**EC442 Macroeconomics**

**Problem Set 1**

**Fall 2013**

# 1 Complete Market and Risk-Sharing

Consider an economy that lasts for one period and consists of two individuals $i = A, B$. There are two state of the worlds, $s = 1, 2$, where state $s$ occurs with probability $\pi_s$ (clearly $\pi_1 + \pi_2 = 1$). Let $y_s^i$ be the endowment of individual $i$ in state $s$. The endowments are given by

$$\begin{aligned} y_1^A &= \theta \\ y_2^A &= 0 \\ y_1^B &= 0 \\ y_2^B &= \theta \end{aligned}$$

The consumers preferences are given by

$$u^i(c) = -\frac{1}{\gamma_i} e^{-\gamma_i c}. \tag{1}$$

These preferences are called CARA preference (where CARA stands for constant absolute risk aversion). In particular: consumption can be negative given those preferences. Note that there are two sources of heterogeneity: The two people differ in their tastes (as $\gamma_A \neq \gamma_B$) and in their income profiles (they receive income in different states and $\pi_1 \neq \pi_2$).

1. A standard measure of risk aversion is the *coefficient of absolute risk aversion* $AR(c) = -\frac{u''(c)}{u'(c)}$. Show that CARA is not a misnomer.

2. Assume that there are complete markets in this economy. Let $\lambda^i$ be the multiplier on the budget constraint of individual $i$. Write down the maximization problem of individual $i$.

   (a) Solve for the optimal consumption level $c_s^i$ as a function of prices $p_s$, state probabilities $\pi_s$ and the multiplier on the budget constraint $\lambda^i$.

   (b) Solve for the optimal consumption level $c_s^i$ as a function the multipliers on the budget constraint $\left[\lambda^A, \lambda^B\right]$ and the aggregate endowment $Y_s = y_s^A + y_s^B$. How does $c_s^i$ depend on the state?

3. Show that the equilibrium prices satisfy
$$\frac{p_1}{p_2} = \frac{\pi_1}{\pi_2}.$$

4. We can normalize one of the prices. Hence, let us normalize $p_1 = 1$. Use the individuals' budget constraint to solve for the consumption levels $c_s^i$ only as a function of aggregate income $Y_s$ and parameters. What determines the inequality of consumption between people (i.e. $\frac{c_s^A}{c_s^B}$)? Does it depend on tastes $\gamma^i$? Does it depend on the differences in the income process? Interpret.

5. [EXTRA CREDIT] Now suppose there is a representative consumer in this economy with preferences
$$u(c) = -\frac{1}{\gamma_R} e^{-\gamma_R c}.$$
What is the level of risk-aversion $\gamma_R$ such that this representative consumer choses $c_s^R = Y_s$ at the equilibrium prices of the heterogeneous economy? Can you give an intuition for this result?

## 2 Neoclassical Growth with a Government

Consider the standard neoclassical growth model with a representative household with preferences

$$U = \int_0^\infty e^{-\rho t} u\left(c\left(t\right), g\left(t\right)\right) dt$$

where $u\left(c, g\right) = ln\left(c\right) + g$, $c$ is per-capita consumption and $g$ is per-capita government expenditure. The production function is given by $Y\left(t\right) = F\left(K\left(t\right), L\left(t\right)\right)$, which satisfies all the standard assumptions. Assume that $g\left(t\right)$ is financed by taxes on investment. In particular, the capital accumulation equation is

$$\dot{k}\left(t\right) = \left(1 - \tau\left(t\right)\right) i\left(t\right) - \delta k\left(t\right),$$

where $i\left(t\right)$ is the investment of the household and $\tau\left(t\right)$ is the tax rate at time $t$. Government expenditure is therefore given by $g\left(t\right) = \tau\left(t\right) i\left(t\right)$. The household takes the path of tax rates $\left[\tau\left(t\right)\right]_{t=0}^{\infty}$ as given.

1. Set up the individual maximization problem, and characterize consumption and investment behavior.
2. Assuming that $\lim_{t\to\infty} \tau\left(t\right) = \overline{\tau}$, characterize the steady state of the economy.
3. What value of $\overline{\tau}$ maximizes the steady-state utility of the representative household? Is this value also the tax rate that would maximize the utility level when the economy starts away from the steady state? Why or why not?

## Growth with Overlapping Generations and Borrowing Constraints

Consider the following OLG economy, where individuals live for *three* periods. The preferences of an individual born at time $t$ are given by

$$U\left(t\right) = log\left(c_Y\left(t\right)\right) + \beta log\left(c_M\left(t+1\right)\right) + \rho ln\left(c_O\left(t+2\right)\right),$$

where $c_Y\left(t\right)$ denotes the level of consumption when the individual is young, $c_M\left(t+1\right)$ when individual is middle-aged and $c_O\left(t+2\right)$ when it is old. Individuals are borne with zero assets. Their labor supply is $\left(0, 1, 0\right)$, i.e. individuals can only work when they are middle-aged. The consumer can borrow and save at interest rate $r\left(t+1\right)$. In particular, if he saves $s$ units at time $t$, he receives $\left(1 + r\left(t+1\right)\right) s$ units in period $t+1$. Let $s_Y\left(t\right)$ and $s_M\left(t+1\right)$ denote the level of savings when the consumer is young and middle-aged respectively (if $s_i\left(\tau\right) < 0$, the individual borrows at time $\tau$).

1. Let the consumers face a sequence of wages and interest rates given by $\left\{w\left(t\right), r\left(t\right)\right\}_t$. State the consumers' maximization problem and solve for $\left(c_Y\left(t\right), c_M\left(t\right), c_O\left(t\right), s_Y\left(t\right), s_M\left(t\right)\right)$ as a function of $\left\{w\left(t\right), r\left(t\right)\right\}_t$ and parameters.
2. Now suppose that young consumers face a borrowing constraint of the form
$$s_Y\left(t\right) \geq -\phi \frac{w\left(t+1\right)}{1 + r\left(t+1\right)}, \tag{2}$$
i.e. they can borrow only up to a fraction $\phi > 0$ of their present discounted value of income. Derive a condition on $\phi$ such that the borrowing constraint is binding. How does this value depend on $\left(w\left(t+1\right), r\left(t+1\right), r\left(t+2\right), \beta, \rho\right)$? Interpret.
3. *For the remainder of this exercise assume that the borrowing constraint (2) is binding.* Consider the aggregate economy at time $t$. On the production side there is a representative firm with a Cobb Douglas production function
$$Y\left(t\right) = K\left(t\right)^{\alpha} L\left(t\right)^{1-\alpha}.$$
The population is constant, i.e. each period $t$, $L$ individuals are born. Assume also that capital fully depreciates after one period (i.e. $\delta = 1$). Each period, there is a market for capital and individuals can borrow from each other. The interest rate will be equal to $r\left(t\right)$ on both markets because of arbitrage. Describe (no math needed) the interactions on the asset market, i.e. who borrows from whom and who saves in which type of assets.

4. Derive the accumulation equation for the capital labor ratio $k(t) = \frac{K(t)}{L(t)}$, i.e. express $k(t+1)$ as a function of past values and solve for the steady-state capital-labor ratio $k^*$.

5. How does the steady-state capital-labor ratio $k^*$ and the steady-state return to capital $f'(k^*)$ depend on $\phi$? What is the intuition? Now suppose that $\phi$ varies across countries (with lower values of $\phi$ corresponding to worse financial markets) and all countries are in their steady-state. Is this model consistent with data?