Solutions to Problem Set 2

EC442 Macroeconomics Part 2 (Per Krusell)

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1 Question 1

(a) The flow value equations for employed workers, unemployed workers on benefits, and unemployed workers on social assistance are, respectively,

$$\begin{aligned} rW(w) &= w - \sigma(W(w) - U) && (1)\\ rU &= b_u + \lambda\left(1 - F(w^*)\right)\mathbb{E}_w\left(W(w) - U|w \geq w^*\right) - \rho\left(U - S\right) && (2)\\ rS &= b_s + \lambda\left(\left(1 - F(w^*)\right)\mathbb{E}_w\left(W(w) - S|w \geq w^*\right) + F(w^*)\left(U - S\right)\right) && (3) \end{aligned}$$

The last equation arises because when unemployed workers on social assistance receive a job offer, they always accept it: in case the wage offer is above the reservation wage for unemployed workers w^*, they take and keep the job; in case it is below w^* they take the job but quit immediately, thus entering the pool of unemployed workers with flow value $U > S$.

(b) Rewriting (3) we get

$$rS = b_s + \lambda\left(U - S\right) + \lambda\left(\left(1 - F(w^*)\right)\mathbb{E}_w\left(W(w) - U|w \geq w^*\right)\right)$$

and plugging in (2) to get rid of the conditional expectation, we obtain

$$U - S = \frac{b_u - b_s}{\lambda + r + \rho} \tag{4}$$

Intuitively, the difference in value between being on benefits and being on social assistance is increasing in the difference between the flow payments, and decreasing in the transition flow probabilities.

Next, rewrite (1)

$$\left(r + \sigma\right)\left(W(w) - U\right) = w - rU$$

and take conditional expectations,

$$\mathbb{E}_w\left(W(w) - U|w \geq w^*\right) = \frac{\mathbb{E}_w\left(w|w \geq w^*\right) - rU}{r + \sigma}$$

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and plug this together with (4) into (2) to get

$$rU = b_u + \lambda\left(1 - F(w^*)\right)\frac{\mathbb{E}_w\left(w|w \geq w^*\right) - rU}{r+\sigma} - \rho\frac{b_u - b_s}{\lambda + r + \rho}. \tag{5}$$

Finally, note that $W(w^*) = U$ and (1) implies that $rU = w^*$, thus (5) simplifies to

$$w^* = b_u + \lambda\left(1 - F(w^*)\right)\frac{\mathbb{E}_w\left(w|w \geq w^*\right) - w^*}{r+\sigma} - \rho\frac{b_u - b_s}{\lambda + r + \rho}.$$

(c) We have that $\lambda\left(1 - F(w^*)\right)\left(\mathbb{E}_w\left(w|w \geq w^*\right) - w^*\right)/\left(r+\sigma\right)$ is decreasing in w^* hence w^* is increasing in b_s. Intuitively, when social assistance is lower, agents will choose a lower reservation wage because the payoff loss from falling into social assistance is higher.

(d) One way to calibrate the flow probabilities would be to match the average duration of remaining in the respective states (e.g. the average duration of being employed is $1/\sigma$ in the model, the average duration of unemployment benefits (conditional on staying unemployed) is $1/\rho$, the average duration of unemployment is $1/(\lambda(1 - F(w^*)))$). One can then pick b_u and b_s to match the average relative sizes of the three pools, or, preferably, directly using data on unemployment benefits and social assistance.

2 Question 2

(a) Flow value equations, assuming that $W > U$ so that job offers get accepted:

$$rW = w - \sigma\left(W - U\right) \tag{6}$$
$$rU = b_u - \frac{A}{2}e^2 + \lambda e\left(W - U\right) \tag{7}$$

(b) Workers choose e in every infinitesimal time interval. Since U is the value function, by the envelope theorem, $\partial W/\partial e = \partial U/\partial e = 0$. Hence, the optimal effort satisfies the first-order condition

$$Ae = \lambda\left(W - U\right) \tag{8}$$

and thus $e = \lambda\left(W - U\right)/A$. Subtracting (7) from (6) yields

$$\left(r+\sigma\right)\left(W - U\right) = w - b_u + \frac{A}{2}e^2 - \lambda e\left(W - U\right)$$

and plugging in (8) yields

$$\frac{A}{2}e^2 + \left(r+\sigma\right)\frac{A}{\lambda}e - \left(w - b_u\right) = 0$$

Since $\left(r+\sigma\right)A/\lambda > 0$, this equation has only one positive solution e^*.

(c) It is easy to show that

$$\frac{\partial e^*}{\partial w} > 0, \quad \frac{\partial e^*}{\partial b_u} < 0, \quad \frac{\partial e^*}{\partial A} < 0$$
$$\frac{\partial e^*}{\partial \lambda} > 0, \quad \frac{\partial e^*}{\partial \sigma} < 0, \quad \frac{\partial e^*}{\partial r} < 0.$$

3 Question 3

(a) Same as in Question 1, but with $\rho = 0$ and $b_u = \theta w$. Hence, the value of unemployment and the reservation wage now depend on the most recent wage w,

$$rW(w) = w - \sigma(W(w) - U(w)) \tag{9}$$

$$rU(w) = \theta w + \lambda \int_{w^*(w)}^{\infty} (W(w') - U(w))\, dF(w') \tag{10}$$

(b) Take the derivative of (10) and use the Leibniz rule to get

$$\begin{aligned} rU'(w) &= \theta + \lambda \left(\int_{w^*(w)}^{\infty} (-U'(w))\, dF(w') - (W(w^*(w)) - U(w)) \right) \\ &= \theta - \lambda \left(1 - F(w^*(w))\right) U'(w) \end{aligned}$$

since $W(w^*(w)) = U(w)$ for all w. Rearranging, we get

$$U'(w) = \frac{\theta}{r + \lambda\left(1 - F(w^*(w))\right)}.$$

and, using (9),

$$W'(w) = \frac{1}{r+\sigma} + \frac{\sigma}{r+\sigma} U'(w) = \frac{1}{r+\sigma} + \frac{\sigma}{r+\sigma} \frac{\theta}{r + \lambda\left(1 - F(w^*(w))\right)}. \tag{11}$$

Armed with these expressions, there are now several ways to write down an equation that determines the reservation wage function $w^*(w)$. For example, write down (9), use $W(w^*(w)) = U(w)$, and take the derivative to get

$$(r+\sigma)\, W'(w) = 1 + \sigma W'(w^*(w)) \frac{dw^*(w)}{dw}.$$

Then plug in (11) to get

$$\frac{\sigma\theta}{r + \lambda\left(1 - F(w^*(w))\right)} = \frac{\sigma}{r+\sigma} \left(1 + \frac{\theta\sigma}{r + \lambda\left(1 - F(w^*(w^*(w)))\right)} \right) \frac{dw^*(w)}{dw}$$

which is a differential equation for $w^*(w)$ with no endogenous variables left.

Alternatively, use partial integration in (10) to obtain

$$\begin{aligned} rU(w) &= \theta w + \lambda \int_{w^*(w)}^{\infty} (W(w') - U(w))\, dF(w') \\ &= \theta w + \lambda \int_{w^*(w)}^{\infty} W'(w') \left(1 - F(w')\right) dw', \end{aligned}$$

hence

$$rU(w) = \theta w + \lambda \int_{w^*(w)}^{\infty} \left(\frac{1 - F(w')}{r+\sigma} + \frac{\sigma}{r+\sigma} \frac{\theta\left(1 - F(w')\right)}{r + \lambda\left(1 - F(w^*(w'))\right)} \right) dw'$$

and with

$$(r+\sigma)\,U(w) = w^*(w) + \sigma U(w^*(w))$$

we get the (rather lengthy) equation

$$\frac{r+\sigma}{r}\theta w + \lambda\frac{r+\sigma}{r}\int_{w^*(w)}^{\infty}\left(\frac{1-F(w')}{r+\sigma} + \frac{\sigma}{r+\sigma}\frac{\theta\,(1-F(w'))}{r+\lambda\,(1-F(w^*(w')))}\right)dw' =$$
$$\left(1+\frac{\sigma}{r}\theta\right)w^*(w) + \lambda\frac{\sigma}{r}\int_{w^*(w^*(w))}^{\infty}\left(\frac{1-F(w')}{r+\sigma} + \frac{\sigma}{r+\sigma}\frac{\theta\,(1-F(w'))}{r+\lambda\,(1-F(w^*(w')))}\right)dw'$$

(c) The steady-state wage distribution can be found by equating the inflow into each infinitesimal wage bracket with the outflow. Let $g_E(w)$ be the density of employed agents with wage w, and $g_U(w)$ the density of unemployed agents with former wage w, then

$$\begin{aligned} g_E(w)\sigma &= \int_{w_{\min}}^{w_{\max}} \lambda g_U(w') f(w') 1_{(w' \geq w^*(w))} dw' \\ \lambda\,(1-F(w^*(w))\,g_U(w) &= \sigma g_E(w) \end{aligned}$$

In this case, the wage distribution is not just the truncated wage offer distribution: the unemployed with lower former wages will have a lower reservation wage, therefore, the steady-state wage distribution will be more dispersed.

4 Second part - Question 1 (Sketch)

(a) The first-order conditions are

$$\begin{aligned} c_t &= c_{t+1} \\ (1-\theta)\,c_t &= \theta w_t\,(1-n_t)\,. \end{aligned}$$

From the intertemporal budget constraint we get

$$0 = a_0 = \sum_{t=0}^{\infty}\beta^t\,(c_t - w_t n_t)$$

and hence $c = wn$ and

$$c_t = \theta w$$

as in problem set 1.

(b) From (IBC) and the first-order conditions we get

$$\frac{1}{1-\beta}c = \sum \beta^t\left(-\frac{1-\theta}{\theta}c\right) + \overline{w} + \beta\underline{w} + \beta^2\overline{w} + \beta^3\overline{w} + \ldots$$

hence

$$c = \frac{\theta}{1+\beta}\,(\overline{w} + \beta\underline{w})\,.$$

Asset holdings, labor supply, etc can be obtained from the intertemporal budget constraint and the first-order conditions.

(c) Analogously to (b),

$$c = \frac{\theta}{1+\beta}\left(\underline{w} + \beta\overline{w}\right)$$

(d) Calculate asset holdings from the IBC,

$$\begin{aligned} a_0 &= 0 \\ a_1 &= \frac{\beta}{1+\beta}\left(\overline{w} - \underline{w}\right) > 0 \\ a_2 &= 0 \\ a_3 &= a_1 \\ &\vdots \end{aligned}$$

Hence, the borrowing constraint

$$a_{t+1} \geq 0$$

will never actually constrain the household.

(e) The consumption level c_0 in (c) is greater than income at $t = 0$, hence the borrowing constraint becomes binding, and we have $a_1 = 0$. From $t = 1$ onwards, the situation is exactly as $t = 0$ in (b), with exactly the same consumption patterns. Hence, the borrowing constraint will only bind at $t = 0$.

(f) Use the FOCs.