Research on Trade and Market Integration

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University of Geneva Job Talk
23 February 2026

Research on Trade Networks: Past, Present, and Future

New quantitative methods to study the formation & role of trade networks

- Informed by (and applied to) new empirical results

- How does trade shape growth and development?
- Applications in a range of different settings/fields:
  (a) Quantify trade and growth in historical episodes (the *past*)
  (b) Role of trade and firm networks in present-day industrial development (the *present*)
  (c) Adaptation of economic systems to environmental risk through trade (the *future*)

Common theoretical foundations, very different applications.

Back-and-forth between theory and empirics.

The *Past*: Quantifying trade and its role in historical episodes

1. “Trade and the End of Antiquity” (with T. Chaney, r&r Econometrica)

2. “Trade and the Origins of the Territorial State” (work in progress)

Outline

Trade and the End of Antiquity

Trade and the Origin of the Territorial State

Other work on Trade Networks

1. Trade and the End of Antiquity

- Antiquity: Roman and Greek civilizations centered around the Mediterranean
- "End of Antiquity": shift in economic activity away from the Mediterranean between 5th and 8th century AD
  - Rise of Northern Europe (Charlemagne, 8th c).
  - "Golden Age" of Islam

$\hookrightarrow$ *Question*: When? How? What role did trade play in this?

- Discussed, among others, by Montesquieu (1734), Voltaire (1756), Gibbon (1789)
- Henri Pirenne (1937) blames the Arab conquests and the emerging Islamic-Christian border for the rupture in the Mediterranean unity

▸ Maps: conquests

This paper: quantitatively investigate changing economic geography

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$\Rightarrow$ Use data on the movement of coins to study the changing economic geography during Late Antiquity.

- Coins are the main medium of exchange during Late Antiquity, particularly for long-distance trade $\rightarrow$ informative about trade
- Coins are well studied & documented by historians and numismatists
- Coins have features that help solve econometric identification problems

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$\Rightarrow$ we interpret coin flows through the lens of a trade model and identify

- bilateral trade flows (i, j, t)
- technology (i, t)
- trade deficits (i, t)
- real consumption per capita (i, t)

in order to better understand what's going on during Late Antiquity

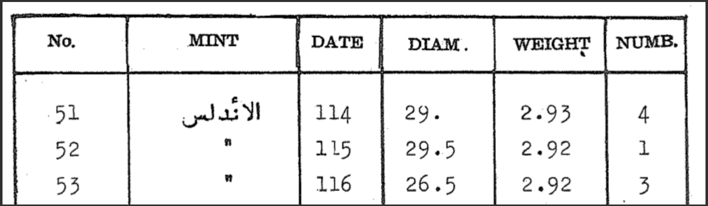
Data: Coins around the Mediterranean, AD 325 to AD 950

Assemble a large dataset of coin finds from around the Mediterranean

1. FLAME (2023) project:
   - "Framing the Late Antique and Early Medieval Economy."
   - Collaboration of $>$60 historians and numismatists.
   - $\sim$200,000 coins with complete records from $\sim$4,600 hoards
   - Pre AD 725.
2. Hand-coded records from numismatic / archaeological literature:
   - 797 coin finds.
   - 100,478 coins.
   - post AD 725.

Data covers most of published literature on hoard records (and more)

Coin hoard data: an example from al ’Ush (1972)



No.	MINT	DATE	DIAM.	WEIGHT	NUMB.
51	الأندلس	114	29.	2.93	4
52	"	115	29.5	2.92	1
53	"	116	26.5	2.92	3

Excerpt of an original publication on the Damascus silver hoard:

- record number (51)
- mint (al-Andalus) *
- mint date (year 114 of the Hijri calendar) *
- diameter (29mm)
- weight (2.93g)
- number of coins with these attributes (4) *

The issuing dynasty (Umayyad) is given in the table headings and the denomination and material (silver *dirham*) is stated in the text. * denotes required attributes

Coin dataset

- Each coin provides the following information:
  1. Mint location ("birthplace"): m
  2. Mint date ("birthdate"): τ
  3. Hoard location ("death place"): h
  4. Terminus post quem, *tpq* ("death date"): $T = \sup \tau$

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  4. Terminus post quem, *tpq* ("death date"): $T = \sup \tau$
- 286,035 coins.
- Time:
  - Mint date $>$ AD 325
  - *tpq* $<$ AD 950.
- Space:
  - Western Europe.
  - Southern Europe.
  - Northern Africa.
  - Middle East++.

Stylized facts

We document 3 main stylized facts:

1. Within hoards, older coins have travelled further.
   $\Rightarrow$ it takes time for coins to travel
2. Distance *and* political borders impede coin flows (gravity).
   $\Rightarrow$ coin flows exhibit similar characteristics as trade flows
3. The geography of coin flows changes sharply around the Arab conquests.

▶ Fact #1: diffusion

▶ Fact #2: gravity

Fact #3: Coin flows before/after the Arab conquests

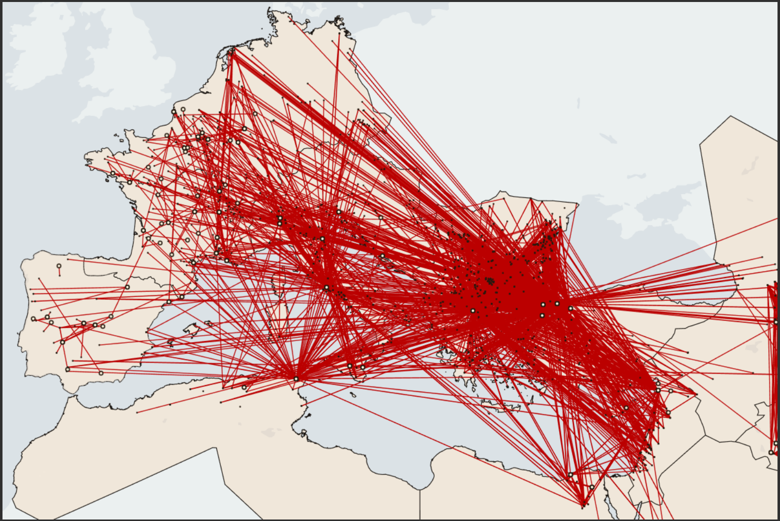

Figure 1: Before the Arab conquests: 450-630 AD

Fact #3: Coin flows before/after the Arab conquests

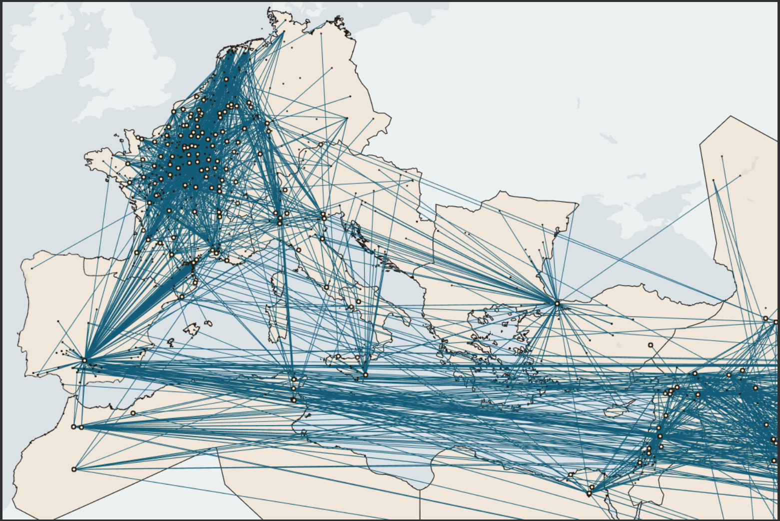

Figure 2: After the Arab conquests: 713-900 AD

▸ Cross-med: reg

▸ Cross-med: plot

Model

Objective:

Quantify impact of trade barriers (and other changes) on market access / trade / welfare

Approach:

Dynamic model of trade flows (gravity), where coins diffuse alongside trade and are thus informative about trade flows.

Key Assumption:

Traders are blind to coin *types* (mint location and date).

$\Rightarrow$ Coins diffuse in proportion to trade flows

$\Rightarrow$ Recover trade *flows* from stocks of coin *types* in hords in different locations/time periods.

Not used/needed for identification: total *quantities* of coins found

Long version

Model components (simplified case: exogenous saving)

N locations with labor endowments L_n, Ricardian trade as in Eaton and Kortum (2002)

- Households finance consumption expenditure using saving $S_n(t-1)$, exogenous newly minted coins $M_n(t)$; end-of-period income is saving for next period
- Goods market clearing, denominated in coins

$$\overbrace{w_i L_i\,[t]}^{income_i[t]} = \sum_n \pi_{ni}\,[t] \overbrace{\Big((1-\lambda)\,w_n L_n\,[t-1] + M_n\,[t]\Big)}^{expenditure_n[t]}, \forall i, t \qquad (1)$$

$$\Leftrightarrow S_i\,[t] = \sum_n \pi_{ni}\,[t] \Big((1-\lambda)\,S_n\,[t-1] + M_n\,[t]\Big), \forall i, t \qquad (2)$$

w_i: wages; L_i: labor; π_{ni}: expenditure shares; λ: coin loss; M_n: minting; S_i: coin stocks

- Fraction π_{ni} of n's expenditure allocated to goods from i

$$\pi_{ni}\,[t] = \frac{\Big(T_i\,[t]\,w_i^{-\theta}\,[t]\Big)\,(d_{ni}\,[t])^{-\theta}}{\sum_k \Big(T_k\,[t]\,w_k^{-\theta}\,[t]\Big)\,(d_{nk}\,[t])^{-\theta}}, \forall i, n, t \qquad (3)$$

T_i: technology; d_{ni}: trade cost; θ: trade elasticity

Composition of coin stocks

- Stock $S_i\left[T\right]$ composed of different coin types, $S_i\left[T\right] = \sum_{m=1}^{N} \sum_{t \leq T} S_{mi}\left[t, T\right]$
- Coins start their 'coin life' when they are minted, $S_{mm}\left[t, t\right] = M_m\left[t\right]$
- Then stocks evolves recursively,

$$S_{mi}\left[t, \tau\right] = (1 - \lambda) \sum_{n=1}^{N} \pi_{ni}\left[\tau\right] S_{mn}\left[t, \tau - 1\right] \tag{4}$$

- Recursive solution in matrix form (coin origin $\times$ coin destination),

$$\mathbf{S}\left[t, T\right] \tag{4'}$$

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- Recursive solution in matrix form (coin origin $\times$ coin destination),

$$\mathbf{S}[t, t+2] = (1 - \lambda)^2 \boldsymbol{M}[t] \boldsymbol{\Pi}[t+1] \boldsymbol{\Pi}[t+2] \tag{4'}$$

Composition of coin stocks

- Stock $S_i\,[T]$ composed of different coin types, $S_i\,[T] = \sum_{m=1}^{N} \sum_{t \leq T} S_{mi}\,[t,\,T]$
- Coins start their 'coin life' when they are minted, $S_{mm}\,[t,\,t] = M_m\,[t]$
- Then stocks evolves recursively,

$$S_{mi}\,[t,\tau] = (1-\lambda) \sum_{n=1}^{N} \pi_{ni}\,[\tau]\, S_{mn}\,[t,\tau-1] \tag{4}$$

- Recursive solution in matrix form (coin origin $\times$ coin destination),

$$\mathbf{S}\,[t,\,T] \;=\; (1-\lambda)^{\,T-t} \boldsymbol{M}\,[t]\,\boldsymbol{\Pi}\,[t+1]\,\boldsymbol{\Pi}\,[t+2] \cdots \boldsymbol{\Pi}\,[T] \tag{4'}$$

Taking the model to the data

- 13 regions around the Mediterranean ▸ details
- 20-year time intervals
- Assume constant λ and estimate as exponential decay parameter in within-hoard age distribution:

$$\hat{\lambda}_{20y} = 0.301$$

  (or 1.7% per year)

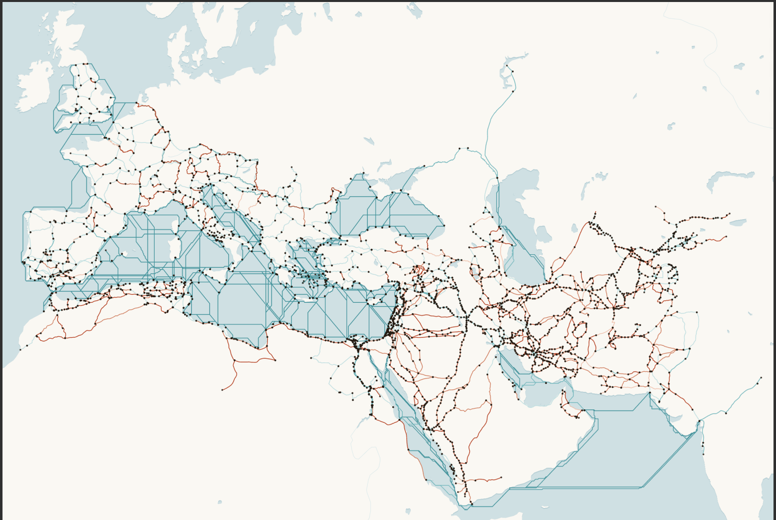
Parameterize trade frictions:

$$d_{ni}(t)^{\theta} = \exp\left(\gamma_0 + \zeta \ln\left(\text{TravelTime}_{ni}\right) + \kappa_1 \text{PoliticalBorder}_{ni}(t) + \kappa_2 \text{ReligiousBorder}_{ni}(t)\right)$$

if $n \neq i$ and $d_{nn}(t) = 1$.

- Full model contains a consumption/saving choice; set $\overline{s_n}(t) = 1.5\%$ (Scheidel, 2020).
- For counterfactuals, assume $\theta = 4$ (Simonovska and Waugh, 2014).

Trade costs *only* depend on travel times (and politics/religion)



Note: Combined geospatial models from Orbis (Scheidel, 2015) and al-Turayya (Romanov and Seydi, 2022). ▸ al-Maqdisi

Maximum likelihood estimation

Assume coins in our data are a random sample of coin types in each location $\times$ time.

- Multinomial distribution of coin types,

$$P\left(\cdots, X_i^{(m,\tau)}(T) = x_i^{(m,\tau)}, \ldots\right) = \frac{N_i(T)!}{\prod_{(m,\tau)} x_i^{(m,\tau)}!} \prod_{(m,\tau)} \left[p_i^{(m,\tau)}(T)\right]^{x_i^{(m,\tau)}}$$

  with the probability of drawing a coin of type (m, τ),

$$p_i^{(m,\tau)}(T) = \frac{S_i^{(m,\tau)}(T)}{\sum_{(m',\tau')} S_i^{(m',\tau')}(T)} = \frac{S_i^{(m,\tau)}(T)}{S_i(T)}.$$

- Likelihood of observing a sample of coins given parameters $\boldsymbol{\theta}$,

$$\ell(X; \boldsymbol{\theta}) = \sum_i \sum_T \sum_{(m,\tau)} x_i^{(m,\tau)} \left[\log S_i^{(m,\tau)}(T; \boldsymbol{\theta}) - \log S_i(T; \boldsymbol{\theta})\right] + \text{constant}$$

Estimation results: Determinants of trade costs

$$\ln\left((d_{ni}[t])^{-\theta}\right) = constant - \underset{(0.02)}{2.98}\ln\left(TravelTime_{ni}\right) - \underset{(0.02)}{0.3}\; PoliticalBorder_{ni}[t] - \underset{(0.11)}{4.05}\; ReligiousBorder_{ni}[t]$$

- Travel time elasticity similar to estimates on ancient trade.
  Roman trade: Flückiger et al. (2022); Bronze Age trade: Barjamovic et al. (2019).
- Political border tax: 8%
  (with $\theta = 4$, $d_{between}/d_{within} = e^{0.3/4} \approx 1.08$)
- Religious border tax: 175%
  (with $\theta = 4$, $d_{between}/d_{within} = e^{4.05/4} \approx 2.75$)
- Anderson and van Wincoop (2003) US-Canada border tax: 49%
  (with $\theta = 4$, $d_{between}/d_{within} = e^{1.59/4} \approx 1.49$)

Detailed estimates

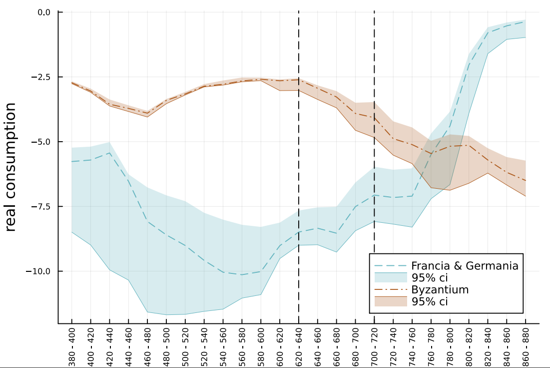
Byzantine Heartlands versus Francia & Germania

Byzantium:

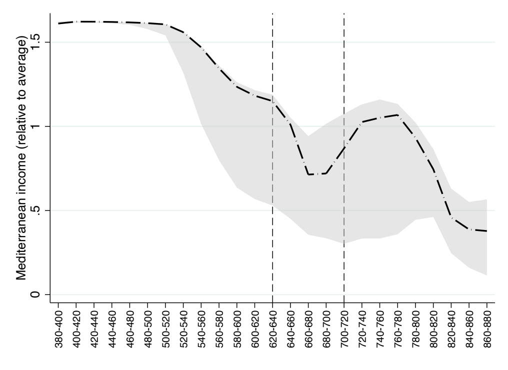
- Stable and wealthy, late 4^{th}–early 7^{th} c.
- Sustained by trade openness and trade deficit (excess minting)
- From early 7^{th} c.: gradual collapse
- Driven by collapse of trade, decline in minting, fall in production capacity

Francia & Germania:

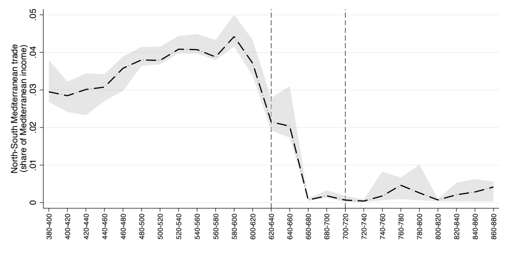
- Steep decline during fall of West Roman Empire
- From early 7^{th} c.: spectacular growth
- One of wealthiest regions by 8^{th} c.



The economic decline of the Mediterranean



Mediterranean income (relative)



North-south Mediterranean trade

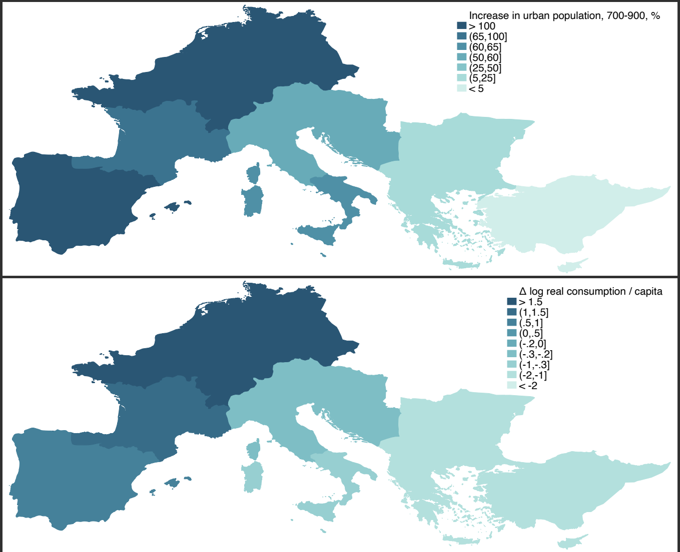
- Mediterranean regions: from 50% *larger* to 50% *smaller* than average
- Decline starts $>$1 century before the birth of Islam
- North-south Mediterranean trade collapses from $\sim$4% to $<$1% after late 7^{th} c.
- Still, contribution of trade collapse to welfare is small for most regions (ACR)

Compare to relative urbanization rates, 700–900 AD

Top: Change in total urban population (urban: > 1k inhabitants), data from Buringh (2021)

Bottom: $\Delta X_n / P_n$

Urbanization and real consumption changes are correlated



Conclusion

Main contribution: framework to estimate trade flows and relative growth for early periods

$\rightarrow$ Book project to expand scope

$\rightarrow$ SNF/FNS grant to develop quantitative estimates for trade and growth for pre-industrial times

Findings:

- Large decline of mediterranean trade following Arab conquests
- But relative decline of mediterranean regions started earlier
- Welfare impacts of trade are swamped by other determinants (tech. change)

Outline

Trade and the End of Antiquity

Trade and the Origin of the Territorial State

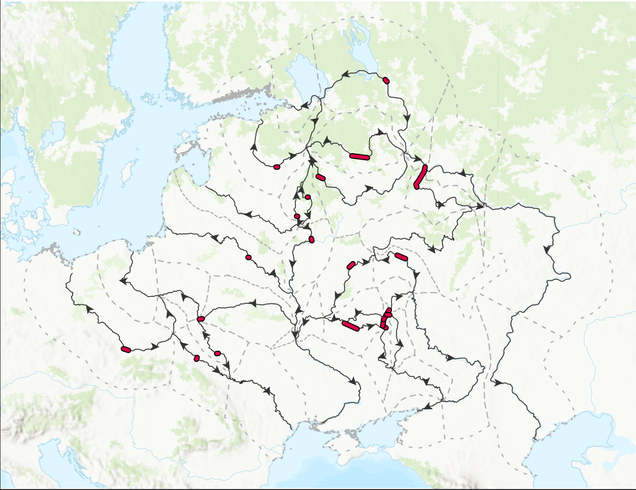
Other work on Trade Networks

How does trade contribute to political change?

Study the role of trade in the formation of early territorial states in 9th/10th century Eastern Europe.

- No formal states (= division of labor, bureaucracy) until 10th century, just tribes
- Eastern Europe was at the center of a long-distance slave trade: slaves & forest products against Arab silver (coins)
- Redirection of trade flows in the 10th century led to the emergence of territorial states:
  - Poland under Piast dynasty ($\sim$ 940s)
  - Volga Bulgaria in Tatarstan ($\sim$ 940s)

Trade routes in Eastern Europe, 9th-10th century



Known trade routes

- over land (in red)
- along rivers (solid black, w. →)

Based on Leontiev and Nosov (2012), Tentiuc (2020), and Channon and Hudson (1995)

Political actors in Eastern Europe, 10th century

- Khazar Khaganate (6th century – 969 AD)
- Volga Bulgaria (8th/9th c – 1236 AD, independence from Khazars in 940s)
- Rus tribes (Ladoga $\sim$ 750, Novgorod $\sim$ 850, conquest of Kyiv $\sim$ 865) / Kyivan Rus (870s onwards)
- Poland (940s)

Around 940 AD: Formation of Polish state under Piast dynasty (Mieszko I.)

Ibn Yaqub writes, around 965:

> *“Regarding the land of Mieszko, this is the largest of the Slavs’ lands. It is abundant with food – meat, honey and crops. He collects taxes in the form of [al–mathaqil al–marqktija] (silver, by weight). and they are used to pay the monthly salaries of his men [warriors], each of whom receives a fixed number. He has three thousand armoured men and they are divided into military units. One hundred of them is equal to a thousand other [warriors]. He gives them clothing, horses, weapons and everything else they need.” [transl., after Kowalski, 2022]*

Hence:

1. Taxation of trade flows on rivers (only source of silver)
2. Silver used to pay warriors
3. Archaeological evidence shows that Poland was heavily and actively involved in the slave trade as well (Adamczyk, 2014, Roach, 2020, Biermann, 2021)

Overview of preliminary results

Use data on coin hoards, 700 – 1000 AD.

1. **Trade causally increases the probability of state formation**

$$(\text{State present})_{it} = \underbrace{\beta}_{<0} \underbrace{(\text{Age of coins in hoards})_{it}}_{\text{Exports}} + \alpha_i + \alpha_t + \varepsilon_{it} \tag{5}$$

   IV's for Exports: changes in market access (Rus-Khazar conflict, shifts in Arab silver coin production)

2. **Trade *shapes* the territorial state**
   Polish border reflects the returns from taxing trade (along rivers) and the risk of expropriation by Vikings
   Model of trade and the spatial extent of the state to quantify forces

First Polish state under Mieszko I

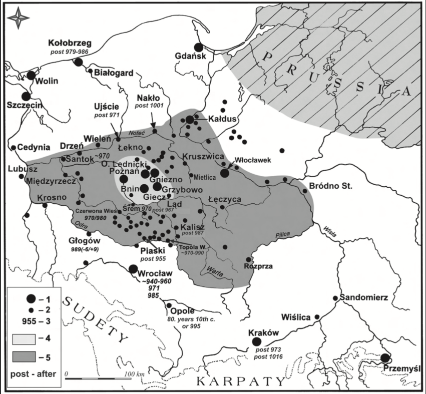


Figure 3: Poland, ca. 960-992, from Kara (2020)

1. Boundary of Poland extends to the rivers, but rarely beyond
   Reach river to tax trade flows
2. State developed away from the sea border
   Sea not taxable. Risk of Viking attacks higher
3. The capital of the Polish state is in the “middle” of the river routes
   Capital location is chosen to maximize consumption
4. State extends further in the south-east
   Cheaper to sell slaves in the south-east (proximity to Arabs)

Outline

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Other work on Trade Networks

The *Present*: Trade Networks in Industrial Development

Research agenda on *frictions in firm-to-firm trade networks*:

1. "The Impact of Contract Enforcement Costs on Value Chains and Aggregate Productivity" (JMP, ReStat 2022)
2. "Misallocation in the Market for Inputs" (with E. Oberfield, QJE 2020)
    - Static distortions from imperfect contract enforcement lead to welfare losses of 0% - 10%.

3. "The Network Origins of Firm Dynamics: Contracting Frictions and Dynamism with Long-Term Relationships" (with E. Oberfield, R. South, M. Waseem, draft available)

The *Future*: The role of trade in the adaptation to changing environmental risks

1. "Firm Adaptation in Production Netowrks" (with C. Balboni, M. Waseem, cond. accept. AER)
    - Climate change increases frequency of extreme weather events. Show evidence that following extreme weather events (floods) in Pakistan, firms *persistently* change their sourcing strategies $\Rightarrow$ Adaptation! Quantification via trade model.

2. "Clearing the Air on the Costs and Benefits of Road Infrastructure" (with C. Balboni, A. Berman, L. Marzano, M. Waseem, work in progress)
    - Urban setting: compare benefits from roads (market integration: trade, commuting) with local environmental externality (air pollution). Integrate atmospheric dispersion model into quant. spatial model.

THANK YOU!

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BACKUP SLIDES

Political changes in the Mediterranean: 600 AD

▸ Back

Political changes in the Mediterranean: 600 AD

▸ Back

Political changes in the Mediterranean: 600 AD

Back

Political changes in the Mediterranean: 600 AD

Back

Political changes in the Mediterranean: 632 AD

Back

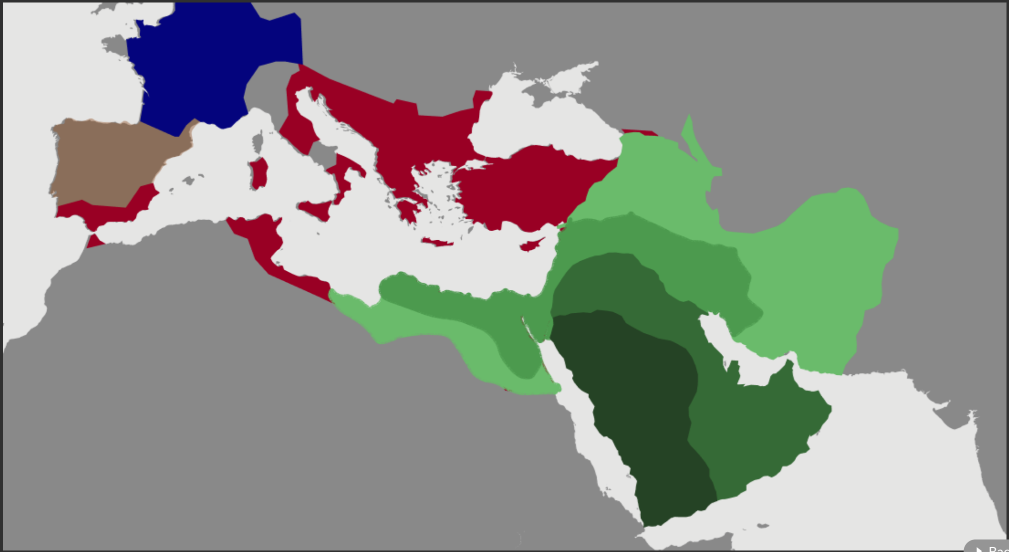
Political changes in the Mediterranean: 634 AD

Back

Political changes in the Mediterranean: 644 AD

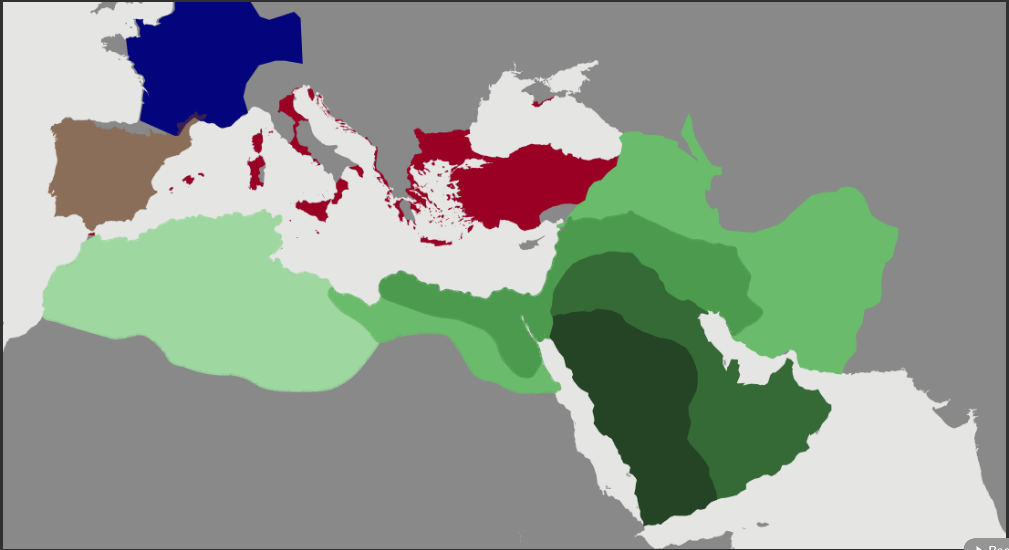
▸ Back

Political changes in the Mediterranean: 661 AD



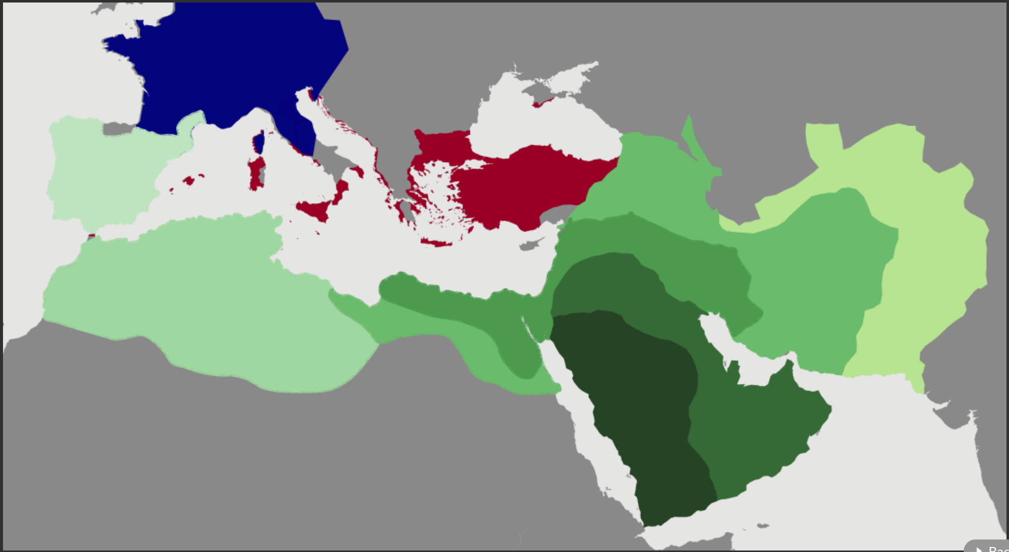
▸ Back

Political changes in the Mediterranean: 661-700 AD



▸ Back

Political changes in the Mediterranean: 750 AD



Fact #1: within a hoard, older coins have travel farther

Table 1: Within-hoard distance travelled and age of coin at deposit

	Dependent variable: Log Distance between Mint and Hoard				
	(1)	(2)	(3)	(4)	(5)
Log Age of Coin	0.146***	0.0831***	0.0749**	0.160***	0.0485**
	(0.044)	(0.026)	(0.031)	(0.043)	(0.020)
Sample				No non-hoards	No non-hoards
Hoard FE	Yes	Yes	Yes	Yes	Yes
Mint $\times$ 50-year-interval FE		Yes			
Mint $\times$ 25-year-interval FE			Yes		Yes
R^2	0.762	0.863	0.869	0.775	0.898
Observations	287243	287029	286873	250156	249830

Age of coin $=$ *tpq* $-$ mint date

$\Rightarrow$ coins diffuse across space over time.

Back

Fact #2: distance and political borders impede coin travels

Construct $1^\circ \times 1^\circ$ cells for mint and hoard locations and calculate flows $\#\text{coins}_{mdh}$

Table 2: Gravity and Border Effects in Coin Flows

	Dependent variable: # Coins$_{mhp}$				Dep. var.: Value$_{mhp}$	
	(1)	(2)	(3)	(4)	(5)	(6)
Log Distance	-1.137***	-1.002***	-1.135***	-0.951***	-1.144***	-0.989***
	(0.12)	(0.13)	(0.10)	(0.076)	(0.075)	(0.068)
Political border		-1.945***		-2.073***		-1.516***
		(0.62)		(0.47)		(0.27)
Hoard Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Mint $\times$ Empire Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample			Gold only	Gold only	Gold and Silver	Gold and Silver
Pseudo-R^2	0.767	0.778	0.808	0.824	0.800	0.810
Observations	217748	217748	57287	57287	146767	146767

Estimating eqn: $\text{count}_{mdh} = \exp\left(\gamma_1 \log \text{distance}_{mh} + \gamma_2 \text{withinBorder}_{dh} + \alpha_{md} + \alpha_h + \varepsilon_{mhd}\right)$

Back

Ancient mines, climate, and cities

Mines and minting output

- Active mine $\rightarrow 4\times$ increase in minting output ($e^{1.4} \approx 4$)
- Robust to region and period fixed effects

Climate and real consumption

- Severe droughts significantly reduce real consumption
- One year of severe drought: -41% ($1 - e^{-0.53}$); -10% with endogenous monetization
- Plausibly causal (exogenous climate variation with region + period FE)

Urbanization

- Consumption estimates qualitatively consistent with urban population changes AD 700–900

Military conflicts & plague

- No systematic association between battles or Justinianic plague and real consumption

Back

References

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Regions

▸ Back

Fact #3: Coin flows before/after the Arab conquests

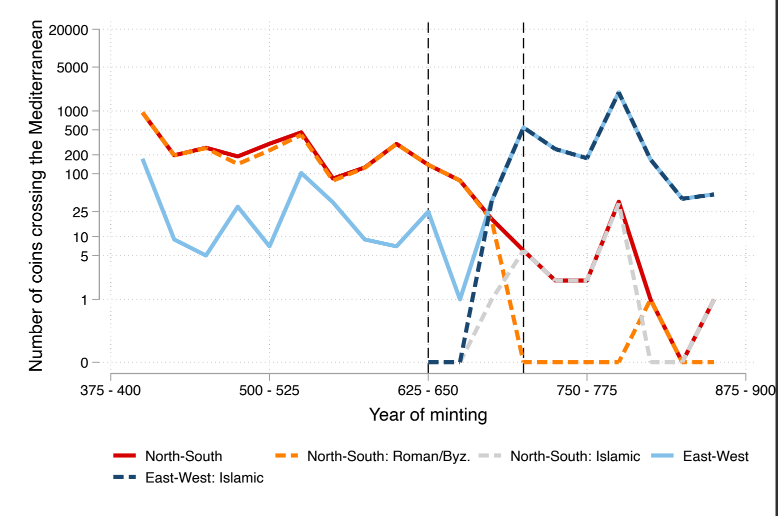


Figure 4: Number of coins flowing across the Mediterranean

Fact #3: Coin flows before/after the Arab conquests

Table 3: The Mediterranean Before and After the Arab Conquest

	Dependent variable: Number of Coins			
	(1)	(2)	(3)	(4)
Crossing Mediterranean × After Conquests	-1.893***	-3.246***	-0.662	-1.736
	(0.48)	(0.53)	(0.63)	(1.27)
Crossing Mediterranean × After Conquests × Islamic Coin		7.267***	4.789***	7.545***
		(0.90)	(0.95)	(0.89)
Crossing Mediterranean × After Conquests × Roman Coin			-3.287***	-2.893***
			(0.75)	(0.61)
Mint Cell × Empire FE	Yes	Yes	Yes	Yes
Mint Cell × Hoard Cell FE	Yes	Yes	Yes	Yes
After Conquests FE	Yes	Yes	Yes	
Mint Cell × After Conquests FE				Yes
Hoard Cell × After Conquests FE				Yes
Estimator	PPML	PPML	PPML	PPML
Observations	10480	10480	10480	6208

Standard errors in parentheses, clustered at the hoard × era and mint × era level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

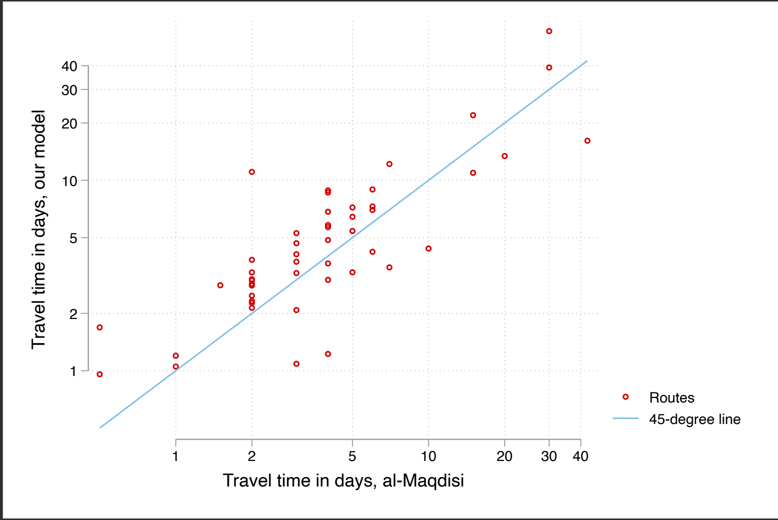
Estimating eqn: $\text{count}_{mdht} = \exp\left(\gamma_1 \text{mediterranean}_{mh} \times \text{after}_t + \cdots + \alpha_{md} + \alpha_{mh} + \varepsilon_{mhdt}\right)$

Back

Gold

Validating Travel Times

Al-Maqdisi (c. 945–991): *The Best Divisions for Knowledge of the Regions*



Back

Intuition for identification of Π_t

When every region mints some new coins in every period ($N \times T$ types of coins)

1. Calibrate savings rate s (to 1.5%)
2. Distribution of coins of type (m, t) in hoards at $t + 1$ inform about trade matrix Π_{t+1}

When some regions sometimes don't mint new coins, older vintages of coins become relevant:

- When distribution of coin shares don't change much over time $\rightarrow$ region does not export much (*conditional on s*)
- When distribution of coin shares changes to become more similar to another region $\rightarrow$ high exports to that region

Let R be the number of coin types (m, t). Then we have $\approx N \times T \times R - 1$ share observations and $N \times N - 1 \times T$ trade matrix entries to identify.

- If $R >> N$, we're pretty safe
- Parameterizing Π further reduces the parameter space

Back

Fact #3: Arab conquests disrupt Mediterranean trade: Gold only

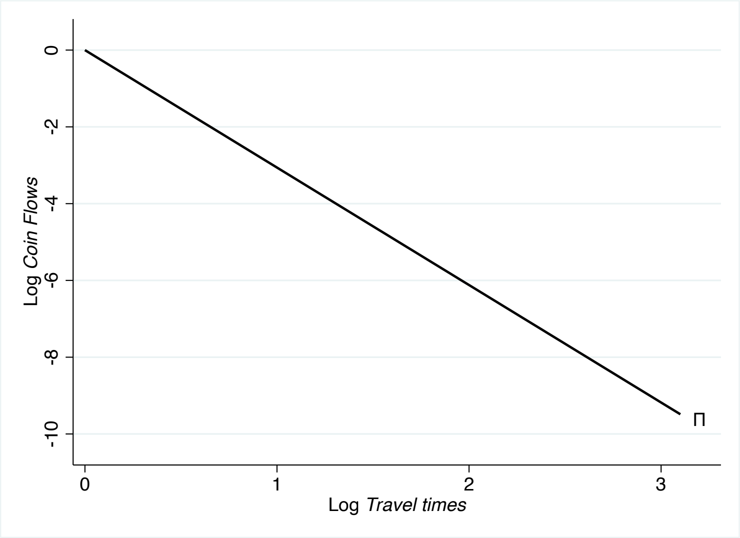
	Dependent variable: Number of Coins				
	(1)	(2)	(3)	(4)	(5)
Log Distance	-1.516***	-1.541***	-1.544***	-1.189***	-1.193***
	(0.13)	(0.13)	(0.13)	(0.15)	(0.15)
Crossing Mediterranean	0.298	0.307	0.320	0.0942	0.122
	(0.40)	(0.39)	(0.39)	(0.31)	(0.31)
Crossing Mediterranean × After Conquests	-1.600**	-2.858***	-1.719**	-2.576***	-3.379***
	(0.70)	(0.68)	(0.69)	(0.98)	(1.13)
Crossing Mediterranean × After Conquests × Islamic Coin		3.020***	1.864**		2.985**
		(0.71)	(0.76)		(1.20)
Crossing Mediterranean × After Conquests × Roman Coin			-1.699		
			(1.04)		
Mint Cell × Empire FE	Yes	Yes	Yes	Yes	Yes
Hoard Cell × After Conquests FE	Yes	Yes	Yes	Yes	Yes
Sample				Gold only	Gold only
Observations	172442	172442	172442	32024	32024

Standard errors in parentheses, clustered at the hoard × era and mint × era level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Before: 400–630; after: 713–950

Back

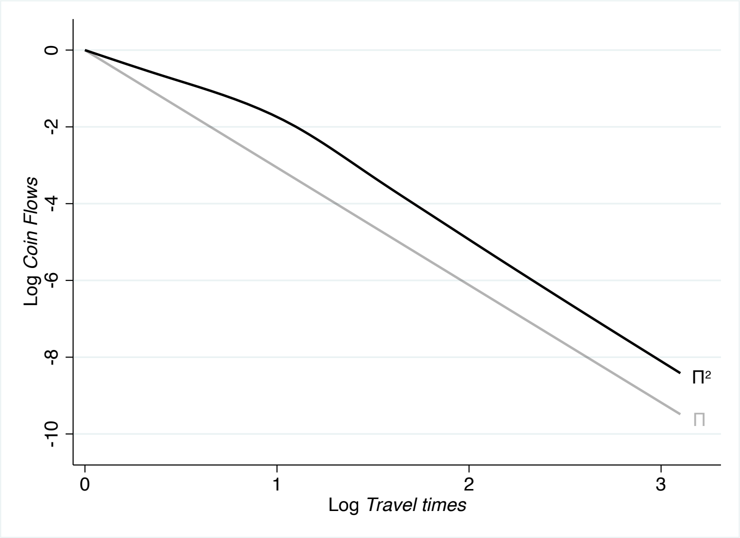
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1 (same as trade flows Π)

Back

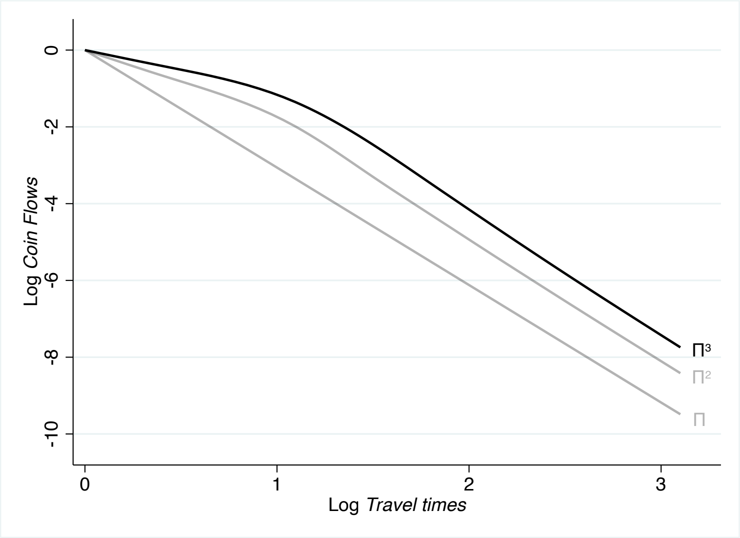
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2

Back

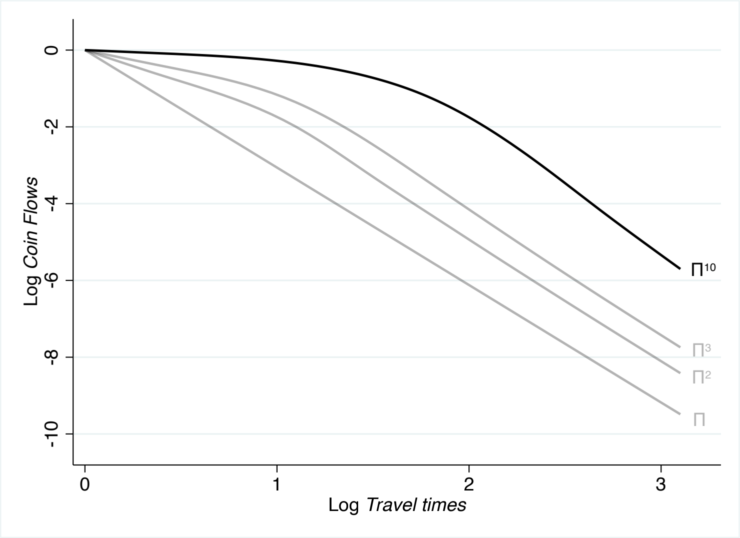
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3

Back

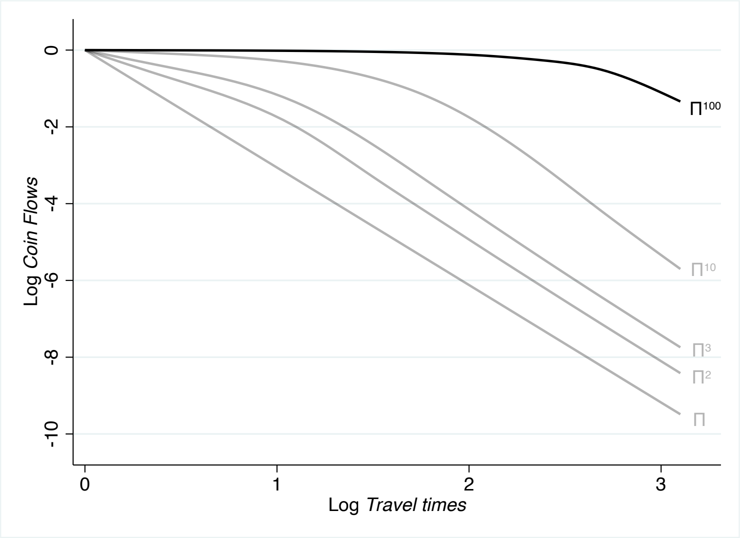
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10

Back

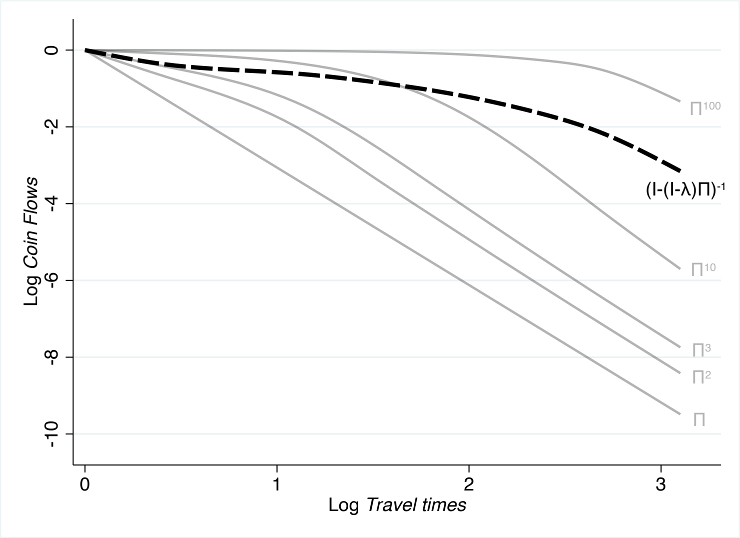
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10, age 100

Back

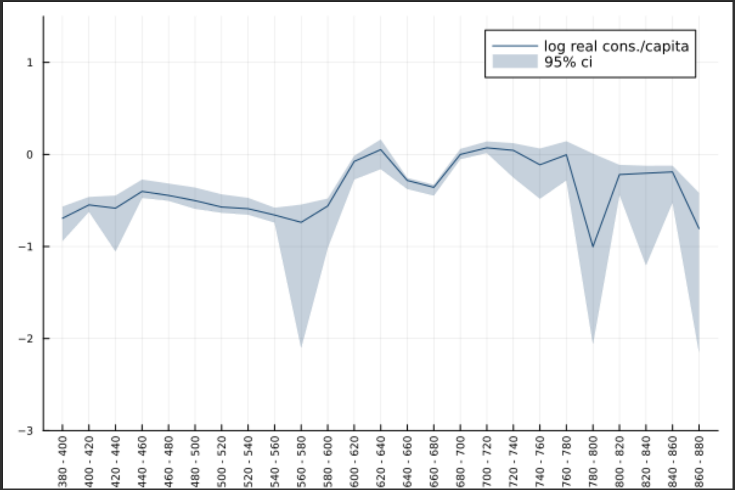
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10, age 100, all ages

Back

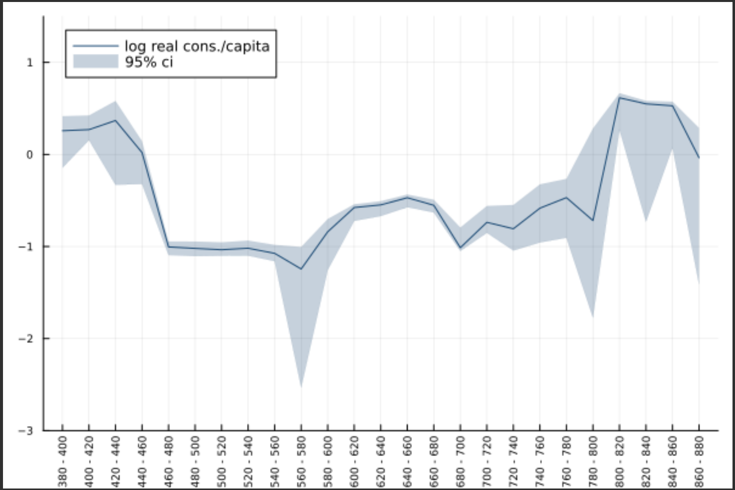
Real consumption per capita (380-880): al-Andalus (Spain)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

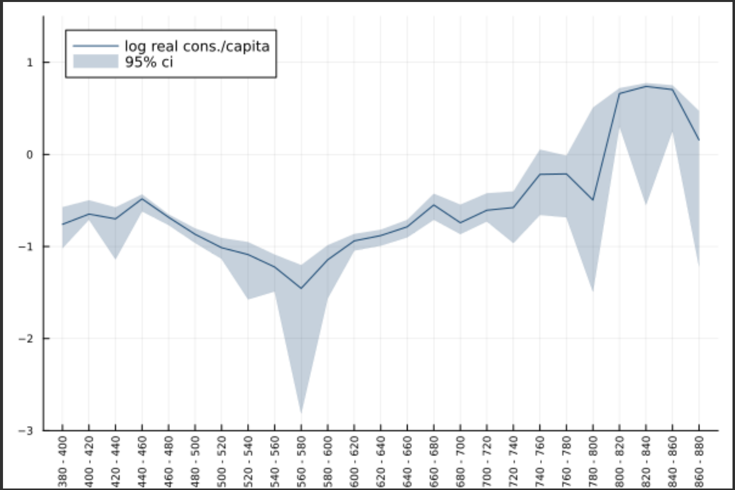
Real consumption per capita (380-880): Aquitaine (South France)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

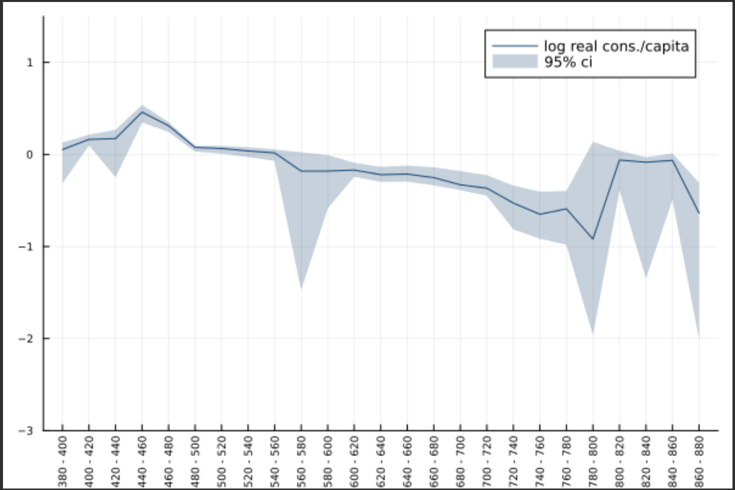
Real consumption per capita (380-880): Francia and Germania



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

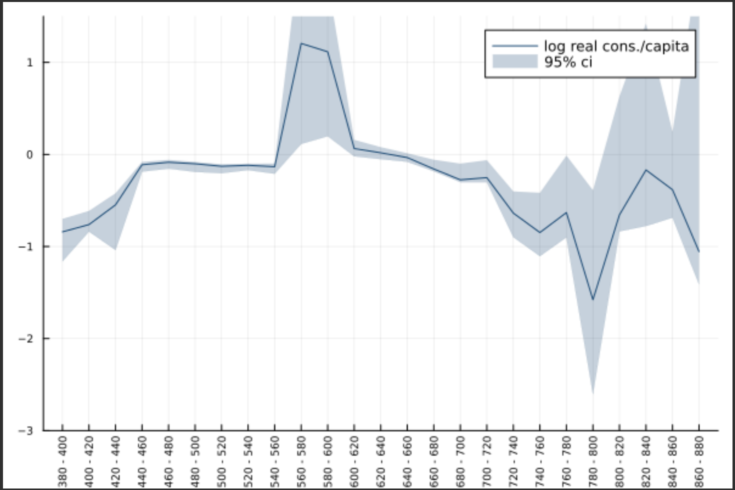
Real consumption per capita (380-880): Northern Italy



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

▸ back

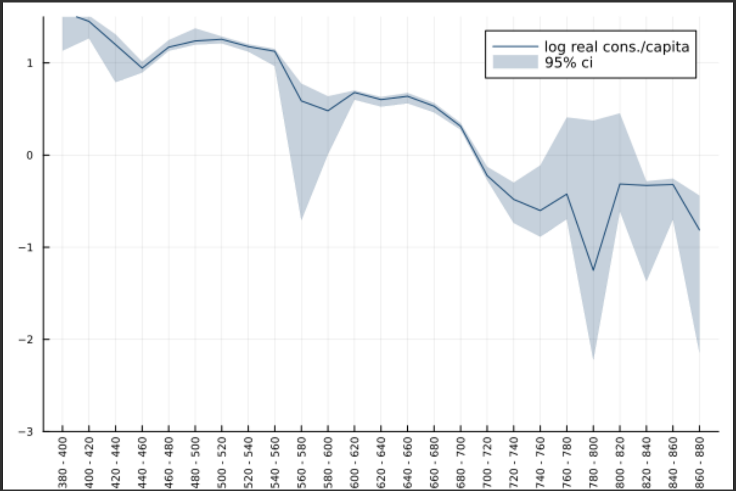
Real consumption per capita (380-880): Southern Italy



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

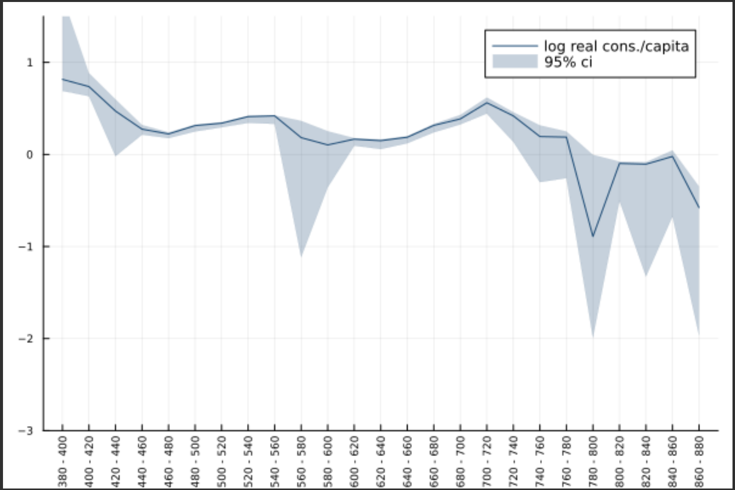
Real consumption per capita (380-880): Byzantine Heartlands



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

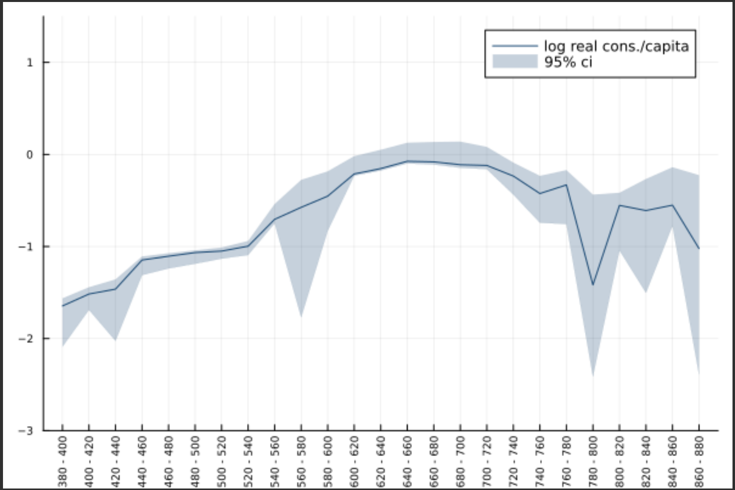
Real consumption per capita (380-880): al-Sham (Greater Syria)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

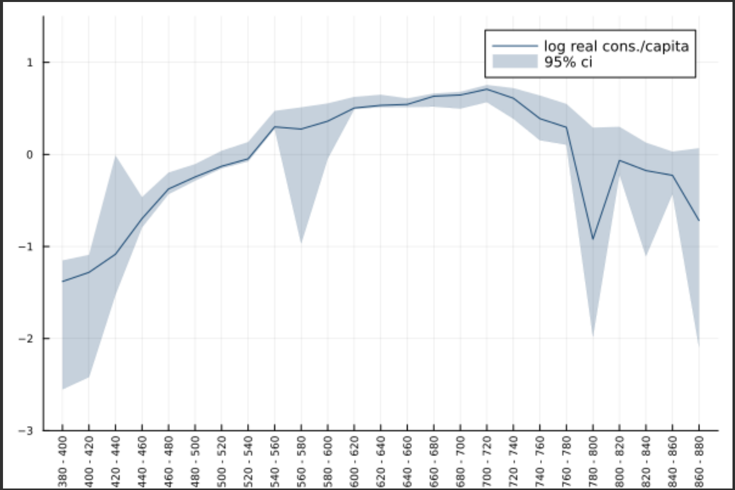
Real consumption per capita (380-880): Northern Syria, Caucasus



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

▸ back

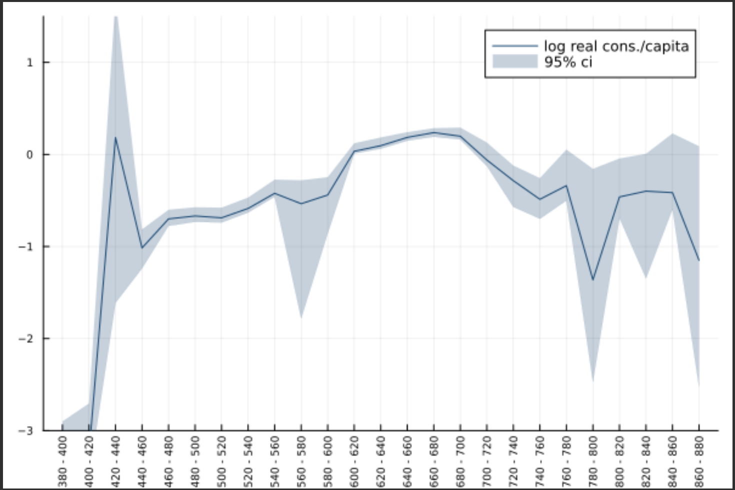
Real consumption per capita (380-880): Iraq, Iran



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

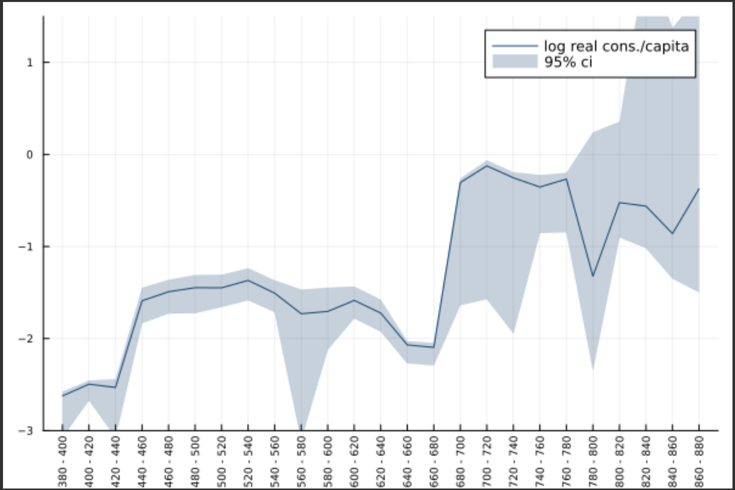
Real consumption per capita (380-880): Eastern Caliphate



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

back

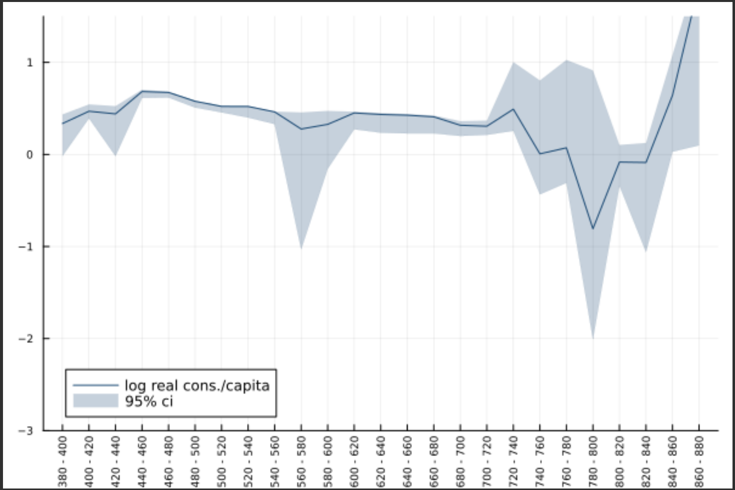
Real consumption per capita (380-880): Arabian Peninsula



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

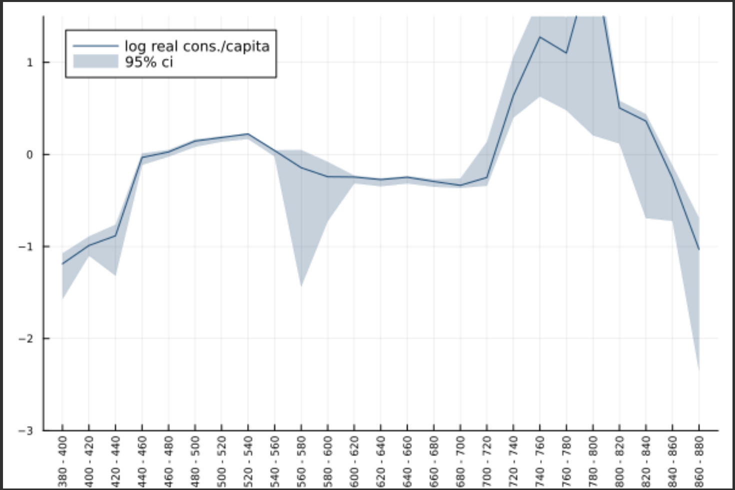
Real consumption per capita (380-880): Misr (Egypt)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

Real consumption per capita (380-880): al-Maghrib



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

Detailed estimates

	Log Trade	
	(1)	(2)
Log Travel Time	-2.98	-3.03
	(0.02)	(0.02)
Political Border	-0.3	-0.49
	(0.02)	(0.02)
Religious Border	-4.05	
	(0.11)	
Religious Border: East		-1.97
		(0.12)
Religious Border: West		-4.59
		(0.22)
Religious Border: Mediterranean		-5.2
		(0.18)
Sample	All	All
Coin Accounting	Number	Number
Observations	4,413	4,413

back

Realized vs counterfactual changes in real consumption per capita

Realized changes, from AD 460-620 to AD 700-900

	Real consumption $\Delta \log \left(\frac{X_n/p_n}{L_n}\right)$		Openness $\Delta \log \left(\pi_{nn}^{-1/\theta}\right)$		Technology $\Delta \log \left(T_n^{1/\theta}\right)$		Trade Deficit $\Delta \log \left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n}\right)$	
al-Andalus (Spain)	0.62	(0.25)	-0.06	(0.04)	0.77	(0.32)	-0.09	(0.18)
Aquitaine (South France)	1.28	(0.23)	-0.05	(0.01)	1.22	(0.23)	0.11	(0.06)
Francia and Germania	1.96	(0.24)	-0.05	(0.01)	1.80	(0.26)	0.20	(0.04)
Northern Italy	-0.31	(0.24)	-0.08	(0.03)	-0.10	(0.26)	-0.13	(0.10)
Southern Italy	-0.20	(0.34)	0.19	(0.18)	-0.94	(0.37)	0.55	(0.40)
Byzantine Heartlands	-1.56	(0.33)	-0.23	(0.14)	-0.44	(0.41)	-0.89	(0.54)
al-Sham (Greater Syria)	-0.32	(0.27)	-0.04	(0.02)	-0.11	(0.29)	-0.17	(0.11)
Northern Syria, Caucasus	0.22	(0.30)	-0.01	(0.03)	0.15	(0.37)	0.08	(0.12)
Iraq, Iran	0.06	(0.27)	-0.00	(0.01)	0.06	(0.29)	-0.00	(0.04)
Eastern Caliphate	0.37	(0.33)	-0.00	(0.00)	0.39	(0.34)	-0.02	(0.04)
Arabian Peninsula	1.16	(0.34)	-0.01	(0.04)	0.66	(0.45)	0.51	(0.26)
Misr (Egypt)	-0.36	(0.72)	0.09	(0.23)	-0.82	(0.50)	0.37	(0.90)
al-Maghrib	0.28	(0.33)	0.13	(0.07)	-0.49	(0.27)	0.65	(0.30)

Normalizations: $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

back

Realized vs counterfactual changes in real consumption per capita

Counterfactual changes relative to to AD 700-900

	Initial $\log\left(\frac{X_n/p_n}{L_n}\right)$		Counterfactual $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$ if:					
	All parameters ad 460-620		Religious border ad 700-900		Technology ad 700-900		Minting ad 700-900	
al-Andalus (Spain)	-0.70	(0.10)	0.09	(0.02)	0.55	(0.10)	1.57	(0.31)
Aquitaine (South France)	-1.04	(0.08)	-0.15	(0.03)	0.99	(0.09)	3.93	(0.30)
Francia and Germania	-1.55	(0.09)	-0.07	(0.02)	1.68	(0.11)	6.17	(0.47)
Northern Italy	0.07	(0.04)	-0.24	(0.05)	-0.24	(0.08)	-0.21	(0.07)
Southern Italy	-0.25	(0.06)	-0.11	(0.02)	-0.60	(0.13)	-0.03	(0.02)
Byzantine Heartlands	1.22	(0.11)	-0.69	(0.08)	-0.57	(0.13)	-1.41	(0.19)
al-Sham (Greater Syria)	0.30	(0.04)	0.04	(0.01)	-0.18	(0.10)	-0.22	(0.08)
Northern Syria, Caucasus	-0.34	(0.11)	0.02	(0.02)	0.15	(0.22)	0.19	(0.19)
Iraq, Iran	0.28	(0.08)	0.01	(0.00)	0.03	(0.08)	0.03	(0.06)
Eastern Caliphate	-0.44	(0.08)	0.01	(0.00)	0.38	(0.16)	0.34	(0.26)
Arabian Peninsula	-1.80	(0.18)	0.26	(0.09)	0.66	(0.40)	2.71	(0.84)
Misr (Egypt)	0.32	(0.07)	0.02	(0.00)	-0.71	(0.24)	-0.09	(0.02)
al-Maghrib	0.12	(0.06)	0.01	(0.00)	-0.46	(0.17)	-0.05	(0.06)

Normalizations: $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

back

Realized vs counterfactual changes in real consumption per capita

Realized changes, from AD 460-620 to AD 700-900

	Real consumption $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$		Openness $\Delta \log\left(\pi_{nn}^{-1/\theta}\right)$		Technology $\Delta \log\left(T_n^{1/\theta}\right)$		Trade Deficit $\Delta \log\left(1+\frac{M_n-\lambda w_n L_n}{w_n L_n}\right)$	
Francia and Germania	1.96	(0.24)	-0.05	(0.01)	1.80	(0.26)	0.20	(0.04)
Byzantine Heartlands	-1.56	(0.33)	-0.23	(0.14)	-0.44	(0.41)	-0.89	(0.54)
Arabian Peninsula	1.16	(0.34)	-0.01	(0.04)	0.66	(0.45)	0.51	(0.26)

Counterfactual changes relative to to AD 700-900

	Initial $\log\left(\frac{X_n/p_n}{L_n}\right)$		Counterfactual $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$ if:					
	All parameters ad 460-620		Religious border ad 700-900		Technology ad 700-900		Minting ad 700-900	
Francia and Germania	-1.55	(0.09)	-0.07	(0.02)	1.68	(0.11)	6.17	(0.47)
Byzantine Heartlands	1.22	(0.11)	-0.69	(0.08)	-0.57	(0.13)	-1.41	(0.19)
Arabian Peninsula	-1.80	(0.18)	0.26	(0.09)	0.66	(0.40)	2.71	(0.84)

Normalizations: $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

back

Setup

Period t, location n: homogeneous mass $L_n(t)$ of workers

- Start with $S_n(t)$ coins saved from period $t-1$ to t.
- Beginning of period t:
  - Fraction $\lambda_n(t)$ of saved coins $S_n(t)$ is lost (turned into hoards $H_n(t) = \lambda_n(t) S_n(t)$)
  - $M_n(t) \geq 0$, fresh new coins minted
- Middle of period t: divide coins into consumption and saving
  - $X_n(t)$, expenditure on consumption
  - $S_n(t+1)$, saving for period $t+1$.
- End of period t:
  - Workers produce and sell goods in exchange for coins.
  - $w_n(t)$, competitive wage, in coins.
  - $Y_n(t) = w_n(t) L_n(t)$, aggregate labor income.

Setup

Location n, period t: homog. mass $L_n(t)$ of workers. Four sub-periods $t_{sub1}, t_{sub2}, t_{sub3}, t_{sub4}$

t_{sub1} Start with $S_n(t)$ coins saved from period $t-1$

t_{sub2} A fraction $\lambda_n(t)$ of those saved coins is lost

$M_n(t) \geq 0$ fresh new coins are minted

t_{sub3} $X_n(t)$, expenditure on consumption

Budget constraint:

$$X_n(t) \leq (1-\lambda_n(t)) S_n(t) + M_n(t)$$

t_{sub4} $L_n(t)$ workers produce and sell goods in exchange for coins

$w_n(t)$, competitive wage, $w_n(t) L_n(t)$, aggregate labor income

$S_n(t+1)$ coins saved for $t+1$

$$\underbrace{(1-\lambda_n(t))\overbrace{S_n(t)}^{t_{sub1}} + M_n(t)}_{t_{sub2}} - \underbrace{X_n(t)}_{t_{sub3}} + \underbrace{w_n(t) L_n(t)}_{t_{sub4}} = \underbrace{S_n(t+1)}_{(t+1)_{sub1}}$$

Intra-temporal allocations

- Fraction π_{ni} of expenditure X_n allocated to goods from i:

$$\pi_{ni}(t) = \frac{T_i(t)\left(w_i(t)\, d_{ni}(t)\right)^{-\theta}}{\sum_k T_k(t)\left(w_k(t)\, d_{nk}(t)\right)^{-\theta}}, \tag{6}$$

as in Eaton and Kortum (2002).

Intertemporal preferences

- Intertemporal utility U_n, within period welfare W_n, ,

$$U_n(t) = \mathbb{E}\left[\sum_{\tau \geq t} \beta^{\tau - t} \ln\left(\frac{X_n(t)}{p_n(t)}\right)\right],$$

$$\text{with } p_n(t) = \gamma\left(\sum_k T_k(t)\left(w_k(t)\, d_{nk}(t)\right)^{-\theta}\right)^{1/\theta}$$

Dynamic optimization

- Optimal coin savings dynamics,

$$\max_{\{S_n(\tau)\}_{\tau \geq t}} \left[\sum_{\tau \geq t} \beta^{\tau - t} \ln \left(\frac{X_n(\tau)}{p_n(\tau)} \right) \right]$$

$$X_n(\tau) = w_n(\tau) L_n(\tau) + M_n(\tau) + (1 - \lambda_n(\tau)) S_n(\tau) - S_n(\tau + 1),$$

$$S_n(\tau + 1) \geq w_n(\tau) L_n(\tau), \forall \tau \geq t,$$

$$\lim_{\tau \to \infty} \beta^{\tau} S_n(\tau + 1) / X_n(\tau) = 0$$

- Dynamic equilibrium wages clear markets,

$$w_i L_i = \sum_n \pi_{ni} (\mathbf{T}, \mathbf{d}; \mathbf{w}) [w_n L_n + M_n + (1 - \lambda_n) S_n - S_n']$$

Savings $S_n(\mathbf{T}, \mathbf{d}, \delta, \mathbf{L}, \mathbf{M}; \mathbf{w})$ depend on parameters and wages, which depend on wages etc.

Optimal consumption/saving

Under log utility:

- price level $p_n(t)$ dynamics irrelevant (i.e. separates out)
- when unconstrained, consumption declines exponentially:

$$\frac{X_n(t+1)}{X_n(t)} = \beta(1-\lambda_n(t)) < 1$$

- when constrained, consume as much as you can:

$$S_n' = w_n(t)L_n$$

Define *net* saving:

$$s_n(t) = \frac{(1-\lambda_n(t))S_n(t) + M_n(t) - X_n(t)}{(1-\lambda_n(t))S_n(t) + M_n(t)}$$

Useful benchmark: zero net savings, $S_n(t+1) = S_n(t), \forall t$

- Wages and trade shares jointly determined,

$$w_i L_i = \sum_n \pi_{ni} \left((1 - \lambda_n) w_n L_n + M_n \right)$$

$$\pi_{ni} = \frac{T_i (w_i d_{ni})^{-\theta}}{\sum_k T_k (w_k d_{nk})^{-\theta}}$$

- Constant aggregate stock of coins in circulation,

$$\sum_n M_n = \lambda_n \sum_n w_n L_n$$

- *Note*: trade deficits as in Dekle, Eaton and Kortum (2007), but endogenous:

$$D_n = X_n - Y_n = M_n - \lambda_n S_n$$

Introducing and tracking coins of different vintages

Coin stocks $S_n(\tau)$ consist of coins of different vintage:

$$S_n(\tau) = \sum_{m=1}^{N} \sum_{t<\tau} S_{mn}(t,\tau)$$

Coins start their life when minted: $S_{mm}(t,t) = M_m(t)$.

Traders are 'blind' to coin types, draw coins with equal probability:

$$S_{mi}(t,\tau+1) = \sum_{n=1}^{N} (1-\lambda_n(\tau))(1-s_n(\tau))\, S_{mn}(t,\tau)\, \pi_{ni}(\tau) + (1-\lambda_i(\tau))\, s_i(\tau)\, S_{mi}(t,\tau), \forall \tau \geq t$$

In compact matrix form:

$$\mathbf{S}(t,T) = \boldsymbol{M}(t) \left(\prod_{\tau=t}^{T-1} (\boldsymbol{I} - \boldsymbol{\lambda}(\tau)) \left((\boldsymbol{I} - \boldsymbol{s}(\tau))\, \boldsymbol{\Pi}(\tau) + \boldsymbol{s}(\tau) \right) \right)$$

▸ Back

Spatial distribution: the (extended) Mediterranean

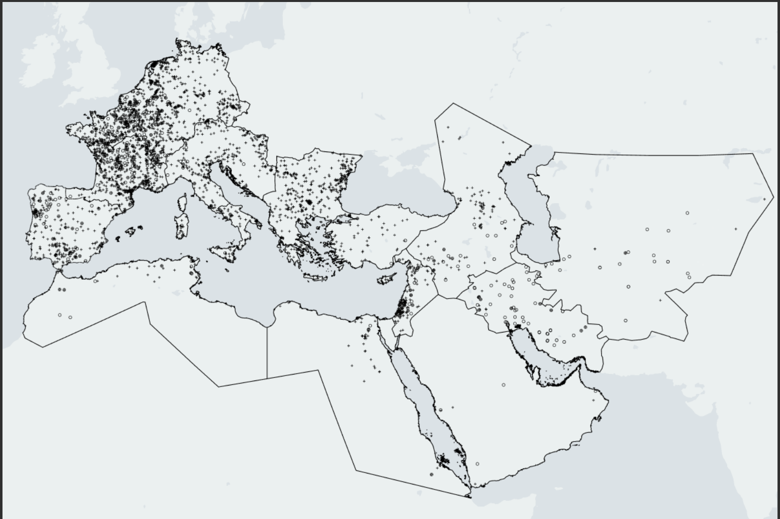


Figure 5: Region Definitions

Distribution of coin “death dates” (*tpq*)

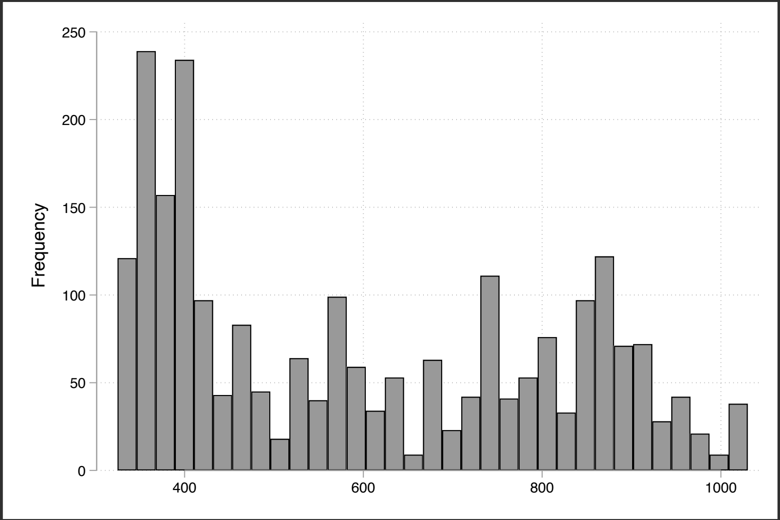


Figure 6: Terminus Post Quem (*tpq*) of hoards

Distribution of coin ages (*tpq* minus mint date)

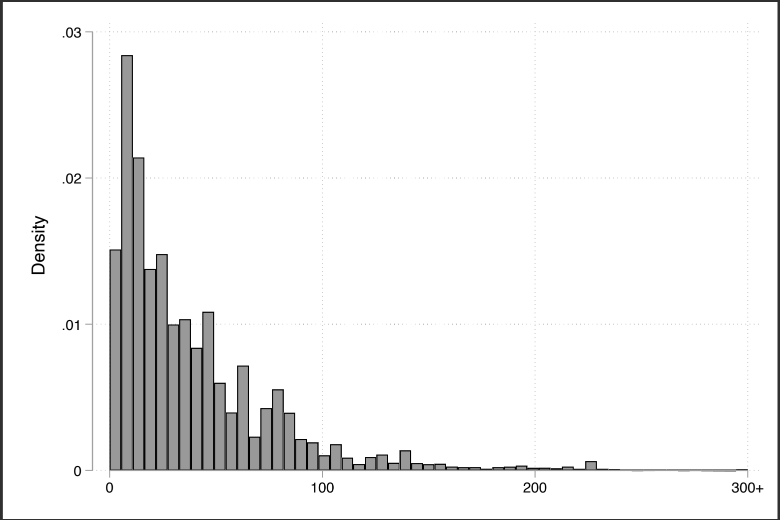


Figure 7: Coin age at time of deposit (*tpq*), in years

Fact #2: distance has a weaker impact on older coin flows

$$\begin{aligned}
\text{logcount}_{mth\tau} = \\
&\sum_{\tau' \in T} \beta_{\tau'} \log \text{distance}_{mh} \\
&\times 1\left(t - \tau = \tau'\right) \\
&+ \alpha_{mt} + \alpha_{h\tau} + \varepsilon_{mth\tau}
\end{aligned}$$

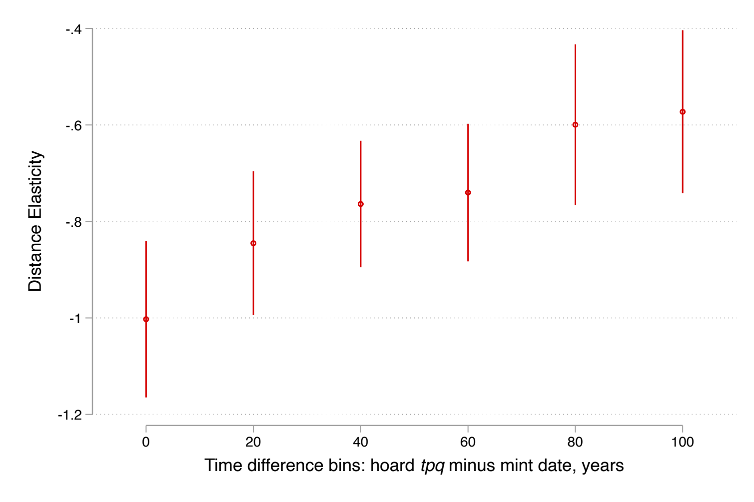
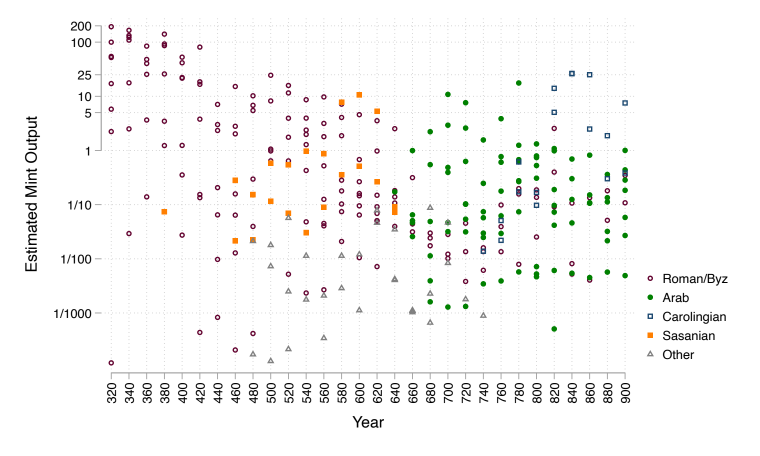


Figure 8: The distance elasticity declines as coins get older

Estimation results: Mint output



Normalization: $M_{n_0}[t_0] = 100$ (Northern Italy, ad 320-40).

Discussion on Byzantine monetary output: Kazhdan (1954), Pennas (1996)

Real consumption per capita: technology, geography, and trade (deficits)

Table 4: Real consumption in the ancient world from ad 460-620 to ad 700-900

	Consumption		Openness		Technology		Trade Deficits	
	Δ log $\left(\frac{X_n/p_n}{L_n}\right)$		Δ log $\left(\pi_{nn}^{-1/\theta}\right)$		Δ log $\left(T_n^{1/\theta}\right)$		Δ log $\left(1+\frac{M_n-\lambda w_n L_n}{w_n L_n}\right)$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
al-Andalus	0.53	(0.08)	-0.04	(0.01)	0.65	(0.10)	-0.08	(0.05)
Francia and Germania	1.99	(0.14)	-0.07	(0.02)	1.94	(0.16)	0.12	(0.04)
Byzantine Heartlands	-1.56	(0.22)	-0.16	(0.06)	-0.74	(0.13)	-0.66	(0.25)
Arabian Peninsula	1.12	(0.28)	-0.02	(0.04)	0.98	(0.37)	0.15	(0.23)

▸ changes and counterfactuals

Welfare and counterfactuals

Real consumption depends on a combination of L and T (that's not separately identified):

$$X_n/p_n = \gamma^{-1} \left(\pi_{nn}\right)^{-1/\theta} \left(L_n T_n^{1/\theta}\right) \left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n}\right)$$

or equivalently in per capita terms

$$\underbrace{\frac{X_n/p_n}{L_n}}_{\text{Real Consumption}} = \gamma^{-1} \underbrace{\left(\pi_{nn}\right)^{-1/\theta}}_{\text{Openness}} \underbrace{\left(T_n\right)^{1/\theta}}_{\text{Technology}} \underbrace{\left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n}\right)}_{\text{Trade Deficit}}$$

Note: T and L are not separately identified. We separate L and T through a Malthusian assumption

$$L_n = T_n \quad \forall n$$

and decompose per-capita real consumption into the three components

Counterfactuals

Table 5: Counterfactual changes in real consumption per capita after ad 700

	Log consumption		Counterfactual log consumption change if:					
	All parameters ad 460-620		Religious border ad 700-900		Technology ad 700-900		Minting ad 700-900	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
al-Andalus	-0.66	(0.03)	0.07	(0.01)	0.53	(0.05)	1.15	(0.18)
Francia and Germania	-1.53	(0.08)	-0.09	(0.02)	1.78	(0.11)	6.70	(0.54)
Byzantine Heartlands	1.17	(0.03)	-0.61	(0.03)	-0.64	(0.06)	-1.34	(0.05)
Arabian Peninsula	-1.50	(0.22)	0.13	(0.04)	1.05	(0.37)	2.03	(0.50)