EC321: Problem Set 6 Question 1

Johannes Boehm

London School of Economics

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Question 1: Debt deflation with credit frictions

Kiyotaki-Moore model of credit markets:

- **(a) Derive farmer's demand for capital.**
- Budget constraint of a farmer is

$$c_{f,t} = a_t k_{f,t-1} + q_t k_{f,t-1} - (1+r) b_{f,t-1} - q_t k_{f,t} + b_{f,t}$$

  where $c_{f,t}$ is farmer's consumption, $k_{f,t}$ is capital owned at the end of period t, $b_{f,t}$ is amount borrowed by farmers between t and $t+1$, and q_t is the price of capital at time t.
- Farmer's collateral constraint:

$$(1+r) b_{f,t} \leq q_{t+1} k_{f,t}$$

  This must bind (see lecture notes).
- Marketability constraint:

$$c_{f,t} \geq (1-\sigma) a_t k_{f,t-1}$$

  i.e. at most a fraction σ of output $a_t k_{f,t-1}$ can be sold. Must also bind in equilibrium.

- Plug the two binding constraints into the budget constraint to eliminate $b_{f,t}$ and $c_{f,t}$. We get

$$k_{f,t} = \frac{\sigma a_t k_{f,t-1} + q_t k_{f,t-1} - (1+r) b_{f,t-1}}{q_t - \frac{q_{t+1}}{1+r}} \equiv \frac{n_t}{d_t}$$

The numerator is net worth n_t: value of capital he owns, $q_t k_{f,t-1}$, plus current income net of consumption, $\sigma a_t k_{f,t-1}$, minus debt repayments, $(1+r)b_{f,t-1}$. The denominator is the down-payment required to purchase a unit of capital: the direct cost of capital is q_t, but each unit of capital can act as collateral for a maximum amount of borrowing $q_{t+1}/(1+r)$.

- **(b) Derive an expression for the farmer's leverage ratio, i.e. the ratio of the value of capital goods to their net worth, in the steady state.**
- Leverage ratio is

$$\frac{q_t k_{f,t}}{n_t} = \frac{q_t}{d_t} = \frac{1}{1 - \frac{q_{t+1}}{q_t} \frac{1}{1+r}}$$

and in steady state the price of capital q_t is constant q,

$$\frac{q k_f}{n} = \frac{1+r}{r}$$

- **(c) Suppose there is an unexpected decrease in the price level of goods, which increases the total value of the repayment $B_{f,t-1} = (1+r)b_{f,t-1}$. What is the direct effect of a 1% change in $B_{f,t-1}$ from its steady state level on the amount of capital $k_{f,t}$ purchased by farmers?**
- Start with the definition of net worth:

$$n_t = \sigma a_t k_{f,t-1} + q_t k_{f,t-1} - (1+r)b_{f,t-1}$$

- Since productivity a is not changing, we have $\Delta a = 0$, and

$$\Delta n_t = \Delta q_t k_{f,t-1} - \Delta B_{f,t-1}.$$

- Since $k_{f,t} = n_t / d_t$,

$$\Delta n_t \approx k_{f,t} \Delta d_t + \Delta k_{f,t} d_t.$$

- From the lecture notes, $d_t = \mu_{t+1}/(1+r)$ where μ_{t+1} is the marginal product of capital of the gatherers, and $\mu_{t+1} = aG'(\bar{K} - k_{f,t})$, with $\bar{K}$ the exogenous supply of capital.

- Denote by η the elasticity of $k_{f,t}$ with respect to d_t,

$$\eta = \frac{k_{f,t}}{d_t} \frac{\partial d_t}{\partial k_{f,t}}$$

so approximately

$$\Delta d_t = \eta \frac{d}{k_f} \Delta k_{f,t}.$$

- Substitute this into the equations above,

$$\Delta k_{f,t} \approx \frac{\Delta n_t}{(1+\eta)\, d}$$

and, substituting for Δn_t, (omit time subscripts since we're in steady state)

$$\frac{\Delta k_{f,t}}{k_f} \approx \frac{1}{1+\eta} \left(\frac{\Delta q_t}{d} - \frac{\Delta B_{f,t-1}}{k_f d} \right)$$

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  and using the collateral constraint (binding)

  $$B_f \frac{d}{q} = k_f d$$

  and $d = (r/(1+r))q$,

  $$\frac{\Delta k_{f,t}}{k_f} \approx \frac{1}{1+\eta}\frac{1+r}{r}\left(\frac{\Delta q_t}{q} - \frac{\Delta B_{f,t-1}}{B_f}\right)$$

- $$\frac{\Delta k_{f,t}}{k_f} \approx \frac{1}{1+\eta}\frac{1+r}{r}\left(\frac{\Delta q_t}{q} - \frac{\Delta B_{f,t-1}}{B_f}\right)$$
  Interpretation: if $B_{f,t-1}$ increases by one percent, then $k_{f,t}$ decreases by $\frac{1}{1+\eta}\frac{1+r}{r}$ percent. If q_t increases by one percent, then $k_{f,t}$ increases by $\frac{1}{1+\eta}\frac{1+r}{r}$ percent.
- Higher leverage ratio $(1+r)/r$ means capital purchases more elastic w.r.t asset prices and real value of debt!

- **(d) What would happen if a fall in the demand for capital goods led to further deflation?**
- This model does not have a Phillips curve, so we need to think outside the model.
- Fall in demand for capital goods causes downward pressure on prices (think: NKPC), this increases the real value of debt.
- Net worth lower, thereby reducing capital demand (collateral constraint). And so on.
- This multiplier effect is in addition to the financial accelerator effect (which goes via q), but only if contracts are in nominal terms.