EC321: Problem Set 3 Question 3

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12 July 2011

Inflation and expectations about future monetary policy

- Take the NKPC

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

and write it down for the next period, $t+1$,

$$\pi_{t+1} = \beta E_{t+1} \pi_{t+2} + \kappa x_{t+1}$$

and plug this into the time t NKPC (for π_{t+1}),

$$\pi_t = \beta E_t \left(\beta E_{t+1} \pi_{t+2} + \kappa x_{t+1} \right) + \kappa x_t$$

then write down the NKPC for period $t+2$ and substitute again, this time for π_{t+2}, to get

$$\pi_t = \beta E_t \left(\beta E_{t+1} \left(\beta E_{t+2} \pi_{t+3} + \kappa x_{t+2} \right) + \kappa x_{t+1} \right) + \kappa x_t$$

and so on. Since $E_t \left(E_{t+j} \pi_{t+j+i} \right) = E_t \pi_{t+j+i}$ for all $i, j > 0$ (this is called the *law of iterated expectations*) the above becomes

$$\pi_t = \beta^3 E_t \pi_{t+3} + \beta^2 \kappa E_t x_{t+2} + \beta \kappa E_t x_{t+1} + \kappa x_t.$$

- Repeating this process ad infinitum we obtain

$$\pi_t = \kappa \left(x_t + \beta E_t x_{t+1} + \beta^2 E_t x_{t+2} + \beta^3 E_t x_{t+3} + \cdots \right)$$

- If the central bank announces at time t that it will reduce the output gap from period $t+2$ onwards, then $E_t x_{t+2}, E_t x_{t+3}, \ldots$ will fall. This means that current π_t will fall.
- If we write down the same equation for π_{t+1},

$$\pi_{t+1} = \kappa \left(x_{t+1} + \beta E_{t+1} x_{t+2} + \beta^2 E_{t+1} x_{t+3} + \cdots \right)$$

  we notice that π_{t+1} will fall as well (by more than π_t).
- If the policy announcement is not credible, however, nothing will happen until the policy is implemented.

- We can do the same this with the IS curve,

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}).$$

Writing this curve for periods $t+1, t+2, \ldots$ and recursively plugging into each other (exercise!) yields

$$x_t = -\sigma \left(E_t \left(i_t - \pi_{t+1} \right) + E_t \left(i_{t+1} - \pi_{t+2} \right) + \cdots \right).$$

Again, we need to use the law of iterated expectations.

- Now if the CB announces higher interest rates in the future, the current output gap will fall (if the announcement is credible).

$$\begin{aligned} \pi_t &= \kappa \left(x_t + \beta E_t x_{t+1} + \beta^2 E_t x_{t+2} + \beta^3 E_t x_{t+3} + \cdots \right) \\ x_t &= -\sigma \left(E_t \left(i_t - \pi_{t+1} \right) + E_t \left(i_{t+1} - \pi_{t+2} \right) + \cdots \right) \end{aligned}$$

- Therefore higher interest rates in the future imply a lower output gap, which in turn implies lower inflation (that was point (b)). Thus the real interest rate increases (Fisher equation).