EC442 Macroeconomics

Problem Set 5

Fall 2013

Congrats - you made it! The final problem set of the growth part of your macro sequences. This problem set is more a “guide to literature” focusing on some of the aspects of misallocation, which I think are really interesting and important. To talk about misallocation we of course have to have multiple firms being active in the market at the same time. Hence, these are all microfounded models. If you do not have time to do all the problems, focus on Problem 3, which is based on Banerjee and Newman (1993). This paper is on the top 5 list of papers in macro-development and growth of every economist I know. It is just really amazing.

Problem 1

This problem revisits the classic model of Lucas (1978), which is a model about the firm size distribution and provides a special case of an aggregation result for the aggregate production function. Hence, it is *the* model to micro-founded macro model and it is worth to go through it once in your life. Let there be L individuals in the economy. Let the aggregate supply of capital be K. Each individual is endowed with a stock of human capital h (hence, total human capital supply is $H = hL$) and a managerial talent z, which is distributed in the population according to some distribution G_Z. Hence, managerial talent z is the only source of heterogeneity. Because we will be assuming perfect markets (i.e. there are no capital market frictions), the distribution of wealth does not matter for the production side of the economy. Each individual can either open a firm and hire capital and labor on a frictionless market or he can supply his human capital to work for the equilibrium wage w. If an individual opens a firm, she is solely involved in running the firm, i.e. she does not supply labor to her own firm. All firms produce the same final good, which we take as the numeraire.

In case the individual opens a firm, the production function is given by

$$y = zg\left(f\left(k,h\right)\right) = z\left(k^{\alpha}h^{1-\alpha}\right)^{\gamma} \text{ where } \gamma < 1, \tag{1}$$

where γ is the so-called span of control parameter. In case the individual supplies her human capital to another firm, her income is given by hw, where w is the wage per unit of human capital. Let the rental rate of capital be equal to r.

(1) Derive the firm’s cost function, show that equilibrium output (given factor prices) is given by

$$y\left(z\right) = \left(\gamma\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{r}\right)^{\alpha}\right)^{\frac{\gamma}{1-\gamma}} z^{\frac{1}{1-\gamma}}. \tag{2}$$

and derive the factor demands $\left(l\left(z\right), k\left(z\right)\right)$ as a function of z and factor prices. What are the profits of a firm with productivity z?

(2) Now solve for the equilibrium. State the equilibrium conditions (there should be three as there are three unknowns).

(3) To solve for the equilibrium in closed form, we need to assume a particular distribution for productivity z. Let us assume that z is Pareto with support $[z_0, \infty)$ and shape parameter η, i.e.

$$P\left(z < \tau\right) = 1 - \left(\frac{z_0}{\tau}\right)^{\eta}. \tag{3}$$

For the remainder assume that η is high enough that the respective moments are well-defined. Express the equilibrium conditions using (3). *Hint: Recall that if z is Pareto on $[z_0, \infty)$ with shape η, the distribution of z^{β} conditional on $z > \tau_0 > z_0$ is pareto on $[\tau_0^{\beta}, \infty)$ with shape $\frac{\eta}{\beta}$. Also recall that $E[z] = \frac{\eta}{\eta-1}z_0$.* Use the equilibrium conditions to solve for the cutoff $\overline{z}$, i.e. the value $\overline{z}$ such that only individuals with $z > \overline{z}$ open a firm (Hint: there is a explicit solution for $\overline{z}$). How does the cutoff depend on aggregate factor supplies K and H? What is the intuition for this result and what assumption drives it? How does $\overline{z}$ depend on α and γ and what is the intuition?

(4) Now solve for equilibrium prices. In particular, show that

$$r = \frac{\alpha}{k}\left(h^{1-\alpha}k^{\alpha}\right)^{\gamma}\Theta z_0 \quad (4)$$

$$w = \frac{1-\alpha}{h}\left(h^{1-\alpha}k^{\alpha}\right)^{\gamma}\Theta z_0 \quad (5)$$

where Θ is a constant.

(5) Express aggregate output Y as a function of aggregate factor supplies K and L. Derive an expression for aggregate TFP.

(6) We are now going to look at the empirical implications of this model, in particular the distribution of employment in the cross section of active firms. Given the equilibrium factor prices, solve for $l(z)$, i.e. the employment of a z-firm as a function of parameters and aggregate factor supplies. How is employment distributed in the cross-section of active firms, i.e. what is the equilibrium employment distribution? How does the average firm size (i.e. mean employment) vary with aggregate factor supplies K and L? Do you think this implication consistent with the facts? What is the size of the smallest firm? Can you give an intuition why there are no "atomistic" firms in this economy, i.e. why is the firm-size bounded away from zero?

Problem 2

In Problem 1 we saw an expression for aggregate TFP based on productivity of individual units in a frictionless market. Given your results above, you can probably anticipate how frictions in the market for inputs affect aggregate TFP. To see a particular example, please read the first part of Ben Molls paper (up to Section 1.6 "The Evolution of Wealth Shares") - it is a very nice example to formalize the effect of imperfect credit markets.

Problem 3

In this exercise, you are asked to study Banerjee and Newman (1993)'s model of occupational choice. The utility of each individual is $(1-\delta)^{-(1-\delta)}\delta^{-\delta}c^{1-\delta}b^{\delta} - z$, where z denotes whether the individual is exerting effort, with cost of effort normalized to 1. Each agent chooses one of four possible occupations. These are (1) subsistence and no work, which leads to no labor income and has a rate of return on assets equal to $\hat{r} < 1/\delta$; (2) work for a wage ν; (3) self-employment, which requires investment I plus the labor of the individual; and (4) entrepreneurship, which requires investment μI plus the employment of μ workers, and the individual becomes the boss, monitoring the workers (and does not take part in production). All occupations other than subsistence involve effort. Let us assume that both entrepreneurship and self-employment generate a rate of return greater than subsistence (i.e., the mean return for both activities is $\overline{r} > \hat{r}$).

(1) Read the Banerjee and Newman (1993) paper. And once you read it, read it again ... yes, it is that good.

(a) Derive the indirect utility function associated with the preferences above. Show that no individual will work as a worker for a wage less than 1.

(b) Assume that $\mu[I(\overline{r}-\hat{r})-1]-1 > I(\overline{r}-\hat{r})-1] > 0$. Interpret this assumption. [Hint: it relates the profitabilities of entrepreneurship and self-employment at the minimum possible wage of 1.]

(c) Suppose that only agents who have wealth $w \geq w^*$ can borrow enough to become self-employed, and only agents who have wealth $w \geq w^{**} > w^*$ can borrow μI to become an entrepreneur. Provide an intuition for these borrowing constraints.

(d) Now compute the expected indirect utility from the four occupations. Show that if $v > \overline{v} \equiv (\mu-1)(\overline{r}-\hat{r})I/\mu$, then self-employment is preferred to entrepreneurship.

(e) Suppose the wealth distribution at time t is given by $G_t(w)$. On the basis of the results in part d, show that the demand for labor in this economy is given by

$$\begin{array}{rcl} x = & = 0 & \text{if } v > \overline{v} \\ x \in & [0, \mu(1 - G_t(w^{**}))] & \text{if } v = \overline{v} \\ x = & \mu(1 - G_t(w^{**})) & \text{if } v < \overline{v}. \end{array}$$

(f) Let $\tilde{v} \equiv (\overline{r} - \hat{r})I > \overline{v}$. Then show that the supply of labor is given by

$$\begin{array}{rcl} s = & 0 & \text{if } v < 1 \\ s \in & [0, G_t(w^*)] & \text{if } v = 1 \\ s = & G_t(w^*) & \text{if } 1 < v < \tilde{v} \\ s \in & [G_t(w^*), 1] & \text{if} v = \tilde{v} \\ s = & 1 & \text{if } v > \tilde{v} \end{array}$$

(g) Show that if $G_t(w^*) > \mu[1 - G_t(w^{**})]$, there is an excess supply of labor and the equilibrium wage rate is $v = 1$. Show that if $G_t(w^*) < \mu[1 - G_t(w^{**})]$, there is an excess demand for labor and the equilibrium wage rate is $v = \overline{v}$.

References

Banerjee, A., and A. F. Newman (1993): "Occupational Choice and the Process of Development," *The Journal of Political Economy*, 101(2), 274–298.

Lucas, R. E. (1978): "On the Size Distribution of Business Firms," *The Bell Journal of Economics*, 9(2), 508–523.