Trade and the End of Antiquity

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What caused the End of Antiquity?

- Antiquity: Roman and Greek civilizations centered around the Mediterranean
- Archaeological evidence points to a shift in economic activity away from the Mediterranean between 5th and 8th century AD (“end of antiquity”)
    - Rise of Northern Europe (Charlemagne).
    - “Golden Age” of Islam

$\hookrightarrow$ *Question*: What caused the End of Antiquity?

- Discussed, among others, by Montesquieu (1734), Voltaire (1756), Gibbon (1789)
- Henri Pirenne (1937) blames the Arab conquests and the emerging Islamic-Christian border for the rupture in the Mediterranean unity
    - Some views challenged by more recent archaeological evidence.

Political changes in the Mediterranean: 600 AD

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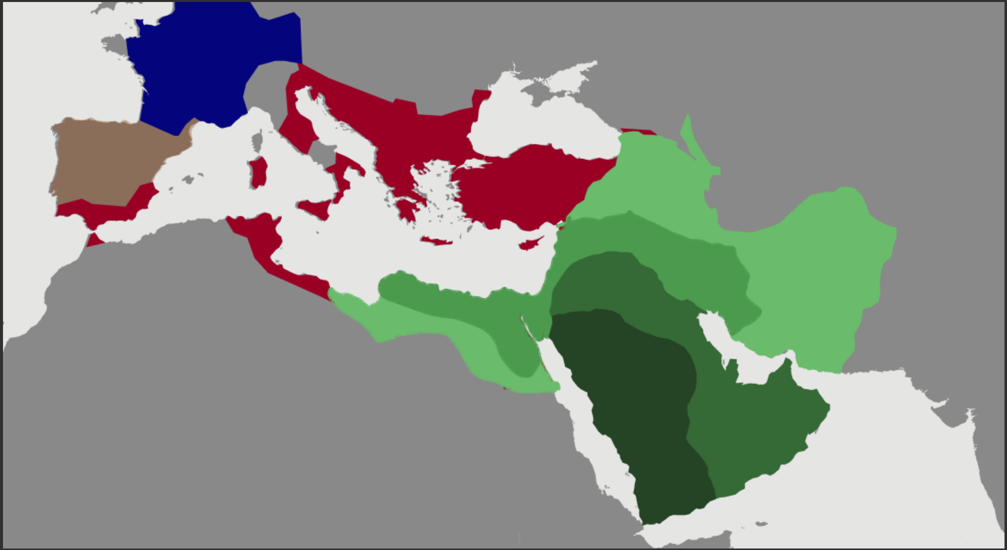
Political changes in the Mediterranean: 632 AD

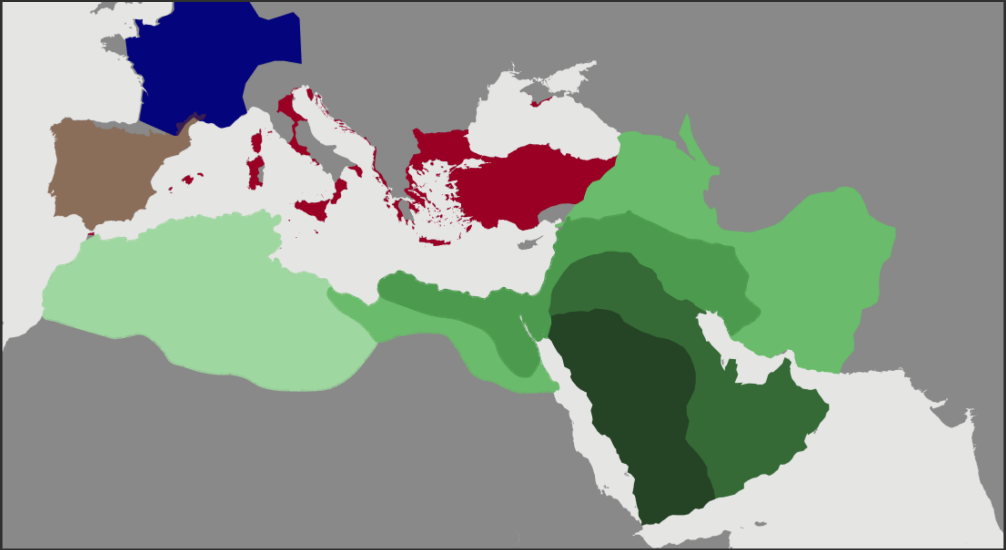
Political changes in the Mediterranean: 634 AD

Political changes in the Mediterranean: 644 AD

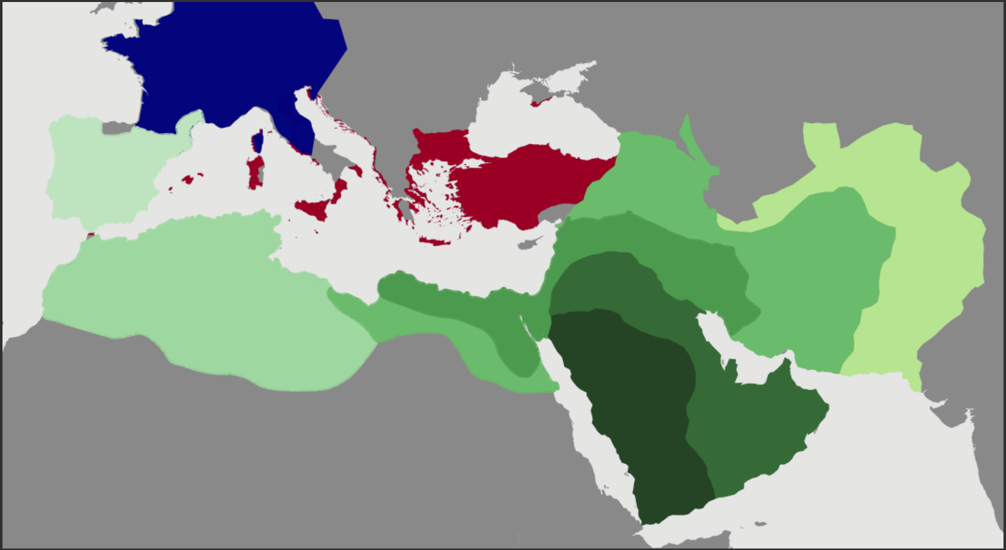
Political changes in the Mediterranean: 661 AD



Political changes in the Mediterranean: 661-700 AD



Political changes in the Mediterranean: 750 AD



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$\Rightarrow$ Use data on the movement of coins to study the changing economic geography during Late Antiquity.

- Coins are the main medium of exchange during Late Antiquity, particularly for long-distance trade $\rightarrow$ informative about trade
- Coins are well studied & documented by historians and numismatists
- Coins have features that help solve econometric identification problems

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- Coins have features that help solve econometric identification problems

$\Rightarrow$ we interpret coin flows through the lens of a trade model and identify

- bilateral trade flows (i, j, t)
- technology (i, t)
- trade deficits (i, t)
- real consumption per capita (i, t)

in order to better understand what's going on during Late Antiquity

Data: Coins around the Mediterranean, AD 325 to AD 950

Assemble a large dataset of coin finds from around the Mediterranean

1. FLAME (2023) project by historians around Princeton
   - ∼200,000 coins with complete records 325–725
2. Hand-coded records from numismatic / archaeological literature:
   - 797 coin finds, ∼100k coins, 725-950

▸ Coin age distribution ▸ Hoard Dates (tpq) ▸ Spatial Distribution

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Example: excerpt from al Ush's (1972) Damascus silver hoard:

No.	MINT	DATE	DIAM.	WEIGHT	NUMB.
51	الأندلس	114	29.	2.93	4
52	"	115	29.5	2.92	1
53	"	116	26.5	2.92	3

Index / Mint (al-Andalus/Cordoba) / Date: 114 AH = 732 AD / Diameter / Weight / Q'tity

▸ Coin age distribution ▸ Hoard Dates (tpq) ▸ Spatial Distribution

Fact #1: within a hoard, older coins have travel farther

Table 1: Within-hoard distance travelled and age of coin at deposit

	Dependent variable: Log Distance between Mint and Hoard				
	(1)	(2)	(3)	(4)	(5)
Log Age of Coin	0.146***	0.0831***	0.0749**	0.160***	0.0485**
	(0.044)	(0.026)	(0.031)	(0.043)	(0.020)
Sample				No non-hoards	No non-hoards
Hoard FE	Yes	Yes	Yes	Yes	Yes
Mint $\times$ 50-year-interval FE		Yes			
Mint $\times$ 25-year-interval FE			Yes		Yes
R^2	0.762	0.863	0.869	0.775	0.898
Observations	287243	287029	286873	250156	249830

Standard errors in parentheses, clustered at the hoard level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Age of coin $=$ *tpq* $-$ mint date

$\Rightarrow$ coins diffuse across space over time.

Fact #2: distance and political borders impede coin travels

Construct $1° \times 1°$ cells for mint and hoard locations and calculate flows count_{mdh}

Table 2: Gravity and Border Effects in Coin Flows

	Dependent variable: # Coins_{mdh}				Dep. var.: Value_{mdh}	
	(1)	(2)	(3)	(4)	(5)	(6)
Log Distance	-1.137***	-1.002***	-1.135***	-0.951***	-1.144***	-0.989***
	(0.12)	(0.13)	(0.10)	(0.076)	(0.075)	(0.068)
Political border		-1.945***		-2.073***		-1.516***
		(0.62)		(0.47)		(0.27)
Hoard Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Mint $\times$ Empire Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample			Gold only	Gold only	Gold and Silver	Gold and Silver
Estimator	PPML	PPML	PPML	PPML	PPML	PPML
Pseudo-R^2	0.767	0.778	0.808	0.824	0.800	0.810
Observations	217748	217748	57287	57287	146767	146767

Standard errors in parentheses, clustered at mint cell $\times$ empire and hoard cell level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

▸ Declining Elasticity

Estimating eqn: $\text{count}_{mdh} = \exp\left(\gamma_1 \log \text{distance}_{mh} + \gamma_2 \text{withinBorder}_{dh} + \alpha_{md} + \alpha_h + \varepsilon_{mhd}\right)$

Fact #3: Coin flows before/after the Arab conquests

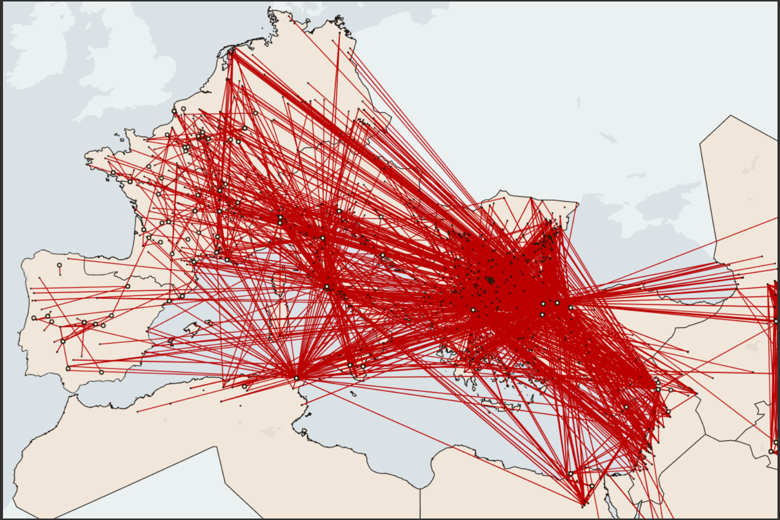


Figure 1: Before the Arab conquests: 450-630 AD

Fact #3: Coin flows before/after the Arab conquests

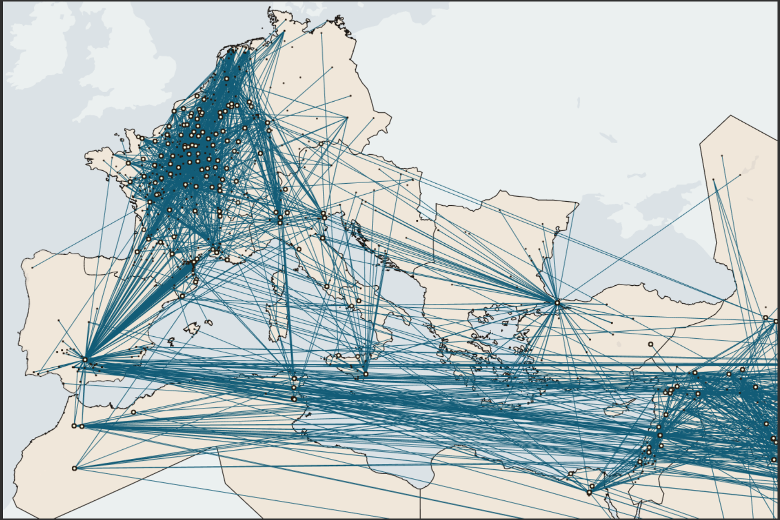


Figure 2: After the Arab conquests: 713-900 AD

▸ Cross-med: reg

▸ Cross-med: plot

Model

Objective:

Quantify impact of trade barriers (and other changes) on market access / trade / welfare

Approach:

Dynamic model of trade flows (gravity), where coins diffuse alongside trade and are thus informative about trade flows.

Key Assumption:

Traders are blind to coin *types* (mint location and date).

- $\Rightarrow$ Coins diffuse in proportion to trade flows
- $\Rightarrow$ Recover trade *flows* from stocks of coin *types* in hords in different locations/time periods.

Not used/needed for identification: total *quantities* of coins found in each location

Setup

Location n, period t: homog. mass $L_n(t)$ of workers. Four sub-periods $t_{sub1}, t_{sub2}, t_{sub3}, t_{sub4}$

t_{sub1} Start with $S_n(t)$ coins saved from period $t-1$

t_{sub2} A fraction $\lambda_n(t)$ of those saved coins is lost

$M_n(t) \geq 0$ fresh new coins are minted

t_{sub3} $X_n(t)$, expenditure on consumption

Cash-in-advance constraint:

$$X_n(t) \leq (1-\lambda_n(t))\, S_n(t) + M_n(t)$$

t_{sub4} $L_n(t)$ workers produce and sell goods in exchange for coins

$w_n(t)$, competitive wage, $w_n(t)\, L_n(t)$, aggregate labor income

$S_n(t+1)$ coins saved for $t+1$

$$\underbrace{(1-\lambda_n(t))\overbrace{S_n(t)}^{t_{sub1}} + M_n(t)}_{t_{sub2}} - \underbrace{X_n(t)}_{t_{sub3}} + \underbrace{w_n(t)\, L_n(t)}_{t_{sub4}} = \underbrace{S_n(t+1)}_{(t+1)_{sub1}}$$

Intra-temporal allocations

- Fraction π_{ni} of expenditure X_n allocated to goods from i:

$$\pi_{ni}(t) = \frac{T_i(t)\left(w_i(t)\, d_{ni}(t)\right)^{-\theta}}{\sum_k T_k(t)\left(w_k(t)\, d_{nk}(t)\right)^{-\theta}}, \tag{1}$$

  as in Eaton and Kortum (2002).

Intertemporal preferences

- Intertemporal utility U_n, within period welfare W_n, ,

$$U_n(t) = \mathbb{E}\left[\sum_{\tau \geq t} \beta^{\tau - t} \ln\left(\frac{X_n(t)}{p_n(t)}\right)\right],$$

$$\text{with } p_n(t) = \gamma\left(\sum_k T_k(t)\left(w_k(t)\, d_{nk}(t)\right)^{-\theta}\right)^{1/\theta}$$

Dynamic optimization

- Optimal coin savings dynamics,

$$\max_{\{S_n(\tau)\}_{\tau \geq t}} \left[\sum_{\tau \geq t} \beta^{\tau - t} \ln \left(\frac{X_n(\tau)}{p_n(\tau)} \right) \right]$$

$$X_n(\tau) = w_n(\tau) L_n(\tau) + M_n(\tau) + (1 - \lambda_n(\tau)) S_n(\tau) - S_n(\tau + 1),$$

$$S_n(\tau + 1) \geq w_n(\tau) L_n(\tau), \forall \tau \geq t,$$

$$\lim_{\tau \to \infty} \beta^{\tau} S_n(\tau + 1) / X_n(\tau) = 0$$

- Dynamic equilibrium wages clear markets,

$$w_i' L_i' = \sum_n \pi_{ni}(\mathrm{T}, \mathrm{d}; \mathrm{w}) \left[w_n L_n + M_n + (1 - \lambda_n) S_n - S_n' \right]$$

Savings $S_n(\mathrm{T}, \mathrm{d}, \boldsymbol{\delta}, \mathrm{L}, \mathrm{M}; \mathrm{w})$ depend on parameters and wages, which depend on wages etc.

Optimal consumption/saving

Under log utility:

- price level $p_n(t)$ dynamics irrelevant (i.e. separates out)
- when unconstrained, consumption declines exponentially:

$$\frac{X_n(t+1)}{X_n(t)} = \beta(1 - \lambda_n(t)) < 1$$

- when constrained, consume as much as you can:

$$S_n' = w_n(t)L_n$$

Define *net* saving:

$$s_n(t) = \frac{(1 - \lambda_n(t))S_n(t) + M_n(t) - X_n(t)}{(1 - \lambda_n(t))S_n(t) + M_n(t)}$$

Introducing and tracking coins of different vintages

Coin stocks $S_n(\tau)$ consist of coins of different vintage:

$$S_n(\tau) = \sum_{m=1}^{N} \sum_{t<\tau} S_{mn}(t,\tau)$$

Coins start their life when minted: $S_{mm}(t,t) = M_m(t)$.

Traders are 'blind' to coin types, draw coins with equal probability:

$$S_{mi}(t,\tau+1) = \sum_{n=1}^{N} (1-\lambda_n(\tau))(1-s_n(\tau)) S_{mn}(t,\tau) \pi_{ni}(\tau) + (1-\lambda_i(\tau)) s_i(\tau) S_{mi}(t,\tau), \forall \tau \geq t$$

In compact matrix form:

$$\mathbf{S}(t,T) = \boldsymbol{M}(t) \left(\prod_{\tau=t}^{T-1} (I - \boldsymbol{\lambda}(\tau)) \left((I - \boldsymbol{s}(\tau)) \boldsymbol{\Pi}(\tau) + \boldsymbol{s}(\tau) \right) \right)$$

▸ Pitfall 1: Store

▸ Pitfall 2: Stocks vs flows

Taking the model to the data

- 13 regions around the Mediterranean ▸ details
- 20-year time intervals
- Assume constant λ and estimate as exponential decay parameter in within-hoard age distribution:

$$\hat{\lambda}_{20y} = 0.301$$

(or 1.7% per year)

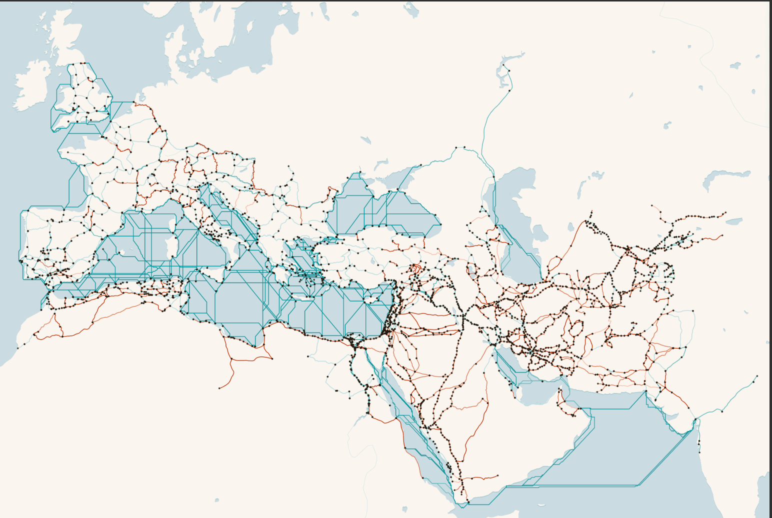
Parameterize trade frictions:

$$d_{ni}(t)^{\theta} = \exp\left(\gamma_0 + \zeta \ln\left(\text{TravelTime}_{ni}\right) + \kappa_1 \text{PoliticalBorder}_{ni}(t) + \kappa_2 \text{ReligiousBorder}_{ni}(t)\right)$$

if $n \neq i$ and $d_{nn}(t) = 1$.

- How to split domestic trade vs saving? Calibrate $\overline{s_n}(t) = 1.5\%$ (Scheidel, 2020).
- For counterfactuals, assume $\theta = 4$ (Simonovska and Waugh, 2014).

Trade costs *only* depend on travel times (and politics/religion)



Note: Combined geospatial models from Orbis (Scheidel, 2015) and al-Ṯurayyā (Romanov and Seydi, 2022). ▸ al-Maqdisi

Maximum likelihood estimation

Assume coins in our data are a random sample of coin types in each location $\times$ time.

- Multinomial distribution of coin types,

$$P\left(\cdots, X_i^{(m,\tau)}(T) = x_i^{(m,\tau)}, \ldots\right) = \frac{N_i(T)!}{\prod_{(m,\tau)} x_i^{(m,\tau)}!} \prod_{(m,\tau)} \left[p_i^{(m,\tau)}(T)\right]^{x_i^{(m,\tau)}}$$

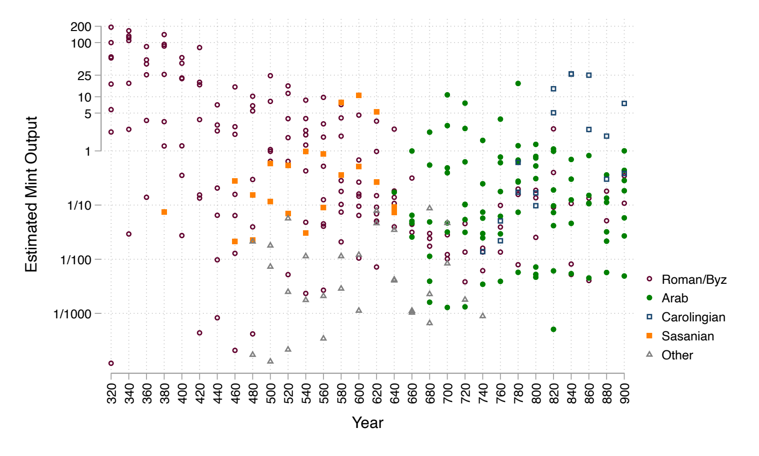
  with the probability of drawing a coin of type (m, τ),

$$p_i^{(m,\tau)}(T) = \frac{S_i^{(m,\tau)}(T)}{\sum_{(m',\tau')} S_i^{(m',\tau')}(T)} = \frac{S_i^{(m,\tau)}(T)}{S_i(T)}.$$

- Likelihood of observing a sample of coins given parameters $\boldsymbol{\theta}$,

$$\ell(X; \boldsymbol{\theta}) = \sum_i \sum_T \sum_{(m,\tau)} x_i^{(m,\tau)} \left[\log S_i^{(m,\tau)}(T; \boldsymbol{\theta}) - \log S_i(T; \boldsymbol{\theta})\right] + \text{constant}$$

Estimation results: Mint output



Normalization: $M_{n_0}[t_0] = 100$ (Northern Italy, AD 320-40).

Discussion on Byzantine monetary output: Kazhdan (1954), Pennas (1996)

Estimation results: Determinants of trade costs

$$\ln\left((d_{ni}[t])^{-\theta}\right) = constant$$
$$- \underset{(0.02)}{2.98} \ln(TravelTime_{ni}) - \underset{(0.02)}{0.3} \; PoliticalBorder_{ni}[t] - \underset{(0.11)}{4.05} \, ReligiousBorder_{ni}[t]$$

- Travel time elasticity similar to estimates on ancient trade.
  Roman trade: Flückiger et al. (2022); Bronze Age trade: Barjamovic et al. (2019).
- Political border tax: 8%
  (with $\theta = 4$, $d_{between}/d_{within} = e^{0.3/4} \approx 1.08$)
- Religious border tax: 175%
  (with $\theta = 4$, $d_{between}/d_{within} = e^{4.05/4} \approx 2.75$)
- Anderson and van Wincoop (2003) US-Canada border tax: 49%
  (with $\theta = 4$, $d_{between}/d_{within} = e^{1.59/4} \approx 1.49$)

Welfare and counterfactuals

Real consumption depends on a combination of L and T (that's not separately identified):

$$X_n/p_n = \gamma^{-1} (\pi_{nn})^{-1/\theta} \left(L_n T_n^{1/\theta}\right) \left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n}\right)$$

or equivalently in per capita terms

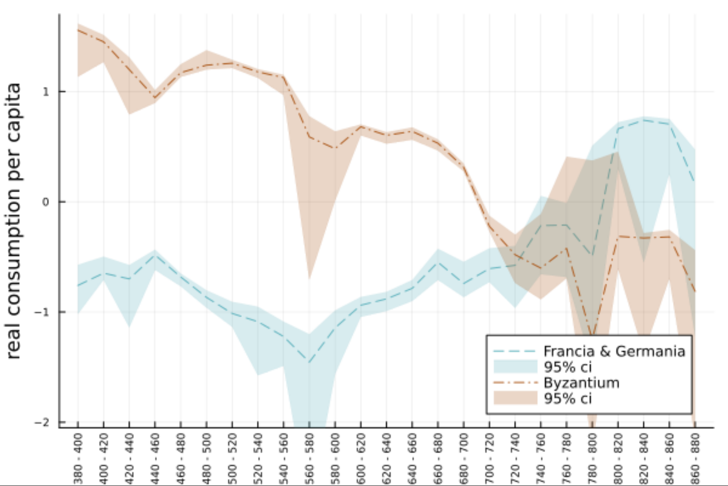
$$\underbrace{\frac{X_n/p_n}{L_n}}_{\text{Real Consumption}} = \gamma^{-1} \underbrace{(\pi_{nn})^{-1/\theta}}_{\text{Openness}} \underbrace{(T_n)^{1/\theta}}_{\text{Technology}} \underbrace{\left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n}\right)}_{\text{Trade Deficit}}$$

Note: T and L are not separately identified. We separate L and T through a Malthusian assumption

$$L_n = T_n \quad \forall n$$

and decompose per-capita real consumption into the three components

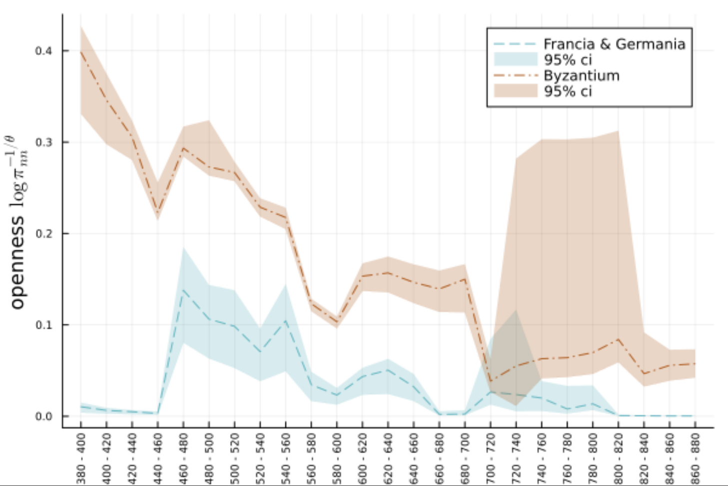
Byzantium vs northern Europe (380-880): *real consumption per capita*



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathsf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

other regions

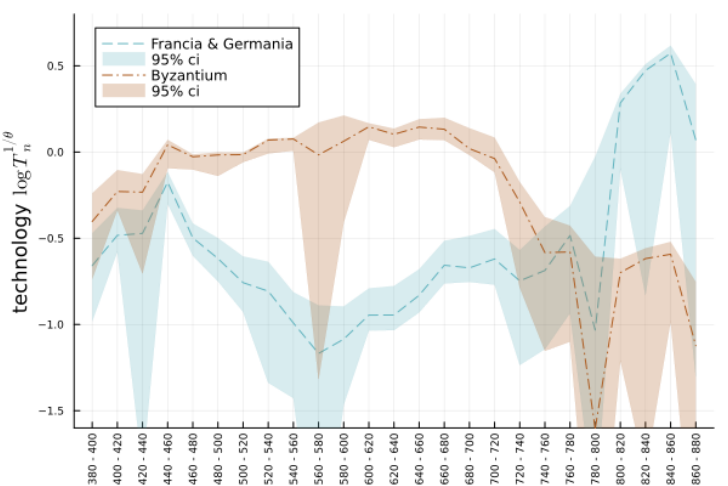
Byzantium vs northern Europe (380-880): *trade openness*



Bootstrapped 95% confidence intervals.

other regions

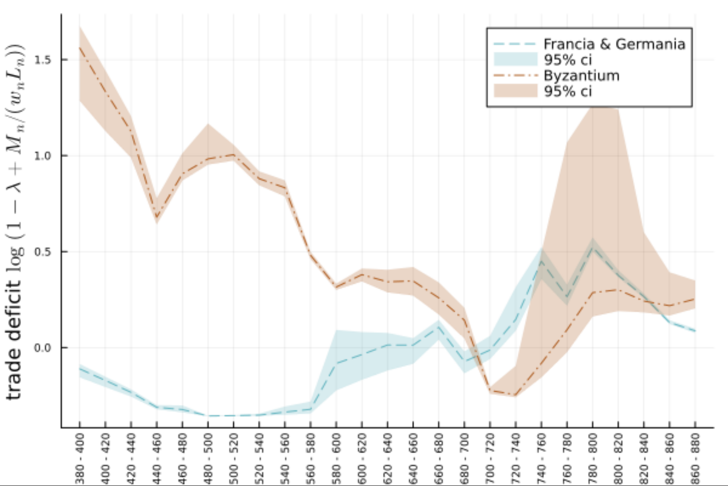
Byzantium vs northern Europe (380-880): *technology*



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathrm{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

other regions

Byzantium vs northern Europe (380-880): *trade deficits*



Bootstrapped 95% confidence intervals.

other regions

Real consumption per capita: technology, geography, and trade (deficits)

Steady state equilibria: AD 460-620 and AD 700-900

	Consumption $\frac{X_n/p_n}{L_n}$		Import share $1-\pi_{nn}$		Technology $T_n^{1/\theta}$		Trade deficits $\frac{X_n}{w_n L_n}=1-\lambda+\frac{M_n}{w_n L_n}$	
	460-620	700-900	460-620	700-900	460-620	700-900	460-620	700-900
Francia and Germania	0.21	1.49	0.18	0.00	0.25	1.52	0.80	0.98
	(0.02)	(0.20)	(0.04)	(0.01)	(0.03)	(0.20)	(0.03)	(0.01)
Byzantine Heartlands	3.40	0.71	0.64	0.12	0.96	0.62	2.73	1.11
	(0.44)	(1.08)	(0.06)	(0.13)	(0.10)	(0.12)	(0.68)	(19.33)
Arabian Peninsula	0.17	0.53	0.17	0.15	0.23	0.44	0.70	1.16
	(0.03)	(0.11)	(0.07)	(0.12)	(0.04)	(0.23)	(0.00)	(0.30)
Average	1.00	1.00	0.20	0.15	0.85	0.92	1.04	1.09
	(0.00)	(0.00)	(0.02)	(0.02)	(0.05)	(0.13)	(0.08)	(1.50)

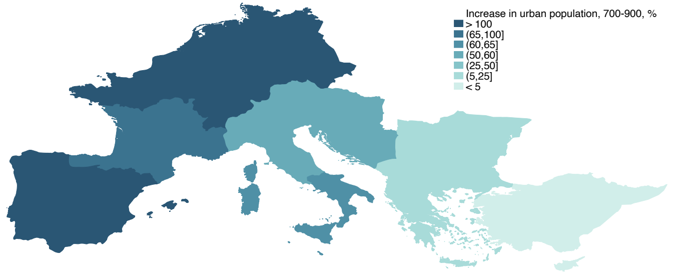
Normalizations: $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\equiv 1, \forall t$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

▸ changes and counterfactuals

Compare to relative urbanization rates, 700–900 AD

Top: Change in total urban population (urban: $>$ 1k inhabitants), data from Buringh (2021)

Bottom: $\Delta W_n / L_n$



Taking stock

- Clear pattern of change in economic geography before vs after conquest
- Trade disruption can account for the relative decline in the eastern Mediterranean
- Change in trade cost *alone* not able to account for urbanization in Muslim Spain, or in Carolingian empire
- In conjunction with changes in technology T_i and mint output, can account for urbanization patterns.

Back to Pirenne:

- Yes, new political and religious borders change market access, quant'ly relevant
- But unlikely to account for entire shift towards north-east
- Seignorage and technical change seem to be more important drivers

Conclusion

"Simply looking at the Mediterranean cannot of course explain everything about a complicated past created by human agents, with varying doses of calculation, caprice and misadventure. But this is a sea that patiently recreates for us scenes from the past, breathing new life into them, locating them under a sky and in a landscape that we can see with our own eyes, a landscape and sky like those of long ago. A moment's concentration or daydreaming, and that past comes back to life."

Fernand Braudel, Les Mémoires de la Méditerranée

THANK YOU!

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BACKUP SLIDES

References

Anderson, James E., and Eric van Wincoop. 2003. "Gravity with Gravitas: A Solution to the Border Puzzle." *American Economic Review*, 93(1): 170–192.

Barjamovic, Gojko, Thomas Chaney, Kerem Coşar, and Ali Hortaçsu. 2019. "Trade, merchants, and the lost cities of the bronze age." *The Quarterly Journal of Economics*, 134(3): 1455–1503.

Eaton, Jonathan, and Samuel Kortum. 2002. "Technology, Geography, and Trade." *Econometrica*, 70(5): 1741–1779.

FLAME. 2023. "Framing the Late Antique and early Medieval Economy." `https://coinage.princeton.edu/`, Accessed: 2023-07-01.

Flückiger, Matthias, Erik Hornung, Mario Larch, Markus Ludwig, and Allard Mees. 2022. "Roman transport network connectivity and economic integration." *The Review of Economic Studies*, 89(2): 774–810.

Gibbon, Edward. 1789. *The history of the decline and fall of the Roman Empire.* Vol. I-VI, London:Strahan & Cadell.

Kazhdan, Alexander. 1954. "Vizantijskie goroda v VII-IX vv." *Sovetskaja arkheologija*, 21: 164–188.

Regions

▸ Back

Fact #3: Coin flows before/after the Arab conquests

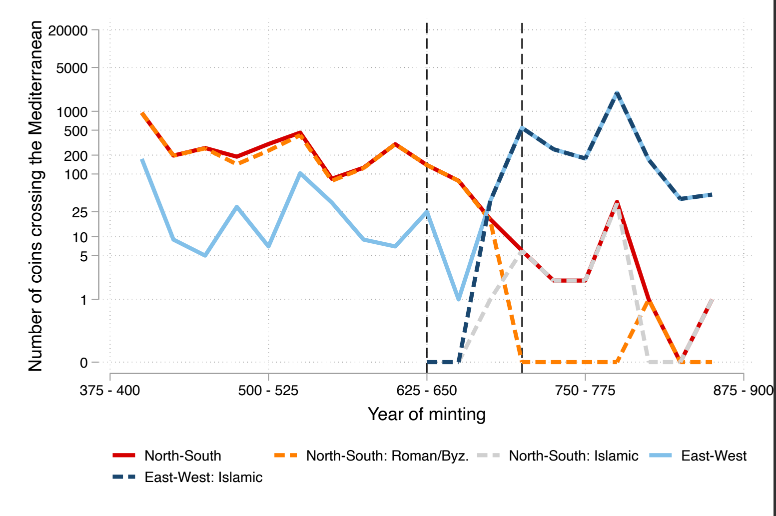


Figure 3: Number of coins flowing across the Mediterranean

Fact #3: Coin flows before/after the Arab conquests

Table 3: The Mediterranean Before and After the Arab Conquest

	Dependent variable: Number of Coins			
	(1)	(2)	(3)	(4)
Crossing Mediterranean × After Conquests	-1.893***	-3.246***	-0.662	-1.736
	(0.48)	(0.53)	(0.63)	(1.27)
Crossing Mediterranean × After Conquests × Islamic Coin		7.267***	4.789***	7.545***
		(0.90)	(0.95)	(0.89)
Crossing Mediterranean × After Conquests × Roman Coin			-3.287***	-2.893***
			(0.75)	(0.61)
Mint Cell × Empire FE	Yes	Yes	Yes	Yes
Mint Cell × Hoard Cell FE	Yes	Yes	Yes	Yes
After Conquests FE	Yes	Yes	Yes	
Mint Cell × After Conquests FE				Yes
Hoard Cell × After Conquests FE				Yes
Estimator	PPML	PPML	PPML	PPML
Observations	10480	10480	10480	6208

Standard errors in parentheses, clustered at the hoard × era and mint × era level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

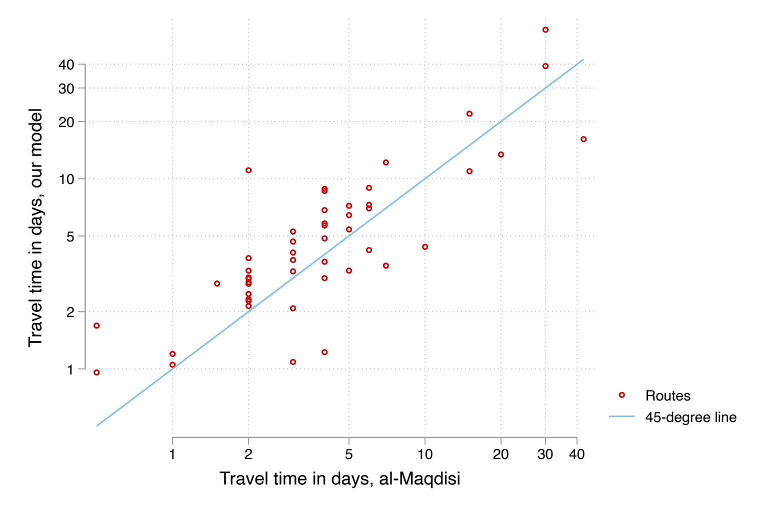
Estimating eqn: $\text{count}_{mdht} = \exp\left(\gamma_1 \text{mediterranean}_{mh} \times \text{after}_t + \cdots + \alpha_{md} + \alpha_{mh} + \varepsilon_{mhdt}\right)$

Back

Gold

Validating Travel Times

Al-Maqdisi (c. 945–991): *The Best Divisions for Knowledge of the Regions*



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Unsolved problems (as of yet)

- *Lucas critique #1*: cost function does not minimize costs

$$\ln\left(\delta_{ni}(t)\right) = \min_{p \in paths(i \to n)} \Bigg(\gamma_0 + \gamma_1 \ln\left(TravelTime_p(t)\right) + \sum_{pb:\text{ all political borders along } p} \gamma_2 PoliticalBorder_{pb}(t) + \sum_{rb:\text{ all religious borders along } p} \gamma_3 ReligiousBorder_{rb}(t) \Bigg)$$

- *Lucas critique #2*: net saving (in δ_{nn}) depends on parameters.
- *Fix for #2*: location-specific intercepts ($\gamma_{0,n}$) and δ_{nn}'s.

For now: constant γ_0, and $\delta_{nn} = 1$...

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Fact #3: Arab conquests disrupt Mediterranean trade: Gold only

	Dependent variable: Number of Coins				
	(1)	(2)	(3)	(4)	(5)
Log Distance	-1.516***	-1.541***	-1.544***	-1.189***	-1.193***
	(0.13)	(0.13)	(0.13)	(0.15)	(0.15)
Crossing Mediterranean	0.298	0.307	0.320	0.0942	0.122
	(0.40)	(0.39)	(0.39)	(0.31)	(0.31)
Crossing Mediterranean × After Conquests	-1.600**	-2.858***	-1.719**	-2.576***	-3.379***
	(0.70)	(0.68)	(0.69)	(0.98)	(1.13)
Crossing Mediterranean × After Conquests × Islamic Coin		3.020***	1.864**		2.985**
		(0.71)	(0.76)		(1.20)
Crossing Mediterranean × After Conquests × Roman Coin			-1.699		
			(1.04)		
Mint Cell × Empire FE	Yes	Yes	Yes	Yes	Yes
Hoard Cell × After Conquests FE	Yes	Yes	Yes	Yes	Yes
Sample				Gold only	Gold only
Observations	172442	172442	172442	32024	32024

Standard errors in parentheses, clustered at the hoard × era and mint × era level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Before: 400–630; after: 713–950

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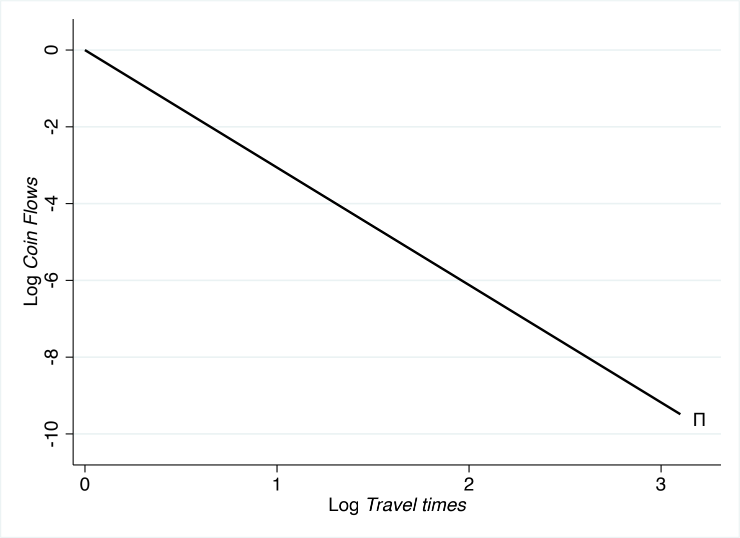
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1 (same as trade flows $\mathbf{\Pi}$)

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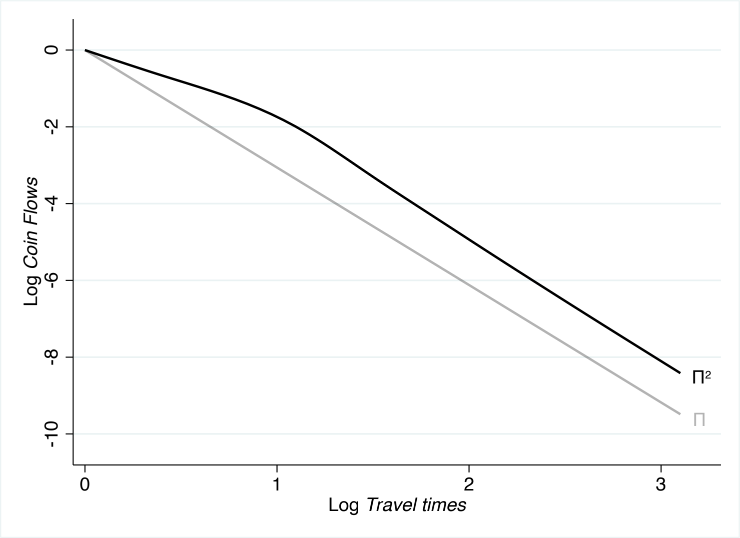
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2

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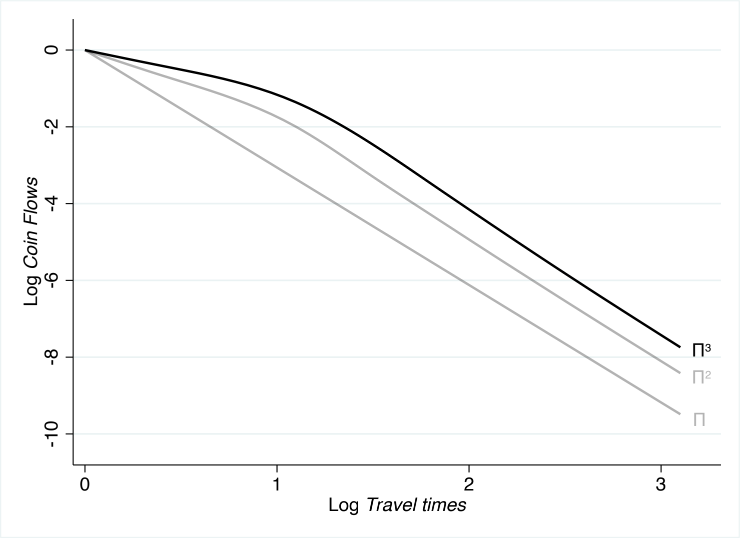
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3

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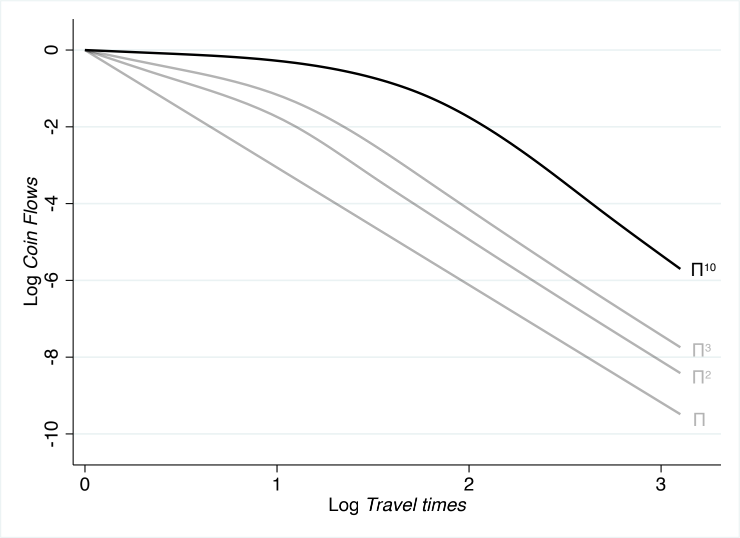
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10

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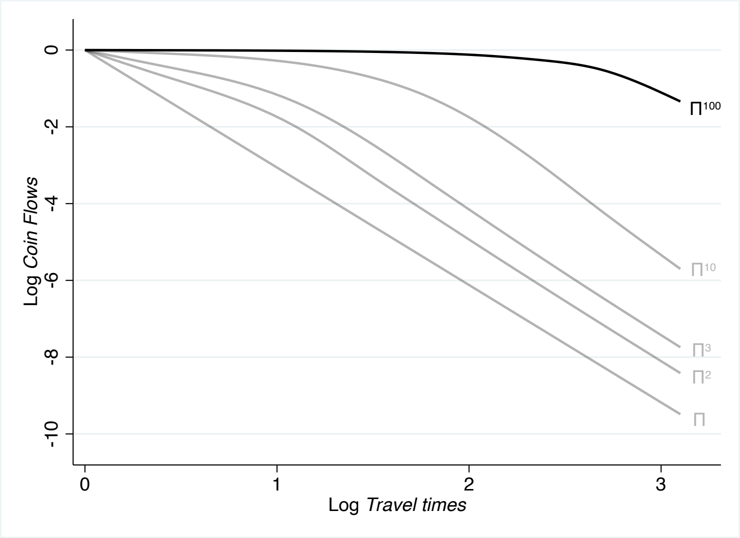
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10, age 100

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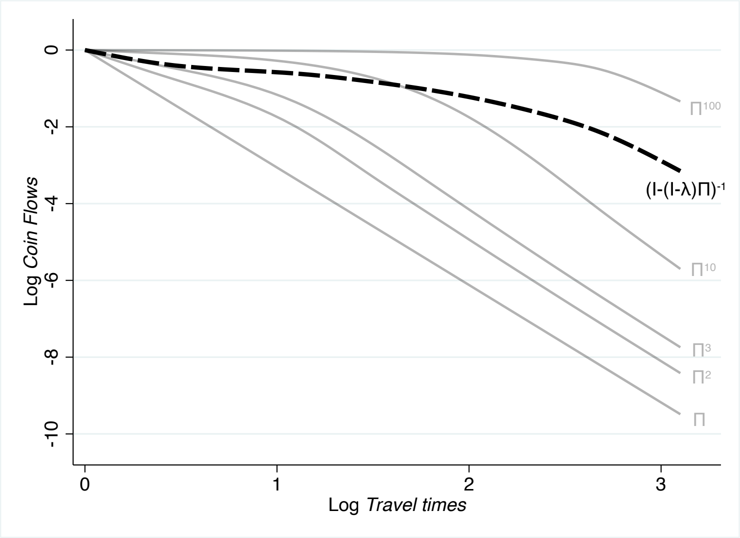
Pitfall #2: stocks vs flows (numerical example)



- Flow of coins: age 1, age 2, age 3, age 10, age 100, all ages

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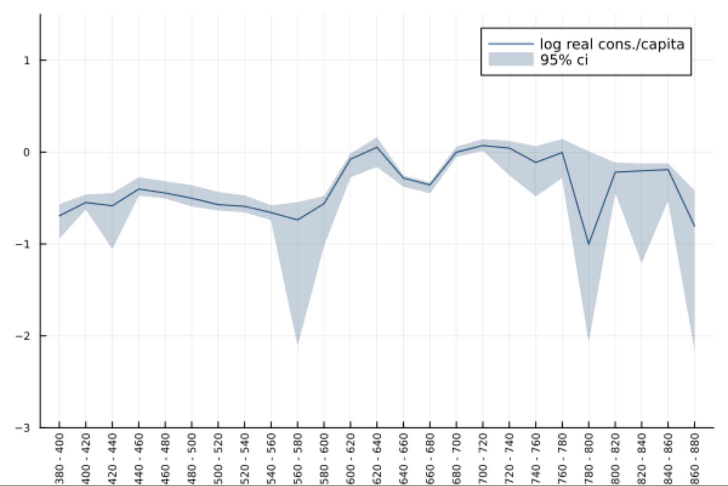
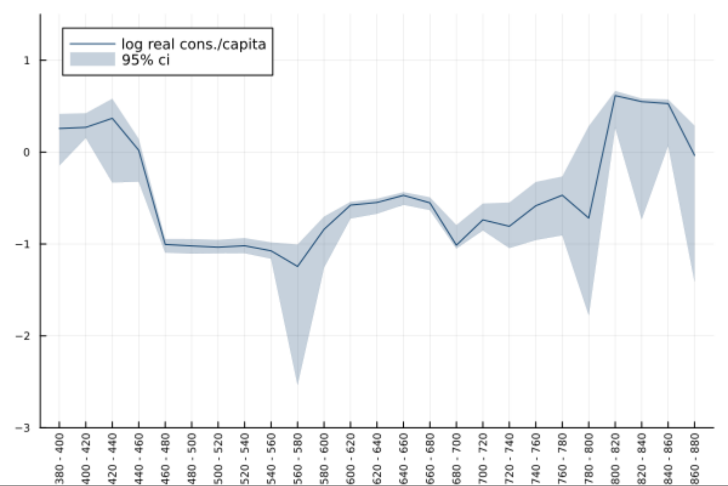
Real consumption per capita (380-880): al-Andalus (Spain)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

▸ back

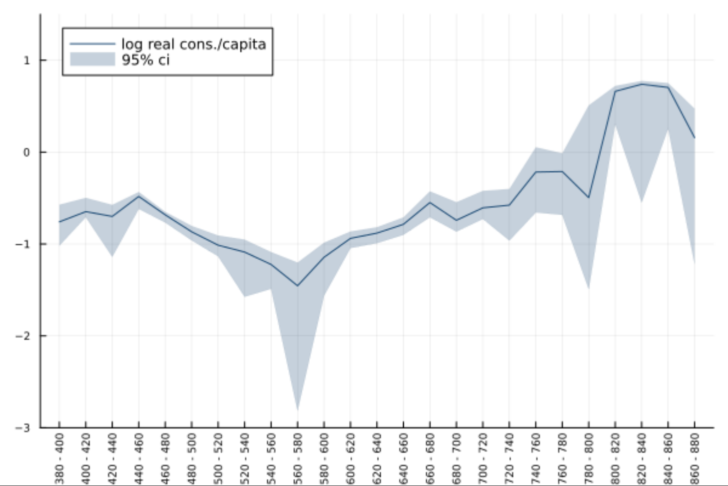
Real consumption per capita (380-880): Aquitaine (South France)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbb{E}_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

back

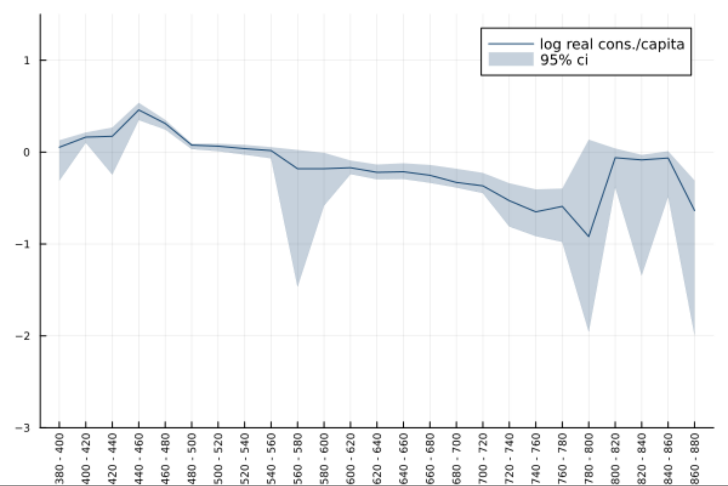
Real consumption per capita (380-880): Francia and Germania



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathrm{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

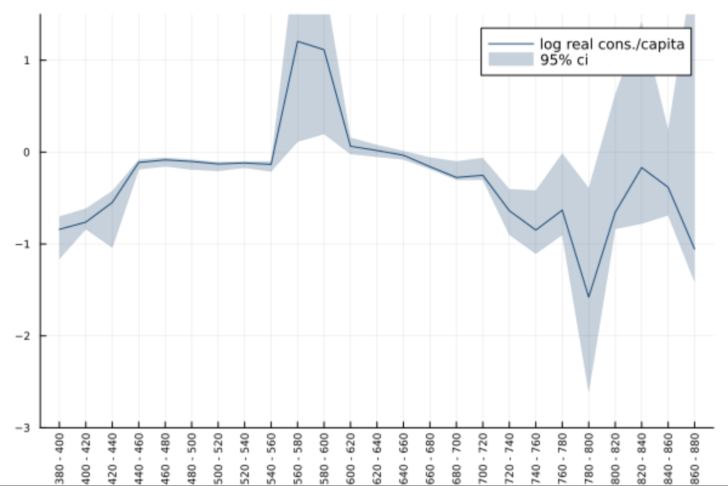
Real consumption per capita (380-880): Northern Italy



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathrm{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

▸ back

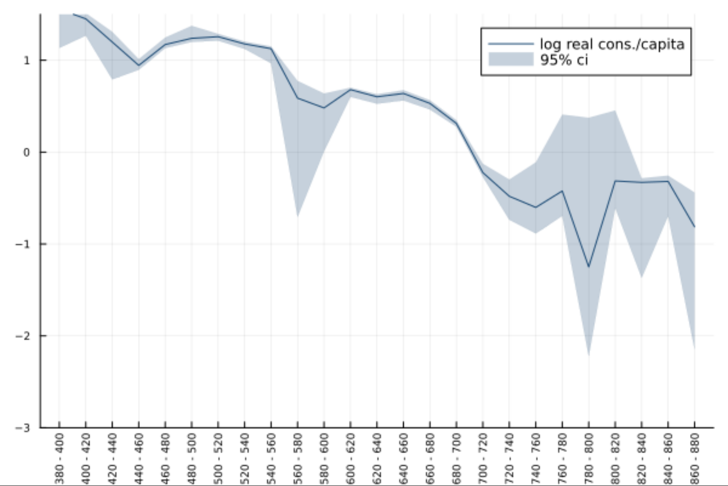
Real consumption per capita (380-880): Southern Italy



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

▸ back

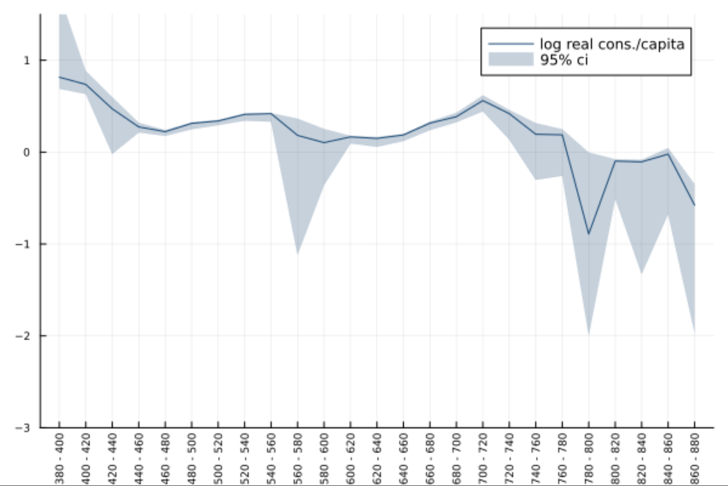
Real consumption per capita (380-880): Byzantine Heartlands



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathrm{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

Real consumption per capita (380-880): al-Sham (Greater Syria)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathrm{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

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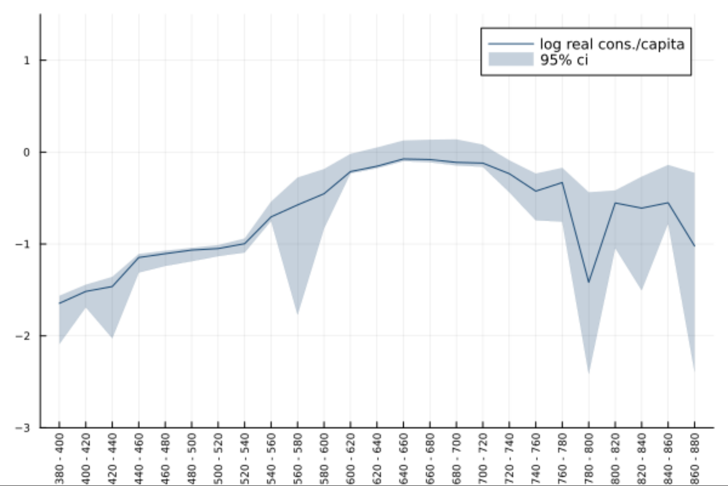
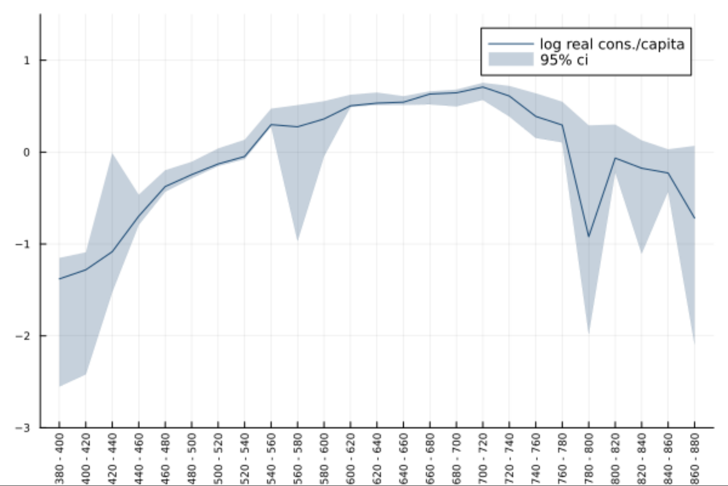
Real consumption per capita (380-880): Northern Syria, Caucasus



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathrm{E}_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

back

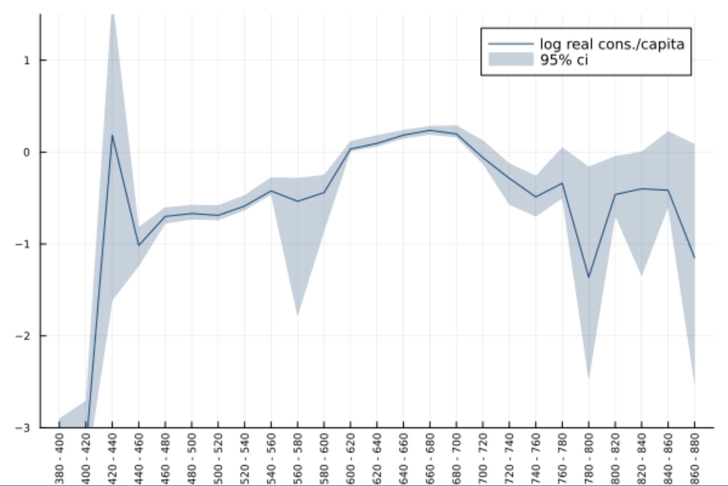
Real consumption per capita (380-880): Iraq, Iran



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbb{E}_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

back

Real consumption per capita (380-880): Eastern Caliphate



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathbf{E}_t \left[\frac{X_n[t]/p_n[t]}{L_n[t]} \right] \equiv 1, \forall t.$

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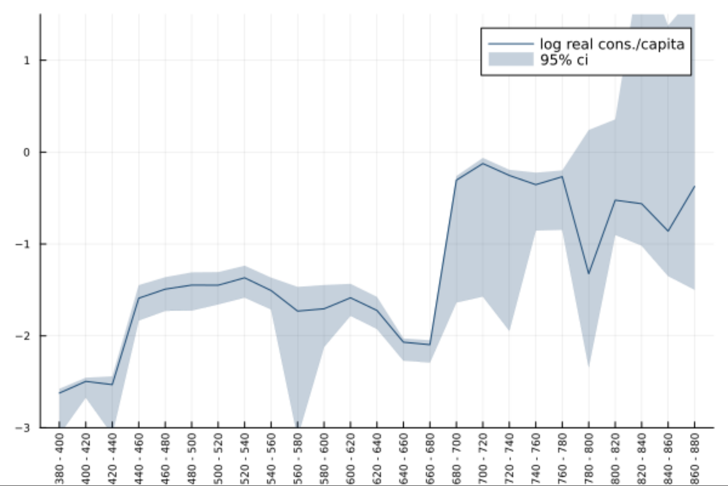
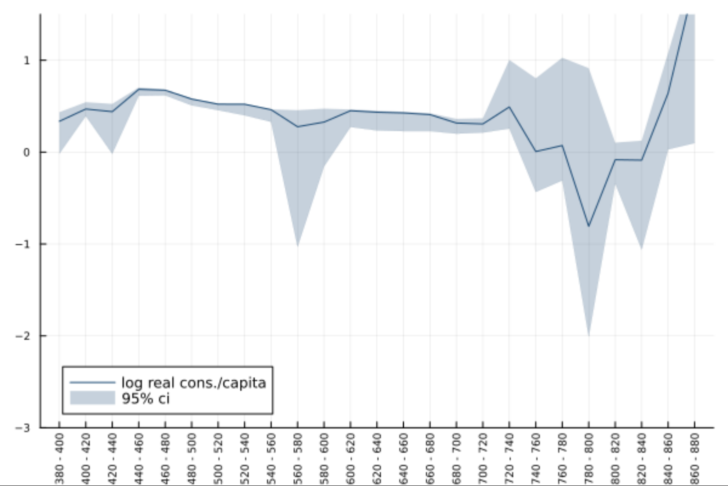
Real consumption per capita (380-880): Arabian Peninsula



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathrm{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

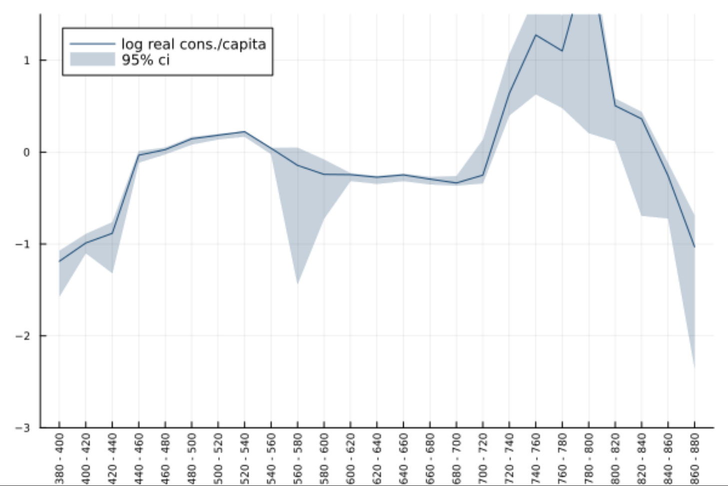
Real consumption per capita (380-880): Misr (Egypt)



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathsf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

back

Real consumption per capita (380-880): al-Maghrib



Bootstrapped 95% confidence intervals. *Normalizations:* $\mathrm{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$

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Realized vs counterfactual changes in real consumption per capita

Realized changes, from AD 460-620 to AD 700-900

	Real consumption $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$		Openness $\Delta \log\left(\pi_{nn}^{-1/\theta}\right)$		Technology $\Delta \log\left(T_n^{1/\theta}\right)$		Trade Deficit $\Delta \log\left(1+\frac{M_n-\lambda w_n L_n}{w_n L_n}\right)$	
al-Andalus (Spain)	0.62	(0.25)	-0.06	(0.04)	0.77	(0.32)	-0.09	(0.18)
Aquitaine (South France)	1.28	(0.23)	-0.05	(0.01)	1.22	(0.23)	0.11	(0.06)
Francia and Germania	1.96	(0.24)	-0.05	(0.01)	1.80	(0.26)	0.20	(0.04)
Northern Italy	-0.31	(0.24)	-0.08	(0.03)	-0.10	(0.26)	-0.13	(0.10)
Southern Italy	-0.20	(0.34)	0.19	(0.18)	-0.94	(0.37)	0.55	(0.40)
Byzantine Heartlands	-1.56	(0.33)	-0.23	(0.14)	-0.44	(0.41)	-0.89	(0.54)
al-Sham (Greater Syria)	-0.32	(0.27)	-0.04	(0.02)	-0.11	(0.29)	-0.17	(0.11)
Northern Syria, Caucasus	0.22	(0.30)	-0.01	(0.03)	0.15	(0.37)	0.08	(0.12)
Iraq, Iran	0.06	(0.27)	-0.00	(0.01)	0.06	(0.29)	-0.00	(0.04)
Eastern Caliphate	0.37	(0.33)	-0.00	(0.00)	0.39	(0.34)	-0.02	(0.04)
Arabian Peninsula	1.16	(0.34)	-0.01	(0.04)	0.66	(0.45)	0.51	(0.26)
Misr (Egypt)	-0.36	(0.72)	0.09	(0.23)	-0.82	(0.50)	0.37	(0.90)
al-Maghrib	0.28	(0.33)	0.13	(0.07)	-0.49	(0.27)	0.65	(0.30)

Normalizations: $\mathrm{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

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Realized vs counterfactual changes in real consumption per capita

Counterfactual changes relative to to AD 700-900

	Initial $\log\left(\frac{X_n/p_n}{L_n}\right)$		Counterfactual $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$ if:					
	All parameters AD 460-620		Religious border AD 700-900		Technology AD 700-900		Minting AD 700-900	
al-Andalus (Spain)	-0.70	(0.10)	0.09	(0.02)	0.55	(0.10)	1.57	(0.31)
Aquitaine (South France)	-1.04	(0.08)	-0.15	(0.03)	0.99	(0.09)	3.93	(0.30)
Francia and Germania	-1.55	(0.09)	-0.07	(0.02)	1.68	(0.11)	6.17	(0.47)
Northern Italy	0.07	(0.04)	-0.24	(0.05)	-0.24	(0.08)	-0.21	(0.07)
Southern Italy	-0.25	(0.06)	-0.11	(0.02)	-0.60	(0.13)	-0.03	(0.02)
Byzantine Heartlands	1.22	(0.11)	-0.69	(0.08)	-0.57	(0.13)	-1.41	(0.19)
al-Sham (Greater Syria)	0.30	(0.04)	0.04	(0.01)	-0.18	(0.10)	-0.22	(0.08)
Northern Syria, Caucasus	-0.34	(0.11)	0.02	(0.02)	0.15	(0.22)	0.19	(0.19)
Iraq, Iran	0.28	(0.08)	0.01	(0.00)	0.03	(0.08)	0.03	(0.06)
Eastern Caliphate	-0.44	(0.08)	0.01	(0.00)	0.38	(0.16)	0.34	(0.26)
Arabian Peninsula	-1.80	(0.18)	0.26	(0.09)	0.66	(0.40)	2.71	(0.84)
Misr (Egypt)	0.32	(0.07)	0.02	(0.00)	-0.71	(0.24)	-0.09	(0.02)
al-Maghrib	0.12	(0.06)	0.01	(0.00)	-0.46	(0.17)	-0.05	(0.06)

Normalizations: $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

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Realized vs counterfactual changes in real consumption per capita

Realized changes, from AD 460-620 to AD 700-900

	Real consumption $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$		Openness $\Delta \log\left(\pi_{nn}^{-1/\theta}\right)$		Technology $\Delta \log\left(T_n^{1/\theta}\right)$		Trade Deficit $\Delta \log\left(1+\frac{M_n-\lambda w_n L_n}{w_n L_n}\right)$	
Francia and Germania	1.96	(0.24)	-0.05	(0.01)	1.80	(0.26)	0.20	(0.04)
Byzantine Heartlands	-1.56	(0.33)	-0.23	(0.14)	-0.44	(0.41)	-0.89	(0.54)
Arabian Peninsula	1.16	(0.34)	-0.01	(0.04)	0.66	(0.45)	0.51	(0.26)

Counterfactual changes relative to to AD 700-900

	Initial $\log\left(\frac{X_n/p_n}{L_n}\right)$		Counterfactual $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$ if:					
	All parameters AD 460-620		Religious border AD 700-900		Technology AD 700-900		Minting AD 700-900	
Francia and Germania	-1.55	(0.09)	-0.07	(0.02)	1.68	(0.11)	6.17	(0.47)
Byzantine Heartlands	1.22	(0.11)	-0.69	(0.08)	-0.57	(0.13)	-1.41	(0.19)
Arabian Peninsula	-1.80	(0.18)	0.26	(0.09)	0.66	(0.40)	2.71	(0.84)

Normalizations: $\mathbf{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t.$ Bootstrapped s.e.'s in parentheses (100 bootstraps).

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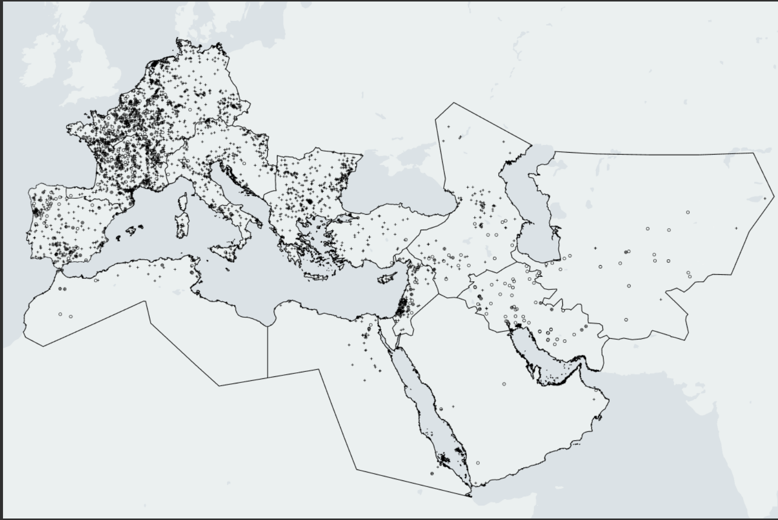
Spatial distribution: the (extended) Mediterranean



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Distribution of coin "death dates" (*tpq*)

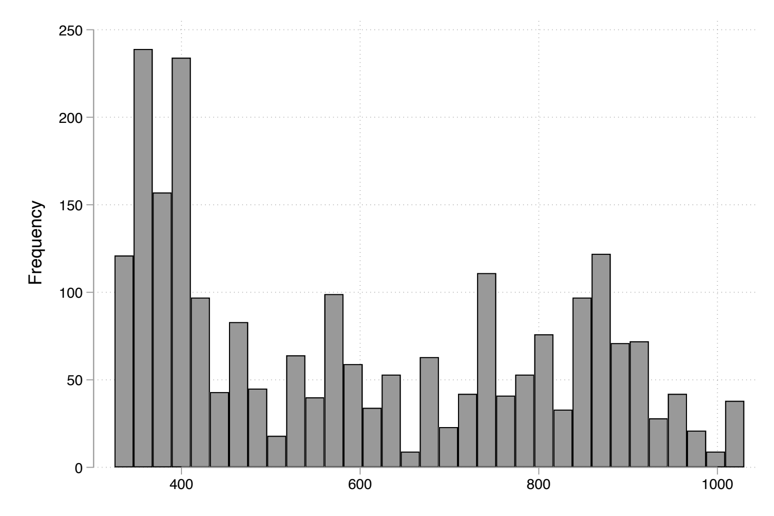

Figure 4: Terminus Post Quem (*tpq*) of hoards

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Distribution of coin ages (*tpq* minus mint date)

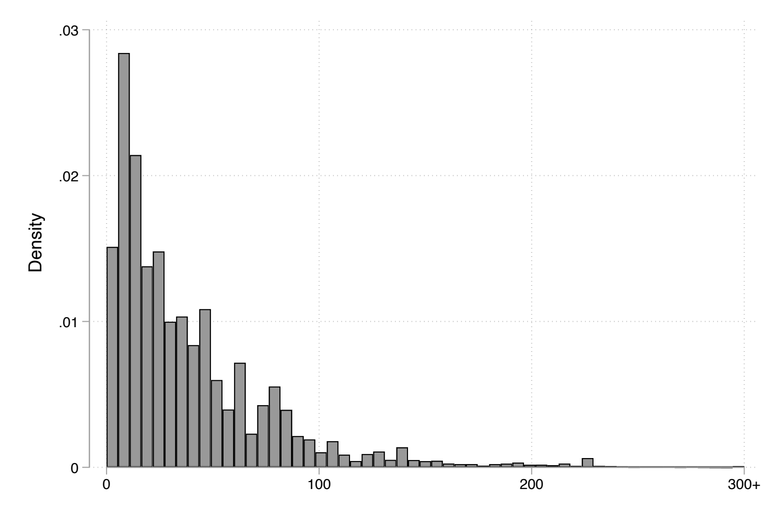

Figure 5: Coin age at time of deposit (*tpq*), in years

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Fact #2: distance has a weaker impact on older coin flows

$$\log \text{count}_{mth\tau} = \sum_{\tau' \in T} \beta_{\tau'} \log \text{distance}_{mh} \times 1\left(t - \tau = \tau'\right) + \alpha_{mt} + \alpha_{h\tau} + \varepsilon_{mth\tau}$$

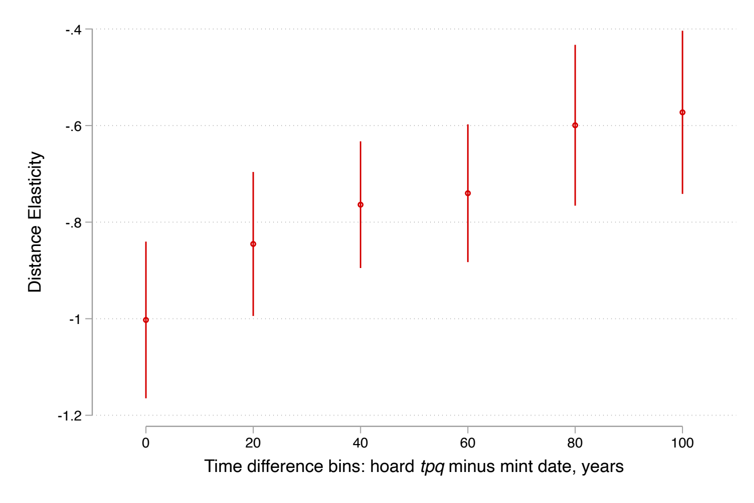

Figure 6: The distance elasticity declines as coins get older

Pitfall #1: medium of exchange vs store of value

- Dynamics with 'saving-augmented' trade shares,

$$\mathsf{S}(t,T) = \boldsymbol{M}(t)\left(\prod_{\tau=t}^{T-1}\Big(\boldsymbol{I}-\boldsymbol{\lambda}(\tau)\Big)\tilde{\boldsymbol{\Pi}}(\tau)\right)$$

- Separate origin, destination, and bilateral terms,

$$\tilde{\pi}_{ni}(\tau) = \alpha_n(\tau)\,\beta_i(\tau)\,\delta_{ni}(\tau)$$

$$\alpha_n = \frac{1}{\sum_k T_k\left(w_k d_{nk}\right)^{-\theta}}, \beta_i = T_i\left(w_i\right)^{-\theta}$$

$$\delta_{ni} = \left(d_{ni}\right)^{-\theta}$$

Pitfall #1: medium of exchange vs store of value

- Dynamics with 'saving-augmented' trade shares,

$$\mathsf{S}(t,T) = \boldsymbol{M}(t)\left(\prod_{\tau=t}^{T-1}\Big(\boldsymbol{I}-\boldsymbol{\lambda}(\tau)\Big)\tilde{\boldsymbol{\Pi}}(\tau)\right)$$

- Separate origin, destination, and bilateral terms,

$$\tilde{\pi}_{ni}(\tau) = \alpha_n(\tau)\,\beta_i(\tau)\,\delta_{ni}(\tau)$$

$$\alpha_n = \frac{1}{\sum_k T_k\,(w_k d_{nk})^{-\theta}},\ \beta_i = T_i\,(w_i)^{-\theta}$$

$$\delta_{ni} = (d_{ni})^{-\theta} \times \begin{cases}(1-s_n) & \text{if } n \neq i\\ (1-s_n) + \dfrac{s_n \sum_k T_k\,(w_k d_{nk})^{-\theta}}{T_n\,(w_n d_{nn})^{-\theta}} & \text{if } n = i.\end{cases}$$

- $\frac{\delta_{nj}}{\delta_{ni}} = \frac{(d_{nj})^{-\theta}}{(d_{ni})^{-\theta}}, \forall n \neq i, j, \forall s_n \geq 0$: no impact on external trade
- $\frac{\delta_{nn}}{\delta_{ni}} > \frac{(d_{nn})^{-\theta}}{(d_{ni})^{-\theta}}, \forall s_n > 0$: net saving mimics home bias in trade!

▶ Back

Pitfall #2: stocks vs flows (steady state math)

- SS: no net saving ($\boldsymbol{s} = 0$), only age (a) matters, not time (t),

$$\mathsf{S}(t, t+a) = \mathsf{S}(a) = M\Big((I - \boldsymbol{\lambda})\,\boldsymbol{\Pi}\Big)^{a}, \forall t$$

- Sum of different vintages (stocks by origin-destination),

$$\sum_{a=0}^{A} \mathsf{S}(a) = M\left(\sum_{a=0}^{A}\Big((I - \boldsymbol{\lambda})\,\boldsymbol{\Pi}\Big)^{a}\right) \underset{A\to+\infty}{=} M\left(I - (I - \boldsymbol{\lambda})\,\boldsymbol{\Pi}\right)^{-1}$$

- Naive gravity on stocks gives Leontief inverse of trade shares!
  $\Rightarrow$ inconsistent estimates of trade elasticities/border effects due to model misspecification

▸ Illustration