Moderated Network Models

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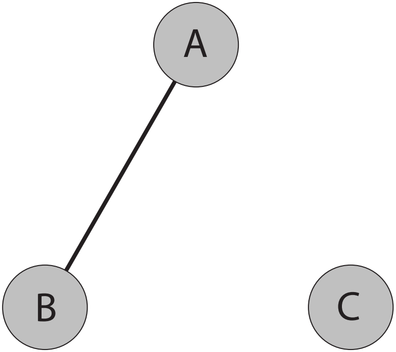
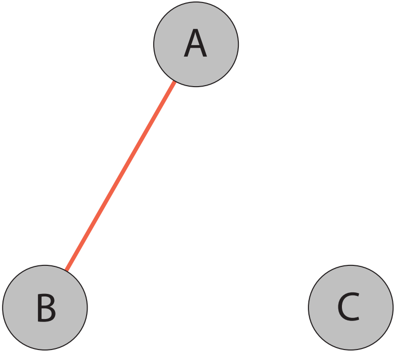
Columbia University, NYC, July 12

Pairwise Network Models



$$P(A, B, C) = \exp\{\dots \alpha_A A + \alpha_B B + \alpha_C C + \beta_{AB} AB \dots\}$$

Pairwise Network Models



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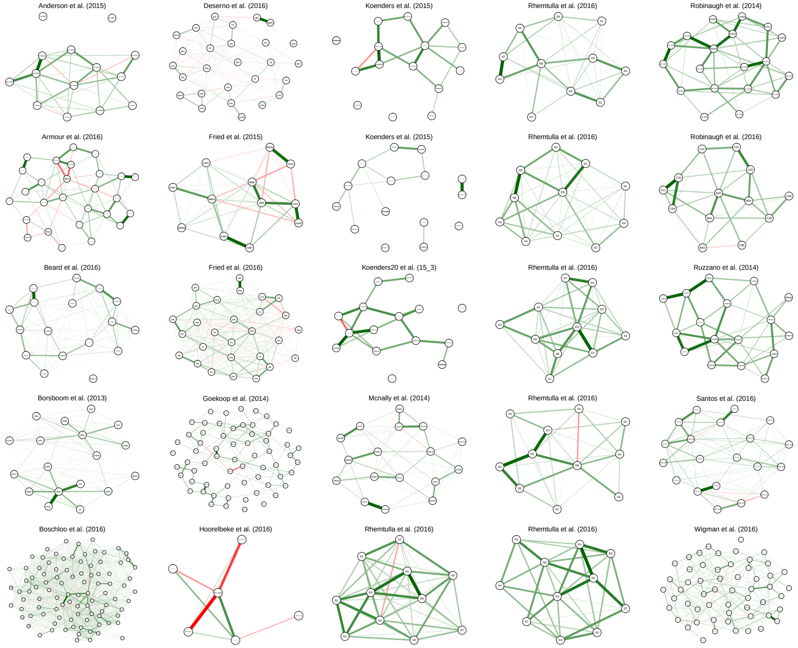

Anderson et al. (2015)
Deserno et al. (2016)
Koenders et al. (2015)
Rhemtulla et al. (2016)
Robinaugh et al. (2014)
Armour et al. (2016)
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Koenders et al. (2015)
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Koenders20 et al. (15_3)
Rhemtulla et al. (2016)
Ruzzano et al. (2014)
Borsboom et al. (2013)
Goekoop et al. (2014)
Mcnally et al. (2014)
Rhemtulla et al. (2016)
Santos et al. (2016)
Boschloo et al. (2016)
Hoorelbeke et al. (2016)
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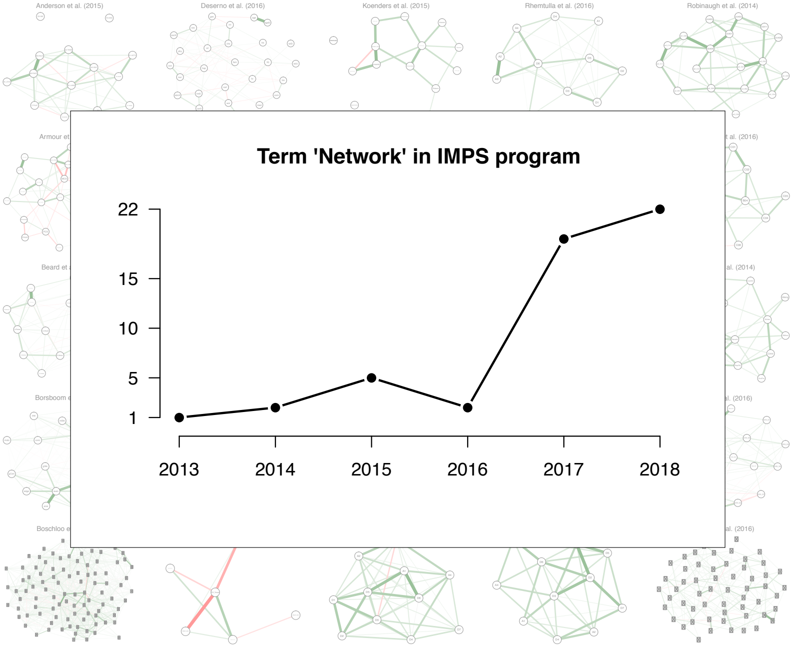
Rhemtulla et al. (2016)
Wigman et al. (2016)


Anderson et al. (2015)
Deserno et al. (2016)
Koenders et al. (2015)
Rhemtulla et al. (2016)
Robinaugh et al. (2014)
Term 'Network' in IMPS program
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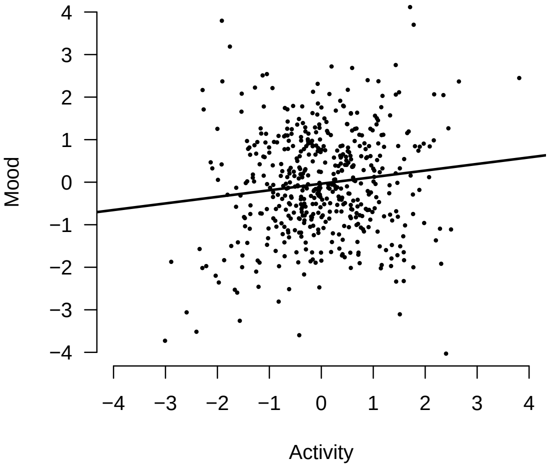
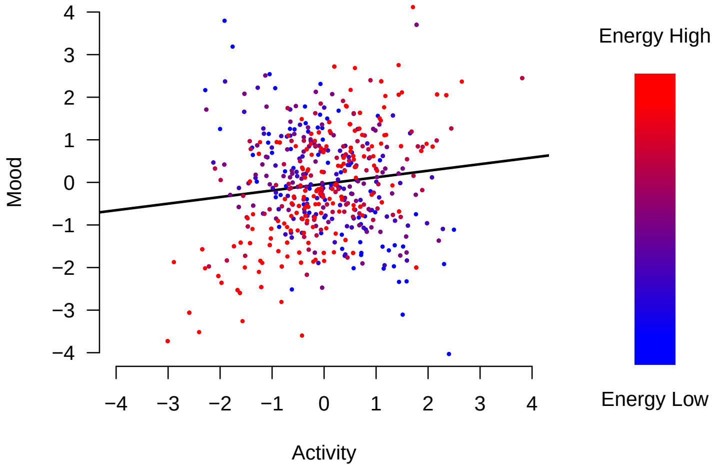
Are there only pairwise interactions in psychology?

Importance of Detecting Moderators

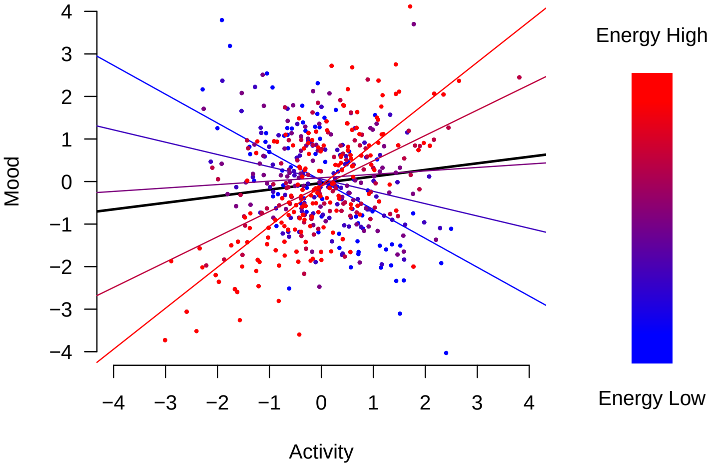


$$\text{Mood} = 0.2 \times \text{Activity} + \epsilon$$

Importance of Detecting Moderators

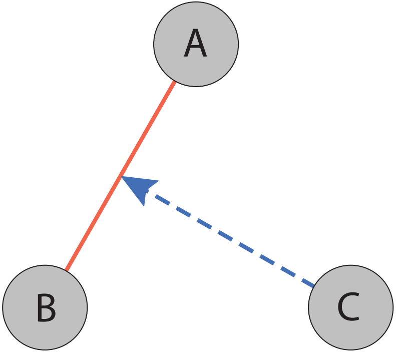


Importance of Detecting Moderators



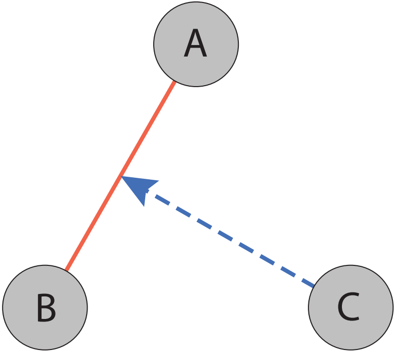
$$\text{Mood} = (0.2 + 0.75 \times \text{Energy}) \times \text{Activity} + \epsilon$$

Moderated Network Models



$$P(A,B,C) = \exp\{\ldots \alpha_A A + \alpha_B B + \alpha_C C + \beta_{AB} AB + \omega_{ABC} ABC \ldots\}$$

From Interactions to Moderation



$$\begin{aligned} P(A, B, C) &= \exp\{\ldots \alpha_A A + \alpha_B B + \alpha_C C + \beta_{AB} AB + \omega_{ABC} ABC \ldots\} \\ &= \exp\{\ldots \alpha_A A + \alpha_B B + \alpha_C C + (\beta_{AB} + \omega_{ABC} C) AB \ldots\} \end{aligned}$$

Moderated Network Model (MNM) Joint Distribution

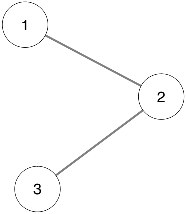
$$P(X) = \exp\left\{ \sum_{i}^{p} \alpha_i \frac{X_i}{\sigma_i} + \sum_{\substack{i,j \in V \\ i \neq j}} \beta_{i,j} \frac{X_i}{\sigma_i} \frac{X_j}{\sigma_j} + \right.$$

$$\left. \sum_{\substack{i,j,q \in V \\ i \neq j \neq q}} \omega_{i,j,q} \frac{X_i}{\sigma_i} \frac{X_j}{\sigma_j} \frac{X_q}{\sigma_q} + \sum_{i}^{p} \frac{X_i^2}{\sigma_i^2} - \Phi(\alpha, \beta, \omega) \right\}$$

- Intercepts
- Pairwise interactions
- Moderation effects / 3-way interactions

Yang et al. (2014); Chen et al. (2014); Haslbeck et al. (2018)

Visualizing MNMs with factor graphs



$$P(X) = \exp\left\{ \dots \sum_{\substack{i,j \in V \\ i \neq j}} \beta_{i,j} \frac{X_i}{\sigma_i} \frac{X_j}{\sigma_j} + \sum_{\substack{i,j,q \in V \\ i \neq j \neq q}} \omega_{i,j,q} \frac{X_i}{\sigma_i} \frac{X_j}{\sigma_j} \frac{X_q}{\sigma_q} + \dots \right\}$$

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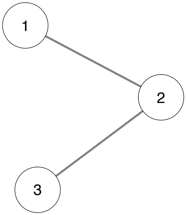
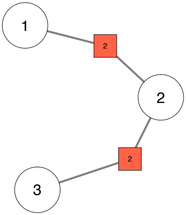
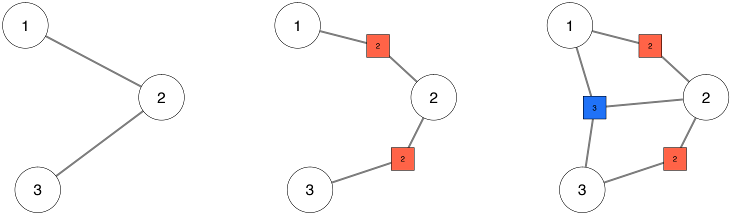




$$P(X) = \exp\left\{ \dots \sum_{\substack{i,j \in V \\ i \neq j}} \beta_{i,j} \frac{X_i}{\sigma_i} \frac{X_j}{\sigma_j} + \sum_{\substack{i,j,q \in V \\ i \neq j \neq q}} \omega_{i,j,q} \frac{X_i}{\sigma_i} \frac{X_j}{\sigma_j} \frac{X_q}{\sigma_q} + \dots \right\}$$

Visualizing MNMs with factor graphs



$$P(X) = \exp\left\{ \dots \sum_{\substack{i,j \in V \\ i \neq j}} \beta_{i,j} \frac{X_i}{\sigma_i} \frac{X_j}{\sigma_j} + \sum_{\substack{i,j,q \in V \\ i \neq j \neq q}} \omega_{i,j,q} \frac{X_i}{\sigma_i} \frac{X_j}{\sigma_j} \frac{X_q}{\sigma_q} + \dots \right\}$$

MNM Conditionals

$$P(X_s|X_{\setminus s}) = \exp\left\{ \alpha_s \frac{X_s}{\sigma_s} + \sum_{\substack{i \in V \\ i \neq s}} \beta_{i,s} \frac{x_i}{\sigma_i} \frac{X_s}{\sigma_s} + \right.$$

$$\left. \sum_{\substack{i,j \in V \\ i \neq j \neq s}} \omega_{i,j,s} \frac{x_i}{\sigma_i} \frac{x_j}{\sigma_j} \frac{X_s}{\sigma_s} + \frac{X_s^2}{\sigma_s^2} - \Phi^*(\alpha, \beta, \omega) \right\}$$

(... some algebra ...)

$$P(X_s|X_{\setminus s}) = \frac{1}{\sqrt{2\pi}} \exp\left\{ \frac{(X_s - \mu_s)^2}{2} \right\}$$

with

$$\mu_s = \alpha_s + \sum_{\substack{i \in V \\ i \neq s}} \beta_{i,s} x_i + \sum_{\substack{i,j \in V \\ i \neq j \neq s}} \omega_{i,j,s} x_i x_j$$

Estimating MNMs via Nodewise Regression

Nodewise estimation:

1. For each $s \in \{1, 2, \ldots, p\}$ estimate nodewise regression $\mathbb{E}[X_s | X_{\setminus s}] = \mu_s$ with (LASSO) ℓ_1-penality
2. Combine estimates across regressions using the OR- or AND-rule

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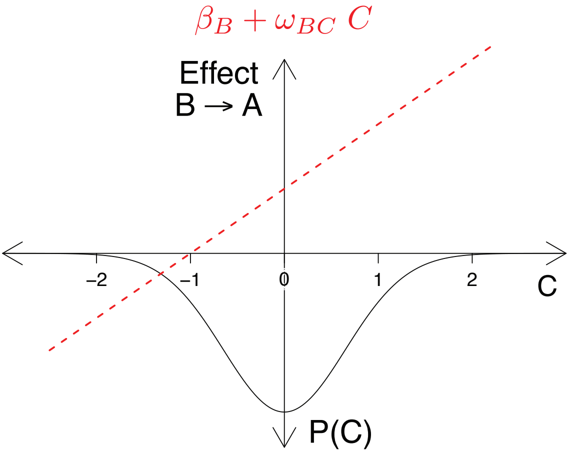
Why ℓ_1-penalty?

1. Reduces variance of estimates
2. Sets small parameter estimates to zero
3. Ensures that the model is identified (e.g., $p = 20$ with all moderation effects requires $n \geq 190$ observations)

Meinshausen & Buehlmann (2006); Haslbeck & Waldorp (2018)

Alternative to detect moderation: Split-sample Methods



E.g.: Network Comparison Test (NCT), Fused Graphical Lasso (FGL)
van Borkulo et al. (2016); Danaher et al. (2014); Costantini et al. (2017)

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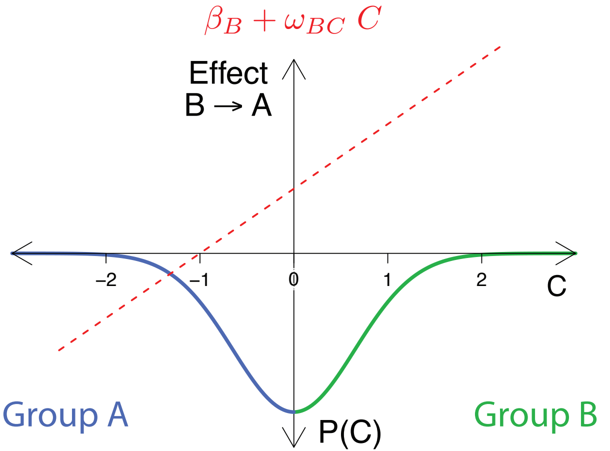

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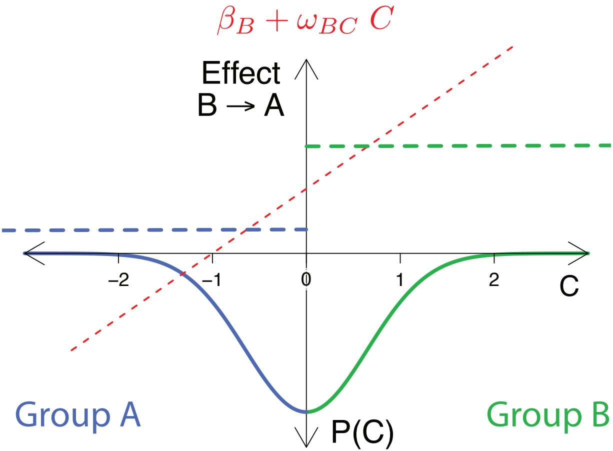

E.g.: Network Comparison Test (NCT), Fused Graphical Lasso (FGL)
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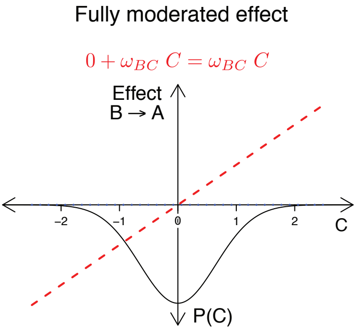
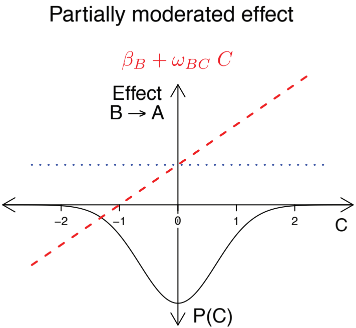
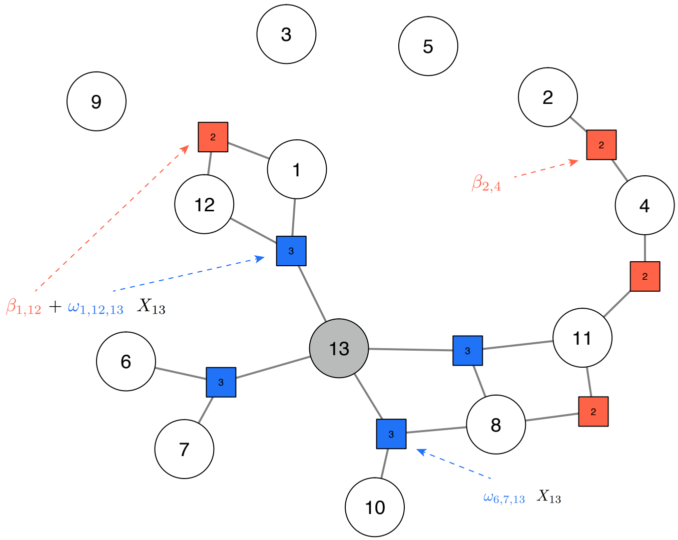
Simulation Setup: Types of Moderation

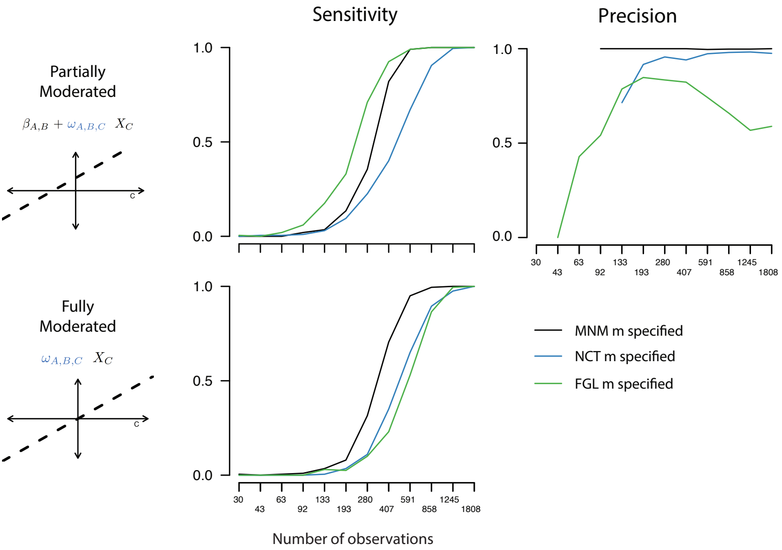




Simulation Setup: The Network Structure



Simulation Results

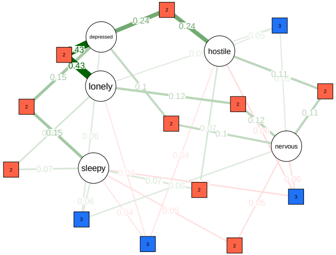


Implementation: R-package *mgm*

```
# Install & load package
install.packages("mgm")
library(mgm)

# Estimate MNM
mgm_mod <- mgm(data = msq_p5,
               type = rep("g", 5
               level = rep(1, 5)
               lambdaSel = "EBIC
               lambdaGam = .5,
               ruleReg = "AND",
               moderators = 1:5,
               scale = TRUE)

# Visualize MNM as factor graph
FactorGraph(object = mgm_mod,
            edge.labels = TRUE,
            labels = colnames(msq_p5))
```



Haslbeck & Waldorp (2018); Haslbeck, Borsboom & Waldorp (2018)

Discussion & Future Directions

1. Applications of MNMs
   - Capture higher order structures
   - Influence of contextual variables on network structure
   - Opens window for interventions on edges
   - Possible explanation for contradictory empirical results
2. Normalizability of MNM Joint Distribution
3. Alternatives to nodewise regression (e.g. Block coordinate algorithm)
4. Model Misspecification & robust estimation
5. Extension to Mixed Graphical Models

Statistics > Methodology

Moderated Network Models

Jonas Haslbeck, Denny Borsboom, Lourens Waldorp

(Submitted on 8 Jul 2018)

Pairwise network models such as the Gaussian Graphical Model (GGM) are a powerful and intuitive way to analyze dependencies in multivariate data. A key assumption of the GGM is that each pairwise interaction is independent of the values of all other variables. However, in psychological research this is often implausible. In this paper, we extend the GGM by allowing each pairwise interaction between two variables to be moderated by (a subset of) all other variables in the model, and thereby introduce a Moderated Network Model (MNM). We show how to construct the MNW and propose an L1-regularized nodewise regression approach to estimate it. We provide performance results in a simulation study and show that MNMs outperform the split-sample based methods Network Comparison Test (NCT) and Fused Graphical Lasso (FGL) in detecting moderation effects. Finally, we provide a fully reproducible tutorial on how to estimate MNMs with the R-package mgm and discuss possible issues with model misspecification.

Subjects: **Methodology (stat.ME)**
Cite as: arXiv:1807.02877 **[stat.ME]**
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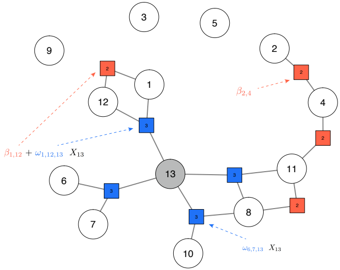
Moderated Network Models: Summary

1. Moderated Network Models extend the GGM with moderation effects
2. Visualization with Factor Graph
3. Outperform split-sample methods in detecting moderation effects
4. Implementation: R-package *mgm*



Website: `www.jonashaslbeck.com` (preprint and slides)
Email: `jonashaslbeck@gmail.com`

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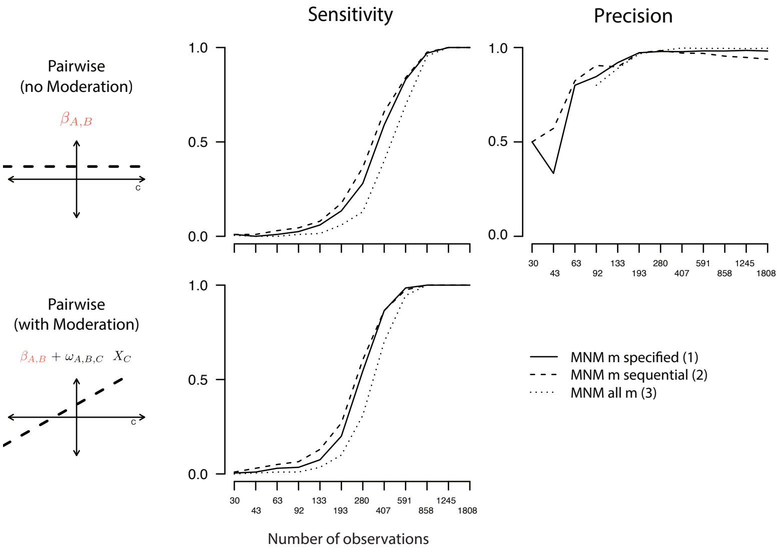
Extra

Simulation Results (Extended)



Simulation Results (Extended)