The product rule of probability - Contribution 1

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1 Stating the rule

$$P(A, B \,|\, I) = P(A \,|\, B,\, I) \times P(B \,|\, I) \tag{1}$$

Qualitatively, the rule (1) states that the probability that two events A and B are true, given the known background information (I), is given by the probability that A is true, given that B is true and the known background information, divided by the probability that B is true.

2 Deducing the rule

The product rule of probability may be deducted from the following definition[2]:

$$P(A \,|\, B, I) = \frac{P(A, B \,|\, I)}{P(B \,|\, I)} \tag{2}$$

The definition stated in equation (2) stands for the conditional probability of an event A given an event B. By multiplying both sides of equation (2) by $P(B \,|\, I)$, we get to:

$$P(A \,|\, B, I) \times P(B \,|\, I) = \frac{P(A, B \,|\, I)}{P(B \,|\, I)} \times P(B \,|\, I)$$

$$P(A, B \,|\, I) = P(A \,|\, B,\, I) \times P(B \,|\, I) \tag{3}$$

3 An application of the rule

The product rule of probability is widely used in genetics, which is a branch of biology and medicine. For instance, when it is necessary to know the probability that two events will occur simultaneously, this rule is very useful. See the following example: suppose that, when a female of a certain species is pregnant, the probability of being pregnant with a female is given by $\frac{1}{2}$, while the probability that an individual of the offspring will have a long tail is given by $\frac{1}{10}$; considering that both events are independent, the probability that a female will give birth to a female with a long tail is given by

$$\frac{1}{2} \times \frac{1}{10} = \frac{1}{20}$$

4 References

1. D. S. Sivia, J. Skilling. *Data Analysis - a Bayesian Tutorial.* Page 5. 2^{nd} Edition. Oxford Science Publications.

2. F. M. Dekking, C. Kraaikamp, H. P. Lopuhaa, L. E. Meester. *A Modern Introduction to Probability and Statistics - Understanding Why and How.* Pages 26 and 27. 1^{st} Edition. Springer.

3. https://www.khanacademy.org/science/high-school-biology/hs-classical-genetics/hs-introduction-to-heredity/a/probabilities-in-genetics