$$q\left(\frac{1}{\theta_H^{-1}+\theta_L^{-1}}\right)$$

2.) Fraction μ is high-skilled

$$rV = -c + \lambda \overset{q\left(\frac{v}{u_H+u_L}\right)}{\cancel{\quad\quad}}\left(\frac{u_H}{u_H+u_L}\cdot J_H + \frac{u_L}{u_H+u_L}\cdot J_L\right)$$

$$rJ_H = p_H - w_H - \sigma(J_H - V)$$

$$rJ_L = p_L - w_L - \sigma(J_L - V) \qquad \theta_H = \frac{v}{u_H}$$

$$rV = -c + \lambda\left(\frac{\theta_H^{-1}}{\theta_H^{-1}+\theta_L^{-1}}\cdot J_H + \frac{\theta_L^{-1}}{\theta_H^{-1}+\theta_L^{-1}}\cdot J_L\right) = 0$$

$$rW_i = w_i - \sigma(W_i - U_i)$$

$$rU_i = b + q\left(\frac{1}{\theta_H^{-1}+\theta_L^{-1}}\right)\cdot\frac{1}{\theta_H^{-1}+\theta_L^{-1}}\cdot\left[W_H - U_H\right] \qquad q(\tilde{\theta})\cdot\theta_i$$

$$V=0 \qquad W_i - U_i = \beta S_i = \beta(W_i - U_i + J_i)$$

$$r(S_i) = \cancel{w_i} - \sigma(W_i - U_i) - b_i - \underbrace{q(\ )\frac{1}{\theta_H^{-1}+\theta_L^{-1}}}_{\lambda(\theta)}\left[W_i - U_i\right]$$

$$+ p_i - \cancel{w_i} - \sigma(J_i - V)$$

$$= p_i - b_i - \sigma(W_i + J_i - U_i) - \left[q(\ )\cdot -\right]\cdot\beta S_i =$$

$$= p_i - b_i - (\sigma + \beta\lambda_w(\theta))\, S_i$$

$$\Rightarrow S_i = \frac{p_i - b_i}{r + \sigma + \beta\lambda_w(\theta)}$$

$J_i = (1-\beta) S_i \qquad V = 0$

$$J_i = \frac{p_i - w_i}{r+\sigma}$$

$$c = q\left(\frac{1}{\theta_H^{-1}+\theta_L^{-1}}\right) \cdot \left[\ldots \frac{p_i - w_i}{r+\sigma} + \ldots \frac{p_L - w_L}{r+\sigma} \right]$$

$$= q(\,) \cdot \left[\ldots (1-\beta) \cdot S_H + \ldots (1-\beta) S_L \right] =$$

$$= q(\,)(1-\beta)\left[\frac{1}{1+\frac{\theta_H}{\theta_L}} S_H + \frac{1}{1+\frac{\theta_L}{\theta_H}} S_L \right]$$

$$= q(\,)(1-\beta)\left[\ldots \frac{p_H - b_H}{r+\sigma+\beta\lambda_W(\theta)} + \ldots \frac{p_L - b_L}{\ldots} \right] =$$

$$= q(\,)(1-\beta) \frac{1}{r+\sigma+\beta q(\theta)\cdot\frac{1}{\theta_H^{-1}+\theta_L^{-1}}} \cdot \left[\frac{\theta_H^{-1}}{\theta_H^{-1}+\theta_L^{-1}} (p_H - b_H) + [1 - \ldots](p_L - b_L) \right]$$

$$\Rightarrow \quad c \cdot \Big(r+\sigma+\underbrace{\beta q(\theta)\cdot\frac{1}{\theta_H^{-1}+\theta_L^{-1}}}_{\lambda_W(\theta)}\Big) = q(\theta)(1-\beta)\cdot[\qquad]$$

$$\Rightarrow \quad \Big(\frac{r+\sigma}{q(\theta)} + \beta \cdot \underbrace{\frac{1}{\theta_H^{-1}+\theta_L^{-1}}}_{\uparrow\,\theta}\Big) \cdot c = (1-\beta)[\qquad]$$

Wage equation with 'mixed' market tightness and surplus

In other market structure:

standard wage equation:

$$\left(\frac{r+\sigma}{q(\theta_i)} + \beta\theta_i\right)c = (1-\beta)(p_i - b)$$

q increasing in θ, hence $p_H \Rightarrow \theta_H > \tilde{\theta} > \theta_L$

when productivity is high, more vacancies will be posted,

$\Rightarrow u \downarrow$

For common market:

$$\dot{u}_H = 0 = \sigma(1-u_H) - q(\tilde{\theta})\cdot\tilde{\theta}\cdot u$$

$$\Rightarrow u_i = \frac{\sigma}{\sigma + q(\tilde{\theta})\tilde{\theta}}$$

$$\dot{u}_i = \sigma(\mu - u_i) - q(\tilde{\theta})\tilde{\theta}_i u_i = 0$$

$$\theta_i^{-1} v = u_i = \frac{\sigma\mu}{\sigma + q(\tilde{\theta})\tilde{\theta}_i}$$

Efficiency:

$$\max_{\theta} \; \underbrace{(1-u)\cdot p}_{\text{employed value}} + ub - \underbrace{u\theta c}_{\substack{\text{search costs} \\ = v\cdot c}}$$

~~s.t.~~

$$\varepsilon_\theta = -\frac{\partial f(\theta)}{\partial f}\cdot\frac{\theta}{f(\theta)}$$

$$\eta = -\frac{\partial \ln f}{\partial \ln \theta}$$

(4)

$$\max_{\theta} \int e^{-rt}\left[p(1-u) + ub - u\theta c\right] dt$$

$$\text{s.t. } \dot{u} = \lambda(1-u) - \theta\cdot f(\theta) u$$

ignore job creation condition

$$\mathcal{H} = p(1-u) + ub - u\theta c \quad - \mu_t\cdot\left[\lambda(1-u) - \theta f(\theta) u\right]$$

$$\mathcal{H}_\theta = 0 = \cancel{p(1-u) + ub} - uc - \mu_t\cdot\left[-\theta f'(\theta) - f(\theta)\right] u$$

$$= \dots + \mu u f(\theta)\cdot\left[-\eta(\theta) + 1\right]$$

$$\mathcal{H}_u = -\dot{\mu}_t + \rho\mu_t = -p + b - \theta c - \mu_t\cdot\left[-\lambda - \theta f(\theta)\right] =$$

$$= -\dot{\mu}_t + \rho\mu_t$$

$$\Rightarrow \frac{\dot{\mu}_t}{\mu_t} = \left[p - b + \theta c \cdot\right]\cdot\frac{u f(\theta)\left[1-\eta(\theta)\right]}{p(1-u) + ub - uc} + \underbrace{\left[-\lambda - \theta f(\theta)\right]}_{= -\frac{\lambda}{u}} + \rho = 0$$

$$\dot{u} = 0 \Rightarrow \lambda(1-u) - \theta f(\theta) u = 0 \Rightarrow \lambda + u(-\lambda - \theta f(\theta)) =$$

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$$\mathcal{H}_u = -\dot{\mu} + r\mu = -[p - b + \theta c] + \mu[\lambda + \theta q(\theta)]$$

$$\mathcal{H}_\theta = 0 = \cancel{\ldots} - uc + \cancel{\mu[\ldots]} \; \mu u q(\theta)[1-\eta(\theta)]$$

$$\Rightarrow \cancel{u}c = \mu \cancel{u} q(\theta)[1-\eta(\theta)]$$

$$\mu = \frac{c}{q(\theta)} \cdot [1-\eta(\theta)]^{-1}$$

$$\Rightarrow \dot{\mu} = 0 \Rightarrow p - b + \theta c = \frac{c}{q(\theta)} \cdot [\lambda + \theta q(\theta) - r][1-\eta(\theta)]^{-1}$$

$$\Rightarrow (1-\eta(\theta))(p-b) + \theta c(1-\eta(\theta)) = \frac{c}{q(\theta)}[\lambda - r + \theta q(\theta)]$$

$$(1-\eta(\theta))(p-b) - \frac{c[\lambda - r + \cancel{\theta q(\theta)} - \cancel{\theta q(\theta)} + \eta(\theta) q(\theta) \theta]}{q(\theta)} = 0$$

social optimality condition

Private optimality condition:

JC curve: $p - w + \frac{(r+\lambda) \cdot c}{q(\theta)} = 0$

wage equation: $w = (1-\beta)z + \beta(1 + c\theta)$

$$\Rightarrow (1-\beta)(p-z) + \frac{(r+\lambda+\beta\theta q(\theta))c}{q(\theta)} = 0$$

$\Rightarrow$ private optimality - social optimality iff

$\beta = \eta(\theta)$ (Hosios condition)

Intuition for Hosios condition:

search externality on both sides:

~~[illegible]~~

if $\alpha = \beta = 0$: fixed number of matches,

P of ~~[illegible]~~ match (for v) is decreasing in θ

=> weight on worker in bargaining has to be zero.

Expected duration of vacancy :

$$\cancel{[illegible]}\ [q(\theta)]^{-1} = \left[A \cdot \left(\frac{u}{v}\right)^{\alpha}\right]^{-1} = A^{-1}$$

Number of matches: $A \cdot v$

Worker's search externality is pretty bad.

=> give him less of the pie to discourage search

if $\alpha = \beta = 1$: $\Pi = A \cdot u$

firm search externality is bad

=> 'tax' firms by giving the workers a lot

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(5) (a) $rV(\ell) = \ell - \sigma(V(\ell) - \mathcal{U})$

(b) $r\mathcal{U} = s + \lambda(u) \cdot \int_{\ell^*}^{\infty} (V(\ell') - \mathcal{U})\, dF(\ell')$

(c) $V(\ell^*) = \mathcal{U}$

(d) $(r+\sigma)\left[V(\ell) - \mathcal{U}\right] = \ell^* - r\mathcal{U}$

$$V(\ell) - \mathcal{U} = \frac{\ell^* - r\mathcal{U}}{r+\sigma}$$

$$\Rightarrow r\mathcal{U} = s + \frac{\lambda(u)}{r+\sigma} \left[\int_{\ell^*}^{\infty} \ell' dF(\ell') - r(1-F(\ell^*))\mathcal{U} \right]$$

$$\left[r(r+\sigma) + r\lambda(u) \cdot (1-F(\ell^*))\right] \mathcal{U} = s \cdot (r+\sigma) + \lambda(u) \int_{\ell^*}^{\infty} \ell' dF(\ell')$$

From (a) and (c)

$$r \cdot V(\ell^*) = \ell^* \Rightarrow r \cdot \mathcal{U} = \ell^*$$

(A) $\rightarrow \left[r(r+\sigma) + \lambda(u) \cdot (1-F(\ell^*))\right] \cancel{r} \cdot \ell^* = s(r+\sigma) + \lambda(u) \int_{\ell^*}^{\infty} \ell' dF(\ell')$

(e) $\dot{u} = \sigma \cdot (1-u) + \lambda(u)(1-F(\ell^*)) \cdot u = 0$

(f) $\lambda(u) = \frac{M(u)}{u}$

$M(u)$ IRS $\Rightarrow M(u) > M'(u) \cdot u$
CRS $M(u) = M'(u) u$
DRS $M(u) < M'(u) u$

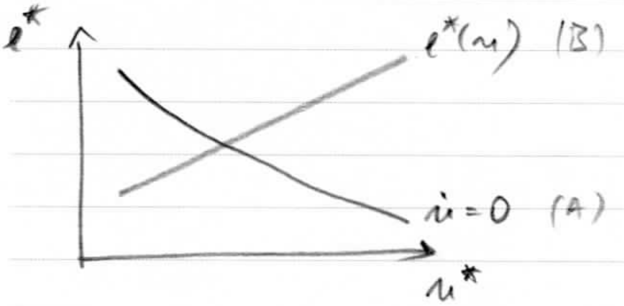
$$\rightarrow \lambda'(u) = \frac{1}{u^2}\left[M'(u) \cdot u - M(u) \right] \quad \begin{matrix} > 0 & \text{IRS} \\ = 0 & \text{CRS} \\ < 0 & \text{DRS} \end{matrix}$$

(g) $\sigma(1-u) + \lambda(u)\left(1-F(\ell^*)\right)\cdot u = 0$

$$1-F(\ell^*) = \frac{\sigma(u-1)}{\lambda(u)\cdot u}$$

increasing

$$F(\ell^*) = 1 - \frac{\sigma(u-1)}{\lambda(u)\cdot u} = 1 - \frac{\sigma(1-\frac{1}{u})}{\lambda(u)}$$



decreasing in u

$F(\ell^*)$ decreasing in u

$\Rightarrow \ell^*$ decreasing in u

B: $\cancel{\sigma}(r+\sigma)\ell^* + \lambda(u)\left(1-F(\ell^*)\right)\cdot\cancel{f}\cdot\ell^* = s(r+\sigma) + \lambda(u)\cdot\int$

$$\Rightarrow \lambda(u)\left[(1-F)\cancel{f}\ell^* - \cancel{\quad}\int\right] = (r+\sigma)\ell^* + \text{const}$$

$$\Rightarrow \lambda(u) = \frac{(r+\sigma)\cancel{\ell^*} + \text{const}/\ell^*}{\left(1-F(\ell^*)\right)\cancel{\ell^*} - \int_{\ell^*}^{\infty}\ell' dF(\ell)/\ell^*}$$

Numerator ↘ in ℓ^*

Denominator: $-f(\ell^*) - \left[\frac{\ell^*\cdot(-\ell^* f(\ell^*)) - [\int]\cdot 1}{\ell^{*2}}\right] =$

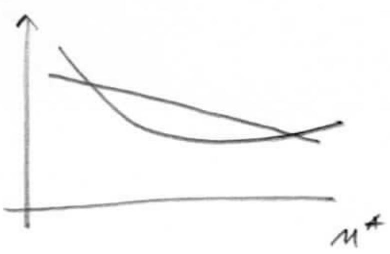
$$= -f(\ell^*) - \left[\frac{-f(\ell^*) - \int_{\ell^*}\ell' f(\ell')d\ell}{\ell^{*2}}\right]$$

$$= + \frac{\int_{\ell^*}^{\infty}\ell' f(\ell')d\ell}{\ell^{*2}} > 0 \quad \text{increasing in } \ell^*$$

λ decreasing in u $\Rightarrow$ u increasing in ℓ^*

$\Rightarrow$ unique equilibrium

(h) ℓ^*



(i) $s = 0$, $r = 0{,}04$

$\lambda(u)\ (1 - F(\ell^*)) = 0{,}8$

$\sigma = 0{,}04$

$$\left[\cancel{0{,}04} \cdot (0{,}08) + 0{,}8 \right] \frac{1}{0{,}04} \cdot \ell^* = 0{,}8 \cdot \underbrace{\frac{1}{1 - F(\ell^*)} \int_{\ell^*}^{\infty} \ell' \, dF(\ell')}_{= \mathbb{E}_\ell (\ell \mid \ell \geq \ell^*)}$$

$\Rightarrow$

$$\frac{\mathbb{E}_\ell (\ell \mid \ell \geq \ell^*)}{\ell^*} = \frac{0{,}08 + \cancel{20}\ 0{,}8}{0{,}8} = 1{,}1$$

$\Rightarrow$ 10% higher