Trade and the End of Antiquity

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8 July 2024

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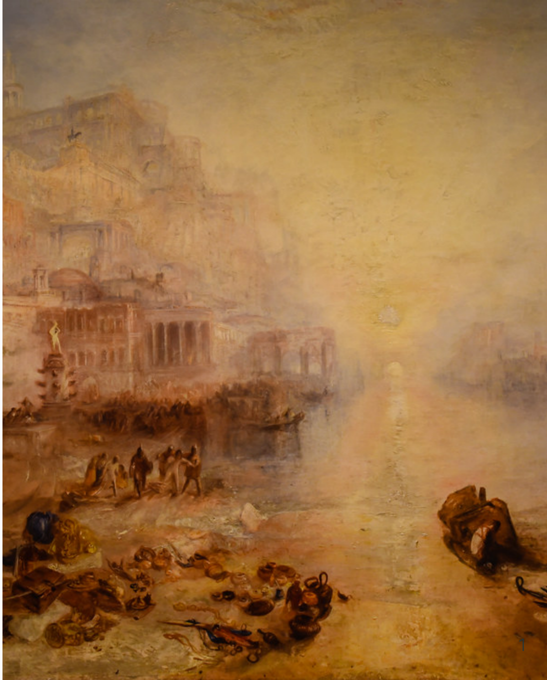

What caused the End of Antiquity?

- Antiquity: Roman and Greek civilizations centered around the Mediterranean
- End of antiquity circa 7th-8th Century AD:
  - Economic activity shifts away from the Mediterranean.
  - Rise of Northern Europe (Charlemagne).

$\hookrightarrow$ *Question*: What caused the End of Antiquity?

- Discussed, among others, by Montesquieu (1734), Voltaire (1756), Gibbon (1789)

Pirenne Hypothesis

Henri Pirenne (1937), *“Mahomet et Charlemagne”*

- Rise of the Islamic Caliphate disrupts Mediterranean trade/exchanges.
- Causes a shift of economic activity away from the Mediterranean.
- Rise of the Carolingian Empire in Northern Europe.

Political changes in the Mediterranean: 600 AD

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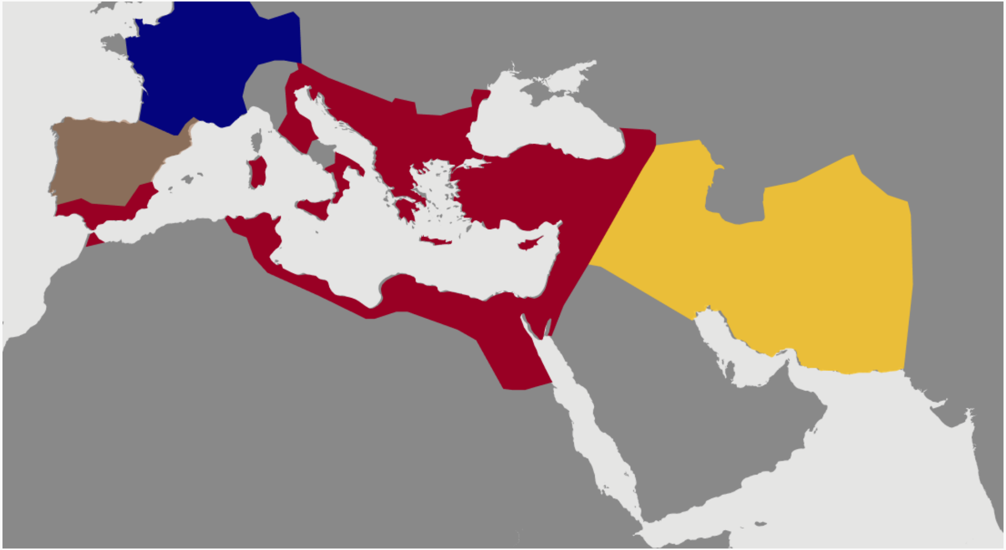
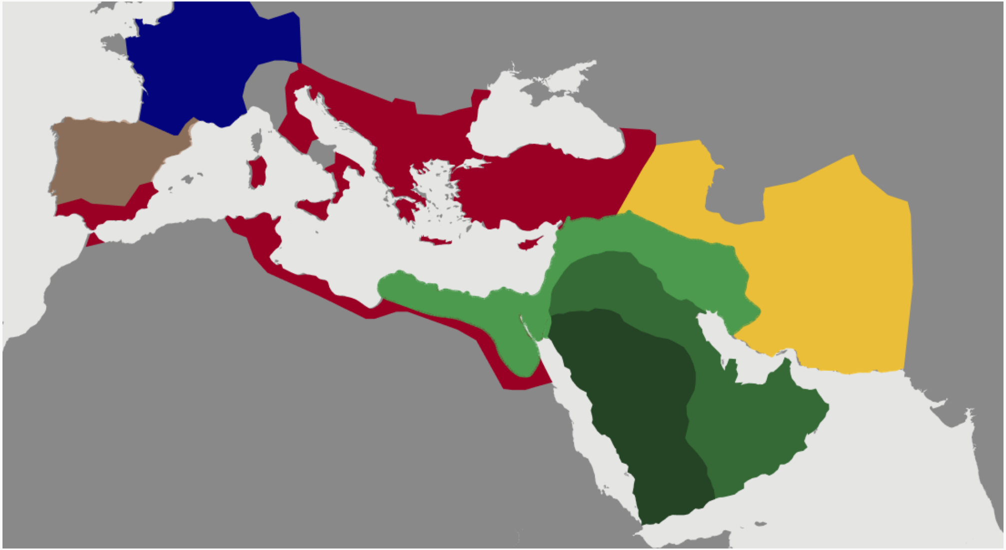


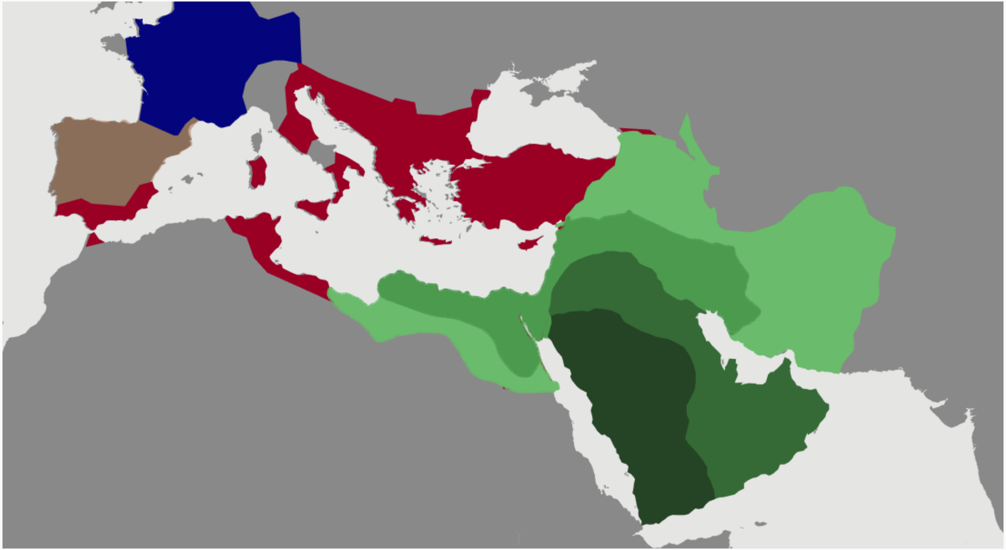
Political changes in the Mediterranean: 632 AD

Political changes in the Mediterranean: 634 AD

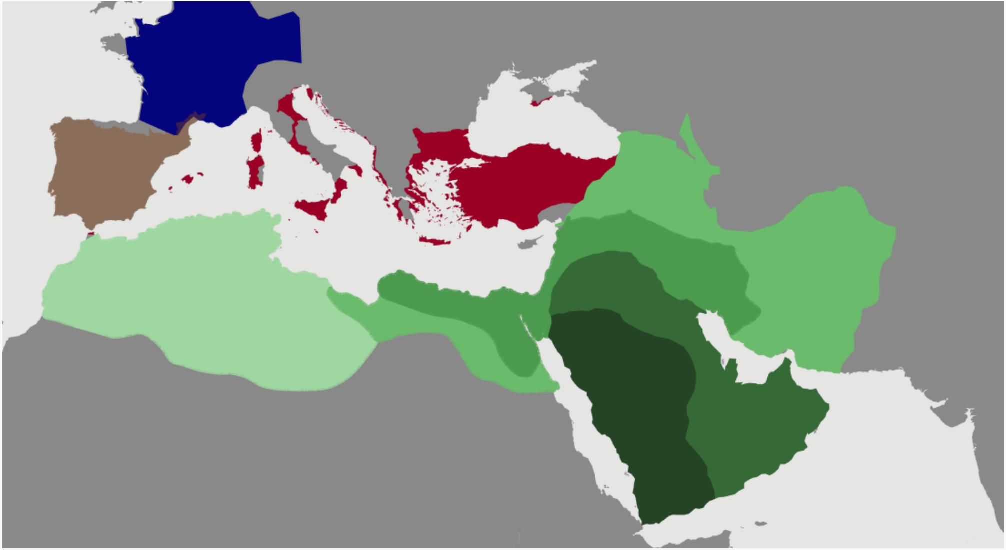
Political changes in the Mediterranean: 644 AD



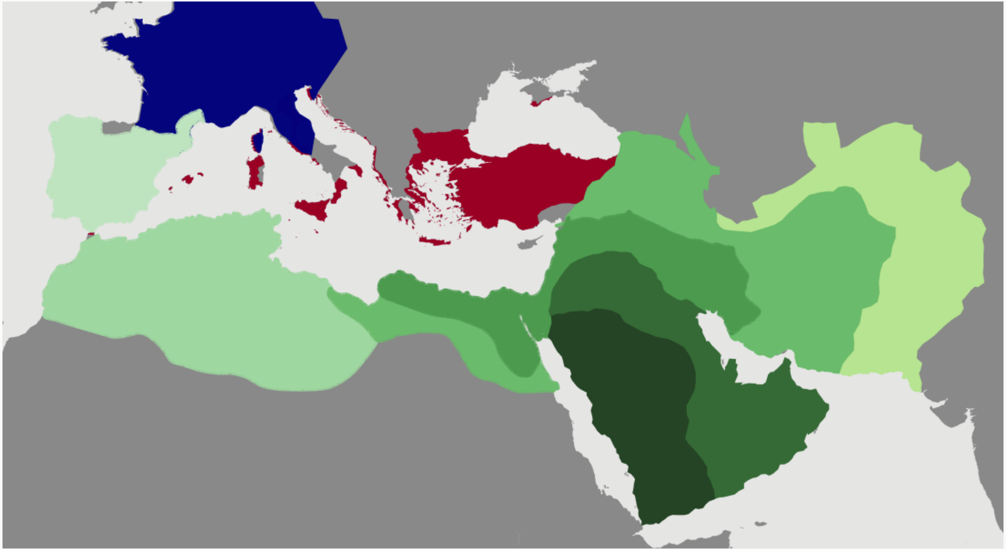
Political changes in the Mediterranean: 661 AD



Political changes in the Mediterranean: 661-700 AD



Political changes in the Mediterranean: 750 AD



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$\Rightarrow$ Use data on the movement of coins to study the changing economic geography during Late Antiquity.

- Coins are the main medium of exchange (precious metals!) during Late Antiquity, particularly for long-distance trade $\rightarrow$ informative about trade
- Coins are well studied & documented by historians and numismatists
- Coins have features that help solve econometric identification problems

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Roadmap:

1. Data description
2. Stylized facts / reduced-form evidence
3. Quantitative Model
    - Identification of trade shares from coin stocks
    - Identification of trade cost changes from origin/destination "sizes"
4. Decomposition of estimated Y/L changes into trade/openness, technology, minting.

Data: Coins around the Mediterranean, AD 325 to AD 950

Assemble a large dataset of coin finds from around the Mediterranean

1. FLAME (2023) project by historians around Princeton
   - $\sim$200,000 coins with complete records 325–725
2. Hand-coded records from numismatic / archaeological literature:
   - 797 coin finds, $\sim$100k coins, 725-950

▸ Coin age distribution ▸ Hoard Dates (tpq) ▸ Spatial Distribution

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Example: excerpt from al Ush's (1972) Damascus silver hoard:

No.	MINT	DATE	DIAM.	WEIGHT	NUMB.
51	الأندلس	114	29.	2.93	4
52	"	115	29.5	2.92	1
53	"	116	26.5	2.92	3

Index / Mint (al-Andalus/Cordoba) / Date: 114 AH = 732 AD / Diameter / Weight / Q'tity

▸ Coin age distribution ▸ Hoard Dates (tpq) ▸ Spatial Distribution

Spatial distribution: the (extended) Mediterranean

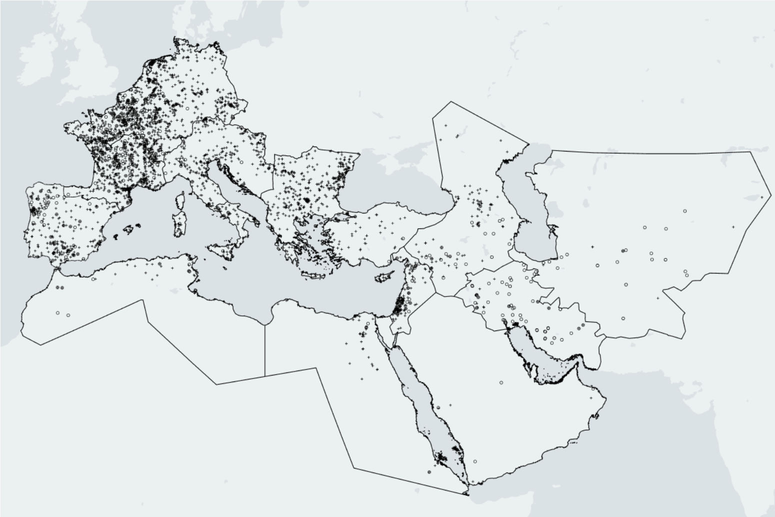
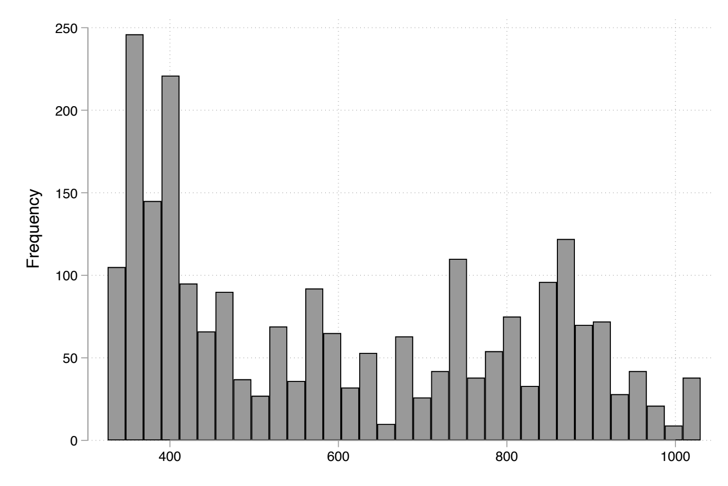


Figure 1: Region Definitions

Distribution of coin “death dates” (*tpq*)

Historians use the date of the most recent object in the hoard to date the hoard: “terminus post quem” (or *tpq*)



Distribution of coin ages (*tpq* minus mint date)

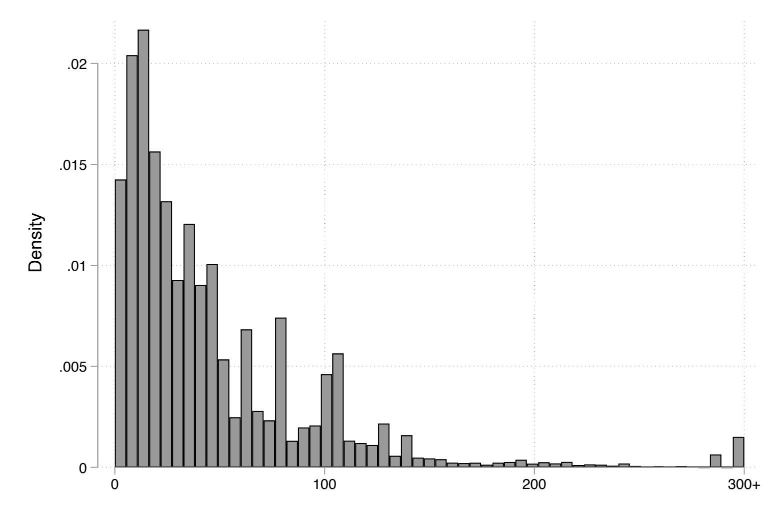


Figure 3: Coin age at time of deposit (*tpq*), in years

Stylized Facts

Fact #1: within a hoard, older coins have travel farther

Table 1: Within-hoard distance travelled and age of coin at deposit

	Dependent variable: Log Distance between Mint and Hoard				
	(1)	(2)	(3)	(4)	(5)
Log Age of Coin	0.160**	0.0942**	0.0882**	0.178**	0.0623**
	(0.050)	(0.025)	(0.031)	(0.049)	(0.020)
Sample				No non-hoards	No non-hoards
Hoard FE	Yes	Yes	Yes	Yes	Yes
Mint × 50-year-interval FE		Yes			
Mint × 25-year-interval FE			Yes		Yes
R^2	0.762	0.863	0.869	0.775	0.899
Observations	287235	287018	286860	250133	249806

Standard errors in parentheses, clustered at the hoard level.
$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Age of coin = hoard deposit date − mint date

⇒ coins diffuse across space over time.

Fact #2: distance and political borders impede coin travels

Construct $1^\circ \times 1^\circ$ cells for mint and hoard locations and calculate flows count_{mdh}

Table 2: Gravity and Border Effects in Coin Flows

	Dependent variable: # Coins_{mdh}					
	(1)	(2)	(3)	(4)	(5)	(6)
Log Distance	-1.138**	-1.002**	-1.140**	-0.955**	-0.727**	-0.694**
	(0.12)	(0.13)	(0.10)	(0.077)	(0.10)	(0.10)
Political border		-1.945**		-2.073**		-1.540**
		(0.62)		(0.47)		(0.41)
Hoard Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Mint × Empire Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample			Gold only	Gold only	Int. Marg. only	Int. Marg. only
Estimator	PPML	PPML	PPML	PPML	PPML	PPML
Pseudo-R^2	0.766	0.778	0.809	0.825	0.737	0.744
Observations	216809	216809	57457	57457	6306	6306

Standard errors in parentheses, clustered at mint cell × empire and hoard cell level.
$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

▸ Declining Elasticity

Estimating eqn: $\text{count}_{mdh} = \exp\left(\gamma_1 \log \text{distance}_{mh} + \gamma_2 \text{withinBorder}_{dh} + \alpha_{md} + \alpha_h + \varepsilon_{mhd}\right)$

Fact #3: Arab conquests disrupt Mediterranean trade

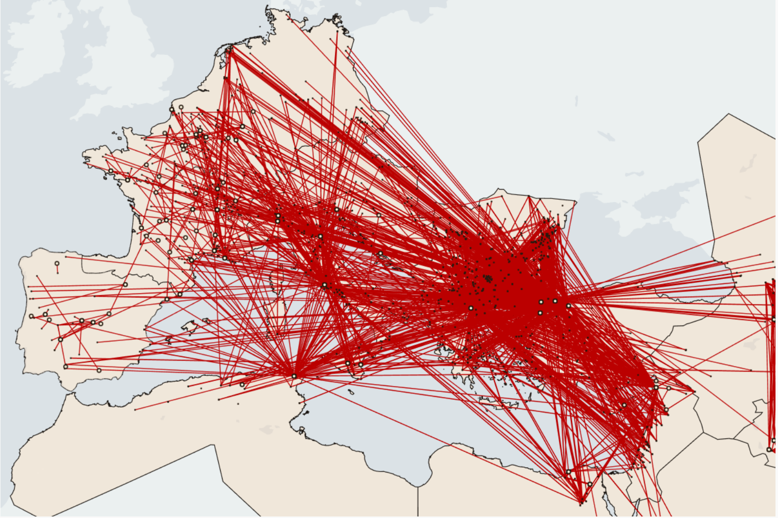


Figure 4: Before the Arab conquests: 450-630 AD

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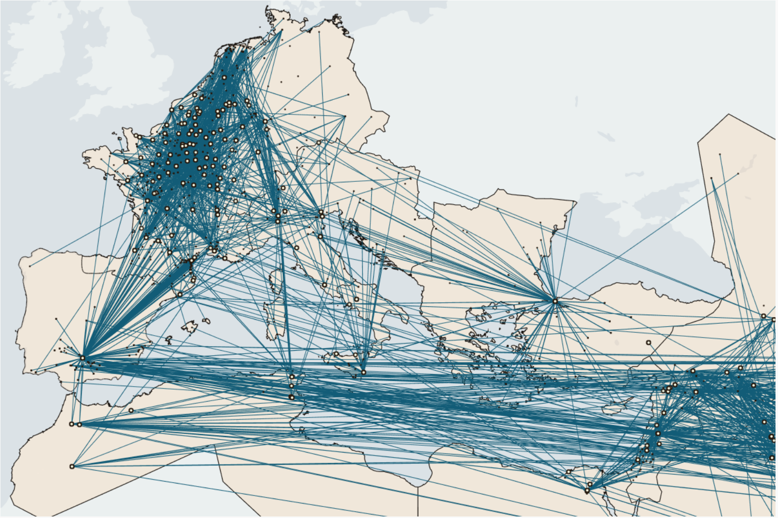


Figure 5: After the Arab conquests: 713-900 AD

Fact #3: Arab conquests disrupt Mediterranean trade

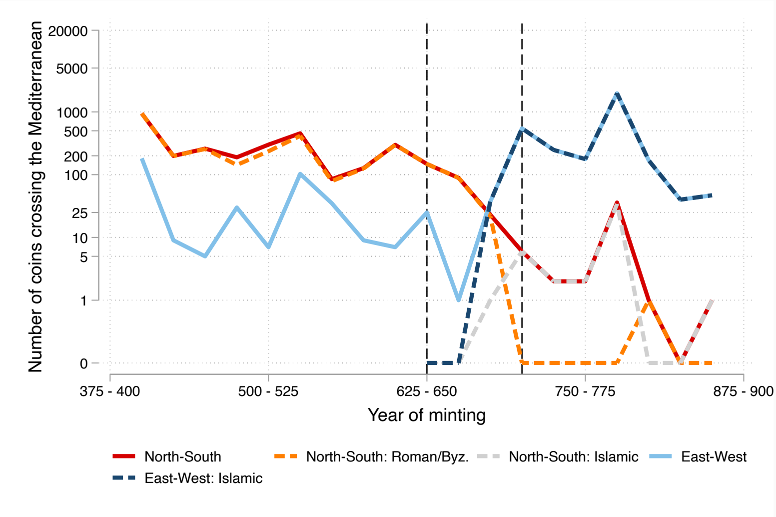


Figure 6: Number of coins flowing across the Mediterranean

Regressions

Model

Model

Objective:
Quantify impact of trade barriers (and other changes) on market access / trade / welfare

Approach:
Dynamic model of trade flows (gravity), where coins diffuse alongside trade and are thus informative about trade flows.

Key Assumption:
Traders are blind to coin *types* (mint location and date).

- $\Rightarrow$ Coins diffuse in proportion to trade flows
- $\Rightarrow$ Recover trade *flows* from stocks of coin *types* in hords in different locations/time periods.

Not used/needed for identification: total *quantities* of coins found in a location/time
$\rightarrow$ identification from *shares* only

Setup

Location n, period t: homog. mass $L_n(t)$ of workers. Four sub-periods $t_{sub1}, t_{sub2}, t_{sub3}, t_{sub4}$

t_{sub1} Start with $S_n(t)$ coins saved from period $t-1$

t_{sub2} A fraction $\lambda_n(t)$ of those saved coins is lost

$M_n(t) \geq 0$ fresh new coins are minted

t_{sub3} $X_n(t)$, expenditure on consumption

Budget constraint:

$$X_n(t) \leq (1-\lambda_n(t))\, S_n(t) + M_n(t)$$

t_{sub4} $L_n(t)$ workers produce and sell goods in exchange for coins

$w_n(t)$, competitive wage, $w_n(t)\, L_n(t)$, aggregate labor income

$S_n(t+1)$ coins saved for $t+1$

$$\underbrace{(1-\lambda_n(t))\overbrace{S_n(t)}^{t_{sub1}} + M_n(t)}_{t_{sub2}} - \underbrace{X_n(t)}_{t_{sub3}} + \underbrace{w_n(t)\, L_n(t)}_{t_{sub4}} = \underbrace{S_n(t+1)}_{(t+1)_{sub1}}$$

Intra-temporal allocations

- Fraction π_{ni} of expenditure X_n allocated to goods from i:

$$\pi_{ni}(t) = \frac{T_i(t)\left(w_i(t)\, d_{ni}(t)\right)^{-\theta}}{\sum_k T_k(t)\left(w_k(t)\, d_{nk}(t)\right)^{-\theta}}, \tag{1}$$

as in Eaton and Kortum (2002).

Intertemporal preferences

- Intertemporal utility U_n, within period welfare W_n, ,

$$U_n(t) = \mathbb{E}\left[\sum_{\tau \geq t} \beta^{\tau - t} \ln\left(\frac{X_n(t)}{p_n(t)}\right)\right],$$

$$\text{with } p_n(t) = \gamma\left(\sum_k T_k(t)\left(w_k(t)\, d_{nk}(t)\right)^{-\theta}\right)^{1/\theta}$$

Dynamic optimization

- Optimal coin savings dynamics,

$$\max_{\{S_n(\tau)\}_{\tau\geq t}} \left[\sum_{\tau\geq t} \beta^{\tau-t} \ln\left(\frac{X_n(\tau)}{p_n(\tau)}\right)\right]$$

$$X_n(\tau) = w_n(\tau) L_n(\tau) + M_n(\tau) + (1-\lambda_n(\tau)) S_n(\tau) - S_n(\tau+1),$$

$$S_n(\tau+1) \geq w_n(\tau) L_n(\tau), \forall \tau \geq t,$$

$$\lim_{\tau\to\infty} \beta^{\tau} S_n(\tau+1)/X_n(\tau) = 0$$

- Dynamic equilibrium wages clear markets,

$$w_i' L_i' = \sum_n \pi_{ni}(\mathbf{T}, \mathbf{d}; \mathbf{w}) \left[w_n L_n + M_n + (1-\lambda_n) S_n - S_n'\right]$$

Savings $S_n(\mathbf{T}, \mathbf{d}, \boldsymbol{\delta}, \mathbf{L}, \mathbf{M}; \mathbf{w})$ depend on parameters and wages, which depend on wages etc.

Optimal consumption/saving

Under log utility:

- price level $p_n(t)$ dynamics irrelevant (i.e. separates out)
- when unconstrained, consumption declines exponentially:

$$\frac{X_n(t+1)}{X_n(t)} = \beta(1-\lambda_n(t)) < 1$$

- when constrained, consume as much as you can:

$$S_n' = w_n(t)L_n$$

Define *net* saving:

$$s_n(t) = \frac{(1-\lambda_n(t))S_n(t) + M_n(t) - X_n(t)}{(1-\lambda_n(t))S_n(t) + M_n(t)}$$

Steady state

- No motive for extra savings, $S_n = w_n L_n$. Hence $s_n(t) = 0$.
- Wages and trade shares jointly determined,

$$w_i L_i = \sum_n \pi_{ni} \left(\left(1 - \lambda_n\right) w_n L_n + M_n \right)$$

$$\pi_{ni} = \frac{T_i \left(w_i d_{ni}\right)^{-\theta}}{\sum_k T_k \left(w_k d_{nk}\right)^{-\theta}}$$

- Constant aggregate stock of coins in circulation,

$$\sum_n M_n = \lambda_n \sum_n w_n L_n$$

- *Note*: trade deficits as in Dekle, Eaton and Kortum (2007), but endogenous:

$$D_n = X_n - Y_n = M_n - \lambda_n S_n$$

Introducing and tracking coins of different vintages

Coin stocks $S_n(\tau)$ consist of coins of different vintage:

$$S_n(\tau) = \sum_{m=1}^{N} \sum_{t<\tau} S_{mn}(t,\tau)$$

Coins start their life when minted: $S_{mm}(t,t) = M_m(t)$.

Traders are 'blind' to coin types, draw coins with equal probability:

$$S_{mi}(t,\tau+1) = \sum_{n=1}^{N} (1-\lambda_n(\tau))(1-s_n(\tau)) S_{mn}(t,\tau)\pi_{ni}(\tau) + (1-\lambda_i(\tau)) s_i(\tau) S_{mi}(t,\tau), \forall \tau \geq t$$

In compact matrix form:

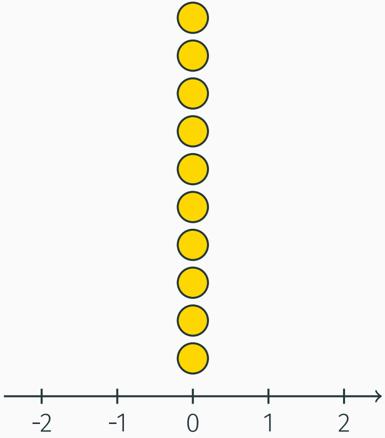
$$\mathbf{S}(t,T) = \boldsymbol{M}(t) \left(\prod_{\tau=t}^{T-1} (\boldsymbol{I} - \boldsymbol{\lambda}(\tau)) \left((\boldsymbol{I} - \boldsymbol{s}(\tau)) \boldsymbol{\Pi}(\tau) + \boldsymbol{s}(\tau) \right) \right)$$

▸ Stocks vs Flows ▸ Medium of Exchange vs Store of Value

Subtlety #1: Gravity with data on stocks vs flows (extreme example)

Assume locations are symmetric on a circle.

- A number of coins get minted at location 0.
- Trade declines *strongly* with distance: coins travel at most a distance of 1
- As time goes by, coins diffuse over space
- Ultimately they will end up pretty evenly dispersed over space



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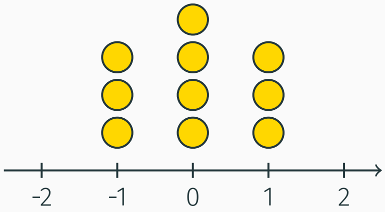

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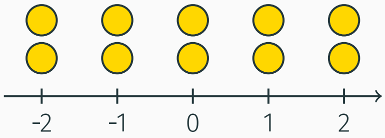

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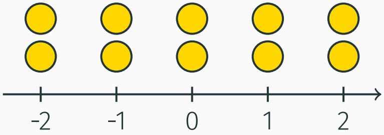
$\Rightarrow$ **Using data on the spatial distribution of *stocks* of coins in a naive gravity regression will underestimate the distance elasticity for per-period trade *flows***



▸ Declining Distance Elasticity in the Data

Subtlety #2: medium of exchange vs store of value

Dynamics with 'saving-augmented' trade shares: $\mathbf{S}(t,T) = \boldsymbol{M}(t)\left(\prod_{\tau=t}^{T-1}\Big(\boldsymbol{I} - \boldsymbol{\lambda}(\tau)\Big)\tilde{\boldsymbol{\Pi}}(\tau)\right)$

	Eaton and Kortum (2002) trade flows	Saving-'augmented' coin flows, $s_n \geq 0$
Trade/coin shares	$\pi_{ni} = \alpha_n \beta_i \delta_{ni}$	$\tilde{\pi}_{ni} = \tilde{\alpha}_n \tilde{\beta}_i \tilde{\delta}_{ni}$
Seller FE	$\beta_i = T_i (w_i)^{-\theta}$	$\tilde{\beta}_i = T_i (w_i)^{-\theta}$
Buyer FE	$\alpha_n = \dfrac{1}{\sum_k \beta_k \delta_{nk}}$	$\tilde{\alpha}_n = \dfrac{1}{\sum_k \tilde{\beta}_k \tilde{\delta}_{nk}}$
Bilateral (internal)	$\delta_{nn} = 1$	$\tilde{\delta}_{nn} = 1$
Bilateral (external)	$\delta_{ni} = \dfrac{(d_{ni})^{-\theta}}{(d_{nn})^{-\theta}}$	$\dfrac{(d_{ni})^{-\theta}}{(d_{nn})^{-\theta}}\left(1 - \dfrac{s_n[t]}{\tilde{\pi}_{nn}[t]}\right)$

(In practice s_n very small, so quantitatively not important)

Taking the model to the data

- 13 regions around the Mediterranean ▸ details
- 20-year time intervals
- Assume constant λ and estimate as exponential decay parameter in within-hoard age distribution:

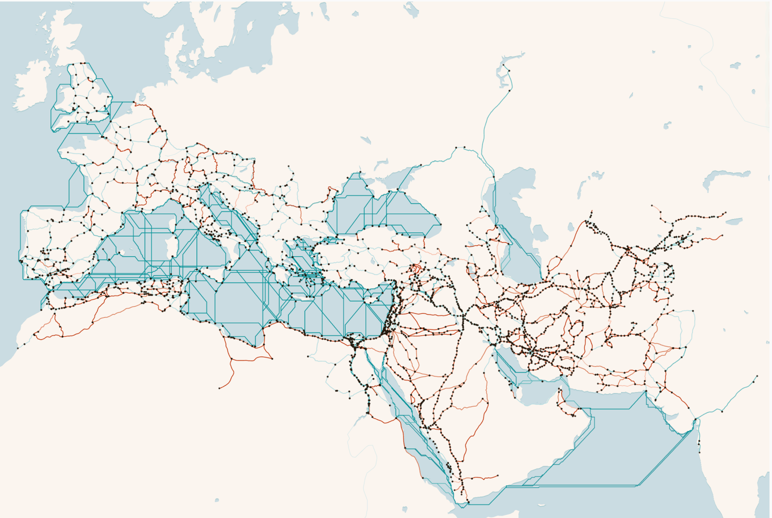
$$\hat{\lambda}_{20y} = 0.3$$

- Parameterize trade frictions,

$$d_{ni}(t)^{\theta} = \exp\left(\gamma_0 + \zeta \ln\left(\text{TravelTime}_{ni}\right) + \kappa_1 \text{PoliticalBorder}_{ni}(t) + \kappa_2 \text{ReligiousBorder}_{ni}(t)\right)$$

  if $n \neq i$ and $d_{nn}(t) = 1$.
- How to split domestic trade vs saving? Calibrate $\overline{s_n}(t) = 1.5\%$ (Scheidel, 2020).
- For counterfactuals, assume $\theta = 4$ (Simonovska and Waugh, 2014).

Trade costs *only* depend on travel times (and politics/religion)



Note: Combined geospatial models from Orbis (Scheidel, 2015) and al-Ṯurayyā (Romanov and Seydi, 2022). ▸ al-Maqdisi

Maximum likelihood estimation

Assume coins in our data are a random sample of coin types in each location $\times$ time.

- Multinomial distribution of coin types,

$$P\left(\cdots, X_i^{(m,\tau)}(T) = x_i^{(m,\tau)}, \ldots\right) = \frac{N_i(T)!}{\prod_{(m,\tau)} x_i^{(m,\tau)}!} \prod_{(m,\tau)} \left[p_i^{(m,\tau)}(T)\right]^{x_i^{(m,\tau)}}$$

  with the probability of drawing a coin of type (m, τ),

$$p_i^{(m,\tau)}(T) = \frac{S_i^{(m,\tau)}(T)}{\sum_{(m',\tau')} S_i^{(m',\tau')}(T)} = \frac{S_i^{(m,\tau)}(T)}{S_i(T)}.$$

- Likelihood of observing a sample of coins given parameters $\boldsymbol{\theta}$,

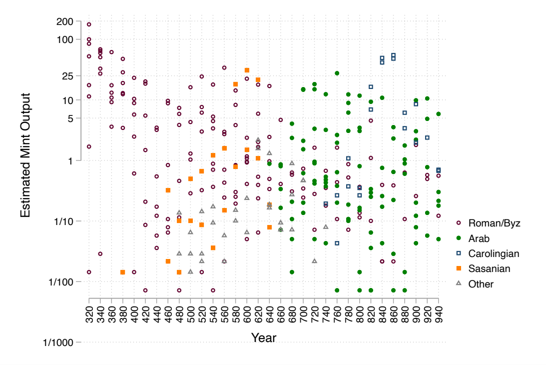
$$\ell(X; \boldsymbol{\theta}) = \sum_i \sum_T \sum_{(m,\tau)} x_i^{(m,\tau)} \left[\log S_i^{(m,\tau)}(T; \boldsymbol{\theta}) - \log S_i(T; \boldsymbol{\theta})\right] + \text{constant}$$

Estimation results

Trade costs:

$$\ln\left(\left(d_{ni}(t)\right)^{-\theta}\right) = constant - 3.04 \ln\left(\text{TravelTime}_{ni}\right) - 0.64\,\text{PoliticalBorder}_{ni}(t) - 3.85\,\text{ReligiousBorder}_{ni}(t)$$

Mint output:



▸ More Specifications

Welfare decomposition

Real consumption ("welfare") depends on a combination of L and T (that's not separately identified):

$$X_n/p_n = \gamma^{-1}\left(\pi_{nn}\right)^{-1/\theta}\left(L_n T_n^{1/\theta}\right)\left(1-\lambda+\frac{M_n}{\left(L_n T_n^{1/\theta}\right)\tilde{\beta}_n^{-1/\theta}}\right)$$

Real consumption per capita:

$$\frac{X_n/p_n}{L_n} = \gamma^{-1}\underbrace{\left(\pi_{nn}\right)^{-1/\theta}}_{\text{Openness}} \times \underbrace{\left(T_n\right)^{1/\theta}}_{\text{Technology}} \times \underbrace{\left(1+\frac{M_n-\lambda w_n L_n}{w_n L_n}\right)}_{\text{Minting (Trade deficit: DEK)}}$$

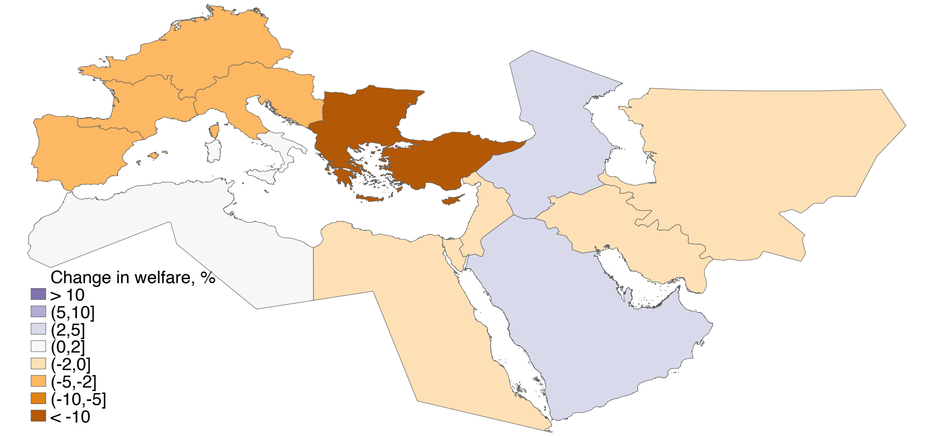
We separate L and T through a Malthusian assumption

$$L_n = T_n \quad \forall n$$

and decompose per-capita welfare into the three components

Welfare change decomposition: Openness

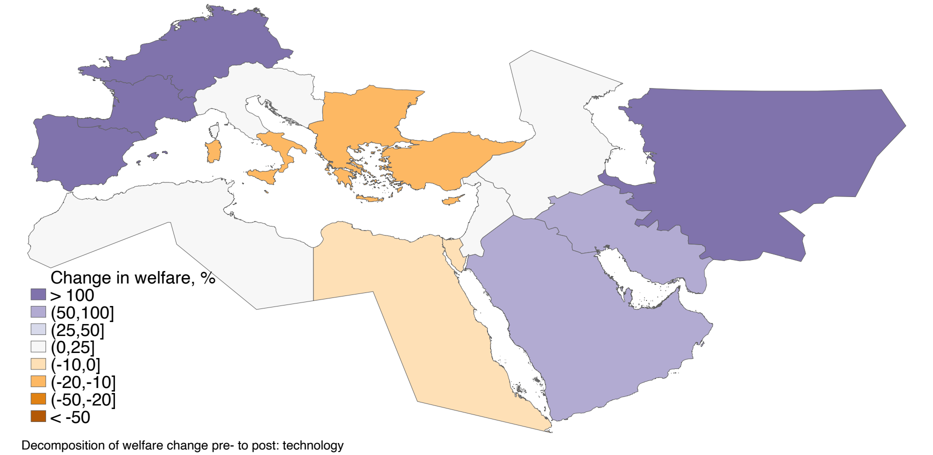
Change in measured openness $(\pi_{nn})^{-1/\theta}$
Byzantium, Spain, Franks moved to periphery.



Decomposition of welfare change pre- to post: openness

Welfare change decomposition: Technology

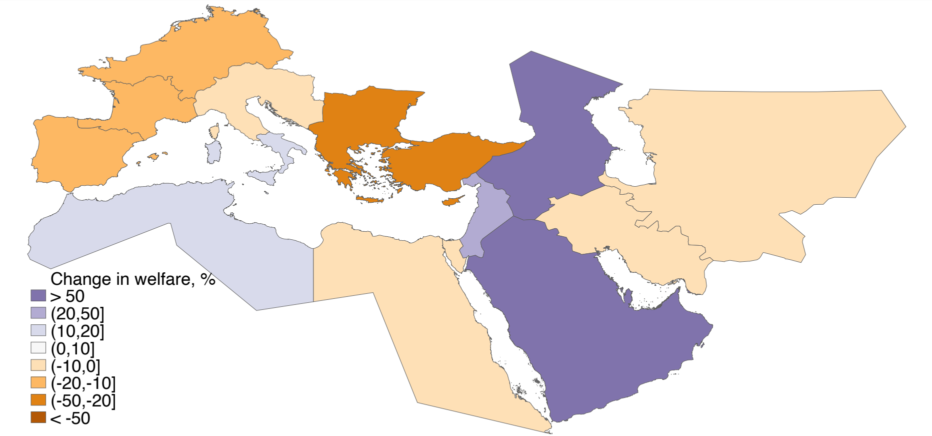
Change in $(T_n)^{1/\theta}$
Technology improvements in Spain (Agriculture) and Francia



Welfare change decomposition: Minting

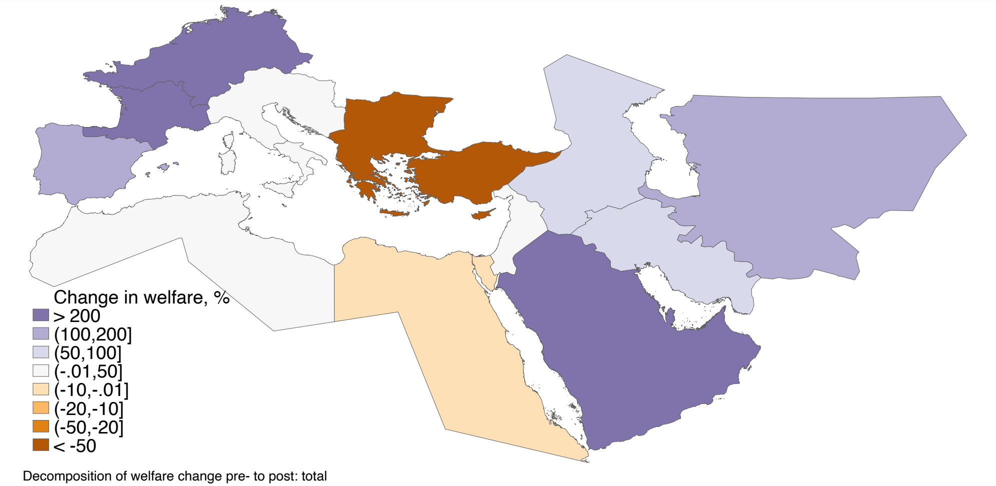
Change in minting over income

Byzantine Mint output declines; Frankish mint output increases (but income increases even more due to technology)



Decomposition of welfare change pre- to post: minting

Welfare change decomposition: Openness + Tech + Minting

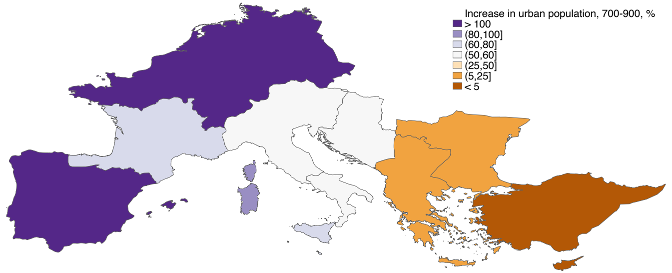


▸ In Table Form ▸ Counterfactuals

Compare to relative urbanization rates, 700–900 AD

Data:
Change in total urban population
(urban: $>$ 1k inhabitants, from Buringh, 2021)

Model:
Change in relative per-capital consumption
$\Delta \left(X_n / p_n \right) / L_n$



Taking stock

- Clear pattern of change in economic geography before vs after conquest
- Trade disruption leads to a significant relative decline of the eastern Mediterranean
- Minting ($\rightarrow$ trade deficits) and in particular change in "technology" quantitatively more important than trade costs
- Jointly, can account for relative urbanization patterns.

Conclusion

"Simply looking at the Mediterranean cannot of course explain everything about a complicated past created by human agents, with varying doses of calculation, caprice and misadventure. But this is a sea that patiently recreates for us scenes from the past, breathing new life into them, locating them under a sky and in a landscape that we can see with our own eyes, a landscape and sky like those of long ago. A moment's concentration or daydreaming, and that past comes back to life."

Fernand Braudel, Les Mémoires de la Méditerranée

THANK YOU!

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BACKUP SLIDES

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Regions

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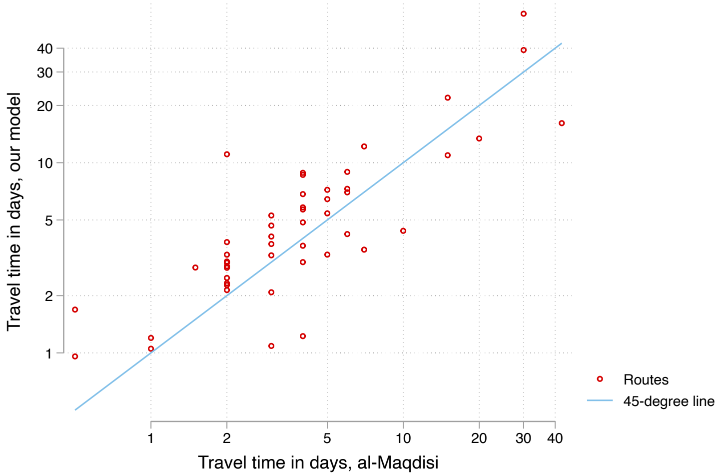
Validating Travel Times

Al-Maqdisi (c. 945–991): *The Best Divisions for Knowledge of the Regions*



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Spatial distribution: the (extended) Mediterranean

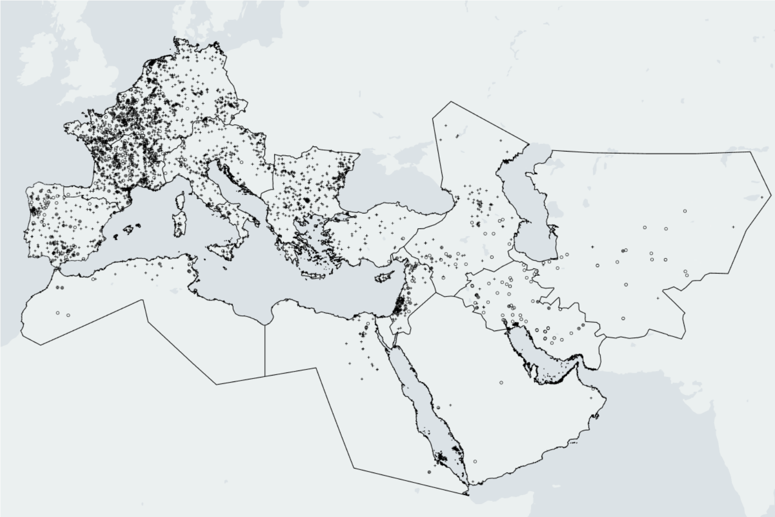

Figure 7: Region Definitions

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Distribution of coin "death dates" (*tpq*)

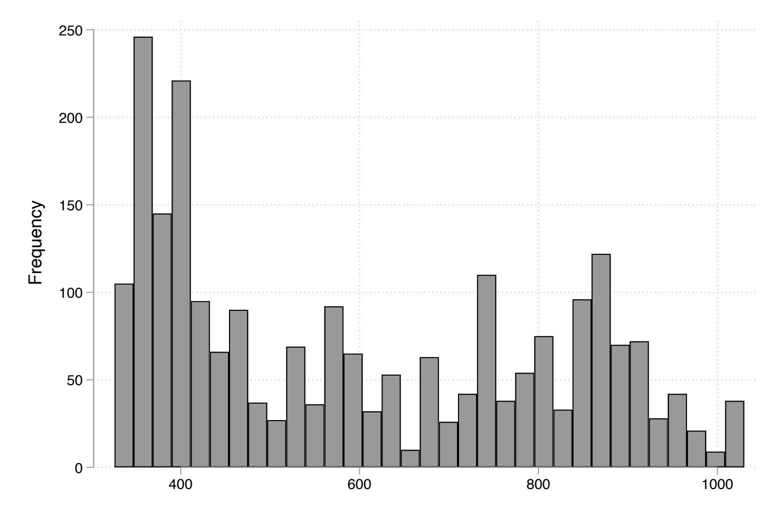

Figure 8: Terminus Post Quem (*tpq*) of hoards

Distribution of coin ages (*tpq* minus mint date)

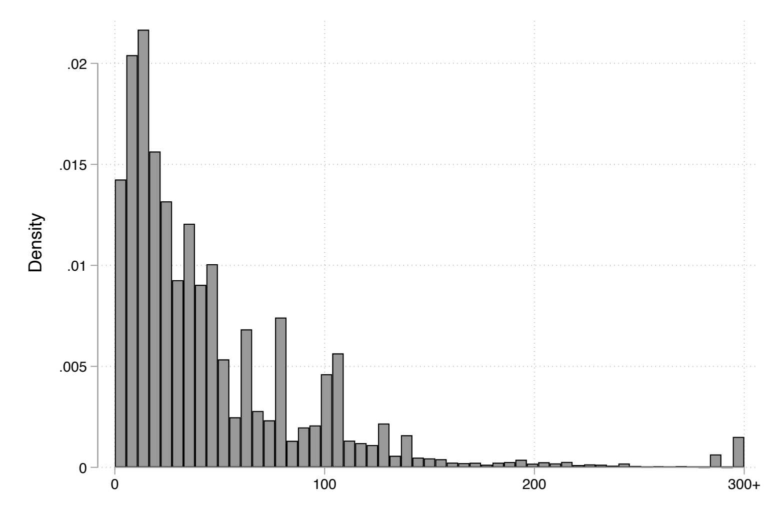

Figure 9: Coin age at time of deposit (*tpq*), in years

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Fact #1: Arab conquests disrupt Mediterranean trade

Table 3: The Mediterranean Before and After the Arab Conquest

	Dependent variable: Number of Coins			
	(1)	(2)	(3)	(4)
Crossing Mediterranean × After Conquests	-1.774**	-3.141**	-0.712	-1.751
	(0.46)	(0.53)	(0.66)	(1.24)
Crossing Mediterranean × After Conquests × Islamic Coin		7.171**	4.835**	8.382**
		(0.91)	(0.97)	(1.15)
Crossing Mediterranean × After Conquests × Roman Coin			-3.108**	-2.976**
			(0.79)	(0.71)
Mint Cell × Empire FE	Yes	Yes	Yes	Yes
Mint Cell × Hoard Cell FE	Yes	Yes	Yes	Yes
After Conquests FE	Yes	Yes	Yes	
Mint Cell × After Conquests FE				Yes
Hoard Cell × After Conquests FE				Yes
Estimator	PPML	PPML	PPML	PPML
Observations	10350	10350	10350	6023

Standard errors in parentheses, clustered at the hoard × era and mint × era level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Estimating eqn: $\text{count}_{mdht} = \exp\left(\gamma_1 \text{mediterranean}_{mh} \times \text{after}_t + \cdots + \alpha_{md} + \alpha_{mh} + \varepsilon_{mhdt}\right)$

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Fact #2: distance has a weaker impact on older coin flows

$$\log \text{count}_{mth\tau} = \sum_{\tau' \in T} \beta_{\tau'} \log \text{distance}_{mh} \times 1(t - \tau = \tau') + \alpha_{mt} + \alpha_{h\tau} + \varepsilon_{mth\tau}$$

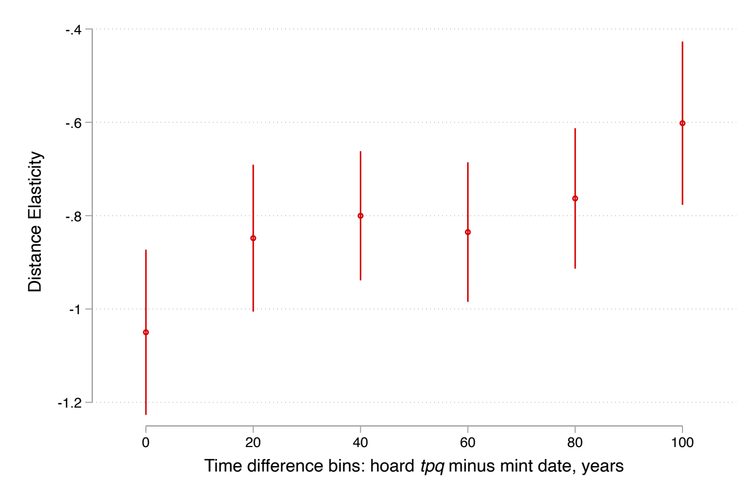

Figure 10: The distance elasticity declines as coins get older

▸ Back ▸ Back to Structural

Fact #3: Arab conquests disrupt Mediterranean trade: Gold only

	Dependent variable: Number of Coins				
	(1)	(2)	(3)	(4)	(5)
Log Distance	-1.484**	-1.512**	-1.517**	-1.159**	-1.163**
	(0.13)	(0.12)	(0.12)	(0.15)	(0.16)
Crossing Mediterranean	0.157	0.172	0.187	-0.126	-0.114
	(0.38)	(0.38)	(0.37)	(0.29)	(0.29)
Crossing Mediterranean × After Conquests	-1.565*	-2.887**	-1.701*	-2.273*	-3.067**
	(0.70)	(0.73)	(0.70)	(0.98)	(1.14)
Crossing Mediterranean × After Conquests × Islamic Coin		3.112**	1.907*		2.865*
		(0.77)	(0.77)		(1.24)
Crossing Mediterranean × After Conquests × Roman Coin			-1.766		
			(1.13)		
Mint Cell × Empire FE	Yes	Yes	Yes	Yes	Yes
Hoard Cell × After Conquests FE	Yes	Yes	Yes	Yes	Yes
Sample				Gold only	Gold only
Observations	167840	167840	167840	28895	28895

Standard errors in parentheses, clustered at the hoard × era and mint × era level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Before: 400–630; after: 713–950

Structural estimation: Robustness

"Religious Border: East":
al-Andalus $\leftrightarrow$ Aquitaine, Francia/Germania

"Religious Border: West":
Byzantine Heartlands $\leftrightarrow$ Eastern Caliphal regions

"Religious Border: Mediterranean":
post conquest across-Mediterranean

	Log Trade Costs			
	(1)	(2)	(3)	(4)
Log Travel Time	3.04	3.06	1.41	1.09
	(0.01)	(0.02)	(0.04)	(0.04)
Political Border	0.64	0.47	2.51	3.19
	(0.02)	(0.02)	(0.05)	(0.05)
Religious Border	3.85		2.94	
	(0.11)		(0.16)	
Religious Border: East		1.99		0.12
		(0.12)		(0.30)
Religious Border: West		4.69		14.63
		(0.21)		(147.22)
Religious Border: Mediterranean		5.20		2.72
		(0.19)		(0.19)
Sample	All	All	Gold/Silver	Gold/Silver
Coin Accounting	Number	Number	Value	Value
Estimator	MLE	MLE	MLE	MLE
Observations	4,389	4,389	2,010	2,010

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Welfare decomposition: By Region

	Consumption $\Delta \log \left(\frac{X_n/p_n}{L_n}\right)$	Openness $\Delta \log \left(\pi_{nn}^{-1/\theta}\right)$	Technology $\Delta \log \left(T_n^{1/\theta}\right)$	Trade Deficit $\Delta \log \left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n}\right)$
al-Andalus (Spain)	0.79	-0.04	0.97	-0.13
Aquitaine (South France)	1.34	-0.05	1.52	-0.13
Francia and Germania	1.63	-0.05	1.87	-0.18
Northern Italy	0	-0.03	0.10	-0.06
Southern Italy	0.03	0.00	-0.11	0.13
Byzantine Heartlands	-0.87	-0.15	-0.11	-0.61
al-Sham (Greater Syria)	0.40	-0.00	0.19	0.21
Northern Syria, Caucasus	0.55	0.04	0.07	0.43
Iraq, Iran	0.44	-0.01	0.50	-0.04
Eastern Caliphate	0.73	-0.00	0.74	-0.01
Arabia	1.26	0.04	0.66	0.56
Misr (Egypt)	-0.03	-0.00	-0.01	-0.02
al-Maghrib	0.35	0.01	0.19	0.15

Normalizations: $\log\left(\frac{X_n/p_n}{L_n}\right) \equiv 0$ for Northern Italy in 460-620 AD and 720-900 AD.

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Counterfactuals

	Initial $\log\left(\frac{X_n/p_n}{L_n}\right)$	Counterfactual $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$ if:		
	All parameters 460-620 AD	Religious border 720-900 AD	Technology 720-900 AD	Minting 720-900 AD
al-Andalus (Spain)	-0.35	0.02	0.20	0.86
Aquitaine (South France)	-0.71	-0.17	0.61	3.26
Francia and Germania	-0.97	-0.15	1.03	4.02
Northern Italy	0	-0.16	0.43	-0.35
Southern Italy	-0.38	-0.02	0.20	-0.01
Byzantine Heartlands	0.95	-0.42	1.33	-1.27
al-Sham (Greater Syria)	0.00	0.03	-0.02	-0.04
Northern Syria, Caucasus	-0.53	0.01	-0.06	0.06
Iraq, Iran	0.08	0.03	0.11	-0.02
Eastern Caliphate	-0.62	0.01	0.25	0.24
Jazirat al-arab and al-Yaman	-1.71	0.00	0.34	1.10
Misr (Egypt)	0.06	0.01	-0.13	-0.06
al-Maghrib	-0.07	0.00	0.25	-0.14

Normalization: $\log\left(\frac{X_n/p_n}{L_n}\right) \equiv 0$ for Northern Italy in 460-620 AD.

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Pitfall #1: medium of exchange vs store of value

- Dynamics with 'saving-augmented' trade shares,

$$\mathsf{S}(t,T) = M(t)\left(\prod_{\tau=t}^{T-1}\Big(I - \boldsymbol{\lambda}(\tau)\Big)\tilde{\boldsymbol{\Pi}}(\tau)\right)$$

- Separate origin, destination, and bilateral terms,

$$\tilde{\pi}_{ni}(\tau) = \alpha_n(\tau)\,\beta_i(\tau)\,\delta_{ni}(\tau)$$

$$\alpha_n = \frac{1}{\sum_k T_k\left(w_k d_{nk}\right)^{-\theta}},\ \beta_i = T_i\left(w_i\right)^{-\theta}$$

$$\delta_{ni} = \left(d_{ni}\right)^{-\theta}$$

Pitfall #1: medium of exchange vs store of value

- Dynamics with 'saving-augmented' trade shares,

$$\mathbf{S}(t,T) = \boldsymbol{M}(t)\left(\prod_{\tau=t}^{T-1}\Big(\boldsymbol{I}-\boldsymbol{\lambda}(\tau)\Big)\tilde{\boldsymbol{\Pi}}(\tau)\right)$$

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$$\tilde{\pi}_{ni}(\tau) = \alpha_n(\tau)\,\beta_i(\tau)\,\delta_{ni}(\tau)$$

$$\alpha_n = \frac{1}{\sum_k T_k\left(w_k d_{nk}\right)^{-\theta}},\ \beta_i = T_i\left(w_i\right)^{-\theta}$$

$$\delta_{ni} = \left(d_{ni}\right)^{-\theta} \times \begin{cases} (1-s_n) & \text{if } n \neq i \\ (1-s_n) + \dfrac{s_n \sum_k T_k\left(w_k d_{nk}\right)^{-\theta}}{T_n\left(w_n d_{nn}\right)^{-\theta}} & \text{if } n = i. \end{cases}$$

- $\frac{\delta_{nj}}{\delta_{ni}} = \frac{\left(d_{nj}\right)^{-\theta}}{\left(d_{ni}\right)^{-\theta}}, \forall n \neq i,j, \forall s_n \geq 0$: no impact on external trade
- $\frac{\delta_{nn}}{\delta_{ni}} > \frac{\left(d_{nn}\right)^{-\theta}}{\left(d_{ni}\right)^{-\theta}}, \forall s_n > 0$: net saving mimics home bias in trade!

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Pitfall #2: stocks vs flows (steady state math)

- SS: no net saving ($\boldsymbol{s} = 0$), only age (a) matters, not time (t),

$$\mathsf{S}(t, t+a) = \mathsf{S}(a) = M\Big((I - \boldsymbol{\lambda})\,\boldsymbol{\Pi}\Big)^{a}, \forall t$$

- Sum of different vintages (stocks by origin-destination),

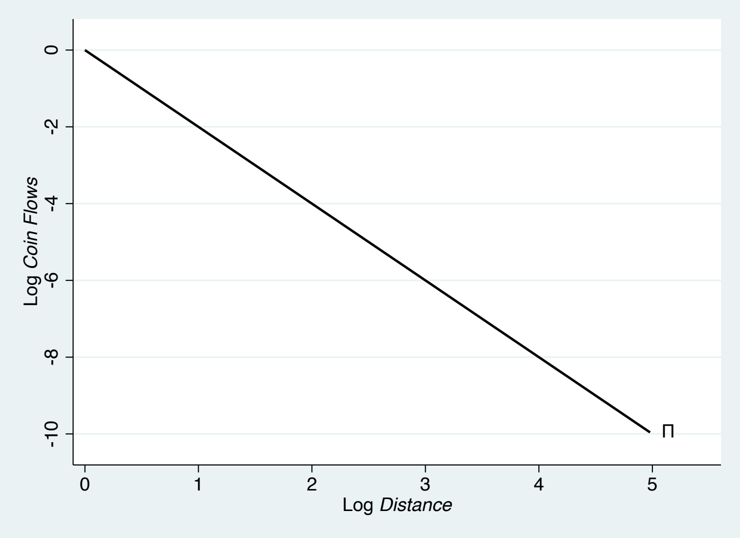
$$\sum_{a=0}^{A} \mathsf{S}(a) = M\left(\sum_{a=0}^{A}\Big((I - \boldsymbol{\lambda})\,\boldsymbol{\Pi}\Big)^{a}\right) \underset{A \to +\infty}{=} M\left(I - (I - \boldsymbol{\lambda})\,\boldsymbol{\Pi}\right)^{-1}$$

- Naive gravity on stocks gives Leontief inverse of trade shares!
  $\Rightarrow$ inconsistent estimates of trade elasticities/border effects due to model misspecification

▸ Illustration

▸ Back

Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1 (same as trade flows $\mathbf{\Pi}$)

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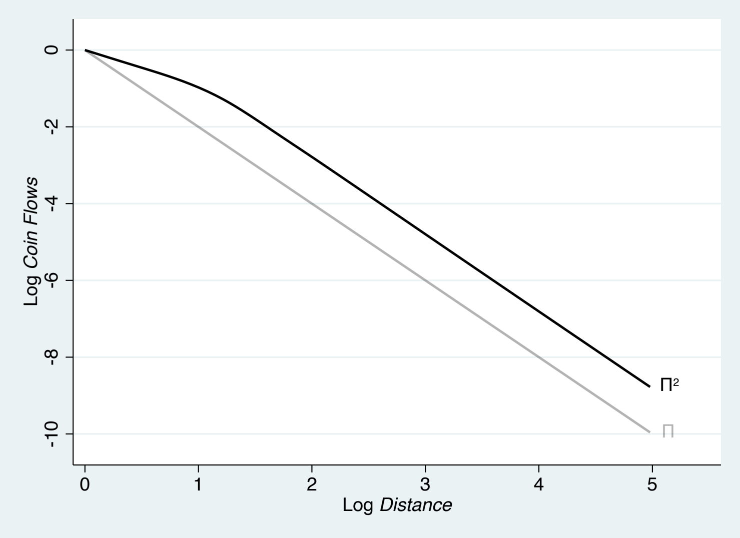
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2

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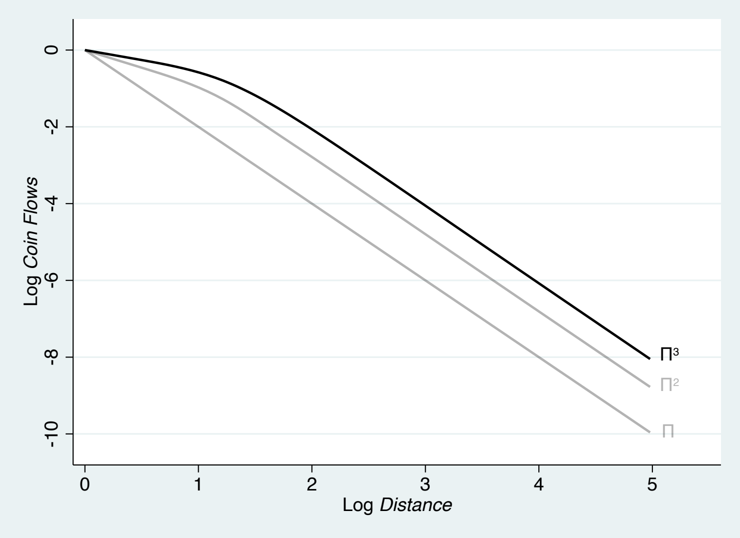
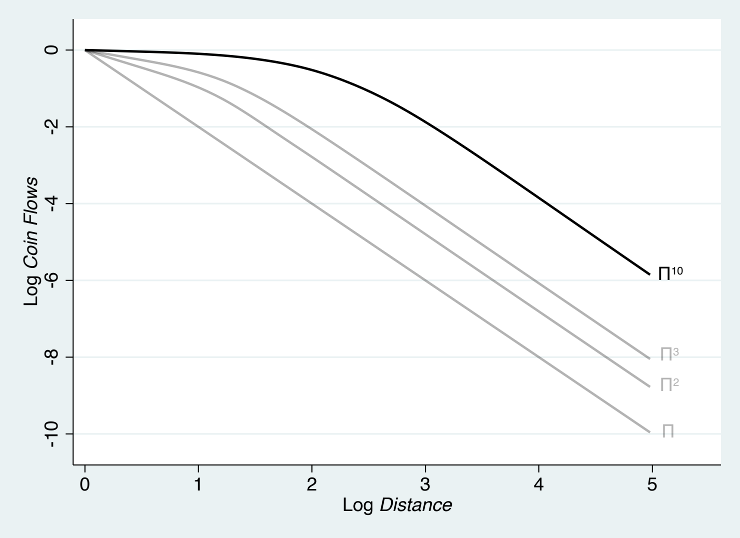
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3

Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10

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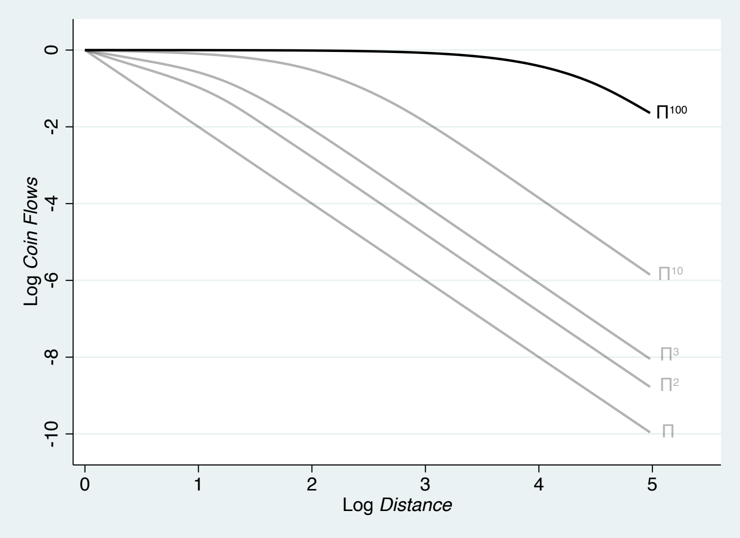
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10, age 100

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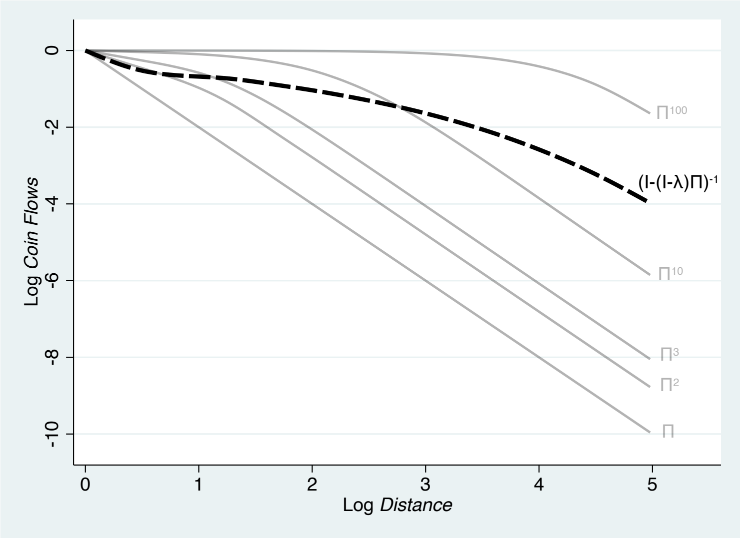
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10, age 100, all ages

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