EC442 Macroeconomics

Problem Set 1

Fall 2013

Important note: This problem set is much shorter than it seems! It just contains a lot of explanations for problem 2.

1 Solow with Investment-Specific Technological Progress

In this exercise, we work through an alternative conception of technology, which will be useful in the next chapter. Consider the basic Solow model in continuous time and suppose that $A(t) = A$, so that there is no technological progress of the usual kind. However, assume that the relationship between investment and capital accumulation is modified to

$$\dot{K}(t) = q(t) I(t) - \delta K(t),$$

where $[q(t)]_{t=0}^{\infty}$ is an exogenously given time-varying path (function). Intuitively, when $q(t)$ is high, the same investment expenditure translates into a greater increase in the capital stock. Therefore we can think of $q(t)$ as the inverse of the relative price of machinery to output. When $q(t)$ is high, machinery is relatively cheaper. It has been documented that the relative prices of durable machinery have been declining relative to output throughout the postwar era. This decline is quite plausible, especially given recent experience with the decline in the relative price of computer hardware and software. Thus we may want to suppose that $\dot{q}(t) > 0$. This exercise asks you to work through a model with this feature based on Greenwood, Hercowitz, and Krusell (1997).

1. Suppose that $\frac{\dot{q}(t)}{q(t)} = \gamma_K > 0$. Show that for a general production function, $F(K, L)$, there exists no BGP.

2. Now suppose that the production function is Cobb-Douglas, $F(K, L) = K^{\alpha} L^{1-\alpha}$, and characterize the unique BGP.

3. Show that this steady-state equilibrium does not satisfy the Kaldor fact of constant K/Y. Is this discrepancy a problem? [Hint: how is K measured in practice? How is it measured in this model?]

2 Taking the Solow Model to the Data

In this exercise we are going to solve the canonical Solow in continuous time and try to see if there is hope to explain the income distribution across countries with the model. As seen in the class: without population growth, there will be no growth in the long-run. Hence, we will not be able to see anything about the time-series behavior (after all: most countries did grow in the last 50 years). Hence, we focus on the cross-section of countries in the year 2010.

Consider the standard Solow environment. Time is continuous, the saving rate is s, production in country i is given by

$$Y_i(t) = F(K_i(t), A_i L_i)$$

where F is a neoclassical production function and A_i is a labor-augmenting technology term. Note that L_i and A_i are assumed to be constant over time but allowed to be different for different countries. Capital depreciates at rate δ. Both the saving rate and the rate of depreciation is assumed to be equal across countries.

1. State the accumulation equation for capital-per-capita $k_i(t) = \frac{K_i(t)}{L_i}$ (i.e. the equation for $\dot{k}_i(t)$) and express the level of output per worker $y_i(t)$, wages $W_i(t)$ and capital returns $R_i(t)$ as a function of $k_i(t)$ and A_i. How do $(y_i(t), W_i(t), R_i(t))$ depend on A_i and $k_i(t)$?

2. Derive the condition for a the steady-state capital-labor ratio k_i^*. How does k_i^* depend on A_i? Taking k_i^* as a function of A_i, how does (y^*, W^*, R^*) depend on A_i?

3. Now suppose that F takes the Cobb-Douglas form, i.e.

$$Y_i(t) = F(K_i(t), A_i L_i) = K_i(t)^{\alpha} (A_i L_i)^{1-\alpha}.$$

Derive the expression for the steady state capital-labor ratio k_i^*. Derive an expression for $ln(y_i(t))$ as a function of $(ln(A_i), ln(k_i(t)))$. Derive an expression of $ln(y_i^*)$ as a function of parameters.

4. Now let us go to the data. The most important cross-country dataset are the Penn World Tables[1]. Go to this website and download the newest version of the data PWT 7.1. In particular, download the data on per-capita GDP (to be more precise "PPP Converted GDP Per Capita (Chain Series), at 2005 constant prices") in 2010 for the 190 countries in the world. You also need capital per worker k_i. As capital per worker is not available "off the shelf" and it is a little messy to construct (check Acemoglu, p.97 how to do it), I already calculated it for you. It is available in the file PWT_k.csv on the website. Capital per worker is only available for 159 countries. Hence, you need to drop the countries without this information.

   (a) To give you a rough idea about the magnitude of income differences across the world, calculate per-capita income relative to the US for the following countries: China, India, France, Vietnam Nigeria. Report your results or plot them in a graph.

   (b) Now we are going to test how well the Solow model does to explain the differences in income across the world. We still assume that $Y(t) = K_i(t)^{\alpha} (A_i L_i)^{1-\alpha}$. Let

$$ln(y_i) = \phi(A_i, ln(k_i))$$

   be the relationship between $ln(y_i)$ and $(A_i, ln(k_i))$ derived in part 3. Suppose that $\alpha = 1/3$.

      i. Assume that technologies are equal across the world, i.e. $A_i = A_j = A^{Solow}$. Hence, the variation in income across countries is fully driven the variation in the capital labor ratio. Let A^{Solow} satisfy

$$(1-\alpha) A^{Solow} = \frac{1}{N} \sum_{i=1}^{N} ln(y_i) - \alpha \frac{1}{N} \sum_{i=1}^{N} ln(k_i),$$

      where N is the number of countries. The predicted income by the model is

$$ln\left(\hat{y}_i^{Solow}\right) = \phi\left(A^{Solow}, ln(k_i)\right).$$

      Plot $ln\left(\hat{y}_i^{Solow}\right)$ against $ln(y_i)$. Does the model do a good job in predicting income differences across the world? Does it over- or underestimate the inequality across countries? What does this tell you about the assumption that $A_i = A_j$?

      ii. Now suppose that we wanted to make the model consistent with data. Given the data on $(ln(y_i), ln(k_i))_i$, find the implied values of log productivity $(ln(A_i))_i$, such that the model fits the data perfectly. Plot $ln(A_i)$ against $ln(y_i)$. How does productivity in rich countries compare the one in poor countries? Did you expect this result from your answer in part (i)?

      iii. Up to now we have taken the data on $(k_i)_i$ taken as given, i.e. we have not used the model's formula for the steady-state. The steady-state capital-labor ratio you found above follows a relationship

$$k_i^* = \psi\left(\left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}, A_i\right), \tag{1}$$

      i.e. the variation of capital across countries is informative about productivity differences. Let $ln\left(\left(\frac{s\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}\right)$ satisfy

$$ln\left(\left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}\right) = \frac{1}{N} \sum_{i=1}^{N} ln(k_i).$$

[1] http://pwt.econ.upenn.edu

Use (1) to find $\left(\tilde{A}_i\right)_i$ such that $k_i^* = k_i$, i.e. that the observed capital-labor ratios are consistent with a steady-state. Now predict per capita income by

$$ln\left(\hat{y}_i^{Full\ Model}\right) = \phi\left(\tilde{A}_i, ln\left(k_i\right)\right).$$

Plot $ln\left(\hat{y}_i^{Full\ Model}\right)$ against $ln\left(y_i\right)$. Does the model do a good job in predicting income differences across the world?

3 The Solow Model with Labor Market Policies

Consider the canonical Solow model discussed above, i.e. population and technology is constant. Output is given by

$$Y\left(t\right) = F\left(K\left(t\right), L\left(t\right)\right) = K\left(t\right)^{\alpha} L\left(t\right)^{1-\alpha}.$$

The saving rate is s and capital depreciates at rate δ. While the capital market is perfect, i.e. capital owners receive a rental rate $R\left(t\right)$ per unit of capital, the labor market is subject to a minimum wage regulation, i.e. workers are not allowed to be paid less than $\overline{w}$. If labor demand at this wage falls short of L, employment is equal to the amount of labor demanded by firms, L^d and the unemployed do not contribute to production and earn zero. Assume that $\overline{w} > (1-\alpha)\left(k^*\right)^{\alpha}$, where $k^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$ is the steady-state capital-labor ratio of the basic Solow model. Characterize the long-run evolution of capital $K\left(t\right)$, starting with some amount of physical capital $K\left(0\right)$, which satisfies $\frac{K(0)}{L} < \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}} = k^*$. [Hint: The correct result looks a bit strange at first sight, so do not think you did something wrong when you get a "weird" result. When you think about though, the result is actually not that weird.]