# Quantifiers

Jon Beattie

February 12, 2018

## 1 Mark each statement as True or False. Justify each answer.

(a) The symbol "$\forall$" means "for every".

(b) The negation of a universal statement is another universal statement.

(c) The symbol "$\ni$" is read "such that".

## 2 Mark each statement True or False. Justify each answer.

(a) The symbol "$\exists$" means "there exist several".

(b) If a variable is used in the antecedent of an implication without being quantified, then the universal quantifier is assumed to apply.

(c) The order in which quantifiers are used affects the truth value.

## 3 Write the negation of each statement.

(a) All the roads in Yellowstone are open.

(b) Some fish are green.

(c) No even integer is prime.

(d) $\exists x < 3 \ni x^2 \geq 10$.

(e) $\forall x$ in $A, \exists y < k \ni 0 < f(y) < f(x)$

(f) If $n > N$, then $\forall x$ in $S$, $\mid f_n(x) - f(x) \mid < \epsilon$.

4 Write the negation of each statement.

(a) Some basketball players at Central High are short.

(b) All of the lights are on.

(c) No bounded interval contains infinitely many integers.

(d) $\exists x$ in $S \ni x \geq 5$.

(e) $\forall x \ni 0 < x < 1, f(x) < 2$ or $f(x) > 5$.

(f) If $x > 5$, then $\exists y > 0 \ni x^2 > 25 + y$.

5 Determine the truth value of each statement, assuming that $x, y$, and $z$ are real numbers.

(a) $\exists x \ni \forall y \exists z \ni x + y = z$.

(b) $\exists x \ni \forall y$ and $\forall z, x + y = z$.

(c) $\forall x$ and $\forall y, \exists z \ni y - z = x$.

(d) $\forall x$ and $\forall y, \exists z \ni xz = y$.

(e) $\exists x \ni \forall y$ and $\forall z, z > y$ implies that $z > x + y$.

(f) $\forall x, \exists y$ and $\exists z \ni z > y$ implies that $z > x + y$.

6 Determine the truth value of each statement, assuming that $x, y$ and $z$ are real numbers.

(a) $\forall x$ and $\forall y, \exists z \ni x + y = z$.

(b) $\forall x \exists y \ni \forall z, x + y = z$.

(c) $\exists x \ni \forall y, \exists z \ni xz = y$.

(d) $\forall x$ and $\forall y, \exists z \ni yz = x$.

(e) $\forall x \exists y \ni \forall z, z > y$ implies that $z > x + y$.

(f) $\forall x$ and $\forall y, \exists z \ni z > y$ implies that $z > x + y$.

7 Below are two strategies for determining the truth value of a statement involving a positive number $x$ and another statement $P(x)$. Find some $x > 0$ such that $P(x)$ is true. Let $x$ be the name for any number greater than 0 and show $P(x)$ is true. For each statement below, indicate which strategy is more appropriate.

(a) $\forall x > 0, P(x)$.

(b) $\exists x > 0 \ni P(x)$.

(c) $\exists x > 0 \ni \neg P(x)$.

(d) $\forall x > 0, \neg P(x)$.

8 Which of the following best identifies $f$ as a constant function, where $x$ and $y$ are real numbers

(a) $\exists x \ni \forall y, f(x) = y$.

(b) $\forall x \exists y \ni f(x) = y$.

(c) $\exists y \ni \forall x, f(x) = y$.

(d) $\forall y \exists x \ni f(x) = y$.

9 Determine the truth value of each statement, assuming $x$ is a real number.

(a) $\exists x$ in $[2, 4] \ni x < 7$.

(b) $\forall x$ in $[2, 4], x < 7$.

(c) $\exists x \ni x^2 = 5$.

(d) $\forall x, x^2 = 5$.

(e) $\exists x \ni x^2 \neq -3$.

(f) $\forall x, x^2 \neq -3$.

(g) $\exists x \ni x \div x = 1$.

(h) $\forall x, x \div x = 1$.

10 Determine the truth value of each statement, assuming $x$ is a real number.

(a) $\exists x$ in $[3, 5] \ni x \geq 4$.

(b) $\forall x$ in $[3, 5], x \geq 4$.

(c) $\exists x \ni x^2 \neq 3$.

(d) $\forall x, x^2 \neq 3$.

(e) $\exists x \ni x^2 = -5$.

(f) $\forall x, x^2 = -5$.

(g) $\exists x \ni x - x = 0$.

(h) $\forall x, x - x = 0$.