G_2 is described in Equation 2, where the first half of the equation is the x component and the second half is the y component. The y component is negative since the double pendulum's origin at $(x,y) = (0,0)$ is assumed at point A.

$$G_2(x,y) = \Big([L_1 \sin\theta + h_2 \sin\phi], [-L_1 \cos\theta - h_2 \cos\phi] \Big) \quad (2)$$

The velocity of point G_2 is found by using the angular velocity of the entire first arm, ω_1, and combining it with the angular velocity of the second arm, ω_2. The components of this velocity are found by multiplying the angular velocities against the distance to point G_2. For the x-direction, we take $-L_1 \cos\theta$ for the first arm and $-h_2 \cos\phi$ for the second arm. Likewise for the y-direction, we take $-L_1 \sin\theta$ and $-h_2 \sin\phi$ for the first and second arms, respectively. These components are oriented negatively since the arm is moving in the negative x- and y-directions. The velocities V_x and V_y for point G_2 are described in Equations 3 and 4.

$$V_x = \frac{dx}{dt} = -L_1 \omega_1 \cos\theta - h_2 \omega_2 \cos\phi \quad (3)$$

$$V_y = \frac{dy}{dt} = -L_1 \omega_1 \sin\theta - h_2 \omega_2 \sin\phi \quad (4)$$

The forces on center of mass point G_2 are found by taking the derivatives of Equations 3 and 4. Newton's second law $\sum \mathbf{F} = m\mathbf{a}$ is used to describe the sum of forces acting on the double pendulum. For the force in the x-direction, the only force applied to G_2 is the force from the upper arm pulling on the lower arm. For the y-direction, the upper arm as well as force due to gravity act on the lower arm. The x- and y-components of force are given by Equations 5 and 6.

$$
\begin{aligned}
F_x &= M_2 \frac{dV_x}{dt} \\
&= -M_2 \Big[L_1 \cos\theta \frac{d\omega_1}{dt} + L_1 \omega_1^2 \sin\theta \\
&\quad + h_2 \cos\phi \frac{d\omega_2}{dt} + h_2 \omega_2^2 \sin\phi \Big]
\end{aligned}
\quad (5)
$$

$$
\begin{aligned}
F_y - M_2 g &= M_2 \frac{dV_y}{dt} \\
&= -M_2 \Big[L_1 \sin\theta \frac{d\omega_1}{dt} - L_1 \omega_1^2 \cos\theta \\
&\quad + h_2 \sin\phi \frac{d\omega_2}{dt} - h_2 \omega_2^2 \cos\phi \Big]
\end{aligned}
\quad (6)
$$

These derived equations are for a generic double pendulum that falls in a vertical fashion. In order to apply this concept to the swing of a baseball bat, we will use the torques on the pendula components. The terminology we will use for the upper and lower components of the pendulum will be the "arm" and "rod", respectively. In a baseball swing, we assume that the upper body musculature applies a torque C_1 to the arm and a torque C_2 to the rod of the pendulum. These torques originate from the equal and opposite muscle and joint reaction forces from the shoulder and elbow joints[3] (points A and B, respectively). The net torque acting on the arm is given using the generic equation $\sum \tau = \mathbf{I} \frac{d\omega}{dt}$, where τ is the net torque, I is the moment of inertia, and ω is the angular velocity. With torques C_1 and C_2 on the arm and rod, the rod exerts an equal and opposite torque $-C_2$ on the arm. There is also a clockwise torque on the arm about point A in the form of $F_x L_1 \cos\theta + F_y L_1 \sin\theta$. Thus, the torque acting on the arm is given by Equation 7.

$$
\begin{aligned}
\sum \tau_{\text{arm}} &= I_1 \frac{d\omega_1}{dt} \\
&= C_1 - C_2 + M_1 g h_1 \sin\theta + F_x L_1 \cos\theta + F_y L_1 \sin\theta
\end{aligned}
\quad (7)
$$

The torque acting on the rod's center of mass is given in Equation 8. The arm applies no torque to the rod. We solve for the torque on the center of mass for the rod since we are interested in this point as it pertains to the transfer of energy (described in the coming sections).

$$
\begin{aligned}
\sum \tau_{\text{rod}} &= I_{c.m.} \frac{d\omega_2}{dt} \\
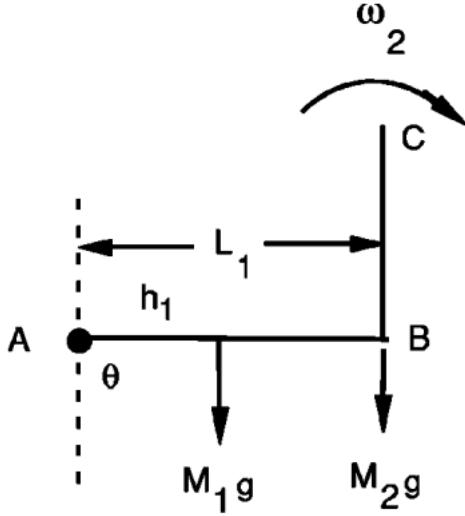&= C_2 + F_x h_2 \cos\phi + F_y h_2 \sin\phi
\end{aligned}
\quad (8)
$$

FIG. 3. Initial conditions of the double pendulum with $\theta = 90°$, $\phi = 180°$, and $\beta = \phi - \theta = 90°$. These initial conditions accurately simulate the baseball swing, since the arms are most often at a 90° angle with the bat right before a swing.

Substituting in Equations 5 and 6, we get a complete description of the torque on the system (Equation 9). The angle β is the difference between ϕ and θ ($\beta = \phi - \theta$), $A = I_1 + M_2L_1^2$, $B = M_2h_2L_1$, and $I_2 = I_{c.m.} + M_2h_2^2$.

$$
\begin{aligned}
C_1 - C_2 = & A\frac{d\omega_1}{dt} + B\cos\beta\frac{d\omega_2}{dt} + B\omega_2^2\sin\beta \\
& - (M_1h_1 + M_2L_1)g\sin\theta \\
C_2 = & I_2\frac{d\omega_2}{dt} + B\cos\beta\frac{d\omega_1}{dt} \\
& - B\omega_1^2\sin\beta - M_2h_2g\sin\phi
\end{aligned}
\tag{9}
$$

It is difficult to parse Equation 9 for context of the nature of our system. To simplify the equation, we use initial conditions that are relevant to our baseball swing. Referring to Figure 3, we have the conditions $\theta = 90°$, $\phi = 180°$, and $\beta = 90°$. We also apply the initial condition that $C_1 = C_2 = 0$, since initially, the upper body musculature is not applying a torque to the arm or the rod components of the pendulum. Applying the initial conditions to Equation 9, we get a simplified version that is more applicable to the baseball swing. This simplified version is presented in Equation 10.

$$
\begin{aligned}
M_1gh_1 - F_yL_1 = & I_1\frac{d\omega_1}{dt} \\
F_xh_2 = & -I_{c.m.}\frac{d\omega_2}{dt}
\end{aligned}
\tag{10}
$$

In the position depicted in Figure 3, the swing has not yet begun, so we are at time $t = 0$. At this time, the angular velocities of the arm and rod are $\omega_1 = \omega_2 = 0$. This means that

Equation 10 can be further simplified into the initial angular accelerations of the arm and rod (given in Equation 11).

$$
\begin{aligned}
\frac{d\omega_1}{dt} &= \frac{g}{A}(M_1h_1 + M_2L_1) \\
\frac{d\omega_2}{dt} &= 0
\end{aligned}
\tag{11}
$$

III. EXPERIMENTAL RESULTS

An experimental double pendulum was constructed in the study by Cross et al.[3] from two 0.30 meter length aluminum bars, the upper bar of mass 109.3 grams and the lower bar of mass 25.0 grams. A mechanical stop was implemented, which kept the β angle between the arm and the rod of the pendulum at 90° for the initial stage of the swing. The mechanical stop implemented is a small bolt placed inbetween the arm and the rod that prevents the β angle from falling below 90°, shown in Figure 4. Contact between the bolt and the end of the arm applies a force F on the rod. An equal and opposite force is incident on the arm at the "hinge" location. These forces do not significantly contribute to the overall acceleration of the rod through its swinging motion. After the arm swings through to about $\theta = 75°$, 15° less than its initial angle, the β angle increases past 90°, thus the bolt loses contact with the rod and this contact force is nonexistent.
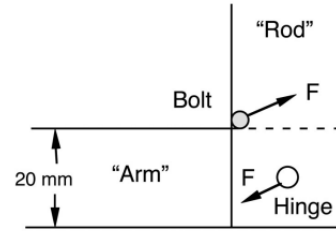

FIG. 4. The stop mechanism that enables an initial β angle of 90° at the initial swing position. The stop mechanism is a rod placed at the hinge of the arm and the rod that applies a force to the rod. An equal and opposite force is applied to the arm.

The stop mechanism is a simulation of actual tendons in the human body that pull on the bone to place joints and limbs in specific positions. In the case of the baseball swing pendulum, the stop mechanism simulates tendons in the forearm and wrist that pull the wrist into a cocked position, holding the bat at a 90° angle to the arms. A visual representation of this is given in Figure 5.

FIG. 5. A visual of the real life $\beta = 90°$ angle between the "arm" and "rod" (for the double pendulum) or arms and bat (for the baseball player).

An extra 51.3 g mass was attached to near the end of the rod to increase the moment of inertia of the rod. This added mass, however, creates an unrealistic scenario in which the arm mass to rod mass ratio is too small. In baseball, a realistic arm to rod mass ratio is around 6:1. This added mass lowers the ratio. The increased moment of inertia was simply desired for experimental variability. Trials of the double pendulum swing were ran under the four following conditions:

- Without the stop mechanism and without the added 51.3 g mass,

- Without the stop and with the added mass,

- With the stop and without the added mass,

- With the stop and with the added mass.

These trials were ran with the afformentioned initial conditions (Figure 3); video software was used to track the (x, y) positions of points of interest for our double pendulum through its swing. The trajectory of point C (the end of the rod) is shown in Figure 6. The chaos that was mentioned earlier in this paper can be seen in Figure 6 to be irrelevant. Since only the first half-cycle of the pendulum swing is representative of the baseball swing, the time t (Equation 1) needed for chaos to become significant is too small. Figure 6 proves that the double pendulum swing is reproducable for this experiment. The graph on the right in Figure 6 is the angular velocity of the arm (ω_1) and the angular velocity of the rod (ω_2) for the same swings used to produce the graph of the trajectories of point C. The velocities can be viewed as "coupled", that is,

they react to each other strongly. When the angular velocity of the arm is at a minimum, the angular velocity of the rod is at a maximum. This means that momentum is being transferred efficiently into the rod.
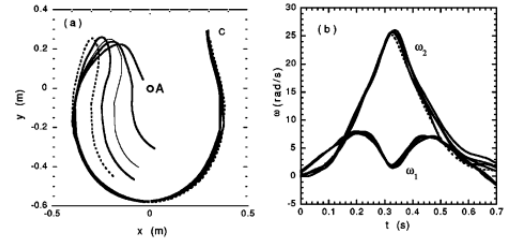

FIG. 6. The trajectory of point C through the full half-cycle swing of the experimental double pentulum. As seen, the trajectories are the same through the greater majority of this motion. The angular velocities of the arm and rod are coupled; the rod is at a peak angular velocity when the arm is at its minimum.

The same trials were ran for the four conditions previously mentioned. These results are shown in Figure 7. For all trial conditions, the maximum rod angular velocity occurs during the point of the minimum arm angular velocity. In the case with no stop mechanism and no extra 51.3 g mass, the minimum arm angular velocity goes to zero and the maximum rod angular velocity is the highest out of all trial conditions. For the trial with no stop mechanism and the extra mass, the minimum angular velocity for the arm dips below zero, which means that the arm travels backwards in its motion as the energy is transferred from arm to rod. This is unrealistic as it relates to the baseball swing since the arms in the baseball swing do not travel backwards; they actually continue moving forward at a decreased velocity (nonzero velocity). In the trial with the stop mechanism and no added mass (the most realistic case due to the accurate arm to rod ratio and stop mechanism), the minimum arm angular velocity does not go to zero. For the trial with the stop mechanism and the added mass, the minimum arm angular velocity dips below zero. There is another moment in this swing where the arm and rod share their minimum and maximum angular velocity, however the arm is now at the maximum and the rod is at the minimum. For both trials with the stop mechanism, the arm and rod have the same angular velocity for about the first 0.2 seconds. This is due to the stop mechanism holding the arm and the rod in the same position relative to one another as they swing through their first cycle. The added mass to the rod increases the coupling between the arm and the rod since the arm and rod swap velocity and direction with one another. If the mass of the rod M_2 were to go to zero, the coupling would also go to zero. If the mass of the arm M_1 were to go to zero, the coupling would increase. This situation is similar to a double pendulum in which the upper arm of the pendulum is a string ($M_1 \approx 0$), which has been proven to have high coupling. Increased coupling between the arm and the rod produces increased angular velocity of the rod, which improves the performance of the rod's ability to hit a ball further. An increased velocity improves the ability for the rod to transfer the maximum amount of momen-

tum into the ball. Unfortunately, the stop mechanism decrease this coupling, however this limitation provides the most realistic swing in context with the baseball swing. The highest angular velocity of the rod also occurs when the angular velocity of the arm goes to zero ($\omega_1 \rightarrow 0$). In the case where $\omega_1 = 0$ at the arm's minimum, none of the kinetic energy is retained in the arm and all of it is transferred to the rod.

FIG. 8. The analytical results for Equation 9, evidently consistent with the experimental results obtained from Figure 7.
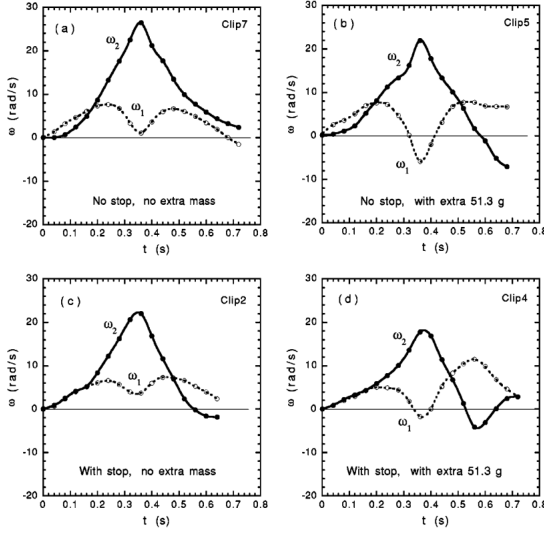

FIG. 7. Trials ran for double pendulum with/without stop mechanism and with/without extra 51.3 g mass. The most realistic condition is the trial ran with the stop mechanism and without the added 51.3 g mass (bottom left).

Equation 9 was used to solve for the angular velocities of the arm and the rod. The analytical solutions are shown in Figure 8. Comparing these analytical results to Figure 7, the two are highly consistent with one another. There are minor discrepancies that could have been improved with better high speed cameras (used to plot nodes on the double pendulum during its swing) or a lower speed pendulum (that would make the need of a high speed camera irrelevant).
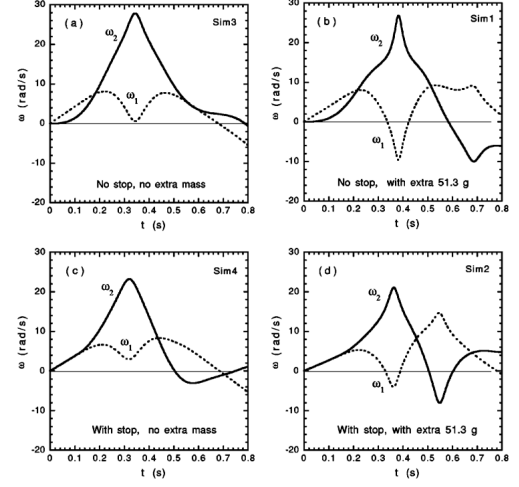
IV.   FORCE ANALYSIS

Finding the cause of rotation of the rod segment of the double pendulum can help with finding the best way to maximize the energy transferred from the arm to the rod. The cause of rotation for the rod segment of the double pendulum can be determined by analyzing the forces acting on it. A first look at the double pendulum might imply that the cause of rotation for the rod segment of the arm is similar to the cause of rotation for the arm of a single pendulum. That is, gravitational force acts on the arm and pulls it through its swing. The double pendulum case is, however, more complicated than that of a single pendulum since the center of mass of the rod (point G_2) rotates around its own axis point B as well as the axis of rotation for the arm, point A. Figure 9 shows the forces acting on the rod for two conditions: one with the stop mechanism and no added mass, the other with the stop mechanism and no added mass. These forces shown in the diagram are of the arm pulling on the rod, corresponding to Equations 5 and 6. All other forces (gravitational force and force due to stop mechanism bolt) are left out of the figure; this will be discussed at the end of this section. Note the direction of motion for the center of mass through the swing of the rod. This direction is perpendicular to the rod itself, making a circular/elliptical motion through the swinging path. The forces incident on this center of mass G_2 are thus, mostly perpendicular to this motion. This means that there is a large centripetal force and torque incident on the rod's center of mass. The centripetal force pulls the center of mass towards the axis of rotation, always pointing inward.
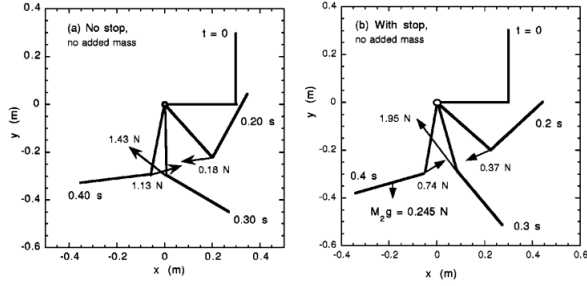
FIG. 9. Forces incident on the center of mass G_2 of the rod. The limitations set on the graph on the right (with a stop mechanism and without the extra mass) is the most realistic scenario in context of the baseball swing.

The total forces acting on the rod's center of mass can be described in Equations 12 and 13. These forces are split into the parallel and perpendicular force since, as previously mentioned, the parallel and perpendicular forces contribute to different actions. It should be noted that these forces are described with their orientation to the direction of motion of center of mass for the rod. The velocity of the center of mass of the rod is broken up into three variables, V for the total velocity of G_2, and V_x and V_y for the x and y components of the total velocity V. The parallel force (Equation 12) is the linear force that pulls the rod's center of mass instantaneously backwards or forwards in relation to the direction of motion of G_2. The first vector shown in Figure 9 (for both conditions) has a negative parallel force since the direction of motion is away from the direction of the force. The remaining vectors have positive parallel forces. The parallel force is set equal to $M_2 \frac{dV}{dt}$ since this force is linear. The perpendicular force (Equation 13) is the larger of the two since the vectors in Figure 9 are, for the most part, parallel to the rod itself (and thus perpendicular to the rod's direction of motion). In Equation 13, the perpendicular force is set equal to $M_2 \frac{V^2}{R}$ since this is the centripetal force that acts to pull the rod's center of mass toward the center of the circular/elliptical motion that it makes during its swing.

$$F_{\parallel} = \frac{F_x V_x + (F_y - M_2 g) V_y}{V} = M_2 \frac{dV}{dt} \quad (12)$$

$$F_{\perp} = \frac{(F_y - M_2 g) V_x + F_x V_y}{V} = M_2 \frac{V^2}{R} \quad (13)$$

The torque that is incident on the center of mass of the rod from the forces on the rod are described in Equations 14 and 15. Equation 14 describes the torque that is produced from the parallel force on the rod's center of mass; Equation 15 describes the torque that is produced from the perpendicular force that is centripetal in nature. Note that the description of these torques as parallel and perpendicular are not in coordination to the motion of direction of G_2, it is simply a description of the torque produced from the different components of force on the rod. The actual directions of these torques are,

according to Figure 9, into the page. In Equations 14 and 15, λ is the angle between the velocity vector V and the vertical axis.

$$\tau_{\parallel} = F_{\parallel} h_2 \sin(\phi - \lambda) \quad (14)$$

$$\tau_{\perp} = F_{\perp} h_2 \cos(\phi - \lambda) \quad (15)$$

The total torque is a sum of Equations 14 and 15 as well as the torque added from the upper body musculature, mentioned earlier in the paper. This total torque is set equal to the product of the moment of inertia of the rod $I_{c.m.}$ and the time derivative of the angular velocity of the rod $\frac{d\omega_2}{dt}$. The total torque is expressed in Equation 16.

$$C_2 + \tau_{\parallel} + \tau_{\perp} = I_{c.m.} \frac{d\omega_2}{dt} \quad (16)$$

The torque components are all shown in Figure 10 through the complete first half-cycle swing of the double pendulum. As is seen in the figure, after about 1 ms, the torque C_2 from the upper body musculature does not contribute to the swing. This is because, in a baseball swing, the upper body musculature is used to initiate the swing, and then from there the nature of the double pendulum swing takes over. At this time, the centripetal force (that makes up the perpendicular force and torque) contributes to all of the force in the system. The $\tau_{\parallel}$ and $\tau_{\perp}$ torques are initially negative due to the stop mechanism. As evident in the figure, the $\tau_{\perp}$ torque is the greatest contributor to the total torque in the system, especially since the events that occur after impact are not of importance (impact occurs around 0.35 seconds) for this study. This is because, after the rod makes impact with the ball, no extra energy can be imparted onto the ball from the rod. Since $\tau_{\perp}$ is of the most significance for this system, optimizing $F_{\perp}$ is of special interest. Thus, the perpendicular force and torque are the primary causes of rotation for the rod. If the vectors in Figure 9 were to all be exactly parallel to the rod at all points through the rods motion, then the torque on the center of mass of the rod would be at its greatest. This means that if a player were able to position his/her arms in the optimal position to pull on the bat (rod) so that the rod was in line with the direction of pull for the entire swing (or at least up until contact), then the rotation of the rod through the swing would be optimized, creating the best chance to impart the greatest amount of energy into the ball. This transfer of energy from the bat to the ball will be discussed in the next section.
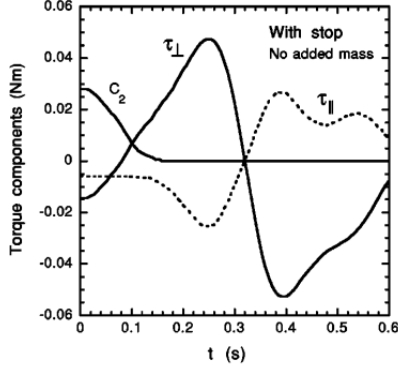
FIG. 10. All three torque components that act on the center of mass of the rod. Impact with the rod and baseball occurs around 0.35 seconds, when $\tau_\parallel$ and $\tau_\perp$ intersect.
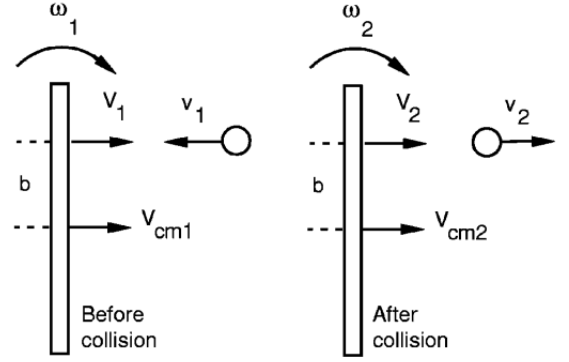

FIG. 11. Interaction between the rod and the ball. Transfer of momentum occurs on impact. Ideally, the rod and the arm components of the double pendulum are in line with one another when this impact occurs.

The transfer of momentum on impact is described in Equation 17, where M is the mass of the rod and m is the mass of the ball. The corresponding angular transfer of momentum is described in Equation 18, where I_{cm} is the moment of inertia for the rod's center of mass.

For the purposes of deriving equations, the effects of gravity on the rod were included. However, since the baseball swing occurs in a primarily horizontal plane, the effect of the upper body musculature and forces from the arm onto the rod are the only significant forces on the rod. Thus, the effects of gravity are ignored for this experiment.

$$MV_{cm1} - mv_1 = MV_{cm2} + mv_2 \qquad (17)$$

$$I_{cm}\omega_1 - mv_1 b = I_{cm}\omega_2 + mv_2 b \qquad (18)$$

The coefficent of restitution, e, describes the energy loss in the ball and ball for the collision (Equation 19). For a collision in which all the energy is transferred from the bat to the ball, $V_{cm2} = \omega_2 = 0$. With a perfect collision for which all the energy *is* transferred to the ball, Equation 20 is the result, solved for the mass of the ball. In Equation 20, $M_e = MI_{cm}/(I_{cm} + Mb^2)$ is the effective mass of the bat.

V. ENERGY TRANSFER

$$e = \frac{v_2 - V_2}{v_1 + V_1} = \frac{v_2 - V_{cm2} - b\omega_2}{v_1 + V_{cm1} + b\omega_1} \qquad (19)$$

Now that the double pendulum has been solved with respect to the arm and rod components, this section will focus on the interaction between the rod and the ball. This interaction is depicted Figure 11, with the impact of the rod and the ball. Before impact occurs, the rod at the point of impact is traveling at a velocity V_1 to the right and the ball is traveling at a velocity v_1. Although the angular velocities before and after the collision are shown (ω_1 and ω_2, respectively), it is assumed that the velocities are linear since only the moments immediately before and after impact are being analyzed. The rod's center of mass travels at velocity V_{cm1}, with a distance b between the center of mass G_2 and the point of impact. The corresponding velocities after the collision for the rod at point of impact, the rod's center of mass, and the ball are V_2, V_{cm2}, and v_2, respectively.

$$m = \frac{M_e}{[e + (1+e)v_1/V_1]} \qquad (20)$$

The effective mass for the baseball bat is the amount of mass that is actually used in the transfer of momentum with respect to how far away from the bat's center of mass the collision occurs. For example, a collision that occurs at the end of the bat results in an effective mass of about 0.4, a huge reduction! So, assuming that the collision occurs exactly at the bat's center of mass and that the coefficent of restitution e is equal to 1 (no energy lost), the mass of the baseball would need to be around 0.5 kg (using Equation 20 for an MLB average ball and bat velocity, v_1 and V_1). The mass of an actual baseball is 0.14 kg. For a more realistic collision (nonperfect) with a coefficent of restitution e equal to 0.5 and effective mass M_e

equal to 0.7 kg, the the mass of the baseball would need to be 0.35 kg (twice as massive as a conventional baseball!). For the same parameters ($e = 0.5$ and $M_e = 0.7$ kg), a bat will only come to rest if the ratio between v_1 and V_1 is 3.0. This is a highly unrealistic ratio meaning that the bat must be swung at an incredibly low speed in relation to the ball, resulting in a low kinetic energy in the bat during the swing. Also, for a realistic arm to rod ratio of about 6:1, the transfer of momentum from the arm to the rod is not all that great. This is shown in Figure 12, where a realistic arm to rod ratio was simulated using Equation 9 for the condition with and without the stop mechanism. Since a perfect transfer of momentum occurs when the bat comes to rest immediately after impact, but the arms do not come to rest at this same point of impact, much of the energy in the swing is still present in the arms of the double pendulum.
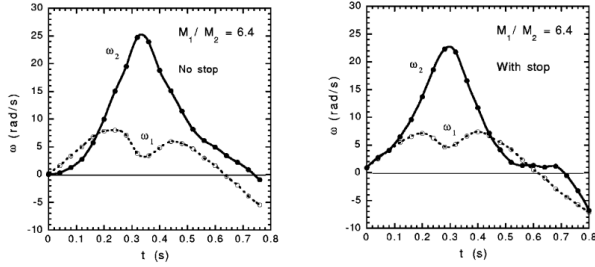

FIG. 12. Simulated swing with an arm to rod ratio of 6.4. The coupling between the arm and rod is too weak at this ratio, meaning that the arm does not come to a full rest at its minimum velocity. This implies that there is not a perfect transfer of momentum from the arm to the rod in a realistic baseball swing.

VI. CONCLUSION

Ultimately, this experiment was performed to decide whether a baseball bat or ball could be designed in such a way that all the energy from the arms and bat could be transferred perfectly to the ball in order to increase the distance the ball travels, and therefore increase on-field performance.

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