Logical Connectives

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1 Mark each statement as True or False. Justify each answer.

(a) In order to be classified as a statement, a sentence must be true.

False; statements must be either true or false, but not both.

(b) Some statements are both true and false.

False; a statement cannot be both true and false.

(c) When statement p is true, then its negation $\neg p$ is false.

True; this is the definition of negation

(d) A statement and its negation may both be false.

False; this violates the definition of negation. The negation of a statement must evaluate to the opposite truth value of the statement.

(e) In mathematical logic, the word "or" has an inclusive meaning.

True; by default, "or" is inclusive. There is a seperate operation for "exclusive or".

2 Mark each statement as True or False. Justify each answer.

(a) In an implication $p \implies q$, statement p is referred to as the proposition.

False; p is referred to as the antecedent. The whole implication is a single proposition.

(b) The only case where $p \implies q$ is false is when p is true and q is false.

True; this is the definition of implication.

(c) "If p, then q is equivalent to "p whenever q".

False; a statement that if false can imply a statement that is true. "p whenever q is actually logical equivalence (*if and only if*).

(d) The negation of a conjunction is the disjunction of the negation of the individual parts.

True; The negation of a conjunction is a disjunction of negations. This is often referred to as DeMorgan's Law.

(e) The negation of $p \implies q$ is $q \implies p$.

False; True will still imply true and false will still imply false. The negation of implication is "p and not q".

3 Write the negation of each statement

(a) M is a cyclic subgroup.

M is not a cyclic subgroup.

(b) The interval [0,3] is finite.

The interval [0,3] is not finite.

(c) The relation R is reflexive and symmetric.

The relation R is neither reflexive or symmetric.

(d) The set S is finite or denumerable.

The set S is infinite and uncountable.

(e) If $x > 3$, then $f(x) > 7$.

$x > 3$ and $f(x) \leq 7$.

(f) If f is continuous and A is connected, then $f(A)$ is connected.

f is continuous and A is connected and $f(A)$ is not connected.

(g) If K is compact, then K is closed and bounded.

K is compact and K is neither closed or bounded.

4 Write the negation of each statement

(a) The relation R is transitive.

The relation R is not transitive.

(b) The set of rational numbers is bounded.

The set of rational numbers is unbounded.

(c) The function f is injective and surjective.

The function f is neither injective or surjective.

(d) $x < 5$ or $x > 7$.

$x \geq 5$ and $x \leq 7$.

(e) If x is in A, then $f(x)$ is not in B.

x is in A and $f(x)$ is in B.

(f) If f is continuous, then $f(S)$ is closed and bounded.

f is continuous and $f(S)$ is open or unbounded.

(g) If K is closed and bounded, then K is compact.

K is closed and bounded and K is not compact.

5 Identify the antecedent and the consequent in each statement.

(a) M has a zero eigenvalue only if M is singular.

Antecedent: M has a zero eigenvalue.

Consequent: M is singular.

(b) Normality is a necessary condition for regularity.

Antecedent: Regularity

Consequent: Normality

(c) A sequence is bounded if it is Cauchy.

Antecedent: A sequence is Cauchy.

Consequent: It is bounded.

(d) If $x = 5$, then $f(x) = 14$.

Antecedent: $x = 5$

Consequent: $f(x) = 14$

6 Identify the antecedent and the consequent in each statement.

(a) $5n$ is odd only if n is odd.

Antecedent: $5n$ is odd.

Consequent: n is odd.

(b) A sequence is convergent provided that it is monotone and bounded.

Antecedent: A sequence is monotone and bounded.

Consequent: It is convergent.

(c) A real sequence is Cauchy whenever it is convergent.

Antecedent: A real sequence is convergent.

Consequent: It is Cauchy

(d) Convergence is a sufficient condition for boundedness.

Antecedent: A sequence is convergent.

Consequent: It is bounded.

7 Construct a truth table for each statement.

(a) $p \implies \neg q$

p	q	$\neg q$	$p \implies \neg q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

(b) $[p \wedge (p \implies q)] \implies q$

p	q	$p \implies q$	$p \wedge (p \implies q)$	$[p \wedge (p \implies q)] \implies q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(c) $[p \implies (q \wedge \neg q)] \iff \neg p$

p	q	$\neg q$	$q \wedge \neg q$	$p \implies (q \wedge \neg q)$	$[p \implies (q \wedge \neg q)] \iff \neg p$	$\neg p$
T	T	F	F	F	T	F
T	F	T	F	F	T	F
F	T	F	F	T	T	T
F	F	T	F	T	T	T

8 Construct a truth table for each statement.

(a) $\neg p \vee q$

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

(b) $p \wedge \neg q$

p	q	$\neg q$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

(c) $[\neg q \wedge (p \implies q)] \implies \neg p$

p	q	$\neg q$	$p \implies q$	$\neg q \wedge (p \implies q)$	$\neg p$	$[\neg q \wedge (p \implies q)] \implies \neg p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F

9 Indicate whether each statement is True or False.

(a) $2 \leq 3$ and 7 is prime.

True

(b) $6 + 2 = 8$ or 6 is prime.

True

(c) 5 is not prime or 8 is prime.

False

(d) If 3 is prime, then $3^2 = 9$.

True

(e) If 3 is not prime, then $3^2 \neq 9$.

True

(f) If $3^2 = 9$, then 3 is not prime.

False

(g) If 6 is even or 4 is odd, then 6 is prime.

False

(h) If $2 < 3$ implies that $4 > 5$, then 8 is prime.

True

(i) If both $2 + 5 = 7$ and $2 \cdot 5 = 7$, then $2^2 + 5^2 = 7^2$.

True

(j) It is not the case that $2 + 3 \neq 5$.

True

10 Indicate whether each statement is True or False.

(a) 5 is odd and 3 is even.

False

(b) 5 is odd or 3 is even.

True

(c) 6 is prime or 8 is odd.

False

(d) If 7 is odd, then $7 + 7 = 10$.

False

(e) If $2 + 2 = 5$, then 5 is prime.

True

(f) If $4 > 5$, then 5 is even.

True

(g) If 5 is odd and 6 is prime, then $5 + 6 = 11$.

True

(h) If $5 \leq 3$ only if 3 is odd, then 5 is even.

True

(i) If both $2 + 5 = 7$ and $2 \cdot 5 = 10$, then $2^2 + 5^2 = 10^2$.

False

(j) It is not the case that 4 is even and 7 is not prime.

True

11 Let p be the statement "Misty is a dog," and let q be the statement "Misty is a cat." Express each of the following statements in symbols.

(a) Misty is not a cat, but she is a dog.

$\neg q \wedge p$

(b) Misty is a dog or a cat, but not both.

$(p \vee q) \wedge \neg(p \wedge q)$

(c) Misty is a dog or a cat, but she is not a cat.

$(p \vee q) \wedge \neg q$

(d) If Misty is not a dog, then Misty is a cat.

$\neg p \implies q$

(e) Misty is a dog iff she is not a cat.

$p \iff \neg q$

12 Let p be the statement "Buford got a C on the exam," and let q be the statement "Buford passed the class." Express each of the following statements in symbols.

(a) Buford did not get a C on the exam, but he passed the class.

$\neg p \wedge q$

(b) Buford got neither a C on the exam nor did he pass the class.

$\neg(p \vee q)$

(c) If Buford passed the class, he did not get a C on the exam.

$q \implies \neg p$

(d) It was necessary for Buford to get a C on the exam in order for him to pass the class.

$q \implies p$

(e) Buford passed the class only if he got a C on the exam.

$q \implies p$

13 Define a new sentential connective $\triangledown$, called *nor*, by the following truth table.

p	q	$p \triangledown q$
T	T	F
T	F	F
F	T	F
F	F	T

(a) Use a truth table to show that $p \triangledown p$ is logically equivalent to $\neg p$.

p	$\neg p$	$p \triangledown p$
T	F	F
F	T	T

(b) Complete a truth table for $(p \triangledown p) \triangledown (q \triangledown q)$.

p	q	$(p \triangledown p)$	$(q \triangledown q)$	$(p \triangledown p) \triangledown (q \triangledown q)$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	F

(c) Which of our basic connectives ($p \wedge q$, $p \vee q$, $p \implies q$, $p \iff q$) is logically equivalent to $(p \triangledown p) \triangledown (q \triangledown q)$?

It is logically equivalent to $p \wedge q$.

14 Use truth tables to verify that each of the following is a tautology. Parts (a) and (b) are called *commutative laws*, parts (c) and (d) are *associative laws*, and parts (e) and (f) are *distributive laws*.

(a) $(p \wedge q) \iff (q \wedge p)$

p	q	$p \wedge q$	$(p \wedge q) \iff (q \wedge p)$	$q \wedge p$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

(b) $(p \vee q) \iff (q \vee p)$

p	q	$p \vee q$	$(p \vee q) \iff (q \vee p)$	$q \vee p$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	T	F

(c) $[p \wedge (q \wedge r)] \iff [(p \wedge q) \wedge r]$

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$	$[p \wedge (q \wedge r)] \iff [(p \wedge q) \wedge r]$	$(p \wedge q) \wedge r$	$p \wedge q$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	F	F	T	F	F
T	F	F	F	F	T	F	F
F	T	T	T	F	T	F	F
F	T	F	F	F	T	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	T	F	F

(d) $[p \vee (q \vee r)] \iff [(p \vee q) \vee r]$

p	q	r	$q \vee r$	$p \vee (q \vee r)$	$[p \vee (q \vee r)] \iff [(p \vee q) \vee r]$	$(p \vee q) \vee r$	$p \vee q$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	F
F	F	F	F	F	T	F	F

(e) $[p \wedge (q \vee r)] \iff [(p \wedge q) \vee (p \wedge r)]$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$[p \wedge (q \vee r)] \iff [(p \wedge q) \vee (p \wedge r)]$	$(p \wedge q) \vee (p \wedge r)$	$p \wedge q$	$p \wedge r$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	F
T	F	T	T	T	T	T	F	T
T	F	F	F	F	T	F	F	F
F	T	T	T	F	T	F	F	F
F	T	F	T	F	T	F	F	F
F	F	T	T	F	T	F	F	F
F	F	F	F	F	T	F	F	F

(f) $[p \vee (q \wedge r)] \iff [(p \vee q) \wedge (p \vee r)]$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$[p \vee (q \wedge r)] \iff [(p \vee q) \wedge (p \vee r)]$	$(p \vee q) \wedge (p \vee r)$	$p \vee q$	$p \vee r$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	F	T	F
F	F	T	F	F	T	F	F	T
F	F	F	F	F	T	F	F	F