

# Trade and the End of Antiquity

## A Quantitative Investigation of the Pirenne Thesis

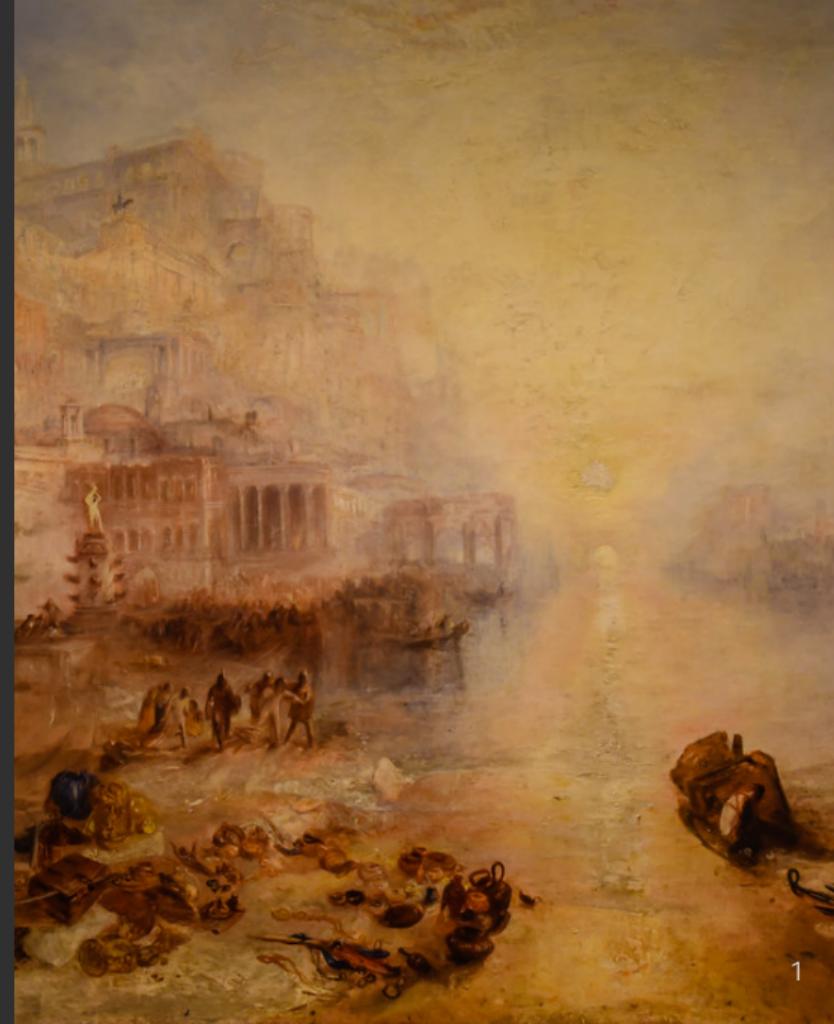
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# What caused the End of Antiquity?

- Antiquity: Roman and Greek civilizations centered around the Mediterranean
- End of antiquity circa 7th-8th Century AD:
  - Economic activity shifts away from the Mediterranean.
  - Rise of Northern Europe (Charlemagne).

→ *Question:* What caused the End of Antiquity?

- Discussed, among others, by Montesquieu (1734), Voltaire (1756), Gibbon (1789)

# Pirenne Hypothesis

Henri Pirenne (1937), “*Mahomet et Charlemagne*”

- Rise of the Islamic Caliphate disrupts Mediterranean trade/exchanges.
- Causes a shift of economic activity away from the Mediterranean.
- Rise of the Carolingian Empire in Northern Europe.



## Political changes in the Mediterranean: 600 AD



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## Political changes in the Mediterranean: 600 AD



## Political changes in the Mediterranean: 632 AD



## Political changes in the Mediterranean: 634 AD



## Political changes in the Mediterranean: 644 AD



# Political changes in the Mediterranean: 661 AD

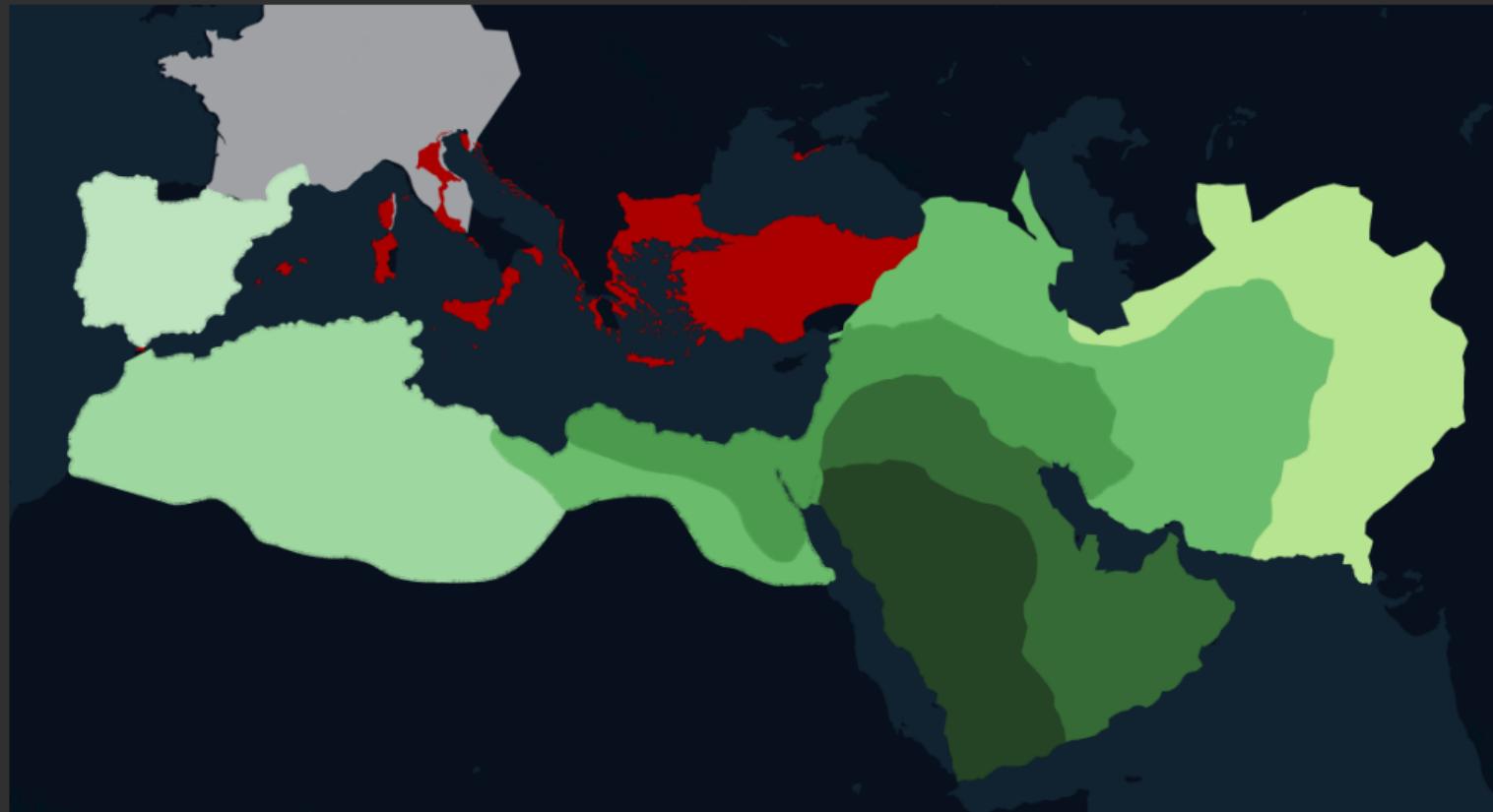


# Political changes in the Mediterranean: 661-700 AD



# Political changes in the Mediterranean:

750 AD



# This paper

We use data on the movement of coins to study the changing economic geography during Late Antiquity

1. Coins and goods/services travel in opposite directions:  
→ Ancient coins record traces of ancient trade flows.
2. Trade flows are shaped by gravity:  
→ (*Reconstructed*) trade flows inform:
  - (a) bilateral trade frictions
  - (b) origin/destination “sizes”
3. Time-varying (*estimated*) trade frictions:  
→ Test + quantify the Pirenne hypothesis.

# Outline

Data

Reduced form evidence

Model

Parameterization and Estimation

# Data: Coins around the Mediterranean, AD 325 to AD 950

Assemble a large dataset of coin finds from around the Mediterranean

## 1. FLAME (2023) project:

- “Framing the Late Antique and Early Medieval Economy.”
- Collaboration of >60 historians and numismatists.
- ~200,000 coins with complete records from ~4,600 hoards
- Pre AD 725.

## 2. Hand-coded records from numismatic / archaeological literature:

- 797 coin finds.
- 100,478 coins.
- post AD 725.

Data covers most of published literature on hoard records (and more)

## Coin hoard data: an example from al 'Ush (1972)

No.	MINT	DATE	DIAM.	WEIGHT	NUMB.
51	الأندلس	114	29.	2.93	4
52	"	115	29.5	2.92	1
53	"	116	26.5	2.92	3

Excerpt of an original publication on the Damascus silver hoard:

- record number (51)
- mint (al-Andalus) \*
- mint date (year 114 of the Hijri calendar) \*
- diameter (29mm)
- weight (2.93g)
- number of coins with these attributes (4) \*

The issuing dynasty (Umayyad) is given in the table headings and the denomination and material (silver *dirham*) is stated in the text. \* denotes required attributes

## Coin dataset

- Each coin provides the following information:
  1. Mint location (“birthplace”):  $m$
  2. Mint date (“birthdate”):  $\tau$
  3. Hoard location (“death place”):  $h$
  4. Terminus post quem,  $tpq$  (“death date”):  $T = \sup \tau$

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  4. Terminus post quem,  $tpq$  (“death date”):  $T = \sup \tau$
- 286,035 coins.
- Time:
  - Mint date > AD 325
  - $tpq$  < AD 950.
- Space:
  - Western Europe.
  - Southern Europe.
  - Northern Africa.
  - Middle East++.

## Spatial distribution: the (extended) Mediterranean



Figure 1: Region Definitions

# Distribution of coin “death dates” ( $tpq$ )

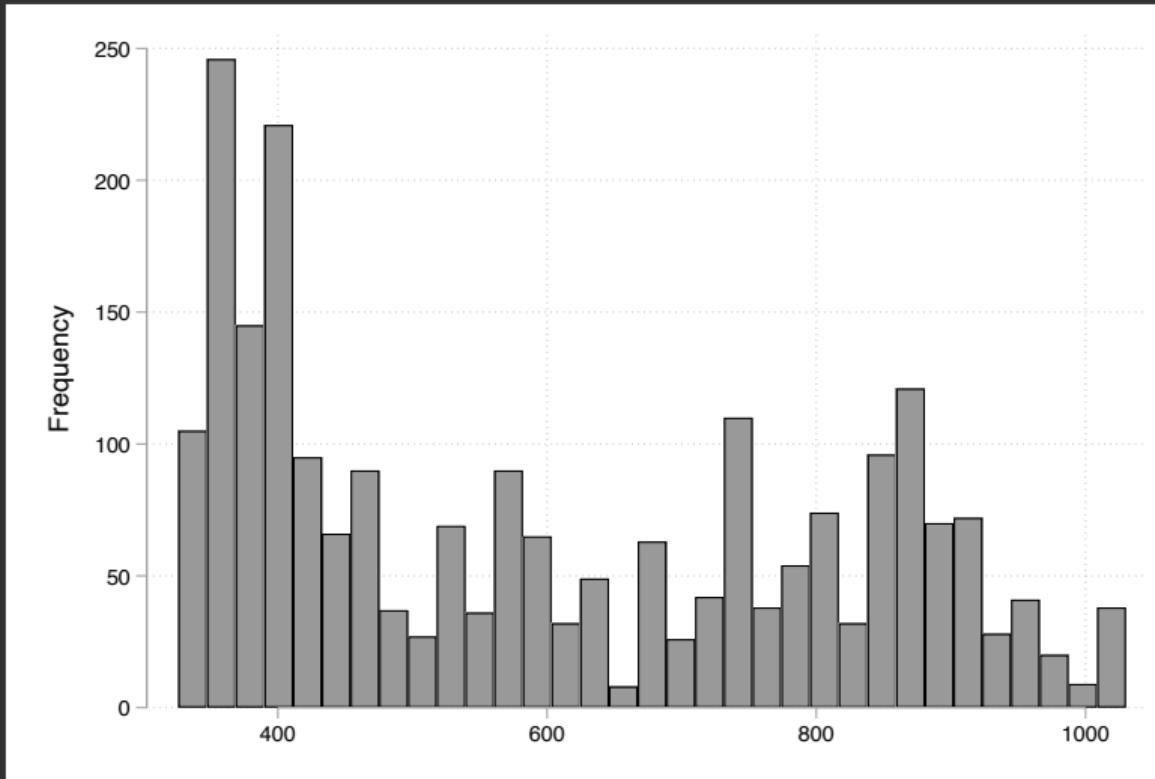


Figure 2: Terminus Post Quem ( $tpq$ ) of hoards

## Distribution of coin ages ( $tpq$ minus mint date)

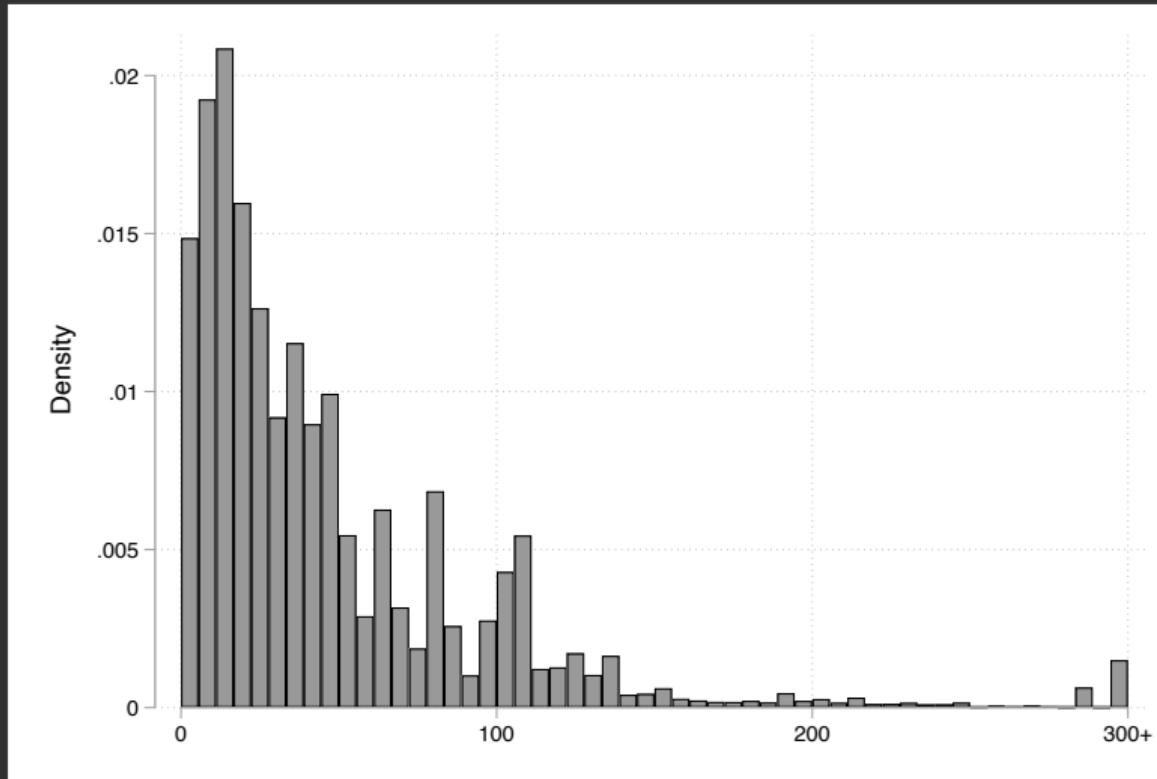


Figure 3: Coin age at time of deposit ( $tpq$ ), in years

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## Stylized facts

We document 3 main stylized facts:

1. Older coins travel further.
2. Distance *and* politics impede coin travels (gravity).
3. The Arab conquest disrupts Mediterranean crossings.

# Fact #1: within a hoard, older coins have travel farther

Table 1: Within-hoard distance travelled and age of coin at deposit

Dependent variable: Log Distance between Mint and Hoard					
	(1)	(2)	(3)	(4)	(5)
Log Age of Coin	0.162** (0.050)	0.0982** (0.026)	0.0910** (0.031)	0.181** (0.050)	0.0650** (0.021)
Sample					No non-hoards No non-hoards
Hoard FE	Yes	Yes	Yes	Yes	Yes
Mint × 50-year-interval FE		Yes			
Mint × 25-year-interval FE					Yes
R <sup>2</sup>	0.767	0.865	0.871	0.783	0.901
Observations	282354	282136	281972	245252	244927

Standard errors in parentheses, clustered at the hoard level.

+  $p < 0.10$ , \*  $p < 0.05$ , \*\*  $p < 0.01$

$$\text{Age of coin} = tpq - \text{mint date}$$

⇒ coins diffuse across space over time.

## Fact #2: distance and political borders impede coin travels

Construct  $1^\circ \times 1^\circ$  cells for mint and hoard locations and calculate flows count<sub>mdh</sub>

**Table 2:** Gravity and Border Effects in Coin Flows

	Dependent variable: # Coins <sub>mdh</sub>			
	(1)	(2)	(3)	(4)
Log Distance	-1.138** (0.12)	-0.984** (0.13)	-0.740** (0.10)	-0.707** (0.10)
Within border dummy		2.446** (0.37)		1.608** (0.41)
Hoard Cell FE	Yes	Yes	Yes	Yes
Mint × Empire Cell FE	Yes	Yes	Yes	Yes
Sample			Int. Marg. only	Int. Marg. only
Estimator	PPML	PPML	PPML	PPML
R <sup>2</sup>				
Observations	215984	215984	6222	6222

Standard errors in parentheses, clustered at mint cell × empire and hoard cell level.

+ p < 0.10, \* p < 0.05, \*\* p < 0.01

Estimating eqn: count<sub>mdh</sub> = exp ( $\gamma_1 \log \text{distance}_{mh} + \gamma_2 \text{withinBorder}_{dh} + \alpha_{md} + \alpha_h + \varepsilon_{mhd}$ )

## Fact #3: Arab conquests disrupt Mediterranean trade



Figure 4: Before the Arab conquests: 450-630 AD

## Fact #3: Arab conquests disrupt Mediterranean trade

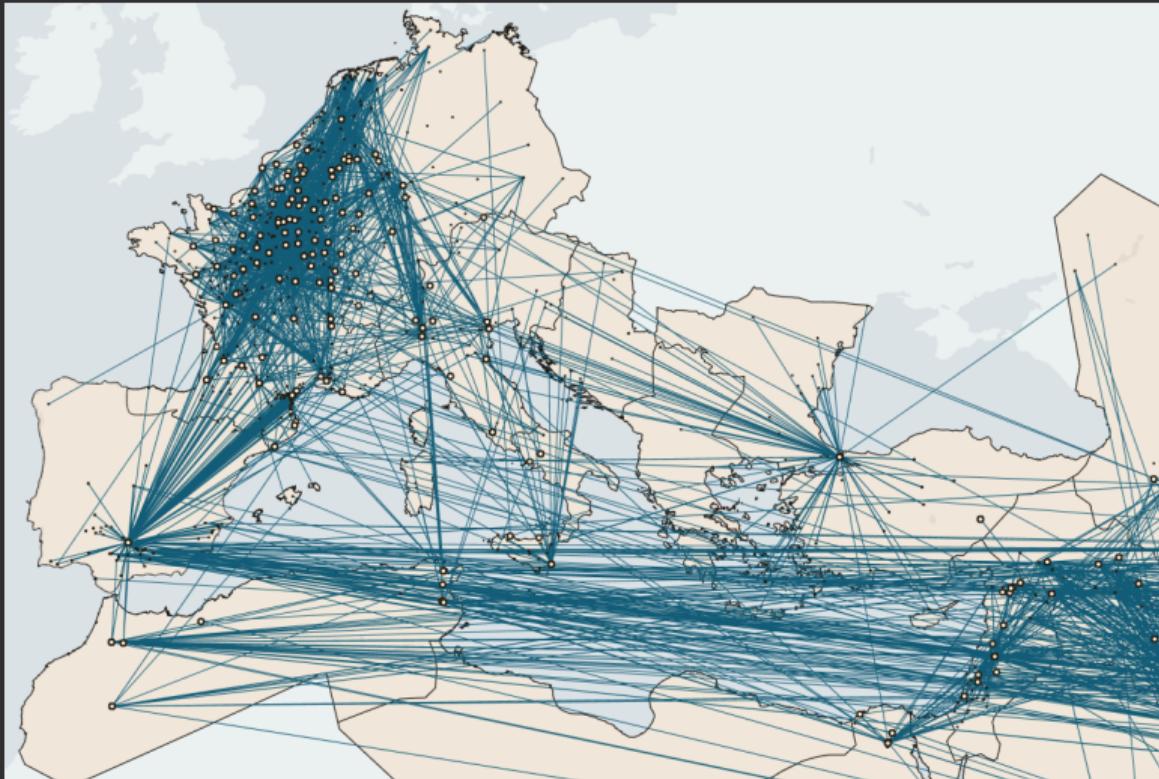


Figure 5: After the Arab conquests: 713-900 AD

## Fact #3: Arab conquests disrupt Mediterranean trade

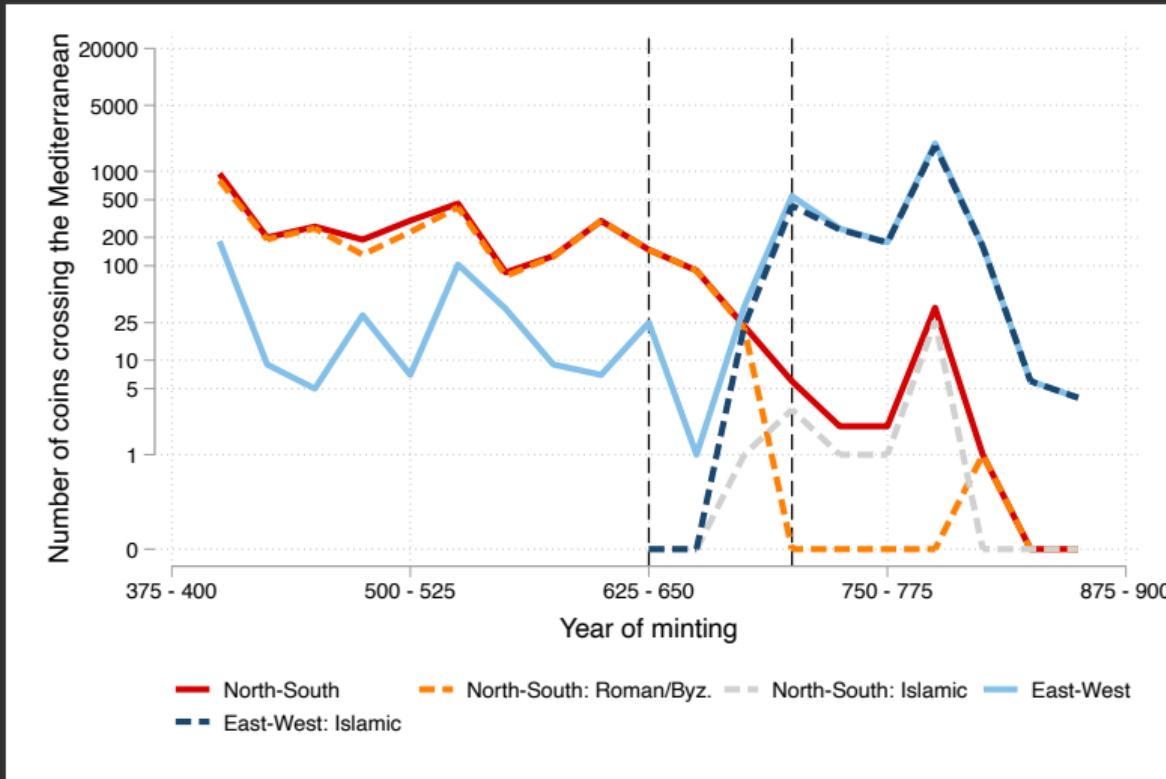


Figure 6: Number of coins flowing across the Mediterranean

## Fact #3: Arab conquests disrupt Mediterranean trade

Table 3: The Mediterranean Before and After the Arab Conquest

	Dependent variable: Number of Coins			
	(1)	(2)	(3)	(4)
Crossing Mediterranean × After Conquests	-1.878** (0.48)	-3.186** (0.55)	-0.820 (0.68)	-2.283 (1.59)
Crossing Mediterranean × After Conquests × Islamic Coin		6.748** (0.86)	4.435** (0.94)	7.614** (0.96)
Crossing Mediterranean × After Conquests × Roman Coin			-3.048** (0.78)	-3.061** (0.69)
Mint Cell × Empire FE	Yes	Yes	Yes	Yes
Mint Cell × Hoard Cell FE	Yes	Yes	Yes	Yes
After Conquests FE	Yes	Yes	Yes	
Mint Cell × After Conquests FE				Yes
Hoard Cell × After Conquests FE				Yes
Estimator	PPML	PPML	PPML	PPML
Observations	9246	9246	9246	4884

Standard errors in parentheses, clustered at the hoard × era and mint × era level.

+  $p < 0.10$ , \*  $p < 0.05$ , \*\*  $p < 0.01$

Estimating eqn:  $\text{count}_{mdht} = \exp(\gamma_1 \text{mediterranean}_{mh} \times \text{after}_t + \dots + \alpha_{md} + \alpha_{mh} + \varepsilon_{mhdt})$

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## Model ingredients

- Ricardian model of trade.  
(Eaton and Kortum, 2002)
- Coins-in-advance economy.
- Minting: coin birth.
- Hoarding: coin death.
- Optimal coins saving.  
(Dekle, Eaton and Kortum, 2007)

## Setup

Location  $n$ , period  $t$ : homog. mass  $L_n(t)$  of workers. Four sub-periods  $t_{sub1}, t_{sub2}, t_{sub3}, t_{sub4}$

$t_{sub1}$  Start with  $S_n[t]$  coins saved from period  $t - 1$

$t_{sub2}$  A fraction  $\lambda_n[t]$  of those saved coins is lost

$M_n[t] \geq 0$  fresh new coins are minted

$t_{sub3}$   $X_n[t]$ , expenditure on consumption

Budget constraint:

$$X_n[t] \leq (1 - \lambda_n[t]) S_n[t] + M_n[t]$$

$t_{sub4}$   $L_n[t]$  workers produce and sell goods in exchange for coins

$w_n[t]$ , competitive wage,  $w_n[t] L_n[t]$ , aggregate labor income

$S_n[t + 1]$  coins saved for  $t + 1$

$$\underbrace{(1 - \lambda_n[t]) \overbrace{S_n[t]}^{t_{sub1}} + M_n[t]}_{t_{sub2}} - \underbrace{X_n[t]}_{t_{sub3}} + \underbrace{w_n[t] L_n[t]}_{t_{sub4}} = \underbrace{S_n[t + 1]}_{(t+1)_{sub1}}$$

## Intra-temporal allocations

- Fraction  $\pi_{ni}$  of expenditure  $X_n$  allocated to goods from  $i$ :

$$\pi_{ni}(t) = \frac{T_i(t)(w_i(t)d_{ni}(t))^{-\theta}}{\sum_k T_k(t)(w_k(t)d_{nk}(t))^{-\theta}}, \quad (1)$$

as in Eaton and Kortum (2002).

## Intertemporal preferences

- Intertemporal utility  $U_n$ , within period welfare  $W_n$ ,

$$U_n(t) = \mathbb{E} \left[ \sum_{\tau \geq t} \beta^{\tau-t} \ln \left( \frac{x_n(\tau)}{p_n(\tau)} \right) \right],$$

$$\text{with } p_n(t) = \gamma \left( \sum_k T_k(t) (w_k(t) d_{nk}(t))^{-\theta} \right)^{1/\theta}$$

# Dynamic optimization

- Optimal coin savings dynamics,

$$\max_{\{S_n(\tau)\}_{\tau \geq t}} \left[ \sum_{\tau \geq t} \beta^{\tau-t} \ln \left( \frac{X_n(\tau)}{p_n(\tau)} \right) \right]$$

$$X_n(\tau) = w_n(\tau)L_n(\tau) + M_n(\tau) + (1 - \lambda_n(\tau))S_n(\tau) - S_n(\tau+1),$$

$$S_n(\tau+1) \geq w_n(\tau)L_n(\tau), \forall \tau \geq t,$$

$$\lim_{\tau \rightarrow \infty} \beta^\tau S_n(\tau+1)/X_n(\tau) = 0$$

- Dynamic equilibrium wages clear markets,

$$w'_i L'_i = \sum_n \pi_{ni}(T, d; w) [w_n L_n + M_n + (1 - \lambda_n) S_n - S'_n]$$

Savings  $S_n(T, d, \delta, L, M; w)$  depend on parameters and wages, which depend on wages etc.

# Optimal consumption/saving

Under log utility:

- price level  $p_n(t)$  dynamics irrelevant (i.e. separates out)
- when unconstrained, consumption declines exponentially:

$$\frac{X_n(t+1)}{X_n(t)} = \beta(1 - \lambda_n(t)) < 1$$

- when constrained, consume as much as you can:

$$S'_n = w_n(t)L_n$$

Steady state: zero net savings,  $S_n(t+1) = S_n(t), \forall t$

- No motive for extra savings,  $S_n = w_n L_n$ .
- Wages and trade shares jointly determined,

$$w_i L_i = \sum_n \pi_{ni} ((1 - \lambda_n) w_n L_n + M_n)$$
$$\pi_{ni} = \frac{T_i (w_i d_{ni})^{-\theta}}{\sum_k T_k (w_k d_{nk})^{-\theta}}$$

- Constant aggregate stock of coins in circulation,

$$\sum_n M_n = \lambda_n \sum_n w_n L_n$$

- Note: trade deficits as in Dekle, Eaton and Kortum (2007),

$$D_n = X_n - Y_n = M_n - \lambda_n S_n$$

# Introducing and tracking coins of different vintages

Coin stocks  $S_n(\tau)$  consist of coins of different vintage:

$$S_n(\tau) = \sum_{m=1}^N \sum_{t < \tau} S_{mn}(t, \tau)$$

Coin stocks start their life when minted:  $S_{mm}(t, t) = M_m(t)$ .

Traders are ‘blind’ to coin types, draw coins with equal probability:

$$S_{mi}(t, \tau + 1) = \sum_{n=1}^N (1 - \lambda_n(\tau)) (1 - s_n(\tau)) S_{mn}(t, \tau) \pi_{ni}(\tau) + (1 - \lambda_i(\tau)) s_i(\tau) S_{mi}(t, \tau), \forall \tau \geq t$$

In steady state,  $s_n = 0$ , and the diffusion equation becomes:

$$\mathbf{S}(t, t + a) = \mathbf{S}(a) = \mathbf{M} \left( (I - \boldsymbol{\lambda}) \mathbf{\Pi} \right)^a, \forall (t, a).$$

Sum of different vintages (stocks by origin-destination),

$$\sum_{a=0}^A \mathbf{S}(a) = \mathbf{M} \left( \sum_{a=0}^A \left( (I - \boldsymbol{\lambda}) \mathbf{\Pi} \right)^a \right) \underset{A \rightarrow +\infty}{=} \mathbf{M} (I - (I - \boldsymbol{\lambda}) \mathbf{\Pi})^{-1}$$

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## Taking the model to the data

- 21 regions around the Mediterranean ▶ details
- 20-year time intervals
- Assume constant  $\lambda$  and estimate as exponential decay parameter in within-hoard age distribution:

$$\hat{\lambda}_{20y} = 0.3$$

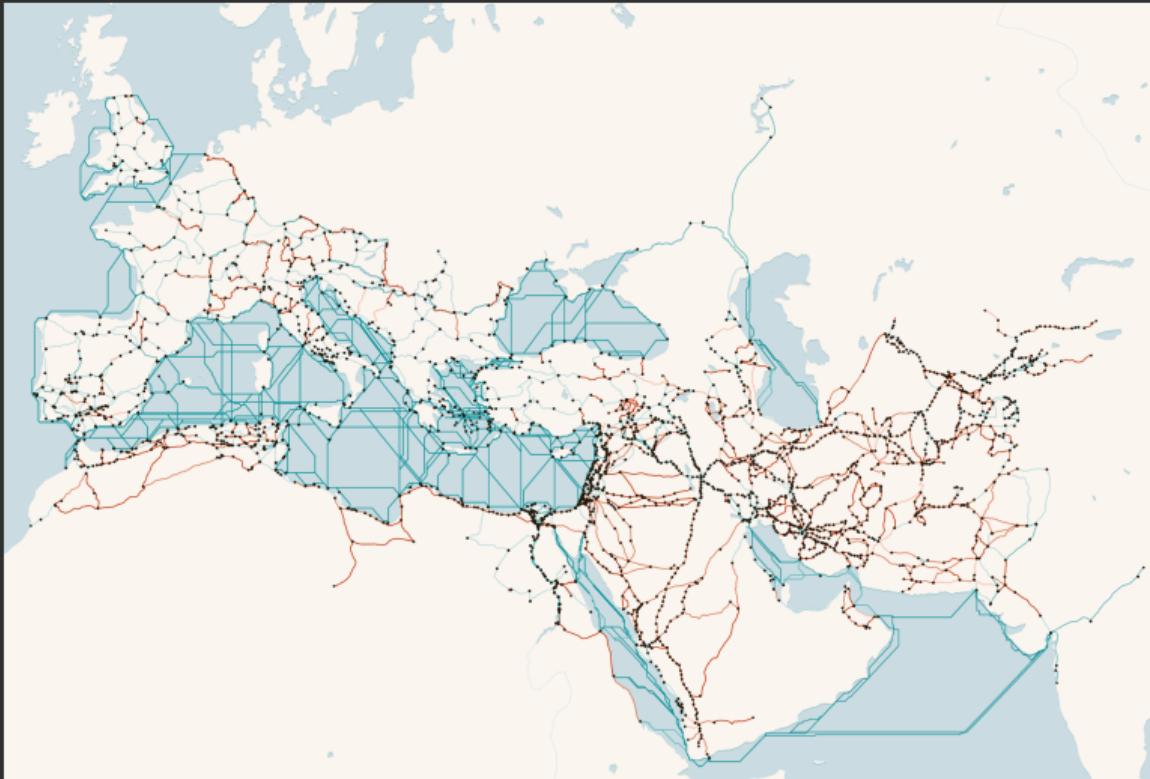
- Parameterize trade frictions,

$$d_{ni}(t)^\theta = \exp(\beta_0 + \beta_1 \ln(\text{TravelTime}_{ni}) + \beta_3 \text{PoliticalBorder}_{ni}(t) + \beta_4 \text{ReligiousBorder}_{ni}(t))$$

- Assume (for now) that we're in steady state for savings:  $s_n(t) = 0$ .
- For counterfactuals, assume  $\theta = 4$  (Simonovska and Waugh, 2014).

▶ Unsolved problems (as of yet)

Trade costs *only* depend on travel times (and politics/religion)



Note: Combined geospatial models from Orbis (Scheidel, 2015) and al-Turayyā (Romanov and Seydi, 2022).



## Maximum likelihood estimation

Assume coins in our data are a random sample of coin types in each location  $\times$  time.

- Multinomial distribution of coin types,

$$P(\dots, X_i^{(m,\tau)}(T) = x_i^{(m,\tau)}, \dots) = \frac{N_i(T)!}{\prod_{(m,\tau)} x_i^{(m,\tau)}!} \prod_{(m,\tau)} [p_i^{(m,\tau)}(T)]^{x_i^{(m,\tau)}}$$

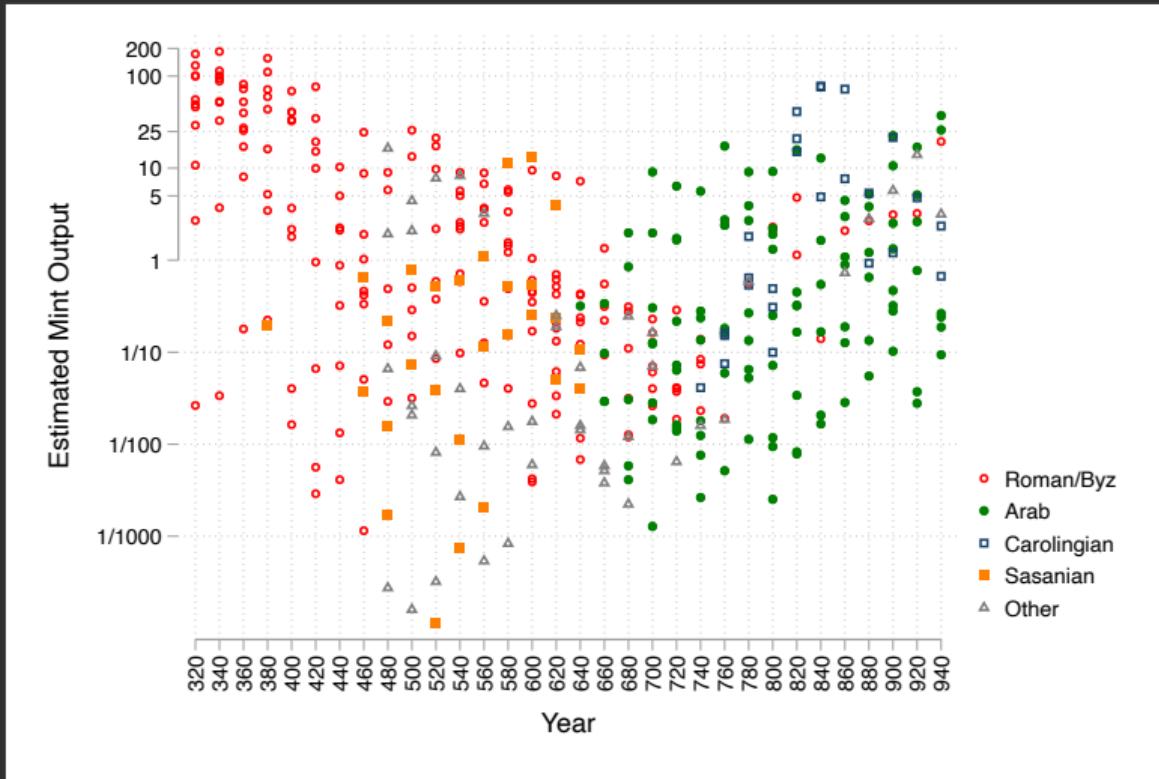
with the probability of drawing a coin of type  $(m, \tau)$ ,

$$p_i^{(m,\tau)}(T) = \frac{S_i^{(m,\tau)}(T)}{\sum_{(m',\tau')} S_i^{(m',\tau')}(T)} = \frac{S_i^{(m,\tau)}(T)}{S_i(T)}.$$

- Likelihood of observing a sample of coins given parameters  $\theta$ ,

$$\ell(X; \theta) = \sum_i \sum_T \sum_{(m,\tau)} x_i^{(m,\tau)} [\log S_i^{(m,\tau)}(T; \theta) - \log S_i(T; \theta)] + \text{constant}$$

## Estimation results: Mint output



## Estimation results: Determinants of trade costs

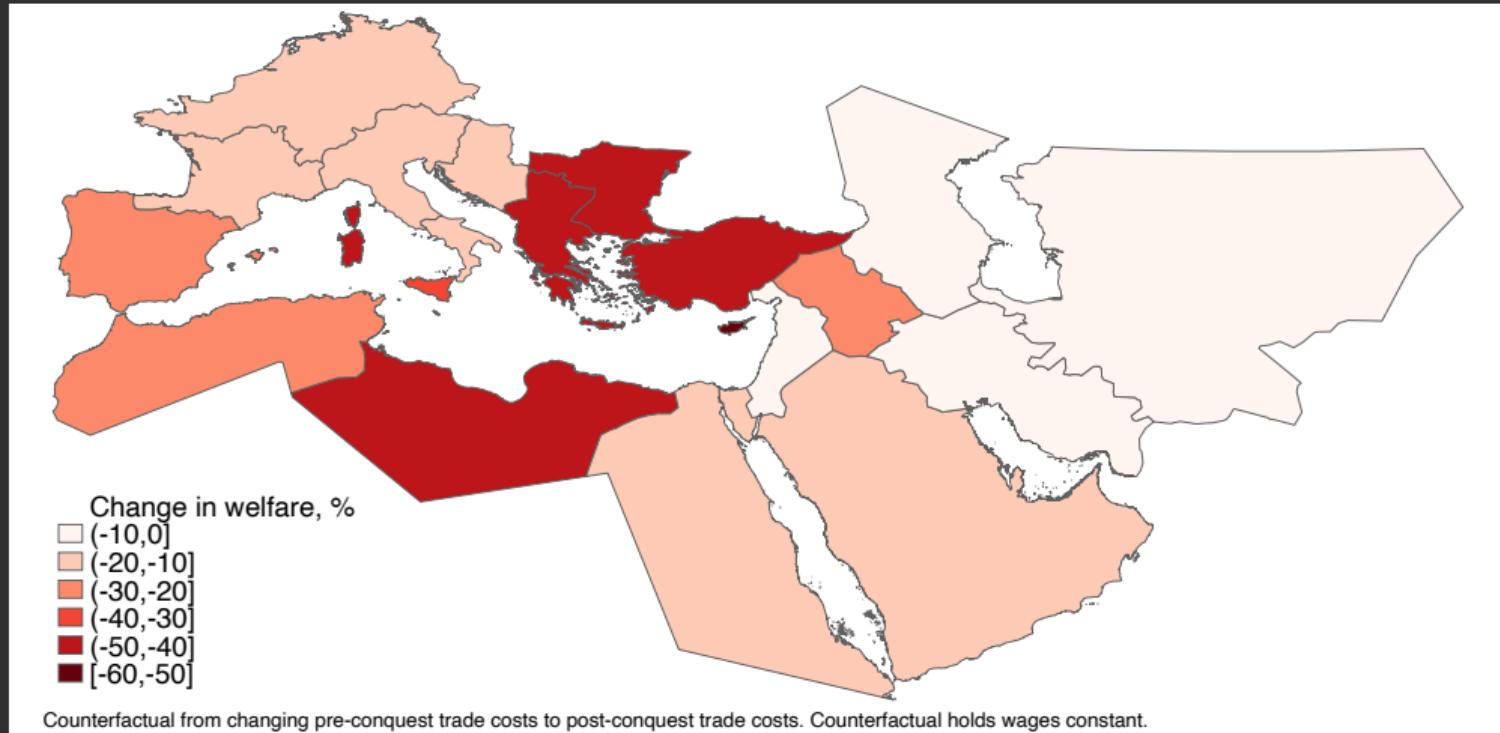
$$\begin{aligned}\theta \ln(\lambda_{ni}(t)) = & -1.81 + 2.22 \ln(TravelTime_{ni}) \\ & + 1.28 PoliticalBorder_{ni}(t) + 2.09 ReligiousBorder_{ni}(t) \quad (2)\end{aligned}$$

Holding fixed multilateral resistance term:

- removing a political border would increase a bilateral trade share by  $\exp(1.28) - 1 \approx 260\%$
- removing a religious border would increase the same trade share by  $\exp(2.09) - 1 \approx 710\%$

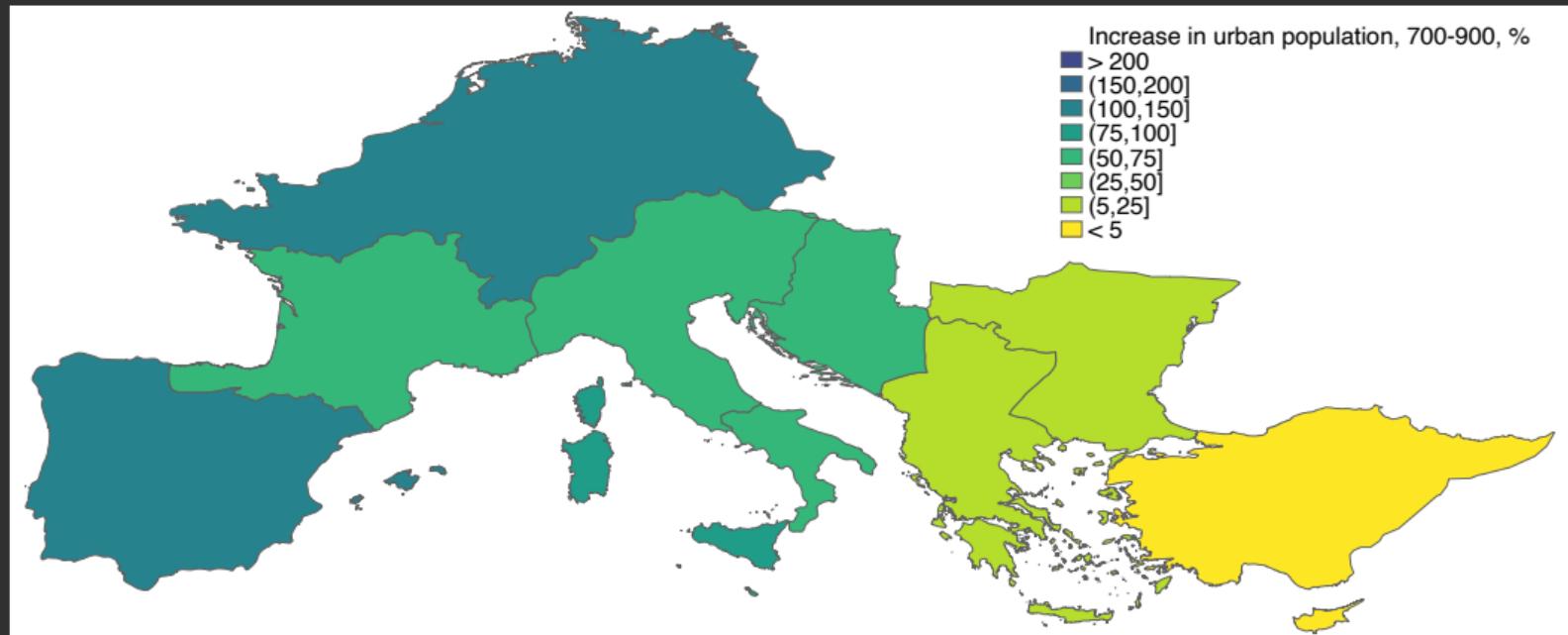
# Changing pre-conquest trade costs to post-conquest: $(d_{ni}^{-\theta})^{pre} \rightarrow (d_{ni}^{-\theta})^{post}$

Holding fixed technology  $T_i$ , wages  $w_i$ , and mint output  $M_i$  at pre-conquest levels, and assume  $\theta = 4$ .



## Compare to relative urbanization rates, 700–900 AD

Change in total urban population (urban: > 1k inhabitants), data from Buringh (2021)



## Taking stock

- Clear pattern of change in economic geography before vs after conquest
- Trade disruption can account for the relative decline in the eastern Mediterranean
- Change in trade cost *alone* not able to account for urbanization in Muslim Spain, or in Carolingian empire
- Potentially in conjunction with changes in technology  $T_i$  ( $\rightarrow$  Spain) or mint output ( $\rightarrow$  Carolingian empire).

# Conclusion

*“Simply looking at the Mediterranean cannot of course explain everything about a complicated past created by human agents, with varying doses of calculation, caprice and misadventure. But this is a sea that patiently recreates for us scenes from the past, breathing new life into them, locating them under a sky and in a landscape that we can see with our own eyes, a landscape and sky like those of long ago. A moment’s concentration or daydreaming, and that past comes back to life.”*

*Fernand Braudel, Les Mémoires de la Méditerranée*

THANK YOU!

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## BACKUP SLIDES

## References

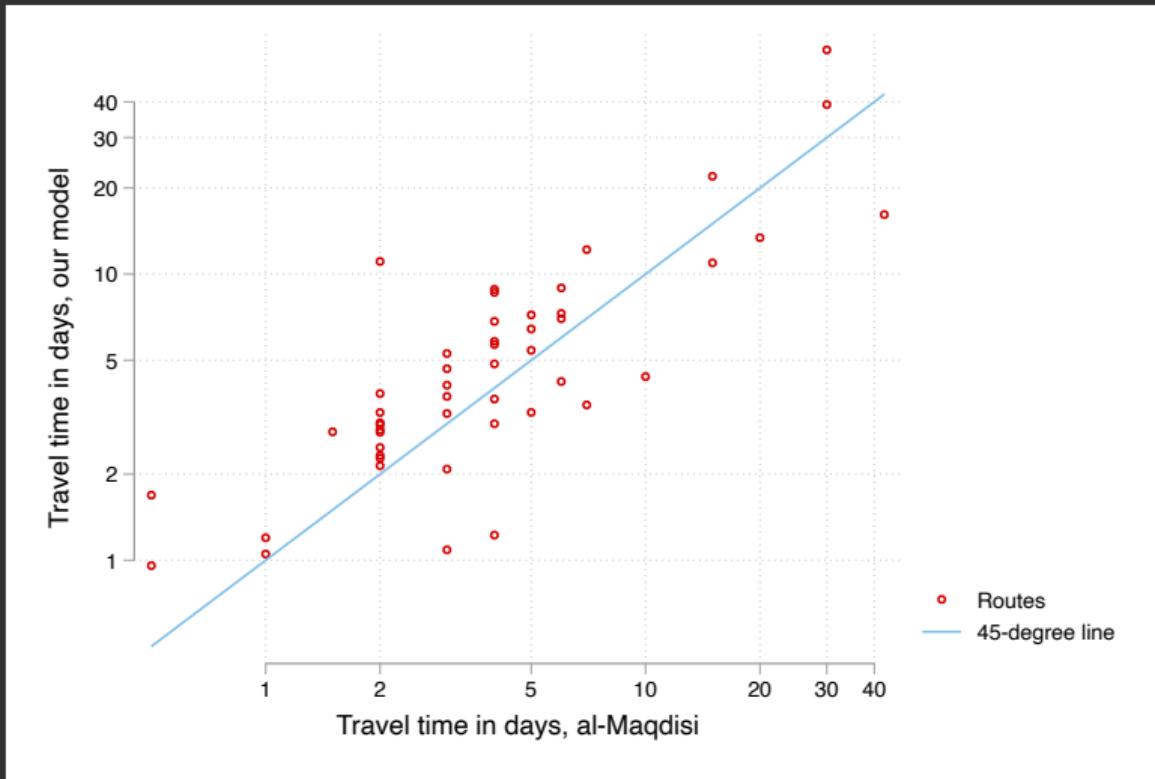
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# Regions



# Validating Travel Times

Al-Maqdisi (c. 945–991): *The Best Divisions for Knowledge of the Regions*



## Unsolved problems (as of yet)

- Lucas critique #1: cost function does not minimize costs

$$\ln(\delta_{ni}[t]) = \min_{p \in paths(i \rightarrow n)} \left( \gamma_0 + \gamma_1 \ln(TravelTime_p[t]) + \sum_{pb: \text{ all political borders along } p} \gamma_2 PoliticalBorder_{pb}[t] + \sum_{rb: \text{ all religious borders along } p} \gamma_3 ReligiousBorder_{rb}[t] \right)$$

- Lucas critique #2: net saving (in  $\delta_{nn}$ ) depends on parameters.
- Fix for #2: location-specific intercepts ( $\gamma_{0,n}$ ) and  $\delta_{nn}$ 's.

For now: constant  $\gamma_0$ , and  $\delta_{nn} = 1$ ...