HOMEWORK 1

Due: Tuesday, February 21

1. Consider an economy with an infinitely lived consumer with preferences

$$\sum_{t=0}^{\infty} \beta^t \left(\log c_t + \psi \log(1 - l_t)\right)$$

and budget constraint

$$c_t + \beta a_{t+1} = a_t + w_t l_t$$

for all t, with $a_0 = 0$ and no restrictions on borrowing.

   (a) Consider, initially, the case $w_t = w$ for all t. Solve this problem fully for the sequences of consumption and leisure as a function of the exogenously given parameters.

   (b) Consider the possibility that some w_t departs from w by a small amount. What is the Frisch (λ-constant) elasticity of labor substitution in this model if the parameters of the model are calibrated so that l_t is equal to 1/3 (one third of the available time is spent working)? How can you use the answer to find how l_t will react to the small change in w_t? Explain.

2. Consider an economy with an infinitely lived representative consumer with preferences

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

where c_t is consumption and l_t is time worked, both at time t, where we assume that $u(c, l) = \log c - B\frac{l^{\psi}}{\psi}$. The resource constraints for all t in the economy are

$$c_t + k_{t+1} = A_t k_t^{\alpha} l_t^{1-\alpha},$$

where k_t is capital (which depreciates fully after use), l_t is labor input, and A_t is an exogenous productivity variable that may vary with t.

   (a) State the social planner's problem for this economy as a dynamic optimization problem. Be clear to specify which variables are choice variables and which are not.

   (b) Derive first-order conditions for this problem.

   (c) Now assume that A_t is moving deterministically: in even periods it is high, $\bar{A}$, and in odd periods it is $\underline{A} < \bar{A}$. Show that the equilibrium allocation is very simple: the rate of saving is constant and hours worked are constant as well. Solve for these constants in terms of the primitive parameters α, β, $\bar{A}$, $\underline{A}$, and ψ.

3. Consider a static economy with a continuum of agents $i \in [0,1]$ each of whom has a utility function

$$\log c(i) - \chi e(i)$$

where $c(i)$ is consumption of agent i, $\chi > 0$ is a parameter, and $e(i) \in \{0,1\}$ is the individual's employment level (i.e., the agent has indivisible labor and experiences disutility χ if and only if he works). Total output in this economy is

$$K^{\alpha} L^{1-\alpha},$$

where K is capital and L is total employment (i.e., $L = \int_0^1 e(0,1)$). Capital is given (not subject to choice), and output is used only for consumption.

(a) Set up the planning problem (the planner gives equal utility weight to all consumers).

(b) Show that it is optimal to give the same consumption level to working agents as to non-working agents.

(c) Find total employment.

(d) As briefly mentioned in class, the planning outcome can be decentralized (the "lottery" model). Here, we can think of $1 - L$ as the non-participation rate—the non-working agents are not unemployed in the sense that they would (strictly) prefer to work. To show this, suppose you consider a given non-employed agent and asks if he would be willing to work at the going wage. He would then be able to consume w more, where w is the competitive wage, but he would experience a utility loss χ from the effort. If one uses the shadow value (marginal utility) of consumption to translate w into utils, how do these two effects compare?

Suppose now instead that there is a "double continuum" $[0,1]^2$ of agents (i,j) with same preferences as above. Here, j refers to a specialized skill; thus, there is a continuum of specialized skills. These are used as imperfect substitutes in aggregate production, which now reads

$$K^{\alpha} \int_0^1 L(j)^{1-\alpha} dj,$$

where $L(j)$ is now total employment in "sector" j (i.e., this is the fraction of workers with skill j who are employed). One could again solve a planning problem for this economy, and it could be implemented as a competitive lottery equilibrium. It is easy to show that it would deliver the same consumption and employment as above (and sectoral outcomes are symmetric).

Now instead consider a monopolistic-competition decentralization where workers in each sector are unionized. The representative firm rents capital from consumers (all consumers are endowed with K units) at rate r and buys specialized labor from each sector j at wage $w(j)$. The union in sector j maximizes welfare of the sector-j workers by choosing how many of them should work and has budget constraint

$c(j) = w(j)L(j) + rK$. In this maximization problem the union internalizes the effect of its employment choice on the wage. Because it is small, it takes the behavior of other unions as given; in particular, it takes r as given.

(e) Set up the profit maximization of a firm and derive the inverse demand function for sector-j labor.

(f) Assuming a symmetric equilibrium, find total consumption and employment and show that they are lower than optimal.

(g) Show that it makes sense to think of the non-working people as unemployed in this case.