1 PS3 Q2

1. Budget constraint
$$c_t + k_{t+1} = (1-\tau)r_t k_t + w_t$$
FOC
$$r_t = \alpha k_t^{\alpha-1}, \quad w_t = (1-\alpha)k_t^{\alpha}$$
also
$$y_t = c_t + g_t + k_{t+1}.$$
Euler equation is
$$c_{t+1} = \beta(1-\tau)r_{t+1}c_t$$
and steady state implies $c_t = c_{t+1}$ and hence
$$1 = (1-\tau)\alpha\beta k^{\alpha-1}$$
and thus capital (and thus output) depends negatively on taxes.

2. Budget constraints:
$$\begin{aligned} c_{1,t} + k_{2,t+1} &= w_1 \\ c_{2,t+1} &= (1-\tau)r_t k_{2,t+1} \end{aligned}$$
Euler equation:
$$\frac{1}{c_{1,t}} = \beta(1-\tau)r_{t+1}\frac{1}{c_{2,t+1}}$$
Factor prices as above. Thus,
$$(1-\tau)r_{t+1}k_{t+1} = \beta(1-\tau)r_{t+1}\left((1-\alpha)k_t^{\alpha} - k_{t+1}\right)$$
and hence τ cancels out.