Logical Connectives

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1 Mark each statement as True or False. Justify each answer.

- In order to be classified as a statement, a sentence must be true.

  False; statements must be either true or false, but not both.

- Some statements are both true and false.

  False; a statement cannot be both true and false.

- When statement p is true, then its negation $\neg p$ is false.

  True; this is the definition of negation

- A statement and its negation may both be false.

  False; this violates the definition of negation. The negation of a statement must evaluate to the opposite truth value of the statement.

- In mathematical logic, the word "or" has an inclusive meaning.

  True; by default, "or" is inclusive. There is a seperate operation for "exclusive or".

2 Mark each statement as True or False. Justify each answer.

- In an implication $p \implies q$, statement p is referred to as the proposition.

  False; p is referred to as the antecedent. The whole implication is a single proposition.

- The only case where $p \implies q$ is false is when p is true and q is false.

  True; this is the definition of implication.

- "If p, then q is equivalent to "p whenever q".

  False; a statement that if false can imply a statement that is true. "p whenever q is actually logical equivalence (*if and only if*).

- The negation of a conjunction is the disjunction of the negation of the individual parts.

  True; The negation of a conjunction is a disjunction of negations. This is often referred to as DeMorgan's Law.

- The negation of $p \implies q$ is $q \implies p$.

  False; True will still imply true and false will still imply false. The negation of implication is "p and not q".

3 Write the negation of each statement

- M is a cyclic subgroup.

  M is not a cyclic subgroup.

- The interval [0,3] is finite.

  The interval [0,3] is not finite.

- The relation R is reflexive and symmetric.

  The relation R is neither reflexive or symmetric.

- The set S is finite or denumerable.

  The set S is infinite and uncountable.

- If $x > 3$, then $f(x) > 7$.

  $x > 3$ and $f(x) \leq 7$.

- If f is continuous and A is connected, then $f(A)$ is connected.

  f is continuous and A is connected and $f(A)$ is not connected.

- If K is compact, then K is closed and bounded.

  K is compact and K is neither closed or bounded.

4 Write the negation of each statement

- The relation R is transitive.

  The relation R is not transitive.

- The set of rational numbers is bounded.

  The set of rational numbers is unbounded.

- The function f is injective and surjective.

  The function f is neither injective or surjective.

- $x < 5$ or $x > 7$.

  $x \geq 5$ and $x \leq 7$.

- If x is in A, then $f(x)$ is not in B.

  x is in A and $f(x)$ is in B.

- If f is continuous, then $f(S)$ is closed and bounded.

  f is continuous and $f(S)$ is open or unbounded.

5 Identify the antecedent and the consequent in each statement.

- M has a zero eigenvalue only if M is singular.

  Antecedent: M has a zero eigenvalue.
  Consequent: M is singular.

- Normality is a necessary condition for regularity.

  Antecedent: Regularity
  Consequent: Normality

- A sequence is bounded if it is Cauchy.

  Antecedent: A sequence is Cauchy.
  Consequent: It is bounded.

- If $x = 5$, then $f(x) = 14$.

  Antecedent: $x = 5$
  Consequent: $f(x) = 14$

6 Identify the antecedent and the consequent in each statement.

- $5n$ is odd only if n is odd.
- A sequence is convergent provided that it is monotone and bounded.
- A real sequence is Cauchy whenever it is convergent.
- Convergence is a sufficient condition for boundedness.

7 Construct a truth table for each statement.

- $p \implies \neg q$
- $[p \wedge (p \implies q)] \implies q$
- $[p \implies (q \wedge \neg q)] \iff \neg p$

8 Construct a truth table for each statement.

- $\neg p \vee q$
- $p \wedge \neg q$
- $[\neg q \wedge (p \implies q)] \implies \neg p$

9 Indicate whether each statement is True or False.

- $2 \leq 3$ and 7 is prime.
- $6 + 2 = 8$ or 6 is prime.
- 5 is not prime or 8 is prime.
- If 3 is prime, then $3^2 = 9$.
- If 3 is not prime, then $3^2 \neq 9$.
- If $3^2 = 9$, then 3 is not prime.
- If 6 is even or 4 is odd, then 6 is prime.
- If $2 < 3$ implies that $4 > 5$, then 8 is prime.
- If both $2 + 5 = 7$ and $2 \cdot 5 = 7$, then $2^2 + 5^2 = 7^2$.
- It is not the case that $2 + 3 \neq 5$.

10 Indicate whether each statement is True or False.

- 5 is odd and 3 is even.
- 5 is odd or 3 is even.
- 6 is prime or 8 is odd.
- If 7 is odd, then $7 + 7 = 10$.
- If $2 + 2 = 5$, then 5 is prime.
- If $4 > 5$, then 5 is even.
- If 5 is odd and 6 is prime, then $5 + 6 = 11$.
- If $5 \leq 3$ only if 3 is odd, then 5 is even.
- If both $2 + 5 = 7$ and $2 \cdot 5 = 10$, then $2^2 + 5^2 = 10^2$.
- It is not the case that 4 is even and 7 is not prime.