Trade and the End of Antiquity

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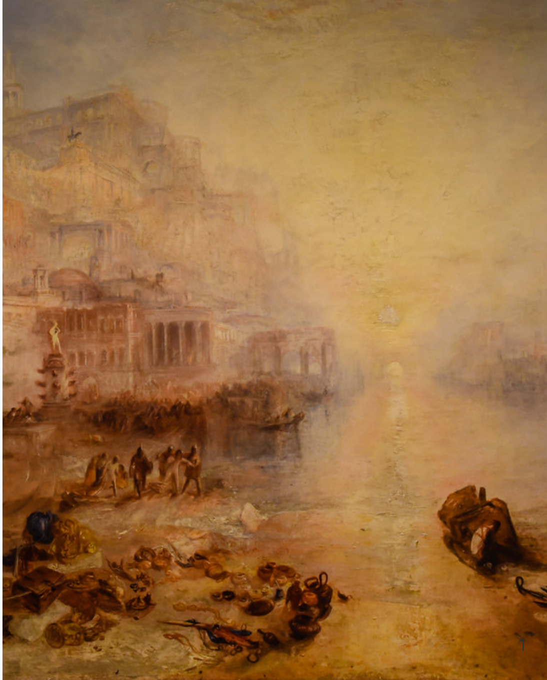

What caused the End of Antiquity?

- Antiquity: Roman and Greek civilizations centered around the Mediterranean
- Archaeological evidence points to a shift in economic activity away from the Mediterranean between 5th and 8th century AD ("end of antiquity")
  - Rise of Northern Europe (Charlemagne).
  - "Golden Age" of Islam

  $\rightarrow$ Origin of "Europe" geographic entity north of Mediterranean

$\hookrightarrow$ *Question*: What caused the End of Antiquity?

- Discussed, among others, by Montesquieu (1734), Voltaire (1756), Gibbon (1789)
- Henri Pirenne (1937) blames the Arab conquests and the emerging Islamic-Christian border for the rupture in the Mediterranean unity
  - Some views challenged by more recent archaeological evidence.

This paper: quantitatively investigate changing economic geography

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$\Rightarrow$ Use data on the movement of coins to study the changing economic geography during Late Antiquity.

- Coins are the main medium of exchange during Late Antiquity, particularly for long-distance trade $\rightarrow$ informative about trade
- Coins are well studied & documented by historians and numismatists
- Coins have features that help solve econometric identification problems

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- Coins have features that help solve econometric identification problems

$\Rightarrow$ we interpret coin flows through the lens of a trade model and identify

- bilateral trade flows (i, j, t)
- technology (i, t)
- trade deficits (i, t)
- real consumption per capita (i, t)

in order to better understand what's going on during Late Antiquity

Data: Coins around the Mediterranean, AD 325 to AD 950

Assemble a large dataset of coin finds from around the Mediterranean

1. FLAME (2023) project by historians around Princeton
   - ∼200,000 coins with complete records 325–725
2. Hand-coded records from numismatic / archaeological literature:
   - 797 coin finds, ∼100k coins, 725-950

▸ Coin age distribution ▸ Hoard Dates (tpq) ▸ Spatial Distribution

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Example: excerpt from al Ush's (1972) Damascus silver hoard:

No.	MINT	DATE	DIAM.	WEIGHT	NUMB.
51	الأندلس	114	29.	2.93	4
52	"	115	29.5	2.92	1
53	"	116	26.5	2.92	3

Index / Mint (al-Andalus/Cordoba) / Date: 114 AH = 732 AD / Diameter / Weight / Q'tity

▸ Coin age distribution ▸ Hoard Dates (tpq) ▸ Spatial Distribution

Fact #1: within a hoard, older coins have travel farther

Table 1: Within-hoard distance travelled and age of coin at deposit

	Dependent variable: Log Distance between Mint and Hoard				
	(1)	(2)	(3)	(4)	(5)
Log Age of Coin	0.146***	0.0831***	0.0749**	0.160***	0.0485**
	(0.044)	(0.026)	(0.031)	(0.043)	(0.020)
Sample				No non-hoards	No non-hoards
Hoard FE	Yes	Yes	Yes	Yes	Yes
Mint $\times$ 50-year-interval FE		Yes			
Mint $\times$ 25-year-interval FE			Yes		Yes
R^2	0.762	0.863	0.869	0.775	0.898
Observations	287243	287029	286873	250156	249830

Standard errors in parentheses, clustered at the hoard level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Age of coin $= tpq -$ mint date

$\Rightarrow$ coins diffuse across space over time.

Fact #2: distance and political borders impede coin travels

Construct $1° \times 1°$ cells for mint and hoard locations and calculate flows count_{mdh}

Table 2: Gravity and Border Effects in Coin Flows

	Dependent variable: # Coins_{mdh}				Dep. var.: Value_{mdh}	
	(1)	(2)	(3)	(4)	(5)	(6)
Log Distance	-1.137***	-1.002***	-1.135***	-0.951***	-1.144***	-0.989***
	(0.12)	(0.13)	(0.10)	(0.076)	(0.075)	(0.068)
Political border		-1.945***		-2.073***		-1.516***
		(0.62)		(0.47)		(0.27)
Hoard Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Mint $\times$ Empire Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample			Gold only	Gold only	Gold and Silver	Gold and Silver
Estimator	PPML	PPML	PPML	PPML	PPML	PPML
Pseudo-R^2	0.767	0.778	0.808	0.824	0.800	0.810
Observations	217748	217748	57287	57287	146767	146767

Standard errors in parentheses, clustered at mint cell $\times$ empire and hoard cell level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Declining Elasticity

Estimating eqn: $\text{count}_{mdh} = \exp\left(\gamma_1 \log \text{distance}_{mh} + \gamma_2 \text{withinBorder}_{dh} + \alpha_{md} + \alpha_h + \varepsilon_{mhd}\right)$

Fact #3: Coin flows before/after the Arab conquests

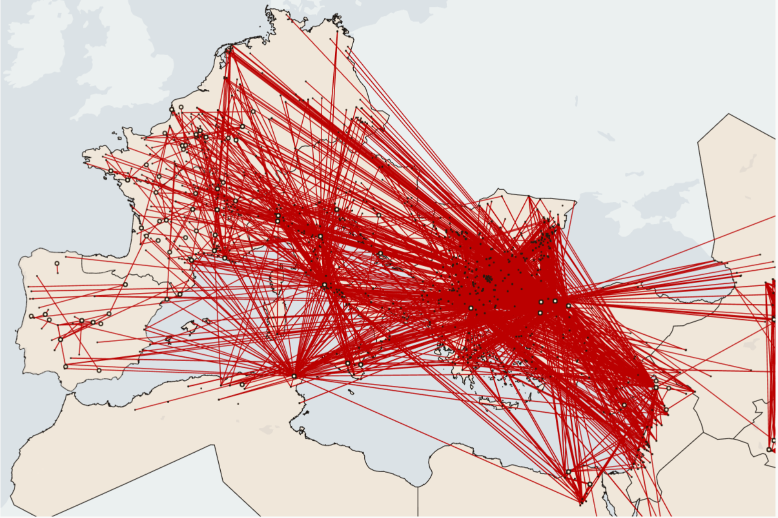

Figure 1: Before the Arab conquests: 450-630 AD

Fact #3: Coin flows before/after the Arab conquests

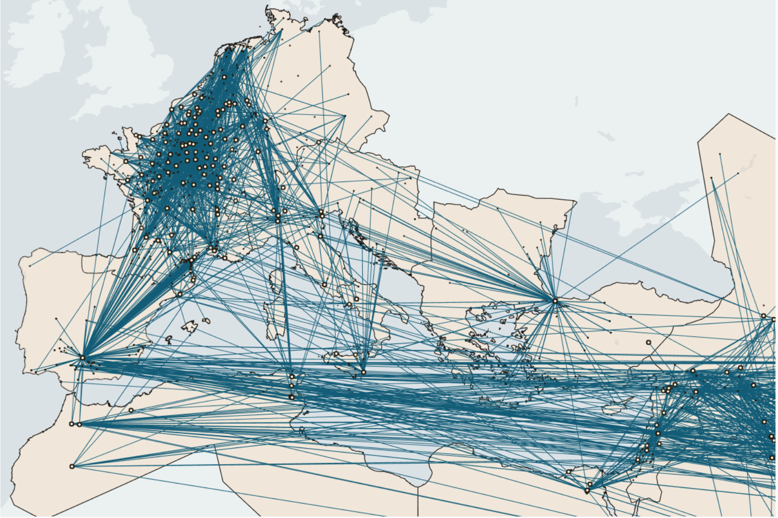

Figure 2: After the Arab conquests: 713-900 AD

▸ Cross-med: reg

▸ Cross-med: plot

Model

Objective:

Quantify impact of trade barriers (and other changes) on market access / trade / welfare

Approach:

Dynamic model of trade flows (gravity), where coins diffuse alongside trade and are thus informative about trade flows.

Key Assumption:

Traders are blind to coin *types* (mint location and date).

- $\Rightarrow$ Coins diffuse in proportion to trade flows
- $\Rightarrow$ Recover trade shares from *shares* of coin *types* in hords in different locations/time periods.

Not used/needed for identification: total *quantities* of coins found in each location

▸ Long Model

Model components (simplified case: exogenous saving)

N locations with labor endowments L_n, Ricardian trade as in Eaton and Kortum (2002)

- Households finance consumption expenditure using saving $S_n(t-1)$, exogenous newly minted coins $M_n(t)$; end-of-period income is saving for next period
- Goods market clearing, denominated in coins

$$\overbrace{w_i L_i\left[t\right]}^{income_i[t]} = \sum_n \pi_{ni}\left[t\right] \overbrace{\left(\left(1-\lambda\right) w_n L_n\left[t-1\right] + M_n\left[t\right]\right)}^{expenditure_n[t]}, \forall i, t \tag{1}$$

$$\Leftrightarrow S_i\left[t\right] = \sum_n \pi_{ni}\left[t\right]\left(\left(1-\lambda\right) S_n\left[t-1\right] + M_n\left[t\right]\right), \forall i, t \tag{2}$$

w_i: wages; L_i: labor; π_{ni}: expenditure shares; λ: coin loss; M_n: minting; S_i: coin stocks

- Fraction π_{ni} of n's expenditure allocated to goods from i

$$\pi_{ni}\left[t\right] = \frac{\left(T_i\left[t\right] w_i^{-\theta}\left[t\right]\right)\left(d_{ni}\left[t\right]\right)^{-\theta}}{\sum_k \left(T_k\left[t\right] w_k^{-\theta}\left[t\right]\right)\left(d_{nk}\left[t\right]\right)^{-\theta}}, \forall i, n, t \tag{3}$$

T_i: technology; d_{ni}: trade cost; θ: trade elasticity

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Composition of coin stocks

- Stock $S_i[T]$ composed of different coin types, $S_i[T] = \sum_{m=1}^{N} \sum_{t \leq T} S_{mi}[t, T]$
- Coins start their 'coin life' when they are minted, $S_{mm}[t, t] = M_m[t]$
- Then stocks evolves recursively,

$$S_{mi}[t, \tau] = (1 - \lambda) \sum_{n=1}^{N} \pi_{ni}[\tau] S_{mn}[t, \tau - 1] \tag{4}$$

- Recursive solution in matrix form (coin origin $\times$ coin destination),

$$\mathsf{S}[t, T] \tag{4'}$$

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$$\mathsf{S}[t, t+1] \ = \ (1 - \lambda) \, M[t] \, \mathbf{\Pi}[t+1] \tag{4'}$$

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- Then stocks evolves recursively,

$$S_{mi}[t, \tau] = (1 - \lambda) \sum_{n=1}^{N} \pi_{ni}[\tau] S_{mn}[t, \tau - 1] \tag{4}$$

- Recursive solution in matrix form (coin origin $\times$ coin destination),

$$\mathsf{S}[t, t+2] \; = \; (1 - \lambda)^2 M[t] \, \mathbf{\Pi}[t+1] \, \mathbf{\Pi}[t+2] \tag{4'}$$

Composition of coin stocks

- Stock $S_i[T]$ composed of different coin types, $S_i[T] = \sum_{m=1}^{N} \sum_{t \leq T} S_{mi}[t, T]$
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- Recursive solution in matrix form (coin origin $\times$ coin destination),

$$\mathsf{S}[t, T] = (1 - \lambda)^{T-t} M[t] \, \mathbf{\Pi}[t+1] \, \mathbf{\Pi}[t+2] \cdots \mathbf{\Pi}[T] \tag{4'}$$

Taking the model to the data

- 13 regions around the Mediterranean ▸ details
- 20-year time intervals
- Assume constant λ and estimate as exponential decay parameter in within-hoard age distribution:

$$\hat{\lambda}_{20y} = 0.301$$

  (or 1.7% per year)

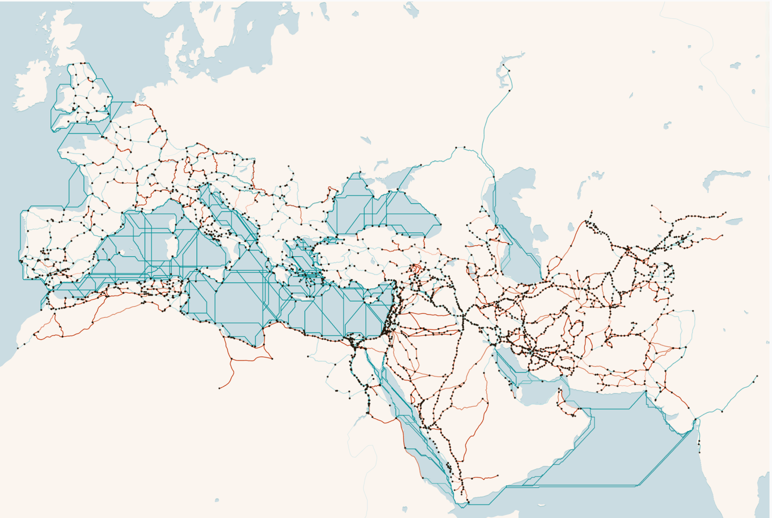
Parameterize trade frictions:

$$d_{ni}(t)^{\theta} = \exp\left(\gamma_0 + \zeta \ln\left(\text{TravelTime}_{ni}\right) + \kappa_1 \text{PoliticalBorder}_{ni}(t) + \kappa_2 \text{ReligiousBorder}_{ni}(t)\right)$$

if $n \neq i$ and $d_{nn}(t) = 1$.

- Full model contains a consumption/saving choice; calibrate $\overline{s_n}(t) = 1.5\%$ (Scheidel, 2020).
- For counterfactuals, assume $\theta = 4$ (Simonovska and Waugh, 2014).

Travel times



Note: Combined geospatial models from Orbis (Scheidel, 2015) and al-Ṯurayyā (Romanov and Seydi, 2022).

▸ al-Maqdisi

Maximum likelihood estimation

Assume coins in our data are a random sample of coin types in each location $\times$ time.

- Multinomial distribution of coin types,

$$P\left(\cdots, X_i^{(m,\tau)}(T) = x_i^{(m,\tau)}, \ldots\right) = \frac{N_i(T)!}{\prod_{(m,\tau)} x_i^{(m,\tau)}!} \prod_{(m,\tau)} \left[p_i^{(m,\tau)}(T)\right]^{x_i^{(m,\tau)}}$$

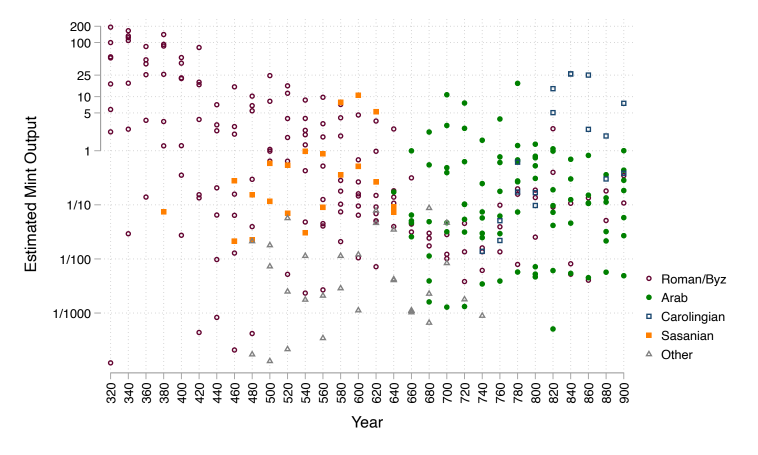
  with the probability of drawing a coin of type (m, τ),

$$p_i^{(m,\tau)}(T) = \frac{S_i^{(m,\tau)}(T)}{\sum_{(m',\tau')} S_i^{(m',\tau')}(T)} = \frac{S_i^{(m,\tau)}(T)}{S_i(T)}.$$

- Likelihood of observing a sample of coins given parameters $\boldsymbol{\theta}$,

$$\ell(X; \boldsymbol{\theta}) = \sum_i \sum_T \sum_{(m,\tau)} x_i^{(m,\tau)} \left[\log S_i^{(m,\tau)}(T; \boldsymbol{\theta}) - \log S_i(T; \boldsymbol{\theta})\right] + \text{constant}$$

Estimation results: Mint output



Normalization: $M_{n_0}[t_0] = 100$ (Northern Italy, AD 320-40).

Discussion on Byzantine monetary output: Kazhdan (1954), Pennas (1996)

Estimation results: Determinants of trade costs

$$\ln\left((d_{ni}[t])^{-\theta}\right) = constant \\ - \underset{(0.02)}{2.98} \ln(TravelTime_{ni}) - \underset{(0.02)}{0.3}\ PoliticalBorder_{ni}[t] - \underset{(0.11)}{4.05} ReligiousBorder_{ni}[t]$$

- Travel time elasticity similar to estimates on ancient trade.
  Roman trade: Flückiger et al. (2022); Bronze Age trade: Barjamovic et al. (2019).
- Political border tax: 8%
  (with $\theta = 4$, $d_{between}/d_{within} = e^{0.3/4} \approx 1.08$)
- Religious border tax: 175%
  (with $\theta = 4$, $d_{between}/d_{within} = e^{4.05/4} \approx 2.75$)
- Anderson and van Wincoop (2003) US-Canada border tax: 49%
  (with $\theta = 4$, $d_{between}/d_{within} = e^{1.59/4} \approx 1.49$)

▸ Mint output estimates

Welfare and counterfactuals

Real consumption depends on a combination of L and T (that's not separately identified):

$$X_n/p_n = \gamma^{-1}\left(\pi_{nn}\right)^{-1/\theta}\left(L_n T_n^{1/\theta}\right)\left(1+\frac{M_n-\lambda w_n L_n}{w_n L_n}\right)$$

or equivalently in per capita terms

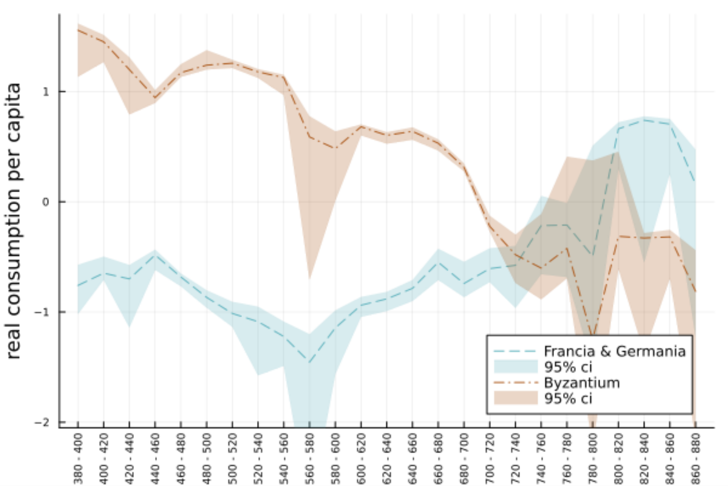
$$\underbrace{\frac{X_n/p_n}{L_n}}_{\text{Real Consumption}} = \gamma^{-1}\underbrace{\left(\pi_{nn}\right)^{-1/\theta}}_{\text{Openness}}\underbrace{\left(T_n\right)^{1/\theta}}_{\text{Technology}}\underbrace{\left(1+\frac{M_n-\lambda w_n L_n}{w_n L_n}\right)}_{\text{Trade Deficit}}$$

Note: T and L are not separately identified. We separate L and T through a Malthusian assumption

$$L_n = T_n \quad \forall n$$

and decompose per-capita real consumption into the three components

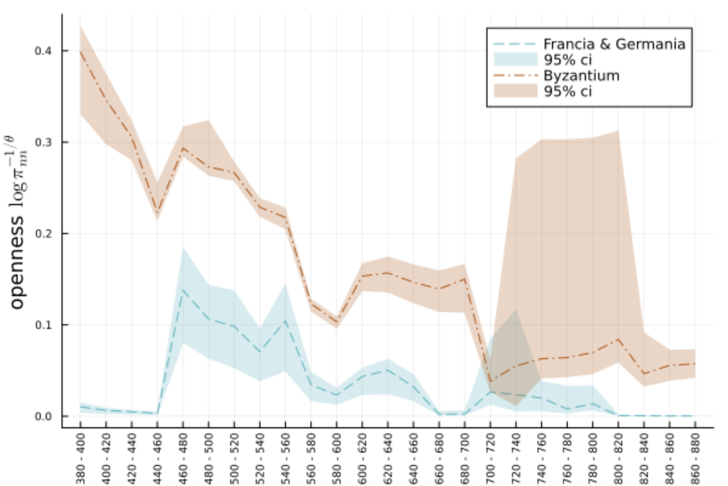
Byzantium vs northern Europe (380-880): *real consumption per capita*



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathsf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

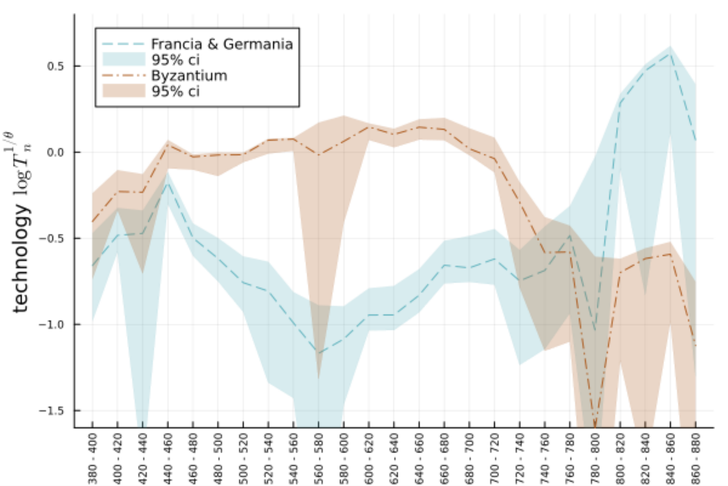
other regions

Byzantium vs northern Europe (380-880): *trade openness*



Bootstrapped 95% confidence intervals.

Byzantium vs northern Europe (380-880): *technology*



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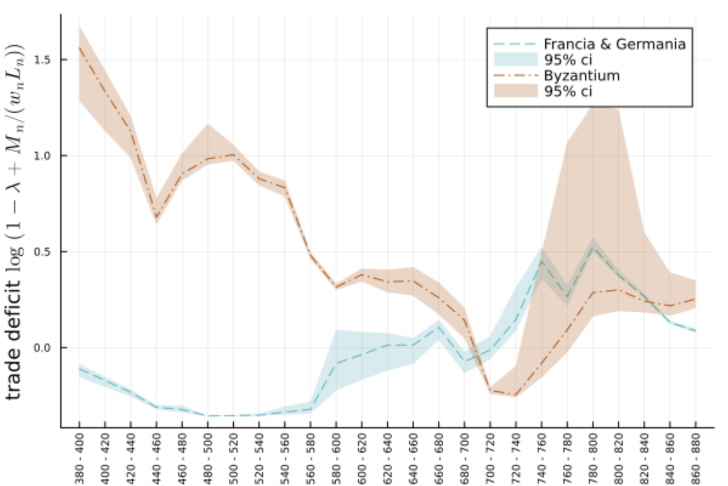
other regions

Byzantium vs northern Europe (380-880): *trade deficits*



Bootstrapped 95% confidence intervals.

other regions

Real consumption per capita: technology, geography, and trade (deficits)

Table 3: Real consumption in the ancient world from AD 460-620 to AD 700-900

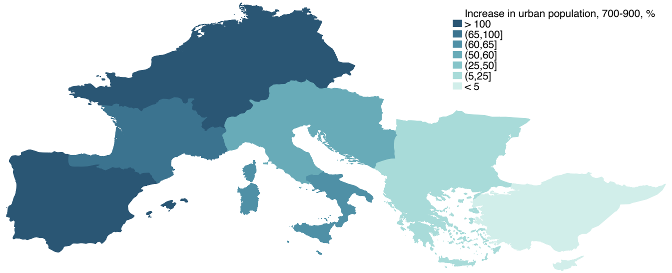
	Consumption		Openness		Technology		Trade Deficits	
	$\Delta \log \left(\frac{X_n / p_n}{L_n} \right)$		$\Delta \log \left(\pi_{nn}^{-1/\theta} \right)$		$\Delta \log \left(T_n^{1/\theta} \right)$		$\Delta \log \left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n} \right)$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
al-Andalus	0.53	(0.08)	-0.04	(0.01)	0.65	(0.10)	-0.08	(0.05)
Francia and Germania	1.99	(0.14)	-0.07	(0.02)	1.94	(0.16)	0.12	(0.04)
Byzantine Heartlands	-1.56	(0.22)	-0.16	(0.06)	-0.74	(0.13)	-0.66	(0.25)
Arabian Peninsula	1.12	(0.28)	-0.02	(0.04)	0.98	(0.37)	0.15	(0.23)

changes and counterfactuals

Compare to relative urbanization rates, 700–900 AD

Top: Change in total urban population (urban: $>$ 1k inhabitants), data from Buringh (2021)

Bottom: $\Delta W_n / L_n$



Results

- Clear pattern of change in economic geography before vs after conquest
- Trade disruption can account for the relative decline in the eastern Mediterranean
- Change in trade cost *alone* not able to account for urbanization in Muslim Spain, or in Carolingian empire
- In conjunction with changes in technology T_i and mint output, can account for urbanization patterns.

Back to Pirenne:

- Yes, new political and religious borders change market access, quant'ly relevant
- But unlikely to account for entire shift towards north-east
- Seignorage and technical change are more important drivers of change

Conclusion

"Simply looking at the Mediterranean cannot of course explain everything about a complicated past created by human agents, with varying doses of calculation, caprice and misadventure. But this is a sea that patiently recreates for us scenes from the past, breathing new life into them, locating them under a sky and in a landscape that we can see with our own eyes, a landscape and sky like those of long ago. A moment's concentration or daydreaming, and that past comes back to life."

Fernand Braudel, Les Mémoires de la Méditerranée

THANK YOU!

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Political changes in the Mediterranean: 600 AD

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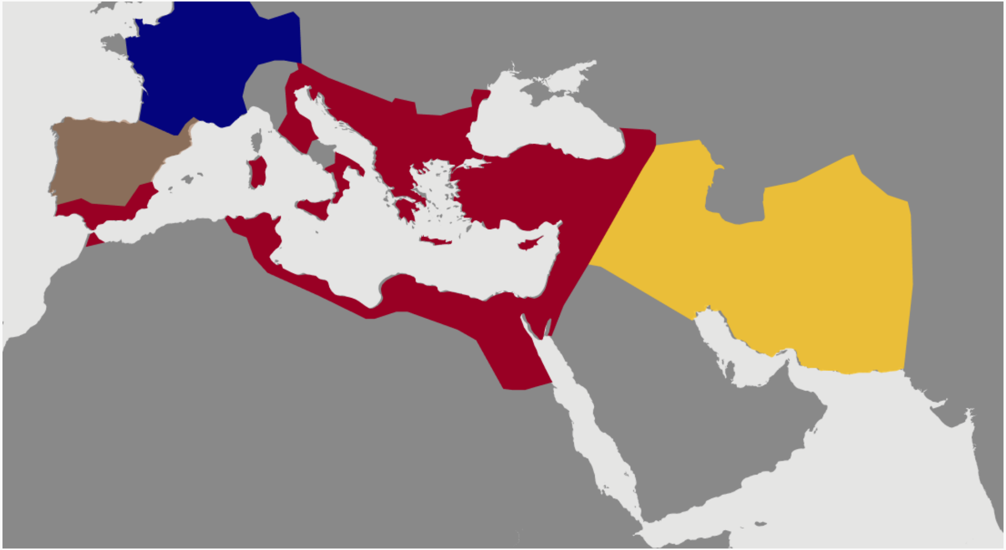

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Political changes in the Mediterranean: 632 AD

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Political changes in the Mediterranean: 634 AD

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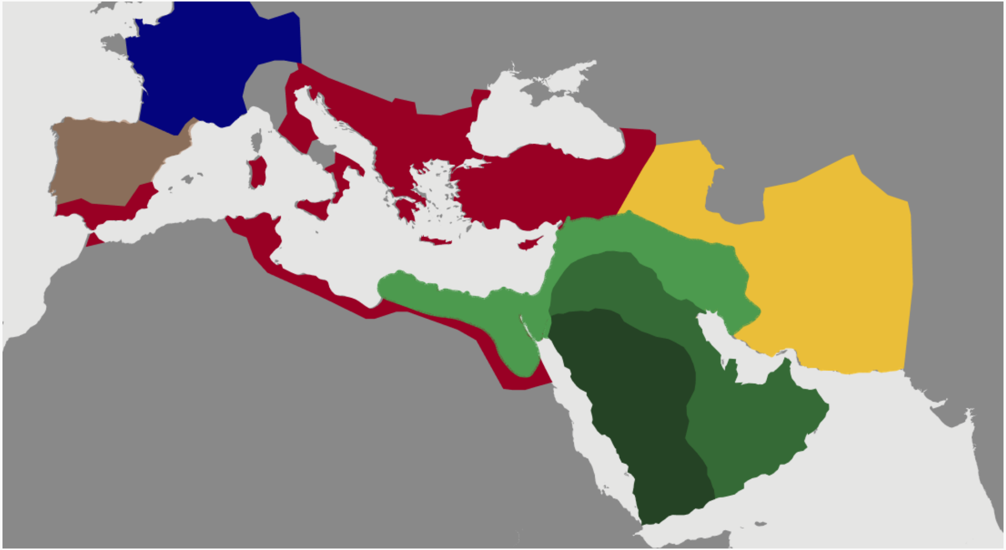
Political changes in the Mediterranean: 644 AD



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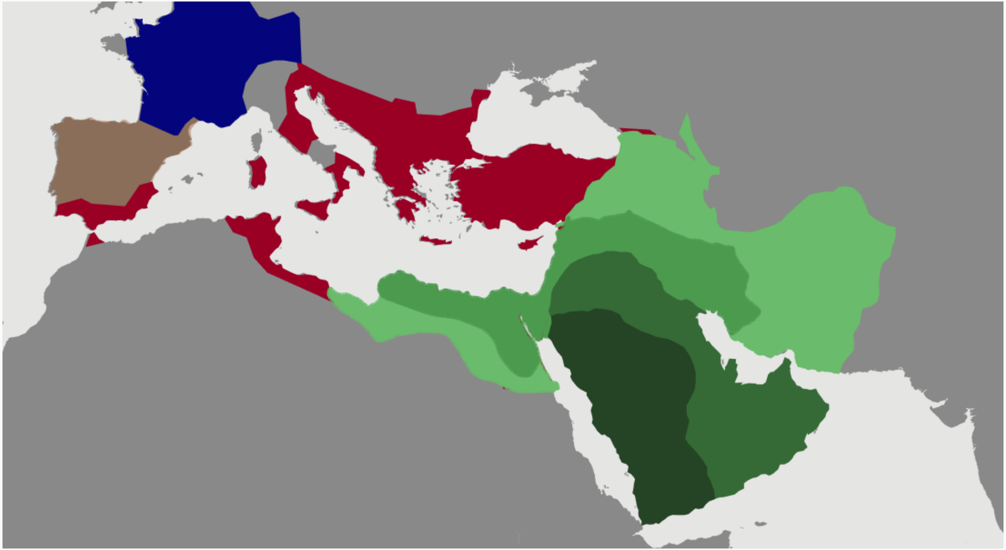
Political changes in the Mediterranean: 661 AD



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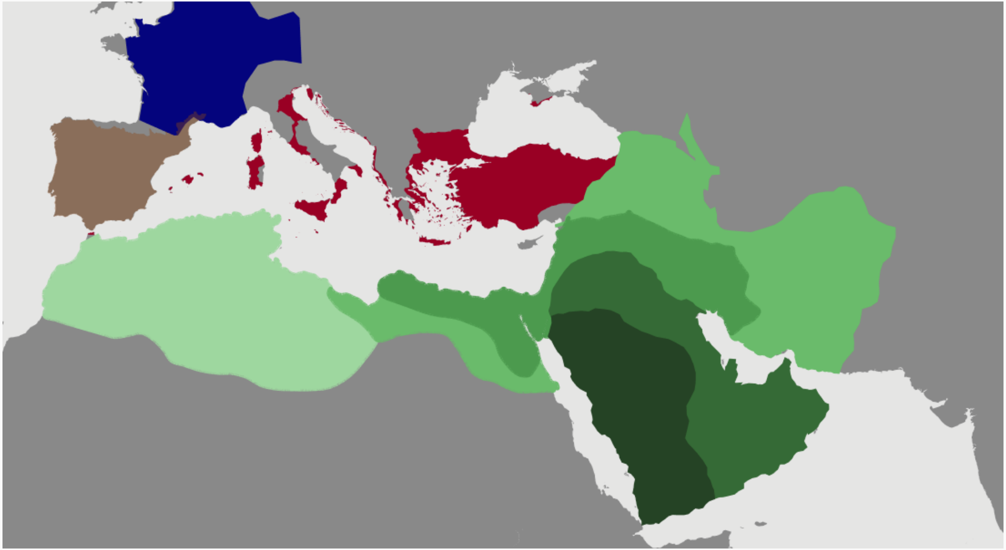
Political changes in the Mediterranean: 661-700 AD



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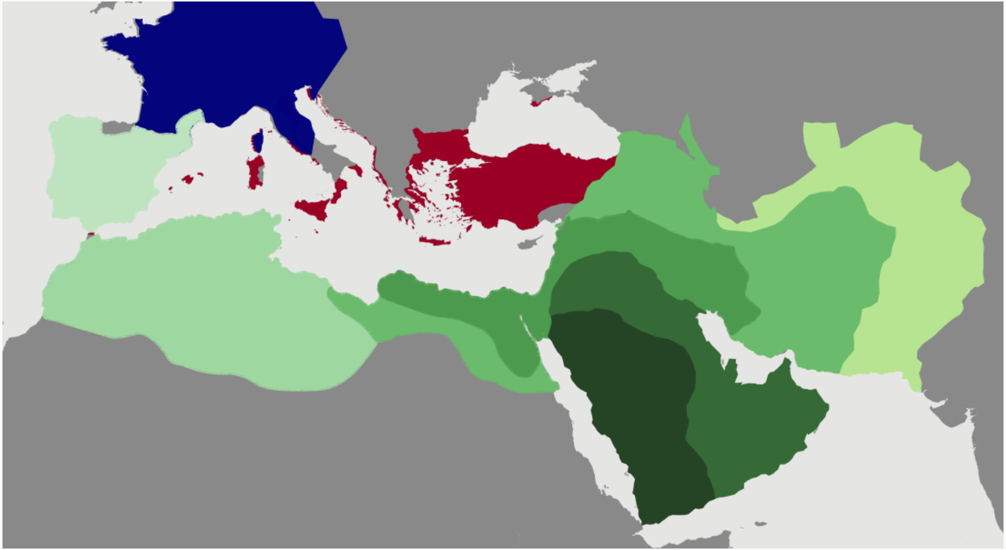
Political changes in the Mediterranean: 750 AD



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Regions

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Fact #3: Coin flows before/after the Arab conquests

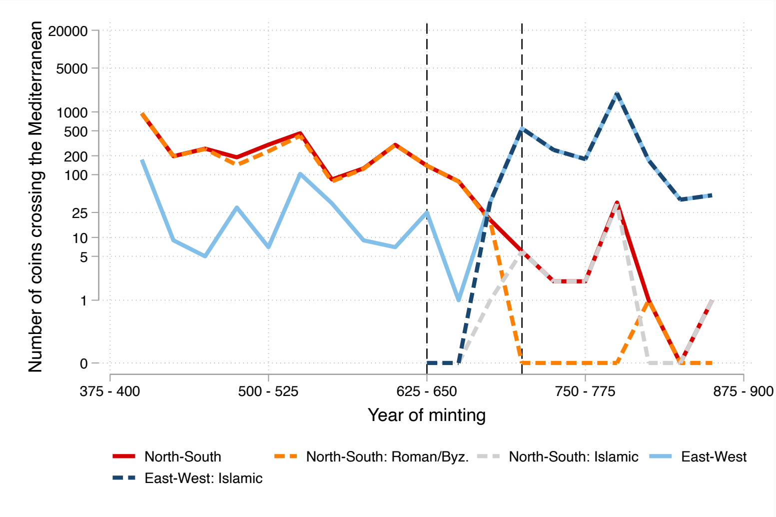


Figure 3: Number of coins flowing across the Mediterranean

Back

Fact #3: Coin flows before/after the Arab conquests

Table 4: The Mediterranean Before and After the Arab Conquest

	Dependent variable: Number of Coins			
	(1)	(2)	(3)	(4)
Crossing Mediterranean × After Conquests	-1.893***	-3.246***	-0.662	-1.736
	(0.48)	(0.53)	(0.63)	(1.27)
Crossing Mediterranean × After Conquests × Islamic Coin		7.267***	4.789***	7.545***
		(0.90)	(0.95)	(0.89)
Crossing Mediterranean × After Conquests × Roman Coin			-3.287***	-2.893***
			(0.75)	(0.61)
Mint Cell × Empire FE	Yes	Yes	Yes	Yes
Mint Cell × Hoard Cell FE	Yes	Yes	Yes	Yes
After Conquests FE	Yes	Yes	Yes	
Mint Cell × After Conquests FE				Yes
Hoard Cell × After Conquests FE				Yes
Estimator	PPML	PPML	PPML	PPML
Observations	10480	10480	10480	6208

Standard errors in parentheses, clustered at the hoard × era and mint × era level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

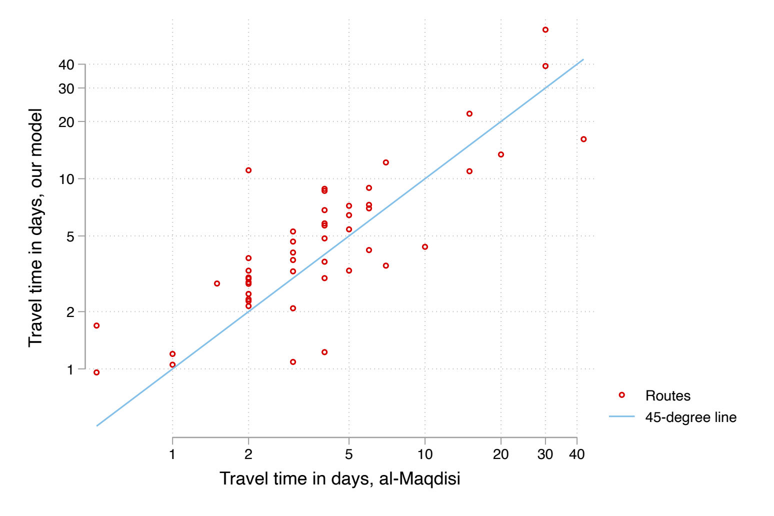
Estimating eqn: $\text{count}_{mdht} = \exp\left(\gamma_1 \text{mediterranean}_{mh} \times \text{after}_t + \cdots + \alpha_{md} + \alpha_{mh} + \varepsilon_{mhdt}\right)$

Back

Gold

Validating Travel Times

Al-Maqdisi (c. 945–991): *The Best Divisions for Knowledge of the Regions*



Back

Unsolved problems (as of yet)

- *Lucas critique #1*: cost function does not minimize costs

$$\ln\left(\delta_{ni}(t)\right) = \min_{p \in paths(i \to n)} \Bigg(\gamma_0 + \gamma_1 \ln\left(TravelTime_p(t)\right) + \sum_{pb\text{: all political borders along } p} \gamma_2 PoliticalBorder_{pb}(t) + \sum_{rb\text{: all religious borders along } p} \gamma_3 ReligiousBorder_{rb}(t) \Bigg)$$

- *Lucas critique #2*: net saving (in δ_{nn}) depends on parameters.
- *Fix for #2*: location-specific intercepts ($\gamma_{0,n}$) and δ_{nn}'s.

For now: constant γ_0, and $\delta_{nn} = 1$...

Fact #3: Arab conquests disrupt Mediterranean trade: Gold only

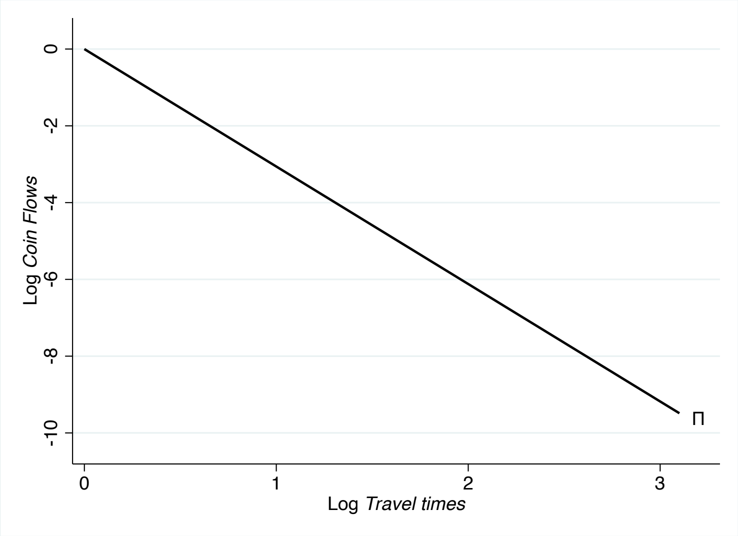
	Dependent variable: Number of Coins				
	(1)	(2)	(3)	(4)	(5)
Log Distance	-1.516***	-1.541***	-1.544***	-1.189***	-1.193***
	(0.13)	(0.13)	(0.13)	(0.15)	(0.15)
Crossing Mediterranean	0.298	0.307	0.320	0.0942	0.122
	(0.40)	(0.39)	(0.39)	(0.31)	(0.31)
Crossing Mediterranean × After Conquests	-1.600**	-2.858***	-1.719**	-2.576***	-3.379***
	(0.70)	(0.68)	(0.69)	(0.98)	(1.13)
Crossing Mediterranean × After Conquests × Islamic Coin		3.020***	1.864**		2.985**
		(0.71)	(0.76)		(1.20)
Crossing Mediterranean × After Conquests × Roman Coin			-1.699		
			(1.04)		
Mint Cell × Empire FE	Yes	Yes	Yes	Yes	Yes
Hoard Cell × After Conquests FE	Yes	Yes	Yes	Yes	Yes
Sample				Gold only	Gold only
Observations	172442	172442	172442	32024	32024

Standard errors in parentheses, clustered at the hoard × era and mint × era level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Before: 400–630; after: 713–950

Back

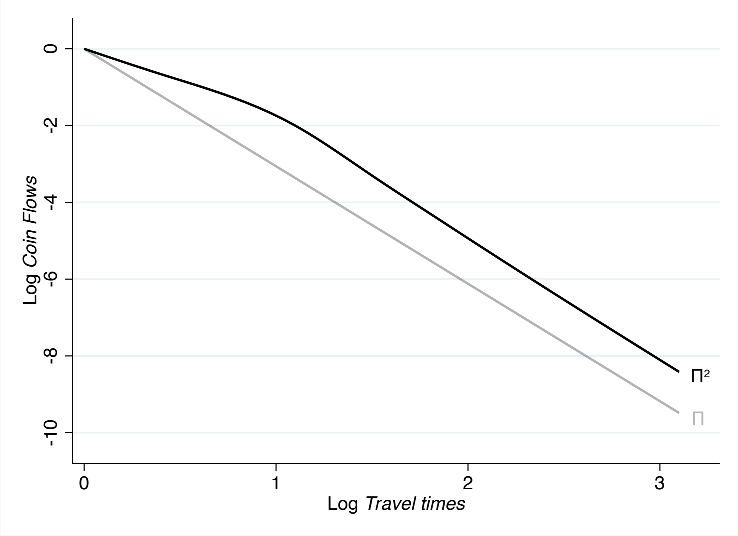
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1 (same as trade flows $\mathbf{\Pi}$)

Back

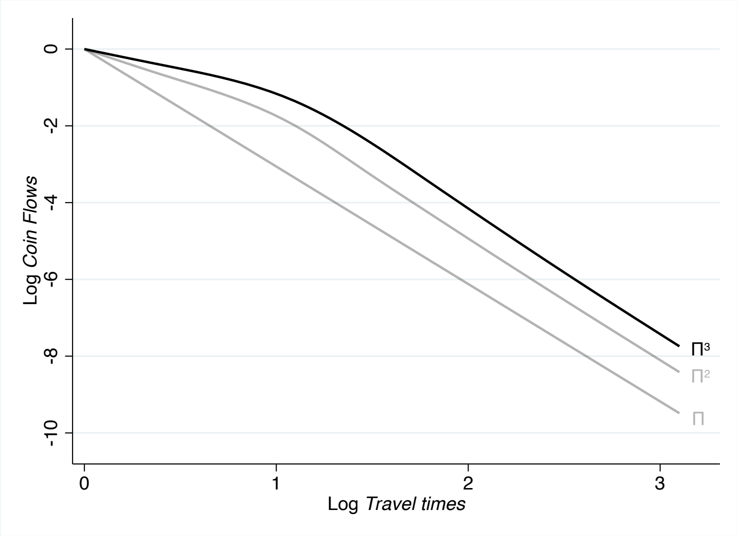
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2

Back

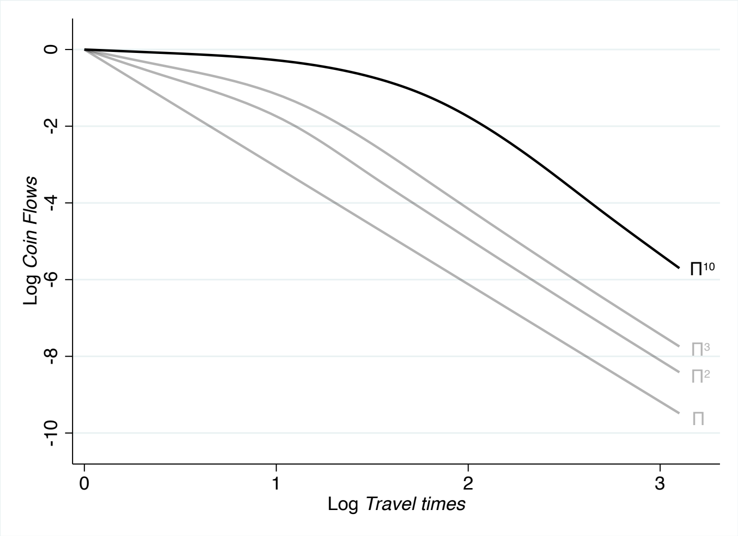
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3

▸ Back

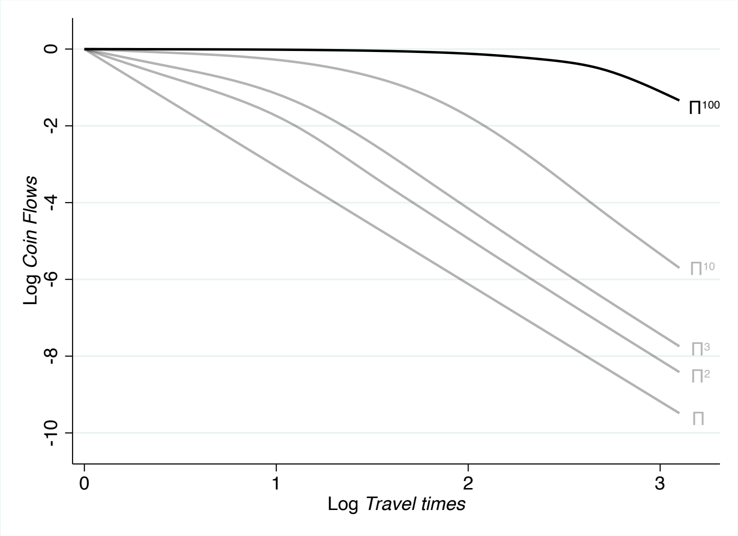
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10

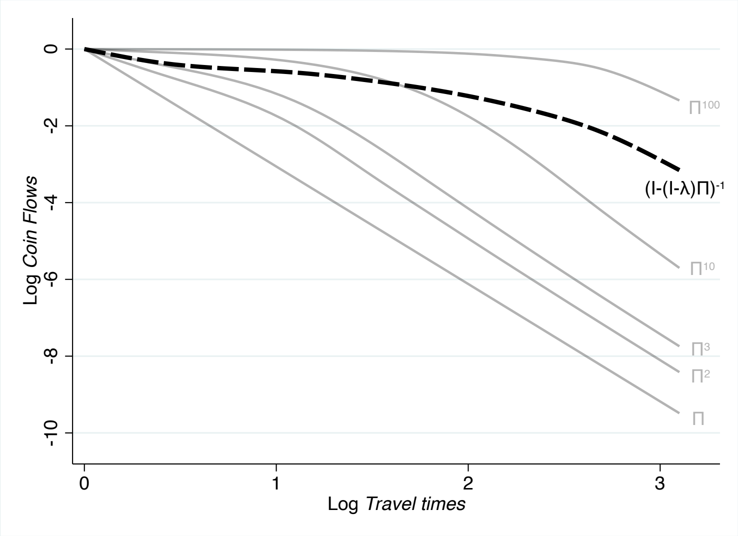
Back

Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10, age 100

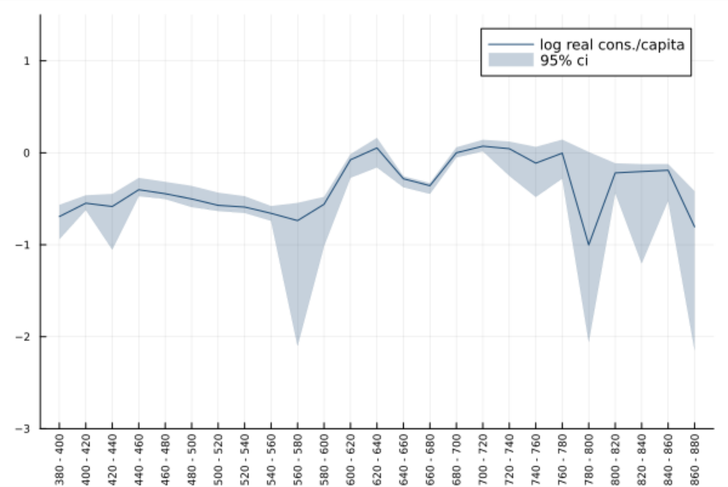
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10, age 100, all ages

Back

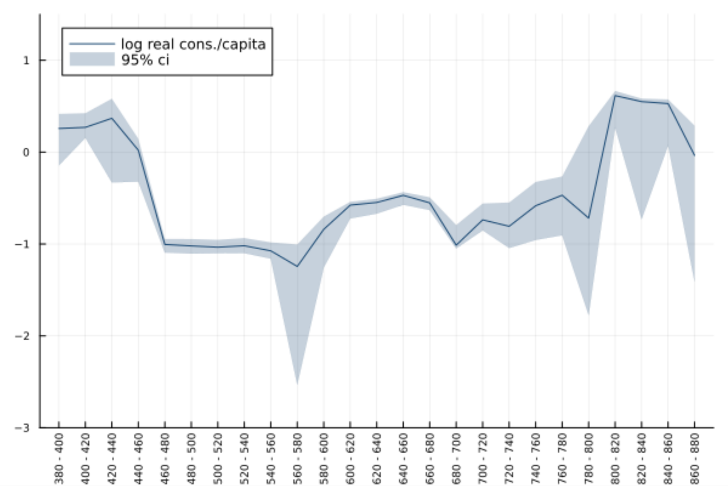
Real consumption per capita (380-880): al-Andalus (Spain)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

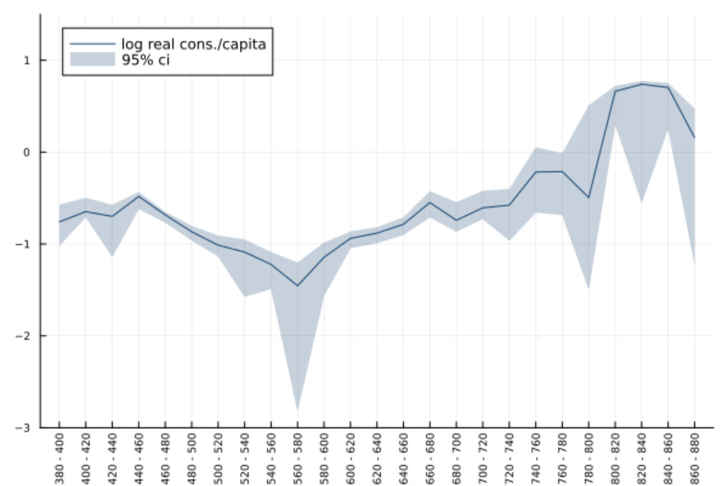
Real consumption per capita (380-880): Aquitaine (South France)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

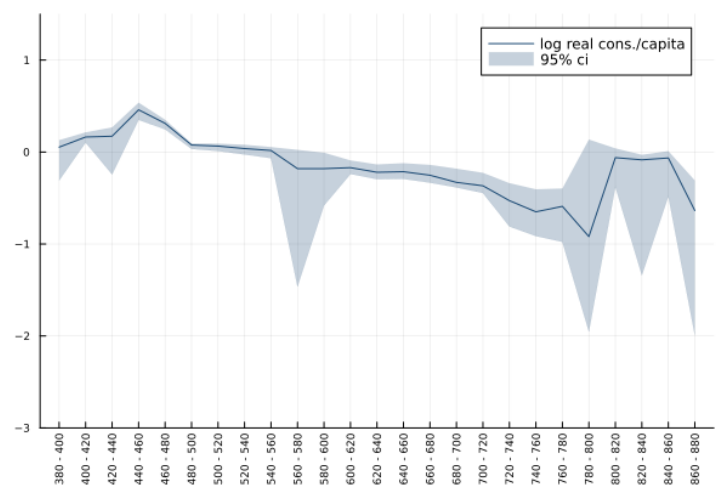
Real consumption per capita (380-880): Francia and Germania



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathsf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

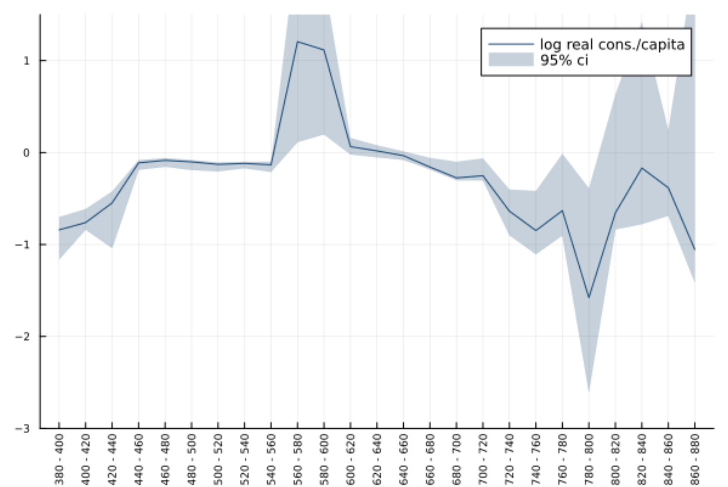
Real consumption per capita (380-880): Northern Italy



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathrm{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

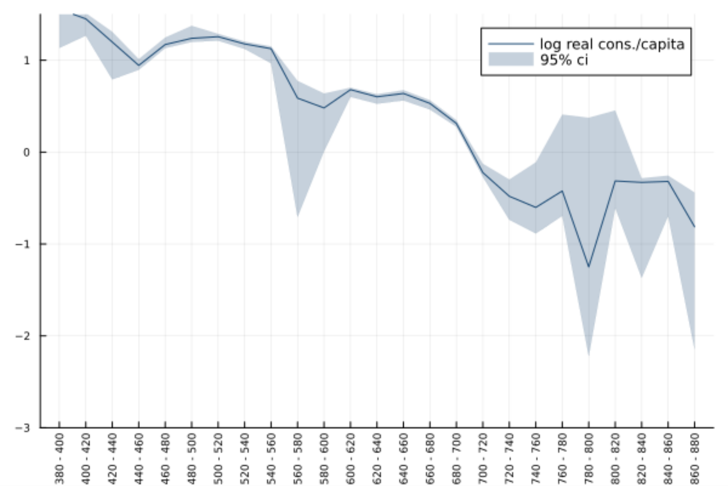
Real consumption per capita (380-880): Southern Italy



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

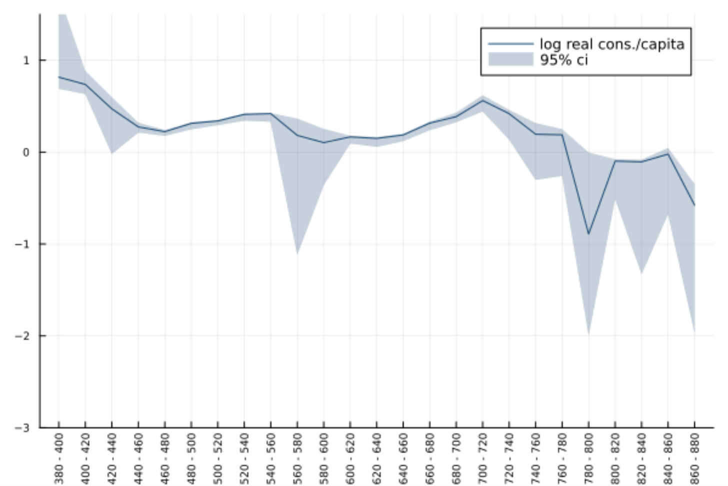
Real consumption per capita (380-880): Byzantine Heartlands



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathsf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

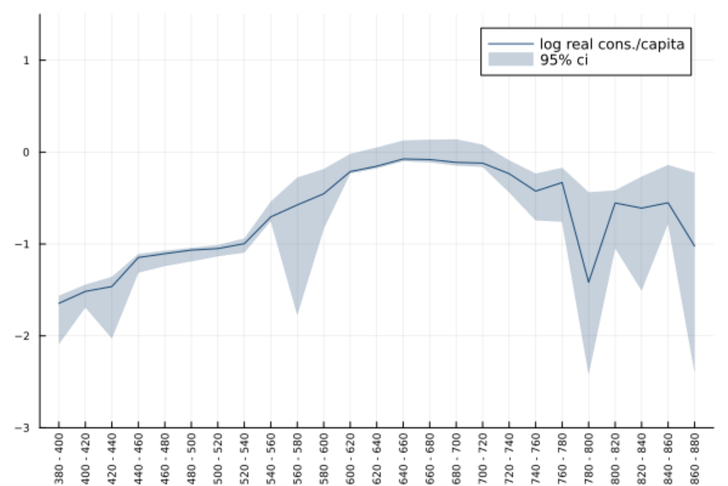
Real consumption per capita (380-880): al-Sham (Greater Syria)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

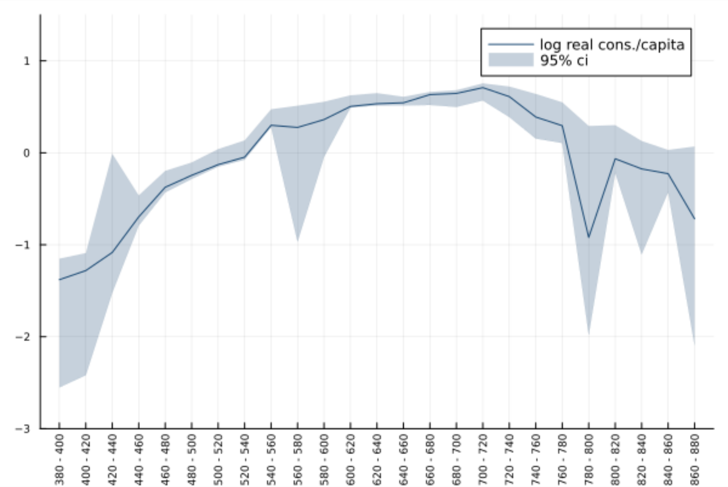
Real consumption per capita (380-880): Northern Syria, Caucasus



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

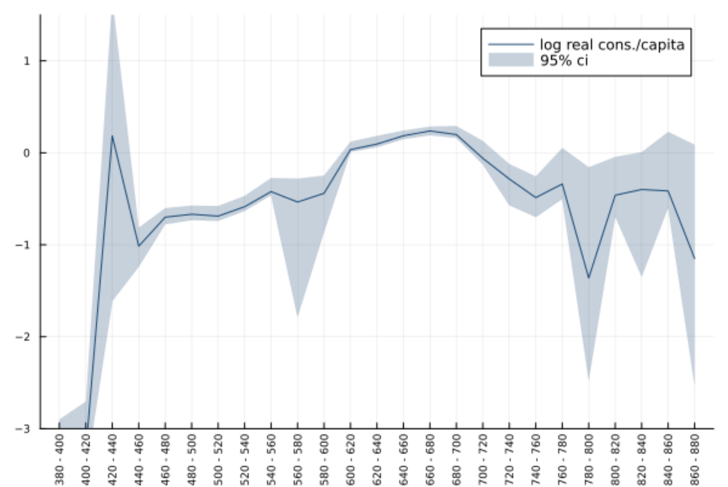
Real consumption per capita (380-880): Iraq, Iran



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

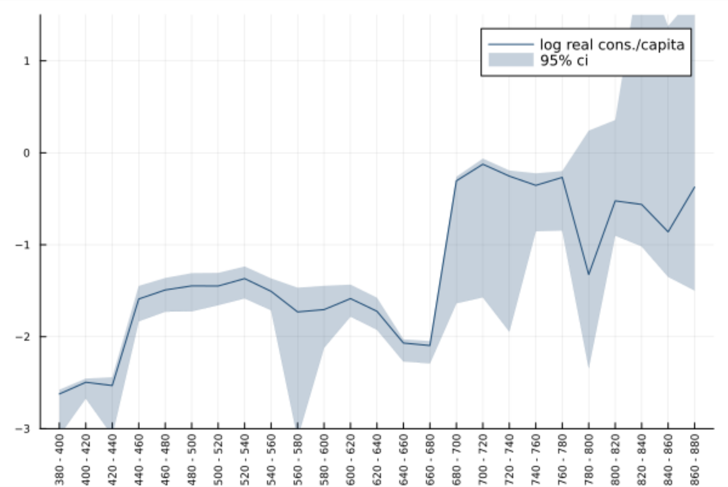
Real consumption per capita (380-880): Eastern Caliphate



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

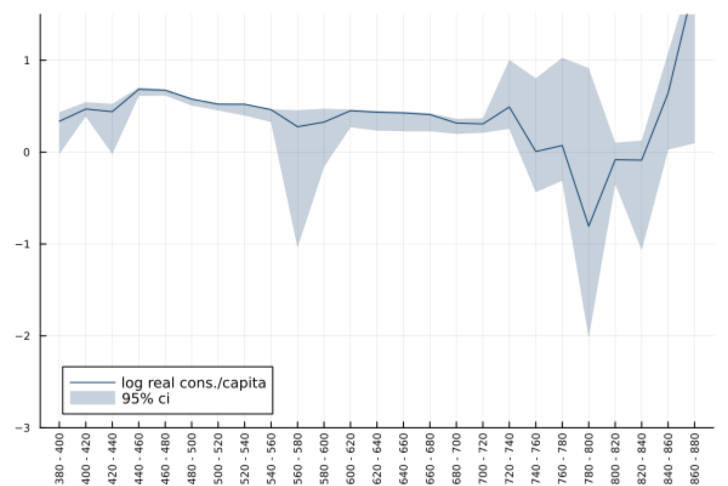
Real consumption per capita (380-880): Arabian Peninsula



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

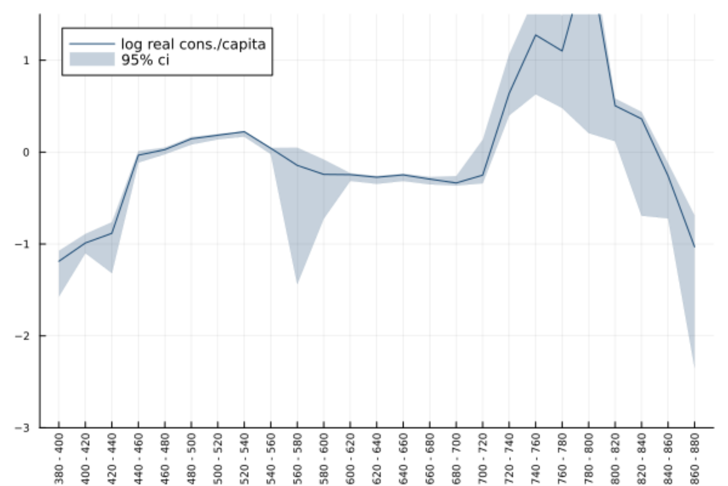
Real consumption per capita (380-880): Misr (Egypt)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathsf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

Real consumption per capita (380-880): al-Maghrib



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathrm{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

Realized vs counterfactual changes in real consumption per capita

Realized changes, from AD 460-620 to AD 700-900

	Real consumption $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$		Openness $\Delta \log\left(\pi_{nn}^{-1/\theta}\right)$		Technology $\Delta \log\left(T_n^{1/\theta}\right)$		Trade Deficit $\Delta \log\left(1+\frac{M_n-\lambda w_n L_n}{w_n L_n}\right)$	
al-Andalus (Spain)	0.62	(0.25)	-0.06	(0.04)	0.77	(0.32)	-0.09	(0.18)
Aquitaine (South France)	1.28	(0.23)	-0.05	(0.01)	1.22	(0.23)	0.11	(0.06)
Francia and Germania	**1.96**	**(0.24)**	**-0.05**	**(0.01)**	**1.80**	**(0.26)**	**0.20**	**(0.04)**
Northern Italy	-0.31	(0.24)	-0.08	(0.03)	-0.10	(0.26)	-0.13	(0.10)
Southern Italy	-0.20	(0.34)	0.19	(0.18)	-0.94	(0.37)	0.55	(0.40)
Byzantine Heartlands	**-1.56**	**(0.33)**	**-0.23**	**(0.14)**	**-0.44**	**(0.41)**	**-0.89**	**(0.54)**
al-Sham (Greater Syria)	-0.32	(0.27)	-0.04	(0.02)	-0.11	(0.29)	-0.17	(0.11)
Northern Syria, Caucasus	0.22	(0.30)	-0.01	(0.03)	0.15	(0.37)	0.08	(0.12)
Iraq, Iran	0.06	(0.27)	-0.00	(0.01)	0.06	(0.29)	-0.00	(0.04)
Eastern Caliphate	0.37	(0.33)	-0.00	(0.00)	0.39	(0.34)	-0.02	(0.04)
Arabian Peninsula	**1.16**	**(0.34)**	**-0.01**	**(0.04)**	**0.66**	**(0.45)**	**0.51**	**(0.26)**
Misr (Egypt)	-0.36	(0.72)	0.09	(0.23)	-0.82	(0.50)	0.37	(0.90)
al-Maghrib	0.28	(0.33)	0.13	(0.07)	-0.49	(0.27)	0.65	(0.30)

Normalizations: $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

▸ back

Realized vs counterfactual changes in real consumption per capita

Counterfactual changes relative to to AD 700-900								
	Initial $\log\left(\frac{X_n/p_n}{L_n}\right)$		Counterfactual $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$ if:					
	All parameters AD 460-620		Religious border AD 700-900		Technology AD 700-900		Minting AD 700-900	
al-Andalus (Spain)	-0.70	(0.10)	0.09	(0.02)	0.55	(0.10)	1.57	(0.31)
Aquitaine (South France)	-1.04	(0.08)	-0.15	(0.03)	0.99	(0.09)	3.93	(0.30)
Francia and Germania	-1.55	(0.09)	-0.07	(0.02)	1.68	(0.11)	6.17	(0.47)
Northern Italy	0.07	(0.04)	-0.24	(0.05)	-0.24	(0.08)	-0.21	(0.07)
Southern Italy	-0.25	(0.06)	-0.11	(0.02)	-0.60	(0.13)	-0.03	(0.02)
Byzantine Heartlands	1.22	(0.11)	-0.69	(0.08)	-0.57	(0.13)	-1.41	(0.19)
al-Sham (Greater Syria)	0.30	(0.04)	0.04	(0.01)	-0.18	(0.10)	-0.22	(0.08)
Northern Syria, Caucasus	-0.34	(0.11)	0.02	(0.02)	0.15	(0.22)	0.19	(0.19)
Iraq, Iran	0.28	(0.08)	0.01	(0.00)	0.03	(0.08)	0.03	(0.06)
Eastern Caliphate	-0.44	(0.08)	0.01	(0.00)	0.38	(0.16)	0.34	(0.26)
Arabian Peninsula	-1.80	(0.18)	0.26	(0.09)	0.66	(0.40)	2.71	(0.84)
Misr (Egypt)	0.32	(0.07)	0.02	(0.00)	-0.71	(0.24)	-0.09	(0.02)
al-Maghrib	0.12	(0.06)	0.01	(0.00)	-0.46	(0.17)	-0.05	(0.06)

Normalizations: $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

back

Realized vs counterfactual changes in real consumption per capita

Realized changes, from AD 460-620 to AD 700-900

	Real consumption $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$		Openness $\Delta \log\left(\pi_{nn}^{-1/\theta}\right)$		Technology $\Delta \log\left(T_n^{1/\theta}\right)$		Trade Deficit $\Delta \log\left(1+\frac{M_n-\lambda w_n L_n}{w_n L_n}\right)$	
Francia and Germania	1.96	(0.24)	-0.05	(0.01)	1.80	(0.26)	0.20	(0.04)
Byzantine Heartlands	-1.56	(0.33)	-0.23	(0.14)	-0.44	(0.41)	-0.89	(0.54)
Arabian Peninsula	1.16	(0.34)	-0.01	(0.04)	0.66	(0.45)	0.51	(0.26)

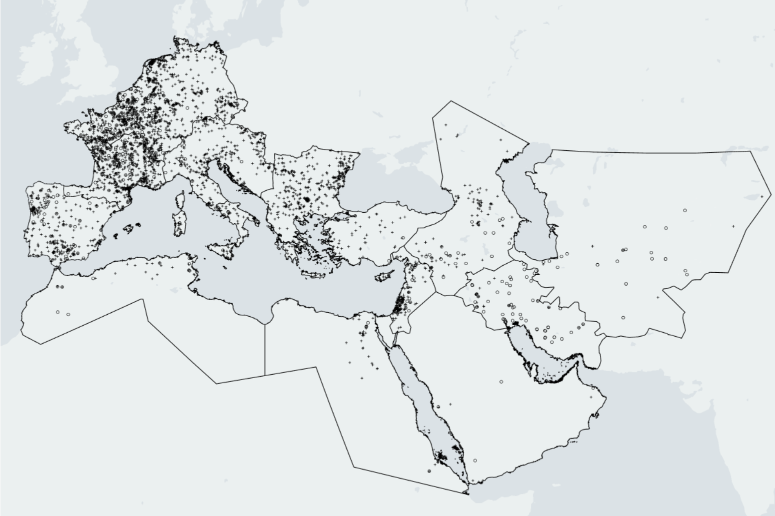
Counterfactual changes relative to to AD 700-900

	Initial $\log\left(\frac{X_n/p_n}{L_n}\right)$		Counterfactual $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$ if:					
	All parameters AD 460-620		Religious border AD 700-900		Technology AD 700-900		Minting AD 700-900	
Francia and Germania	-1.55	(0.09)	-0.07	(0.02)	1.68	(0.11)	6.17	(0.47)
Byzantine Heartlands	1.22	(0.11)	-0.69	(0.08)	-0.57	(0.13)	-1.41	(0.19)
Arabian Peninsula	-1.80	(0.18)	0.26	(0.09)	0.66	(0.40)	2.71	(0.84)

Normalizations: $\mathrm{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$ Bootstrapped s.e.'s in parentheses (100 bootstraps).

back

Spatial distribution: the (extended) Mediterranean



Back

Distribution of coin "death dates" (*tpq*)

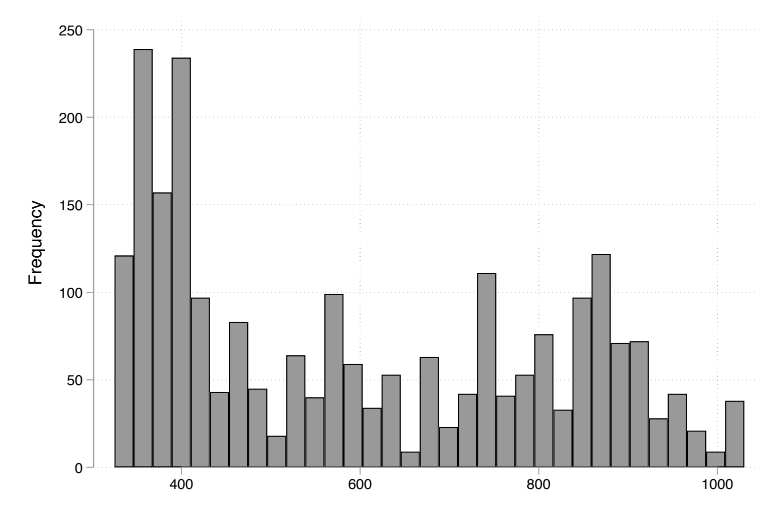


Figure 4: Terminus Post Quem (*tpq*) of hoards

▸ Back

Distribution of coin ages (*tpq* minus mint date)

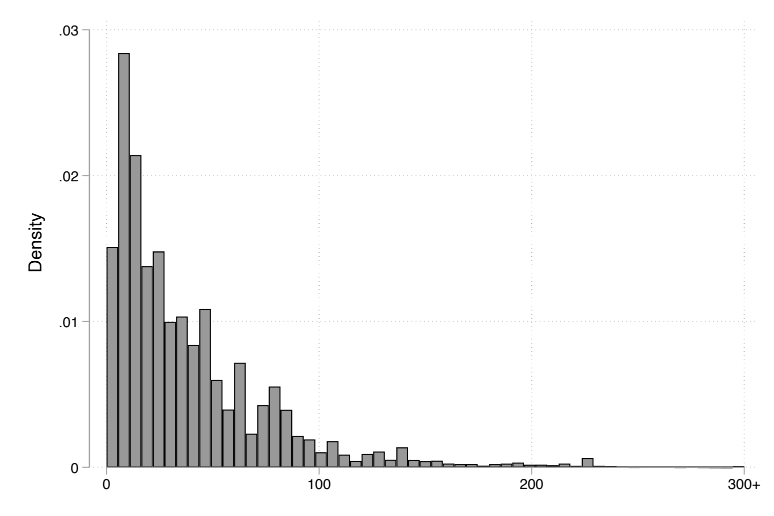


Figure 5: Coin age at time of deposit (*tpq*), in years

Back

Fact #2: distance has a weaker impact on older coin flows

$$\log \text{count}_{mth\tau} = \sum_{\tau' \in T} \beta_{\tau'} \log \text{distance}_{mh} \times 1(t-\tau=\tau') + \alpha_{mt} + \alpha_{h\tau} + \varepsilon_{mth\tau}$$

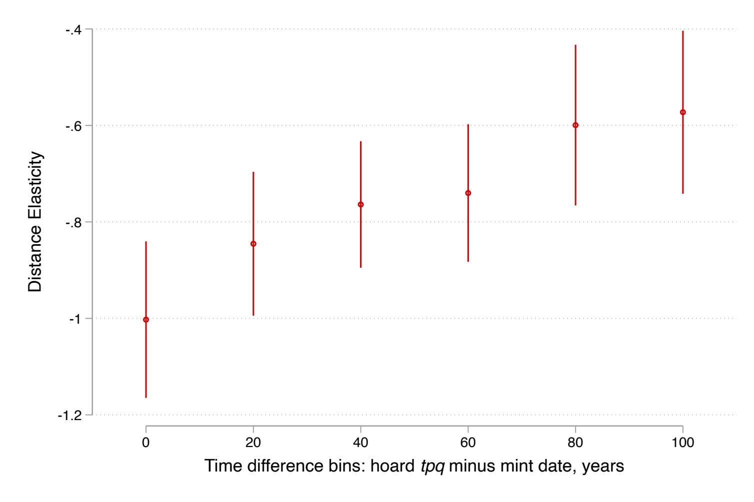


Figure 6: The distance elasticity declines as coins get older

Pitfall #1: medium of exchange vs store of value

- Dynamics with 'saving-augmented' trade shares,

$$\mathsf{S}(t,T) = \boldsymbol{M}(t)\left(\prod_{\tau=t}^{T-1}\Big(\boldsymbol{I}-\boldsymbol{\lambda}(\tau)\Big)\tilde{\boldsymbol{\Pi}}(\tau)\right)$$

- Separate origin, destination, and bilateral terms,

$$\tilde{\pi}_{ni}(\tau) = \alpha_n(\tau)\,\beta_i(\tau)\,\delta_{ni}(\tau)$$

$$\alpha_n = \frac{1}{\sum_k T_k (w_k d_{nk})^{-\theta}}, \beta_i = T_i (w_i)^{-\theta}$$

$$\delta_{ni} = (d_{ni})^{-\theta}$$

Pitfall #1: medium of exchange vs store of value

- Dynamics with 'saving-augmented' trade shares,

$$\mathbf{S}(t,T) = \boldsymbol{M}(t)\left(\prod_{\tau=t}^{T-1}\Big(\boldsymbol{I}-\boldsymbol{\lambda}(\tau)\Big)\tilde{\boldsymbol{\Pi}}(\tau)\right)$$

- Separate origin, destination, and bilateral terms,

$$\tilde{\pi}_{ni}(\tau) = \alpha_n(\tau)\,\beta_i(\tau)\,\delta_{ni}(\tau)$$

$$\alpha_n = \frac{1}{\sum_k T_k\left(w_k d_{nk}\right)^{-\theta}},\ \beta_i = T_i\left(w_i\right)^{-\theta}$$

$$\delta_{ni} = \left(d_{ni}\right)^{-\theta} \times \begin{cases} (1-s_n) & \text{if } n \neq i \\ (1-s_n) + \dfrac{s_n \sum_k T_k\left(w_k d_{nk}\right)^{-\theta}}{T_n\left(w_n d_{nn}\right)^{-\theta}} & \text{if } n = i. \end{cases}$$

- $\frac{\delta_{nj}}{\delta_{ni}} = \frac{\left(d_{nj}\right)^{-\theta}}{\left(d_{ni}\right)^{-\theta}}, \forall n \neq i, j, \forall s_n \geq 0$: no impact on external trade
- $\frac{\delta_{nn}}{\delta_{ni}} > \frac{\left(d_{nn}\right)^{-\theta}}{\left(d_{ni}\right)^{-\theta}}, \forall s_n > 0$: net saving mimics home bias in trade!

Back

Pitfall #2: stocks vs flows (steady state math)

- SS: no net saving ($\boldsymbol{s} = 0$), only age (a) matters, not time (t),

$$\mathsf{S}(t, t+a) = \mathsf{S}(a) = \boldsymbol{M}\left((\boldsymbol{I} - \boldsymbol{\lambda})\,\boldsymbol{\Pi}\right)^{a}, \forall t$$

- Sum of different vintages (stocks by origin-destination),

$$\sum_{a=0}^{A} \mathsf{S}(a) = \boldsymbol{M}\left(\sum_{a=0}^{A}\left((\boldsymbol{I} - \boldsymbol{\lambda})\,\boldsymbol{\Pi}\right)^{a}\right) \underset{A \to +\infty}{=} \boldsymbol{M}\left(\boldsymbol{I} - (\boldsymbol{I} - \boldsymbol{\lambda})\,\boldsymbol{\Pi}\right)^{-1}$$

- Naive gravity on stocks gives Leontief inverse of trade shares!
  $\Rightarrow$ inconsistent estimates of trade elasticities/border effects due to model misspecification

▸ Illustration

▸ Back

Setup

Location n, period t: homog. mass $L_n(t)$ of workers. Four sub-periods $t_{sub1}, t_{sub2}, t_{sub3}, t_{sub4}$

t_{sub1} Start with $S_n(t)$ coins saved from period $t-1$

t_{sub2} A fraction $\lambda_n(t)$ of those saved coins is lost

$M_n(t) \geq 0$ fresh new coins are minted

t_{sub3} $X_n(t)$, expenditure on consumption

Cash-in-advance constraint:

$$X_n(t) \leq (1-\lambda_n(t)) S_n(t) + M_n(t)$$

t_{sub4} $L_n(t)$ workers produce and sell goods in exchange for coins

$w_n(t)$, competitive wage, $w_n(t) L_n(t)$, aggregate labor income

$S_n(t+1)$ coins saved for $t+1$

$$\underbrace{(1-\lambda_n(t))\overbrace{S_n(t)}^{t_{sub1}} + M_n(t)}_{t_{sub2}} - \underbrace{X_n(t)}_{t_{sub3}} + \underbrace{w_n(t) L_n(t)}_{t_{sub4}} = \underbrace{S_n(t+1)}_{(t+1)_{sub1}}$$

Intra-temporal allocations

- Fraction π_{ni} of expenditure X_n allocated to goods from i:

$$\pi_{ni}(t) = \frac{T_i(t)\left(w_i(t)\, d_{ni}(t)\right)^{-\theta}}{\sum_k T_k(t)\left(w_k(t)\, d_{nk}(t)\right)^{-\theta}}, \tag{5}$$

as in Eaton and Kortum (2002).

Intertemporal preferences

- Intertemporal utility U_n, within period welfare W_n, ,

$$U_n(t) = \mathbb{E}\left[\sum_{\tau \geq t} \beta^{\tau - t} \ln\left(\frac{X_n(t)}{p_n(t)}\right)\right],$$

$$\text{with } p_n(t) = \gamma \left(\sum_k T_k(t) \left(w_k(t) d_{nk}(t)\right)^{-\theta}\right)^{1/\theta}$$

Dynamic optimization

- Optimal coin savings dynamics,

$$\max_{\{S_n(\tau)\}_{\tau\geq t}} \left[\sum_{\tau\geq t} \beta^{\tau-t} \ln\left(\frac{X_n(\tau)}{p_n(\tau)}\right)\right]$$

$$X_n(\tau) = w_n(\tau) L_n(\tau) + M_n(\tau) + (1-\lambda_n(\tau)) S_n(\tau) - S_n(\tau+1),$$

$$S_n(\tau+1) \geq w_n(\tau) L_n(\tau), \forall \tau \geq t,$$

$$\lim_{\tau\to\infty} \beta^{\tau} S_n(\tau+1)/X_n(\tau) = 0$$

- Dynamic equilibrium wages clear markets,

$$w_i' L_i' = \sum_n \pi_{ni}(\mathbf{T}, \mathbf{d}; \mathbf{w}) \left[w_n L_n + M_n + (1-\lambda_n) S_n - S_n'\right]$$

Savings $S_n(\mathbf{T}, \mathbf{d}, \boldsymbol{\delta}, \mathbf{L}, \mathbf{M}; \mathbf{w})$ depend on parameters and wages, which depend on wages etc.

Optimal consumption/saving

Under log utility:

- price level $p_n(t)$ dynamics irrelevant (i.e. separates out)
- when unconstrained, consumption declines exponentially:

$$\frac{X_n(t+1)}{X_n(t)} = \beta(1-\lambda_n(t)) < 1$$

- when constrained, consume as much as you can:

$$S_n' = w_n(t)L_n$$

Define *net* saving:

$$s_n(t) = \frac{(1-\lambda_n(t))S_n(t) + M_n(t) - X_n(t)}{(1-\lambda_n(t))S_n(t) + M_n(t)}$$

Introducing and tracking coins of different vintages

Coin stocks $S_n(\tau)$ consist of coins of different vintage:

$$S_n(\tau) = \sum_{m=1}^{N} \sum_{t<\tau} S_{mn}(t,\tau)$$

Coins start their life when minted: $S_{mm}(t,t) = M_m(t)$.

Traders are 'blind' to coin types, draw coins with equal probability:

$$S_{mi}(t,\tau+1) = \sum_{n=1}^{N} (1-\lambda_n(\tau))(1-s_n(\tau)) S_{mn}(t,\tau)\pi_{ni}(\tau) + (1-\lambda_i(\tau)) s_i(\tau) S_{mi}(t,\tau), \forall \tau \geq t$$

In compact matrix form:

$$\mathbf{S}(t,T) = \boldsymbol{M}(t) \left(\prod_{\tau=t}^{T-1} (\boldsymbol{I} - \boldsymbol{\lambda}(\tau)) \left((\boldsymbol{I} - \boldsymbol{s}(\tau)) \boldsymbol{\Pi}(\tau) + \boldsymbol{s}(\tau) \right) \right)$$

▸ Back

▸ Pitfall 1: Store

▸ Pitfall 2: Stocks vs flows