Growth and the Fragmentation of Production

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Motivation

Since Adam Smith, economists have been postulating a link between specialization and productivity

In the context of supply chains: how is value chain broken down into work done by different plants?

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This paper: study specialization in value chain *among plants* and growth

1. Empirical facts about organization and performance using manufacturing data from India
    - macro: vertical specialization $\Leftrightarrow$ income per capita
    - micro: vertical specialization $\Leftrightarrow$ plant size
2. Simple model, full quantitative model
3. (in the future) estimation + counterfactuals

Literature

Division of labor and productivity:

- **Theory:** Young (1928), Stigler (1951), Rosen (1978), Baumgardner (1988), Becker and Murphy (1992), Rodriguez-Clare (1996), Chaney and Ossa (2013)
- **Empirical evidence:** Baumgardner (1988), Brown (1992), Garicano and Hubbard (2009), Duranton and Jayet (2011), Tian (2018), Hansman et al. (2020), Chor et al. (2020), Bergeaud et al. (2021)

Smithian Growth:

- Boreland and Yang (1991), Kelly (1997), Legros, Newman, Proto (2014), Menzio (2020)

Indian Trade Liberalization:

- Panagariya (2004), Sivadasan (2009), Khandelwal and Topalova (2010), Goldberg et al. (2010), Peter and Ruane (2020)

Manufacturing Plants in India

Data: **Indian Annual Survey of Industries**, 1989/90-2014/15 (with gaps)

- Plant-level panel survey of formal manufacturing plants
  - All plants that have 100+ employees
  - 1/5 of all plants between 20 (10 if using power) and 100 employees

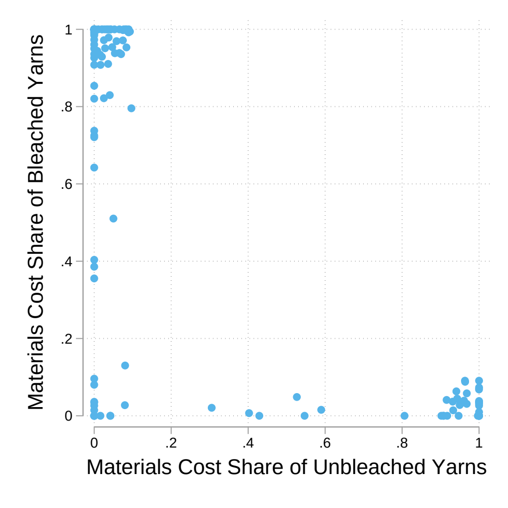
Most important part of the survey:

- Quantities, unit values & 5-digit product codes for all manufacturing output and intermediate inputs (domestic and imported)

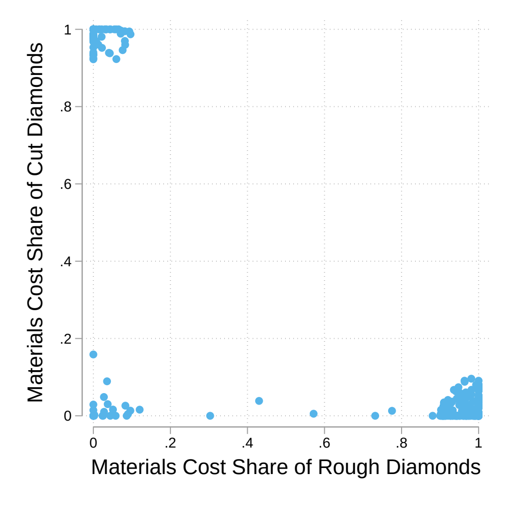
Example product codes: Silk yarn, bleached (61222), beryllium copper wire (72246), aluminium ingots (73107)

	min	p25	p50	p75	max	count
# 5-digit Inputs	1	1	3	5	117	595460

Within narrow industries, firms use different inputs



(a) Input mixes for Bleached Cotton Cloth (63303)



(b) Input mixes for Polished Diamonds (92104)

Measuring the vertical span of production (Boehm & Oberfield, 2020)

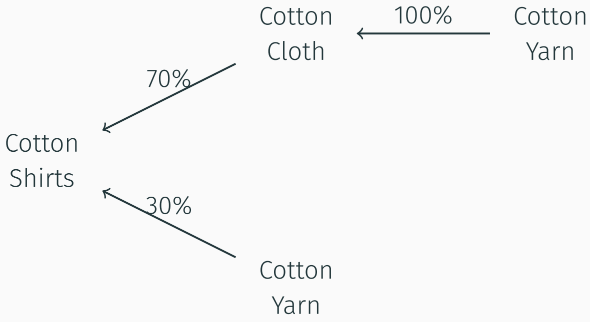
Two steps:

1. Define a **vertical distance** $d_{\omega\hat{\omega}}$ of an input from an output (varies at product-pair level)
    - Rough diamonds are more distant from polished diamonds than cut diamonds
    - Similar to upstreamness of Alfaro et al. (2019)

2.

Vertical Distance of inputs from output – Intuition

1. For a given product ω, construct the materials cost shares of industry ω on each input
2. Recursively construct the cost shares of the input industries (and inputs' inputs, etc...), excluding all products that are further downstream.
3. Vertical distance between ω and ω' is the average number of steps between ω and ω', weighted by the product of the cost shares.



$\Rightarrow$ Shirts $\leftarrow$ Cloth: 1; Shirts $\leftarrow$ Yarn: $0.3 \times 1 + 0.7 \times 1.0 \times 2 = 1.7$

Vertical Distance of inputs from output – Examples

Table 1: Vertical distance examples for 63428: *Cotton Shirts*

Input group	Average vertical distance
Fabrics Or Cloths	1.67
Yarns	2.78
Raw Cotton	3.55

Table 2: Vertical distance examples for 73107: *Aluminium Ingots*

ASIC code	Input description	Vertical distance
73105	Aluminium Casting	1.23
73104	Aluminium Alloys	1.46
73103	Aluminium	1.92
22301	Alumina (Aluminium Oxide)	2.92
31301	Caustic Soda (Sodium Hydroxide)	3.81
23107	Coal	3.85
22304	Bauxite, raw	3.93

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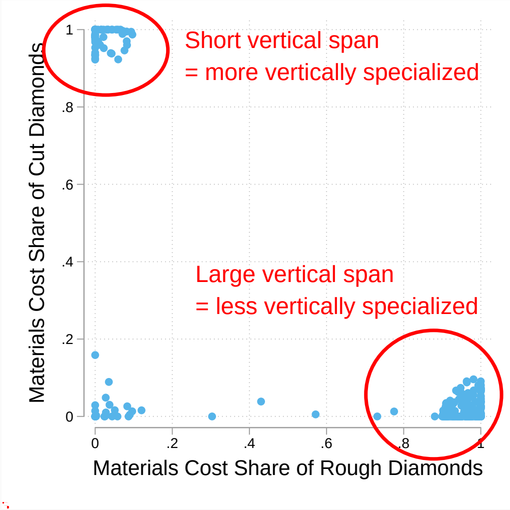
    - Rough diamonds are more distant from polished diamonds than cut diamonds
    - Similar to upstreamness of Alfaro et al. (2019)
2. Construct each plant's **vertical span**: how far are the plant's inputs from the output?

$$\text{span}_{jt} = \sum_{\hat{\omega}} \frac{X_{j\hat{\omega}}}{\sum_{\tilde{\omega}} X_{j\tilde{\omega}}} d_{\omega\hat{\omega}}$$

Long and short vertical span

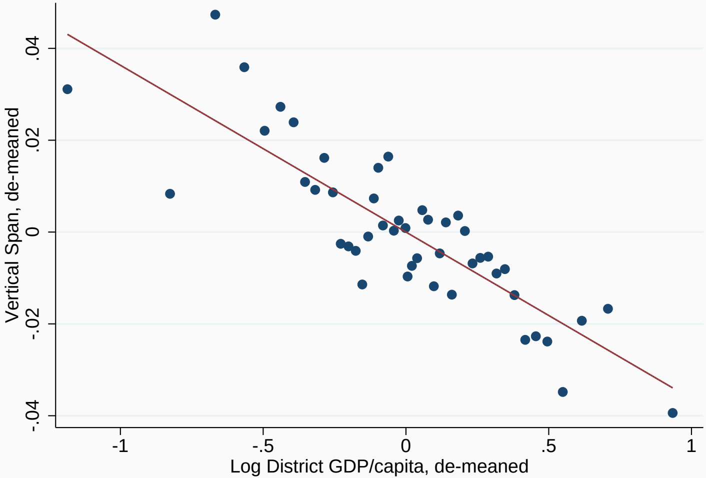
Figure 1: Input mixes for Polished Diamonds (92104)



Motivational Facts about Vertical Specialization

Fact 1: In richer districts, plants are more specialized (shorter vertical span)

Within industry × year:



Binscatter with n=50. Y and X variable de-meaned by industry x year. SP plants only.

▸ Single-plant

Fact 2: Increased vertical specialization positively correlated with state growth

Within plant, over time:

	Dependent variable: Vertical Span		
	(1)	(2)	(3)
Log GDP/capita$_{st}$	-0.0716*	-0.0601*	-0.0551*
	(0.028)	(0.026)	(0.026)
Year FE	Yes	Yes	
Plant FE	Yes	Yes	
5-digit Industry FE		Yes	
5-digit Industry $\times$ Year FE			Yes
Plant $\times$ 5-digit Industry FE			Yes
R^2	0.592	0.656	0.808
Observations	270003	269399	163668

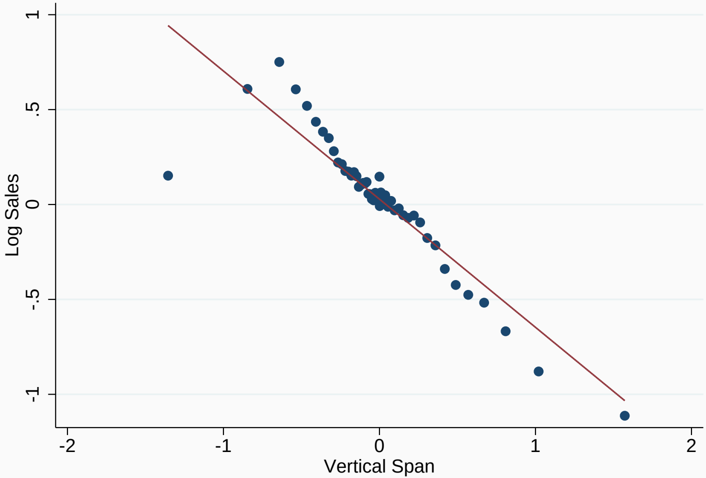
Standard errors in parentheses, clustered at the state $\times$ 5-dgt industry level. SP plants only.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Single-plant

Bin Scatter

Fact 3: More specialized plants (shorter span) are larger

Plants with higher sales tend to have shorter vertical span (within industry $\times$ year)



SP plants only. Regression includes industry x year FE's.

Single-plant

Table

Other cross-sectional covariates

	Dependent variable: Vertical Span					
	(1)	(2)	(3)	(4)	(5)	(6)
Materials Share of Cost	-0.250**			-0.119**		
	(0.018)			(0.015)		
Importer Dummy		-0.163**			-0.0143**	
		(0.0094)			(0.0055)	
Share of R-Inputs in Materials Cost			-0.260**			-0.181**
			(0.021)			(0.021)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes			
Plant x Industry FE				Yes	Yes	Yes
R^2	0.310	0.309	0.322	0.774	0.765	0.773
Observations	332356	353694	347548	173141	186641	181958

Standard errors in parentheses, clustered at the 5-dgt industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Fact 4: Sales growth is correlated with increased vertical specialization

	Dependent variable: Δ log Sales			
	(1)	(2)	(3)	(4)
Δ Vertical Span	-0.0655**	-0.0445**	-0.0284*	-0.0259*
	(0.0082)	(0.0087)	(0.013)	(0.011)
Year FE	Yes			
Product × Year FE		Yes	Yes	Yes
Plant FE			Yes	
Plant × Product FE				Yes
R^2	0.00819	0.149	0.432	0.431
Observations	120436	111244	83026	74707

Changes within plant-products

Standard errors in parentheses, clustered at the state-industry level.

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SP plants only.

Indian Trade Liberalization to get at causality

- **Until end of 80s:** India in near-autarky
  - Import licensing system
  - Very high tariffs. Large variation (up to 355%), average $\sim$80%. Was set in the 1950s.
- **July 1991:** Balance of Payments crisis. Removal of import licensing system, starts cutting tariffs.
- **1992-1997:** Tariffs come down to average of 35%, ending up fairly uniform. ▸ Tariffs
- $\Rightarrow$ **tariff change was determined in the 50's**
- $\Rightarrow$ **tariff changes are uncorrelated with 1992 industry characteristics** (Khandelwal and Topalova, 2010: "as exogenous to the state of the industries as a researcher might hope for").

See also Panagariya (2004), Sivadasan (2009), Khandelwal and Topalova (2010), Goldberg et al. (2010).

▸ Tariff details

Tariff changes act as demand & supply shocks

	Dependent variable: Δ log Sales	
	(1)	(2)
Δ log Output Tariff	0.172^{+}	0.252^{**}
	(0.090)	(0.094)
$\Delta \log(1+\bar{\tau}^{\text{input}}_{\omega t})$		-0.229^{+}
		(0.12)
Year-Pair FE	Yes	Yes
R^2	0.0625	0.0626
Observations	104996	104985

Standard errors in parentheses, clustered at the state $\times$ industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

$\Rightarrow$ We are going to use **changes in import tariffs** in the output good as a **demand shifter**

▸ 90s

Demand shocks affect vertical specialization

	Dependent variable: Δ Vertical Span			
	(1)	(2)	(3)	(4)
Δ log Sales	-0.0158**	-0.0160**	-0.0326*	-0.0536*
	(0.0020)	(0.0020)	(0.013)	(0.025)
$\Delta \log(1 + \bar{\tau}^{\text{input}}_{j\omega t})$		-0.0170		-0.0473
		(0.020)		(0.044)
Year FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.00207	0.00208	0.0000365	-0.00419
Observations	123666	123666	90115	90115

Changes within plant-products
Standard errors in parentheses, clustered at the state-industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Columns (3), (4) instrument Δ log sales by the change in the log output tariff.

Smith (1776): "The division of labour is limited by the extent of the market"

▸ Levels ▸ 90s

Other empirical results

- **Vertical specialization comes with a reduction in the number of intermediate inputs**
  Demand shocks $\Rightarrow$ Sales $\Rightarrow$ # Inputs
  ▸ # Inputs ▸ Inverse HHI
- **Tariff supply & demand shocks affect entry.**
  Lower output tariffs decreases the number of plants
  Lower input tariffs increases the number of plants.
  ▸ Table
- **Input tariffs affect input adoption.**
  Lower input tariffs lead to an increased probability of plants using that input.
  ▸ Table

Taking stock

- Vertical span changes with demand $\Rightarrow$ Production is non-homothetic
- Young (1928): Economies of scale? Increasing Returns? Network Externalities?

Key questions for growth: Are there increasing returns? What determines economies of scale? How to identify them?

Unit Costs

Trying to find evidence for network economies

Plant j in industry ω

Trying to find evidence for network economies

Plant j in industry ω

$\uparrow$

j's upstream industries $\hat{\omega}$

Trying to find evidence for network economies

Plant j in industry ω

$\hat{\omega}$'s downstream industries, excluding ω

j's upstream industries $\hat{\omega}$

Trying to find evidence for network economies

1.) output tariff ↘

$\hat{\omega}$'s downstream industries, excluding ω

Plant j in industry ω

j's upstream industries $\hat{\omega}$

Trying to find evidence for network economies

1.) output tariff $\searrow$

$\hat{\omega}$'s downstream industries, excluding ω

Plant j in industry ω

j's upstream industries $\hat{\omega}$

2.) number of producers $\searrow$

Trying to find evidence for network economies

sales impact?

Plant j in industry ω

1.) output tariff ↘

$\hat{\omega}$'s downstream industries, excluding ω

j's upstream industries $\hat{\omega}$

2.) number of producers ↘

Upstream entry and sales

	Dependent variable: log Sales					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg. log #Producers in Upstream Ind.	0.0466**	0.0383**	0.0384**	0.0383*	0.0613**	0.0618**
	(0.0041)	(0.0050)	(0.0050)	(0.017)	(0.017)	(0.017)
$\log(1+\bar{\tau}_{j\omega t}^{\text{input}})$			0.0280			0.0293
			(0.085)			(0.085)
Year FE	Yes			Yes		
Industry $\times$ Year FE		Yes	Yes		Yes	Yes
Plant $\times$ Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.942	0.952	0.952	0.00183	0.000631	0.000621
Observations	215805	199039	198727	215805	199039	198727

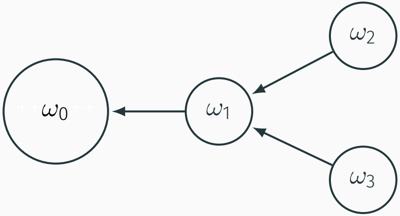
Standard errors in parentheses, clustered at the industry-year level.

$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

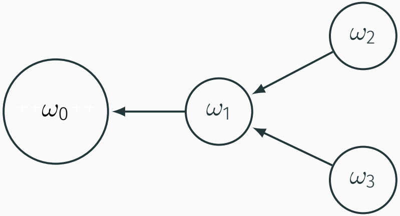
▶ 90s

▶ Sales

Simple Model

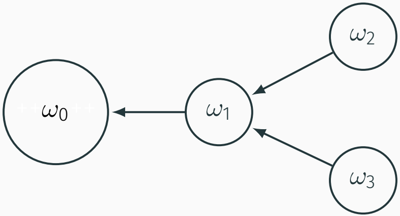


Firm produces good ω_0. Two (perf. substitutable) ways of producing:



Firm produces good ω_0. Two (perf. substitutable) ways of producing:

- Buying ω_1 from a supplier ('shirts from cloth')
- Buying ω_2 and ω_3 from suppliers ('shirts from yarn & dye')



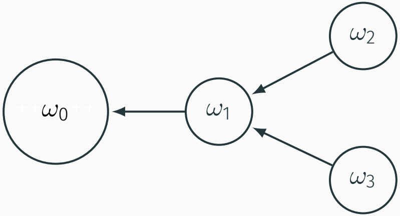
Firm produces good ω_0. Two (perf. substitutable) ways of producing:

1. Buying ω_1 from a supplier ('shirts from cloth')

Firms search for ω_1 suppliers. Search effort h_1.

Cost of production: $c_{j\omega_0} = \frac{1}{q_j} w^{\alpha_l^0} \left(\tilde{c}_j^1\right)^{1-\alpha_l^0}$ $\quad\quad$ $\tilde{c}_j^1 = \mathbf{min}_{s \in S_j^1} \frac{\text{price}_s}{\text{match-specific prod}_{js}}$

Arrival rate of supplier matches + match-specific productivity so that $\tilde{c}_j^1 \sim EV(h_1 v_1, \zeta)$



Firm produces good ω_0. Two (perf. substitutable) ways of producing:

2. **Buying ω_2 and ω_3 from suppliers ('shirts from yarn & dye')**

Firms search for ω_2, ω_3 suppliers. Search efforts h_2, h_3

Cost of production: $c_{j\omega_0} = \frac{1}{q_j} w^{\alpha_l^0} \Big(\underbrace{\frac{1}{b_j} w^{\alpha_l^1} (\tilde{c}_j^2)^{\alpha_2^1} (\tilde{c}_j^3)^{\alpha_3^1}}_{\sim EV\left((h_2 v_2)^{\alpha_2^1} (h_3 v_3)^{\alpha_3^1}, \zeta\right)} \Big)^{1-\alpha_l^0}$

$$\tilde{c}_j^2 \sim EV(h_2 v_2, \zeta), \qquad \tilde{c}_j^3 \sim EV(h_3 v_3, \zeta), \qquad \chi(\log b_j) = \frac{\Gamma(1-\zeta i t)}{\Gamma(1-\alpha_2^1 \zeta i t)\Gamma(1-\alpha_3^1 \zeta i t)}$$

Search problem

- Firm born with productivity q_j, make search choice based only on that.
- Profits from sales to households, isoelastic demand, isoelastic search costs:

$$\max_{\{h\}_i} \quad \mathbb{E}\left(\pi_j | q_j, \{h\}_i\right) - \sum_{i=1,2,3} \frac{k}{1+\gamma} h_i^{1+\gamma}$$

$$A_{\omega_0} q^{\varepsilon-1} \mathbb{E}\left(c_j | q_j, \{h\}_i\right)^{1-\varepsilon} - \sum_{i=1,2,3} \frac{k}{1+\gamma} h_i^{1+\gamma}$$

$$A_{\omega_0} q^{\varepsilon-1} \left[h_1 v_1 + (h_2 v_2)^{\alpha_2^1} (h_3 v_3)^{\alpha_3^1}\right]^{(1-\alpha_l^0)\frac{\varepsilon-1}{\zeta}} - \sum_{i=1,2,3} \frac{k}{1+\gamma} h_i^{1+\gamma}$$

- **Nonhomotheticity:** return from searching in upstream industries (i.e. 2, 3) is more concave than in downstream industry (1).

  $\Rightarrow$ Plants born with high q will be more likely to be vertically specialized (use ω_1 rather than ω_2, ω_3). **Size** $\leftrightarrow$ **Span relationship** in the data

Nonhomotheticity

- A firm with a higher Hicks-neutral productivity q_j will search more in all markets
- But if the elasticity of substitution *across nests* is higher than *within nests* then $\log h_1 \nearrow$ more than $\log h_2 \nearrow$ (or $\log h_3 \nearrow$).
  - Why? When organizational forms are substitutable, x_{ω_1} is more elastic than $(x_{\omega_2}, x_{\omega_3})$ bundle
  - Searching more in upstream industries would increase $(x_{\omega_2}, x_{\omega_3})$ by less, since extra labor also needs to get hired (compared to increase in x_1 from searching more in $\omega - 1$)

Proposition

Under the optimal search effort, the probability of using ω_1 is

- *increasing in Hicks-neutral productivity q,*
- *increasing in the final consumer's demand for ω_0*

Full Model

Demand & entry

Large number of industries ω, each with continuum of firms producing differentiated varieties

Consumption: Representative household has standard nested CES preferences

$$u = \left(\sum_{\omega} \delta_{\omega}^{\frac{1}{\eta}} u_{\omega}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \qquad u_{\omega} = \left(\int_{J_{\omega}} u_{\omega j}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \varepsilon > \eta > 1$$

Market Structure: Firms sell to firms further downstream, and to final consumers.

- Firms price at marginal cost when selling to other firms*
- Firms are monopolistically competitive when selling to final consumers.

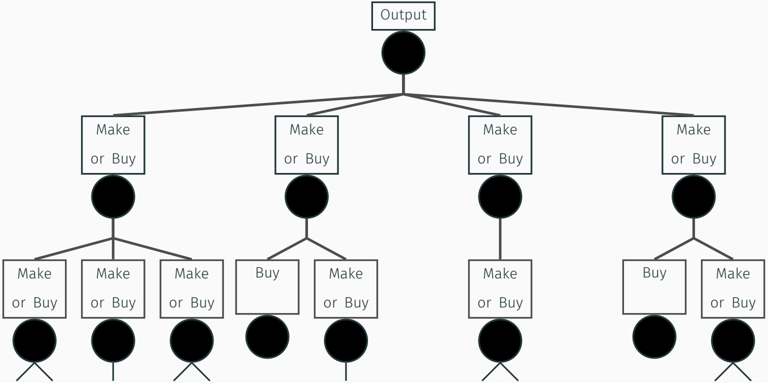
Entry: Representative entrepreneur chooses

$$\max \sum_{\omega} J_{\omega} \bar{\pi}_{\omega} \qquad \text{subject to} \qquad \left(\sum_{\omega} J_{\omega}^{\frac{1+\beta}{\beta}}\right)^{\frac{\beta}{1+\beta}} \leq 1$$

This nests free entry ($\beta \to \infty$) and inelastic entry ($\beta = 0$) as special cases. Assume $\beta < \infty$.

Production: technology menu

Each firm produces using **production modules** that make up a **production tree** (of finite depth):



The firm faces a **make-or-buy decision** for each non-leaf production module.

Production modules: make-or-buy decision

A firm's **effective unit cost of input** $\tilde{\omega}$ (in production tree) is

$$c_{j\tilde{\omega}} = \min\{c^o_{j\tilde{\omega}}, c^i_{j\tilde{\omega}}\}$$

1. **Buy input from supplier**
   - Search effort yields set of potential suppliers, $S_{j\tilde{\omega}}$
   - For each $s \in S_{j\tilde{\omega}}$: price p_s and match-specific productivity z_{js}

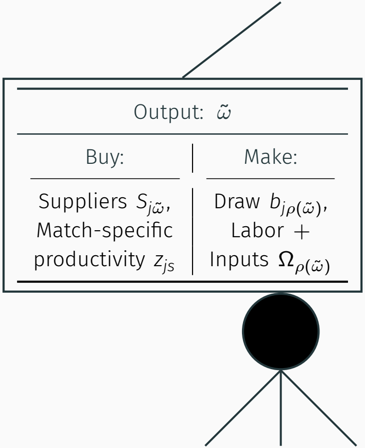
$$c^o_{j\tilde{\omega}} = \min_{s \in S_{j\tilde{\omega}}} \frac{p_j}{z_{js}}$$

2. **Produce in-house** using a production module, $\rho(\tilde{\omega})$
   - Module-specific productivity draw, $b_{j\rho(\tilde{\omega})}$
   - Module prod. fct. is Cobb-Douglas in labor and inputs $\hat{\Omega}_{\rho(\tilde{\omega})}$

$$c^i_{j\tilde{\omega}} = \frac{1}{b_{j\rho(\tilde{\omega})}} w^{\alpha_\ell^{\rho(\tilde{\omega})}} \prod_{\hat{\omega} \in \hat{\Omega}_{\rho(\tilde{\omega})}} c_{j\hat{\omega}}^{\alpha_{\hat{\omega}}^{\rho(\tilde{\omega})}}$$

Firm's effective unit cost for its output ω is

$$c_{j\omega} = \frac{1}{q_j} w^{\alpha_\ell^{\rho(\omega)}} \prod_{\hat{\omega} \in \hat{\Omega}_{\rho(\omega)}} c_{j\hat{\omega}}^{\alpha_{\hat{\omega}}^{\rho(\omega)}}$$



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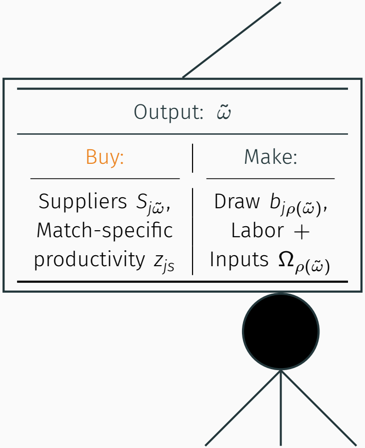

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Firm's effective unit cost for its output ω is

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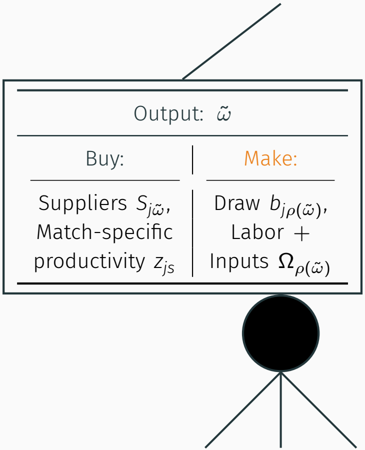
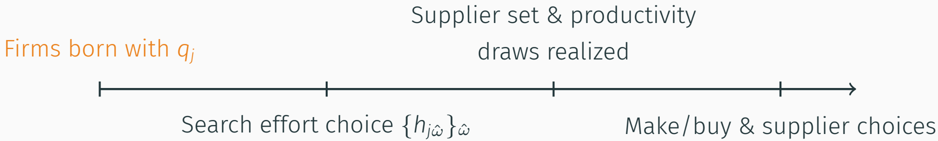


Timing & functional form assumptions



1. **Firms born** with q_j.
   Assume distribution of birth productivities has sufficiently thin tail

2. **Search.** Assume isoelastic and additive search cost, then the firm chooses search efforts $\{h_{j\hat{\omega}}\}_{\hat{\omega}}$ to maximize

$$\max_{\{h_{j\hat{\omega}}\}_{\hat{\omega}\in\hat{\Omega}^{\infty}_{\rho(\omega)}}} E\left[\pi_j | q_j, \{h_{j\hat{\omega}}\}_{\hat{\omega}}\right] - \sum_{\hat{\omega}} \frac{k}{1+\gamma} h_{j\hat{\omega}}^{1+\gamma}$$

Timing & functional form assumptions

Supplier set & productivity draws realized

Firms born with q_j

Search effort choice $\{h_{j\hat{\omega}}\}_{\hat{\omega}}$

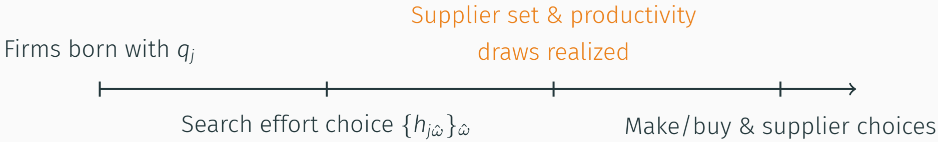
Make/buy & supplier choices

1. **Firms born** with q_j.
   Assume distribution of birth productivities has sufficiently thin tail

2. **Search.** Assume isoelastic and additive search cost, then the firm chooses search efforts $\{h_{j\hat{\omega}}\}_{\hat{\omega}}$ to maximize

$$\max_{\{h_{j\hat{\omega}}\}_{\hat{\omega}\in\hat{\Omega}^{\infty}_{\rho(\omega)}}} E\left[\pi_j | q_j, \{h_{j\hat{\omega}}\}_{\hat{\omega}}\right] - \sum_{\hat{\omega}} \frac{k}{1+\gamma} h_{j\hat{\omega}}^{1+\gamma}$$

Timing & functional form assumptions



3. **Productivity/supplier draws.** If firm j chooses search effort $h_{j\hat{\omega}}$ for input in the production tree, number of supplier with match-specific productivity greater than z is Poisson with mean

$$h_{j\hat{\omega}} m(J_{\hat{\omega}}) z^{-\zeta}$$

Log of module/task productivity $b_{j\rho}$ drawn from dist with characteristic function

$$\frac{\Gamma(1-\zeta i t)}{\prod_{\hat{\omega}\in\hat{\Omega}_\rho}\Gamma(1-\alpha^{\rho}_{\hat{\omega}}\zeta i t)}$$

Distribution is backward engineered to help with discrete choice.

4. **Make/buy & supplier choice** to minimize ex-post cost ($\Leftrightarrow$ maximize profit)

Timing & functional form assumptions

Firms born with q_j

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   Distribution is backward engineered to help with discrete choice.

4. Make/buy & supplier choice to minimize ex-post cost ($\Leftrightarrow$ maximize profit)

With functional form assumptions

Normalize $w = 1$. Then the **distribution of unit cost of an input** $\tilde{\omega}$ conditional on $\{h_{j\hat{\omega}}\}$ is Weibull:

$$P\left(c_{j\tilde{\omega}} > c | \{h_{j\hat{\omega}}\}\right) = e^{-T_{j\rho(\tilde{\omega})}c^{\zeta}}$$

where

$$T_{j\rho(\tilde{\omega})} = \begin{cases} \prod_{\hat{\omega}\in\hat{\Omega}_{\rho(\tilde{\omega})}} \left(h_{j\hat{\omega}}v_{\hat{\omega}} + T_{j\rho(\hat{\omega})}\right)^{\alpha^{\rho}_{\hat{\omega}}}, & \text{input nodes} \\ \prod_{\hat{\omega}\in\hat{\Omega}_{\rho(\tilde{\omega})}} \left(h_{j\hat{\omega}}v_{\hat{\omega}}\right)^{\alpha^{\rho}_{\hat{\omega}}}, & \text{terminal production modules (leaves)} \end{cases}$$

where $v_{\hat{\omega}} \equiv m(J_{\hat{\omega}}) \int_0^{\infty} c^{-\zeta} dF_{\hat{\omega}}(c)$ indexes the cost distribution of suppliers.

$\Rightarrow$ Conditional on requiring input $\tilde{\omega}$, the probability that the firm uses a supplier for it is

$$\frac{h_{j\tilde{\omega}}v_{\tilde{\omega}}}{h_{j\tilde{\omega}}v_{\tilde{\omega}} + T_{j\rho(\tilde{\omega})}}.$$

Demand shocks in ω

Proposition

If δ_ω increases (= positive demand shock),

- *more entry in industry ω: $J_\omega \nearrow$*
- *the price level in industry ω falls: $p_\omega \searrow$ (and $v_\omega \nearrow$)*
- *For each input $\hat{\omega}$, the probability of buying it (rather than making it) increases.*

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- *For each input $\hat{\omega}$, the probability of buying it (rather than making it) increases.*

Intuition:

(1) $\delta_\omega \nearrow \Rightarrow \bar{\pi}_\omega \nearrow \Rightarrow J_\omega$

(2) $\delta_\omega \nearrow \Rightarrow \bar{\pi}_\omega \nearrow \Rightarrow$ search efforts $\nearrow \Rightarrow p_\omega \searrow$

(3) Firms shift search effort toward more downstream suppliers

Demand shocks in upstream industry $\hat{\omega} \in \hat{\Omega}_{\rho(\omega)}$

Proposition

If $\delta_{\hat{\omega}}$ increases ($=$ positive demand shock in the upstream industry), then if γ is sufficiently large (search effort not too elastic):

- *more entry in industry $\hat{\omega}$: $J_{\hat{\omega}} \nearrow$, $v_{\hat{\omega}} \nearrow$*
- *the fraction of ω firms buying $\hat{\omega}$ increases*
- *total sales in industry ω increase*

Demand shocks in upstream industry $\hat{\omega} \in \hat{\Omega}_{\rho(\omega)}$

Proposition

If $\delta_{\hat{\omega}}$ increases (= positive demand shock in the upstream industry), then if γ is sufficiently large (search effort not too elastic):

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- *the fraction of ω firms buying $\hat{\omega}$ increases*
- *total sales in industry ω increase*

Intuition:

(1) As before
- More matches $m\left(J_{\hat{\omega}}\right) \nearrow$
- firms in $\hat{\omega}$ search more $\Rightarrow$ lower cost

(2) $v_{\hat{\omega}} \nearrow$ but $p_{\omega} \searrow$. If γ sufficiently large, $v_{\hat{\omega}} \nearrow$ dominates for all q.

(3) $v_{\omega} \nearrow$ and $p_{\omega} \searrow$, and demand elastic

Going forward

- **Differentiated vs Standardized Inputs** (preliminary) empirical patterns driven by use of differentiated inputs
- **Profits from firm-to-firm trade**
  - Account explicitly for demand shocks from downstream sectors
  - What is internalized?
- **Identification of scale economies** through h and m

Going forward

- **Differentiated vs Standardized Inputs** (preliminary) empirical patterns driven by use of differentiated inputs
- **Profits from firm-to-firm trade**
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  - What is internalized?
- **Identification of scale economies** through h and m

Conclusion:

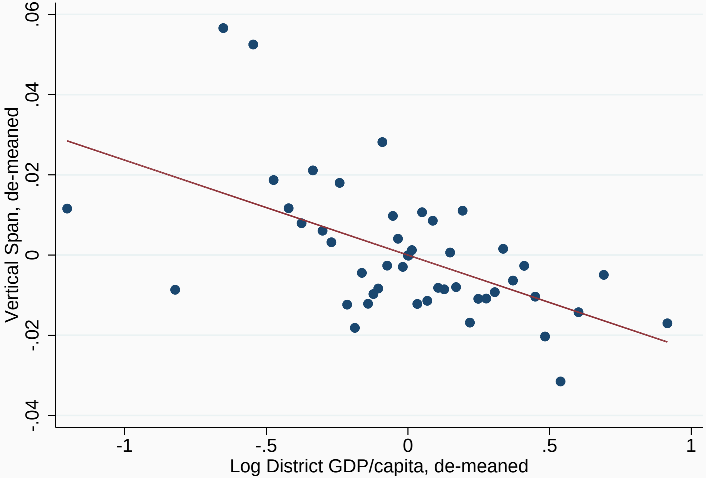
Indian Microdata suggests

- Internal economies of scale from search
- Possibly external economies of scale through matching process

Overall, try to make progress on quantitative models of growth. How important is "Smithian" growth?

Fact 1: In richer districts, plants are more specialized (shorter vertical span)

Within industry × year:



Binscatter with n=50. Y and X variable de-meaned by industry x year. SP plants only.

Back

Fact 2: Increased vertical specialization positively correlated with state growth

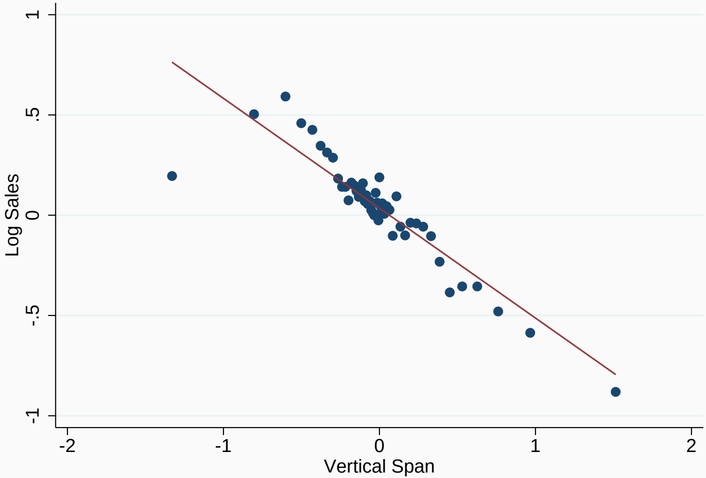
Within plant, over time:

	Dependent variable: Vertical Span		
	(1)	(2)	(3)
Log GDP/capita$_{st}$	-0.0552	-0.0647	-0.0741^{+}
	(0.048)	(0.045)	(0.043)
Year FE	Yes	Yes	Yes
Plant FE	Yes	Yes	
5-digit Industry FE		Yes	
Plant $\times$ 5-digit Industry FE			Yes
R^2	0.644	0.720	0.780
Observations	95727	94754	61073

Standard errors in parentheses, clustered at the state $\times$ 5-dgt industry level. SP plants only.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Fact 3: More specialized plants (shorter span) are larger

Plants with higher sales tend to have shorter vertical span (within industry × year)

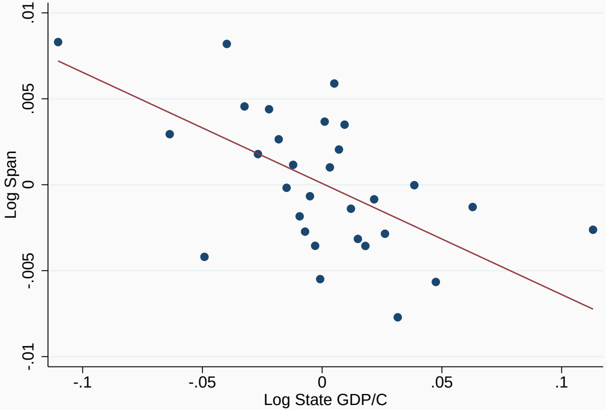


SP plants only. Regression includes industry x year FE's.

Back

Fact 2: Increased vertical specialization is positively correlated with state growth

Within plant×industry, year:



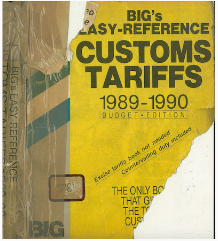
Binscatter with n=30. SP plants only. Variables de-meaned by plant-industry and year.

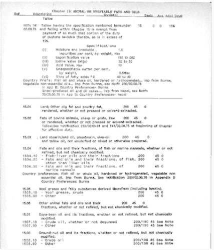
Back

Details on tariff construction

We use UNCTAD tariffs, complemented by hand-digitized effective tariff rates for early years of the liberalization (1990-1996).

- Exclude agricultural tariffs (which changed in response to domestic shocks)
- Exclude mechanical & electrical machinery (HS headings 84, 85): long list of exceptions





Chapter 15: ANIMAL OR VEGETABLE FATS AND OILS

Ref Description Duties→ Basic Aux Addl Total

Tallow

Ntfn 141 02.08.76 Tallow having the specification mentioned hereunder and falling within Chapter 15 is exempt from payment of so much that portion of the duty of customs leviable thereon, as is in excess of 15%. 15 0 0 15%

Specifications

(i)	Moisture and insoluble, impurities per cent, by weight, Max	1.0
(ii)	Saponification value	192 to 202
(iii)	Iodine Value (Wijs)	32 to 50
(iv)	Acid Value, Max	10
(v)	Unsaponifiable matter per cent, by weight,	0.5Max
(vi)	Titre of fatty acids °C	40 to 45

Country Prefs: Fish oil and whale oil, hardened or hydrogenated... imp from Burma, Vegetable non-essential oils... imp from Burma, see Notfn 200/02.08.76 in App B: Country Preferences- Burma
Ghani-produced oil and oil cakes.... imp from Nepal, see Notfn 70/25.03.78 in App D: Country Preferences- Nepal

Ref	Description	Basic	Aux	Addl
15.01	Lard; Other pig fat and poultry fat, rendered, whether or not pressed or solvent-extracted.	200	45	0
15.02	Fats of bovine animals, sheep or goats, raw or rendered, whether or not pressed or solvent-extracted. Tallow:See Notification 213/02.09.81 and 141/02.08.75 at beginning of Chapter for effective duty.	200	45	0
15.03	Lard stearin,lard oil, oleostearin, oleo-oil and tallow oil, not emulsified or mixed or otherwise prepared.	200	45	0
15.04	Fats and oils and their fractions, of fish or marine mammals, whether or not refined, but not chemically modified.			
1504.10	- Fish-liver oils and their fractions	200	45	0
1504.20	- Fats and oils and their fractions, of fish, other than liver oils	200	45	0
1504.30	- Fats and oils and their fractions, of marine mammals	200	45	0

Country preferences: Fish oil or whale oil, hardened or hydrogenated, vegetable non essential oil. Imp from Burma. See Notification 200/02.08.76 in Appendix B Country Preferences: Burma

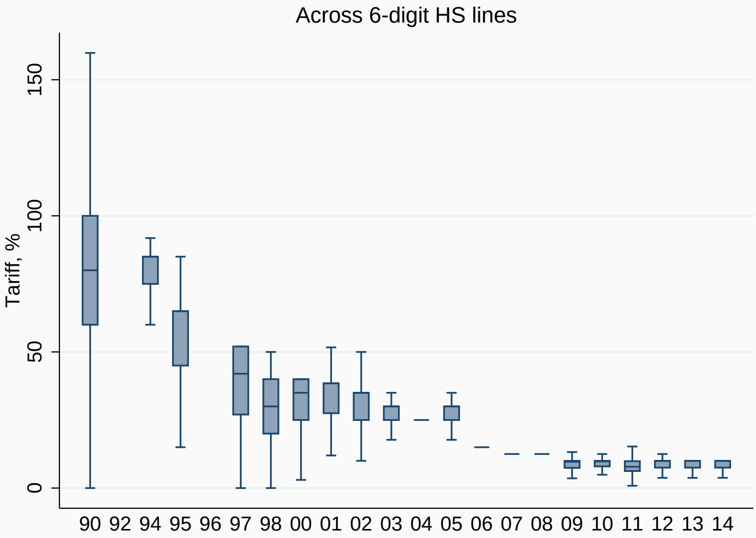
Ref	Description	Basic	Aux	Addl
15.05	Wool grease and fatty substances derived therefrom (including lanolin).			
1505.10	- Wool grease, crude	200	45	0
1505.90	- Other	200	45	0
15.06	Other animal fats and oils and their fractions, whether or not refined, but not chemically modified.	200	45	0
15.07	Soya-bean oil and its fractions, whether or not refined, but not chemically modified.			
1507.10	- Crude oil, whether or not degummed	200/190	45	See Note
1507.90	- Other	200/190	45	See Note
15.08	Ground-nut oil and its fractions, whether or not refined, but not chemically modified.			
1508.10	- Crude oil	200/190	45	See Note
1508.90	- Other	200/190	45	See Note

Plants with shorter span are larger: Details

	Dependent variable: Log Sales				
	(1)	(2)	(3)	(4)	(5)
Vertical Span	-0.719**	-0.670**	-0.431**	-0.432**	-0.193**
	(0.024)	(0.023)	(0.034)	(0.034)	(0.015)
Age				0.00616**	-0.00314**
				(0.0012)	(0.00068)
Log Employment					1.067**
					(0.015)
Year FE	Yes	Yes	Yes	Yes	Yes
5-digit Industry FE	Yes	Yes			
District FE		Yes			
Industry × District × Year FE			Yes	Yes	Yes
R^2	0.394	0.440	0.700	0.701	0.859
Observations	353659	295094	140610	136831	136608

Standard errors in parentheses, clustered at the 5-dgt industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Import Tariffs, India, 1990-2014



Changes since 1990: tariffs and sales

	Dep. var.: Δ^t_{1990} log Sales	
	(1)	(2)
$\Delta^t_{1990} \log(1+\tau^{\text{output}}_{\omega t})$	1.394^{+}	1.635^{*}
	(0.75)	(0.79)
$\Delta^t_{1990} \log(1+\bar{\tau}^{\text{input}}_{\omega t})$		-1.227
		(0.77)
Year FE	Yes	Yes
R^2	0.0873	0.0927
Observations	2376	2376

Standard errors in parentheses, clustered at the state $\times$ industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Changes since 1990: vertical span and demand

	Dependent variable: Δ_{1990}^{t} Vertical Span	
	(1)	(2)
Δ_{1990}^{t} log Sales	-0.127^{+}	-0.147^{+}
	(0.075)	(0.077)
Δ_{1990}^{t} $\log(1+\bar{\tau}_{it}^{\text{input}})$		0.208
		(0.23)
Year FE	Yes	Yes
R^2	-0.139	-0.192
Observations	2179	2179

Standard errors in parentheses, clustered at the state $\times$ industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Columns (1), (2) instrument Δ_{1990}^{t} log sales by the change in the log output tariff since 1990.

Back

Vertical span and demand, in changes

	Dependent variable: Vertical Span			
	(1)	(2)	(3)	(4)
Log Sales	-0.0191**	-0.0190**	-0.472+	-0.296+
	(0.0020)	(0.0020)	(0.25)	(0.17)
$\log(1+\bar{\tau}_{j\omega t}^{\text{input}})$		-0.0870**		-0.123*
		(0.026)		(0.050)
Year FE	Yes	Yes	Yes	Yes
Plant $\times$ Product FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.765	0.765	-0.878	-0.327
Observations	186628	186628	137387	137387

Standard errors in parentheses, clustered at the state-industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Columns (3), (4) instrument log sales by the log output tariff.

Back

Demand $\nearrow \Rightarrow$ firms reduce the actual number of inputs

	Dependent variable: # Inputs			
	(1)	(2)	(3)	(4)
Log Sales	0.0477**	0.0479**	-1.321*	-0.674*
	(0.0033)	(0.0032)	(0.64)	(0.34)
$\log(1 + \bar{\tau}_{j\omega t}^{\text{input}})$		-0.244**		-0.369**
		(0.086)		(0.10)
Year FE	Yes	Yes	Yes	Yes
Plant $\times$ Product FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.871	0.872	-6.543	-1.816
Observations	188868	188803	138938	138898

Changes within plant-products
Standard errors in parentheses, clustered at the state-industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Back

Firms are more likely to adopt inputs when faced with a cost decrease

	Dependent variable: Input Used Dummy $\mathbf{1}(X_{j\hat{\omega}t} > 0)$	
	(1)	(2)
$\log(1 + \tau_{it})$	-0.0506**	-0.0373**
	(0.0067)	(0.0071)
Year FE	Yes	Yes
Plant × Input FE	Yes	Yes
Plant × Product FE		Yes
R^2	0.337	0.361
Observations	2460831	2454899

Standard errors in parentheses.
$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Back

Supply and demand shifters determine entry

	Dependent variable: log Producers $\lvert J\rvert_{d\omega t}$	
	(1)	(2)
$\log(1+\bar{\tau}_{it}^{\text{input}})$	-0.108**	-0.0496**
	(0.025)	(0.015)
$\log(1+\tau_{it}^{\text{output}})$	0.186**	0.251**
	(0.021)	(0.013)
Year FE	Yes	
State FE	Yes	
Industry FE	Yes	
State $\times$ Year FE		Yes
State $\times$ Industry FE		Yes
R^2	0.481	0.844
Observations	548180	537013

Standard errors in parentheses.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

The left-hand side is the log number of producers of a good ω at time t in state d.

Back

Firms reduce the effective number of inputs when demand $\nearrow$

	Dependent variable: Inverse Input HHI			
	(1)	(2)	(3)	(4)
Log Sales	0.0101	0.0104	-1.888^{+}	-1.055^{+}
	(0.0072)	(0.0069)	(1.03)	(0.59)
$\log(1+\bar{\tau}_{j\omega t}^{\text{input}})$		-0.411^{+}		-0.498*
		(0.25)		(0.24)
Year FE	Yes	Yes	Yes	Yes
Plant $\times$ Product FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.807	0.808	-3.437	-1.076
Observations	192809	192809	142270	142270

Changes within plant-products
Standard errors in parentheses, clustered at the state-industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Effective # of inputs measured by the inverse of the HHI of cost shares. Results similar for

Back

Sample of 1990 plants: upstream industry size and sales

	Dependent variable: log Sales					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg. log #Producers in Upstream Ind.	0.0655**	0.0560**	0.0551**	0.0201	0.119**	0.115**
	(0.013)	(0.018)	(0.018)	(0.043)	(0.044)	(0.044)
$\log(1+\bar{\tau}^{\text{input}}_{j\omega t})$			0.540*			0.519*
			(0.26)			(0.26)
Year FE	Yes			Yes		
Industry × Year FE		Yes	Yes		Yes	Yes
Plant × Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.916	0.943	0.943	0.00262	-0.000638	0.000690
Observations	13683	9768	9757	13683	9768	9757

Standard errors in parentheses, clustered at the industry-year level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Sample: all SP plants observed in 1990
(except (3) and (6) which further condition on $t \leq 2000$)

Upstream industry size and sales

	Dependent variable: log Sales					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg. log Sales in Upstream Ind.	0.00367**	0.00251**	0.00251**	0.00642*	0.00930**	0.00936**
	(0.00034)	(0.00038)	(0.00038)	(0.0029)	(0.0026)	(0.0026)
$\log(1+\bar{\tau}^{\text{input}}_{j\omega t})$			0.0241			0.0193
			(0.085)			(0.086)
Year FE	Yes			Yes		
Industry × Year FE		Yes	Yes		Yes	Yes
Plant × Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.942	0.952	0.952	0.000572	-0.00304	-0.00311
Observations	215805	199039	198727	215805	199039	198727

Standard errors in parentheses, clustered at the industry-year level.

$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Back

Fact 4: Sales growth is correlated with increased vertical specialization

	Dependent variable: Δ log Sales			
	(1)	(2)	(3)	(4)
Δ Vertical Span	-0.0693**	-0.0668**	-0.0577**	-0.0546**
	(0.0085)	(0.0085)	(0.011)	(0.011)
Δ R-Share in Materials	-0.0242*	-0.0247*	-0.0270^{+}	-0.0346*
	(0.012)	(0.012)	(0.015)	(0.014)
Δ Vertical Span × Δ R-Share in Materials	-0.0359*	-0.0408*	-0.0549*	-0.0544*
	(0.016)	(0.016)	(0.025)	(0.023)
Constant	0.194**	0.194**	0.181**	0.171**
	(0.0046)	(0.0025)	(0.0015)	(0.00030)
Year FE	Yes	Yes	Yes	Yes
Industry FE		Yes		
Plant FE			Yes	
Plant × Product FE				Yes
R^2	0.00825	0.0409	0.305	0.314
Observations	116199	115643	89440	80377

Unit Costs and Tariff changes

	Dependent variable: Δ^t_{1990} log Unit Cost	
	(1)	(2)
Δ^t_{1990} log$(1+\tau^{\text{output}}_{it})$	-0.789**	-0.949**
	(0.10)	(0.17)
Δ^t_{1990} log$(1+\bar{\tau}^{\text{input}}_{j\omega t})$		0.226
		(0.17)
Year FE	Yes	Yes
R^2	0.0566	0.0583
Observations	920	916

Standard errors in parentheses, clustered at the state $\times$ industry level.
$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Generalizations

Extreme value math extends to any finite "production tree"

- Any (finite) number of inputs in each stage
- Any (finite) depth of the tree

Conditional on search effort choices, the distributions of input unit costs are EV

Search choices depend on Hicks-neutral productivity and upstream cost distributions

$\Rightarrow$ solve search problem recursively starting with most upstream (leaf) nodes

Full Model:

- (Imperfectly) elastic entry into industries ω on a large "production tree"
- Positive profits from sales to households, marginal cost pricing to firms further downstream
- Firms born with Hicks-neutral q. Increasing returns to scale through input search.
- Potentially network economies through arrival rate of draws also depending on upstream sector characteristics.

Conclusion

Discrete Choice Math

- Lowest cost way of acquiring good $\omega - 1$

$$\min\left\{\min_{s\in S_1}\frac{p_s}{z_s}\ ,\ \frac{1}{b_2}\min_{s\in S_2} w^{1-\alpha}\left(\frac{p_s}{z_s}\right)^{\alpha}\right\}$$

- Arrival of suppliers with $z_s > z$ is Poisson with arrival rate $\propto z^{-\zeta}$

$$\min_{s\in S_1}\frac{p_s}{z_s} \sim Weibull(scale_1, \zeta) \tag{1}$$

$$\min_{s\in S_2} w^{1-\alpha}\left(\frac{p_s}{z_s}\right)^{\alpha} \sim Weibull\left(scale_2, \frac{\zeta}{\alpha}\right)$$

(3)

Back

Discrete Choice Math

- Lowest cost way of acquiring good $\omega - 1$

$$\min \left\{ \min_{s \in S_1} \frac{p_s}{z_s} \, , \, \frac{1}{b_2} \min_{s \in S_2} w^{1-\alpha} \left(\frac{p_s}{z_s} \right)^{\alpha} \right\}$$

- Arrival of suppliers with $z_s > z$ is Poisson with arrival rate $\propto z^{-\zeta}$

$$\min_{s \in S_1} \frac{p_s}{z_s} \sim Weibull(scale_1, \zeta) \tag{1}$$

$$\min_{s \in S_2} w^{1-\alpha} \left(\frac{p_s}{z_s} \right)^{\alpha} \sim Weibull \left(scale_2, \frac{\zeta}{\alpha} \right) \tag{2}$$

$$\frac{1}{b_2} \min_{s \in S_2} w^{1-\alpha} \left(\frac{p_s}{z_s} \right)^{\alpha} \sim Weibull \left(scale_3, \zeta \right) \tag{3}$$

- Follows from:

$$Z \sim \text{standard exponential}, \; Y \sim \alpha\text{-Stable} \qquad \Rightarrow \qquad (Z/Y)^{\alpha} \sim Z$$

Back

Nested CES Example

Imagine the production function was a Nested CES:

$$y_j = q\left\{(A_1 h_1 x_1)^{\frac{\eta-1}{\eta}} + \left[(A_0 l)^{\frac{\phi-1}{\phi}} + (A_2 h_2 x_2)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}}$$

Proposition

If $\gamma \geq \eta - 2$ and $\gamma \geq \phi - 2$, then

$$\frac{d\ln h_1}{d\ln q} > \frac{d\ln h_2}{d\ln q} \qquad \textit{iff} \qquad \eta > \phi$$

Our setting is a special case with $\eta \to \infty$ and $\phi \to 1$.

Where does the nonhomotheticity come from?

- Imagine a production function where search effort is factor-augmenting.

$$\max_{h_1,h_2} \delta g \left\{ C\left(w, \frac{p_1}{h_1}, \frac{p_2}{h_2} \right) \right\} - \frac{h_1^{1+\gamma}}{1+\gamma} - \frac{h_2^{1+\gamma}}{1+\gamma}$$

- *Levels* of optimal search effort are determined by cost shares:

$$0 = -\delta g' \frac{p_i}{h_i^2} C_i \left(w, \frac{p_1}{h_1}, \frac{p_2}{h_2} \right) - h_i^{\gamma}$$

- Relative elasticity of h_1 vs h_2 is therefore determined by relative *elasticity* of cost shares ... and these are encoded in the Morishima elasticities of substitution σ_{21}, σ_{12}
- If γ sufficiently large, $d \log h_1 / d \log q > d \log h_2 / d \log q$ iff $\sigma_{21} > \sigma_{12}$.
- In particular that's satisfied when there is perfect substitutability between a nested and non-nested production function:

$$y_j = \begin{cases} q_j f(l_{j0}, x_{j1}) & \text{or} \\ q_j f(l_{j0}, g(l_{j1}, x_{j2})) \end{cases}$$

(assuming g is imperfectly substitutable...)