EC442 Macroeconomics

Problem Set 3

Fall 2013

1 Problem 1: Growth and Labor Supply

This exercise is a special treat for you excelling in Per's part - it integrates labor supply into the neoclassical growth model. In particular, the representative household has a utility function given by

$$\int_0^\infty e^{-\rho t} u\left(c\left(t\right), 1 - l\left(t\right)\right) dt,$$

where $l\left(t\right)$ denotes the households' supply of labor. Assume that the production function is $Y\left(t\right) = F\left(K\left(t\right), A\left(t\right) l\left(t\right)\right)$ and $A\left(t\right) = e^{gt} A\left(0\right)$.

1. Set up the current-value Hamiltonian which the household solves, taking wages and interest rates as given, and determine the necessary and sufficient conditions for the allocation of consumption over time and the leisure-labor trade-off.
2. Set up the current-value Hamiltonian for a planner maximizing the utility of the representative household, and derive the necessary and sufficient conditions for a solution.
3. Show that the two problems are equivalent given competitive markets.
4. Show that along a BGP, the utility function needs to have a representation of the form

$$u\left(c, l\right) = \begin{cases} \gamma_1 \frac{c(t)^{1-\theta}}{1-\theta} h\left(1 - l\right) & \text{for } \theta \neq 1 \\ \gamma_2 ln\left(c\right) + \gamma_3 h\left(1 - l\right) & \text{for } \theta = 1 \end{cases}$$

   for some $h\left(1 - l\right)$ with $h'\left(1 - l\right) > 0$ and constants γ_i. [Hint: to simplify you may assume that the intertemporal elasticity of substitution for consumption, $\varepsilon_u \equiv -\frac{u_{cc}c}{u_c}$ is only a function of c.] Provide an intuition for this functional form in terms of income and substitution effects.

2 To Tax or Not To Tax

Consider an economy with aggregate production function $Y\left(t\right) = AK\left(t\right)^\alpha L\left(t\right)^{1-\alpha}$. All markets are competitive, the labor supply is normalized to 1, capital fully depreciates after use, and the government imposes a linear tax on capital income at the rate τ and uses the proceeds for government consumption. Consider two alternative demographic structures:

1. Agents are infinitely lived, with preferences $\sum_{t=0}^\infty \beta^t ln\left(c\left(t\right)\right)$
2. Agents work in the first period and consume the capital income from their savings in the second period (an OLG model). The preferences of a generation born at time t are

$$ln\left(c_1\left(t\right)\right) + \beta ln\left(c_2\left(t+1\right)\right)$$

Characterize the equilibria in these two economies, and show that in the first economy, taxation reduces output, while in the second, it does not. Interpret this result. In light of this result discuss the applicability of models that try to explain income differences across countries with differences in the rates of capital income taxation.

Problem 3: Growth with Non-Life-cycle Consumers

It is hard to believe, but rumor has it that some consumers out there do not solve their Hamiltonian before deciding on their consumption expenditures. Even more shockingly, they simply adopt an ad-hoc rule-of-thumb consumption behavior where they simply consume their entire labor-income each period and do not save. In this exercise we are going to study the long-run implications of the presence of such ghastly, non-optiming individuals. In particular, suppose the economy consists of a continuum of individuals of size 1. A fraction μ of these consumers are the *savers*, which behave according to the neoclassical model, i.e. they choose consumption and savings to maximize their utility function

$$U = \sum_{t=0}^{\infty} \beta^t u\left(c^S(t)\right),$$

where $c^S(t)$ is the per-capita consumption level of a saver. The remaining $1-\mu$ individuals are *hand-to-mouth consumers*, who simply set their consumption $c^{HM}(t)$ equal to their income. Each individual has one unit of labor efficiency units, that she can supply to the market (and hence aggregate labor supply is equal to 1). Technology is given by a representative firm with Cobb-Douglas technology, i.e.

$$Y(t) = K(t)^{\alpha} L(t)^{1-\alpha}$$

and the aggregate resource constraint is

$$K(t+1) + C(t) = Y(t) + (1-\delta) K(t),$$

where

$$C(t) = C^S(t) + C^{HM}(t) = \mu c^S(t) + (1-\mu) c^{HM}(t)$$

is aggregate consumption and $C^S(t)$ and $C^{HM}(t)$ is aggregate consumption of the savers and hand-to-mouth consumers respectively. The initial capital stock $K(0)$ is entirely owned by the savers.

1. Define an equilibrium in this economy. In particular, state the budget constraints faced by savers and hand-to-mouth consumers and derive the static optimality conditions for the firms.

2. Show that in equilibrium, labor income and consumption of the group of hand-to-mouth consumers is a constant fraction λ of aggregate output $Y(t)$ (i.e. $C^{HM}(t) = \lambda Y(t)$). Solve for λ. How does λ depend on α? Interpret.

3. Now solve for the steady-state capital-level in this economy. How does it compare to the one in the standard neoclassical growth model? How does it depend on μ?

4. Now suppose that $u\left(c^S\right) = ln\left(c^S\right)$ and that capital depreciates fully, i.e. $\delta = 1$. Guess and verify that in equilibrium capital accumulates according to

$$K(t+1) = sK(t)^{\alpha}.$$

   Determine the constant s. How does the introduction of hand-to-mouth consumers affect the equilibrium dynamics of capital relative to the neoclassical model? [HINT: SHOW THAT THE GUESS $K(t+1) = sK(t)^{\alpha}$ SOLVES THE NECESSARY CONDITIONS FOR AN EQUILIBRIUM AND FIND s.]