The Double Pendulum

Creating the Perfect Baseball Bat

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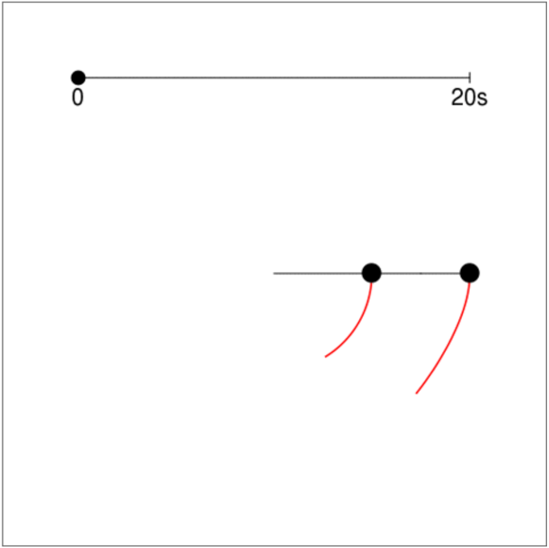
Scope

The Double Pendulum



The Double Pendulum

Chaos increases exponentially

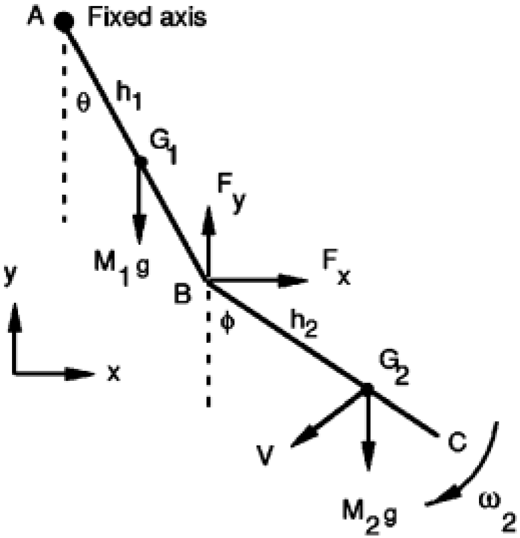
$$\Delta x(t) \sim \Delta x(t_0) e^{\lambda t} \tag{1}$$

The Double Pendulum

Chaos is not as prevalent in a baseball swing, due to the swing only occuring in the first half cycle of a double pendulum

Can we design a “perfect” baseball bat?

Equations of Motion



Equations of Motion

Coordinates (x, y) of G_2, with respect to origin A:

$$L_1 \sin\theta + h_2 \sin\phi - L_1 \cos\theta - h_2 \cos\phi \tag{2}$$

With V as the velocity of G_2, components of V are given as:

$$V_x = \frac{dx}{dt} = -L_1\omega_1 \cos\theta - h_2\omega_2 \cos\phi \tag{3}$$

$$V_y = \frac{dy}{dt} = -L_1\omega_1 \sin\theta - h_2\omega_2 \sin\phi \tag{4}$$

Equations of Motion

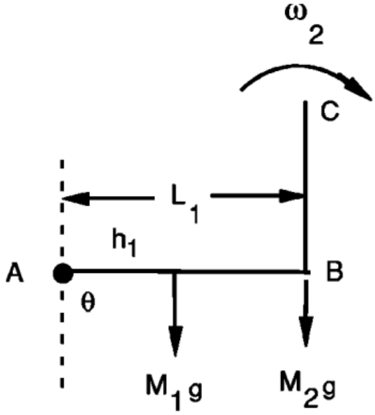
Forces arm exerts on rod:

$$\begin{aligned} F_x &= M_2 \frac{dV_x}{dt} \\ &= -M_2 \left[L_1 \cos\theta \frac{d\omega_1}{dt} + L_1 \omega_1^2 \sin\theta + h_2 \cos\phi \frac{d\omega_2}{dt} + h_2 \omega_2^2 \sin\phi \right] \end{aligned} \quad (5)$$

$$\begin{aligned} F_y - M_2 g &= M_2 \frac{dV_y}{dt} \\ &= -M_2 \left[L_1 \sin\theta \frac{d\omega_1}{dt} - L_1 \omega_1^2 \cos\theta + h_2 \sin\phi \frac{d\omega_2}{dt} - h_2 \omega_2^2 \cos\phi \right] \end{aligned}$$

Equations of Motion

Torque from muscles, C_1 is applied on arm and C_2 is applied on the rod.
The arm applies an equal and opposite torque $-C_2$ to the rod.
Initial conditions: $\theta = 90°$, $\phi = 180°$, and $\beta = \theta - \phi = 90°$.



Equations of Motion

When C_1 and C_2 equal zero, initial angular accelerations are

$$\frac{d\omega_1}{dt} = (M_1 h_1 + M_2 L_1) g / A \tag{7}$$

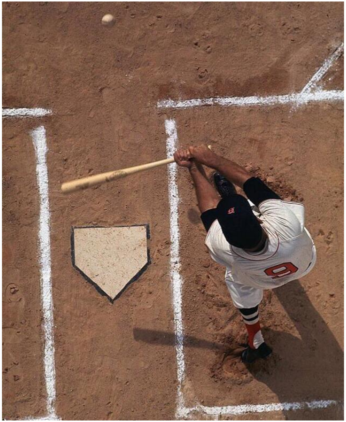
$$\frac{d\omega_2}{dt} = 0 \tag{8}$$

Rod acts as point mass at the end of the arm since it has no initial angular acceleration, so the total moment of inertia of the arm and rod is

$$I_1 + M_2 L_2^2 = A \tag{9}$$

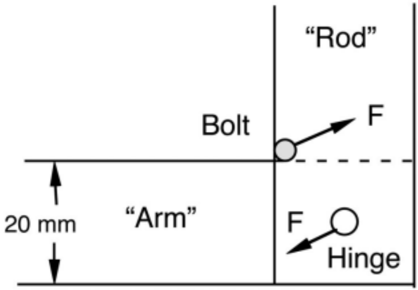
Equations of Motion

In practice, initial stage of swing is in "wrist-cock" position. After arm has rotated to about $\theta = 45^\circ$, torque from centripetal force is large enough to swing the rod without assistance from the wrist.



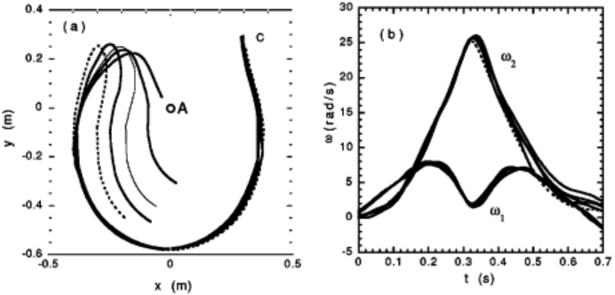
Experimental Results

With the stop mechanism, β cannot exceed 90° and the pendulum initially rotates as a rigid body



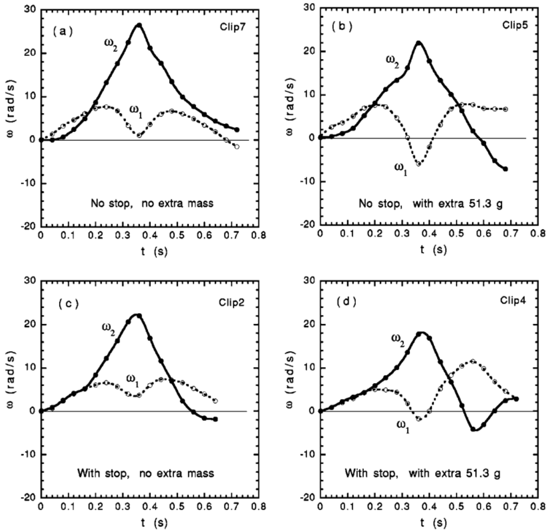
Experimental Results

Chaotic motion is highly dependent on the initial conditions of the arm and rod, however this chaos develops mostly after the first half cycle of motion.



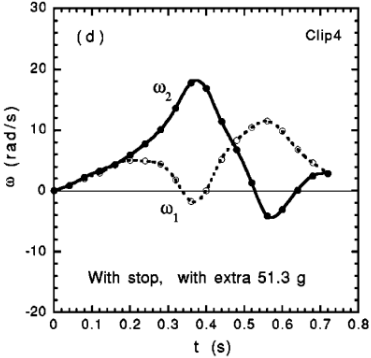
The area of interest with the double pendulum swing is the point where the lower segment of the rod reaches its maximum speed and shortly afterward.

Experimental Results



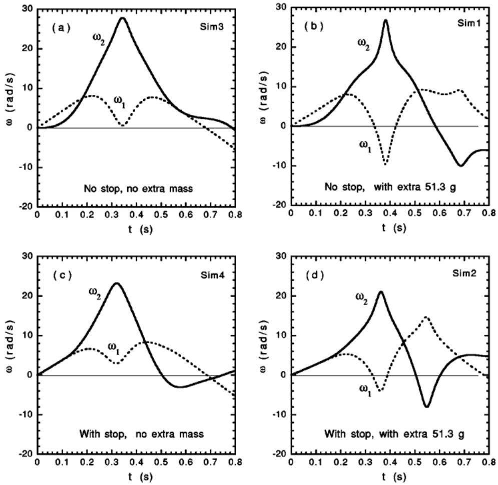
Experimental Results

- $\omega_1 = \omega_2$ for first 0.2 seconds with stop
- Angular speed of arm is at minimum when angular speed of rod is at maximum (angular momentum is transferred efficiently)



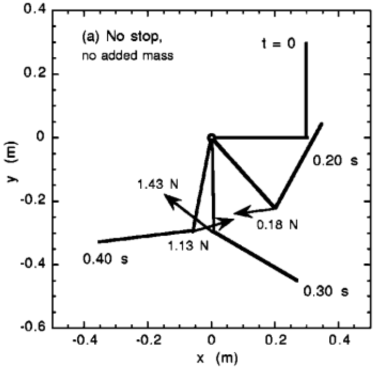
Experimental Results

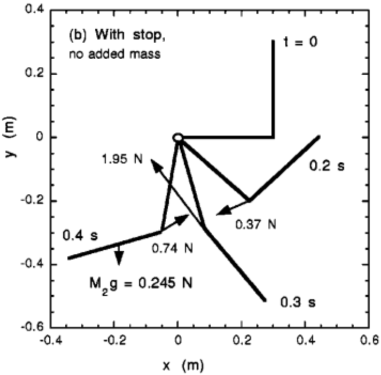
Experimental and analytical results are consistent



Torque Analysis

Forces are parallel to the rod and perpendicular to the rod's center of mass



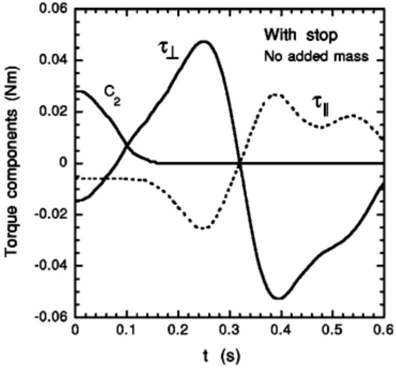


Gravitational force is very weak relative to the other forces in play

Torque Analysis

$$F_{\parallel} = \frac{F_x V_x + (F_y - M_2 g) V_y}{V} = M_2 \frac{dV}{dt} \tag{10}$$

$$F_{\perp} = \frac{(F_y - M_2 g) V_x + F_x V_y}{V} = M_2 \frac{V^2}{R} \tag{11}$$



Torque Analysis

$$\tau_{\parallel} = F_{\parallel} h_2 \sin(\phi - \lambda) \tag{12}$$

$$\tau_{\perp} = F_{\perp} h_2 \cos(\phi - \lambda) \tag{13}$$

$$C_2 + \tau_{\parallel} + \tau_{\perp} = I_{c.m.} \frac{d\omega_2}{dt} \tag{14}$$

Energy Transfer

Maximum speed of the rod results when the pendulum arm comes to a temporary stop at the instance both pendula components are vertical.

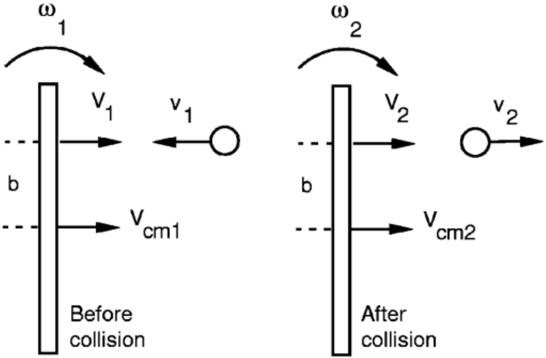
Effective mass, M_e, equals M for impact at center of mass. M_e is less than M for impact away from the center of mass.
e.g.: $\frac{M_e}{M} = 0.4$ near the tip of bat

Energy Transfer

Why does energy not get transferred efficiently from bat to ball?

- Large difference in mass between striking implement and ball
- Driving force is primarily supplied from arms and torso

Energy Transfer



$$MV_{c.m.,1} - mv_1 = MV_{c.m.,2} + mv_2 \tag{15}$$

$$I_{c.m.}\omega_1 - mv_1 b = I_{c.m.}\omega_2 + mv_2 b \tag{16}$$

Energy Transfer

$e =$ coefficient of restitution
$e = 1$ for perfectly elastic collision

$$e = \frac{v_2 - V_2}{v_1 + V_1} = \frac{v_2 - V_{c.m.,2} - b\omega_2}{v_1 + V_{c.m.,1} + b\omega_1} \tag{17}$$

If all energy from bat is transferred to ball, then $V_{c.m.,2} = \omega_2 = 0$

$$m = \frac{M_e}{[e + (1 + e)v_1/V_1]} \tag{18}$$

Context

$v_1 = V_1$, $e = 0.5$, $M_e = 0.7kg$, mass of baseball $m = 0.35kg$ (standard baseball mass $m = 0.14kg$)

For standard ball mass $m = 0.14kg$, $v_1/V_1 \approx 3$

Conclusion

A striking implement and ball could be chosen to transfer all energy from arms to the ball (not feasible for most sports)

What to look forward to

Want to look into center of percussion