The Double Pendulum

Creating the Perfect Baseball Bat

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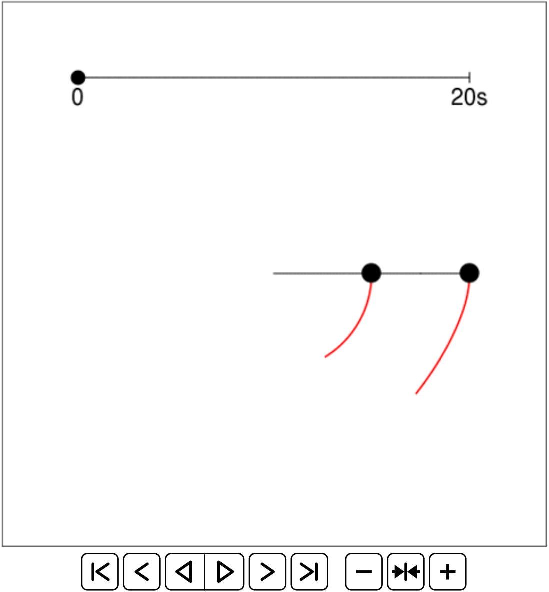
Scope

1. Introduction
2. Equations of Motion

The Double Pendulum



The Double Pendulum

Chaos increases exponentially

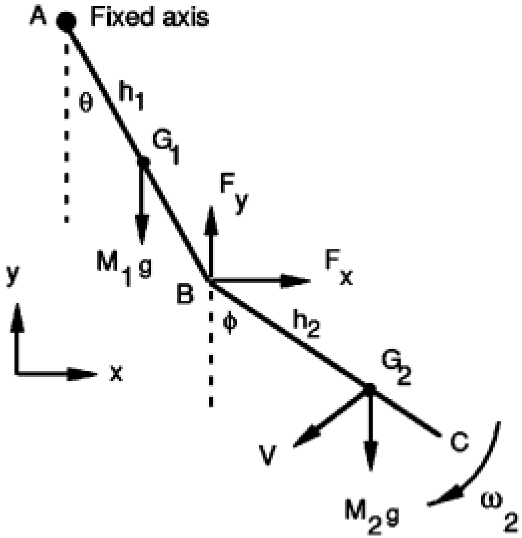
$$\Delta x(t) \sim \Delta x(t_0) e^{\lambda t} \tag{1}$$

The Double Pendulum

Chaos is not as prevalent in a baseball swing, due to the swing only occuring in the first half cycle of a double pendulum

Can we design a “perfect” baseball bat?

Equations of Motion



Equations of Motion

Coordinates (x, y) of G_2, with respect to origin A:

$$L_1 \sin\theta + h_2 \sin\phi - L_1 \cos\theta - h_2 \sin\phi \tag{1}$$

With V as the velocity of G_2, components of V are given as:

$$V_x = \frac{dx}{dt} = -L_1\omega_1 \cos\theta - h_2\omega_2 \cos\phi \tag{2}$$

$$V_y = \frac{dy}{dt} = -L_1\omega_1 \sin\theta - h_2\omega_2 \sin\phi \tag{3}$$

Equations of Motion

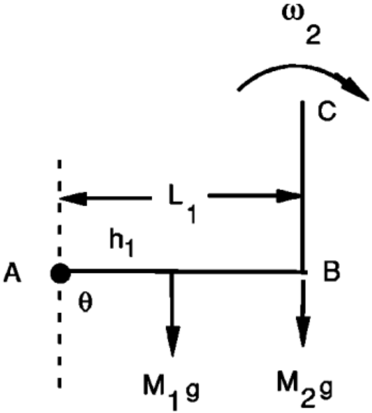
Forces arm exerts on rod:

$$
\begin{aligned}
F_x &= M_2 \frac{dV_x}{dt} \\
&= -M_2 \left[L_1 \cos\theta \frac{d\omega_1}{dt} + L_1 \omega_1^2 \sin\theta + h_2 \cos\phi \frac{d\omega_2}{dt} + h_2 \omega_2^2 \sin\phi \right]
\end{aligned}
$$

$$
\begin{aligned}
F_y - M_2 g &= M_2 \frac{dV_y}{dt} \\
&= -M_2 \left[L_1 \sin\theta \frac{d\omega_1}{dt} - L_1 \omega_1^2 \cos\theta + h_2 \sin\phi \frac{d\omega_2}{dt} - h_2 \omega_2^2 \cos\phi \right]
\end{aligned}
$$

Equations of Motion

Torque from muscles, C_1 is applied on arm and C_2 is applied on the rod.
The arm applies an equal and opposite torque $-C_2$ to the rod.
Initial conditions: $\theta = 90^\circ$, $\phi = 180^\circ$, and $\beta = \theta - \phi = 90^\circ$.



Equations of Motion

When C_1 and C_2 equal zero, initial angular accelerations are

$$\frac{d\omega_1}{dt} = (M_1 h_1 + M_2 L_1) g / A \tag{6}$$

$$\frac{d\omega_2}{dt} = 0 \tag{7}$$