

# Quantifiers

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1 Mark each statement as True or False. Justify each answer.

- (a) The symbol “ $\forall$ ” means “for every”.
- (b) The negation of a universal statement is another universal statement.
- (c) The symbol “ $\exists$ ” is read “such that”.

2 Mark each statement True or False. Justify each answer.

- (a) The symbol “ $\exists$ ” means “there exist several”.
- (b) If a variable is used in the antecedent of an implication without being quantified, then the universal quantifier is assumed to apply.
- (c) The order in which quantifiers are used affects the truth value.

3 Write the negation of each statement.

- (a) All the roads in Yellowstone are open.
- (b) Some fish are green.
- (c) No even integer is prime.
- (d)  $\exists x < 3 \ni x^2 \geq 10$ .
- (e)  $\forall x \text{ in } A, \exists y < k \ni 0 < f(y) < f(x)$
- (f) If  $n > N$ , then  $\forall x \text{ in } S, |f_n(x) - f(x)| < \epsilon$ .

4 Write the negation of each statement.

- (a) Some basketball players at Central High are short.
- (b) All of the lights are on.
- (c) No bounded interval contains infinitely many integers.
- (d)  $\exists x \text{ in } S \ni x \geq 5$ .
- (e)  $\forall x \ni 0 < x < 1, f(x) < 2 \text{ or } f(x) > 5$ .
- (f) If  $x > 5$ , then  $\exists y > 0 \ni x^2 > 25 + y$ .

5 Determine the truth value of each statement, assuming that  $x, y$ , and  $z$  are real numbers.

- (a)  $\exists x \ni \forall y \exists z \ni x + y = z$ .
- (b)  $\exists x \ni \forall y \text{ and } \forall z, x + y = z$ .
- (c)  $\forall x \text{ and } \forall y, \exists z \ni y - z = x$ .
- (d)  $\forall x \text{ and } \forall y, \exists z \ni xz = y$ .
- (e)  $\exists x \ni \forall y \text{ and } \forall z, z > y \text{ implies that } z > x + y$ .
- (f)  $\forall x, \exists y \text{ and } \exists z \ni z > y \text{ implies that } z > x + y$ .