HOMEWORK 3

Due: Lucia

1. "I wonder", by Sixto Rodriguez (from the movie Searching for Sugar Man), was the song played during class, outside in the street, on Tuesday—a great song, and a great movie. Anyhow: "I wonder" if you know when the homework is due?

2. Consider a matching model like the Pissarides-85 model but with the following change: there are two kinds of workers, high-skilled and low-skilled. A high-skilled worker produces p_h when matched with a firm and a low-skilled worker produces $p_l < p_h$. All workers have the same utility flow at home.

   There are two possible labor-market settings here:

   - There is one frictional labor market for each kind of worker: low-skilled workers search in one and high-skilled workers search in the other, and firms are free to enter in either one (vacancy-posting costs are identical across markets and so are the matching functions). This case corresponds to the assumption that firms can identify skill ex ante, without meeting the worker (i.e., the skill is like an easily verifiable university degree).
   - All workers match in the same friction-ridden pool. I.e., firms may either bump into a high-skilled or a low-skilled worker.

   (a) Define and characterize steady-state equilibria in each case. You don't need to solve explicitly for tightness but you need to derive equations determining it. In the second market arrangement, assume that the parameters are such that the surplus is positive in all matches (it could be otherwise that the surplus is negative for matches between the firm and the low-skill worker: it would be better to separate so that the firm can later match with a more productive worker).

   (b) Compare outcomes and interpret. Are wages/unemployment the same? If not, why not?

   (c) In the data, unemployment rates are higher for workers with low education. Can the model(s) here be used to account for this fact?

3. Using the Pissarides-85 model (with a Cobb-Douglas matching function), analyze the effects of a per-capita subsidy to all employed workers financed by a(n equal) tax on all workers, employed as well as unemployed (thus, employed workers both receive a subsidy and pay a tax, though the net is a subsidy). The government budget has to balance in each instant.

(a) How are the steady-state levels of θ and u determined? Derive the equations that determine them and discuss how the subsidy influences the outcome qualitatively. Interpret.

(b) Are the transitional dynamics are as simple in this model as they are when is zero? Defend your answer.

4. Consider the Pissarides-85 model again and assume that the matching function is $Au^{\alpha}v^{1-\alpha}$. Determine the efficiency properties of the model for the two cases $\alpha = \beta = 0$ and $\alpha = \beta = 1$ and interpret the results intuitively.

5. Consider a continuous-time search-matching model with the following features.

   - There are men and women: men search for women and vice versa. They both live forever, have linear utility, and discount at a net rate r.
   - When a man and a woman meet, a "love shock", ℓ, is realized; love is assumed mutual and symmetric and is measured in utils. I.e., when ℓ is high, both parties realize that marrying (forming a match and remaining together) would be beneficial because they would both receive flow utility ℓ during the duration of the marriage.
   - The love shock is drawn from an exogenous distribution $F(\ell)$.
   - Marriages are (exogenously) broken up at a rate σ.
   - Unmarried women and men meet someone of the opposite sex with flow probability $\lambda(u)$, where u is the (steady-state) fraction of men who are unmarried (and of women who are unmarried).
   - An unmarried man or woman experiences flow utility s.
   - There is a matching function mapping the number of searchers on each side of the market into a number of formed matches. Let this function be denoted $M(u)$, using the symmetry assumption. Hence, the probability that a man meets a woman (and vice versa) is $\lambda(u) = M(u)/u$.

   Because women and men are treated symmetrically here, you do not state separate equations below for the two sexes. Also, assume that we are in a steady state.

   (a) State the flow value equation for a married individual with love outcome ℓ; use $V(\ell)$ to denote the value of this married individual and U the value of an unmarried individual.

   (b) State the flow value equation for an unmarried individual.

   (c) State an equation, using the value functions, defining the cutoff love value, ℓ^*, making a man and a woman indifferent between marrying and not marrying.

   (d) Use the equations above to derive an equation, denoted (A), that, given any value for the (endogenous) variable u, determines ℓ^*. (There should be no other endogenous variables or functions in your equation.)

(e) State another equation, denoted (B), that, given any value for the cutoff ℓ^*, determines the steady-state level of u. (There should be no other endogenous variables or functions in your equation.)

(f) What can be said about the function $\lambda(u)$ if the matching function has (i) increasing returns to scale, (ii) decreasing returns to scale, and (iii) constant returns to scale? I.e., how does λ vary with u in each of these cases?

(g) Prove, based on equations (A) and (B), that if (i) the matching function has decreasing or constant returns to scale and (ii) there is a steady-state equilibrium in the marriage market, then this steady-state equilibrium is unique.

(h) Argue (but do not formally prove), based on equations (A) and (B), that there may be multiple equilibria in the marriage market when the matching function has sufficiently strong increasing returns to scale. Give intuition for how this outcome would be possible.

(i) Suppose that $s = 0$ and that r is 0.04 (a yearly rate). Suppose, furthermore, that in this economy, we observe that the probability that an individual who is unmarried in year t remains unmarried in year $t + 1$ is 0.2. Suppose that we also observe that the probability that a married couple breaks up, from year t to year $t + 1$, is 0.04. Use this information to compute the average percentage "gain from marriage", defined as how many percent higher the average flow happiness (as computed by the love value) of a married individual is than that of an individual whose marriage is borderline, i.e., an individual with love outcome ℓ^*. Suggest a change in the assumptions underlying this matching model that would raise this number.