

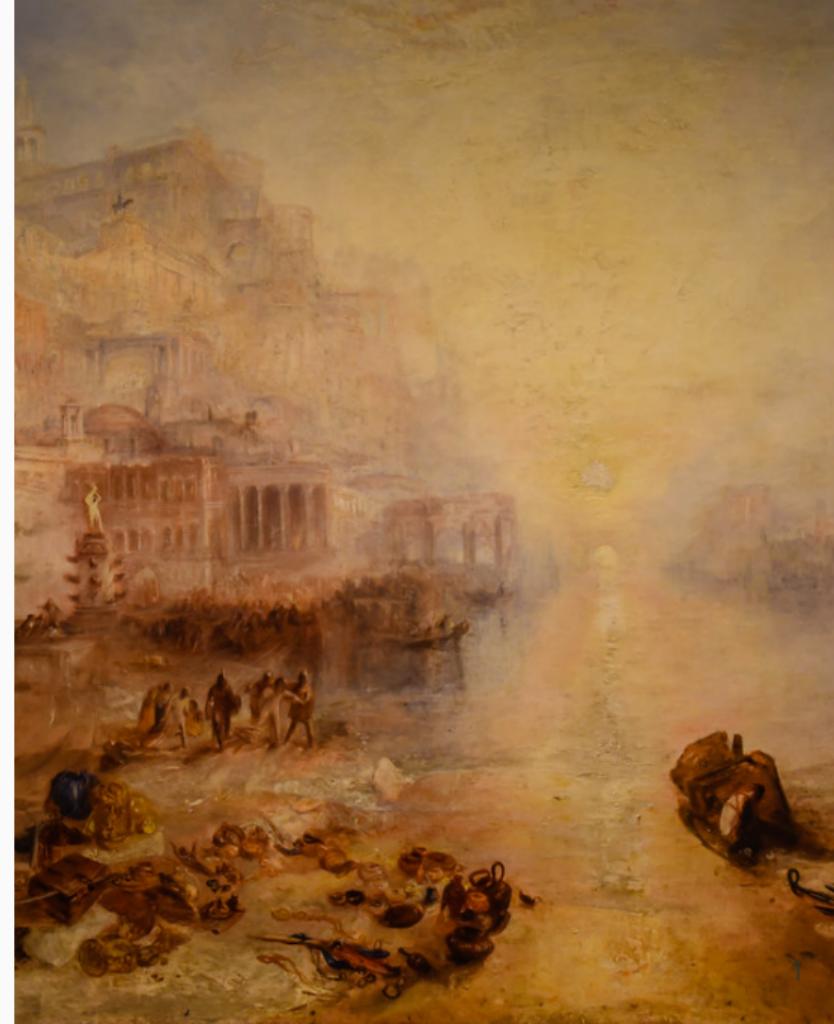
Trade and the End of Antiquity

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What caused the End of Antiquity?

- Antiquity: Roman and Greek civilizations centered around the Mediterranean
 - Archaeological evidence points to a shift in economic activity away from the Mediterranean between 5th and 8th century AD (“end of antiquity”)
 - Rise of Northern Europe (Charlemagne).
 - “Golden Age” of Islam
 - Origin of “Europe” geographic entity north of Mediterranean
- *Question:* What caused the End of Antiquity?
- Discussed, among others, by Montesquieu (1734), Voltaire (1756), Gibbon (1789)
 - Henri Pirenne (1937) blames the Arab conquests and the emerging Islamic-Christian border for the rupture in the Mediterranean unity
 - Some views challenged by more recent archaeological evidence.

This paper: quantitatively investigate changing economic geography

Challenge: virtually no production/consumption/trade data

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⇒ Use data on the movement of coins to study the changing economic geography during Late Antiquity.

- Coins are the main medium of exchange during Late Antiquity, particularly for long-distance trade → informative about trade
- Coins are well studied & documented by historians and numismatists
- Coins have features that help solve econometric identification problems

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- Coins have features that help solve econometric identification problems

⇒ we interpret coin flows through the lens of a trade model and identify

- bilateral trade flows (i, j, t)
- technology (i, t)
- trade deficits (i, t)
- real consumption per capita (i, t)

in order to better understand what's going on during Late Antiquity

Data: Coins around the Mediterranean, AD 325 to AD 950

Assemble a large dataset of coin finds from around the Mediterranean

1. FLAME (2023) project by historians around Princeton
 - ~200,000 coins with complete records 325–725
2. Hand-coded records from numismatic / archaeological literature:
 - 797 coin finds, ~100k coins, 725–950

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Example: excerpt from al Ush's (1972) Damascus silver hoard:

No.	MINT	DATE	DIAM.	WEIGHT	NUMB.
51	الأندلس	114	29.	2.93	4
52	"	115	29.5	2.92	1
53	"	116	26.5	2.92	3

Index / Mint (al-Andalus/Cordoba) / Date: 114 AH = 732 AD / Diameter / Weight / Q'tity

Fact #1: within a hoard, older coins have travel farther

Table 1: Within-hoard distance travelled and age of coin at deposit

Dependent variable: Log Distance between Mint and Hoard					
	(1)	(2)	(3)	(4)	(5)
Log Age of Coin	0.146*** (0.044)	0.0831*** (0.026)	0.0749** (0.031)	0.160*** (0.043)	0.0485** (0.020)
Sample				No non-hoards	No non-hoards
Hoard FE	Yes	Yes	Yes	Yes	Yes
Mint × 50-year-interval FE		Yes			
Mint × 25-year-interval FE			Yes		Yes
R ²	0.762	0.863	0.869	0.775	0.898
Observations	287243	287029	286873	250156	249830

Standard errors in parentheses, clustered at the hoard level.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Age of coin = $tpq - \text{mint date}$

⇒ coins diffuse across space over time.

Fact #2: distance and political borders impede coin travels

Construct $1^\circ \times 1^\circ$ cells for mint and hoard locations and calculate flows count_{mdh}

Table 2: Gravity and Border Effects in Coin Flows

	Dependent variable: # Coins _{mdh}				Dep. var.: Value _{mdh}	
	(1)	(2)	(3)	(4)	(5)	(6)
Log Distance	-1.137*** (0.12)	-1.002*** (0.13)	-1.135*** (0.10)	-0.951*** (0.076)	-1.144*** (0.075)	-0.989*** (0.068)
Political border		-1.945*** (0.62)		-2.073*** (0.47)		-1.516*** (0.27)
Hoard Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Mint × Empire Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample		Gold only	Gold only	Gold and Silver	Gold and Silver	
Estimator	PPML	PPML	PPML	PPML	PPML	PPML
Pseudo-R ²	0.767	0.778	0.808	0.824	0.800	0.810
Observations	217748	217748	57287	57287	146767	146767

Standard errors in parentheses, clustered at mint cell × empire and hoard cell level.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

► Declining Elasticity

Estimating eqn: count_{mdh} = exp ($\gamma_1 \log \text{distance}_{mh} + \gamma_2 \text{withinBorder}_{dh} + \alpha_{md} + \alpha_h + \varepsilon_{mhd}$)

Fact #3: Coin flows before/after the Arab conquests

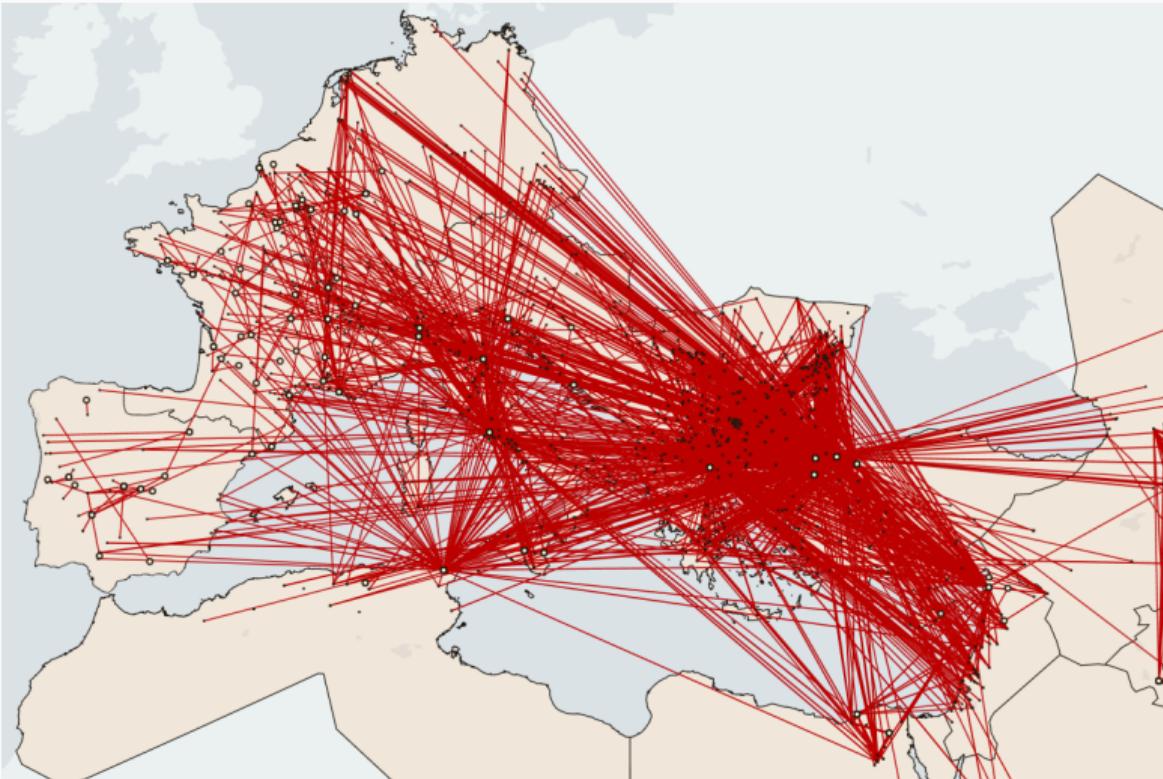


Figure 1: Before the Arab conquests: 450-630 AD

Fact #3: Coin flows before/after the Arab conquests

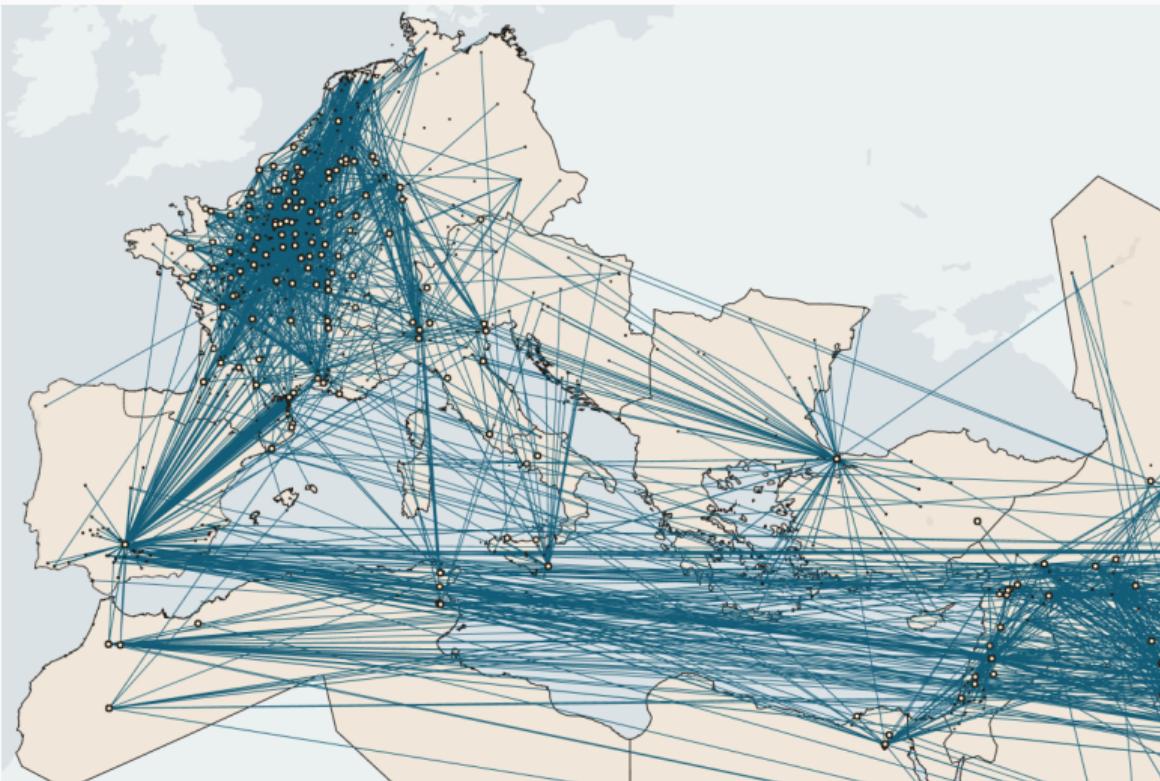


Figure 2: After the Arab conquests: 713-900 AD

▶ Cross-med: reg

▶ Cross-med: plot

Model

Objective:

Quantify impact of trade barriers (and other changes) on market access / trade / welfare

Approach:

Dynamic model of trade flows (gravity), where coins diffuse alongside trade and are thus informative about trade flows.

Key Assumption:

Traders are blind to coin types (mint location and date).

- ⇒ Coins diffuse in proportion to trade flows
- ⇒ Recover trade flows from stocks of coin types in hords in different locations/time periods.

Not used/needed for identification: total *quantities* of coins found in each location

Setup

Location n , period t : homog. mass $L_n(t)$ of workers. Four sub-periods $t_{sub1}, t_{sub2}, t_{sub3}, t_{sub4}$

t_{sub1} Start with $S_n(t)$ coins saved from period $t - 1$

t_{sub2} A fraction $\lambda_n(t)$ of those saved coins is lost

$M_n(t) \geq 0$ fresh new coins are minted

t_{sub3} $X_n(t)$, expenditure on consumption

Cash-in-advance constraint:

$$X_n(t) \leq (1 - \lambda_n(t)) S_n(t) + M_n(t)$$

t_{sub4} $L_n(t)$ workers produce and sell goods in exchange for coins

$w_n(t)$, competitive wage, $w_n(t)L_n(t)$, aggregate labor income

$S_n(t+1)$ coins saved for $t + 1$

$$\underbrace{(1 - \lambda_n(t)) \overbrace{S_n(t)}^{t_{sub1}} + M_n(t)}_{t_{sub2}} - \underbrace{X_n(t)}_{t_{sub3}} + \underbrace{w_n(t) L_n(t)}_{t_{sub4}} = \underbrace{S_n(t+1)}_{(t+1)_{sub1}}$$

Intra-temporal allocations

- Fraction π_{ni} of expenditure X_n allocated to goods from i :

$$\pi_{ni}(t) = \frac{T_i(t)(w_i(t)d_{ni}(t))^{-\theta}}{\sum_k T_k(t)(w_k(t)d_{nk}(t))^{-\theta}}, \quad (1)$$

as in Eaton and Kortum (2002).

Intertemporal preferences

- Intertemporal utility U_n , within period welfare W_n ,

$$U_n(t) = \mathbb{E} \left[\sum_{\tau \geq t} \beta^{\tau-t} \ln \left(\frac{x_n(\tau)}{p_n(\tau)} \right) \right],$$

$$\text{with } p_n(t) = \gamma \left(\sum_k T_k(t) (w_k(t) d_{nk}(t))^{-\theta} \right)^{1/\theta}$$

Dynamic optimization

- Optimal coin savings dynamics,

$$\max_{\{S_n(\tau)\}_{\tau \geq t}} \left[\sum_{\tau \geq t} \beta^{\tau-t} \ln \left(\frac{X_n(\tau)}{p_n(\tau)} \right) \right]$$

$$X_n(\tau) = w_n(\tau)L_n(\tau) + M_n(\tau) + (1 - \lambda_n(\tau))S_n(\tau) - S_n(\tau+1),$$

$$S_n(\tau+1) \geq w_n(\tau)L_n(\tau), \forall \tau \geq t,$$

$$\lim_{\tau \rightarrow \infty} \beta^\tau S_n(\tau+1)/X_n(\tau) = 0$$

- Dynamic equilibrium wages clear markets,

$$w'_i L'_i = \sum_n \pi_{ni}(T, d; w) [w_n L_n + M_n + (1 - \lambda_n) S_n - S'_n]$$

Savings $S_n(T, d, \delta, L, M; w)$ depend on parameters and wages, which depend on wages etc.

Optimal consumption/saving

Under log utility:

- price level $p_n(t)$ dynamics irrelevant (i.e. separates out)
- when unconstrained, consumption declines exponentially:

$$\frac{X_n(t+1)}{X_n(t)} = \beta(1 - \lambda_n(t)) < 1$$

- when constrained, consume as much as you can:

$$S'_n = w_n(t)L_n$$

Define *net saving*:

$$s_n(t) = \frac{(1 - \lambda_n(t))S_n(t) + M_n(t) - X_n(t)}{(1 - \lambda_n(t))S_n(t) + M_n(t)}$$

Introducing and tracking coins of different vintages

Coin stocks $S_n(\tau)$ consist of coins of different vintage:

$$S_n(\tau) = \sum_{m=1}^N \sum_{t < \tau} S_{mn}(t, \tau)$$

Coin stocks start their life when minted: $S_{mm}(t, t) = M_m(t)$.

Traders are ‘blind’ to coin types, draw coins with equal probability:

$$S_{mi}(t, \tau + 1) = \sum_{n=1}^N (1 - \lambda_n(\tau)) (1 - s_n(\tau)) S_{mn}(t, \tau) \pi_{ni}(\tau) + (1 - \lambda_i(\tau)) s_i(\tau) S_{mi}(t, \tau), \forall \tau \geq t$$

In compact matrix form:

$$\mathbf{S}(t, T) = \mathbf{M}(t) \left(\prod_{\tau=t}^{T-1} (\mathbf{I} - \boldsymbol{\lambda}(\tau)) \left((\mathbf{I} - \mathbf{s}(\tau)) \boldsymbol{\Pi}(\tau) + \mathbf{s}(\tau) \right) \right)$$

► Pitfall 1: Store

► Pitfall 2: Stocks vs flows

Taking the model to the data

- 13 regions around the Mediterranean ▶ details
- 20-year time intervals
- Assume constant λ and estimate as exponential decay parameter in within-hoard age distribution:

$$\hat{\lambda}_{20y} = 0.301$$

(or 1.7% per year)

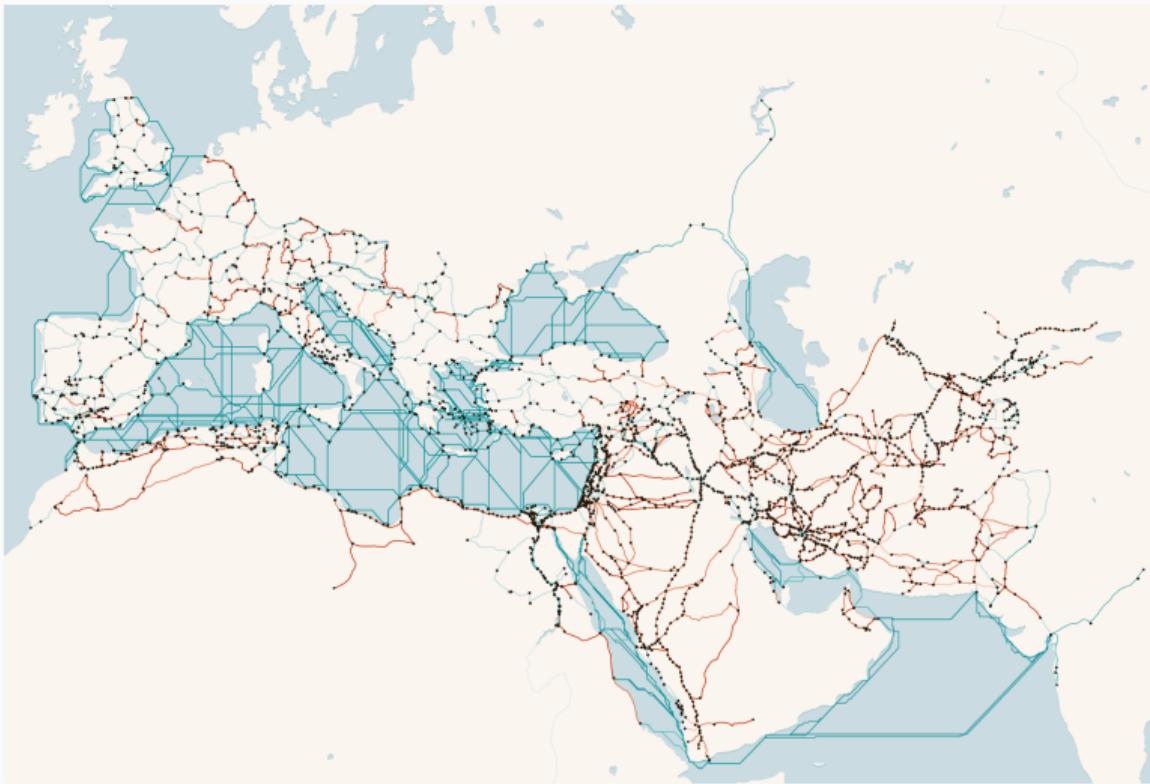
Parameterize trade frictions:

$$d_{ni}(t)^\theta = \exp(\gamma_0 + \zeta \ln(\text{TravelTime}_{ni}) + \kappa_1 \text{PoliticalBorder}_{ni}(t) + \kappa_2 \text{ReligiousBorder}_{ni}(t))$$

if $n \neq i$ and $d_{nn}(t) = 1$.

- How to split domestic trade vs saving? Calibrate $\bar{s}_n(t) = 1.5\%$ (Scheidel, 2020).
- For counterfactuals, assume $\theta = 4$ (Simonovska and Waugh, 2014).

Trade costs *only* depend on travel times (and politics/religion)



Note: Combined geospatial models from Orbis (Scheidel, 2015) and al-Turayyā (Romanov and Seydi, 2022).



Maximum likelihood estimation

Assume coins in our data are a random sample of coin types in each location \times time.

- Multinomial distribution of coin types,

$$P(\dots, X_i^{(m,\tau)}(T) = x_i^{(m,\tau)}, \dots) = \frac{N_i(T)!}{\prod_{(m,\tau)} x_i^{(m,\tau)}!} \prod_{(m,\tau)} [p_i^{(m,\tau)}(T)]^{x_i^{(m,\tau)}}$$

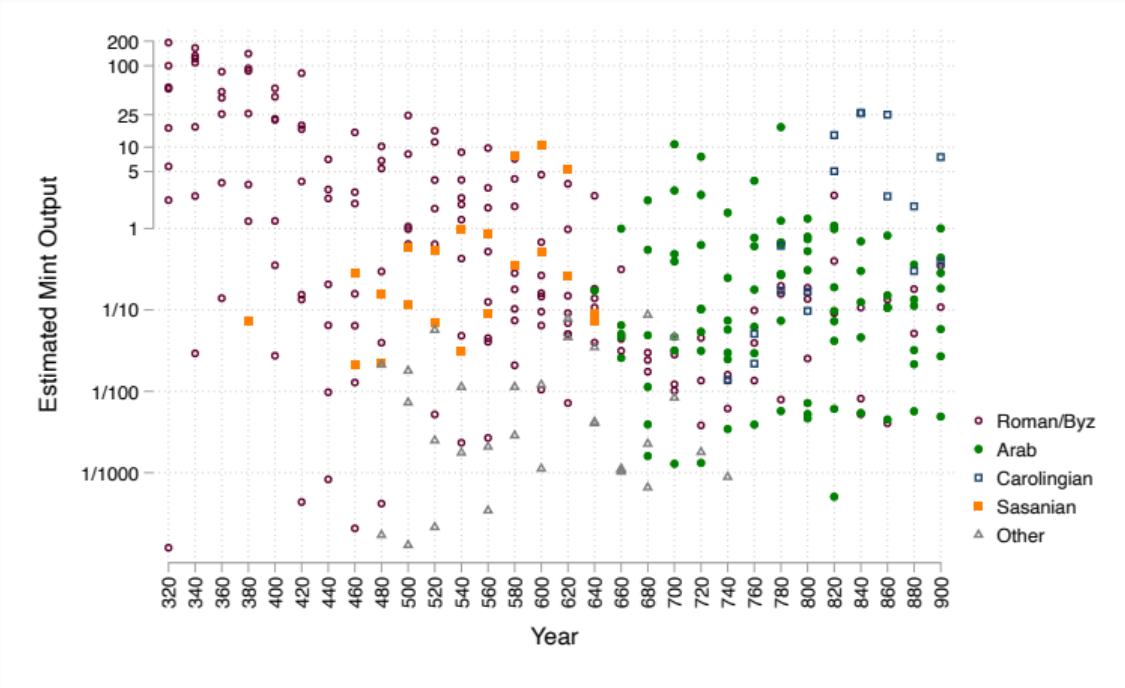
with the probability of drawing a coin of type (m, τ) ,

$$p_i^{(m,\tau)}(T) = \frac{S_i^{(m,\tau)}(T)}{\sum_{(m',\tau')} S_i^{(m',\tau')}(T)} = \frac{S_i^{(m,\tau)}(T)}{S_i(T)}.$$

- Likelihood of observing a sample of coins given parameters θ ,

$$\ell(X; \theta) = \sum_i \sum_T \sum_{(m,\tau)} x_i^{(m,\tau)} [\log S_i^{(m,\tau)}(T; \theta) - \log S_i(T; \theta)] + \text{constant}$$

Estimation results: Mint output



Normalization: $M_{n_0}[t_0] = 100$ (Northern Italy, AD 320-40).

Discussion on Byzantine monetary output: Kazhdan (1954), Pennas (1996)

Estimation results: Determinants of trade costs

$$\ln((d_{ni}[t])^{-\theta}) = \text{constant} - 2.98 \underset{(0.02)}{\ln(TravelTime_{ni})} - 0.3 \underset{(0.02)}{\ln(PoliticalBorder_{ni}[t])} - 4.05 \underset{(0.11)}{\ln(ReligiousBorder_{ni}[t])}$$

- Travel time elasticity similar to estimates on ancient trade.
Roman trade: Flückiger et al. (2022); Bronze Age trade: Barjamovic et al. (2019).
- Political border tax: 8%
(with $\theta = 4$, $d_{between}/d_{within} = e^{0.3/4} \approx 1.08$)
- Religious border tax: 175%
(with $\theta = 4$, $d_{between}/d_{within} = e^{4.05/4} \approx 2.75$)
- Anderson and van Wincoop (2003) US-Canada border tax: 49%
(with $\theta = 4$, $d_{between}/d_{within} = e^{1.59/4} \approx 1.49$)

Welfare and counterfactuals

Real consumption depends on a combination of L and T (that's not separately identified):

$$X_n/p_n = \gamma^{-1} (\pi_{nn})^{-1/\theta} (L_n T_n^{1/\theta}) \left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n}\right)$$

or equivalently in per capita terms

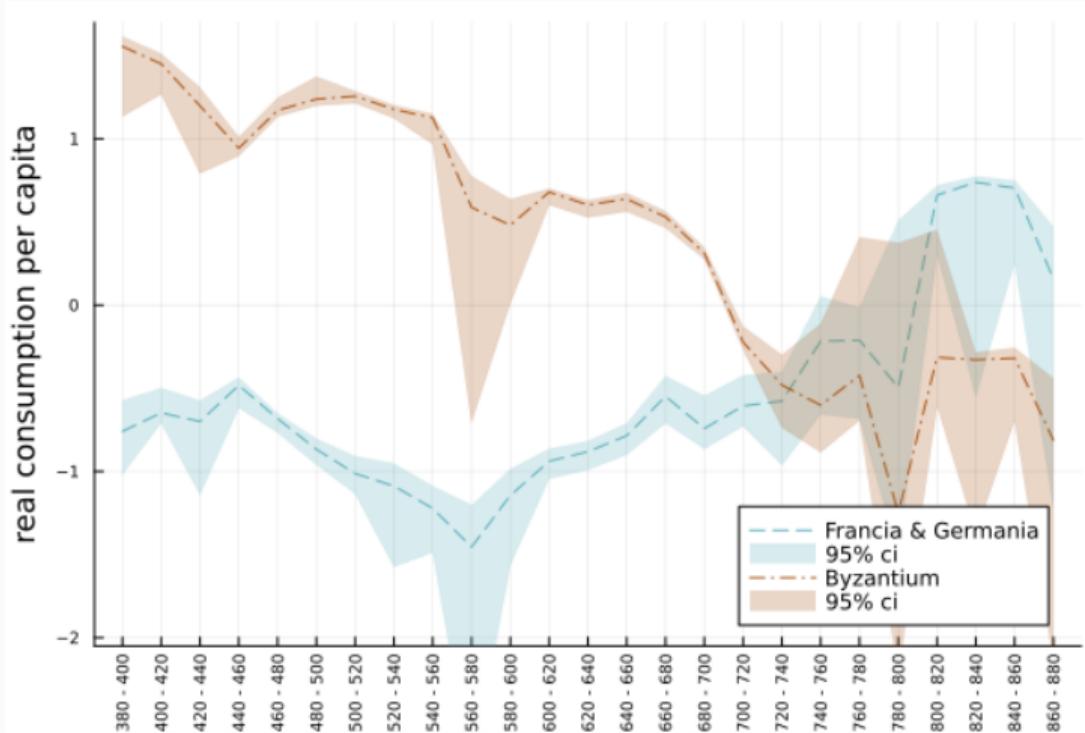
$$\underbrace{\frac{X_n/p_n}{L_n}}_{\text{Real Consumption}} = \underbrace{\gamma^{-1} (\pi_{nn})^{-1/\theta}}_{\text{Openness}} \underbrace{(T_n)^{1/\theta}}_{\text{Technology}} \underbrace{\left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n}\right)}_{\text{Trade Deficit}}$$

Note: T and L are not separately identified. We separate L and T through a Malthusian assumption

$$L_n = T_n \quad \forall n$$

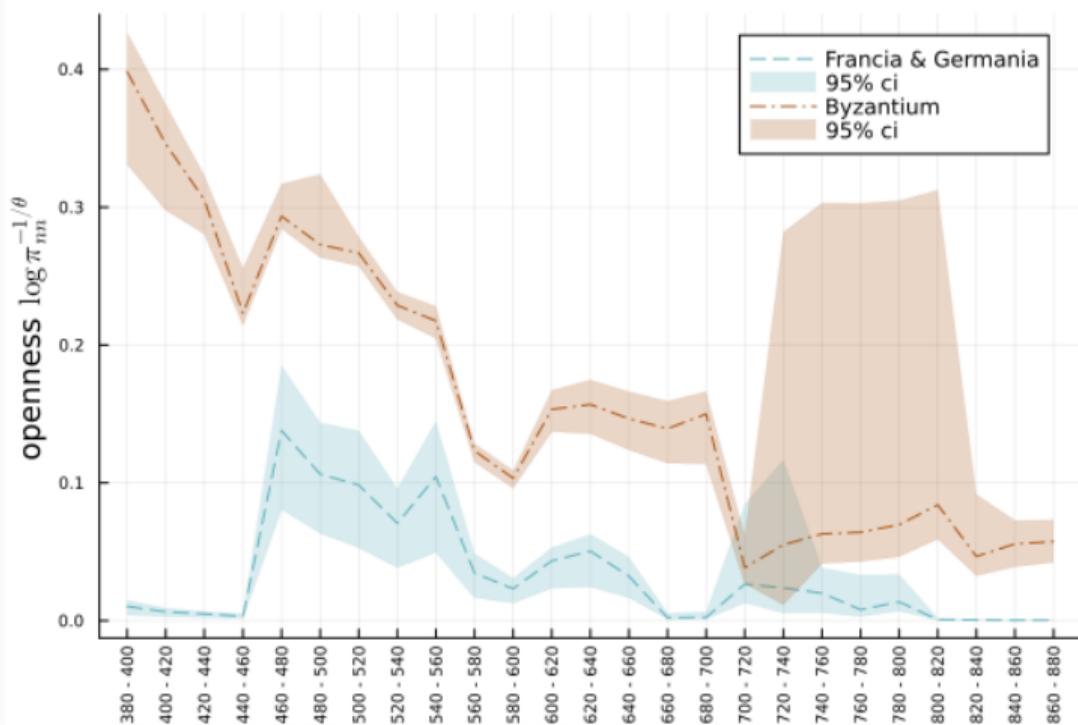
and decompose per-capita real consumption into the three components

Byzantium vs northern Europe (380-880): *real consumption per capita*



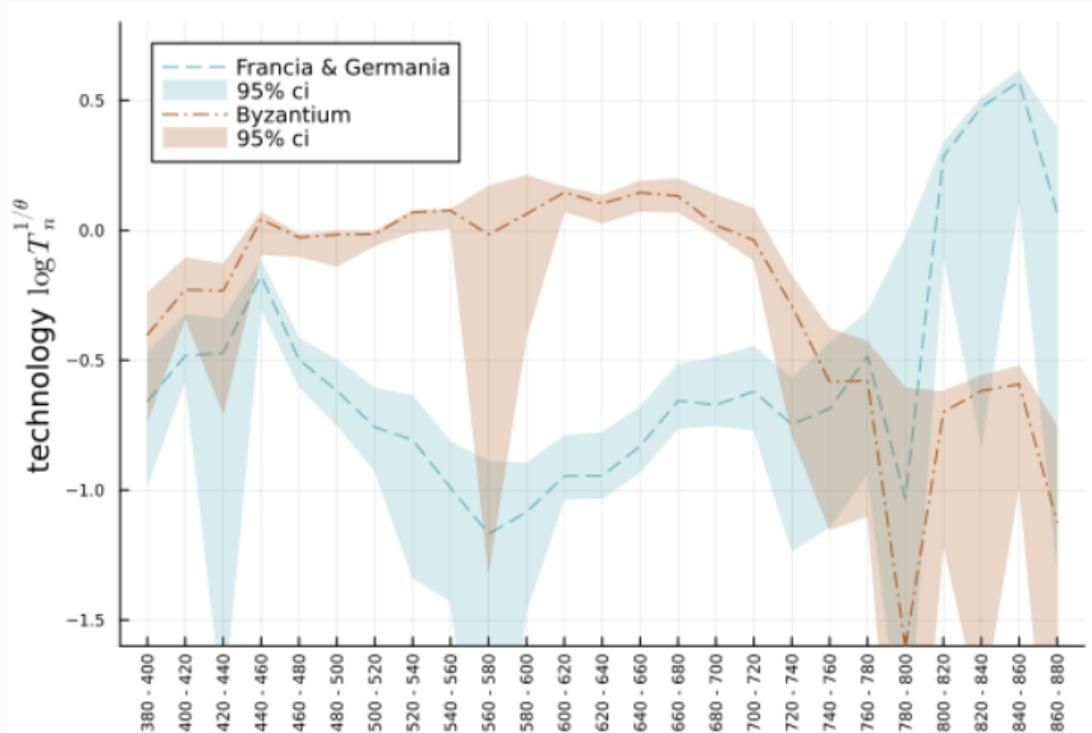
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Byzantium vs northern Europe (380-880): trade openness



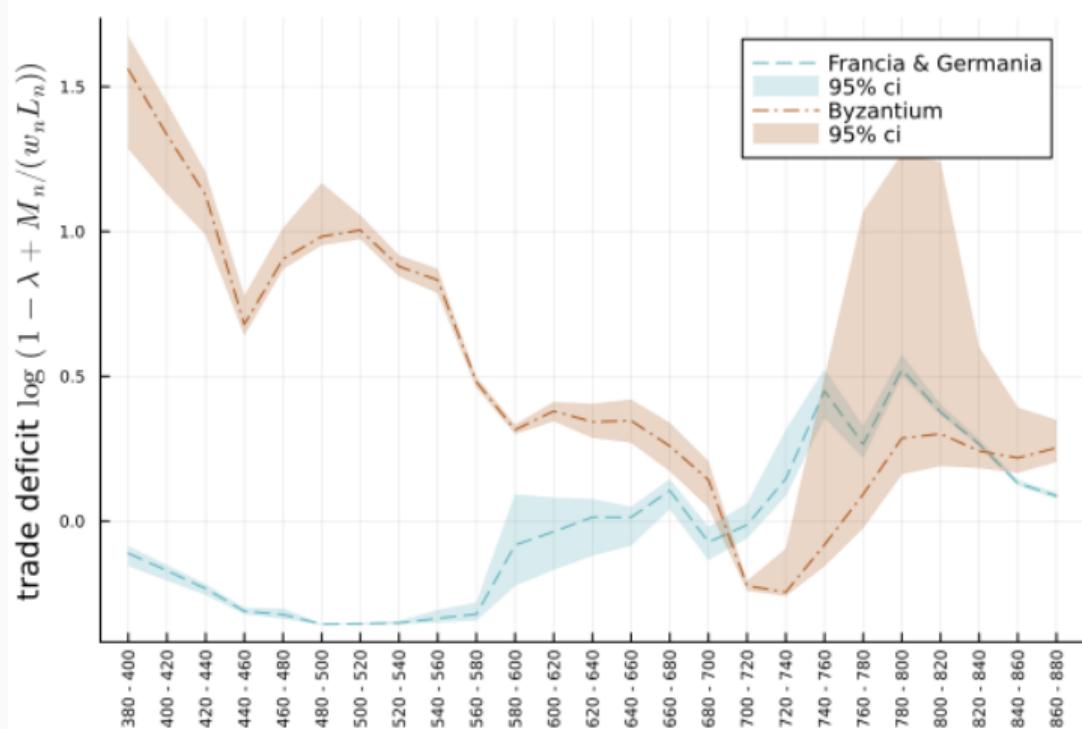
Bootstrapped 95% confidence intervals.

Byzantium vs northern Europe (380-880): technology



Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Byzantium vs northern Europe (380-880): trade deficits



Bootstrapped 95% confidence intervals.

Real consumption per capita: technology, geography, and trade (deficits)

Table 3: Real consumption in the ancient world from AD 460-620 to AD 700-900

	Consumption $\Delta \log \left(\frac{X_n / P_n}{L_n} \right)$ (1)	Openness $\Delta \log \left(\pi_{nn}^{-1/\theta} \right)$ (3)	Technology $\Delta \log \left(T_n^{1/\theta} \right)$ (5)	Trade Deficits $\Delta \log \left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n} \right)$ (7)	
	(2)	(4)	(6)	(8)	
al-Andalus	0.53 (0.08)	-0.04 (0.01)	0.65 (0.10)	-0.08 (0.05)	
Francia and Germania	1.99 (0.14)	-0.07 (0.02)	1.94 (0.16)	0.12 (0.04)	
Byzantine Heartlands	-1.56 (0.22)	-0.16 (0.06)	-0.74 (0.13)	-0.66 (0.25)	
Arabian Peninsula	1.12 (0.28)	-0.02 (0.04)	0.98 (0.37)	0.15 (0.23)	

Compare to relative urbanization rates, 700–900 AD

Top: Change in total urban population (urban: > 1k inhabitants), data from Buringh (2021)



Bottom: $\Delta W_n / L_n$



Taking stock

- Clear pattern of change in economic geography before vs after conquest
- Trade disruption can account for the relative decline in the eastern Mediterranean
- Change in trade cost *alone* not able to account for urbanization in Muslim Spain, or in Carolingian empire
- In conjunction with changes in technology T_i and mint output, can account for urbanization patterns.

Back to Pirenne:

- Yes, new political and religious borders change market access, quant'ly relevant
- But unlikely to account for entire shift towards north-east
- Seignorage and technical change seem to be more important drivers

Conclusion

“Simply looking at the Mediterranean cannot of course explain everything about a complicated past created by human agents, with varying doses of calculation, caprice and misadventure. But this is a sea that patiently recreates for us scenes from the past, breathing new life into them, locating them under a sky and in a landscape that we can see with our own eyes, a landscape and sky like those of long ago. A moment’s concentration or daydreaming, and that past comes back to life.”

Fernand Braudel, Les Mémoires de la Méditerranée

THANK YOU!

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BACKUP SLIDES

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Political changes in the Mediterranean: 600 AD



▶ Back

Political changes in the Mediterranean: 600 AD



▶ Back

Political changes in the Mediterranean: 600 AD



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Political changes in the Mediterranean: 600 AD



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Political changes in the Mediterranean: 632 AD



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Political changes in the Mediterranean: 634 AD



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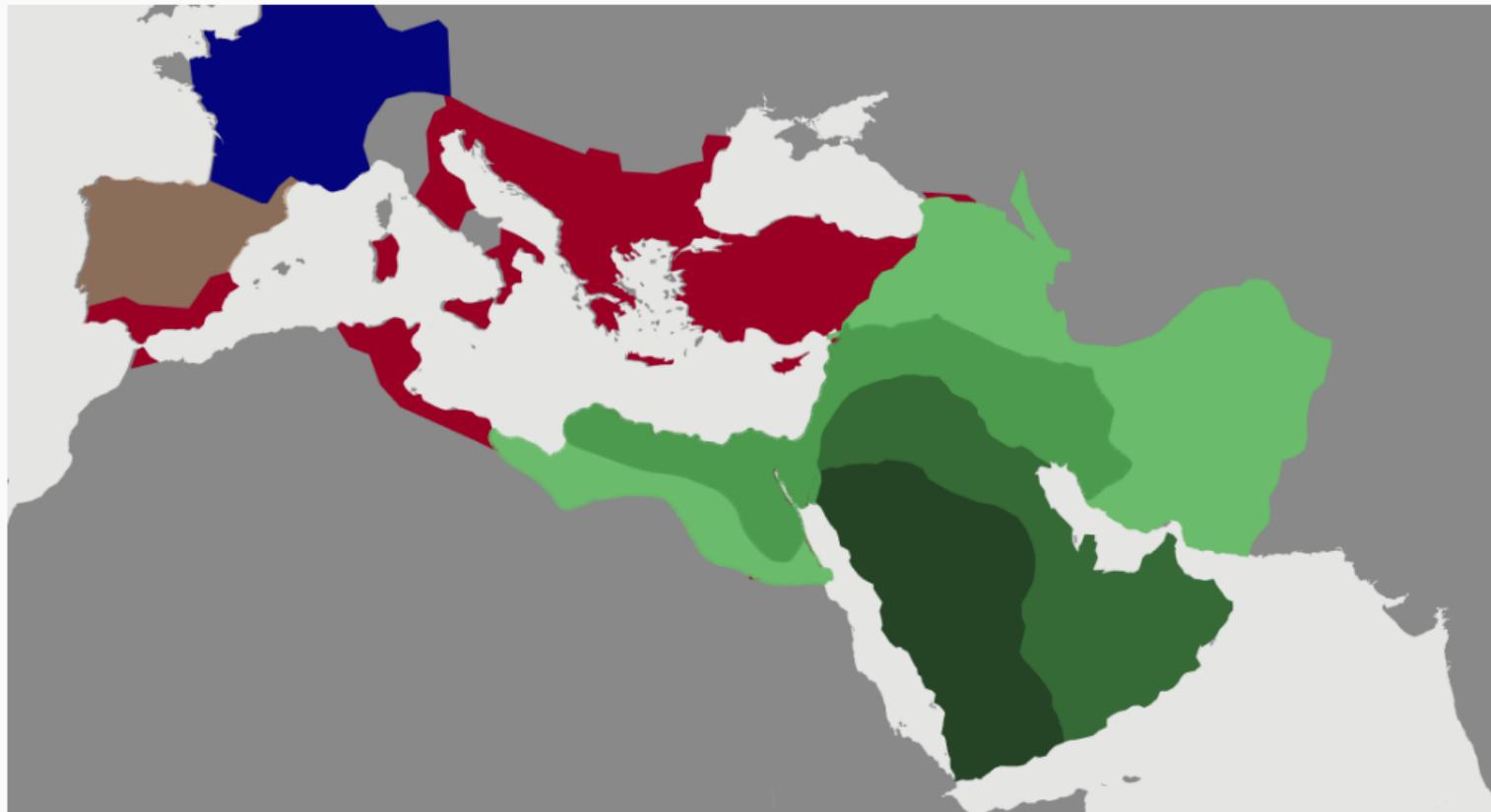
Political changes in the Mediterranean: 644 AD



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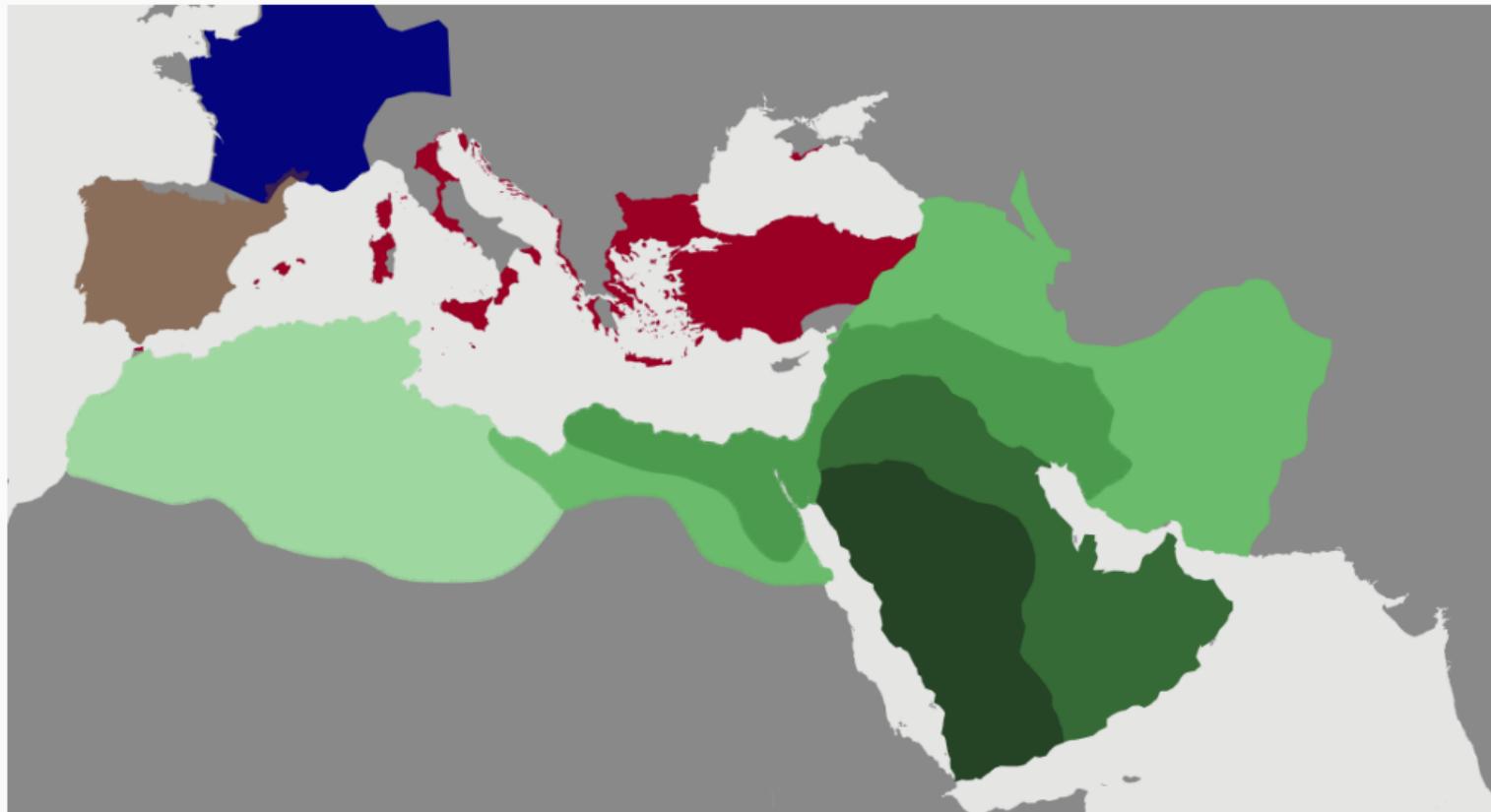
Political changes in the Mediterranean:

661 AD



▶ Back

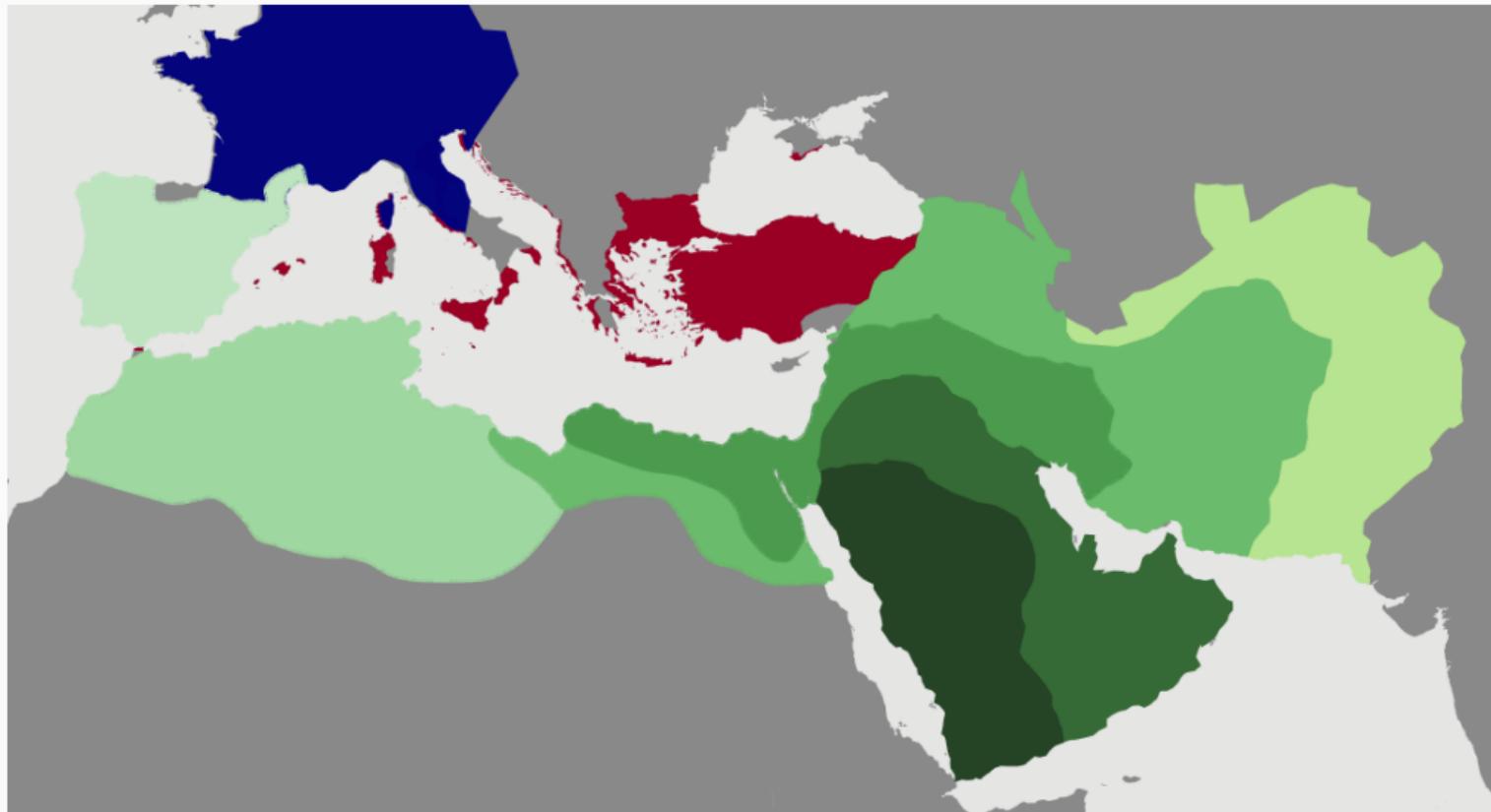
Political changes in the Mediterranean: 661-700 AD



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Political changes in the Mediterranean:

750 AD



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Regions



Fact #3: Coin flows before/after the Arab conquests

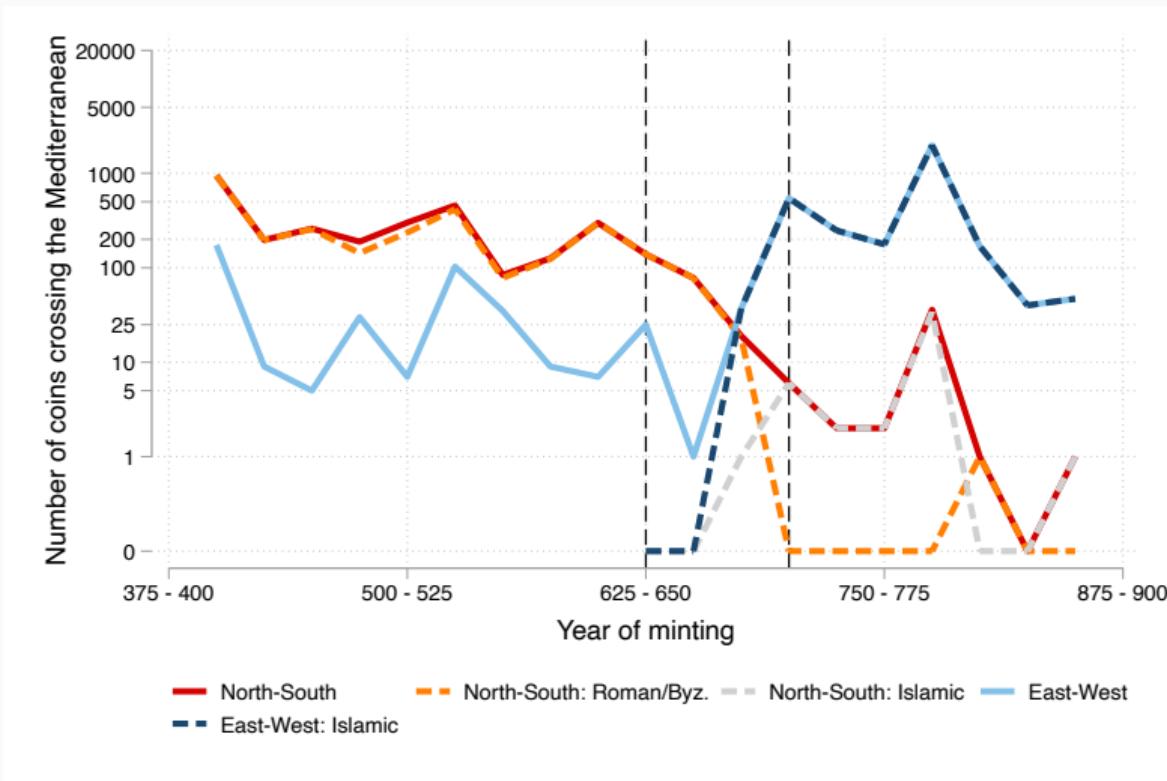


Figure 3: Number of coins flowing across the Mediterranean

Fact #3: Coin flows before/after the Arab conquests

Table 4: The Mediterranean Before and After the Arab Conquest

	Dependent variable: Number of Coins			
	(1)	(2)	(3)	(4)
Crossing Mediterranean × After Conquests	-1.893*** (0.48)	-3.246*** (0.53)	-0.662 (0.63)	-1.736 (1.27)
Crossing Mediterranean × After Conquests × Islamic Coin		7.267*** (0.90)	4.789*** (0.95)	7.545*** (0.89)
Crossing Mediterranean × After Conquests × Roman Coin			-3.287*** (0.75)	-2.893*** (0.61)
Mint Cell × Empire FE	Yes	Yes	Yes	Yes
Mint Cell × Hoard Cell FE	Yes	Yes	Yes	Yes
After Conquests FE	Yes	Yes	Yes	
Mint Cell × After Conquests FE				Yes
Hoard Cell × After Conquests FE				Yes
Estimator	PPML	PPML	PPML	PPML
Observations	10480	10480	10480	6208

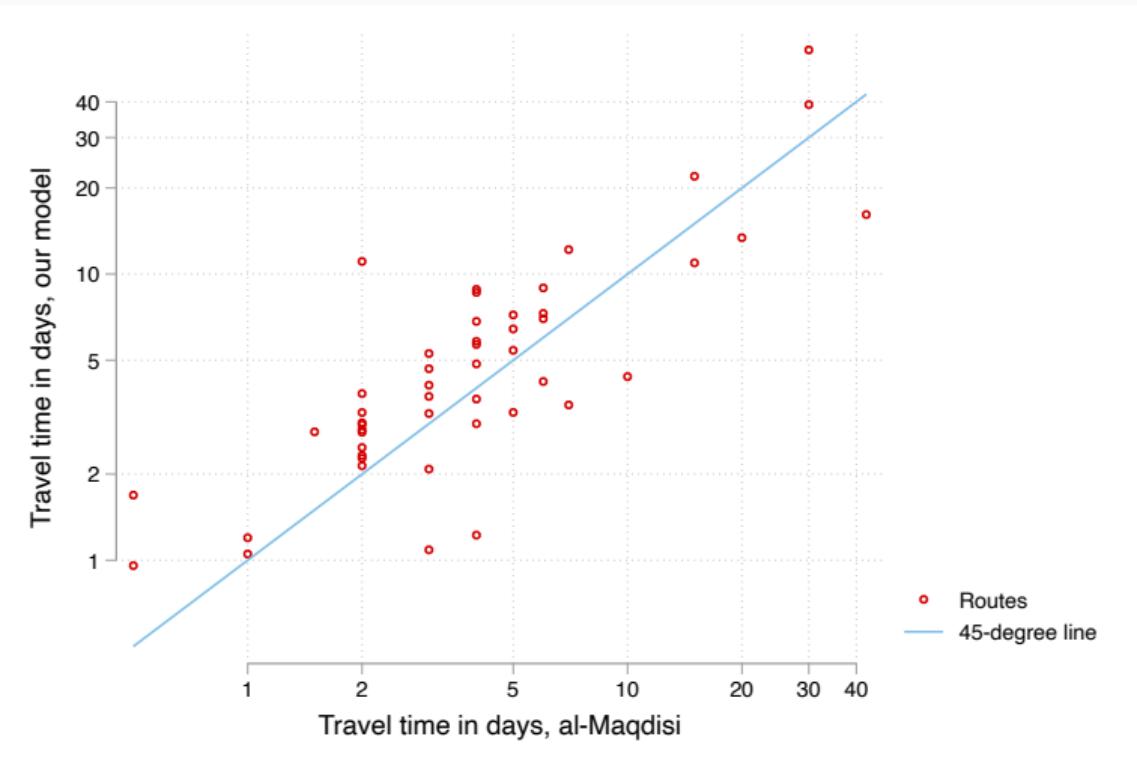
Standard errors in parentheses, clustered at the hoard × era and mint × era level.

+ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Estimating eqn: $\text{count}_{mdht} = \exp(\gamma_1 \text{mediterranean}_{mh} \times \text{after}_t + \dots + \alpha_{md} + \alpha_{mh} + \varepsilon_{mhdt})$

Validating Travel Times

Al-Maqdisi (c. 945–991): *The Best Divisions for Knowledge of the Regions*



Unsolved problems (as of yet)

- *Lucas critique #1*: cost function does not minimize costs

$$\ln(\delta_{ni}(t)) = \min_{p \in paths(i \rightarrow n)} \left(\gamma_0 + \gamma_1 \ln(TravelTime_p(t)) + \sum_{pb: \text{ all political borders along } p} \gamma_2 PoliticalBorder_{pb}(t) + \sum_{rb: \text{ all religious borders along } p} \gamma_3 ReligiousBorder_{rb}(t) \right)$$

- *Lucas critique #2*: net saving (in δ_{nn}) depends on parameters.
- *Fix for #2*: location-specific intercepts ($\gamma_{0,n}$) and δ_{nn} 's.

For now: constant γ_0 , and $\delta_{nn} = 1$...

Fact #3: Arab conquests disrupt Mediterranean trade: Gold only

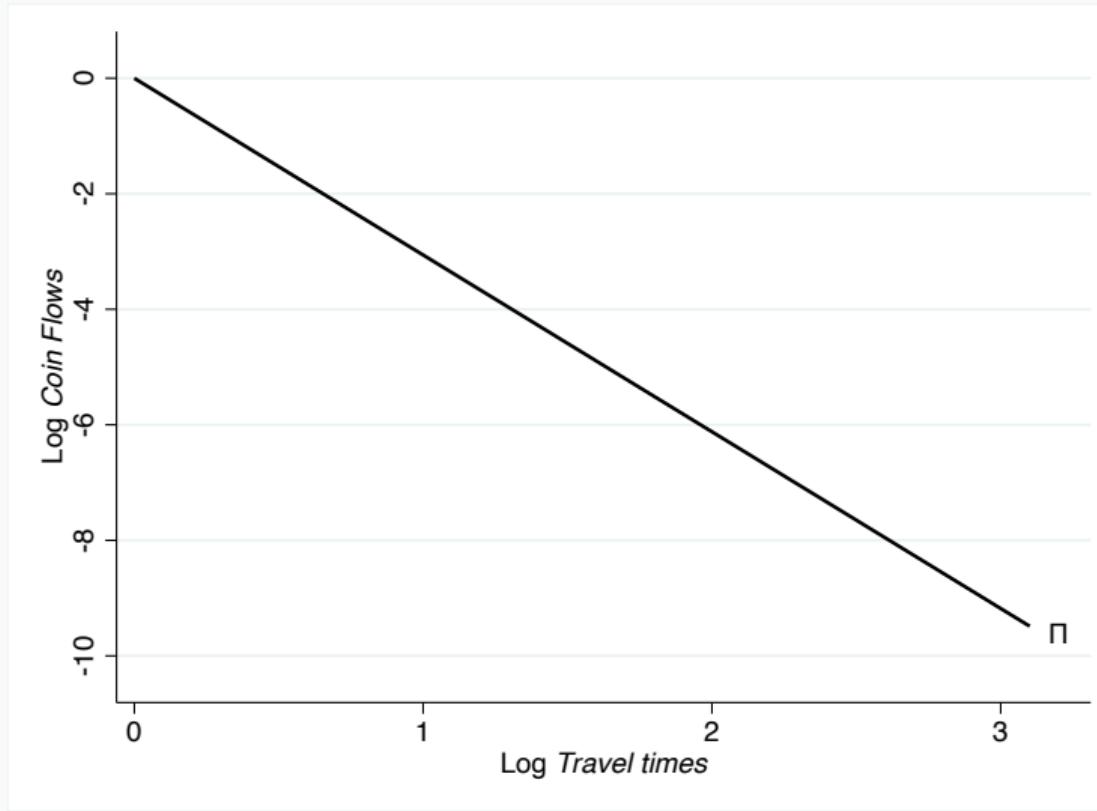
	Dependent variable: Number of Coins				
	(1)	(2)	(3)	(4)	(5)
Log Distance	-1.516*** (0.13)	-1.541*** (0.13)	-1.544*** (0.13)	-1.189*** (0.15)	-1.193*** (0.15)
Crossing Mediterranean	0.298 (0.40)	0.307 (0.39)	0.320 (0.39)	0.0942 (0.31)	0.122 (0.31)
Crossing Mediterranean × After Conquests	-1.600** (0.70)	-2.858*** (0.68)	-1.719** (0.69)	-2.576*** (0.98)	-3.379*** (1.13)
Crossing Mediterranean × After Conquests × Islamic Coin		3.020*** (0.71)	1.864** (0.76)		2.985** (1.20)
Crossing Mediterranean × After Conquests × Roman Coin			-1.699 (1.04)		
Mint Cell × Empire FE	Yes	Yes	Yes	Yes	Yes
Hoard Cell × After Conquests FE	Yes	Yes	Yes	Yes	Yes
Sample				Gold only	Gold only
Observations	172442	172442	172442	32024	32024

Standard errors in parentheses, clustered at the hoard × era and mint × era level.

+ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

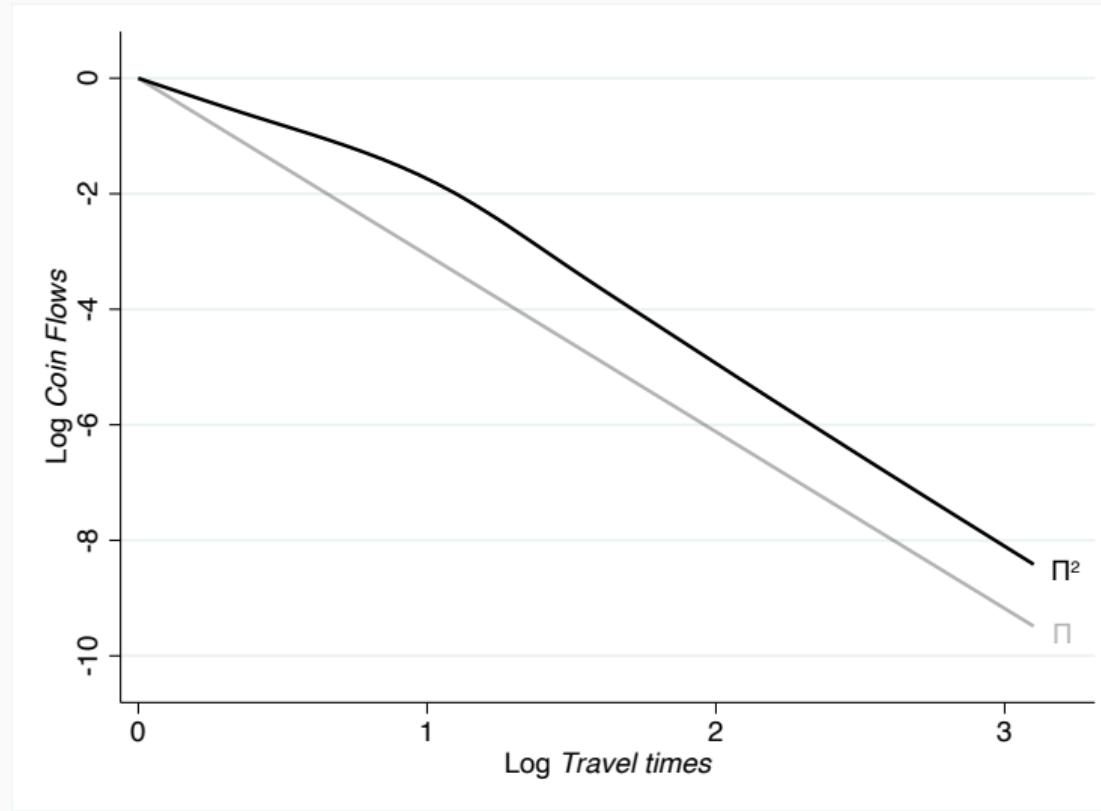
Before: 400–630; after: 713–950

Pitfall #2: stocks vs flows (numerical example)



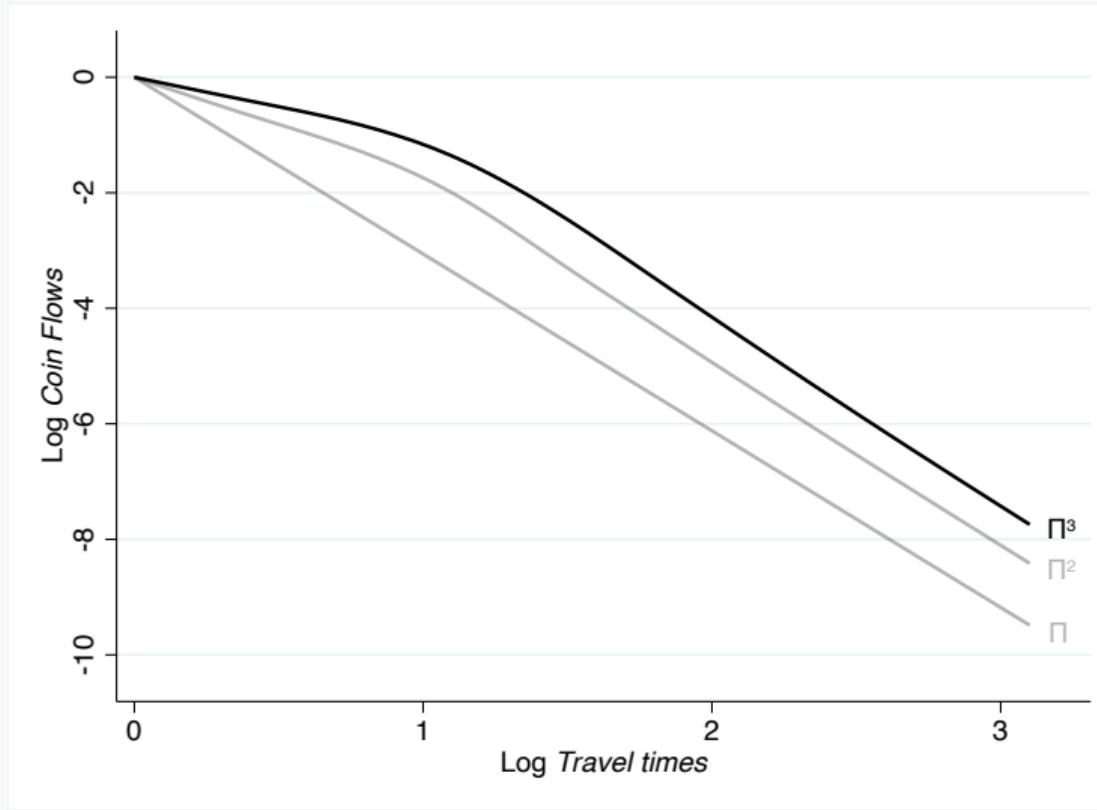
- Flow of coins: age 1 (same as trade flows Π)

Pitfall #2: stocks vs flows (numerical example)



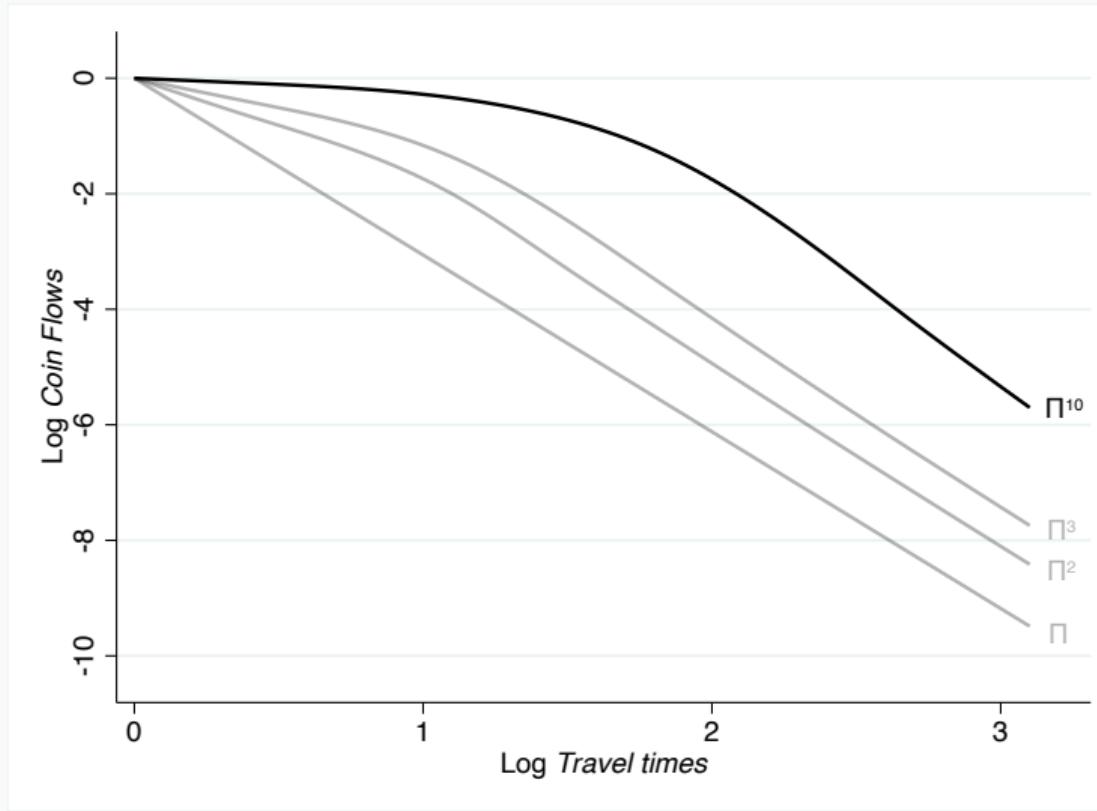
- Flow of coins: age 1, age 2

Pitfall #2: stocks vs flows (numerical example)



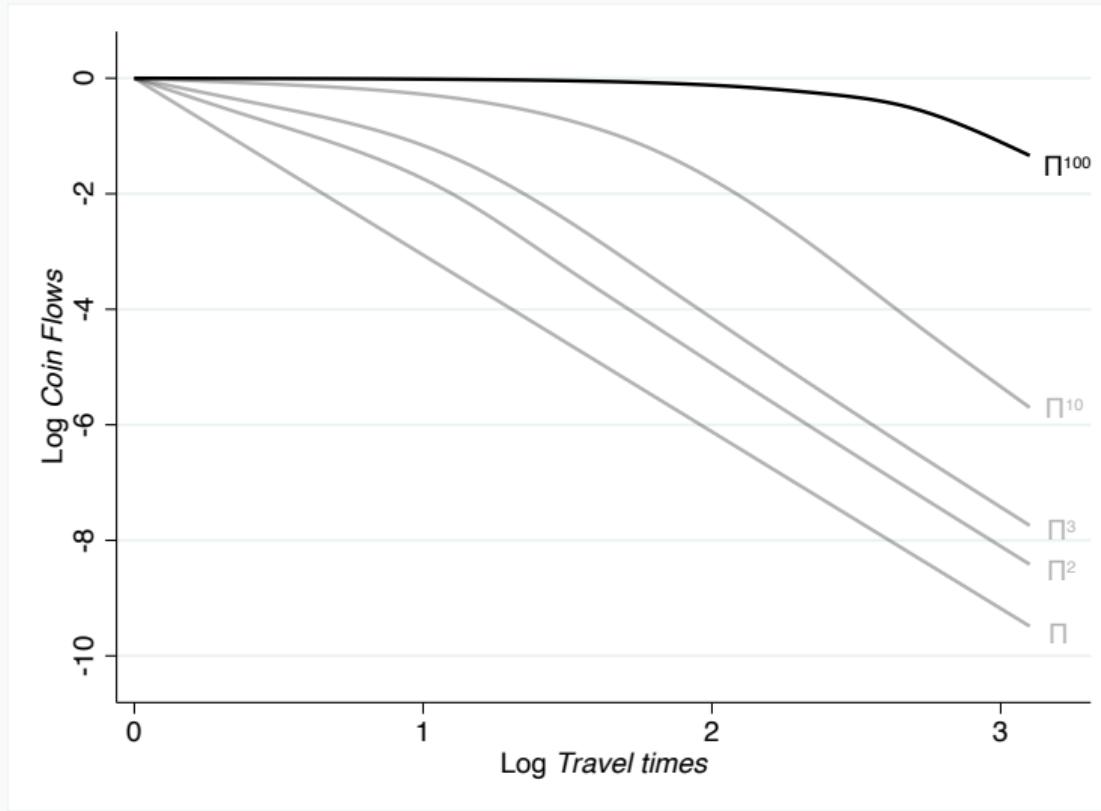
- Flow of coins: age 1, age 2, age 3

Pitfall #2: stocks vs flows (numerical example)



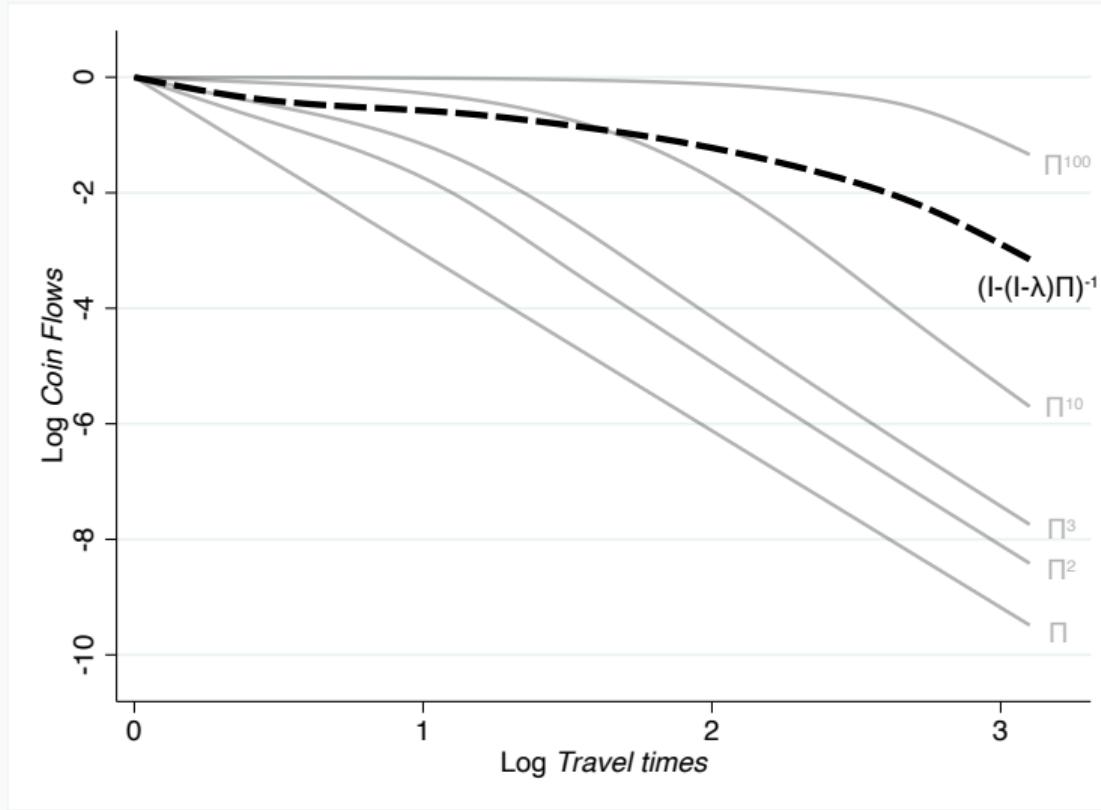
- Flow of coins: age 1, age 2, age 3, age 10

Pitfall #2: stocks vs flows (numerical example)



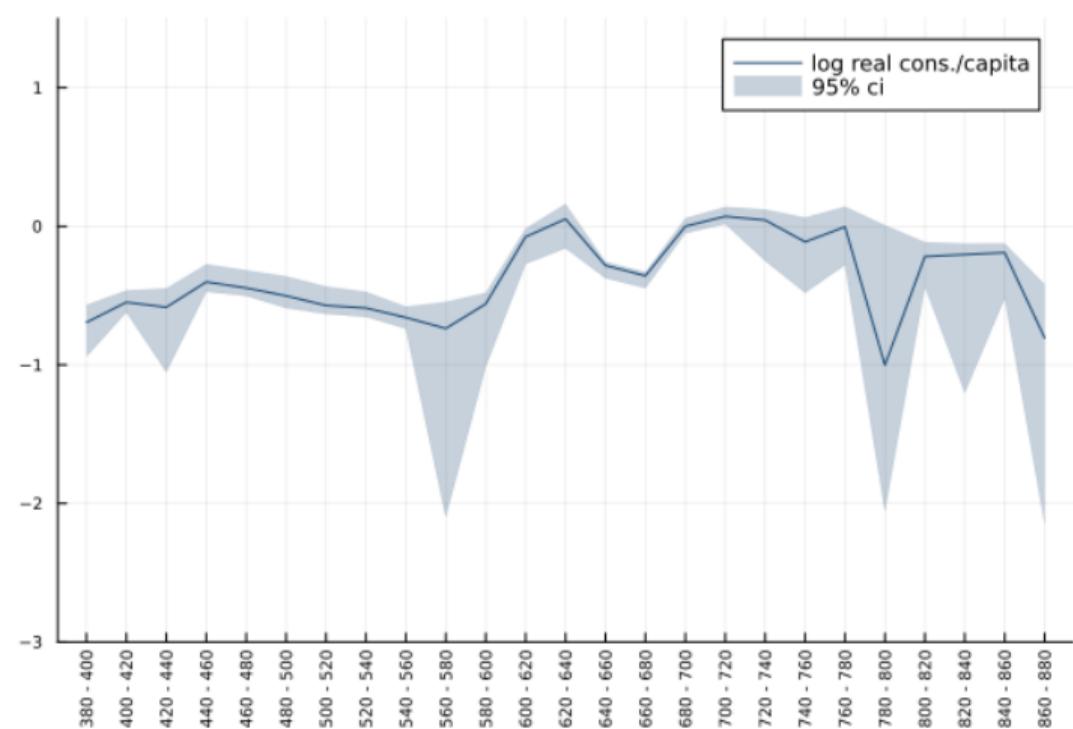
- Flow of coins: age 1, age 2, age 3, age 10, age 100

Pitfall #2: stocks vs flows (numerical example)



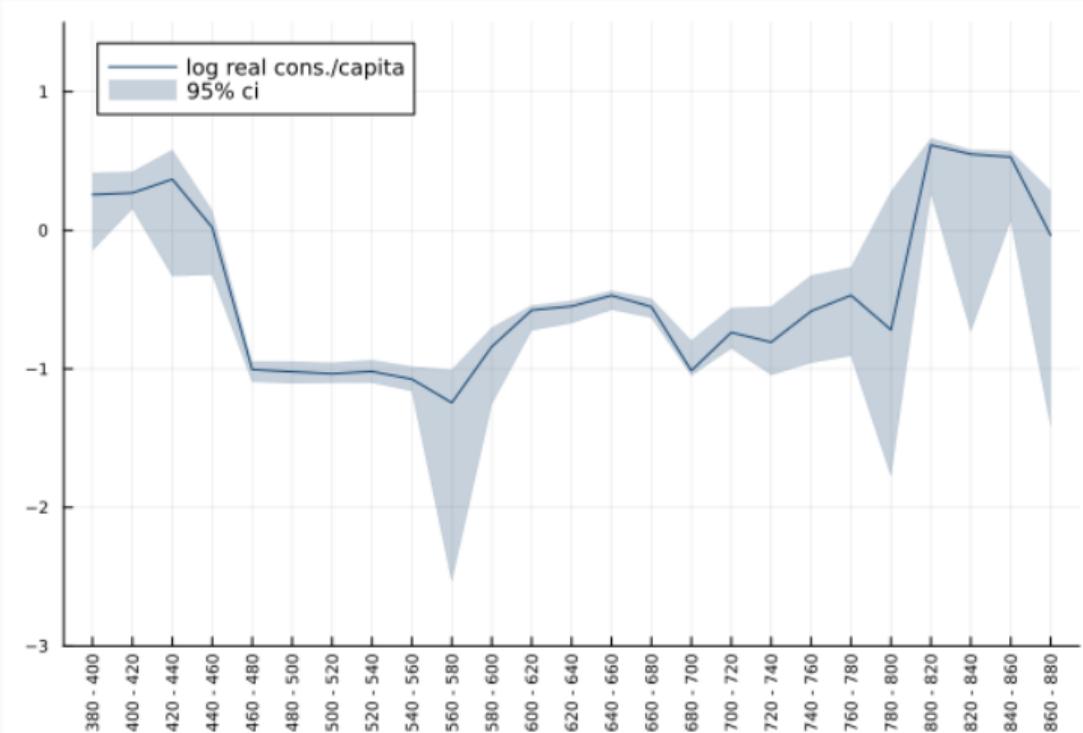
- Flow of coins: age 1, age 2, age 3, age 10, age 100, all ages

Real consumption per capita (380-880): al-Andalus (Spain)



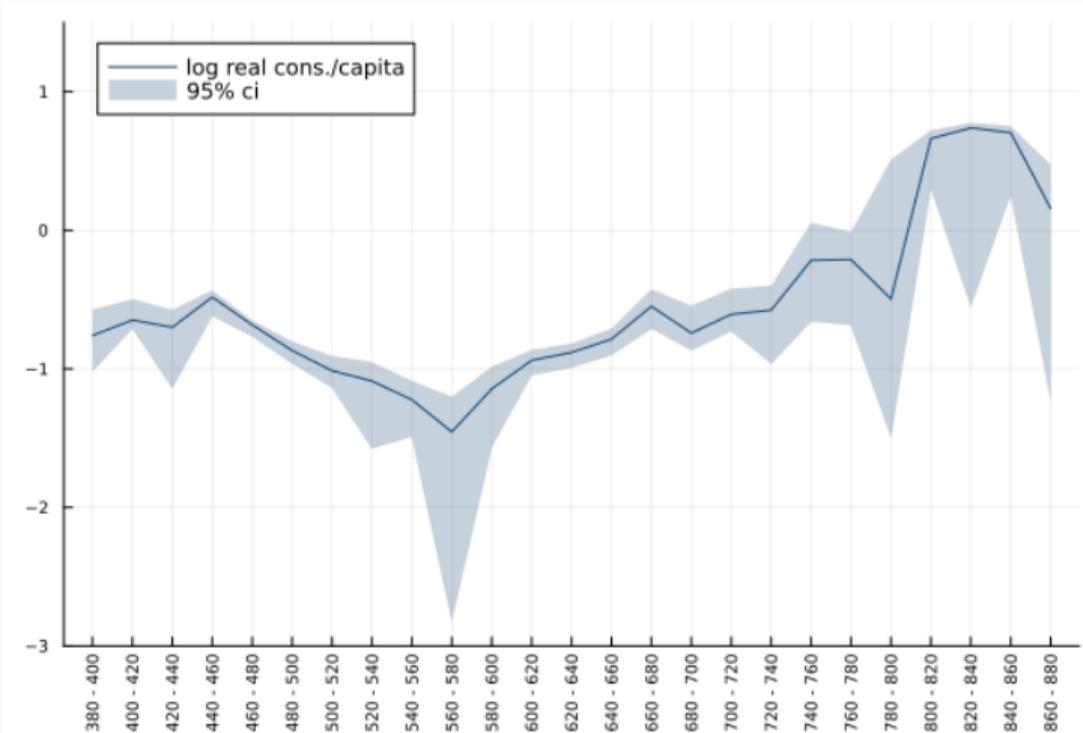
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): Aquitaine (South France)



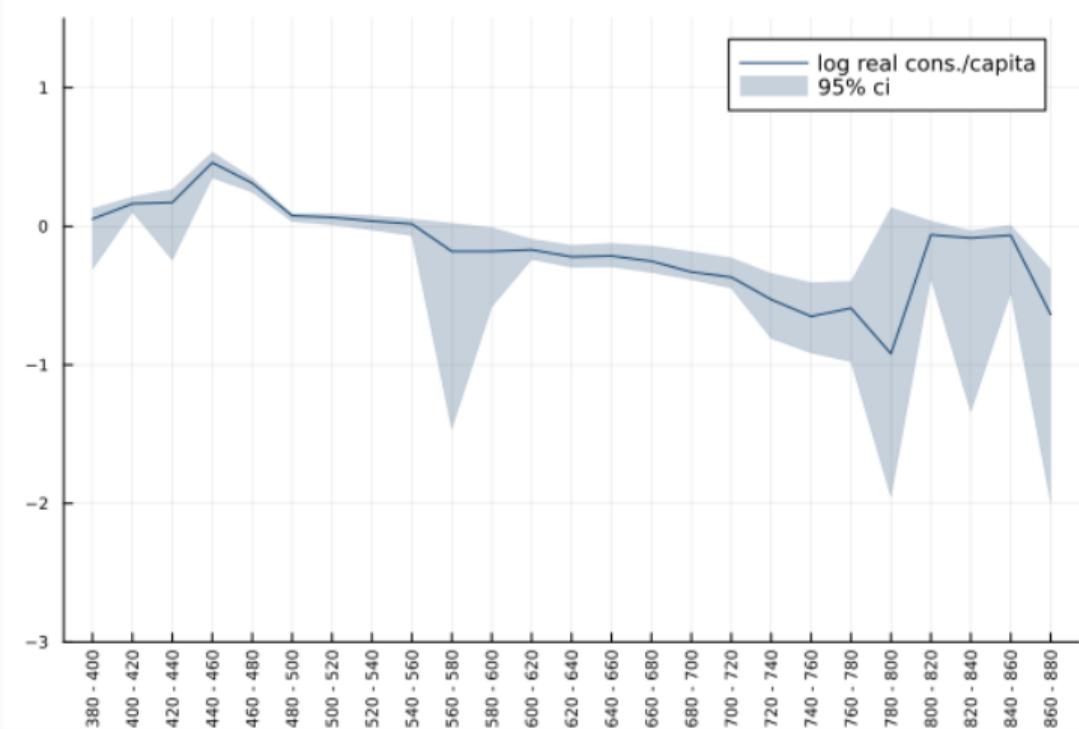
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): Francia and Germania



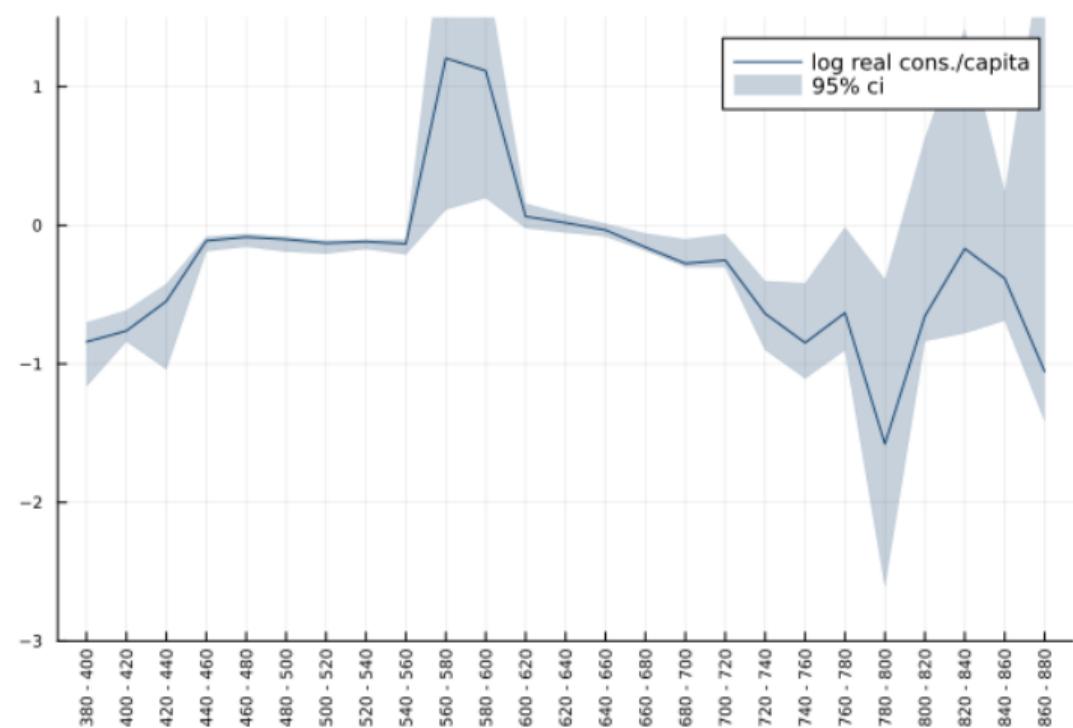
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): Northern Italy



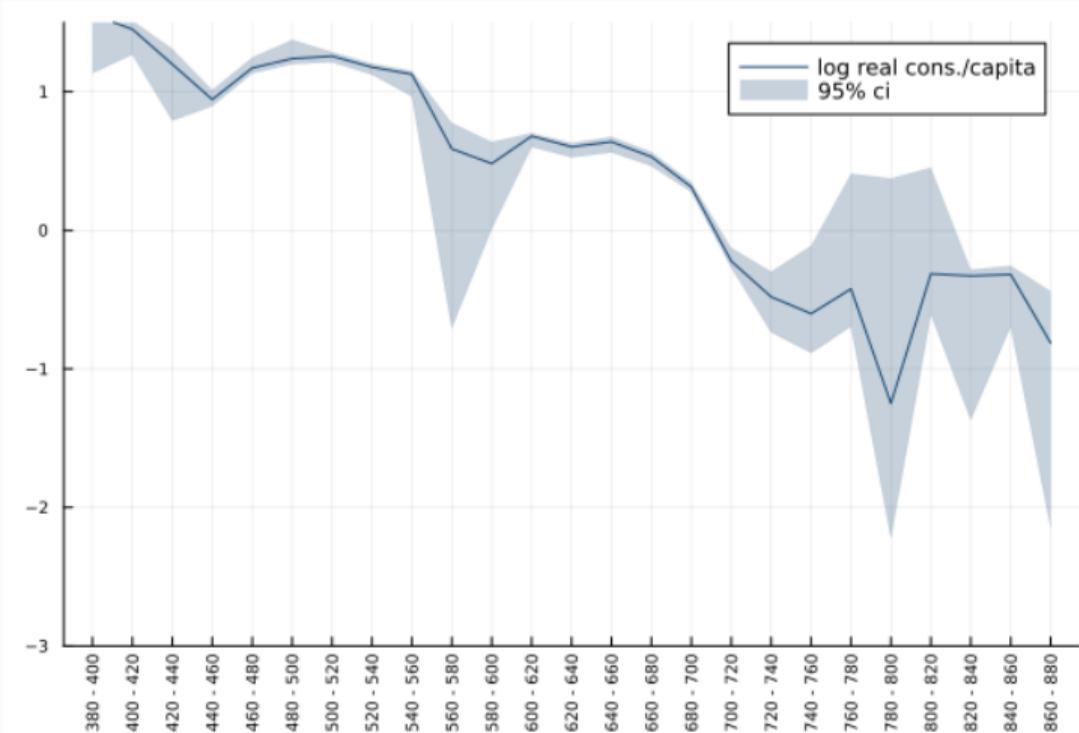
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): Southern Italy



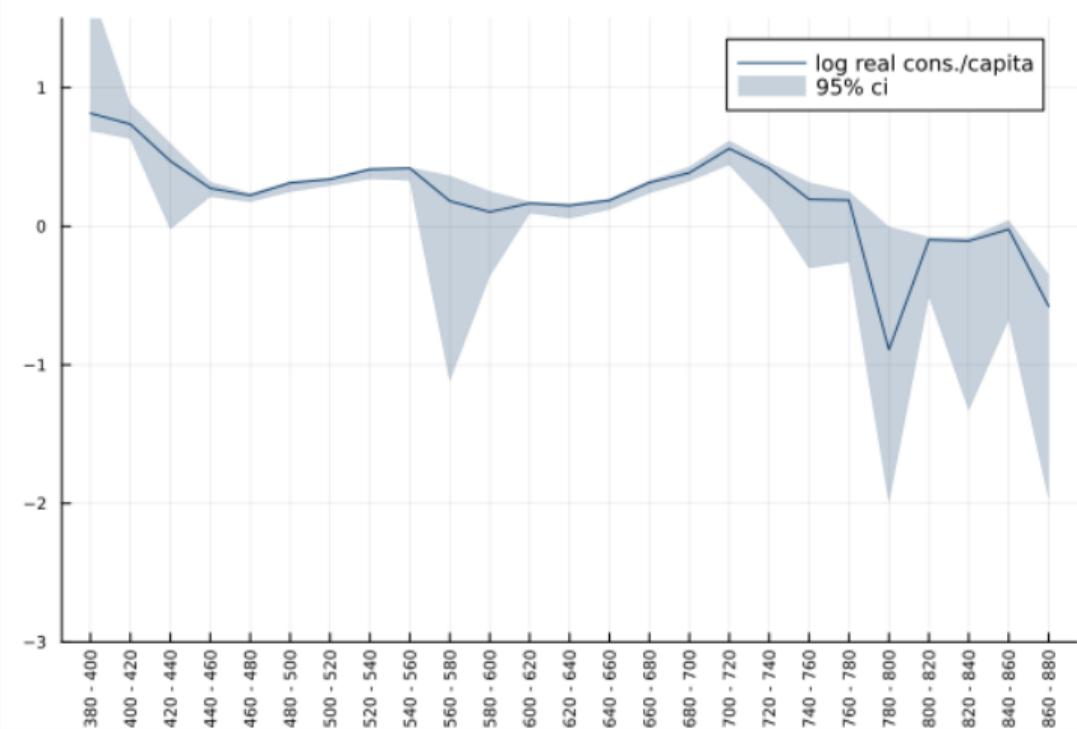
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/P_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): Byzantine Heartlands



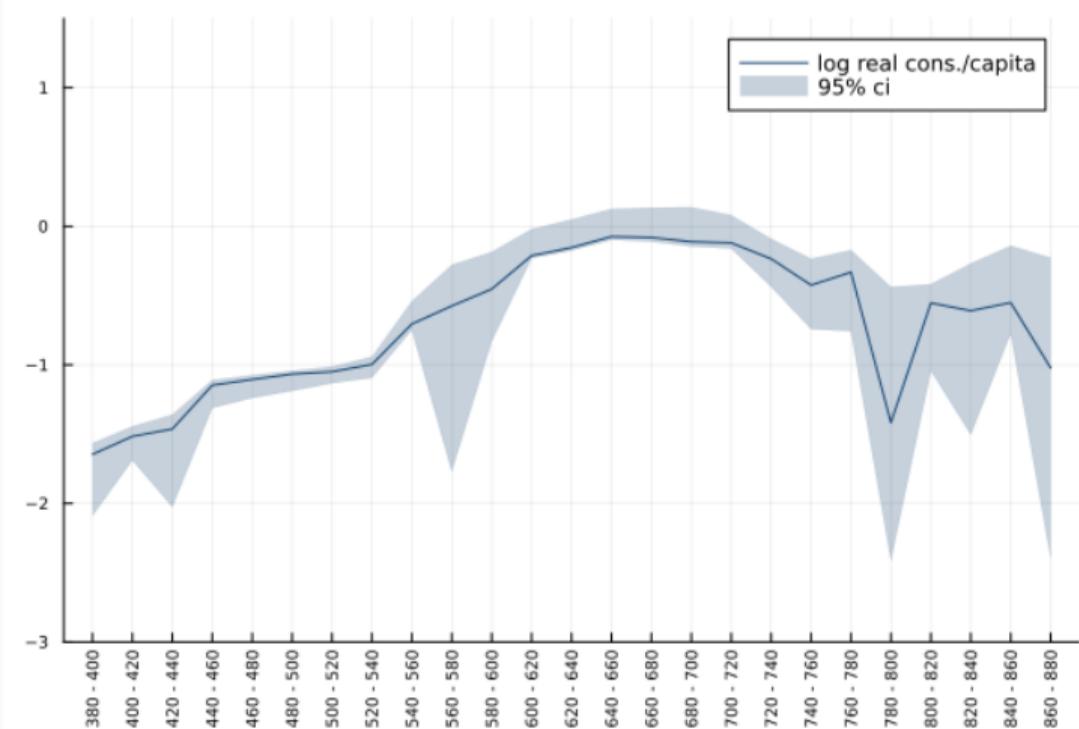
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): al-Sham (Greater Syria)



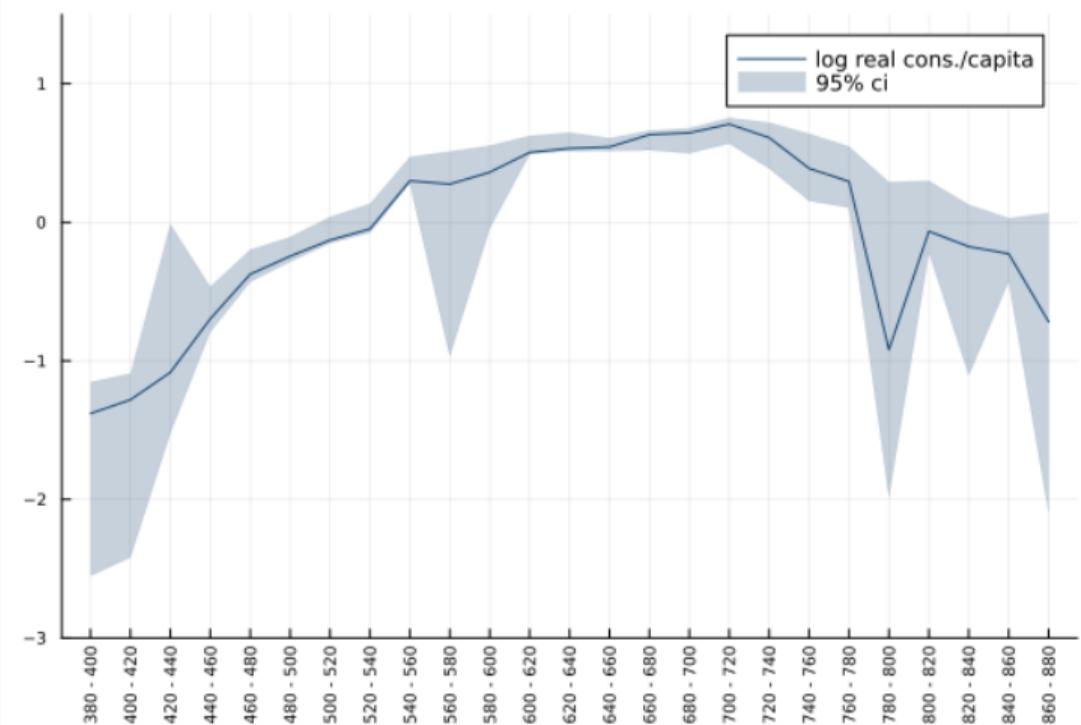
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): Northern Syria, Caucasus



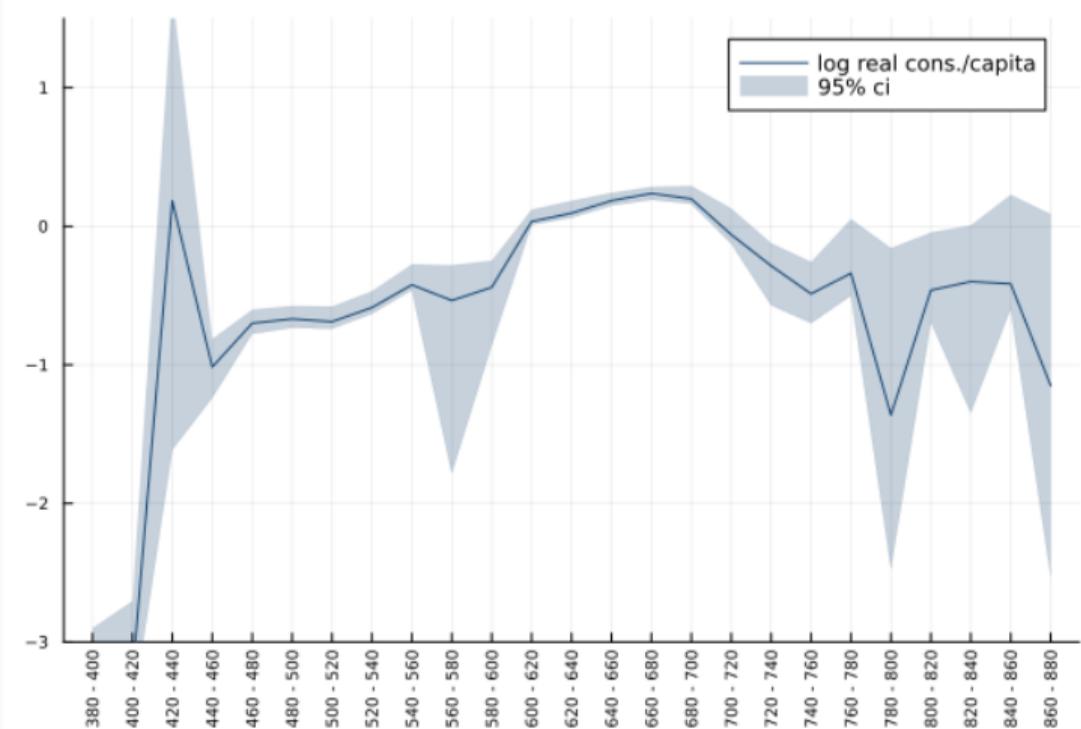
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): Iraq, Iran



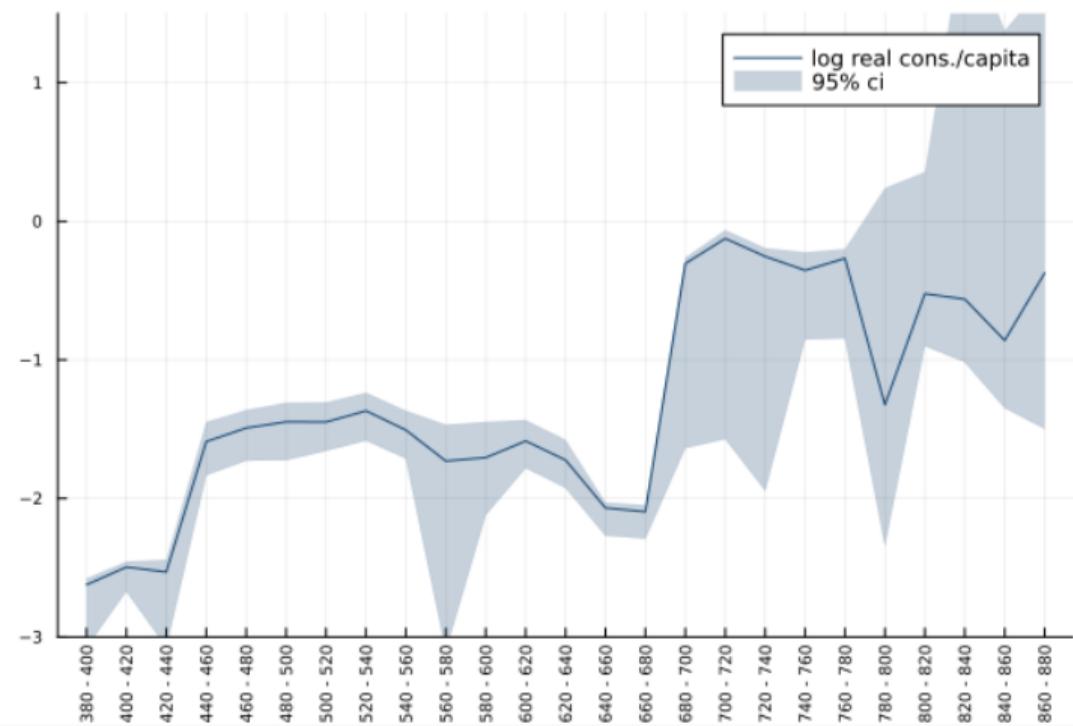
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): Eastern Caliphate



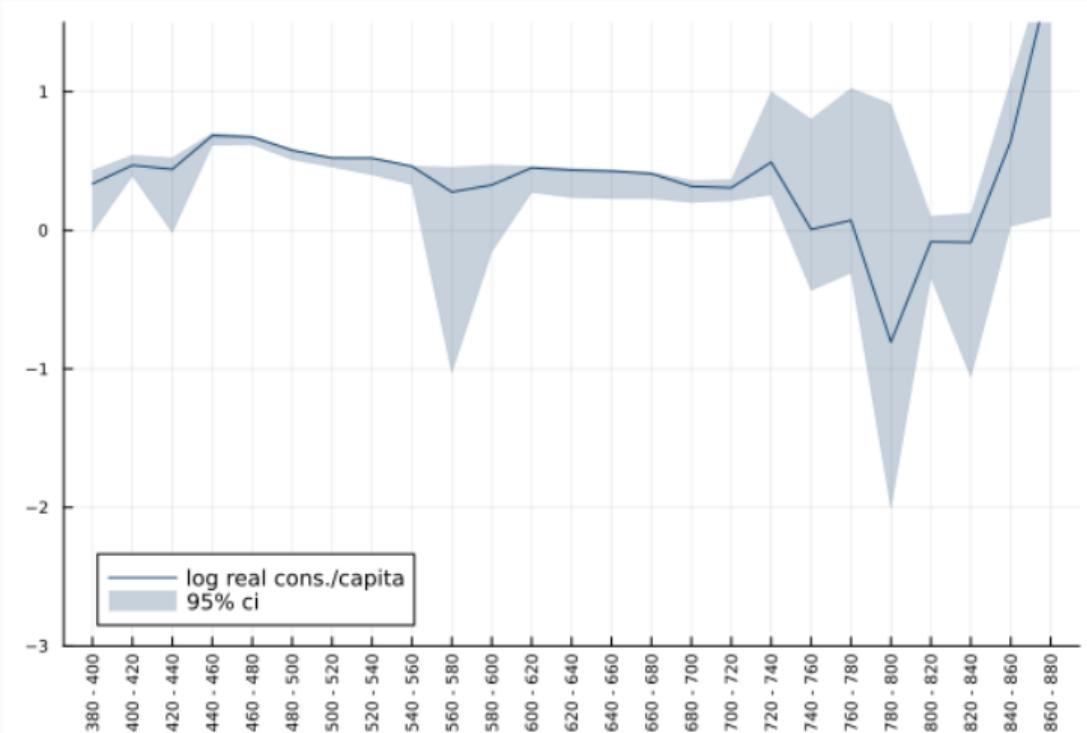
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): Arabian Peninsula



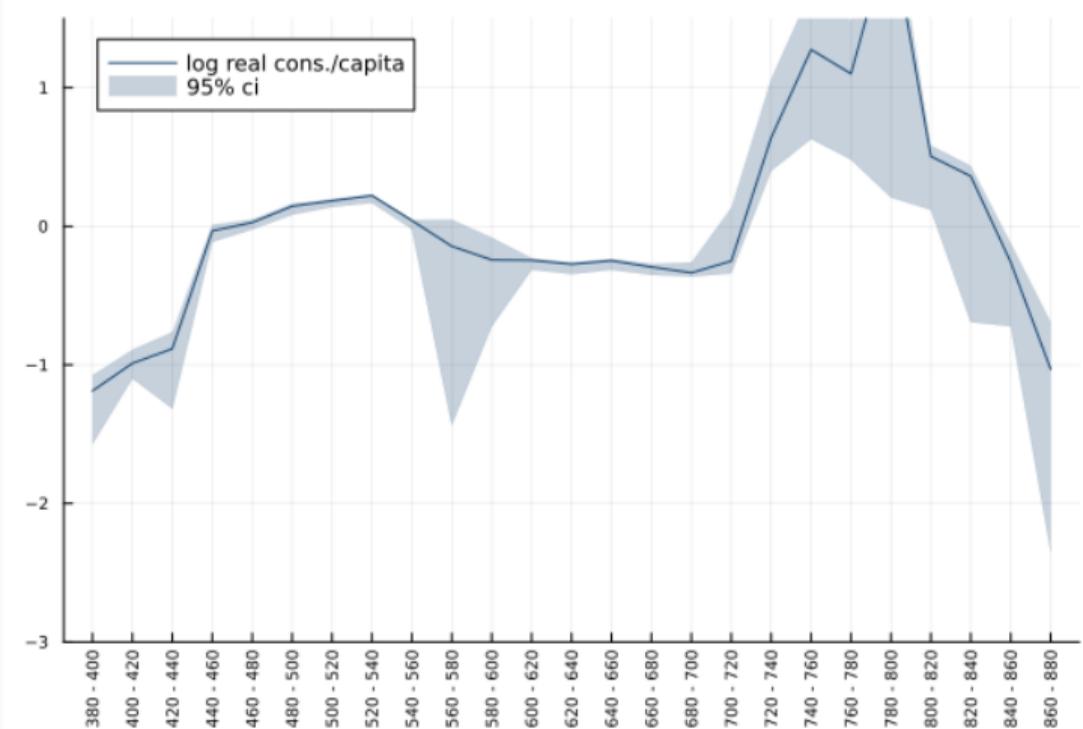
Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): Misr (Egypt)



Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Real consumption per capita (380-880): al-Maghrib



Bootstrapped 95% confidence intervals. Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

Realized vs counterfactual changes in real consumption per capita

Realized changes, from AD 460-620 to AD 700-900

	Real consumption $\Delta \log \left(\frac{X_n/p_n}{L_n} \right)$	Openness $\Delta \log \left(\pi_{nn}^{-1/\theta} \right)$	Technology $\Delta \log \left(T_n^{1/\theta} \right)$	Trade Deficit $\Delta \log \left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n} \right)$
al-Andalus (Spain)	0.62 (0.25)	-0.06 (0.04)	0.77 (0.32)	-0.09 (0.18)
Aquitaine (South France)	1.28 (0.23)	-0.05 (0.01)	1.22 (0.23)	0.11 (0.06)
Francia and Germania	1.96 (0.24)	-0.05 (0.01)	1.80 (0.26)	0.20 (0.04)
Northern Italy	-0.31 (0.24)	-0.08 (0.03)	-0.10 (0.26)	-0.13 (0.10)
Southern Italy	-0.20 (0.34)	0.19 (0.18)	-0.94 (0.37)	0.55 (0.40)
Byzantine Heartlands	-1.56 (0.33)	-0.23 (0.14)	-0.44 (0.41)	-0.89 (0.54)
al-Sham (Greater Syria)	-0.32 (0.27)	-0.04 (0.02)	-0.11 (0.29)	-0.17 (0.11)
Northern Syria, Caucasus	0.22 (0.30)	-0.01 (0.03)	0.15 (0.37)	0.08 (0.12)
Iraq, Iran	0.06 (0.27)	-0.00 (0.01)	0.06 (0.29)	-0.00 (0.04)
Eastern Caliphate	0.37 (0.33)	-0.00 (0.00)	0.39 (0.34)	-0.02 (0.04)
Arabian Peninsula	1.16 (0.34)	-0.01 (0.04)	0.66 (0.45)	0.51 (0.26)
Misr (Egypt)	-0.36 (0.72)	0.09 (0.23)	-0.82 (0.50)	0.37 (0.90)
al-Maghrib	0.28 (0.33)	0.13 (0.07)	-0.49 (0.27)	0.65 (0.30)

Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

back

Realized vs counterfactual changes in real consumption per capita

Counterfactual changes relative to AD 700-900

	Initial $\log\left(\frac{X_n/p_n}{L_n}\right)$		Counterfactual $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$ if:					
	All parameters		Religious border AD 700-900		Technology AD 700-900		Minting AD 700-900	
	AD 460-620		AD 700-900		AD 700-900		AD 700-900	
al-Andalus (Spain)	-0.70	(0.10)	0.09	(0.02)	0.55	(0.10)	1.57	(0.31)
Aquitaine (South France)	-1.04	(0.08)	-0.15	(0.03)	0.99	(0.09)	3.93	(0.30)
Francia and Germania	-1.55	(0.09)	-0.07	(0.02)	1.68	(0.11)	6.17	(0.47)
Northern Italy	0.07	(0.04)	-0.24	(0.05)	-0.24	(0.08)	-0.21	(0.07)
Southern Italy	-0.25	(0.06)	-0.11	(0.02)	-0.60	(0.13)	-0.03	(0.02)
Byzantine Heartlands	1.22	(0.11)	-0.69	(0.08)	-0.57	(0.13)	-1.41	(0.19)
al-Sham (Greater Syria)	0.30	(0.04)	0.04	(0.01)	-0.18	(0.10)	-0.22	(0.08)
Northern Syria, Caucasus	-0.34	(0.11)	0.02	(0.02)	0.15	(0.22)	0.19	(0.19)
Iraq, Iran	0.28	(0.08)	0.01	(0.00)	0.03	(0.08)	0.03	(0.06)
Eastern Caliphate	-0.44	(0.08)	0.01	(0.00)	0.38	(0.16)	0.34	(0.26)
Arabian Peninsula	-1.80	(0.18)	0.26	(0.09)	0.66	(0.40)	2.71	(0.84)
Misr (Egypt)	0.32	(0.07)	0.02	(0.00)	-0.71	(0.24)	-0.09	(0.02)
al-Maghrib	0.12	(0.06)	0.01	(0.00)	-0.46	(0.17)	-0.05	(0.06)

Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t$. Bootstrapped s.e's in parentheses (100 bootstraps).

back

Realized vs counterfactual changes in real consumption per capita

Realized changes, from AD 460-620 to AD 700-900

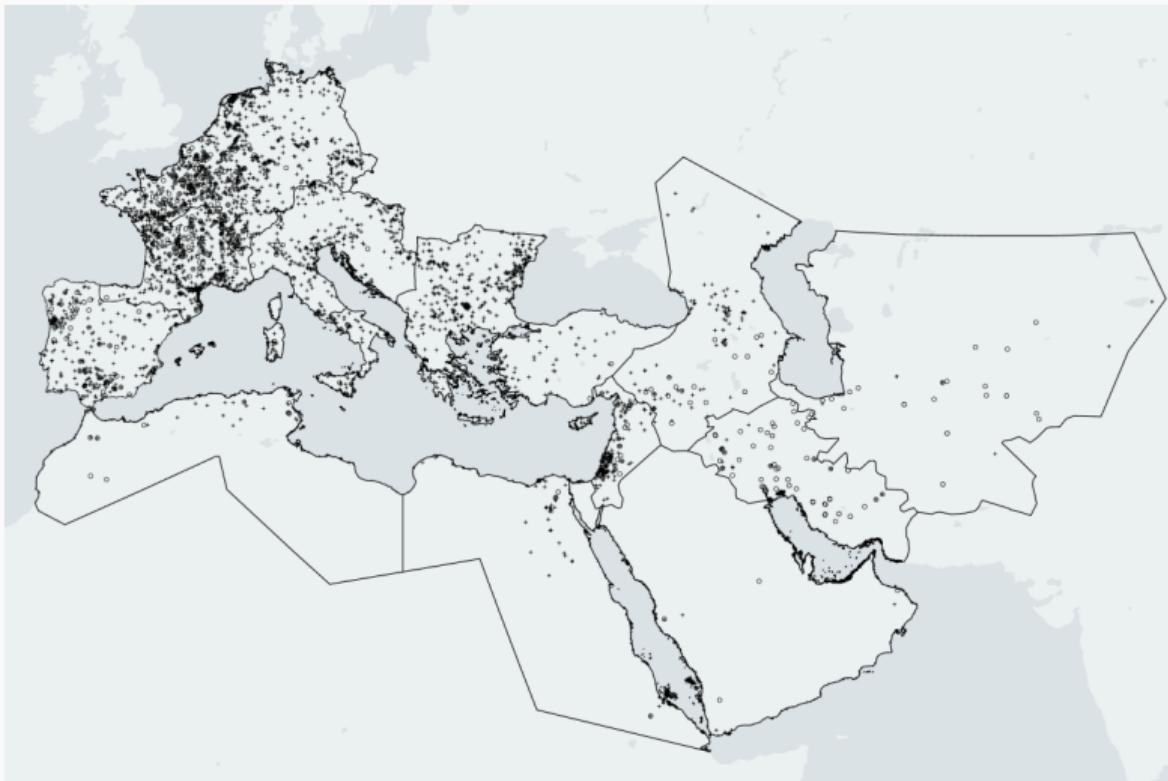
	Real consumption $\Delta \log \left(\frac{X_n/p_n}{L_n} \right)$	Openness $\Delta \log \left(\pi_{nn}^{-1/\theta} \right)$	Technology $\Delta \log \left(T_n^{1/\theta} \right)$	Trade Deficit $\Delta \log \left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n} \right)$
Francia and Germania	1.96 (0.24)	-0.05 (0.01)	1.80 (0.26)	0.20 (0.04)
Byzantine Heartlands	-1.56 (0.33)	-0.23 (0.14)	-0.44 (0.41)	-0.89 (0.54)
Arabian Peninsula	1.16 (0.34)	-0.01 (0.04)	0.66 (0.45)	0.51 (0.26)

Counterfactual changes relative to AD 700-900

	Initial $\log \left(\frac{X_n/p_n}{L_n} \right)$	Counterfactual $\Delta \log \left(\frac{X_n/p_n}{L_n} \right)$ if:					
		All parameters AD 460-620		Religious border AD 700-900		Technology AD 700-900	
Francia and Germania	-1.55 (0.09)	-0.07 (0.02)	1.68 (0.11)	6.17 (0.47)			
Byzantine Heartlands	1.22 (0.11)	-0.69 (0.08)	-0.57 (0.13)	-1.41 (0.19)			
Arabian Peninsula	-1.80 (0.18)	0.26 (0.09)	0.66 (0.40)	2.71 (0.84)			

Normalizations: $E_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

Spatial distribution: the (extended) Mediterranean



Distribution of coin “death dates” (tpq)

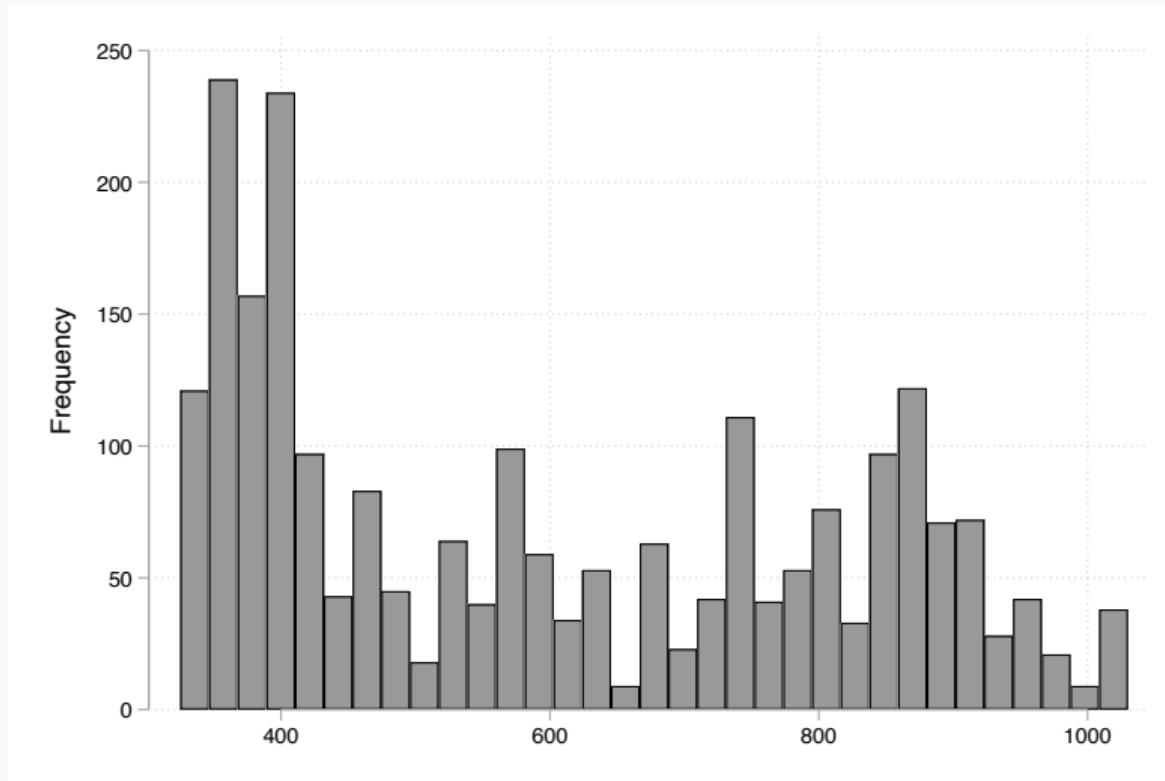


Figure 4: Terminus Post Quem (tpq) of hoards

Distribution of coin ages (tpq minus mint date)

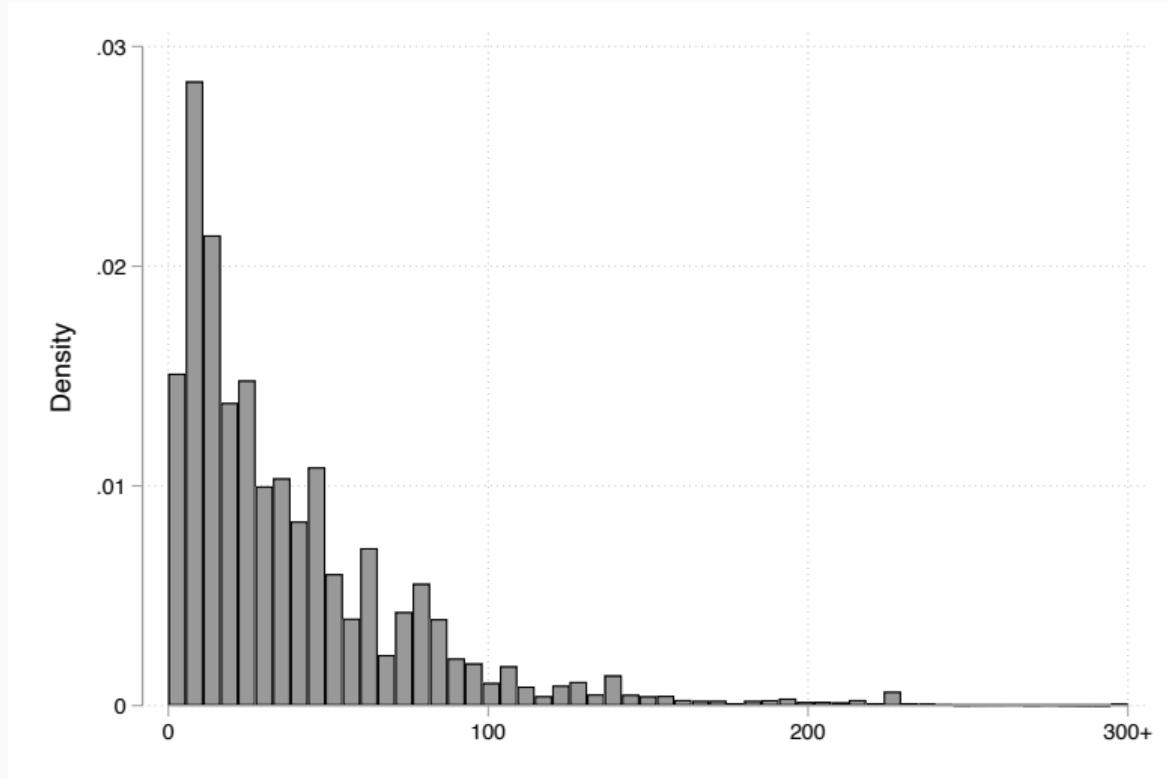


Figure 5: Coin age at time of deposit (tpq), in years

Fact #2: distance has a weaker impact on older coin flows

$\text{logcount}_{mth\tau} =$

$$\sum_{\tau' \in T} \beta_{\tau'} \log \text{distance}_{mh} \times 1(t - \tau = \tau')$$

$$+ \alpha_{mt} + \alpha_{h\tau} + \varepsilon_{mth\tau}$$

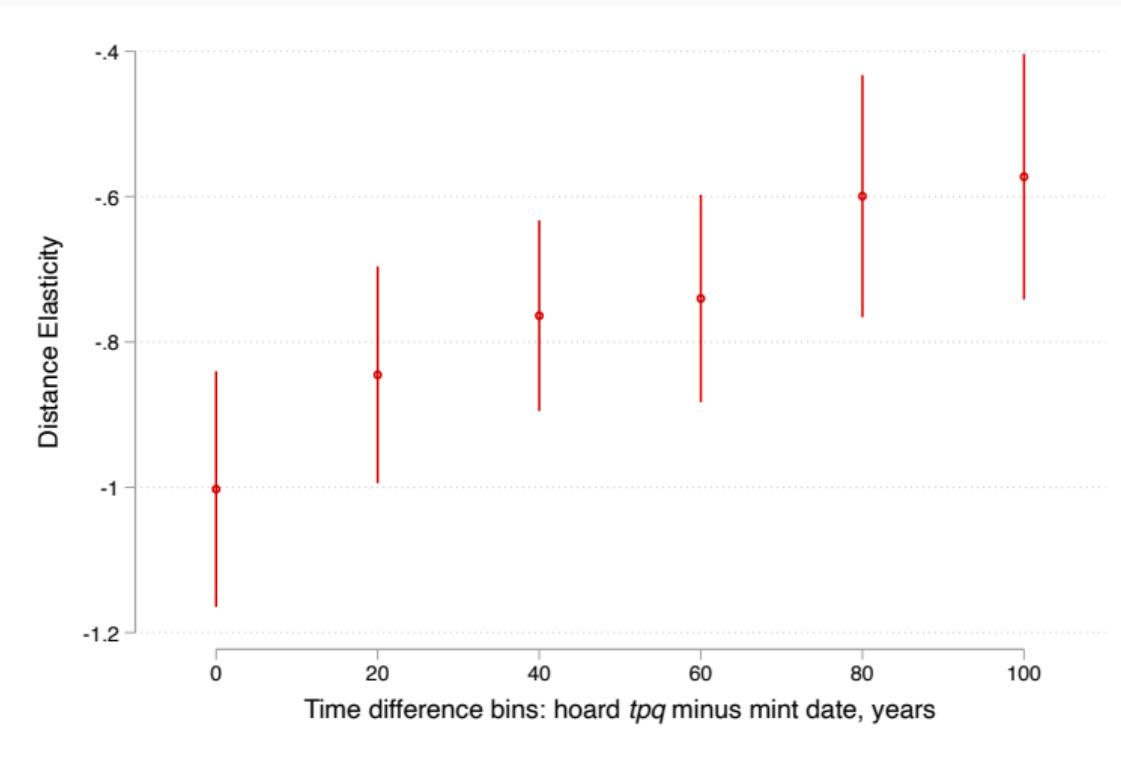


Figure 6: The distance elasticity declines as coins get older

Pitfall #1: medium of exchange vs store of value

- Dynamics with ‘saving-augmented’ trade shares,

$$S(t, T) = M(t) \left(\prod_{\tau=t}^{T-1} (I - \lambda(\tau)) \tilde{\Pi}(\tau) \right)$$

- Separate origin, destination, and bilateral terms,

$$\tilde{\pi}_{ni}(\tau) = \alpha_n(\tau) \beta_i(\tau) \delta_{ni}(\tau)$$

$$\alpha_n = \frac{1}{\sum_k T_k (w_k d_{nk})^{-\theta}}, \quad \beta_i = T_i (w_i)^{-\theta}$$

$$\delta_{ni} = (d_{ni})^{-\theta}$$

Pitfall #1: medium of exchange vs store of value

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$$\tilde{\pi}_{ni}(\tau) = \alpha_n(\tau) \beta_i(\tau) \delta_{ni}(\tau)$$

$$\alpha_n = \frac{1}{\sum_k T_k (w_k d_{nk})^{-\theta}}, \quad \beta_i = T_i (w_i)^{-\theta}$$

$$\delta_{ni} = (d_{ni})^{-\theta} \times \begin{cases} (1 - s_n) & \text{if } n \neq i \\ (1 - s_n) + \frac{s_n \sum_k T_k (w_k d_{nk})^{-\theta}}{T_n (w_n d_{nn})^{-\theta}} & \text{if } n = i. \end{cases}$$

- $\frac{\delta_{nj}}{\delta_{ni}} = \frac{(d_{nj})^{-\theta}}{(d_{ni})^{-\theta}}, \forall n \neq i, j, \forall s_n \geq 0$: no impact on external trade

- $\frac{\delta_{nn}}{\delta_{ni}} > \frac{(d_{nn})^{-\theta}}{(d_{ni})^{-\theta}}, \forall s_n > 0$: net saving mimics home bias in trade!

Pitfall #2: stocks vs flows (steady state math)

- SS: no net saving ($s = 0$), only age (a) matters, not time (t),

$$S(t, t+a) = S(a) = M \left((I - \lambda) \Pi \right)^a, \forall t$$

- Sum of different vintages (stocks by origin-destination),

$$\sum_{a=0}^A S(a) = M \left(\sum_{a=0}^A \left((I - \lambda) \Pi \right)^a \right) \underset{A \rightarrow +\infty}{=} M (I - (I - \lambda) \Pi)^{-1}$$

- Naive gravity on stocks gives Leontief inverse of trade shares!
⇒ inconsistent estimates of trade elasticities/border effects due to model misspecification

▶ Illustration

▶ Back