0.1 PS7 Q3

Household borrowing constraint:

$$c \leq y + \frac{pH}{1+r} \tag{1}$$

Why this constraint and not the more obvious $c \leq y + pH$? Assume the bank can seize the house if the household does not repay the loan in period 2. If a household borrows more than $pH/(1+r)$ in period 1, the value of the loan in period 2 will be more than pH, in which case the household has a strategic incentive to default. Thus, the bank will be unwilling to lend the household more than $pH/(1+r)$ in period 1.

Assume that preferences are such that the borrowing constraint is binding, i.e.

$$c_1 = y_1 + \frac{pH}{1+r}$$

From the intertemporal budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + \frac{pH}{1+r}$$

we can calculate c_2:

$$c_2 = y_2$$

Consider now a negative shock to the value of the housing stock in the first period, $\Delta pH < 0$. Again, assume that the preferences are such that the household ends up being constrained[1] by the borrowing constraint (1). As above, we get

$$\begin{aligned} c_1 &= y_1 + \frac{pH - \Delta pH}{1+r} \\ c_2 &= y_2 \end{aligned}$$

This means that $\Delta c_1 = \Delta pH/(1+r)$ and $\Delta c_2 = 0$, i.e. the impact of the shock is entirely on consumption in the first period. If there is no borrowing constraint (or it is not binding), the household smoothes the impact of the shock over both periods (as discussed in class).

[1] This is the case if the preferences are homothetic in c_1, c_2.