EC442 Macro for MRes Students, MT 2013

Prof. Per Krusell

Solutions for Homework 5

Based on Georg Graetz's solutions

**Question 1**

(a)

The FOCs imply that both consumption and labor supply are constant over time, in particular

$$\psi c = w(1-l).$$

The life-time budget constraint is

$$a_0 = \frac{1}{1-\beta}(c - wl),$$

so given $a_0 = 0$, we have $l = 1/(1+\psi)$ and $c = wl$.

(b)

Let $\lambda_t$ be the Lagrange multiplier of the budget constraint, then the intra-temporal optimality condition is

$$\psi = \lambda_t w_t (1 - l_t).$$

Log-differentiating with respect to the wage, keeping $\lambda_t$ constant, yields

$$\frac{1}{1-l_t}\frac{\partial l_t}{\partial w_t} = \frac{1}{w_t},$$

hence the Frisch elasticity is

$$\varepsilon \equiv \frac{w_t}{l_t}\frac{\partial l_t}{\partial w_t} = \frac{1-l_t}{l_t}.$$

Evaluated at the steady-state with $l = 1/3$, the Frisch elasticity equals 2. Holding the marginal utility of wealth, and hence consumption $c_t$ constant, the intra-temporal optimality condition then says that a one-percent increase in the wage must be matched by a one-percent decrease in leisure. Since leisure is twice as much as time spent working, the labor supply increases by two percent.

Notice that the parameter $\psi$ here equals the Frisch elasticity at the steady state, but in general the Frisch elasticity is not constant for this utility function.

## Question 2

(a)

$$\max_{\{c_t,k_t,l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

$$\text{s.t. } c_t + k_{t+1} = A_t k_t^{\alpha} l_t^{1-\alpha}, \; k_0 \text{ given.}$$

(b)
INTRA

$$B l_t^{\psi} = (1-\alpha)\frac{y_t}{c_t}$$

Euler

$$\frac{1}{c_t} = \frac{\alpha\beta}{c_{t+1}} \frac{y_{t+1}}{k_{t+1}}$$

(c)
Regardless of what the (deterministic) time path of TFP looks like, the constant saving rate $\alpha\beta$ is optimal, as can be verified by plugging this solution into the above FOCs. A complete proof would involve checking that the transversality condition is satisfied. This is sufficient because the problem is "well-behaved" and thus the policy function is unique. The constant level of labor supply is given by $l = [(1-\alpha)/(B(1-\alpha\beta))]^{1/\psi}$.

Unlike in the previous exercise, here changes in consumption are proportional to changes in income, while the labor supply does not respond at all. This is because of the large changes in permanent income, which are due to big movements of the interest rate induced by fluctuations in technology. This mechanism depends crucially on the assumption of full depreciation.

## Question 3

(a)
The planner decides how much consumption to give to employed and unemployed agents, and how many agents should be employed, taking into account the resource constraint.

$$\max_{c_e,c_u,L} L(\log c_e - \chi) + (1-L)\log c_u$$

$$\text{s.t. } Lc_e + (1-L)c_u \leq K^{\alpha} L^{1-\alpha}, \; K \text{ given.}$$

(b)
The FOCs yield $c_e = c_u = 1/\lambda$ and $\chi/\lambda = (1-\alpha)K^{\alpha}L^{-\alpha}$.

(c)
By the resource constraint, $1/\lambda = K^{\alpha}L^{1-\alpha}$, hence $L = (1-\alpha)/\chi$.

(d)

In the decentralized equilibrium, the wage equals the marginal product of labor. The gain in utility is the wage times the marginal utility of consumption. The marginal cost is $\chi$. These two are equal in equilibrium.

(e)

The marginal product of labor equals the wage, $w(j) = (1-\alpha)K^{\alpha}L(j)^{-\alpha}$.

(f)

The union solves

$$\max_{c(j),L(j)} \log c(j) - \chi L(j)$$

$$\text{s.t. } c(j) \leq w(j)L(j) + rK = (1-\alpha)K^{\alpha}L(j)^{1-\alpha} + rK.$$

This yields $L(j) = (1-\alpha)^2/\chi$, which is lower than in the competitive equilibrium. Therefore consumption is also lower.

(g)

Since consumption and labor are lower than before, the marginal utility and the wage are higher, while the marginal cost of working is the same. Now the marginal benefit of working outweighs the marginal cost, so the non-working agents would like to work, and in this sense they can be considered unemployed. (It can be shown that the marginal benefit equals $\chi/(1-\alpha) > \chi$.)