HOMEWORK 4

Due: next Friday

1. Consider an economy with two infinitely-lived consumer-workers: one high-skilled and one low-skilled. There is capital accumulation as in the optimal growth model, with a constant-returns-to-scale, strictly quasi-concave and twice differentiable production function $F(k, n_h, n_l)$, where n_s and n_l are the inputs of skilled and unskilled labor, respectively, and a capital depreciation rate of δ. High- and low-skilled workers have identical, typical time-additive preferences with period utility $u(c)$, so that leisure is not valued, and discount factor β; both workers inelastically supply 1 unit of labor (so that $n_h = n_l = 1$ in any efficient allocation).

   In a competitive equilibrium, firms hire labor of both kinds and rent capital in competitive markets. Both workers can borrow and save unrestrictedly (but subject to a no-Ponzi game constraint). Their initial endowments of capital are k_{h0} and k_{l0}, respectively.

   (a) Define a sequential competitive equilibrium with a proportional tax rate, $\tau > 0$, on capital income, with a balanced government budget period by period and a lump-sum transfer of any proceeds from taxing capital income.

   (b) Define a steady state for this economy. Hint: there is a 1-dimensional continuum of steady states. Interpret the fact that there is

   (c) Compare the steady-state capital stock for $\tau > 0$ to that for $\tau = 0$.

   (d) How do steady-state relative wages of the two workers vary across $\tau > 0$ and $\tau = 1$ for the following two cases: (i) $F(k, n_h, n_l) = k^\alpha g(n_h, n_l)^{1-\alpha}$, where g is a CES aggregate; (ii) $F(k, n_h, n_l) = (k + n_l)^\alpha n_h^{1-\alpha}$? Provide intuition.

   (e) What is the marginal effect on present-value utility of each of the two workers when τ (a tax rate applied on capital income at all points in time) is increased from 0 to an infinitesimal number above zero? (I.e., think of taking the derivative of the indirect utility function of the two workers with respect to τ. You should not do any explicit math here but rather try to provide a heuristic answer.)

2. Consider a standard Pissarides-85 model in steady state. Assume a Cobb-Douglas matching function.

   (a) Suppose the productivity of all jobs rises permanently, from p to $p + \Delta$. What would happen to labor-market tightness over time as a result of this event? You are not supposed to derive an explicit answer but give qualitative properties of an answer.

(b) Suppose there is a credible announcement that the productivity of all jobs will rise permanently from p to $p + \Delta$, beginning in 1 year from the announcement. What would happen to labor-market tightness over time as a result of this event? Again, you should not derive an explicit answer. Instead simply qualitatively compare (using your intuition for how the model works) your answer to your answer to the first sub-question.

(c) Suppose the matching-function productivity rises permanently to a new level. What would happen to labor-market tightness and unemployment over time as a result of this event?

3. Consider an economy with a continuum of islands. Each island is indexed by its productivity level p, distributed with a cumulative function $F(p)$. On each island there is a large number of firms operating a linear technology in labor, $y = pn$, where n is the number of workers employed. In every period, there is a perfectly competitive spot market for labor on every island. An employed worker is subject to exogenous separations at rate σ: the flow probability with which a worker is kicked off the island. Upon separation, she enters the unemployment pool (literally, a big swimming pool). Unemployed workers search for employment and at rate λ_u they find an island, drawn randomly from F. Upon reaching an island, the worker can decide not to join that island but to keep searching. First, find the steady-state distribution of wages across islands. Second, derive an equation determining the reservation wage for an unemployed worker. How does it compare to the reservation wage facing a worker in the standard partial-equilibrium McCall search model?