Trade and the End of Antiquity

A Quantitative Investigation of the Pirenne Thesis

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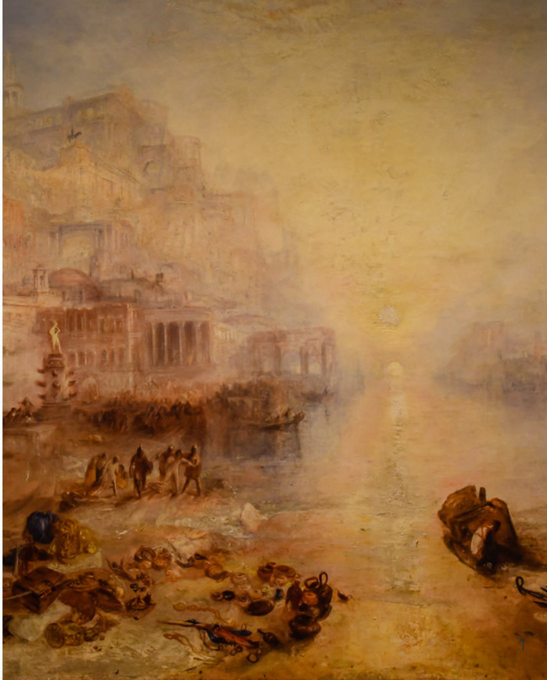

What caused the End of Antiquity?

- Antiquity: Roman and Greek civilizations centered around the Mediterranean
- End of antiquity circa 7th-8th Century AD:
  - Economic activity shifts away from the Mediterranean.
  - Rise of Northern Europe (Charlemagne).

$\hookrightarrow$ *Question*: What caused the End of Antiquity?

- Discussed, among others, by Montesquieu (1734), Voltaire (1756), Gibbon (1789)

Pirenne Hypothesis

Henri Pirenne (1937), *“Mahomet et Charlemagne”*

- Rise of the Islamic Caliphate disrupts Mediterranean trade/exchanges.
- Causes a shift of economic activity away from the Mediterranean.
- Rise of the Carolingian Empire in Northern Europe.

Political changes in the Mediterranean: 600 AD

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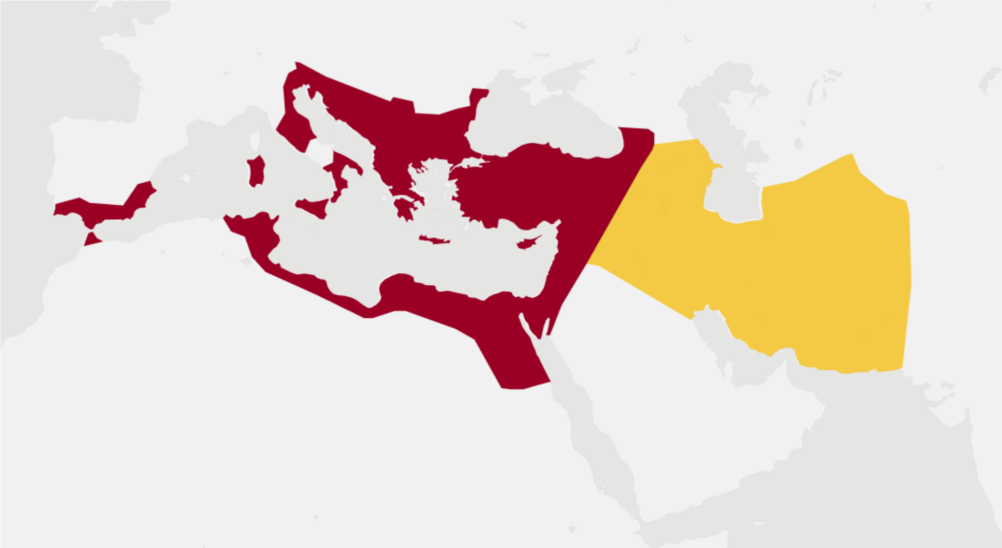
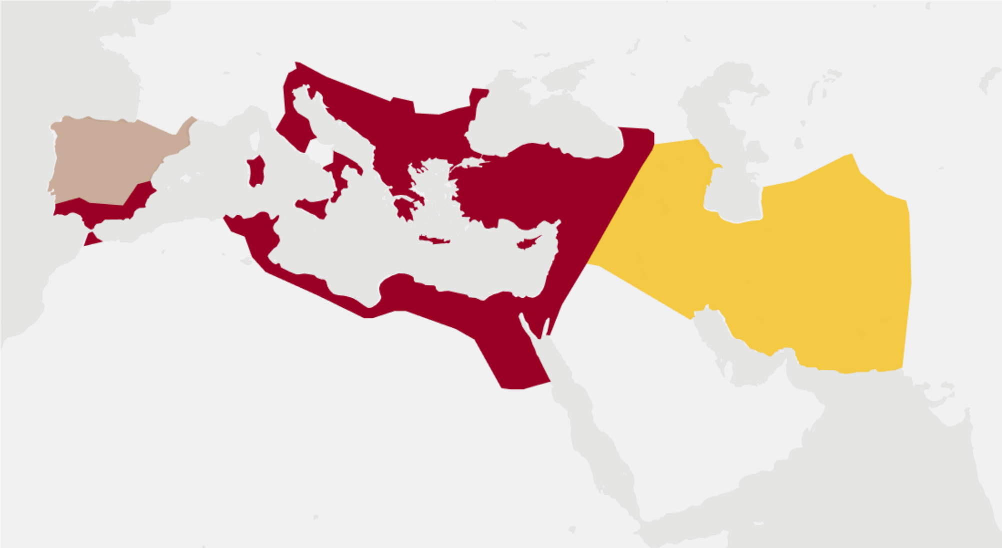
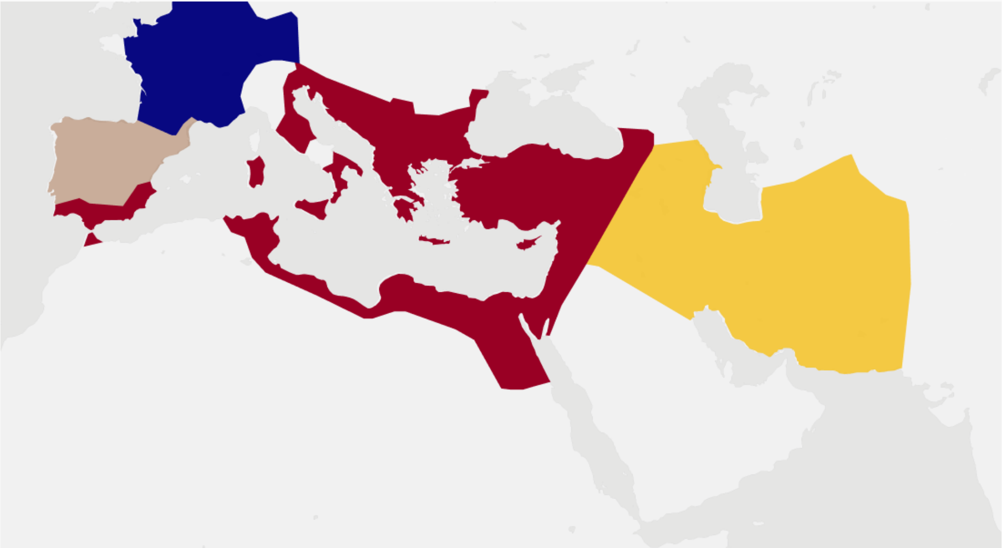


Political changes in the Mediterranean: 600 AD



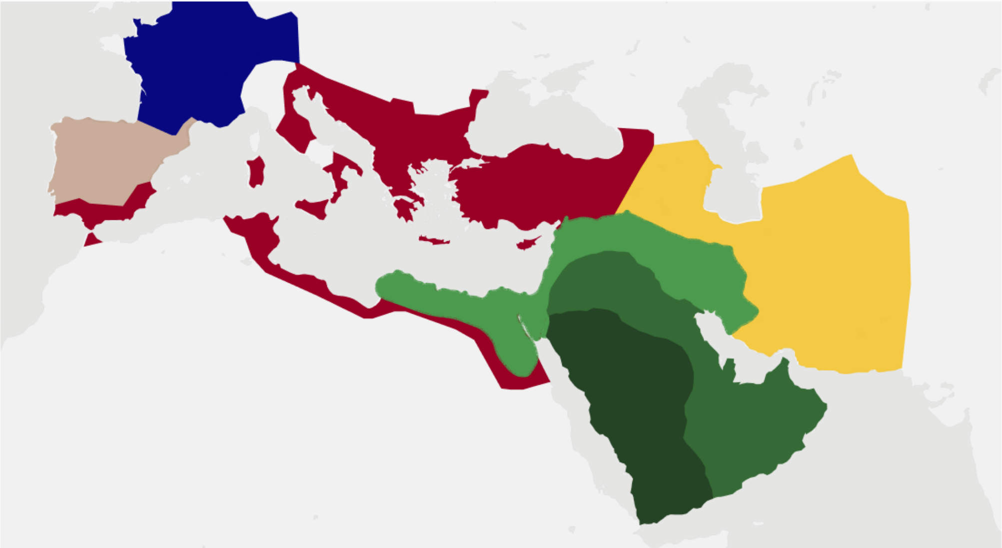
Political changes in the Mediterranean: 600 AD



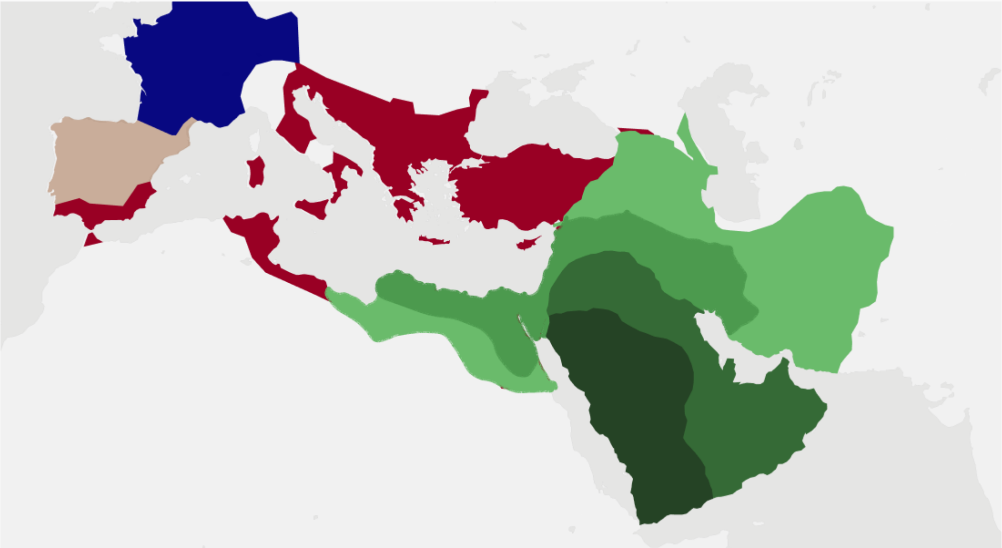
Political changes in the Mediterranean: 632 AD

Political changes in the Mediterranean: 634 AD

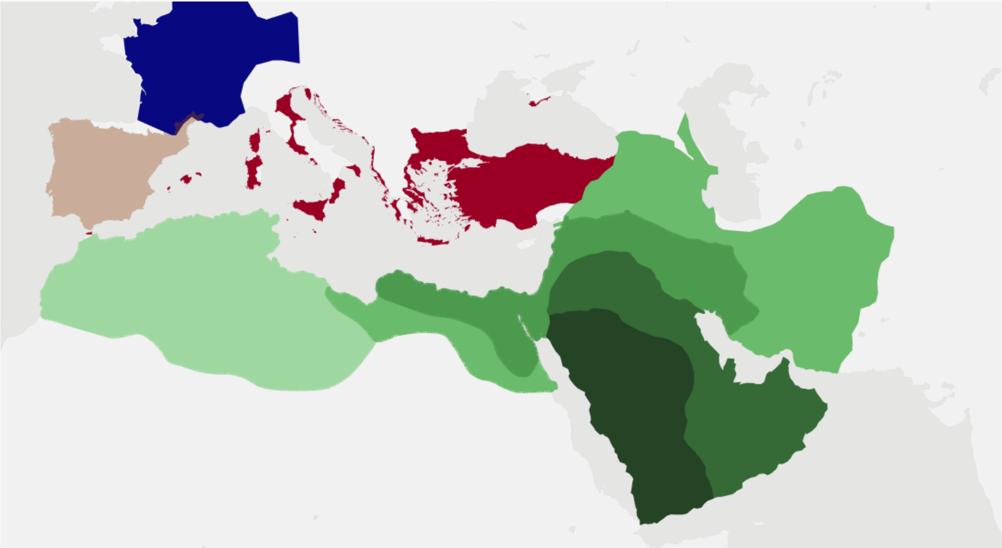
Political changes in the Mediterranean: 644 AD



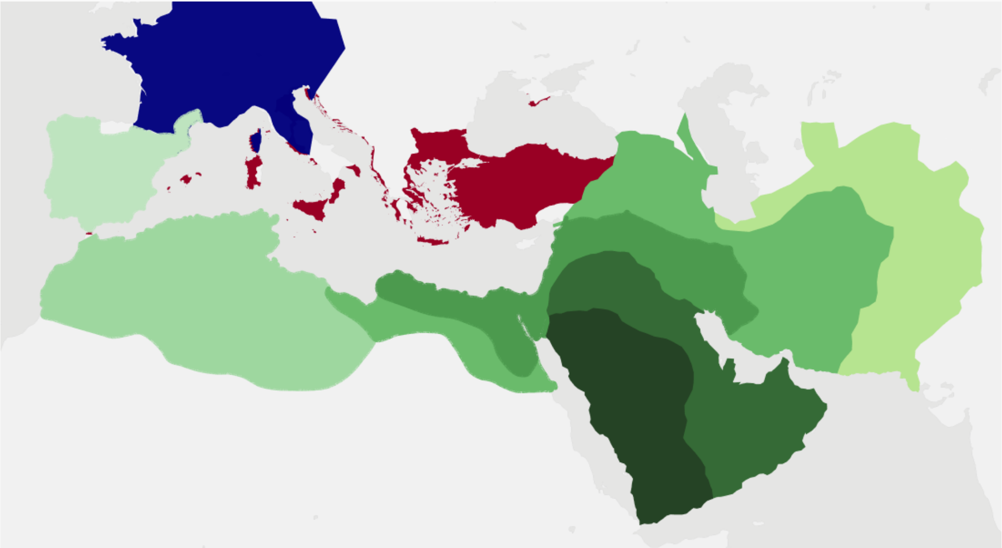
Political changes in the Mediterranean: 661 AD



Political changes in the Mediterranean: 661-700 AD



Political changes in the Mediterranean: 750 AD



This paper: quantitatively investigate changing economic geography

Challenge: virtually no production/consumption/trade data

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$\Rightarrow$ Use data on the movement of coins to study the changing economic geography during Late Antiquity.

- Coins are the main medium of exchange during Late Antiquity, particularly for long-distance trade $\rightarrow$ informative about trade
- Coins are well studied & documented by historians and numismatists
- Coins have features that help solve econometric identification problems

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Roadmap:

1. Data description
2. Stylized facts / reduced-form evidence
3. Quantitative Model
    - Identification of trade shares from coin stocks
    - Identification of trade cost changes from origin/destination "sizes"
4. Decomposition of estimated Y/L changes into trade/openness, technology, minting.

Data: Coins around the Mediterranean, AD 325 to AD 950

Assemble a large dataset of coin finds from around the Mediterranean

1. FLAME (2023) project by historians around Princeton
   - $\sim$200,000 coins with complete records 325–725
2. Hand-coded records from numismatic / archaeological literature:
   - 797 coin finds, $\sim$100k coins, 725-950

▸ Coin age distribution ▸ Hoard Dates (tpq) ▸ Spatial Distribution

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Example: excerpt from al Ush's (1972) Damascus silver hoard:

No.	MINT	DATE	DIAM.	WEIGHT	NUMB.
51	الأندلس	114	29.	2.93	4
52	"	115	29.5	2.92	1
53	"	116	26.5	2.92	3

Index / Mint (al-Andalus/Cordoba) / Date: 114 AH = 732 AD / Diameter / Weight / Q'tity

▸ Coin age distribution ▸ Hoard Dates (tpq) ▸ Spatial Distribution

Fact #1: Arab conquests disrupt Mediterranean trade

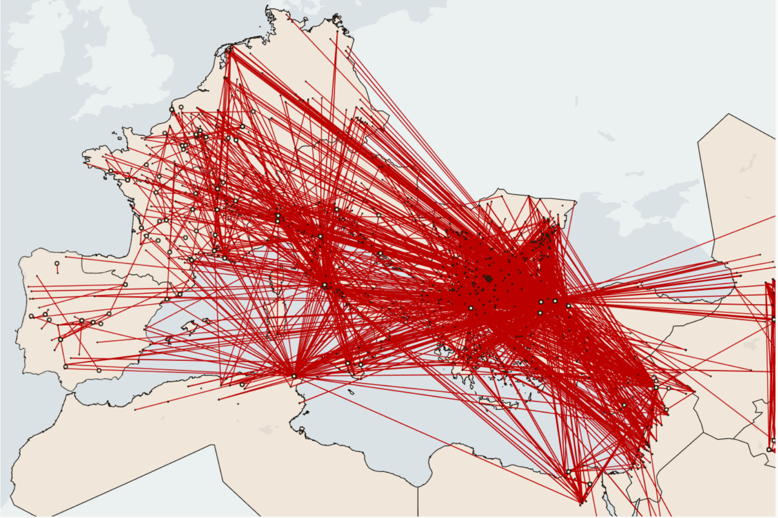


Figure 1: Before the Arab conquests: 450-630 AD

Fact #1: Arab conquests disrupt Mediterranean trade

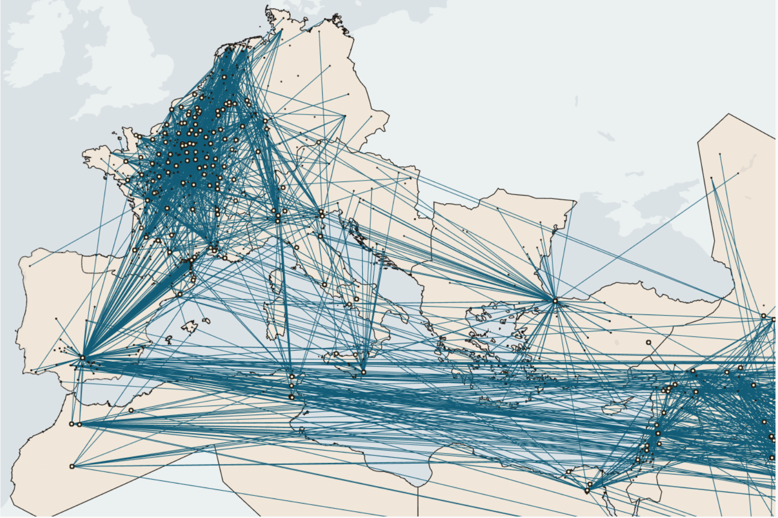


Figure 2: After the Arab conquests: 713-900 AD

Fact #1: Arab conquests disrupt Mediterranean trade

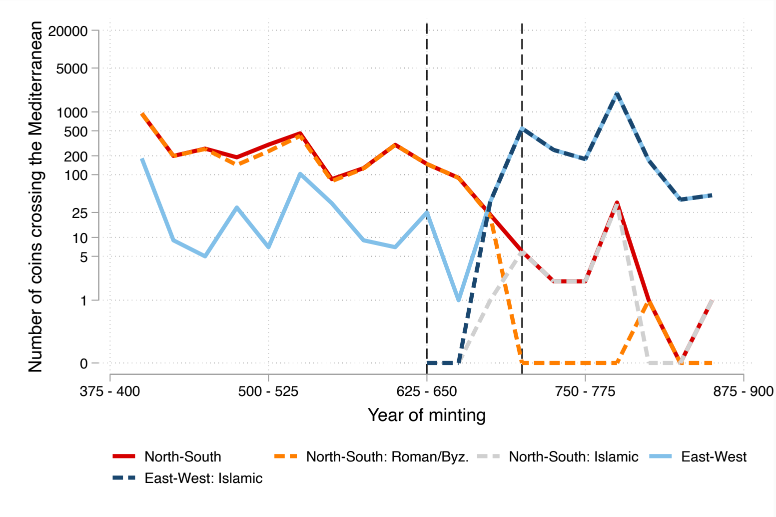


Figure 3: Number of coins flowing across the Mediterranean

▸ Regressions

Fact #2: distance and political borders impede coin travels

Construct $1^\circ \times 1^\circ$ cells for mint and hoard locations and calculate flows count_{mdh}

Table 1: Gravity and Border Effects in Coin Flows

	Dependent variable: # Coins_{mdh}					
	(1)	(2)	(3)	(4)	(5)	(6)
Log Distance	-1.138**	-1.002**	-1.140**	-0.955**	-0.727**	-0.694**
	(0.12)	(0.13)	(0.10)	(0.077)	(0.10)	(0.10)
Political border		-1.945**		-2.073**		-1.540**
		(0.62)		(0.47)		(0.41)
Hoard Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Mint $\times$ Empire Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample			Gold only	Gold only	Int. Marg. only	Int. Marg. only
Estimator	PPML	PPML	PPML	PPML	PPML	PPML
Pseudo-R^2	0.766	0.778	0.809	0.825	0.737	0.744
Observations	216809	216809	57457	57457	6306	6306

Standard errors in parentheses, clustered at mint cell $\times$ empire and hoard cell level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Declining Elasticity

Estimating eqn: $\text{count}_{mdh} = \exp\left(\gamma_1 \log \text{distance}_{mh} + \gamma_2 \text{withinBorder}_{dh} + \alpha_{md} + \alpha_h + \varepsilon_{mhd}\right)$

Fact #3: within a hoard, older coins have travel farther

Table 2: Within-hoard distance travelled and age of coin at deposit

	Dependent variable: Log Distance between Mint and Hoard				
	(1)	(2)	(3)	(4)	(5)
Log Age of Coin	0.160**	0.0942**	0.0882**	0.178**	0.0623**
	(0.050)	(0.025)	(0.031)	(0.049)	(0.020)
Sample				No non-hoards	No non-hoards
Hoard FE	Yes	Yes	Yes	Yes	Yes
Mint $\times$ 50-year-interval FE		Yes			
Mint $\times$ 25-year-interval FE			Yes		Yes
R^2	0.762	0.863	0.869	0.775	0.899
Observations	287235	287018	286860	250133	249806

Standard errors in parentheses, clustered at the hoard level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Age of coin = hoard deposit date − mint date

$\Rightarrow$ coins diffuse across space over time.

Model

Objective:

Quantify impact of trade barriers (and other changes) on market access / trade / welfare

Approach:

Dynamic model of trade flows (gravity), where coins diffuse alongside trade and are thus informative about trade flows.

Key Assumption:

Traders are blind to coin *types* (mint location and date).

- $\Rightarrow$ Coins diffuse in proportion to trade flows
- $\Rightarrow$ Recover trade *flows* from stocks of coin *types* in hords in different locations/time periods.

Not used/needed for identification: total *quantities* of coins found in a location/time
$\rightarrow$ identification from *shares* only

Setup

Location n, period t: homog. mass $L_n(t)$ of workers. Four sub-periods $t_{sub1}, t_{sub2}, t_{sub3}, t_{sub4}$

t_{sub1} Start with $S_n(t)$ coins saved from period $t-1$

t_{sub2} A fraction $\lambda_n(t)$ of those saved coins is lost

$M_n(t) \geq 0$ fresh new coins are minted

t_{sub3} $X_n(t)$, expenditure on consumption

Budget constraint:

$$X_n(t) \leq (1-\lambda_n(t))\, S_n(t) + M_n(t)$$

t_{sub4} $L_n(t)$ workers produce and sell goods in exchange for coins

$w_n(t)$, competitive wage, $w_n(t)\, L_n(t)$, aggregate labor income

$S_n(t+1)$ coins saved for $t+1$

$$\underbrace{(1-\lambda_n(t))\overbrace{S_n(t)}^{t_{sub1}} + M_n(t)}_{t_{sub2}} - \underbrace{X_n(t)}_{t_{sub3}} + \underbrace{w_n(t)\, L_n(t)}_{t_{sub4}} = \underbrace{S_n(t+1)}_{(t+1)_{sub1}}$$

Intra-temporal allocations

- Fraction π_{ni} of expenditure X_n allocated to goods from i:

$$\pi_{ni}(t) = \frac{T_i(t)\left(w_i(t)\, d_{ni}(t)\right)^{-\theta}}{\sum_k T_k(t)\left(w_k(t)\, d_{nk}(t)\right)^{-\theta}}, \tag{1}$$

as in Eaton and Kortum (2002).

Intertemporal preferences

- Intertemporal utility U_n, within period welfare W_n, ,

$$U_n(t) = \mathbb{E}\left[\sum_{\tau \geq t} \beta^{\tau - t} \ln\left(\frac{X_n(t)}{p_n(t)}\right)\right],$$

$$\text{with } p_n(t) = \gamma\left(\sum_k T_k(t)\left(w_k(t)\, d_{nk}(t)\right)^{-\theta}\right)^{1/\theta}$$

Dynamic optimization

- Optimal coin savings dynamics,

$$\max_{\{S_n(\tau)\}_{\tau \geq t}} \left[\sum_{\tau \geq t} \beta^{\tau - t} \ln \left(\frac{X_n(\tau)}{p_n(\tau)} \right) \right]$$

$$X_n(\tau) = w_n(\tau) L_n(\tau) + M_n(\tau) + (1 - \lambda_n(\tau)) S_n(\tau) - S_n(\tau + 1),$$

$$S_n(\tau + 1) \geq w_n(\tau) L_n(\tau), \forall \tau \geq t,$$

$$\lim_{\tau \to \infty} \beta^{\tau} S_n(\tau + 1) / X_n(\tau) = 0$$

- Dynamic equilibrium wages clear markets,

$$w_i' L_i' = \sum_n \pi_{ni}(\mathbf{T}, \mathbf{d}; \mathbf{w}) \left[w_n L_n + M_n + (1 - \lambda_n) S_n - S_n' \right]$$

Savings $S_n(\mathbf{T}, \mathbf{d}, \boldsymbol{\delta}, \mathbf{L}, \mathbf{M}; \mathbf{w})$ depend on parameters and wages, which depend on wages etc.

Optimal consumption/saving

Under log utility:

- price level $p_n(t)$ dynamics irrelevant (i.e. separates out)
- when unconstrained, consumption declines exponentially:

$$\frac{X_n(t+1)}{X_n(t)} = \beta(1 - \lambda_n(t)) < 1$$

- when constrained, consume as much as you can:

$$S_n' = w_n(t)L_n$$

Define *net* saving:

$$s_n(t) = \frac{(1 - \lambda_n(t))S_n(t) + M_n(t) - X_n(t)}{(1 - \lambda_n(t))S_n(t) + M_n(t)}$$

Steady state: zero net savings, $S_n(t+1) = S_n(t), \forall t$

- No motive for extra savings, $S_n = w_n L_n$. Hence $s_n(t) = 0$.
- Wages and trade shares jointly determined,

$$w_i L_i = \sum_n \pi_{ni} \left(\left(1 - \lambda_n\right) w_n L_n + M_n \right)$$

$$\pi_{ni} = \frac{T_i \left(w_i d_{ni}\right)^{-\theta}}{\sum_k T_k \left(w_k d_{nk}\right)^{-\theta}}$$

- Constant aggregate stock of coins in circulation,

$$\sum_n M_n = \lambda_n \sum_n w_n L_n$$

- *Note*: trade deficits as in Dekle, Eaton and Kortum (2007), but endogenous:

$$D_n = X_n - Y_n = M_n - \lambda_n S_n$$

Introducing and tracking coins of different vintages

Coin stocks $S_n(\tau)$ consist of coins of different vintage:

$$S_n(\tau) = \sum_{m=1}^{N} \sum_{t<\tau} S_{mn}(t,\tau)$$

Coins start their life when minted: $S_{mm}(t,t) = M_m(t)$.

Traders are 'blind' to coin types, draw coins with equal probability:

$$S_{mi}(t,\tau+1) = \sum_{n=1}^{N} (1-\lambda_n(\tau))(1-s_n(\tau))\, S_{mn}(t,\tau)\, \pi_{ni}(\tau) + (1-\lambda_i(\tau))\, s_i(\tau)\, S_{mi}(t,\tau), \forall \tau \geq t$$

In compact matrix form:

$$\mathbf{S}(t,T) = \boldsymbol{M}(t) \left(\prod_{\tau=t}^{T-1} (\boldsymbol{I} - \boldsymbol{\lambda}(\tau)) \left((\boldsymbol{I} - \mathbf{s}(\tau))\, \boldsymbol{\Pi}(\tau) + \mathbf{s}(\tau) \right) \right)$$

Taking the model to the data

- 14 regions around the Mediterranean ▸ details
- 20-year time intervals
- Assume constant λ and estimate as exponential decay parameter in within-hoard age distribution:

$$\hat{\lambda}_{20y} = 0.3$$

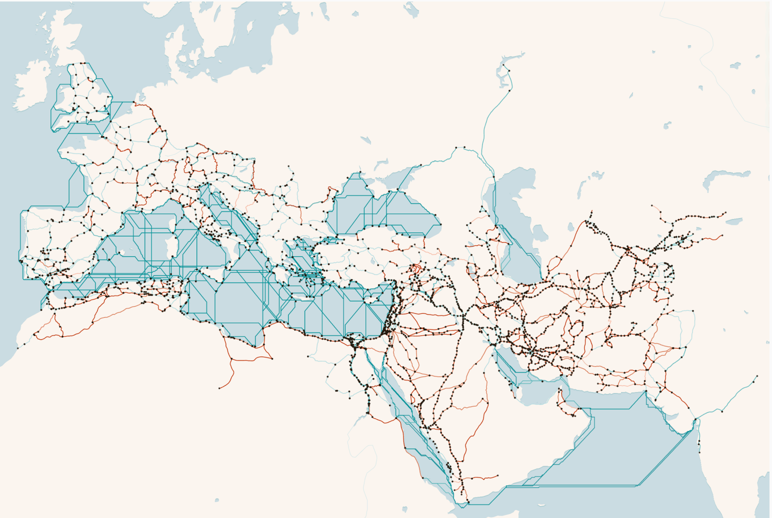
- Parameterize trade frictions,

$$d_{ni}(t)^{\theta} = \exp\left(\gamma_0 + \zeta \ln\left(\text{TravelTime}_{ni}\right) + \kappa_1 \text{PoliticalBorder}_{ni}(t) + \kappa_2 \text{ReligiousBorder}_{ni}(t)\right)$$

if $n \neq i$ and $d_{nn}(t) = 1$.

- How to split domestic trade vs saving? Calibrate $\overline{s_n}(t) = 1.5\%$ (Scheidel, 2020).
- For counterfactuals, assume $\theta = 4$ (Simonovska and Waugh, 2014).

Trade costs *only* depend on travel times (and politics/religion)



Note: Combined geospatial models from Orbis (Scheidel, 2015) and al-Ṯurayyā (Romanov and Seydi, 2022). ▸ al-Maqdisi

Maximum likelihood estimation

Assume coins in our data are a random sample of coin types in each location $\times$ time.

- Multinomial distribution of coin types,

$$P\left(\cdots, X_i^{(m,\tau)}(T) = x_i^{(m,\tau)}, \ldots\right) = \frac{N_i(T)!}{\prod_{(m,\tau)} x_i^{(m,\tau)}!} \prod_{(m,\tau)} \left[p_i^{(m,\tau)}(T)\right]^{x_i^{(m,\tau)}}$$

  with the probability of drawing a coin of type (m, τ),

$$p_i^{(m,\tau)}(T) = \frac{S_i^{(m,\tau)}(T)}{\sum_{(m',\tau')} S_i^{(m',\tau')}(T)} = \frac{S_i^{(m,\tau)}(T)}{S_i(T)}.$$

- Likelihood of observing a sample of coins given parameters $\boldsymbol{\theta}$,

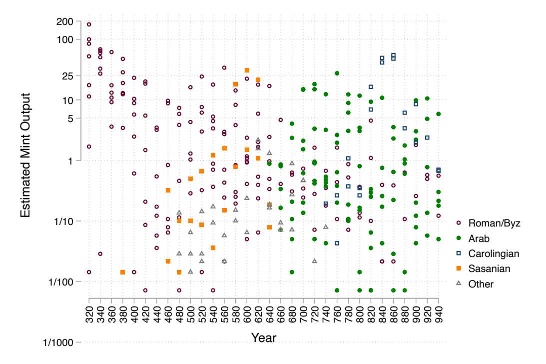
$$\ell(X; \boldsymbol{\theta}) = \sum_i \sum_T \sum_{(m,\tau)} x_i^{(m,\tau)} \left[\log S_i^{(m,\tau)}(T; \boldsymbol{\theta}) - \log S_i(T; \boldsymbol{\theta})\right] + \text{constant}$$

Estimation results

Trade costs:

$$\ln\left(\left(d_{ni}(t)\right)^{-\theta}\right) = constant - 3.01\ln\left(\text{TravelTime}_{ni}\right) - 0.63\,\text{PoliticalBorder}_{ni}(t) - 3.74\,\text{ReligiousBorder}_{ni}(t)$$

Mint output:



Welfare decomposition

Real consumption ("welfare") depends on a combination of L and T (that's not separately identified):

$$X_n/p_n = \gamma^{-1}\left(\pi_{nn}\right)^{-1/\theta}\left(L_n T_n^{1/\theta}\right)\left(1-\lambda+M_n\left(L_n T_n^{1/\theta}\right)\tilde{\beta}_n^{-1/\theta}\right)$$

Real consumption per capita:

$$\frac{X_n/p_n}{L_n} = \gamma^{-1}\underbrace{\left(\pi_{nn}\right)^{-1/\theta}}_{\text{Openness}}\times\underbrace{\left(T_n\right)^{1/\theta}}_{\text{Technology}}\times\underbrace{\left(1-\lambda+\frac{M_n}{w_n L_n}\right)}_{\text{Minting}}$$

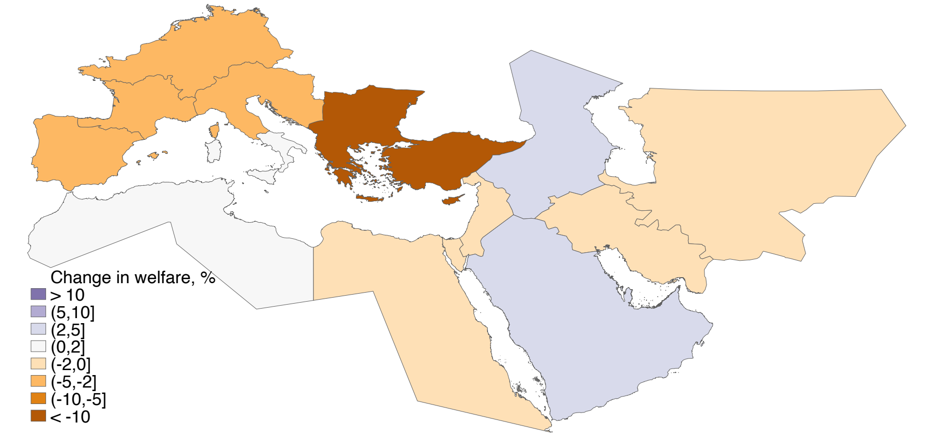
We separate L and T through a Malthusian assumption

$$L_n = T_n \quad \forall n$$

and decompose per-capita welfare into the three components

Welfare change decomposition: Openness

Change in measured openness $(\pi_{nn})^{-1/\theta}$
Byzantium, Spain, Franks moved to periphery.

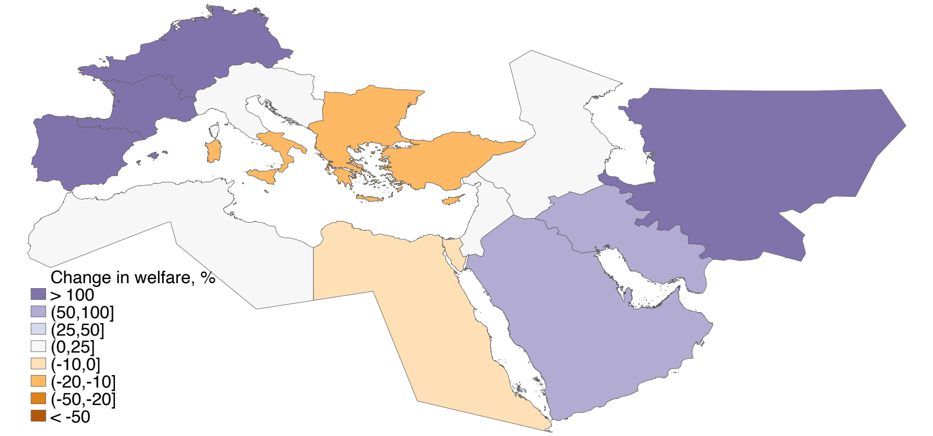


Decomposition of welfare change pre- to post: openness

Welfare change decomposition: Technology

Change in $(T_n)^{1/\theta}$

Technology improvements in Spain (Agriculture) and Francia

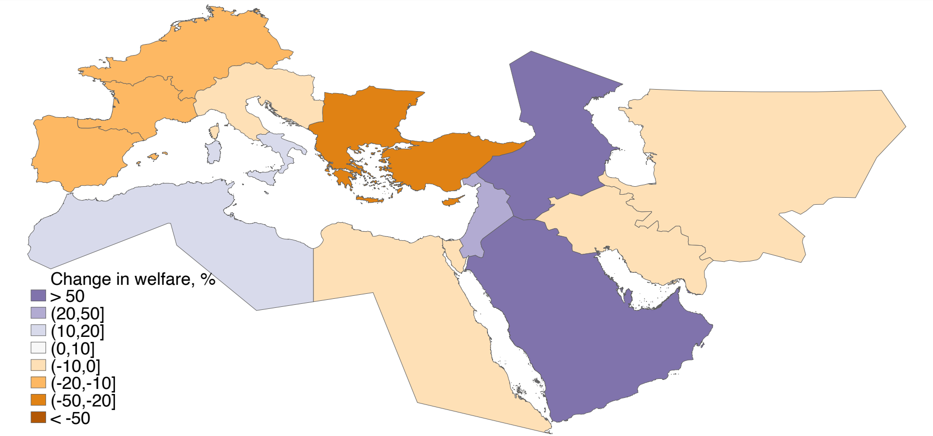


Decomposition of welfare change pre- to post: technology

Welfare change decomposition: Minting

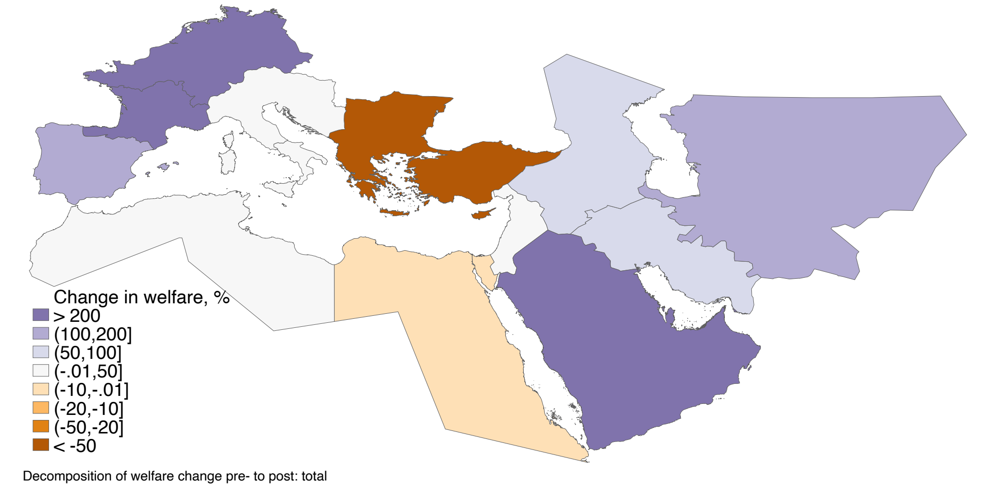
Change in minting over income

Byzantine Mint output declines; Frankish mint output increases (but income increases even more due to technology)



Decomposition of welfare change pre- to post: minting

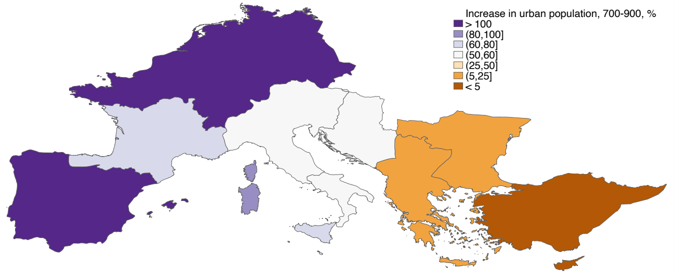
Welfare change decomposition: Openness + Tech + Minting



Compare to relative urbanization rates, 700–900 AD

Top: Change in total urban population (urban: > 1k inhabitants), data from Buringh (2021)

Bottom: $\Delta \frac{X_n / p_n)}{L_n}$



Taking stock

- Clear pattern of change in economic geography before vs after conquest
- Trade disruption leads to a significant relative decline of the eastern Mediterranean
- Minting ($\rightarrow$ trade deficits) and in particular change in "technology" quantitatively more important than trade costs
- Jointly, can account for relative urbanization patterns.

Conclusion

"Simply looking at the Mediterranean cannot of course explain everything about a complicated past created by human agents, with varying doses of calculation, caprice and misadventure. But this is a sea that patiently recreates for us scenes from the past, breathing new life into them, locating them under a sky and in a landscape that we can see with our own eyes, a landscape and sky like those of long ago. A moment's concentration or daydreaming, and that past comes back to life."

Fernand Braudel, Les Mémoires de la Méditerranée

THANK YOU!

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BACKUP SLIDES

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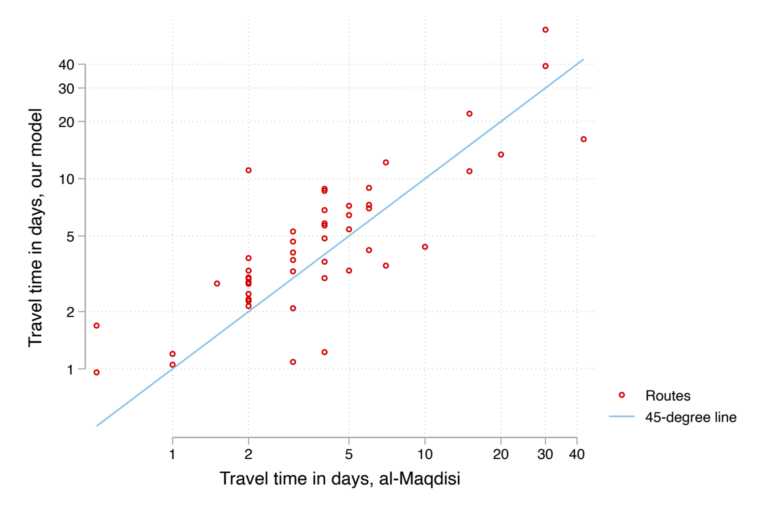
Regions

Back

Validating Travel Times

Al-Maqdisi (c. 945–991): *The Best Divisions for Knowledge of the Regions*



Back

Spatial distribution: the (extended) Mediterranean

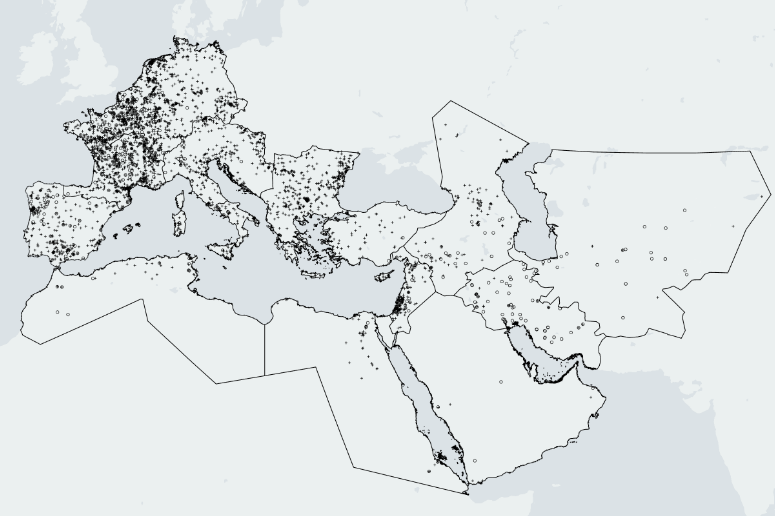


Figure 4: Region Definitions

Back

Distribution of coin “death dates” (*tpq*)

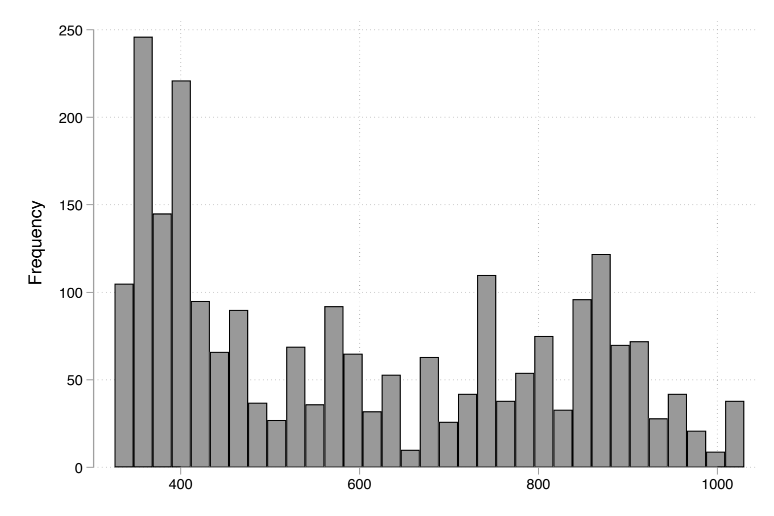


Figure 5: Terminus Post Quem (*tpq*) of hoards

Distribution of coin ages (*tpq* minus mint date)

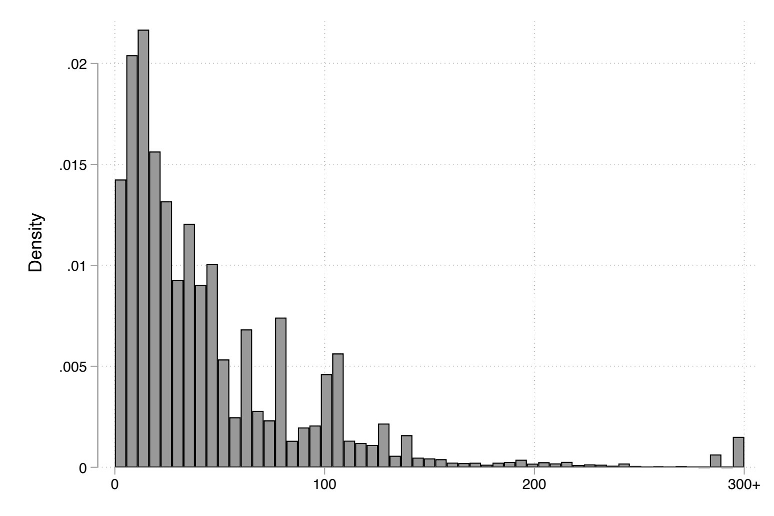


Figure 6: Coin age at time of deposit (*tpq*), in years

▸ Back

Fact #1: Arab conquests disrupt Mediterranean trade

Table 3: The Mediterranean Before and After the Arab Conquest

	Dependent variable: Number of Coins			
	(1)	(2)	(3)	(4)
Crossing Mediterranean × After Conquests	-1.774**	-3.141**	-0.712	-1.751
	(0.46)	(0.53)	(0.66)	(1.24)
Crossing Mediterranean × After Conquests × Islamic Coin		7.171**	4.835**	8.382**
		(0.91)	(0.97)	(1.15)
Crossing Mediterranean × After Conquests × Roman Coin			-3.108**	-2.976**
			(0.79)	(0.71)
Mint Cell × Empire FE	Yes	Yes	Yes	Yes
Mint Cell × Hoard Cell FE	Yes	Yes	Yes	Yes
After Conquests FE	Yes	Yes	Yes	
Mint Cell × After Conquests FE				Yes
Hoard Cell × After Conquests FE				Yes
Estimator	PPML	PPML	PPML	PPML
Observations	10350	10350	10350	6023

Standard errors in parentheses, clustered at the hoard × era and mint × era level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Estimating eqn: $\text{count}_{mdht} = \exp\left(\gamma_1 \text{mediterranean}_{mh} \times \text{after}_t + \cdots + \alpha_{md} + \alpha_{mh} + \varepsilon_{mhdt}\right)$

Back

Fact #2: distance has a weaker impact on older coin flows

$$\log \text{count}_{mth\tau} = \sum_{\tau' \in T} \beta_{\tau'} \log \text{distance}_{mh} \times 1(t-\tau=\tau') + \alpha_{mt} + \alpha_{h\tau} + \varepsilon_{mth\tau}$$

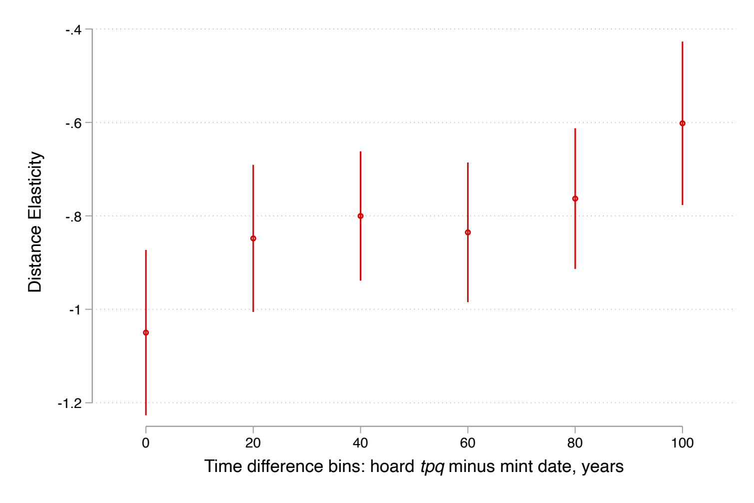


Figure 7: The distance elasticity declines as coins get older

Back

Unsolved problems (as of yet)

- *Lucas critique #1*: cost function does not minimize costs

$$\ln\left(\delta_{ni}\left(t\right)\right) = \min_{p \in paths(i \to n)} \Big(\gamma_0 + \gamma_1 \ln\left(TravelTime_p\left(t\right)\right) + \sum_{pb\text{: all political borders along } p} \gamma_2 PoliticalBorder_{pb}\left(t\right) + \sum_{rb\text{: all religious borders along } p} \gamma_3 ReligiousBorder_{rb}\left(t\right)\Big)$$

- *Lucas critique #2*: net saving (in δ_{nn}) depends on parameters.
- *Fix for #2*: location-specific intercepts ($\gamma_{0,n}$) and δ_{nn}'s.

For now: constant γ_0, and $\delta_{nn} = 1$...

Back

Fact #3: Arab conquests disrupt Mediterranean trade: Gold only

	Dependent variable: Number of Coins				
	(1)	(2)	(3)	(4)	(5)
Log Distance	-1.484**	-1.512**	-1.517**	-1.159**	-1.163**
	(0.13)	(0.12)	(0.12)	(0.15)	(0.16)
Crossing Mediterranean	0.157	0.172	0.187	-0.126	-0.114
	(0.38)	(0.38)	(0.37)	(0.29)	(0.29)
Crossing Mediterranean × After Conquests	-1.565*	-2.887**	-1.701*	-2.273*	-3.067**
	(0.70)	(0.73)	(0.70)	(0.98)	(1.14)
Crossing Mediterranean × After Conquests × Islamic Coin		3.112**	1.907*		2.865*
		(0.77)	(0.77)		(1.24)
Crossing Mediterranean × After Conquests × Roman Coin			-1.766		
			(1.13)		
Mint Cell × Empire FE	Yes	Yes	Yes	Yes	Yes
Hoard Cell × After Conquests FE	Yes	Yes	Yes	Yes	Yes
Sample				Gold only	Gold only
Observations	167840	167840	167840	28895	28895

Standard errors in parentheses, clustered at the hoard × era and mint × era level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Before: 400–630; after: 713–950

Back

Pitfall #1: medium of exchange vs store of value

- Dynamics with 'saving-augmented' trade shares,

$$\mathsf{S}(t,T) = M(t)\left(\prod_{\tau=t}^{T-1}\Big(I - \boldsymbol{\lambda}(\tau)\Big)\tilde{\boldsymbol{\Pi}}(\tau)\right)$$

- Separate origin, destination, and bilateral terms,

$$\tilde{\pi}_{ni}(\tau) = \alpha_n(\tau)\,\beta_i(\tau)\,\delta_{ni}(\tau)$$

$$\alpha_n = \frac{1}{\sum_k T_k\left(w_k d_{nk}\right)^{-\theta}},\ \beta_i = T_i\left(w_i\right)^{-\theta}$$

$$\delta_{ni} = \left(d_{ni}\right)^{-\theta}$$

Pitfall #1: medium of exchange vs store of value

- Dynamics with 'saving-augmented' trade shares,

$$\mathbf{S}(t,T) = \boldsymbol{M}(t)\left(\prod_{\tau=t}^{T-1}\Big(\boldsymbol{I}-\boldsymbol{\lambda}(\tau)\Big)\tilde{\boldsymbol{\Pi}}(\tau)\right)$$

- Separate origin, destination, and bilateral terms,

$$\begin{aligned}
\tilde{\pi}_{ni}(\tau) &= \alpha_n(\tau)\,\beta_i(\tau)\,\delta_{ni}(\tau)\\
\alpha_n &= \frac{1}{\sum_k T_k (w_k d_{nk})^{-\theta}},\ \beta_i = T_i (w_i)^{-\theta}\\
\delta_{ni} &= (d_{ni})^{-\theta} \times \begin{cases}(1-s_n) & \text{if } n \neq i\\ (1-s_n) + \dfrac{s_n \sum_k T_k (w_k d_{nk})^{-\theta}}{T_n (w_n d_{nn})^{-\theta}} & \text{if } n = i.\end{cases}
\end{aligned}$$

- $\frac{\delta_{nj}}{\delta_{ni}} = \frac{(d_{nj})^{-\theta}}{(d_{ni})^{-\theta}}, \forall n \neq i, j, \forall s_n \geq 0$: no impact on external trade
- $\frac{\delta_{nn}}{\delta_{ni}} > \frac{(d_{nn})^{-\theta}}{(d_{ni})^{-\theta}}, \forall s_n > 0$: net saving mimics home bias in trade!

Back

Pitfall #2: stocks vs flows (steady state math)

- SS: no net saving ($\boldsymbol{s} = 0$), only age (a) matters, not time (t),

$$\mathsf{S}\left(t, t+a\right) = \mathsf{S}\left(a\right) = \boldsymbol{M}\Big(\left(\boldsymbol{I} - \boldsymbol{\lambda}\right)\boldsymbol{\Pi}\Big)^{a}, \forall t$$

- Sum of different vintages (stocks by origin-destination),

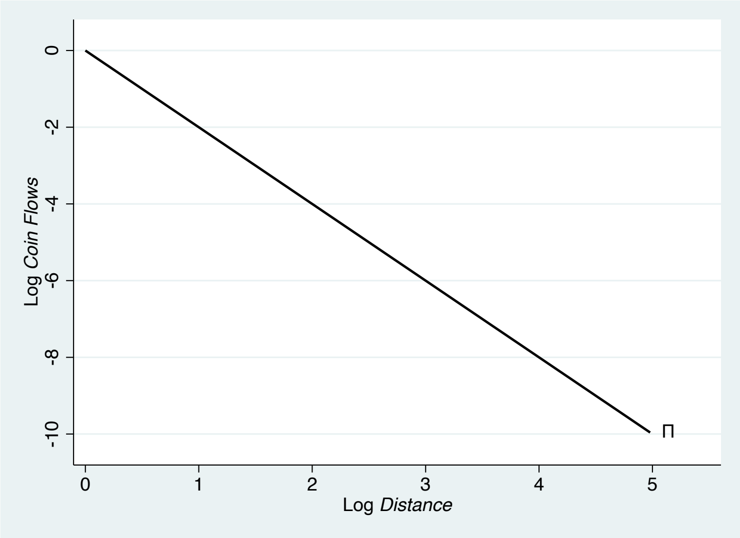
$$\sum_{a=0}^{A} \mathsf{S}\left(a\right) = \boldsymbol{M}\left(\sum_{a=0}^{A}\Big(\left(\boldsymbol{I} - \boldsymbol{\lambda}\right)\boldsymbol{\Pi}\Big)^{a}\right) \underset{A \to +\infty}{=} \boldsymbol{M}\left(\boldsymbol{I} - \left(\boldsymbol{I} - \boldsymbol{\lambda}\right)\boldsymbol{\Pi}\right)^{-1}$$

- Naive gravity on stocks gives Leontief inverse of trade shares!
  $\Rightarrow$ inconsistent estimates of trade elasticities/border effects due to model misspecification

▸ Illustration

▸ Back

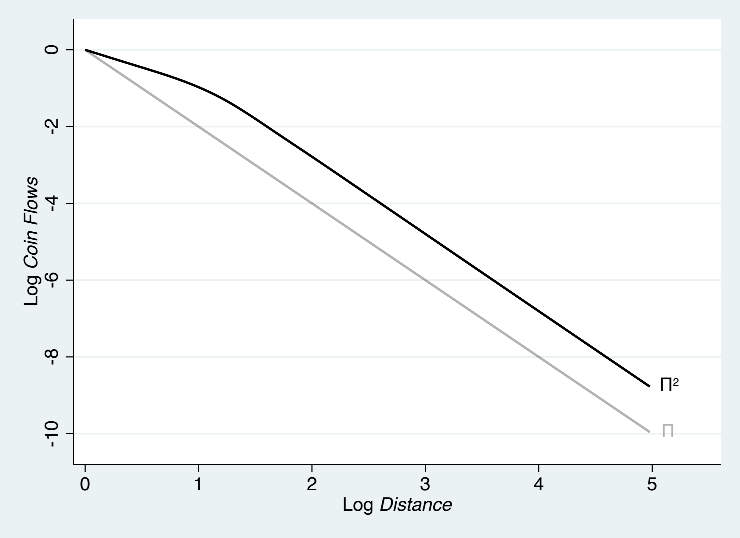
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1 (same as trade flows $\mathbf{\Pi}$)

Back

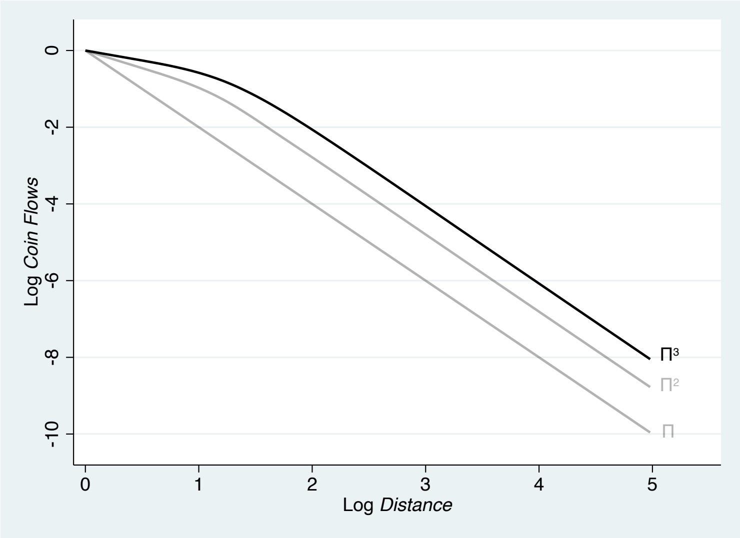
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2

Back

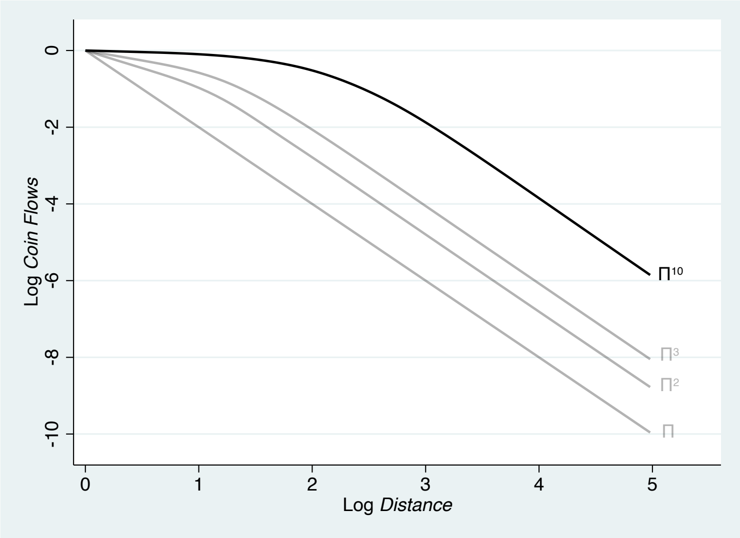
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3

Back

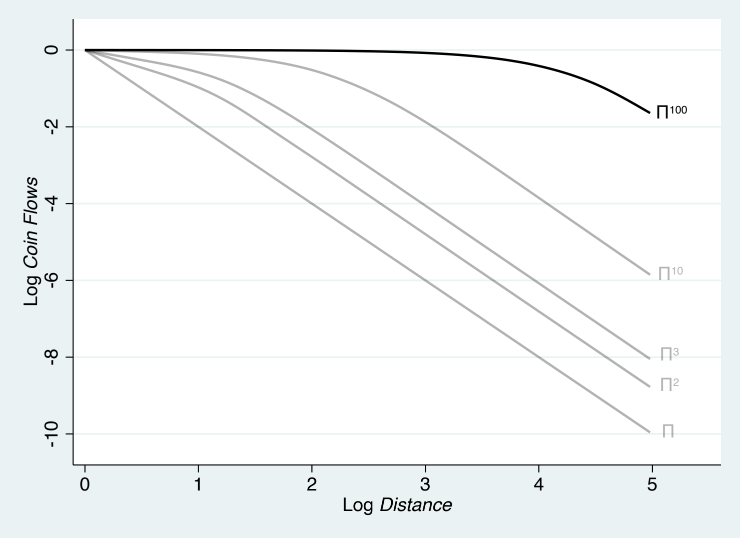
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10

Back

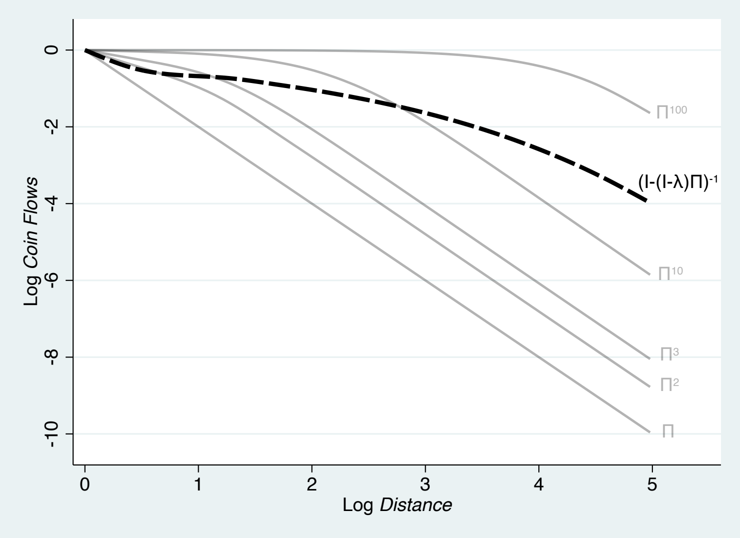
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10, age 100

Back

Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10, age 100, all ages

Back