EC442 Macroeconomics

Problem Set 4

Fall 2013

1 CES-Demand, Externalities and Varieties

Consider the CES demand model. where preferences are given by

$$U(c_1, ..., c_N, y) = u(C, y),$$

where $C = \left(\sum_{i=1}^{N} c_i^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$ and y is an outside good, which is the numeraire. The budget constraint is given by

$$\sum_{i=1}^{N} p_i c_i + y = m,$$

where m denotes income. Varieties can be produced at constant marginal costs ψ and firms act as monopolistic competitors, i.e. they do not internalize the effect of their pricing decision on aggregate outcomes. Introducing a new variety entails a fixed cost μ. Suppose that

$$u(C, y) = C + \frac{1}{1-\alpha} y^{1-\alpha}, \text{ with } \alpha < 1.$$

1. Consider the allocation determined by a social planner who also controls prices. Characterize the number of varieties that a social planner would choose to maximize the utility of the representative household in this case.

2. Suppose that prices are given by the firms' profit maximizing monopoly price. Characterize the number of varieties that the social planner would choose to maximize the utility of the representative household in this case.

3. Characterize the equilibrium number of varieties (at which all monopolistically competitive variety producers make zero profits) and compare this number with the answers to the previous two parts. Explain the sources of differences between the equilibrium and the social planner's solution in each case.

2 Expanding Varieties and Scale Effects I

Consider the baseline lab-equipment model seen in class and presented in Acemoglu (2009), "Introduction to Modern Economic Growth", Section 13.1. Modify the innovation possibilities frontier to be

$$\dot{N}(t) = \eta N(t)^{-\phi} Z(t), \text{ with } \phi > 1.$$

1. Define an equilibrium and characterize the market clearing factor prices and determine the free-entry condition.

2. Show that without population growth, there will be no sustained growth in this economy.

3. Now consider population growth at the exponential rate n, and show that this model generates sustained equilibrium growth.

3 Expanding Varieties and Scale Effects II

Consider the following model. Population at time t is $L(t)$ and grows at the constant rate n (i.e., $\dot{L}(t) = nL(t)$). All agents have preferences given by

$$\int_0^\infty e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt$$

where C is consumption of the final good produced as

$$Y(t) = \left(\int_0^{N(t)} y(\nu,t)^\beta d\nu \right)^{1/\beta},$$

where $y(\nu,t)$ is the amount of intermediate good ν used in production at time t, and $N(t)$ is the number of intermediate goods at time t . The production function of each intermediate is $y(\nu,t) = l(\nu,t)$, where $l(\nu,t)$ is labor allocated to this good at time t. New goods are produced by allocating workers to R&D, with the production function

$$\dot{N}(t) = \eta N(t)^\phi L_R(t), \text{ with } \phi \leq 1,$$

where $L_R(t)$ is labor allocated to R&D at time t. Labor market clearing requires that $\int_0^{N(t)} l(\nu,t)\, d\nu + L_R(t) = L(t)$. Risk-neutral firms hire workers for R&D. A firm that discovers a new good becomes the monopoly supplier, with a perfectly enforced patent.

1. Characterize the BGP in the case where $\phi = 1$ and $n = 0$ and show that there are no transitional dynamics. Why is this? Why does the long-run growth rate depend on θ? Why does the growth rate depend on L? Do you find this dependence plausible?
2. Suppose that $\phi = 1$ and $n > 0$. What happens? Interpret.
3. Characterize the BGP when $\phi < 1$ and $n > 0$. Does the growth rate depend on L? Does it depend on n? Why? Do you think that the configuration $\phi < 1$ and $n > 0$ is more plausible than the one with $\phi = 1$ and $n = 0$?

4 Patent Protection (Disclaimer: this problem is not easy ... but quite interesting)

Consider the baseline lab-equipment model seen in class .... with one little difference. A firm that invents a new machine receives a patent, which expires at the Poisson rate ι, i.e. each instant there is a chance ι that the Patent expires. Once the patent expires, that machine is produced competitively and is supplied to final good producers at marginal cost.

1. Characterize the BGP equilibrium in this case and show how the growth rate depends on ι [Hint: notice that there will be two different machine varieties supplied at different prices at every point in time. Let $N_M(t)$ and $N_C(t) = N(t) - N_M(t)$ be the set of machines with existing and expired patent protection respectively. Look for a BGP where $\frac{N_M(t)}{N(t)}$ will be constant].
2. What is the value of ι that maximizes the equilibrium rate of economic growth? Interpret.
3. Show that a policy of $\iota = 0$ does not necessarily maximize social welfare at time $t = 0$. Why?