

The Double Pendulum

Creating the Perfect Baseball Bat

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Scope

1 Introduction

2 Equations of Motion

3 Experimental Results

The Double Pendulum

The Double Pendulum

Chaos increases exponentially

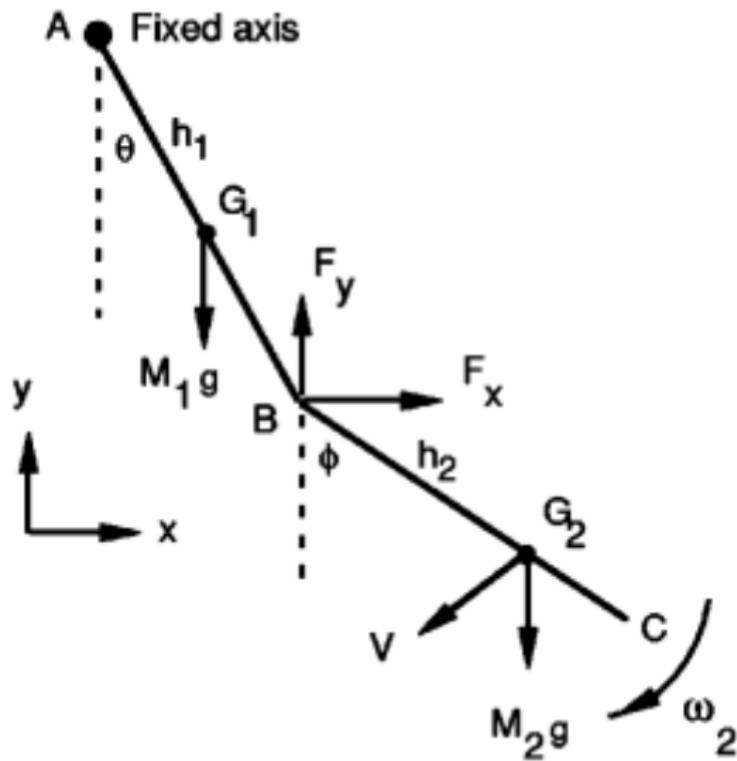
$$\Delta x(t) \sim \Delta x(t_0) e^{\lambda t} \quad (1)$$

The Double Pendulum

Chaos is not as prevalent in a baseball swing, due to the swing only occurring in the first half cycle of a double pendulum

Can we design a “perfect” baseball bat?

Equations of Motion



Equations of Motion

Coordinates (x, y) of G_2 , with respect to origin A :

$$L_1 \sin \theta + h_2 \sin \phi - L_1 \cos \theta - h_2 \sin \phi \quad (1)$$

With V as the velocity of G_2 , components of V are given as:

$$V_x = \frac{dx}{dt} = -L_1 \omega_1 \cos \theta - h_2 \omega_2 \cos \phi \quad (2)$$

$$V_y = \frac{dy}{dt} = -L_1 \omega_1 \sin \theta - h_2 \omega_2 \sin \phi \quad (3)$$

Equations of Motion

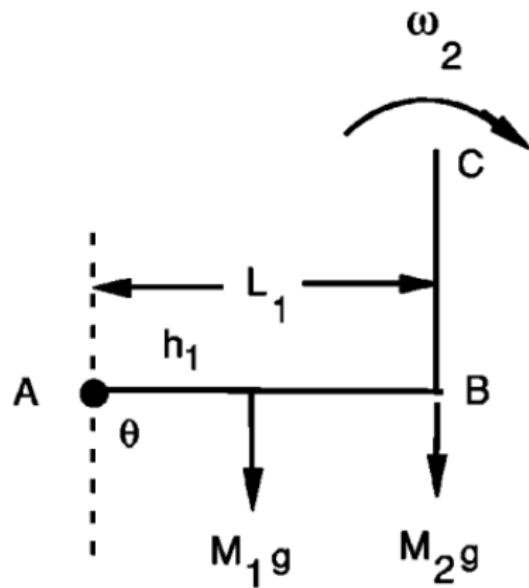
Forces arm exerts on rod:

$$\begin{aligned} F_x &= M_2 \frac{dV_x}{dt} \\ &= -M_2 \left[L_1 \cos \theta \frac{d\omega_1}{dt} + L_1 \omega_1^2 \sin \theta + h_2 \cos \phi \frac{d\omega_2}{dt} + h_2 \omega_2^2 \sin \phi \right] \end{aligned}$$

$$\begin{aligned} F_y - M_2 g &= M_2 \frac{dV_y}{dt} \\ &= -M_2 \left[L_1 \sin \theta \frac{d\omega_1}{dt} - L_1 \omega_1^2 \cos \theta + h_2 \sin \phi \frac{d\omega_2}{dt} - h_2 \omega_2^2 \cos \phi \right] \end{aligned}$$

Equations of Motion

Torque from muscles, C_1 is applied on arm and C_2 is applied on the rod.
The arm applies an equal and opposite torque $-C_2$ to the rod.
Initial conditions: $\theta = 90^\circ$, $\phi = 180^\circ$, and $\beta = \theta - \phi = 90^\circ$.



Equations of Motion

When C_1 and C_2 equal zero, initial angular accelerations are

$$\frac{d\omega_1}{dt} = (M_1 h_1 + M_2 L_1)g/A \quad (6)$$

$$\frac{d\omega_2}{dt} = 0 \quad (7)$$

Rod acts as point mass at the end of the arm since it has no initial angular acceleration, so the total moment of inertia of the arm and rod is

$$I_1 + M_2 L_2^2 = A \quad (1)$$

Equations of Motion

In practice, initial stage of swing is in "wrist-cock" position. After arm has rotated to about $\theta = 45^\circ$, torque from centripetal force is large enough to swing the rod without assistance from the wrist.



Experimental Results

With the spot mechanism, β cannot exceed 90° and the pendulum initially rotates as a rigid body

