Misallocation in the Market for Inputs: Enforcement and the Organization of Production*

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Abstract

The strength of contract enforcement determines how firms source inputs and organize production. Using microdata on Indian manufacturing plants, we show that production and sourcing decisions appear systematically distorted in states with weaker enforcement. Specifically, we document that in industries that tend to rely more heavily on relationship-specific intermediate inputs, plants in states with more congested courts shift their expenditures away from intermediate inputs and have a greater vertical span of production. To quantify the impact of these distortions on aggregate productivity, we construct a model in which plants have several ways of producing, each with different bundles of inputs. Weak enforcement exacerbates a holdup problem that arises when using inputs that require customization, distorting both the intensive and extensive margins of input use. The equilibrium organization of production and the network structure of input-output linkages arise endogenously from the producers' simultaneous cost minimization decisions. We identify the structural parameters that govern enforcement frictions from cross-state variation in the first moments of producers' cost shares. A set of counterfactuals show that enforcement frictions lower aggregate productivity to an extent that is relevant on the macro scale.

KEYWORDS: Production Networks, Intermediate Inputs, Misallocation, Productivity, Contract Enforcement, Value Chains
JEL: E23, O11, F12

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1 Introduction

Weak contract enforcement hinders firm-to-firm trade and distorts production decisions. For example, a manager who cannot rely on courts for timely and cheap enforcement may need to purchase low-quality substitutes from a family member, vertically integrate the production process, or switch to a different technique altogether that avoids the bottleneck input. Regardless of the chosen alternative she will find herself producing at a higher cost. Collectively, the micro distortions induced by weak enforcement alter the equilibrium network structure of production and reduce aggregate productivity.

This paper studies theoretically and empirically how weak legal institutions—more precisely, slow contract enforcement due to congestion of the courts—shape the organization of production. We develop a framework that allows us to use detailed micro production data to quantify the impact of these frictions on aggregate productivity.

We study contract enforcement frictions in the context of the Indian manufacturing sector. India is a country with infamously slow and congested courts: the World Bank (2016) currently ranks India 172nd (out of 190) when it comes to the enforcement of contracts, behind countries such as the Democratic Republic of Congo (171st) and Zimbabwe (165th). Around 6m of the 22m pending cases are older than five years, and while India's Law Commission has been advocating vast reforms for several decades, these reforms have not been implemented, and pendency ratios have not decreased. At the same time, India's liberalization and growth has spurred demand for timely enforcement of contracts.

Using plant-level data from India's Annual Survey of Industries, we document several facts about how court congestion alters plants' input choices. While there is an enormous amount of heterogeneity in the input bundles plants use even within narrowly defined (5-digit) industries,[1] the bundles differ in systematic ways related to the quality of courts. To focus on these differences, we differentiate between inputs that are relatively homogeneous and standardized from those that require customization or are relationship specific, using the classification from Rauch (1999). Users (or potential users) of relationship-specific inputs are most likely to benefit from better formal enforcement of supplier contracts.

Our first fact is that in states where courts are more congested, plants in industries that typically rely on relationship-specific intermediate inputs have systematically lower cost shares of intermediate inputs. Second, we show that, where courts are slower, the composition of plants' intermediate input bundles is tilted toward homogeneous inputs. Third, we construct a measure of the vertical span of production of plants designed to capture the number of sequential production steps performed by the plant. We show that, where courts are more congested, plants in industries that typically rely on relationship-specific inputs tend to have larger vertical spans of production; that

[1]Some of this heterogeneity reflects different organizational and technological choices. As an example, a plant that produces frozen chicken may purchase live chicken and slaughter and freeze them; or it may purchase chicken feed, and raise, slaughter, and freeze the chicken on the same vertically integrated plant. Other examples indicate horizontal technological choices, e.g., aluminum can be produced from bauxite or from aluminum scrap.

is, more sequential production steps are performed within plants.

We employ a variety of strategies to alleviate concerns that these patterns arise from omitted factors or through reverse causality. We control for a range of fixed effects and interactions with state and industry characteristics to ensure that court congestion is not standing in for other state characteristics. We also confirm these results using an instrumental variable strategy that exploits the historical origins and structure of the Indian judiciary. Newly created courts tend to be fast and accumulate backlogs over time. As a result, the oldest High Courts, which were set up by the British in the 19th century, are the most congested, and newer courts, which have often been created because of new states being carved out of existing ones, are typically faster. We therefore use the age of the High Court as an instrument for its congestion, and argue that the nature of the political events surrounding the creation of new courts makes it less likely that the resulting congestion is correlated with unobserved determinants of plants' input mixes.

We then construct a model to interpret these facts and to quantitatively evaluate the ramifications of these distortions for aggregate productivity. Our model is written to speak to the patterns of intermediate input use among Indian manufacturing plants. We see that plants use different mixes of intermediate inputs to produce the same output. It is likely that much of this heterogeneity—reflecting different organizational forms, technology, or variation in input prices—would arise even in the absence of distortions. The model incorporates a rich set of organizational, technological, and sourcing decisions so that we do not conflate distortions with other sources of heterogeneity.

Our model is a multi-industry general-equilibrium model of heterogeneous firms and intermediate input linkages that form between them. Firms face a menu of technology/organizational choices ("recipes") and draw suppliers along with match-specific productivities. Both primary inputs and relationship-specific inputs are subject to distortions that reflect weak contract enforcement. Each firm chooses the production technique and suppliers that minimize cost. The effective cost of an input depends on the match-specific productivity, the supplier's marginal cost, and the distortion. We model the enforcement distortion for each potential supplier as randomly drawn to reflect the idea that formal enforcement may only sometimes be relevant at the margin.[2] Weak enforcement has a direct impact on producers that use inputs that require contract enforcement, but may also lead firms to switch to suppliers with a higher cost or to an entirely different production technique with a different set of inputs. We think of changing the organization of production to increase the vertical span as one such option.

To make quantitative statements, we structurally estimate technological parameters and distributions of wedges that distort the use of relationship-specific intermediate inputs. Our identification strategy rests on two key properties of our model. First, the model has the implication that, among firms that, in equilibrium, use the same production recipe, *average* cost shares of each input among firms are invariant to factor prices, but depend on distortions.[3] This is a weaker implication than

[2] For example, formal enforcement may not be necessary if the buyer and supplier are engaged in a long-term relationship, are related, or share other social ties.

[3] As in Houthakker (1955), the aggregate production function among those firms is Cobb Douglas even though

the assumption typically made in the misallocation literature, which is that each plant's cost share is invariant to factor prices (Cobb-Douglas) and that all heterogeneity in cost shares is due to distortions. This property allows identification of distortions from variation in average cost shares across regions. We also make the conservative assumption that weak contract enforcement does not distort the use of homogeneous inputs. As a result, the structural estimating equations take a simple form that is similar to the motivating reduced-form regressions. The identified structural wedges on relationship-specific intermediate inputs are therefore correlated with the observed congestion in the regional courts, in line with the results from the reduced-form regressions.

Our results suggest that courts may be important in shaping aggregate productivity. Having estimated the model parameters, we conduct counterfactuals to investigate the role of contracting frictions. For each state we ask how much aggregate productivity of the manufacturing sector would rise if court congestion were reduced to be in line with the least congested state. On average across states, the boost to productivity is roughly 4%, and the gains for the states with the most congested courts are roughly 8%. Distortions on relationship-specific inputs are only imperfectly explained by our measured congestion, and if one were to halve the distortions on relationship-specific inputs, the average state would see a 7.1% increase in productivity (up to more than 20% for the most distorted state).

Our model builds on recent models of firm linkages in general equilibrium that include Oberfield (2018), Eaton, Kortum and Kramarz (2015), Lim (2017), Lu, Mariscal and Mejia (2013), Chaney (2014), Acemoglu and Azar (2017), Taschereau-Dumouchel (2017), and Tintelnot et al. (2017)[4], and uses aggregation techniques pioneered by Houthakker (1955) and Jones (2005). We model the technology choice and choice of organization concurrently with the sourcing decision, motivated by evidence that increased access to intermediate inputs has a productivity-enhancing effect (e.g. Pavcnik (2002), Khandelwal and Topalova (2011), Goldberg et al. (2010), Bas and Strauss-Kahn (2015)). As in Grossman and Helpman (2002), one producer's choice of organization depends on the industry environment and the choices of other producers.

Our paper is also closely related to the literature on misallocation in developing countries (Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Hopenhayn (2014)). Several papers have studied distortions to the use of intermediate inputs, e.g., Jones (2013), Bartelme and Gorodnichenko (2015), Boehm (2018), Fadinger, Ghiglino and Teteryatnikova (2016), Bigio and La'O (2016), Caprettini and Ciccone (2015), Liu (2017), Caliendo, Parro and Tsyvinski (2017), Osotimehin and Popov (2020), and Baqaee and Farhi (2020). These papers typically posit industry-level production functions and use industry-level data. Our approach of identifying wedges from factor shares (in our case, intermediate input expenditure shares) extends the work of Hsieh and Klenow (2009) along three key dimensions. First, we relate the estimated wedges to the quality of Indian state-

there may be considerable cross-sectional dispersion in cost shares. While Houthakker (1955) assumed that individual production functions are Leontief, we show that this result extends to any constant returns to scale production function in which inputs are complements.

[4]These are also closely related to models of global value chains and global sourcing such as Costinot, Vogel and Wang (2012), Fally and Hillberry (2015), Antràs and de Gortari (2017), Antras, Fort and Tintelnot (2017).

level institutions, which allows us draw policy conclusions from our exercise. Second, we confront the fact that firms produce in very different ways even in narrowly defined industries by explicitly modeling this heterogeneity; we allow firms to choose among several types of technologies (recipes) in the theory and identify these recipes in the data through the application of techniques from statistics/data mining.[5] Third, we identify wedges from systematic differences in first moments, which helps to alleviate concerns about mismeasurement being interpreted as misallocation.[6] In fact, our model predicts that, even in the absence of distortions, firms that use the same broad technology would use inputs with varying intensities.

The paper is related to the literatures on legal institutions and economic development (La Porta et al. (1997), Djankov et al. (2003), Acemoglu and Johnson (2005), Nunn (2007), Levchenko (2007), Acemoglu, Antràs and Helpman (2007), Laeven and Woodruff (2007) among many others). Ponticelli and Alencar (2016) and Chemin (2012) argue that better courts reduce financial frictions. Amirapu (2017) shows that where district courts in India are more congested, firms in industries that relied on relationship-specific inputs grew faster. Johnson, McMillan and Woodruff (2002) provide survey evidence that reduced trust in courts makes firms that rely on relationship-specific inputs less likely to switch suppliers. By embedding a contracting friction into a general equilibrium model, we explore its quantitative importance for aggregate outcomes. Boehm (2018) characterizes the impact of weak enforcement on aggregate productivity, using cross-country differences in input-output tables to show that weak legal institutions have a larger impact on industry pairs that are more vulnerable to holdup problems.

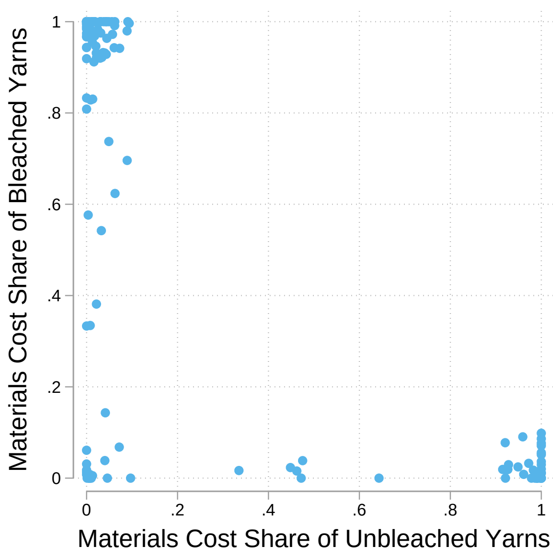
2 Input Use among Indian Manufacturing Plants

2.1 Intermediate Input Use

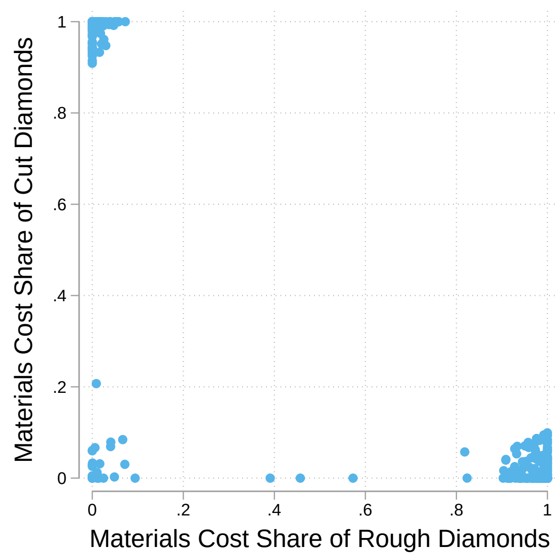
We use data from the 2000/01 to 2012/13 rounds of the Annual Survey of Industry (ASI), the official annual survey of India's formal manufacturing sector. The ASI is a panel that covers all establishments with more than 100 employees, and, every year, a fifth of all establishments with more than 20 employees (or more than 10 if they use power). The ASI's unique feature is that it contains detailed product-level information on each plant's intermediate inputs and outputs. Product codes are at the 5-digit level, of which there are around 5,200 codes in their classification. The product classification remains largely unchanged during the years 2000/01 to 2009/10. The rounds 2010/11 to 2012/13 use a different (albeit similar) product classification, and we bring product-level data to the classification of the earlier years using the official concordance table published by the Ministry of Statistics. Appendix A contains more details on the data and a description of our sample.

[5] Similarly, Asturias and Rossbach (2019) cluster Chilean manufacturing firms based on their capital, labor, and intermediate input usage, and argue that if the clusters are interpreted as representing different technologies, ignoring this heterogeneity would lead to an overstatement of the extent of misallocation.

[6] See Bils, Klenow and Ruane (2017) and Rotemberg and White (2017).



(a) Input mixes for Bleached Cotton Cloth (63303)



(b) Input mixes for Polished Diamonds (92104)

Figure 1 Heterogeneity in input mixes within narrow industries

The figure shows the variation in plant's input mixes for single-product plants that produce only bleached cotton cloth (left panel) or polished diamonds (right panel). Each dot represents a plant-year observation; the coordinates on the horizontal and vertical axes correspond to the materials cost share of two different types of intermediate inputs. For example, a plant on the top left of the right panel produces polished diamonds entirely from cut diamonds (therefore doing just the polishing); an plant on the bottom right produces polished diamonds entirely from rough diamonds (therefore doing both cutting and polishing themselves). Observations on the bottom left mostly produce their output from unbleached cloth (left panel) or industrial diamonds (right panel). Points have been jittered to improve readability.

One striking feature of the data is that even in narrowly defined industries, plants produce using very different input bundles. Figure 1 shows two examples that are particularly clear. Among respective producers of bleached cotton cloth and polished diamonds, output is made using different sets of inputs. While we believe that much of the heterogeneity in organization and input bundles is not associated with inefficiencies and would arise naturally, Section 2.3 below shows that some of the differences are systematically related to court congestion.

Intermediate inputs vary in their degree to which buyers and sellers are subject to hold-up problems. Producers of goods that are tailored to a particular buyer ("relationship-specific") may find that buyers refuse to pay for the supplied good, knowing that they are useless to anyone but themselves (Iyer and Schoar (2008)). We use the Rauch (1999) classification that divides goods into homogeneous goods (those that are traded on organized exchanges or for which a reference price exists), and relationship-specific goods (the remainder). Holdup problems are more likely to arise with relationship-specific inputs. At the same time, timely and cheap enforcement of contracts in a court of justice is a way to alleviate these holdup problems. Hence, firms rely more heavily on judicial institutions to enforce supplier contracts when trading goods belonging to the latter category (Johnson, McMillan and Woodruff (2002)).

2.2 Court Congestion in India

Among all ills of the Indian judicial system, its slowness is perhaps the most apparent one. As of 2017, about nine percent of pending cases in district courts and six percent of pending cases in High Courts are older than ten years.[7] Some cases make international headlines, such as in 2010, when the Bhopal District Court convicted eight executives for death by negligence during the 1984 Bhopal gas leak which killed thousands of people. The conviction took place some 25 years after the disaster; one of the eight executives had already passed away, and the remaining seven appealed the conviction.[8]

The slowness of the Indian courts is at least partly due to the uneven distribution of workload across its three tiers.[9] The lowest tier is the Subordinate (District) Courts, which have courthouses in district capitals and major cities.[10] The next tier are the High Courts, of which there generally exists one for each state, and which have both appellate and original jurisdiction over cases originating from their state (and sometimes an adjacent union territory). High Courts also administer subordinate courts in their jurisdiction. The highest tier is the Supreme Court of India. All three tiers are heavily congested, with district courts facing the additional problems that judges are often inexperienced and are regarded as being less able or willing to make the right decision. While con-

[7]Figures for district courts are from the National Judicial Data Grid (2017). Figures for High Courts are based on authors' calculations from the Daksh data (see below).

[8]"Painfully slow justice over Bhopal," Financial Times, June 7, 2010.

[9]See Robinson (2016) for an overview of the Indian judiciary. Hazra and Debroy (2007) discuss its problems in relation to economic development.

[10]Districts are the administrative divisions below states. Between 2001 and 2010 there were around 620 districts and 28 states in India. Union territories are small administrative divisions (typically cities or islands) that are under the rule of the federal government, as opposed to states, which have their own government.

tract cases between firms should, in principle, be filed at the district level, litigants typically bypass this step by claiming an infringement of their fundamental rights or appealing to the constitution of India, in which case they are permitted to file the claim directly at a high court.[11] High Court judges, often taking a dim view of the subordinate judiciary, tend to accommodate this practice. The result is that the Indian judiciary is relatively heavy in its upper levels, with only the simplest cases being dealt with in the subordinate courts.[12] For better or worse, it is the quality of the higher judiciary that determines whether and how contracts can be enforced.

We construct a measure of court congestion from microdata on pending civil cases in High Courts, which the Indian NGO Daksh collects from causelists and other court records (Narasappa and Vidyasagar (2016)). These records show the status and age of pending and recently disposed cases, along with characteristics of the case, such as the act under which the claim was filed or a case type categorization. Our measure of high court congestion is the average age of pending civil cases in each court, at the end of the calendar year 2016. Whenever a high court has jurisdiction over two states and a separate bench in each of them (such as the Bombay High Court, which has jurisdiction over Maharashtra and Goa), we construct the statistic by state. We prefer this measure over existing measures of the speed of enforcement, such as pendency ratios published by the High Courts, which suffer from the problem that different high courts measure pendencies in vastly different ways (as recently emphasized by the Law Commission of India (2014)).

The average age of pending civil cases varies substantially across high courts – from less than one year in Goa and Sikkim, to about four and a half years in Uttar Pradesh and West Bengal. The cross-state average is two and a half years. These differences can be seen in Figure 2.

The problems of the Indian judiciary are not a recent phenomenon, and have not gone unnoticed. Throughout the modern history of India as an independent nation, the Law Commission of India has pointed out the enormous backlogs and arrears of cases (14th report, 1958, 79th report, 1979, 120th report, 1987, and 245th report, 2014), and suggested a plethora of policies to alleviate the situation. The vast majority of these proposals have not been adopted, and the few exceptions seem to have had little impact. Overall, the backlogs have slowly but continually accumulated.

The main explanation for why court speed varies so much across states lies in the history of India's political subdivisions. The first high courts (Madras, Bombay, and Calcutta) were set up by the British in the 1861 Indian High Courts Act, and served as the precursor for India's post-independence high courts. Upon independence, India was divided into a number of federated states, with the Constitution of India (1947) mandating a high court for each state. Throughout the twentieth century and beyond, India has frequently subdivided its states, often because of ethno-nationalist movements. These subdivisions were often accompanied with new high courts being set up, which then start without any existing backlog of cases.[13] The age of the high court

[11] Some High Courts, such as the High Courts of Bombay, Calcutta, Madras, and Delhi, even allow civil cases to be filed directly whenever the claim exceeds a certain value.

[12] Between 2010 and 2012, about 40% of all disposed cases in subordinate courts were related to traffic tickets, another seven percent related to bounced cheques (Law Commission of India (2014)).

[13] Table VII in the data appendix summarizes the reasons for the high court being set up, or the state being formed.

is hence a strong determinant of its speed of enforcement (see Figure 2). We will later use the age of high courts as an instrument for its level of congestion.

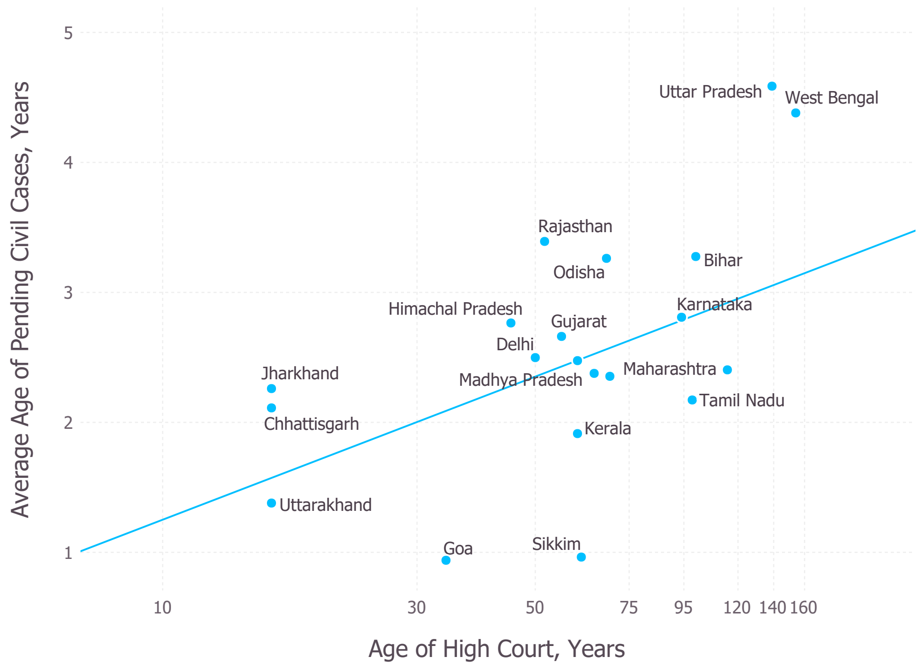


Figure 2 Age of the High Court and Speed of Enforcement

2.3 Motivating Facts

We first turn to documenting the correlation between court congestion and plants' intermediate input use. For the sake of comparability we restrict our attention here to single-product plants.[14]

Fact 1 *In states with more congested courts, cost shares of intermediate inputs are relatively lower in industries that tend to rely more on relationship-specific intermediate inputs.*

Table I shows regressions of the plants' materials cost share on an interaction of court congestion (as measured by the average age of pending cases in the high court of the state in which the plant is located[15]) and the industry's reliance on relationship-specific inputs.[16] The interaction term

[14]One difficulty that arises when studying multi-product plants is that we do not observe which inputs are used to produce each product. Nevertheless, the results in this section are quantitatively similar when we include multi-product plants and assign the plant to the category of its highest-revenue product.

[15] In principle, firms can bring their cases to any court; the court then decides whether it has jurisdiction over the case. We believe that in the case of India this practice is limited, as state borders are often also language borders. More generally, this would be a form of measurement error in the independent variable that would bias the coefficient towards zero.

[16] Following Nunn (2007), we measure an industry's reliance on relationship-specific inputs at the national level by computing the fraction of intermediate input expenditures spent on relationship-specific inputs across all plants in the industry. See Appendix A.1 for details.

has a negative and significant coefficient, showing that plants' materials cost shares decline more steeply with court congestion in industries that tend to rely more heavily on relationship-specific inputs.[17] The magnitude in column (1) indicates that for each additional year of court congestion, plants' materials share of cost declines by 1.67 percentage points more in industries that rely on relationship-specific inputs than in industries that rely on standardized inputs.

A primary concern in this specification is that court congestion is standing in for the level of development, or that the level of development is correlated with the relative productivity of industries that rely on relationship-specific inputs. Column (1) includes district fixed effects. Column (2) controls for the interaction of relationship specificity with district income per capita and column (3) adds controls for the interaction of relationship specificity with a variety of state characteristics including measures of trust, corruption, linguistic fragmentation, and fragmentation by caste. While the coefficients (reported in Appendix C) suggest that ethnolinguistic homogeneity facilitates the use of relationship-specific inputs, this appears to be orthogonal to court congestion. Finally, columns (4) to (6) employ an instrumental variables strategy that we discuss below in Section 2.4.

Table I Materials Shares and Court Congestion (Fact 1)

	Dependent variable: Materials Expenditure in Total Cost					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	-0.0167**	-0.0129*	-0.0118*	-0.0156^{+}	-0.0201*	-0.0212**
	(0.0046)	(0.0051)	(0.0053)	(0.0085)	(0.0082)	(0.0078)
LogGDPC * Rel. Spec.		0.0114	0.0102		0.00710	0.00556
		(0.0086)	(0.0091)		(0.0095)	(0.0096)
Rel. Spec. × State Controls			Yes			Yes
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.480	0.482	0.484	0.480	0.482	0.484
Observations	208527	199544	196748	208527	199544	196748

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"Rel. Spec. × State Controls" are interactions of trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity. Sample consists of single-product plants only.

Fact 2 *In states with more congested courts, intermediate input bundles are tilted towards standardized intermediate inputs.*

Our first fact related court congestion to how plants divided their expenditures between intermediate and primary inputs. We next study how the composition of plants' intermediate input baskets covaries with court congestion. Table II shows that in states where courts are faster, plants' intermediate input baskets are tilted towards relationship-specific intermediate inputs. This cor-

[17] In principle firms may file claims in the courts of any state—it is then up to the court to decide whether it has jurisdiction over the contract. Our results suggest that plants typically find it costly to use courts outside their state.

relation remains statistically significant when controlling for district income per capita and other state characteristics.

Table II Input Mix and Court Congestion (Fact 2)

	Dependent variable: $X_j^R/(X_j^R + X_j^H)$					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg age of Civil HC cases	-0.00547*	-0.00621**	-0.00530*	-0.0144**	-0.0146**	-0.0167**
	(0.0022)	(0.0023)	(0.0024)	(0.0044)	(0.0044)	(0.0045)
Log district GDP/capita		-0.00389	-0.00384		-0.00912+	-0.00980+
		(0.0045)	(0.0046)		(0.0051)	(0.0051)
State Controls			Yes			Yes
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.441	0.446	0.449	0.441	0.446	0.449
Observations	225590	204031	199339	225590	204031	199339

The dependent variable is the share of relationship-specific inputs in total materials cost of plant j. "State Controls" are trust, language herfindahl, caste herfindahl, and corruption. The sample consists of single-product plants only. Standard errors in parentheses, clustered at the state × industry level.
$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$.

Fact 3 *In states with more congested courts, plants in industries that tend to rely more on relationship-specific intermediate inputs have larger vertical spans of production.*

A low materials share suggests that a plant may be doing more consecutive steps in the production process themselves. For example, a car producer that assembles components may also manufacture those components in the same facility. The regressions in Table III show how court congestion is related to the vertical span of production, i.e. to how many consecutive production steps are performed within the plant. We first construct a measure of the "vertical distance" between an output good ω to an input ω'. This is intended to capture the typical number of "steps" between the use of ω' and the production of ω, where we define a step to be the activity performed by a single plant.[18] Finally, for each single-product plant, our measure of vertical span is the expenditure-weighted average of the distance from the plant's output ω to its intermediate inputs:

$$\text{verticalSpan}_j = \sum_{\hat{\omega}\in\Omega} \frac{X_{j\hat{\omega}}}{\sum_{\tilde{\omega}\in\Omega} X_{j\tilde{\omega}}} \text{verticalDistance}_{\omega\hat{\omega}}$$

where $X_{j\hat{\omega}}$ is plant j's expenditure on input $\hat{\omega}$ and ω is j's output. A longer vertical span indicates that the plant uses inputs that are typically further upstream, and suggests that the plant is performing more "steps" in-house. Table III shows that plants' vertical spans of production increase

[18] We construct national input-output tables using our plant-level data. For each output good ω and input good ω', we take a weighted average of the number of steps along any path from ω' to ω, weighted by the product of the input-output shares along that path, excluding any path which cycles. This measure is similar to $Upstreamness_{ij}$ of Alfaro et al. (2015). Appendix B gives the precise mathematical definition of vertical distance.

more sharply with court congestion in industries that tend to rely more heavily on relationship-specific inputs.

Table III Vertical Span of Plants and Court Congestion (Fact 3)

	Dependent variable: Vertical Span					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	0.0195^{+}	0.0269^{*}	0.0280^{*}	0.0292	0.0314^{+}	0.0368^{*}
	(0.011)	(0.012)	(0.012)	(0.019)	(0.018)	(0.018)
LogGDPC * Rel. Spec.		0.0464^{*}	0.0288		0.0491^{*}	0.0330
		(0.022)	(0.024)		(0.023)	(0.024)
Rel. Spec. × State Controls			Yes			Yes
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.443	0.451	0.453	0.443	0.451	0.453
Observations	163334	156191	154021	163334	156191	154021

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"Rel. Spec. × State controls" are interactions of trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity. Sample consists of single-product plants only.

2.4 Endogeneity, and the Historical Determinants of Indian Court Efficiency

The main caveat in the above regressions is the concern that there are unobserved covariates of court congestion that may also affect the cost of plants' inputs, and thereby their input shares. The simplest version is reverse causality. In principle, the bias from reverse causality could be positive or negative. Suppose that a state had, for exogenous reasons, many plants that produced using relationship-specific inputs. The disputes that arise may cause the courts to be congested. Or alternatively, the state may respond to the disputes that arise by spending resources to reduce congestion. Either of these would be problematic for interpreting the regressions as a causal relationship.

While we believe reverse causality is unlikely to arise—the fraction of cases related to firm-to-firm trade is relatively low[19]—it is difficult to rule out other factors that may influence both court congestion and usage of relationship-specific inputs.

We therefore employ an instrumental variables strategy that uses the historical determinants of congestion. As discussed in Section 2.2, courts have been continually accumulating backlogs throughout the 20th century. At certain points in time, however, states were split or reorganized, mostly in response to ethno-nationalist movements. In the course of these reorganizations, new high courts were set up, which initially started with a clean slate but were, like existing courts, understaffed and started accumulating backlogs. The time since their founding—the court's age—is

[19]The fraction of cases that is related to the enforcement of supplier contracts is hard to pinpoint exactly because courts classify cases very broadly, and in different ways across states. We know, however, that they account for less than 5%, 14%, and 7% of the pending cases in the High Courts of Allahabad, Mumbai, and Kolkata, respectively.

therefore a strong predictor for the current backlog, which in turn determines the present-day speed of enforcement. Our instrumental variable for the speed of enforcement is hence the (log) age of the high court, and the instrument for an interaction of an industry-level variable with court speed is the interaction of the industry-level variable with the log age of the court. Figure 2 in Section 2.2 shows the strong correlation between court age and speed of enforcement.

Columns (4) to (6) of Tables I, II, and III repeat the regressions while instrumenting for the speed of enforcement. The point estimates of the coefficient of the interaction term is usually slightly larger than the OLS estimates.

There are a few reasons that the exclusion restriction may be violated. We argue that two candidates would lead us to conclude that the true relationships are *stronger* than reported in the IV regressions. First, new states tend to be relatively poor and have low state capacity. Thus the usual concern that a high level of development causes firms to use more sophisticated technologies that use relationship-specific inputs would cause the newer states to have higher use of homogeneous inputs. Alternatively, it may be that when a state splits, many firms lose their suppliers and must switch. It may be easy to find a new supplier of homogeneous inputs, whereas it might be harder to find a supplier of relationship-specific inputs. This channel would also cause newer states to be more intensive in homogeneous inputs. In either case, the true relationship would be stronger than reported. A third reason that the exclusion restriction might be violated is that newly formed states might be more ethnically homogeneous, so that newer courts might be correlated with better informal enforcement. This is a particular concern here because ethno-nationalist conflicts are a primary reason that states split. Nevertheless, we can control for states' ethnic fragmentation—or more specifically the interaction of relationship-specificity with various measures of ethnic-fragmentation—as we do in columns (3) and (6) of Tables I, II, and III. Controlling for these has little impact on the main coefficients of interest.

In addition to the cross-sectional regressions shown above, we bring evidence from time variation in the degree of court congestion in Appendix C.2. We first show results from two instances in which high courts set up new and fast benches in remote areas during our sample period. In addition, we employ variation in court pendency ratios over time. In both cases we include fixed effects to identify the relationship from time variation in court congestion only. While neither experiment is perfect—with the court expansion the sample is small and some estimates are imprecise, while with pendency ratios it is not clear what drives the variation—the results are consistent with our baseline cross-sectional evidence.

A final concern is that an industry's reliance on relationship-specific inputs is correlated with other industry characteristics such as capital intensity, skill intensity, upstreamness, or tradability. In Appendix C.1 we show that the estimates are robust to controlling for the respective interactions of court congestion with each of these industry characteristics. More broadly, in Appendix C we show a number of additional results about the relationship between court congestion and establishment characteristics (age, size, number of products, and import ratios), and more robustness checks.

2.5 Inferring Properties of Contracting Frictions

What do these regressions tell us about the form of contracting frictions? The literature on contracting frictions (e.g., Antràs (2003)) has emphasized that holdup problems may result in transactions in which the seller shades on the quantity or quality of the inputs. These can be modeled in different ways that would show up differently in the data.

If distortions resulted in an inefficiently low quantity of the input, the buyer's shadow value of the input would be above the seller's marginal cost. In this formulation, the distortion would be observationally equivalent to a positive wedge as in Hsieh and Klenow (2009). One testable implication of this formulation is that the distortion should raise the buyer's ratio of revenue to total cost. In Appendix D.1, we explore this relationship in our setting. We find that plants that are subject to larger wedges—those in industries that tend to use relationship-specific inputs in states with congested courts—have *lower* revenue-cost ratios, in contrast to the prediction from a quantity distortion.

Distortions that result in an inefficiently low quality of the input can take several forms, depending on the relationship between quality and quantity. The simplest form is that quality is quantity-augmenting, in which case the distortion would be equivalent to a higher effective price of the input (i.e. an iceberg cost). In this case, the impact on cost shares depends on the elasticity of substitution between distorted and undistorted inputs. If primary inputs and intermediate inputs were substitutes, a higher effective price of the input would cause the cost share of intermediates to fall. However, we believe the evidence does not support an elasticity of substitution between primary and intermediate inputs that is larger than unity. We know of two estimates of long-run plant level elasticities of substitution between materials and primary inputs, Oberfield and Raval (2014) and Appendix B.3 of Atalay (2017). Each find elasticities that are slightly less than unity. Further, we can investigate this elasticity in the context of Indian manufacturing plants. In Appendix D.4 we use upstream contracting distortions as a shifter of the seller's costs. We find evidence against an elasticity greater than one, in line with what the existing literature finds.

Finally, it could be that quality and quantity enter the production function in different ways. For example, it may be that the seller needs to customize the good for the buyer and can do so inexpensively, and the buyer can do the customization herself but less efficiently than the seller. In that case, if the friction causes the seller to insufficiently customize the good, the buyer's cost will rise because the wrong producer is doing the customization. Further, part of this effective expenditure on the distorted input will appear in the data as an expenditure by the buyer on primary factors–the primary factors used by the buyer to finish the customization of the good. Thus the expenditure share on intermediates (and especially on relationship-specific intermediates) will be lower when distortions are more prevalent, even if the elasticity of substitution between primary and intermediate inputs is weakly less than one. This last remaining way of modeling distortions is consistent with what we find in the regressions of materials cost shares (Table I) and the mix of distorted vs undistorted materials inputs (Table II). We will therefore use it in the subsequent model.[20]

3 Model

The previous section showed that imperfect contract enforcement alters the production decisions of manufacturing firms in India in systematic ways. We next aim to quantify the impact of weak enforcement on the productivity of the manufacturing sector.

A commonly used approach to quantify the role of factor market frictions, pioneered by Hsieh and Klenow (2009), is to posit that plants in the same industry use a common Cobb-Douglas production function. This allows them to use dispersion of cost shares within industries to infer dispersion in marginal products of inputs. What gives this approach traction is the implication that in the absence of distortions, plants would have the same cost shares regardless of the factor prices they face. Deviations from this cost share would then be interpreted as the result of a distortion. This approach, however, extends poorly to the question of finding distortions in the use of intermediate inputs, where, as we have seen, variation in factor use along both intensive and extensive margin is widespread and unlikely to be driven entirely by distortions.[21] At the same time, departing from Cobb-Douglas would require reliable information on input prices that are comparable across plants and across locations, as well as a way to disentangle distortions from differences in the level and factor bias of productivity. This is especially important in the context of intermediate inputs, where such information is typically unavailable or unreliable.

In this section we develop a model which we will use to evaluate the impact of contracting frictions. We will make two key identifying assumptions. First, our assumptions will imply that, at a certain level of aggregation, factor shares will be invariant to factor prices. This implication is related to, but weaker than, those made by Hsieh and Klenow (2009), which is that factor shares are invariant to factor prices *at the firm level*. Second, we assume that imperfect contract enforcement distorts the use of relationship-specific inputs and of labor, but not the use of homogeneous inputs.[22] These two aspects of our model enable us to identify distortions in factor markets from factor shares alone (in our case, *average* factor shares, thereby not requiring price data), while still allowing plants to have production functions that deviate from Cobb-Douglas, and therefore have varying factor shares for reasons that are unrelated to distortions. We then estimate the importance of distortions on the use of relationship-specific intermediate inputs by looking at these average factor shares. This ties back to the reduced-form evidence we have shown above: that patterns of plants' expenditure shares differ systematically across states, and in a way that is correlated with a potential source of distortions.

[20]It could be that the buyer and seller are equally good at customization, in which case the friction simply leads to a different division of labor without raising the buyers marginal cost. However, we find that distortions reduce entry (see Appendix D.2), suggesting that they do increase marginal cost.

[21]We attempt such an exercise in Appendix I. There are several conceptual and practical obstacles. In any case, the observed variation in input use would lead to enormous implied distortions.

[22]If contracting frictions also distorted the use of homogeneous inputs, the gains from reducing contracting friction would be larger than those we report.

The model gives a prominent role to two additional features which we may be important for our quantification of contracting frictions: firm-to-firm trade and technology/organizational choice. First, there are many ways to avoid contracting frictions. Suppose a firm needed to use an input that required customization and therefore gave rise to a holdup problem. In the face of weak formal contract enforcement, the firm might buy the intermediate input from a relative or rely on the repeated interactions of a long-term relationship. Such decisions, however, may come at a cost. A family member may not be the optimal supplier of an intermediate input, and if a firm is in a long-term relationship, it may pass up using new, more cost-effective inputs in order to remain in that long-term relationship.[23] Such a firm's production cost is higher than it would be with better contract enforcement. Nevertheless, since the firm avoids the holdup problem, its expenditure shares will not be distorted. The higher production cost is an *indirect* consequence of weak formal enforcement. We can infer this indirect cost by incorporating this type of decision in the model, and estimating how easily firms can substitute across suppliers of the same input.

Second, as discussed in Section 2.1, even in narrowly defined industries, firms produce in qualitatively different ways. As a simple example, consider two plants that both produce polished diamonds, one that buys cut diamonds and another that buys rough diamonds and cuts and polishes them. It may well be the case that the latter's decision to have a larger vertical span of production and do both cutting and polishing was a consequence of a distortion. The two plants will use different production functions and simply comparing the two plants' expenditure shares will not give a direct measure of the size of the distortion. The empirical implementation of our model must therefore be able to account for these differences and incorporate them into the counterfactual simulation.

3.1 The Environment

There is a set of industries Ω. For industry $\omega \in \Omega$, there is a mass of firms with measure J_ω that produce differentiated varieties. There is a representative household that inelastically provides a mass of labor with measure L and has nested CES preferences over all varieties in each industry, maximizing consumption of the bundle U defined as

$$\begin{aligned} U &= \left[\sum_{\omega\in\Omega} \upsilon_\omega^{\frac{1}{\eta}} U_\omega^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\ U_\omega &= \left[\int_0^{J_\omega} u_{\omega j}^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

where $u_{\omega j}$ is consumption of variety j in industry ω, υ_ω reflects the household's taste for goods in industry ω, η is the elasticity of substitution across industries, and ε is the elasticity of substitution

[23] Lim (2017) documents that in each year, firms switch roughly 40% of suppliers, and Lu, Mariscal and Mejia (2013) document that ubiquitous switching of imported inputs among importers, suggesting gains from taking advantage of new opportunities that arise. Johnson, McMillan and Woodruff (2002) show that those who distrust courts are less likely to switch suppliers, suggesting that weak enforcement inhibits this.

across varieties within each industry.

There are several different ways to produce a good using different combinations of inputs.These different ways correspond to different production functions, which we call *recipes*. Denote the set of recipes to produce a good ω by ϱ_ω. Each recipe $\rho \in \varrho_\omega$ describes a production function $G_{\omega\rho}(\cdot)$ that can be used by any firm in industry ω to produce its good using labor l and a particular bundle of inputs $\hat{\Omega}^\rho$. We assume:

Assumption 1 *For any recipe $\rho \in \varrho_\omega$, the production function $G_{\omega\rho}$ exhibits constant returns to scale and all inputs $(l, \hat{\Omega}^\rho)$ are complements.*

Note that the different $G_{\omega\rho}$ may differ in their shape and their factor intensities, and that they need not be Leontief.

Firm j in industry ω has random sets of productivity and supplier draws $\Phi_{j\rho}$ for each recipe $\rho \in \varrho_\omega$. We call these draws *techniques*. Each technique $\phi \in \Phi_{j\rho}$ is characterized by (i) a set of potential suppliers for each intermediate input, $\{S_{\hat{\omega}}(\phi)\}_{\hat{\omega}\in\hat{\Omega}^\rho}$, (ii) for each of those suppliers $s \in S_{\hat{\omega}}(\phi)$ an input-augmenting productivity and a distortion (which we discuss further below), and (iii) a labor-augmenting productivity $b_l(\phi)$. The input-augmenting productivity for supplier $s \in S_{\hat{\omega}}(\phi)$ consists of a match-specific component z_s that is specific to the supplier and a component $b_{\hat{\omega}}(\phi)$ that is common to all suppliers of input $\hat{\omega}$ in the set $S_{\hat{\omega}}(\phi)$.

Suppose that j produced its good using a technique of recipe ρ which used labor and intermediate inputs $\hat{\Omega}^\rho = \{\hat{\omega}_1, ..., \hat{\omega}_n\}$. If it chose to employ l units of labor and purchase $\{x_{s_1}, ..., x_{s_n}\}$ units of intermediate inputs from respective suppliers $s_1 \in S_{\hat{\omega}_1}, ..., s_n \in S_{\hat{\omega}_n}$, its output would be

$$y_j = G_{\omega\rho}\left(b_l l, b_{\hat{\omega}_1} z_{s_1} x_{s_1}, ..., b_{\hat{\omega}_n} z_{s_n} x_{s_n}\right).$$

Each technique is specific to the firm producing the output (the "buyer") and to the potential suppliers that might provide the intermediate inputs. In equilibrium, the firm chooses to produces using the technique and suppliers that is most cost-effective, which depends on the prices those suppliers charge and the input-augmenting productivities of the technique.

Terms of trade among firms determine their choices of inputs, productions decisions, and productivity. We also assume that sales of goods for intermediate use are priced at the supplier's marginal cost.[24] Firms engage in monopolistic competition when selling to the representative household, and remit all profits to the household.

First, nature chooses the sets of techniques available to each firm. Then all firms simultaneously set prices and make their production decisions (i.e. choices of technique $\phi \in \bigcup_{\rho\in\varrho_\omega} \Phi_{j\rho}$, suppli-

[24] One interpretation of this assumption is that in firm-to-firm trade, buyers have all of the bargaining power. For example, in a simpler environment, Oberfield (2018) characterizes an alternative market structure in which firm-to-firm trade is governed by bilateral trading contracts specifying a buyer, a supplier, a quantity of the supplier's good to be sold to the buyer and a payment. Given a contracting arrangement, each entrepreneur makes her remaining production decisions to maximize profit. The economy is in equilibrium when the arrangement is such that no countable coalition of entrepreneurs would find it mutually beneficial to deviate by altering terms of trade among members of the coalition and/or dropping contracts with those not in the coalition. The terms of trade described here are one particular equilibrium in which buyers have all of the bargaining power.

ers, and inputs $l, x_{s_1}, \ldots, x_{s_n}$) to minimize cost, taking into account the decisions of others. All firms have perfect information about the economy's production possibilities and about other firms' choices. The probability distribution governing the set of techniques with which firm j can produce ($\{\Phi_{j\rho}\}_{\rho \in \varrho_\omega}$) will be described below.

3.2 Contracting Enforcement

Enforcement of contracts facilitates the use of inputs that require customization and the use of labor. Imperfect enforcement introduces wedges between the effective cost to the buyer and the payment to the supplier. If an input requires customization, the supplier can shirk and provide a good that is imperfectly customized to the buyer. If this happens, the buyer needs to use extra labor to correct the defect. This is wasteful because the supplier has an absolute advantage in performing the customization. For each supplier, the buyer draws a random cost of enforcing the contract, which, by modulating the threat of enforcement, affects the equilibrium performance of the supplier and hence the extra labor the buyer needs to use to correct the defect. We discuss the full microfoundation in Appendix E.

Formally, for each potential supplier of a relationship-specific input $\hat{\omega}$ there is a random input wedge $t_x \in [1, \infty)$, drawn from a distribution with CDF $T(t_x)$.[25] If the supplier's price is p_s per unit, the cost to buyer is $t_x p_s$, with p_s paid to the supplier and $(t_x - 1)p_s$ spent on extra labor. We model wedges as random and match-specific because, for some suppliers (e.g., family members, those with whom the buyer is in a long-term relationship) the possibility of informal enforcement may mitigate any hold-up problem; for others, the hold-up problem may be more severe.

The distribution $T(t_x)$ summarizes the quality of enforcement. Perfect enforcement of contracts would imply that $t_x = 1$ for all suppliers. As discussed earlier, courts are not the only way to enforce contracts; contracts could be enforced informally through social punishments or reputation. t_x should be interpreted as the wedges that prevail after all forms of enforcement are exhausted.[26]

For completeness, we include the possibility that imperfect enforcement raises the cost of labor as well. If production is subject to the labor wedge t_l, then the firm needs to hire t_l workers to obtain one efficiency unit of labor, so that the effective cost of labor to the firm is wt_l. For simplicity we assume that t_l is the same across all firms.[27]

[25] As we show in appendix Appendix E, there is a one-to-one mapping between the enforcement cost and the equilibrium input wedge t_x.

[26] For example, if formal enforcement would leave the wedge t_x^{formal} while informal enforcement would leave the wedge t_x^{informal}, then the parties would use whichever form of enforcement is better, i.e., $t_x = \min\{t_x^{\text{formal}}, t_x^{\text{informal}}\}$ (and similarly for t_l). The argument extends in the obvious way if there are multiple ways of enforcing contracts informally. Improving the quality of courts would reduce the wedges t_x^{formal}, and might alter the effectiveness of informal enforcement mechanisms if it worsens the informal arrangements that can be sustained.

[27] Our counterfactuals focus on changes in the distribution of distortions that impede the use of relationship-specific intermediate inputs (T). Our identification strategy does not recover the labor wedge t_l.

3.3 Production Decisions

For each technique, firm j draws a set of potential suppliers to provide each input. Each potential supplier $s \in S_{\hat{\omega}}(\phi)$ comes with an input-augmenting productivity draw and a wedge, so that the effective cost of using that supplier would be $\frac{t_{xs}p_s}{b_{\hat{\omega}}(\phi)z_s}$, which includes the cost of the extra labor needed to customize the input. If j used technique ϕ, it would choose to use the supplier that delivered the lowest effective cost for input $\hat{\omega}$, so that its effective cost of that input would be:

$$\lambda_{\hat{\omega}}(\phi) \equiv \min_{s \in S_{\hat{\omega}}(\phi)} \frac{t_{xs}(\phi)p_s}{b_{\hat{\omega}}(\phi)z_s(\phi)}.$$

Similarly, the effective cost of labor when using technique ϕ is $\lambda_l(\phi) = \frac{t_l w}{b_l(\phi)}$. For the remainder, we normalize the wage to unity, $w = 1$.

The unit cost delivered by a technique depends on the effective cost of each input. Let $\mathcal{C}_{\omega\rho}(\cdot)$ be the unit cost function that is the dual of the production function $G_{\omega\rho}$, so that j's cost of producing one unit of output using technique ϕ would be $\mathcal{C}_{\omega\rho}\left(\lambda_l(\phi), \{\lambda_{\hat{\omega}}(\phi)\}_{\hat{\omega}\in\hat{\Omega}^\rho}\right)$. Minimizing cost across all techniques, j's unit cost is

$$\min_{\rho \in \varrho(\omega)} \min_{\phi \in \Phi_{\omega j\rho}} \mathcal{C}_{\omega\rho}\left(\lambda_l(\phi), \{\lambda_{\hat{\omega}}(\phi)\}_{\hat{\omega}\in\hat{\Omega}^\rho}\right)$$

In words, firm j's unit cost equals that of the technique that delivers the lowest cost across all techniques of all recipes.

In this section, we specialize to particular functional form assumptions. As we show below, with these assumptions, the model aggregates easily and allows us to use a transparent strategy to identify contracting frictions. The set of techniques available to each firm is random and governed by the following assumptions about the distributions of input-augmenting productivities.

Assumption 2 *For a firm in industry ω,*

a. *Each supplier in the set $S_{\hat{\omega}}(\phi)$ is uniformly drawn from all firms that produce $\hat{\omega}$.*

b. *For each technique ϕ that uses input $\hat{\omega}$, the number of suppliers in $S_{\hat{\omega}}(\phi)$ for whom the match-specific component of productivity is greater than z follows a Poisson distribution with mean*

$$z^{-\zeta_{\hat{\omega}}}, \text{ with } \zeta_{\hat{\omega}} = \begin{cases} \zeta_R, & \hat{\omega} \in \hat{\Omega}^\rho_R \\ \zeta_H, & \hat{\omega} \in \hat{\Omega}^\rho_H \end{cases}$$

c. *Consider recipe $\rho \in \varrho_\omega$ which uses labor and the input bundle $\hat{\Omega}^\rho = (\hat{\omega}_1, ..., \hat{\omega}_n)$. For each plant, The number of techniques to produce using that recipe for which the common components of input-augmenting productivities strictly dominate*[28] *$b_l, b_{\hat{\omega}_1}, b_{\hat{\omega}_2}, ..., b_{\hat{\omega}_n}$ follows a Poisson distribution with mean*

$$B_{\omega\rho} b_l^{-\beta_l^\rho} b_{\hat{\omega}_1}^{-\beta_{\hat{\omega}_1}^\rho} ... b_{\hat{\omega}_n}^{-\beta_{\hat{\omega}_n}^\rho}.$$

[28] We say that a vector $(x_0, x_1, ..., x_n)$ strictly dominates the vector $(y_0, y_1, ..., y_n)$ if $x_0 > y_0$, $x_1 > y_1$,...,$x_n > y_n$.

d. There is a constant γ such that for each ω and each recipe $\rho \in \varrho_\omega$, $\beta_l^\rho + \beta_{\hat{\omega}_1}^\rho + ... + \beta_{\hat{\omega}_n}^\rho = \gamma$.

e. The following parameter restrictions hold for each $\hat{\omega}$: $\gamma > \varepsilon - 1$, $\gamma > \zeta_{\hat{\omega}} > \beta_{\hat{\omega}}^\rho$ where $\zeta_{\hat{\omega}}$ is ζ_R if $\hat{\omega}$ is relationship-specific or ζ_H if $\hat{\omega}$ is homogeneous.

Assumption 2b implies that above any threshold, the match-specific components of productivity follow a power law.[29,30] One implication is that the industry-level elasticity of substitution across groups of suppliers of the same input is $\zeta_{\hat{\omega}} + 1$. When there is more dispersion in these match-specific components of productivity (low $\zeta_{\hat{\omega}}$), a buyer is less likely to switch suppliers in response to changes in the supplier's price because it is likely that there is a larger gap between the best and second-best suppliers of an input. ζ_R will play a role quantitatively because it determines the likelihood that a buyer will have a close substitute if it faces a holdup problem with its best supplier of an input.

Assumption 2c says that the common components of input-augmenting productivities of a technique follow independent power laws. $B_{\omega\rho}$ summarizes the level of these productivity draws. We take these to be primitives, although a deeper model might model them as resulting endogenously from directed search or from the diffusion of technologies across entrepreneurs that know each other.

Assumption 2d says that for each recipe, the sum of the power law exponents is the same, equal to γ. We will show later that the industry-level elasticity of substitution across techniques is $\gamma + 1$.[31] The parameter restrictions are necessary to keep utility finite.

It will be useful to decompose the power law exponents into two parts. For recipe ρ, define

$$\alpha_L^\rho = \frac{\beta_l^\rho}{\gamma}, \qquad \alpha_{\hat{\omega}}^\rho \equiv \frac{\beta_{\hat{\omega}}^\rho}{\gamma}, \qquad \hat{\omega} \in \hat{\Omega}^\rho$$

Note that this implies that $\alpha_L^\rho + \sum_{\hat{\omega} \in \hat{\Omega}} \alpha_{\hat{\omega}}^\rho = 1$. Further, for some results, it will be useful to define $\alpha_R^\rho = \sum_{\hat{\omega} \in \hat{\Omega}_R^\rho} \alpha_{\hat{\omega}}^\rho$ and $\alpha_H^\rho = \sum_{\hat{\omega} \in \hat{\Omega}_H^\rho} \alpha_{\hat{\omega}}^\rho$.

[29] This type of functional form assumption goes back to at least Houthakker (1955), and versions of it are also used by Kortum (1997), Jones (2005), Oberfield (2018), and Buera and Oberfield (2020). Note that the expected number of potential suppliers for an input is unbounded. Formally, an economy satisfying Assumption 2b can be thought of as the limit of a sequence of economies that satisfy more standard assumptions. Consider an economy in which firms were restricted to use only suppliers with a match-specific productivity greater than $\underline{z}$. Then the number of potential suppliers for each input of a technique would be given by a Poisson distribution with mean $\underline{z}^{-\zeta}$ and the match-specific productivity for each supplier would be drawn from a Pareto distribution with CDF $1 - (z/\underline{z})^{-\zeta}$. An economy satisfying Assumption 2b can be thought of as the limit of such an economy as $\underline{z} \to 0$. In this limit, the number of suppliers for each input of a technique grows arbitrarily large, but the match-specific productivity associated with any single supplier is drawn from an arbitrarily poor distribution. The limit is well behaved because the probability of drawing a supplier with match-specific productivity greater than z does not change as $\underline{z} \to 0$.

[30] In principle, we could have allowed the level of the match-specific component of productivity draws to vary by input-output pair and recipe, or $Z_{\omega\rho\hat{\omega}} z^{-\zeta_\omega}$, reflecting the idea that industries are often concentrated geographically or ethnically, which may imply that a given output industry may face an unusually high number of good suppliers in the input industry relative to other output industries. However, it turns out that Assumptions 2b and 2c imply that any variation in $\{Z_{\omega\rho\hat{\omega}}\}$ would be absorbed into the constant $B_{\omega\rho}$, so we simply normalize each $Z_{\omega\rho\hat{\omega}}$ to unity.

[31] It would be straightforward to allow different industries to have different values of γ. However, as we show below, our counterfactuals are insensitive to the value of γ. We therefore leave the γ constant across industries to reduce notational clutter.

With these assumptions in hand, we now characterize the equilibrium. All proofs are contained in Appendix F.

Proposition 1 *Under Assumptions 1 and 2, the fraction of firms with unit cost greater than c among those in industry ω is*

$$e^{-(c/C_\omega)^\gamma}$$

where

$$\begin{aligned} C_\omega &= \left\{ \sum_{\rho \in \varrho_\omega} \kappa_{\omega\rho} B_{\omega\rho} \left((t_x^*)^{\alpha_R^\rho} t_l^{\alpha_L^\rho} \prod_{\hat{\omega} \in \hat{\Omega}^\rho} C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho} \right)^{-\gamma} \right\}^{-\frac{1}{\gamma}} \\ t_x^* &= \left(\int_1^\infty t_x^{-\zeta_R} dT(t_x) \right)^{-1/\zeta_R} \end{aligned} \tag{1}$$

and $\kappa_{\omega\rho}$ is a constant that depends on technological parameters.

Proposition 1 shows that the distribution of cost among firms within each industry takes the simple form of a Weibull distribution with shape parameter γ and scale determined by C_ω, which we call the cost index for industry ω. (1) relates industry ω's cost index to that of the industries that provide the inputs for each recipe and to t_x^* and t_l, which summarize the impact of imperfect enforcement on those that produce the inputs used in recipe ρ. t_x^* accounts for both the direct impact of the wedges—the wasted resources from holdup problems—and the indirect impact: wedges might cause firms to switch to a supplier with higher cost or lower productivity, or to a different technique altogether.

(1) is a system of equations that implicitly determines each industry's cost index, $\{C_\omega\}_{\omega\in\Omega}$. Proposition 2 shows that these are sufficient to characterize aggregate productivity.

Proposition 2 *Under Assumptions 1 and 2, the household's aggregate consumption is*

$$U = \left\{ \sum_{\omega \in \Omega} \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \Gamma \left(1 - \frac{\varepsilon - 1}{\gamma} \right)^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta} \right\}^{\frac{1}{\eta-1}} L$$

We next turn to industry-level expenditure shares. The next proposition characterizes the aggregate share of total expenditures (on both intermediate inputs and labor) that is spent on each input among all firms that use a particular recipe.

Proposition 3 *Suppose that assumptions 1 and 2 hold. Among firms that, in equilibrium, produce using recipe ρ:*

- *the average and aggregate shares of expenditures spent on inputs from $\hat{\omega} \in \hat{\Omega}_R^\rho$ are both $\frac{\alpha_{\hat{\omega}}^\rho}{t_x}$,*
- *the average and aggregate shares of expenditures spent on inputs from $\hat{\omega} \in \hat{\Omega}_H^\rho$ are $\alpha_{\hat{\omega}}^\rho$,*

- *the average and aggregate shares of expenditures spent on labor are* $\alpha_L^\rho + (1 - \frac{1}{\bar{t}_x})\alpha_R^\rho$,

where $\bar{t}_x \equiv \left[\int_1^\infty t_x^{-1} d\tilde{T}(t_x)\right]^{-1}$ *and* $\tilde{T}(t_x) \equiv \frac{\int_1^{t_x} t^{-\zeta_R} dT(t)}{\int_1^\infty t^{-\zeta_R} dT(t)}$.

Proposition 3 provides relatively simple expressions for the average and aggregate cost shares of each input among those that choose to use a particular recipe. These properties will be central to our identification procedure. While there is micro-level heterogeneity in the cost shares among those using a particular recipe, the aggregate factor shares among those firms depends only on technological parameters, not on the relative prices of the inputs. Thus at the recipe level, there is a Cobb-Douglas aggregate production function. This extends the celebrated aggregation result of Houthakker (1955) who derived a similar result under the assumption that individual production functions are Leontief.[32] We require only that the production function exhibits constant returns to scale and that all inputs are complements.[33]

Imperfect enforcement, on the other hand, reduces the expenditure share of relationship-specific inputs. The buyer's production decisions depend each input's effective cost, whereas the expenditures reflect the actual payment to each supplier. Recall that imperfect enforcement means that the buyer's effective expenditure on a relationship-specific input is spent partly on payments to the supplier for the input and partly on labor to customize the good.[34]

3.4 Counterfactuals

The quality of contract enforcement can be summarized by the distribution of wedges T. Suppose that the quality of enforcement changed in such a way that the distribution of wedges changed from T to T'. How would this impact aggregate productivity? Taking J_ω and $B_{\omega\rho}$ as primitives, the following proposition shows how one can compute the impact of such a change.[35]

[32] Jones (2005) builds on Houthakker (1955) but derives a different type of result. Jones first shows that if a single plant draws many Leontief production functions where factor augmenting productivities are drawn from independent Pareto distributions, then the envelope of those production functions is Cobb-Douglas. He then shows numerically that the result extends beyond Leontief to CES production functions when the factors are complements. Note that these are not aggregation results; these results apply at the level of a single firm. Lagos (2006) and Mangin (2017) also build on Houthakker (1955) incorporating labor market search, while Growiec (2013) extends the argument of Jones to microfound an aggregate CES production function.

[33] Why complements? When inputs are complements, when the price of an input is higher, there are two offsetting effects on the industry cost share. The higher price raises that cost share on that input for any firm that uses the input. At the same time, firms that use that input more intensively are likely to shrink or switch to a technique that uses the input less intensively. When factor-augmenting productivities are drawn from independent Pareto distributions, these offset exactly and factor shares are unchanged. If inputs were substitutes, the two effects would push in the same direction, so that if the price of an input rose, its industry cost share would fall. Mathematically, if inputs were substitutes then the constant $\kappa_{\omega\rho}$ would diverge, as the arrival rate of techniques that deliver cost lower than c would be infinite for any c.

[34] The wedge due to imperfect enforcement and input-augmenting productivity affect a firm's unit cost in the same way. It is important, however, to model them separately because they affect expenditure shares in different ways. Larger wedges tend to reduce the share of expenditures on that input because some of the effective cost is paid to labor; lower input-augmenting productivities do not.

[35] An interesting alternative exercise is asking what would happen if $\{J_\omega\}$ and $\{B_{\omega\rho}\}$ also responded to the change in T.

Let HH_ω be the share of the household's expenditure on goods from industry ω in the current equilibrium. Among those of type ω, let $R_{\omega\rho}$ be the share of total revenue of those that use recipe ρ in the current equilibrium.

Proposition 4 *If the quality of enforcement changed so that the distribution of wedges changes from T to T', the change in household utility would be*

$$\frac{U'}{U} = \left(\sum_{\omega} HH_\omega \left(\frac{C'_\omega}{C_\omega} \right)^{1-\eta} \right)^{\frac{1}{\eta-1}}$$

and the change in industry efficiencies would satisfy the following system of equations

$$\left(\frac{C'_\omega}{C_\omega} \right)^{-\gamma} = \sum_{\rho \in \varrho_\omega} R_{\omega\rho} \left[\left(\frac{t_x^{*\prime}}{t_x^*} \right)^{\alpha_R^\rho} \prod_{\hat{\omega} \in \hat{\Omega}^\rho} \left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}} \right)^{\alpha_{\hat{\omega}}^\rho} \right]^{-\gamma} \tag{2}$$

The first part of the proposition states that to know how a change in court quality affects aggregate productivity, it is sufficient to know only the changes in industry cost indices, $\frac{C'_\omega}{C_\omega}$. In turn, the change in each industry's cost index depends on the weighted average over input bundles of the change in the cost index of the industries that supply inputs along with the change t_x^*, the summary statistic for the industry of direct and indirect impact of the wedges that distort production using relationship-specific inputs. (2) describes a system of equations that implicitly characterizes these changes in cost indices.

While Proposition 4 describes exactly how a change in enforcement would alter welfare, it is instructive to study a perturbation of the distribution of wedges to show which features of the economy are important for determining the first-order impact of a change in the quality of enforcement.

Corollary 1 *The marginal welfare impact of a change in court quality is*

$$d \log U = -\sum_{\omega \in \Omega} HH_\omega d \log C_\omega$$

and the change in industry efficiencies can be summarized by the following system of equations:

$$d \log C_\omega = \sum_{\rho \in \varrho_\omega} R_{\omega\rho} \left[\alpha_R^\rho d \log t_x^* + \sum_{\hat{\omega} \in \hat{\Omega}^\rho} \alpha_{\hat{\omega}}^\rho d \log C_{\hat{\omega}} \right]$$

One implication is that, to a first order, the change in utility resulting from a change in the quality of enforcement does not depend on γ or η.[36]

[36] This can be viewed as an application of the envelope theorem. For small changes T, one can compute the impact on aggregate productivity holding fixed other choices, i.e., holding fixed the technique each firm uses. Thus γ, which regulates substitution across techniques, does not matter to a first order.

4 Identification and Estimation

Our main counterfactual of interest is how aggregate productivity and the organization of production would change if the quality of enforcement improved. We do this in several steps. We first parameterize the model using information from the ASI under the assumption that the quality of enforcement varies by state. We then project the implied quality of enforcement for each state on our measures of court congestion. Finally, we compute the gains from reducing congestion to the level prevailing in the least congested state.

Our most important identifying assumption is that weak enforcement may introduce a wedge in the use of inputs that require customization and in the use of labor, but not in the use of standardized inputs. Given our scheme for identification, we view this as a conservative assumption. If the use of standardized inputs were also distorted by weak contract enforcement, then all of the wedges would be larger than the ones we infer.

The following proposition shows a set of moments that we can use in a GMM procedure to estimate the model parameters

Proposition 5 *Let s_{Rj}, s_{Hj}, s_{Lj} be firm j's spending on relationship-specific inputs, homogeneous inputs, and labor respectively as shares of its revenue. Under assumptions 1 and 2, the first moments of revenue shares among firms that produce ω that, in equilibrium, use recipe ρ satisfy:*

$$\begin{aligned} \mathbb{E}\left[\bar{t}_x \frac{s_{Rj}}{\alpha_R^\rho} - \frac{s_{Hj}}{\alpha_H^\rho}\right] &= 0 \\ \mathbb{E}\left[\frac{s_{Lj} + s_{Rj}}{\alpha_L^\rho + \alpha_R^\rho} - \frac{s_{Hj}}{\alpha_H^\rho}\right] &= 0 \end{aligned}$$

Assumption 3 *We impose that the following objects are the same across states: (i) the form of the production function for each recipe $\{G_{\omega\rho}\}$; (ii) the power law exponents for the input-augmenting productivity draws for techniques of each recipe $\{\beta_l^\rho, \beta_{\hat{\omega}_1}^\rho, ..., \beta_{\hat{\omega}_n}^\rho\}$, and (iii) the power law exponents for the match-specific productivity draws, ζ_R and ζ_H.*

We allow all other features of preferences and technology to vary freely across states. This includes absolute and comparative advantages in recipes, $\{B_{\omega\rho}\}$, (ii) the measure of firms of each type $\{J_\omega\}$, (iii) the households tastes, $\{v_\omega\}$, and most importantly, (iv) the quality of contract enforcement, T, and t_l.

We also impose a parametric form for the stochastic wedges that a firm draws for each supplier of a relationship-specific input. In particular, the wedge is drawn from a Pareto distribution, where the shape parameter is specific to a state.[37]

[37]The Pareto distributions has the attractive property that is is closed under minimization. Following the discussion in Section 3.2, contracts might be enforced formally or informally. If the probability that the formal wedge is greater than t_x is $t_x^{-\tau_d^{\text{formal}}}$ and the probability that the informal wedge is greater than t_x is $t_x^{-\tau_d^{\text{informal}}}$, then the probability that the effective wedge is greater than t_x is $t_x^{-\tau_d}$, where $\tau_d = \tau_d^{\text{formal}} + \tau_d^{\text{informal}}$.

Assumption 4 *The distribution of wedges in state d is $T_d(t_x) = 1 - t_x^{-\tau_d}$ for $t_x \geq 1$.*

Our algorithm for identification is thus as follows:

1. Identify recipes, estimate technology parameters $\left\{\alpha_L^\rho, \alpha_H^\rho, \alpha_R^\rho\right\}_{\rho \in \varrho_\omega, \omega \in \Omega}$, and distortions to the cost of relationship-specific inputs for each state, $\bar{t}_x^d$. We use an iterative procedure to ensure that our recipe classification is consistent with the possibility of distortions that vary across states.

   (a) Start with an initial guess of $\bar{t}_x^d$ for each state d.

   (b) Identify recipes from plant's cost shares (see next section for details), taking out the distortion to the cost shares of relationship-specific inputs $\bar{t}_x^d$.

   (c) Use Proposition 5 to estimate the production parameters that are common across locations $\left\{\alpha_L^\rho, \alpha_H^\rho, \alpha_R^\rho\right\}_{\rho \in \varrho_\omega, \omega \in \Omega}$ and a new set of the state specific variables, $\{\bar{t}_x^d\}$.

   (d) Go back to step 1b until the $\bar{t}_x^d$ have converged.

2. Compute t_x^* for each state. Assumption 4 implies that $\bar{t}_x = 1 + \frac{1}{\zeta_R + \tau_d}$ and $t_x^* = \left(\frac{\tau_d + \zeta_R}{\tau_d}\right)^{1/\zeta_R}$. We estimate ζ_R externally, and then use this along with our estimates of $\bar{t}_x$ to compute t_x^*.

3. For the counterfactual, we also need values of the industry-level output elasticities of each input for each recipe, $\{\alpha_{\hat{\omega}}^\rho\}$. To do this, we pool data across states to estimate the remaining production function parameters, $\alpha_{\hat{\omega}}$, by using the aggregate expenditures. For example, if the sourced good $\hat{\omega}$ is relationship-specific, then $\alpha_{\hat{\omega}}^\rho$ is equal α_R^ρ multiplied by the ratio of total expenditure on input $\hat{\omega}$ by those that use recipe ρ to total expenditure on relationship-specific inputs.

4. For each state-recipe, directly measure the share of industry ω revenue earned by firms that, in equilibrium, use recipe ρ, $\{R_{\omega\rho}\}$. Similarly, directly measure for each state the share of final demand spent on industry ω, $\{HH_\omega\}$.

5. Calibrate η and γ externally.

In implementing this algorithm, we make several auxiliary assumptions that, in principle, could be relaxed. First, we assume that there is no trade across state borders. While it would be fairly straightforward to incorporate interstate trade, we lack the relevant data.[38] A second assumption is that the recipes used by multi-product firms and the distribution of wedges facing them are the same as those of single-product firms. This type of assumption, while strong, is standard in the literature, as we lack the data that indicates which inputs are used in the production of which

[38] To this point there is no comprehensive, publicly available data about cross-state trade in goods. The conventional wisdom has been that interstate trade is minimal, although the 2016-17 Economic Survey of India's Ministry of Finance challenges this conventional wisdom. In Appendix G.4 we explore how incorporating interstate trade might alter our counterfactuals. We make the assumption that 10% of inputs are sourced from out of state. We find that this has a minor quantitative impact on our counterfactual results.

products. It allows us to estimate wedge distribution parameters and the α's using single-product plants only, while still being able to make statements about the whole formal manufacturing sector by including multi-product plants when we calculate revenue and expenditure shares $R_{\omega\rho}$ and HH_{ω}. Third, we treat service inputs and energy inputs as primary inputs.

4.1 Defining recipes

One of the salient facts of the Indian manufacturing data is that even within narrow industries, plants use vastly different combinations of intermediate inputs to produce the same output. Our model provides a natural way to think of these input-output combinations as different recipes $\rho \in \rho(\omega)$ that could be used to produce the same output ω. In order to estimate the model from the microdata, we need a procedure that classifies each plant-year observation into which recipe the plant is using.

The idea that guides our classification is that, for a given output good, similar input mixes should belong to the same recipe. We describe each plant j's input mix by the vector of its input expenditure shares, $(m_{j\hat{\omega}})_{\hat{\omega}\in\Omega} = (X_{j\hat{\omega}}/\sum_{\hat{\omega}'} X_{j\hat{\omega}'})_{\hat{\omega}\in\Omega}$. Graphically, each vector corresponds to a point in the $|\Omega|$-dimensional hypercube that is lying on the hyperplane where the sum of all coordinates equals to one. Our task is to find plants with similar input mixes, i.e. clouds of points that are close to each other (according to some metric). In statistics, this task is known as *cluster analysis*, and there is a large number of algorithms that classify clusters based on distance, density, shape, and other criteria. Looking at the input mixes of plants in many different industries (see the examples of bleached cotton cloth and polished diamonds in Figure 1) convinced us that these clusters do exist and have a meaningful economic interpretation.

Our preferred method is due to Ward (1963), and constructs clusters recursively, starting with the partition where every cluster is a singleton. In each step, two clusters are merged to minimize the sum of squared errors:

$$\min_{\rho_n \geq \rho_{n-1}} \sum_{\rho\in\rho_n} \sum_{j\in\rho} \sum_{\hat{\omega}} \left(m_{j\hat{\omega}} - \overline{m}_{\rho\hat{\omega}}\right)^2$$

where the ρ_n are partitions of J_ω, and in each step ρ_n runs over all partitions that are coarser than ρ_{n-1}. This method constructs a hierarchical set of partitions of J_ω: one for each desired number of clusters.

Our implementation of the clustering procedure to identify recipes uses not only the five-digit materials shares, but also three-digit shares to allow for the possibility of misclassification of inputs within three-digit baskets. We determine the number of potential recipes using the prediction strength method of Tibshirani and Walther (2005). Similar to cross-validation, the prediction strength method divides the sample into two subsamples (A and B) and assesses the predictive power of clusters obtained from each subsample. We choose a threshold parameter of 0.95, meaning that on average at least 95% of pairs of points in B are correctly predicted when assigning observations to clusters based on the nearest cluster centroids from A (the observations are assigned the same cluster or different clusters as when clustering the observations in B). While the

Table IV Statistics on products and recipes

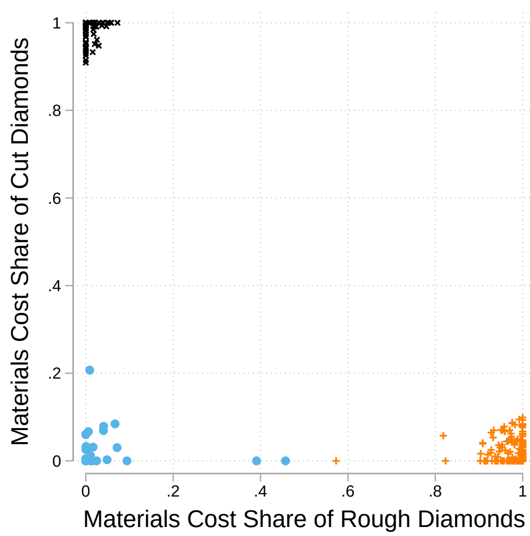
	Count
Products (5-digit ASIC)	4,530
Products with $\geq$ 3 plants	3,573
Products with $\geq$ 5 plants	3,034
Recipes	26,776
Recipes with $\geq$ 3 plants	6,280
Recipes with $\geq$ 5 plants	2,574
Avg. plants per recipe	8.2
SD plants per recipe	79.4

"Products" are the 5-digit product codes in our data, "Recipes" are the output from our clustering procedure. Plant counts include only single-product plants.

Table V Summary statistics on recipes

	Mean	Std. Dev.	Min	Max
Cost share of L	.40	.22	.0002	.999
Cost share of X_R	.27	.28	0	.999
Cost share of X_H	.33	.30	0	.998
Number of inputs with cost share > 1%	4.4	4.6	1	37
Number of inputs with cost share > 0.1%	6.4	12.6	1	205

Figure 3 Recipe Classification Example: Polished Diamonds (92104)



The figure shows the result from the recipe classification procedure (after correcting for wedged) for "polished diamonds" (92104). Observations that are classified as belonging to different recipes are tagged with different markers.

number of clusters is monotonically decreasing in this threshold parameter, the relative number of clusters for each industry is driven by the shape of the point clouds in the data. [39] Table IV shows statistics on the number and size of clusters (recipes) that result from using this procedure.

[39]In Appendix G.2 we show results for varying degrees of recipe fineness.

Broadly speaking, within the identified recipes, most of the expenditures of the plants are on the same set of inputs.[40]

Once we have defined recipes, we assign plants to belong to a recipe with a probability that is proportional to the inverse Euclidean distance to the recipe center:

$$P\left(j \in \rho\right) = \frac{\frac{1}{\sqrt{\sum_{\omega'}\left(m_{j\omega'} - \overline{m}_{\rho\omega'}\right)^2}}}{\sum_{\rho' \in \rho(\omega)} \frac{1}{\sqrt{\sum_{\omega'}\left(m_{j\omega'} - \overline{m}_{\rho'\omega'}\right)^2}}} \tag{3}$$

We use these assigned probabilities as weights in the estimation below.

4.2 Estimation

We estimate the $\bar{t}^d_x$, α^ρ_R, α^ρ_H, and α^ρ_L from the moment conditions in Proposition 5 using our algorithm described above. To identify the level of $\bar{t}_x$, we set the smallest $\bar{t}_x$ to one, thereby making the assumption that the least distorted state is undistorted.[41] We also exclude state-recipe pairs where the average share of relationship-specific inputs in sales exceeds that of homogeneous inputs by a factor of one hundred (and vice versa).

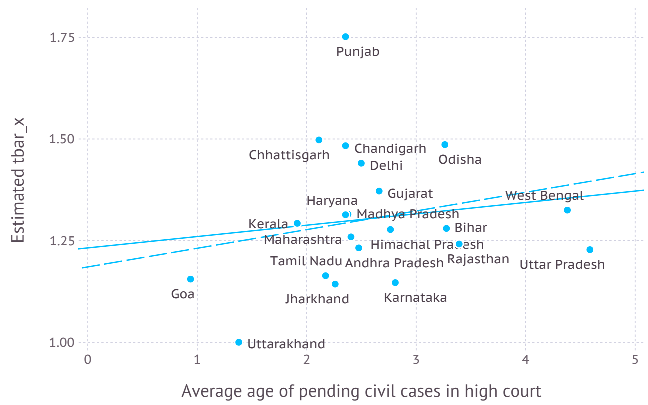
Figure 4 shows the estimated $\bar{t}^d_x$ and their correlation with high court congestion as measured by the average age of pending civil cases. Frictions are large: $\bar{t}_x$ exceeds 1.75 in the most heavily distorted state, and is close to 1.5 in several others. Some of that variation is explained by the congestion of courts. In states with slower courts, firms face larger distortions (higher $\bar{t}_x$) when sourcing relationship-specific intermediate inputs. The solid line is the fit of an OLS regression of $\bar{t}_x$ on court congestion; the dashed line the fit of an IV regressions where we instrument court congestion using the log age of the high court. The estimated IV slope coefficient is similar to the one in the OLS. This relationship between $\bar{t}_x$ and the age of pending court cases is closely related to the fact that intermediate input bundles are tilted towards homogeneous inputs in states where enforcement is weak (Fact 2 in Section 2.3). The main difference here (beyond the fact that the $\bar{t}_x$ are coming from a nonlinear regression) is that the $\bar{t}_x$ are identified from within-recipe variation in the input mix, whereas Fact 2 is about within-product variation.

Before proceeding, it is worth discussing how we separate heterogeneity in production technology (i.e. recipes) from distortions. Given a set of recipes and a classification of plants into which recipes they use, identification of the α's and $\bar{t}_x$'s fundamentally comes from the assumption that the

[40] Our model imposes that all firms using the same recipe use the exact same bundle of inputs, but our classification algorithm does not. Formally, this can be understood in the context or our model as imposing that all plants that we classify as using the same empirical recipe actually use recipes with the same α_R, α_H, and α_L. We believe that this is a relatively innocuous assumption, because inputs that are not being used by a plant but are used by some other plants using the same empirical recipe account for very low cost shares: on average (across recipes, firms, and inputs) for around 0.32% of the recipe's materials expenditure. It is likely that some of the differences in input bundles, especially those arising from inputs with very small cost shares, are due to respondents occasionally omitting unimportant inputs.

[41] We view this as conservative. Given the expenditure shares we see in the data, more severe distortions (smaller $\bar{t}_x$) would raise the estimated output elasticities of relationship-specific inputs (higher α^ρ_R). This would amplify the responses to changes in enforcement.

Figure 4 Correlation between $\bar{t}_x$ and court congestion



The figure shows the correlation between $\bar{t}^d_x$ and the average age of pending civil cases in the state's high court. The solid line is the OLS regression line; the dashed line is fit of an IV regression where the age of pending cases is instrumented using the log age of the high court.

parameters that govern contracting frictions (the $\bar{t}_x$'s) vary by state, whereas the recipe technology parameters (the α's) are the same across states but vary across recipes.[42] Similar to the problem of estimation with grouped fixed effect studied by Bonhomme and Manresa (2015), we face the problem of estimating these parameters jointly with the number of recipes and the assignment of plant observations into recipes, i.e., jointly with the clustering procedure with a choice of the number of clusters, and like Bonhomme and Manresa (2015), we do this by iterating between a clustering procedure and parameter estimation, so that, in our context, the recipes account for the possibility of distortions.

The theoretical results in Bonhomme and Manresa (2015) and Moon and Weidner (2015), who study similar estimation procedures in closely related models, suggest that our estimation procedure has the following properties: First, if we choose the right number of recipes, the estimated distortions will be consistent as the number of plants grows large. Second, if we choose too few recipes, the estimated distortions may be biased, as the difference in recipe usage across states might be misinterpreted as distortions. Third, if we choose too many recipes, the estimated distortions will be consistent, as the additional recipes act like irrelevant extra regressors in a least squares regressions. While our model is not nested by either of those, we confirm these large sample properties, as well as study the small-sample behavior of our estimator, using Monte Carlo simulations in Appendix G.3.

[42]This is analogous to a non-linear panel regression. In a panel regression, one might estimate individual fixed effects and time fixed effects. Here we are estimating state fixed effects (the distortions) and recipe fixed effects (technology parameters). The fact that the moment conditions are non-linear does not change the basic logic.

4.3 Estimating and calibrating the remaining parameters

To perform a counterfactual where we assess the aggregate impact of a change in the wedge distribution T, Proposition 4 tells us that we need to know the change in the moment t_x^* of the wedge distribution that is relevant for the industry's cost distribution, which depends on the parameter ζ_R and, under our parameterization of the wedge distribution, on its Pareto tail τ_d. We also need to know the parameters $\alpha_{\hat{\omega}}^{\rho}$, the within-industry sales shares $R_{\omega\rho}$ of each recipe, the household's expenditure shares HH_ω, and the elasticities γ and η.

4.3.1 Estimating ζ

The parameter ζ_R, which allows us to back out τ_d from $\bar{t}_x$, also governs the elasticity of substitution across sets of suppliers of the same input. While our data does not indicate the identity of the supplier of an input, it does indicate whether it was purchased from a foreign or domestic supplier. We can thus estimate ζ_R using these two groups for each input-output pair:

$$\log\left(\frac{X_{\omega\hat{\omega}t}^{DOM}}{X_{\omega\hat{\omega}t}^{IMP}}\right) = \zeta \log(1+\iota_{\hat{\omega}t}) + \lambda_t + \lambda_{\omega\hat{\omega}} + \eta_{\omega\hat{\omega}t}$$

where $\iota_{\hat{\omega}t}$ is the import tariff on $\hat{\omega}$ at time t, and the λ's are fixed effects. Table VI shows the results for this regression. We use the point estimate of 0.218 for ζ_R.[43]

Table VI Estimating ζ

	Dependent variable: $\log(X_{\omega\hat{\omega}t}^{DOM}/X_{i\omega\hat{\omega}t}^{IMP})$		
	(1)	(2)	(3)
$\log(1+\iota_{\hat{\omega}t})$	0.617	0.218	1.209*
	(0.44)	(0.77)	(0.52)
Industry $\times$ Input FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Level	5-digit	5-digit	5-digit
Sample	All inputs	R only	H only
R^2	0.601	0.580	0.623
Observations	23692	12002	11690

Robust errors in parentheses, clustered at the state $\times$ industry level. Sample
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Notes: Dependent variable is the log ratio of total expenditure on domestically sourced to total imported intermediate inputs of type $\hat{\omega}$ among producers of ω at time t. We only use census plants (which are surveyed every year) to reduce artificial fluctuations that result from sampling.

[43] Choosing a low ζ_R is conservative; see the discussion in 4.5 below. We conduct sensitivity checks in Appendix G.1.

4.3.2 Calculating the $\alpha_{\hat{\omega}}$ from α_R and α_H

The elasticities $\alpha_{\hat{\omega}}$ can be recovered as the product of the input-type elasticity (α_R^ρ or α_H^ρ) and the average cost share of plants that produce using recipe ρ:

$$\alpha_{\hat{\omega}}^\rho = \alpha_R^\rho \frac{\overline{m}_{\rho\hat{\omega}}}{\sum_{\omega' \in \Omega^R} \overline{m}_{\rho\hat{\omega}'}} \text{ if } \hat{\omega} \text{ relationship-specific,} \qquad \alpha_{\hat{\omega}}^\rho = \alpha_H^\rho \frac{\overline{m}_{\rho\hat{\omega}}}{\sum_{\omega' \in \Omega^H} \overline{m}_{\rho\hat{\omega}'}} \text{ if } \hat{\omega} \text{ homogenous.}$$

4.3.3 Accounting for multi-product plants

We target the demand aggregator's expenditure shares HH_ω and the recipe revenue shares $R_{\omega\rho}$ to represent the aggregate of India's formal manufacturing sector, which includes both single- and multi-product plants. We calculate HH_ω separately for each state as total sales of ω (from both single-product and multi-product plants) minus total intermediate consumption (less imports) of ω (or zero, if this difference is negative), divided by the sum of this difference over all products.

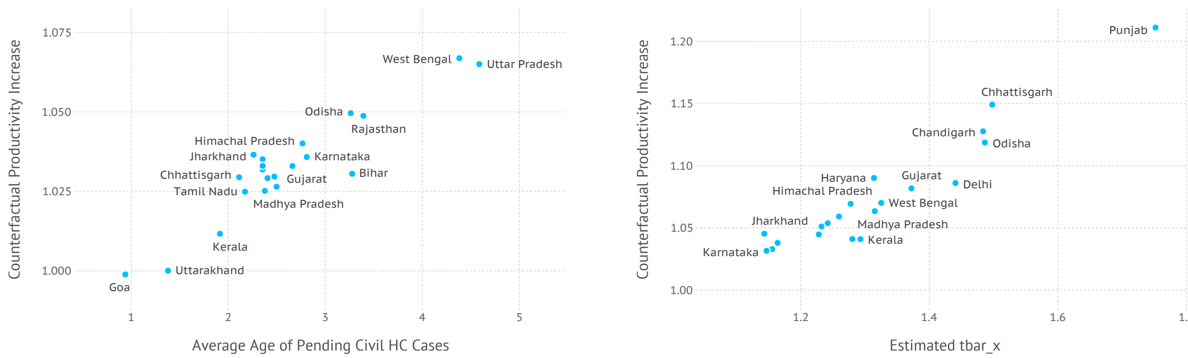
To calculate the recipe revenue shares $R_{\omega\rho}$, we need to assign the revenue of multi-product plants to a particular recipe. We assume that multi-product plants produce each of their products ω using one or more recipes $\rho \in \rho_\omega$, with each of them accounting for a fraction $r_{j\omega\rho}$ of sales of ω. We then estimate the $r_{j\omega\rho}$ by minimizing the Euclidean distance between the firm's observed vector of materials cost shares $m_{j\omega}$ and expected cost shares (which depend on the $r_{j\omega\rho}$ as well as on $\bar{t}_x^d$). We then construct the recipe sales shares $R_{\omega\rho}$ using both the output of single-product-plants as well as the estimated output of multi-product plants. This strategy is model-consistent, but requires that all recipes are being used to some extent by single-product plants (otherwise we would not be able to detect them in the estimation/clustering procedure). Moreover, when calculating $R_{\omega\rho}$, we pool observations across states and years, but weigh each plant-year observation by the inverse of the number of times the plant shows up in the ASI. This weighting allows us to construct parameters that better represent the aggregate of India's formal manufacturing sector.

Finally, we calibrate $\eta = 1$ and $\gamma = 1$; we show in Appendix G.1 that our counterfactuals are insensitive to these parameters. Inputs which do not show up in our data as outputs (predominantly agricultural and mineral commodities) are assumed to have unchanged productivity distributions in the counterfactual simulation.

4.4 Counterfactuals

We then perform two counterfactuals. In the first one, we reduce $\bar{t}_x^d$ for each state by the amount that is implied by the IV regression in Figure 4, down to a point where the average age of pending cases is one year (which is roughly the level of congestion enjoyed by the best state, Goa). Using our estimate for ζ_R, we back out τ_d at the original and counterfactual level, and compute the change in the welfare-relevant moment $t_x^* = \left(\frac{\tau_d + \zeta_R}{\tau_d}\right)^{1/\zeta_R}$. We then compute the corresponding change in the household's utility aggregate as given by Proposition 4.

The left panel of Figure 5 shows the corresponding counterfactual increase in the household's consumption aggregate U. The numbers are in the range of zero to seven percent, suggesting



(a) Improving court speed to level of best court

(b) Halving wedges

Figure 5 Counterfactual increases in aggregate productivity

The figure shows the counterfactual increase in C when the wedges on relationship-specific inputs are reduced. In the left panel we reduce $\bar{t}_x$ according to the fraction of $\bar{t}_x$ that is explained by court congestion in a linear IV regression (Figure 4); in the right panel we cut the $\bar{t}_x$ in half.

that the gains from improving court speed can be large. That said, there are several sources of uncertainty around these estimates. The correlation between court congestion and identified wedges $\bar{t}_x$ is a correlation at the state-level and is therefore not very precisely estimated. Furthermore, the wedges $\bar{t}_x$ are an estimate themselves, and this moment will be less precisely estimated in states where we have fewer plant observations.

In the second counterfactual we reduce the magnitude of distortions in such a way that the wedges $\bar{t}_x$ are half as large as in the estimates. This counterfactual is independent from the correlation of wedges with the measured congestion in the high courts, and would thus reflect an improvement in the overall quality of contract enforcement. The right panel of Figure 5 shows the counterfactual increase in U for each state. On average U would increase by 7.1 percent and for the most distorted states by more than ten percent. These welfare gains are more precisely estimated than those for the first counterfactual; however, they are still subject to random variation in the estimated wedge moments $\bar{t}_x$ that comes from the fact that we do not observe the universe of smaller plants, just a random sample of them.

4.5 Direct and Indirect Costs of Distortions

The model captures the idea that buyer-supplier relationships differ in their susceptibility to imperfect contract enforcement. Some buyer-supplier pairs are able to enforce contracts informally, or transact in a way that avoids holdup problems. To capture this in a parsimonious way, we assume that the buyer draws a distortion for each potential supplier from a distribution that deteriorates when formal enforcement is less reliable.

Our estimation procedure identifies $\bar{t}_x$ for each state, which is the (harmonic) average of the *realized* distortions. This summarizes the direct cost of weak enforcement on users of relationship-specific inputs. t_x^* captures the direct and indirect cost of the distortions. We infer the indirect cost by estimating ζ_R, which indexes the probability that a firm has a comparable alternative supplier. Formally, Jensen's inequality implies that $\bar{t}_x \leq t_x^*$, which implies that the $\bar{t}_x$ is a lower bound for the total impact, and that this lower bound is attained only if the distribution of distortions T is degenerate, i.e., distortion draws are deterministic. In that case, a firm faces the same wedge for all suppliers, and could not avoid a distortion by substituting to an alternative supplier; as a result, there would be no indirect cost.

The respective contributions of the direct and indirect impacts depend on both the value of ζ_R and the levels of the distortion. Given our estimates and the range of $\bar{t}_x$ that we observe, the direct impact comprises more much of the impact of weak enforcement, but the indirect impacts are nontrivial. For the average state, the indirect impact accounts for $\frac{t_x^* - \bar{t}_x}{t_x^*} = 6\%$ of the total impact, and for the most distorted state the indirect impact accounts for about 23%.

4.6 The Role of Heterogeneity

An alternative approach that is common in studies that measure misallocation using input-output tables is to posit industry-level Cobb-Douglas production functions and back out distortions from differences across states (or countries) in industry-level cost shares.[44] In our view, our approach has several advantages over this alternative.

First, a cursory look at the data indicates that plants are producing using different technologies (i.e., some appear to be more vertically integrated than others). We believe that allowing for several recipes facilitates making apples-to-apples comparisons when measuring the direct impact of distortions. Our identification strategy relies on comparing inputs expenditures among plants that, in equilibrium, use the same recipe. With the industry-level approach, differences in recipe composition across states could lead researchers to infer spurious distortions.

Second, the industry-level approach would miss the indirect impact of the distortions. Given our identification strategy and conditioning on the data, the fact that wedges differ across suppliers implies indirect productivity losses due to plant's switching to alternative suppliers to avoid a distorted input. This raises the implied productivity loss from imperfect enforcement. Our identification strategy consists of measuring $\{\bar{t}_x\}$ and then using these along with our functional form assumptions to compute t_x^*. In the industry-level approach there is typically only a direct impact.

Third, as discussed earlier, there is quite a bit of heterogeneity in cost shares across plants. This leads to the following situation: there are occasionally state-industry pairs that are dominated by a small number of plants, and the aggregate cost shares among these plants happen to deviate quite a bit from the industry average. If we used the industry-level cost shares to back out a wedge for that state-industry, we would conclude that the state-industry is severely distorted. In contrast, our estimating equations treat each individual plant as an observation, and the model has a structural error term for the cost share of each plant which stems from different productivity and

[44] Our model nests such a model as a special case when there is a single recipe per industry and the distribution of wedges T is degenerate so that all suppliers of relationship-specific inputs face the same wedge.

cost draws. Thus our approach allows for the possibility that such a state-industry has an extreme cost share because of its draw of a technique; we are not forced to conclude that the distortion for the state-industry is severe.

4.7 Entry and Exit

In our benchmark model, we assumed that the number of producers of each product is given exogenously. Of course we would expect that if the formal enforcement improved, profitability would increase more in industries that rely more heavily on relationship-specific inputs. We do, in fact observe that as formal enforcement deteriorates, the number of firms decreases relatively more in the industries that rely more heavily on relationship-specific inputs (see Table XXVII in Appendix D.2). Thus it is likely the quality of enforcement affects firms' entry and exit decisions.

How would our estimates differ if firms could endogenously enter and exit? Our estimates of contracting frictions and technology would be unchanged because the strategy places no restrictions on the determinants of the set of firms. Nevertheless, entry and exit may be important for the counterfactuals we conduct. Exactly how depends on the specifics of entry and exit and the particular counterfactual. Consider the following two possibilities. First suppose firms were committed to enter, but could choose which industry to enter. Then the additional adjustment after formal enforcement improved would raise aggregate productivity even more than a benchmark model suggests.[45] Second, suppose that firms were committed to producing in a particular industry but choose which state to locate in. Here, if formal enforcement improves in one state, aggregate productivity in that state would rise as firms for other states moved in. However, aggregate productivity in other states would fall as firms moved out. This suggests that the response to nation-wide improvement in formal enforcement would differ from the response to a single state improving enforcement. Further, systematic differences across states in how the number of firms responds to cross-sectional difference in legal enforcement may not be informative about the economy's response to a nationwide improvement.

4.8 Interpretation of our estimates

Our estimates suggest that lowering the degree of congestion in high courts to the level enjoyed by the most efficient states would bring about productivity increases of on average four percent. This estimate is likely to be a lower bound for role of courts in enforcing contracts between buyers and suppliers for several reasons. First, even the best Indian courts are probably still worse than good courts in the developed world, in particular along dimensions that are not captured by our court quality measure (such as objectivity and incorruptibility). Second, if contracting frictions are also present for homogeneous intermediate inputs, our estimates of the productivity gains from improv-

[45] In Appendix G.5, we posit such a model and study how our counterfactual exercises might change. We show that if the household's elasticity of substitution across industries (η) is one, as we have assumed in the our calibration, the improvement in aggregate productivity is the same as our baseline.

ing courts are biased downwards.[46] Third, hold-up problems are particularly severe for services inputs, which account for a substantial fraction of cost in some industries; hence court improvement could lead to substantial additional cost reductions (Boehm, 2018). Formal enforcement is also likely to be important for the smallest firms that may be less able to rely on reputation. Moreover, courts matter for economic outcomes beyond their role in mitigating holdups with suppliers, notably by improving contracting between managers and workers (Besley and Burgess, 2004, and Bloom, Sadun and Reenen, 2012), and improving access to financing (Visaria, 2009).

5 Conclusion

This paper studies the organization of production in the Indian manufacturing sector, and how it relates to the quality of formal contract enforcement institutions. We make two main points: one about the within-industry heterogeneity and measurement of the organization of production, and a second one about how institutions shape intermediate input use and aggregate productivity.

First, we show that there is considerable amount of heterogeneity in technology and organization even within narrowly-defined industries. These differences should be reflected in the level of aggregation at which researchers assume a homogeneous shape of the production function. We argue that information on input use can be helpful in understanding differences in organization and technology within industries: a plant that produces cotton cloth from raw cotton has a long vertical span of production and performs both spinning and weaving, whereas a plant that produces cotton cloth from cotton yarn will only perform the weaving, and might therefore have different factor intensities. We provide a simple and flexible way of finding groups of firms that use the same production function (which we call *recipes*) using a statistical clustering algorithm.

Second, we find that the organization of production is shaped by the contracting environment. We find that slow enforcement of contracts impedes the use of relationship-specific materials. As a result, firms tilt their input basket towards the use of more standardized inputs, for which spot markets exist, and for which enforcement by courts in not necessary. We develop a multi-industry general equilibrium model where firms source multiple inputs and choose the organizational form optimally to minimize the cost of production. We estimate the relative distortions associated with the use of relationship-specific inputs from first moments of cost shares, which, compared to the standard approaches of estimating input wedges, is more robust to potentially imprecise measurement of input use. Our results suggest that distortions associated with poor courts are sizable and that improving courts would increase welfare: reducing the average age of pending cases by a year would, on average, increase a state's aggregate productivity by about two percent.

[46] That said, Johnson, McMillan and Woodruff (2002) bring survey evidence that the role of courts in determining supplier switching is about five times as large for relationship-specific inputs than for standardized inputs, suggesting that most of the productivity gains are correctly accounted for.

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Appendix

A Data Appendix

A.1 Data Sources and Variable Definitions

- *Plant-level data:* Our plant data is India's Annual Survey of Industries (ASI), published by the Central Statistics Office, Ministry of Statistics and Program Implementation. The data is at annual frequency, each reporting year starts on April 1st and ends on March 31st. Our data covers the years 2000/01 to 2012/13. The input and output product codes for 2008/09 and 2009/10 are different to the ones from earlier years; we created a concordance using the product names (which often match perfectly), and concord the few remaining ones by hand. The years 2010/11 to 2012/13 use the NPCMS product classification, which we convert to ASIC 2007/08 product codes using the concordance published by the Ministry.

- *Total cost:* Sum of the user cost of capital, the total wage bill, energy, services, and materials inputs. Total cost is set to be missing if and only if the user cost of capital, the wage bill, or total materials are missing. The user cost of capital is constructed using the perpetual inventory method as in the Appendix of Greenstreet (2007), using depreciation rates of 0%, 5%, 10%, 20%, and 40% for land, buildings, machinery, transportation equipment, and computers & software, respectively. Capital deflators are from the Ministry's wholesale price index, and the nominal interest rate is the India Bank Lending Rate, from the IMF's International Financial Statistics (on average about 11%).

- *Materials expenditure in total cost:* (as used in Table I and subsequent tables) Total expenditure on intermediate inputs which are associated with a product code (that excludes services and most energy inputs) divided by total cost (see above).

- *Pending High Court cases:* From Daksh India (www.dakshindia.org). Daksh collects and updates pending cases by scraping High Court websites. Cases were retrieved on 11 June 2017. To eliminate biases resulting from possible delays in the digitisation, we exclude all cases that were filed after 1/1/2017. We divide cases into civil and criminal based on state-specific case type codes (which are part of the case identifiers), and a correspondence between case types and whether they are civil or criminal cases (from High Courts, where available).

- *Rauch classification of goods*: From James Rauch's website, for 5-digit SITC codes. Concorded from SITC codes to ASIC via the SITC-CPC concordance from UNSTATS, and the NPCMS-ASIC concordance from the Indian Ministry of Statistics (NPCMS is based on CPC codes).

- *Dependence on relationship-specific inputs, by industry:* (as used in Table I) Following Nunn (2007): total expenditure of single-product plants in an industry on relationship-specific inputs (according to the concorded Rauch classification), by 3-digit industry, divided by total expenditures on intermediate inputs that are associated with a 5-digit product code (which excludes services and most energy intermediate inputs).

- *Gross domestic product per capita, by district:* District domestic product was assembled from various state government reports, for the year 2005 (to maximize coverage). Missing for Goa and Gujarat and some union territories, and for some individual districts in the other states. Population data from the 2001 and 2011 Census of India, interpolated to 2005 assuming a constant population growth rate in each district. Whenever district domestic product per capita was unavailable, we used gross state domestic product per capita, as reported by the Ministry of Statistics and Program Implementation.

- *Vertical Span:* See Appendix B. Due to the change in product classification from ASIC to NPCMS after 2010, we construct vertical distance only using the pre-2011 years.

- *Trust:* Fraction of respondents that answer "Most people can be trusted" in the World Value Survey's trust question: "Generally speaking, would you say that most people can be trusted or that you need to be very careful in dealing with people?" Data from waves 4, 5, and 6 of the World Values Survey, except for Himachal Pradesh, Puducherry, and Uttarakhand, which are only available in wave 6. Data is missing for Goa and the UT's, except for Delhi and Puducherry.

- *Language:* Data on population by mother tongue in each state, from the 2001 Census of India, as published by the Office of the Registrar General and Census Commissioner.
- *Caste:* Data on caste from the 2014 round of CMIE's Consumer Pyramids Survey (as made available by ICPSR), covering all states of India except the northeastern states and the UT's of Andaman and Nicobar, Lakshadweep, Dadra and Nagar Haveli and Daman and Diu. Observations have been weighted to be representative at the level of a "homogeneous region", which is "a set of neighbouring districts that have similar agro-climatic conditions, urbanisation levels and female literacy" (CPS 2014 User guide). Data covers 364 castes and caste categories. Herfindahls are constructed at the level of a homogeneous region, and mapped to districts.[47]
- *Corruption:* Number of self-reported bribes per 1,000 inhabitants. Data on bribes is the full history of 35,391 self-reported bribes paid from IPaidABribe.com (as of 03/28/2018), which we aggregate at the state level. Population by state is from the 2011 Census of India.
- *Capital Intensity:* Average plant-level cost share of capital inputs (user cost of capital, see above), by 5-digit industry.
- *Wage Premium:* Average plant-level wage bill divided by the total number of man-days worked, by 5-digit industry.
- *Contract Labor Share:* Average plant-level cost share of contract (non-permanent) workers in the total wage bill, by 5-digit industry.
- *Upstreamness:* Upstreamness as defined by Antràs et al. (2012) of 3-digit industries, calculated by year and averaged across years. Some goods show up mostly as inputs and not as outputs because as outputs they fall outside of the scope of the ASI (chiefly agricultural and mining goods); for these inputs total observed intermediate consumption exceeds total observed production by plants in our sample. We set total production of these goods equal to total intermediate consumption (hence assuming zero sales to final goods consumers).[48] The resulting variable looks very similar to those constructed by Antràs et al. (2012) for industrialized countries.
- *Tradability:* Weighted average freight rates of industry's inputs, by 5-digit industry (using only single-product plants). Freight rates are the "average trade-weighted freight rates" by 2-digit SITC codes, from Hummels (1999), concorded to ASIC codes using the SITC-ASIC concordance that we used for the Rauch classification as well.
- *Household consumption shares HH_ω:* Define total net production as the total production of ω (from the ASI, pooled across all years within each state, with each plant-year observation weighted by the inverse of the number of times the plant shows up in the ASI), minus the total consumption of ω as intermediate inputs by ASI firms (constructed and weighted analogously). If total net production of a good is negative, we set it equal to zero. The value of HH_ω is then the fraction of total net production of ω is the sum of total net production of all goods $\omega' \in \Omega$.
- *Recipe revenue shares $R_{\omega\rho}$:* Share of sales of ω using recipe ρ in total sales of ω. The sales of ω using ρ of a single-product plant j are the sales of ω by j multiplied by the probability that j produces using ρ (equation (3)). To construct the sales of ω using ρ of a multi-product plant j, we choose plant-specific recipe shares to minimize the Euclidean distance of the plant's vector of observed cost shares from the weighted average of the recipes' mean cost shares $\overline{m}_{\rho\omega}$, where the weights are the plant-specific recipe shares (subject to the constraint that weights have to sum up to one for each product ω). To construct $R_{\omega\rho}$, we weigh all plant-year observations by the inverse of the number of times the plant shows up in the ASI, pooling across all states and years.

[47]We are grateful to Renuka Sané and CMIE for helping us get a description of the homogeneous regions.

[48]While this may make these goods look more upstream than they actually are, the biases incurred are likely to be small: minerals are usually not directly sold to households; agricultural goods are either very upstream in the value chain of processed foods, or directly sold to households, but do not appear "in the middle" of the value chain.

A.2 Sample

For the linear regressions in Section 2, the sample consists of all plants are reported as operating, produce a single 5-digit product, and have materials shares in output strictly between zero and two. We also drop the few observations from Sikkim and the Seven Sister States in North East India (Assam, Meghalaya, Manipur, Mizoram, Nagaland, and Tripura; Arunachal Pradesh is not covered by the ASI) because they have a different sampling methodology in the ASI, and Jammu and Kashmir, because coverage of its court cases is inadequate, and because many federal laws do not apply to it due to its special status within the union.

For the structural estimation, we also remove observations where the shares of relationship-specific materials, homogeneous materials, or labor in sales exceed two, and observations where sales or non-materials expenditures are non-positive. To construct the households consumption shares HH_{ω} and the recipe sales shares $R_{\omega\rho}$ in the counterfactual, we weigh each plant-year observation by the inverse of the number of times the plant shows up in our sample.

The regression to estimate ζ (Table VI) is the only one where we identify parameters from time variation. Our sample to construct the expenditure aggregates consists of all census plants (which are surveyed every year). We restrict the sample in this way to eliminate fluctuations in the expenditure ratios that arise artificially from changes in the sample.

A.3 Details on High Court and State creation

Table VII Details on High Court Creation

Name	Jurisdiction over States/UT's	Year founded	Created with State	Notes on High Court creation	Reasons for State creation
Allahabad High Court	Uttar Pradesh	1866	no	Created as HC of Judicature of the Northwestern Provinces by the Indian High Courts Act 1861.	
High Court of Judicature at Hyderabad	Andhra Pradesh, Telangana	1956	yes	Created when Andhra Pradesh was created as part of the State Reorganization Act 1956	Creation of Andhra Pradesh was triggered by the independence movement of the Telugu-speaking population of Madras Presidency.
Mumbai High Court	Goa, Dadra and Nagar Haveli,Daman and Diu, Maharashtra	1862	no	Created by the Indian High Courts Act 1861.	
Kolkata High Court	Andaman and Nicobar Islands,West Bengal	1862	no	Created by the Indian High Courts Act 1861.	
Chhattisgarh High Court	Chhattisgarh	2000	yes	Created when Chhattisgarh state was carved out of Mandhya Pradesh in 2000 (Madhya Pradesh Reorganisation Act)	Chhattisgarh was a separate division in the Central Provinces under British rule. Demand for a separate state goes back to the 1920s.
Delhi High Court	Delhi	1966	no	At the time of independence, Punjab HC had jurisdiction for Delhi. With the State Reorganisation Act 1956, Punjab merged with Pepsu. Given Delhi's importance as capital, it was decided that they should have their own HC.	
Gauhati High Court	Arunachal Pradesh, Assam,Mizoram, Nagaland *	1948	yes	Created as HC of Assam in 1948, with the Indian constitution; renamed Gauhati HC in 1971 with the North East Areas Reorganization Act. Lost jurisdiction over Meghalaya, Manipur, and Tripura in 2013	
Gujarat High Court	Gujarat	1960	yes	Created when Gujarat split from Bombay State with the Bombay Reorganisation Act 1960.	Gujarat was created following the demand of Gujarati-speaking people for their own state (Mahagujarat movement).
Himachal Pradesh High Court	Himachal Pradesh	1971	yes	Created with Himachal Pradesh becoming a state (and therefore should have a separate HC under the Indian constitution)	
Jammu and Kashmir High Court	Jammu and Kashmir	1928	no	Created by the Maharaja in 1928. Special status: Laws passed by the Indian parliament generally do not apply to J&K (except foreign policy, communication, defense).	
Jharkhand High Court	Jharkhand	2000	yes	Created when Jharkhand state was carved out of Bihar in 2000 (Bihar Reorganisation Act)	Jharkhand was richer in natural resources than the rest of Bihar; Jharkhand Mukti Morcha independence movement, and political considerations by ruling parties.

Name	Jurisdiction over States/UT's	Year founded	Created with State	Notes on High Court creation	Reasons for State creation
Karnataka High Court	Karnataka	1884	no	Founded by the British as the Chief Court of Mysore in 1884.	
Kerala High Court	Kerala, Lakshadweep	1956	yes	Created when Kerala was created as part of the State Reorganization Act 1956	Idea of Kerala was to combine Malayalam-speaking regions.
Madhya Pradesh High Court	Madhya Pradesh	1936	no	Established as Nagpur High Court by King George V through a Letters Patent on 2 Jan 1936. Moved to its present location at Jabalpur with the State Reorganization Act 1956.	
Chennai High Court	Puducherry, Tamil Nadu	1862	no	Created by the Indian High Courts Act 1861.	
Odisha High Court	Odisha/Orissa	1948	no	Orissa was split off from Bihar in 1936, but did not get its own high court until the drafting of the Indian constitution (1948).	
Patna High Court	Bihar	1916	yes	Created when Bihar and Orissa were split off from Bengal Presidency.	Bengal nationalism and the undoing of the 1905 Partition of Bengal.
Punjab and Haryana High Court	Chandigarh, Haryana, Punjab	1947	yes	Created with the independence of India in 1947 (former HC of Punjab in Lahore was mostly relevant for modern-day Pakistan)	
Rajasthan High Court	Rajasthan	1949	yes	Created with the foundation of Rajasthan (1948-1950)	
Sikkim High Court	Sikkim	1955	no	Established by the Maharaja of Sikkim in 1955, became an Indian High Court in 1975 when Sikkim joined India and the monarchy was abolished	
Uttarakhand High Court	Uttarakhand	2000	yes	Created when Uttaranchal was carved out of Uttar Pradesh	Uttarakhand Kranti Dal independence movement.
High Court of Mumbai, Goa Bench	Goa, Daman and Diu, Dadra and Nagar Haveli	1982	no	Prior to the HC, a Judicial Commissioner's court existed in Goa. HC was established to safeguard the judge's independence (see Bombay HC at Goa website).	
Manipur High Court	Manipur	2013	no	Parties in Manipur demanded their own high court	
Meghalaya High Court	Meghalaya	2013	no	Parties in Meghalaya demanded their own high court	
High Court Of Tripura	Tripura	2013	no	Parties in Tripura demanded their own high court	

B Vertical Span and Vertical Distance Measures

B.1 Definition

Let $X_{j\omega'}$ be the expenditure of plant j on $\omega' \in \Omega$, then define for products $\omega, \omega' \in \Omega$, and a set $B \subset \Omega$

$$\beta^B_{\omega\omega'} = \frac{\sum_{j \in J_\omega} X_{j\omega'}}{\sum_{j \in J_\omega} \sum_{\omega'' \in \Omega \setminus B} X_{j\omega''}}$$

if $\omega' \notin B$, and $\beta^B_{\omega\omega'} = 0$ otherwise. $\beta^B_{\omega\omega'}$ is the share of ω' in industry ω's materials basket that excludes inputs from B.

Denote by $A^n_{\omega\omega'}$ the set of $(n+1)$-tuples $(\omega^{(0)}, \omega^{(1)}, \ldots, \omega^{(n)}) \in \Omega^{n+1}$ such that

$$\omega^{(0)} = \omega, \quad \omega^{(n)} = \omega', \tag{4}$$

$$\omega^{(i)} \neq \omega^{(j)} \quad \forall (i,j) \in \{0, \ldots, n\}^2, i \neq j. \tag{5}$$

Intuitively, $A^n_{\omega\omega'}$ is the set of all possible non-circular product chains of length n between ω and ω'. Then let

$$\delta_{\omega\omega'} = \sum_{n=1}^{\infty} \sum_{a \in A^n_{\omega\omega'}} \frac{\lambda(a)}{\sum_{n'=1}^{\infty} \sum_{a' \in A^{n'}_{\omega\omega'}} \lambda(a')} \cdot n$$

where

$$\lambda : A^n_{\omega\omega'} \to \mathbb{R}, \ \lambda(\omega^{(0)}, \omega^{(1)}, \ldots, \omega^{(n)}) = \prod_{i=1}^{n} \beta^{\{(\omega^{(0)}, \omega^{(1)}, \ldots, \omega^{(i-1)})\}}_{\omega^{(i-1)}\omega^{(i)}}$$

is the share of ω' in ω's input mix through a given product chain. Hence, $\delta_{\omega\omega'}$ is the average number of production steps between an output ω and an input ω', weighted by the overall expenditure of each product chain in industry ω's mix of materials, and excluding any circular product chains.

The plant-level measure of the vertical span of production, and the left-hand side of Table III, is then the average distance of its inputs from the output, weighted by each inputs' share in the plants materials expenditure: let $j \in J_\omega$, then

$$\text{verticalSpan}_j = \sum_{\omega' \in \Omega} \frac{X_{j\omega'}}{\sum_{\omega'' \in \Omega} X_{j\omega''}} \delta_{\omega\omega'}.$$

To understand why we exclude circular product chains, consider the following example: Some plants sell aluminum and use aluminum scrap as an input, whereas some other plants use aluminum as an input and sell aluminum scrap. Thus in the production of aluminum scrap, aluminum would show up as an input one stage away, three stages away, five stages away, etc.

When we see a plant selling aluminum scrap and using aluminum as an input, we believe that it is the distance of one that is relevant, not the distances of three, five, etc. In other words, we believe the plant is turning the aluminum into scrap, but not turning the aluminum into scrap then back to aluminum and then back to scrap. Therefore we believe the circular part of the production chain is not relevant for constructing a plant's distance to its inputs.

B.2 Examples

Table VIII shows the average vertical distance of several input groups (defined as all inputs that contain the strings “fabric”/“cloth”, “yarn”, or “cotton, raw” in their description”) from the output “cotton shirts”. Fabrics and cloths are closest to the final output; yarns, which are used in the production of cloths and fabrics, are further upstream, and raw cotton inputs are even further upstream.

Table IX shows vertical distances between aluminium ingots as an output, and several intermediate inputs. Aluminium ingots can be made both by recycling castings and alloys, but also by casting molten aluminium. The latter also serves as an intermediate input in the production of castings and alloys, and is hence vertically more distant than the inputs which undergo recycling. Aluminium itself is produced from aluminium oxide using electrolytic reduction (Hall-Heroult process). In turn, aluminium oxide is produced by dissolving bauxite in caustic soda at high temperatures (hence the coal inputs).

Table VIII Vertical distance examples for 63428: Cotton Shirts

Input group	Average vertical distance
Fabrics Or Cloths	1.67
Yarns	2.78
Raw Cotton	3.55

Table IX Vertical distance examples for 73107: Aluminium Ingots

ASIC code	Input description	Vertical distance
73105	Aluminium Casting	1.23
73104	Aluminium Alloys	1.46
73103	Aluminium	1.92
22301	Alumina (Aluminium Oxide)	2.92
31301	Caustic Soda (Sodium Hydroxide)	3.81
23107	Coal	3.85
22304	Bauxite, raw	3.93

C Additional Reduced Form Results

C.1 Controlling for Interactions with State and Industry Characteristics

Tables X, XI, and XII show the main regressions of materials cost shares, input mixes, and vertical span with a full set of controls. See Appendix A for definitions of the variables.

Tables XIII and XV also include interactions of industry characteristics with court quality. Tables XIV and XVI show IV regressions where the court quality $\times$ industry characteristic interactions are instrumented by the interaction of log court age and the corresponding industry characteristic.

Table X Additional Controls – Materials Cost Share

	Dependent variable: Materials Expenditure in Total Cost			
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	-0.0167**	-0.0118*	-0.0156^{+}	-0.0212**
	(0.0046)	(0.0053)	(0.0085)	(0.0078)
LogGDPC * Rel. Spec.		0.0102		0.00556
		(0.0091)		(0.0096)
Trust * Rel. Spec.		0.0300		0.0353
		(0.038)		(0.038)
Language HHI * Rel. Spec.		0.0610		0.0612
		(0.040)		(0.040)
Caste HHI * Rel. Spec.		0.106*		0.0990^{+}
		(0.051)		(0.052)
Corruption * Rel. Spec.		0.0641		0.0508
		(0.097)		(0.097)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.480	0.484	0.480	0.484
Observations	208527	196748	208527	196748

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table XI Additional Controls – Vertical Span

	Dependent variable: Vertical Span			
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	0.0195^{+}	0.0280*	0.0292	0.0368*
	(0.011)	(0.012)	(0.019)	(0.018)
LogGDPC * Rel. Spec.		0.0288		0.0330
		(0.024)		(0.024)
Trust * Rel. Spec.		-0.0939		-0.0984
		(0.090)		(0.091)
Language HHI * Rel. Spec.		-0.0742		-0.0743
		(0.092)		(0.092)
Caste HHI * Rel. Spec.		-0.182		-0.176
		(0.12)		(0.12)
Corruption * Rel. Spec.		0.481*		0.494*
		(0.24)		(0.24)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.443	0.453	0.443	0.453
Observations	163334	154021	163334	154021

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table XII Additional Controls – Input Mix

	Dependent variable: $X_j^R/(X_j^R + X_j^H)$			
	(1)	(2)	(3)	(4)
Avg age of Civil HC cases	-0.00547*	-0.00530*	-0.0144**	-0.0167**
	(0.0022)	(0.0024)	(0.0044)	(0.0045)
Log district GDP/capita		-0.00384		-0.00980^{+}
		(0.0046)		(0.0051)
Trust		-0.00740		-0.00160
		(0.018)		(0.019)
Language HHI		-0.0553**		-0.0567**
		(0.021)		(0.022)
Caste HHI		-0.0428		-0.0525^{+}
		(0.028)		(0.029)
Corruption		-0.0676		-0.0844^{+}
		(0.044)		(0.045)
5-digit Industry FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.441	0.449	0.441	0.449
Observations	225590	199339	225590	199339

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table XIII Materials Cost Share: Industry Characteristic Interactions

	Dependent variable: Materials Expenditure in Total Cost					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	-0.0117*	-0.00987*	-0.0118*	-0.0112*	-0.0105^{+}	-0.00675
	(0.0052)	(0.0049)	(0.0053)	(0.0053)	(0.0054)	(0.0049)
Capital Intensity * Avg. age of cases	-0.110**					-0.0624^{+}
	(0.038)					(0.033)
Ind. Wage Premium * Avg. age of cases		-0.00146				-0.00180*
		(0.0011)				(0.00088)
Ind. Contract Worker Share * Avg. age of cases			-0.00116			0.0207
			(0.030)			(0.027)
Upstreamness * Avg. age of cases				0.00289^{+}		0.00363*
				(0.0015)		(0.0015)
Tradability * Avg. age of cases					-0.00104^{+}	-0.00150**
					(0.00058)	(0.00051)
State × Rel. Spec. Controls	Yes	Yes	Yes	Yes	Yes	Yes
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.484	0.484	0.484	0.484	0.484	0.484
Observations	196748	196748	196748	196748	196735	196735

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"State × Rel. Spec. controls" are interactions of GDP/capita, trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

Table XIV Materials Cost Share: Industry Characteristic Interactions (IV)

	Dependent variable: Materials Expenditure in Total Cost					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	-0.0211**	-0.0155*	-0.0211**	-0.0194*	-0.0217**	-0.0149*
	(0.0078)	(0.0074)	(0.0077)	(0.0080)	(0.0078)	(0.0075)
Capital Intensity * Avg. age of cases	-0.0262					-0.00252
	(0.065)					(0.062)
Ind. Wage Premium * Avg. age of cases		-0.00406*				-0.00343*
		(0.0018)				(0.0016)
Ind. Contract Worker Share * Avg. age of cases			0.0311			0.00893
			(0.040)			(0.041)
Upstreamness * Avg. age of cases				0.00641*		0.00593+
				(0.0032)		(0.0031)
Tradability * Avg. age of cases					0.00108	0.000551
					(0.00100)	(0.0010)
State × Rel. Spec. Controls	Yes	Yes	Yes	Yes	Yes	Yes
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.484	0.484	0.484	0.484	0.483	0.483
Observations	196748	196748	196748	196748	196735	196735

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"State × Rel. Spec. controls" are interactions of GDP/capita, trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

Table XV Vertical Span: Industry Characteristic Interactions

	Dependent variable: Vertical Span					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	0.0279^{*}	0.0232^{+}	0.0278^{*}	0.0275^{*}	0.0295^{*}	0.0247^{*}
	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)
Capital Intensity * Avg. age of cases	0.0279					0.0255
	(0.073)					(0.074)
Ind. Wage Premium * Avg. age of cases		0.00357^{+}				0.00299
		(0.0021)				(0.0021)
Ind. Contract Worker Share * Avg. age of cases			-0.0263			0.00432
			(0.026)			(0.033)
Upstreamness * Avg. age of cases				-0.00447		-0.00363
				(0.0036)		(0.0036)
Tradability * Avg. age of cases					-0.00118^{*}	-0.000961
					(0.00059)	(0.00079)
State × Rel. Spec. Controls	Yes	Yes	Yes	Yes	Yes	Yes
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.453	0.453	0.453	0.453	0.453	0.453
Observations	154021	154021	154021	154021	154011	154011

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"State × Rel. Spec. controls" are interactions of GDP/capita, trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

Table XVI Vertical Span: Industry Characteristic Interactions (IV)

	Dependent variable: Vertical Span					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	0.0351^{*}	0.0211	0.0362^{*}	0.0360^{*}	0.0388^{*}	0.0257
	(0.018)	(0.019)	(0.018)	(0.018)	(0.018)	(0.019)
Capital Intensity * Avg. age of cases	0.259^{+}					0.370^{*}
	(0.15)					(0.17)
Ind. Wage Premium * Avg. age of cases		0.0110^{**}				0.00927^{*}
		(0.0042)				(0.0043)
Ind. Contract Worker Share * Avg. age of cases			-0.0483			0.110^{+}
			(0.047)			(0.060)
Upstreamness * Avg. age of cases				-0.00405		0.00258
				(0.0070)		(0.0070)
Tradability * Avg. age of cases					-0.00397^{**}	-0.00544^{**}
					(0.0013)	(0.0018)
State $\times$ Rel. Spec. Controls	Yes	Yes	Yes	Yes	Yes	Yes
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.453	0.453	0.453	0.453	0.453	0.453
Observations	154021	154021	154021	154021	154011	154011

Standard errors in parentheses, clustered at the state $\times$ industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
"State $\times$ Rel. Spec. controls" are interactions of GDP/capita, trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

C.2 Time variation in court quality

The regressions in the main text use variation in the average age of pending cases in High Courts to identify the impact of court congestion on input use. The microdata that is underlying the construction of this measure is available for one point in time only, meaning the regressions are exploiting entirely cross-sectional variation. In this section we try to use two different time-varying measures of court quality.

C.2.1 Creation of new High Court benches

Our first set of results using time variation in court congestion exploits two episodes where High Courts created new benches in cities further away from the main bench, with the specific aim of increasing access to justice in these remote areas. Two new benches of the Karnataka High Court were set up Dharwad and Gulbarga in July 2008. They have jurisdiction over Belgaum, Balgakot, Koppal, Gadag, Dharwad, Uttara Kannada, Haveri, and Bellary (Dharwad bench), and Bijapur, Gulbarga, Bidar, and Raichur (Gulbarga bench). Similarly, in July 2004, the Chennai High Court set up a bench in Madurai, which has jurisdiction over Kanniyakumari, Tirunelveli, Tuticorin, Madurai, Dindigul, Ramanathapuram, Virudhunagar, Sivaganga, Pudukkottai, Thanjavur, Tiruchirappalli and Karur districts.[49]

The regressions in Table XVII look at whether wedges on relationship-specific inputs have decreased differentially in districts that are under the jurisdiction of the new benches. The coefficients are not very precisely estimated, since the districts with new benches account for few (about 6% on average) of the plants in the respective states. Nevertheless, the results are qualitatively consistent with those from the main text.

Table XVII Identification from Time Variation: Diff-in-Diff

	X^R/Sales	$s_R - s_H$	Materials/TotalCost	Vert. Distance
	(1)	(2)	(3)	(4)
(New Bench in District)$_d \times$ (Post)$_t$	0.0126**	0.00960	-0.00305	0.00678
	(0.0043)	(0.0076)	(0.0033)	(0.010)
(New Bench in District)$_d \times$ (Post)$_t \times$ (Rel.Spec)$_\omega$			0.0142	-0.0764*
			(0.010)	(0.031)
Plant $\times$ Product FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
R^2	0.832	0.824	0.906	0.813
Observations	80427	74696	78462	77995

Figures 6 and 7 show the relative changes in the input mix in treated vs. untreated districts before and after the new high court benches were installed.

C.2.2 Pendency Ratios

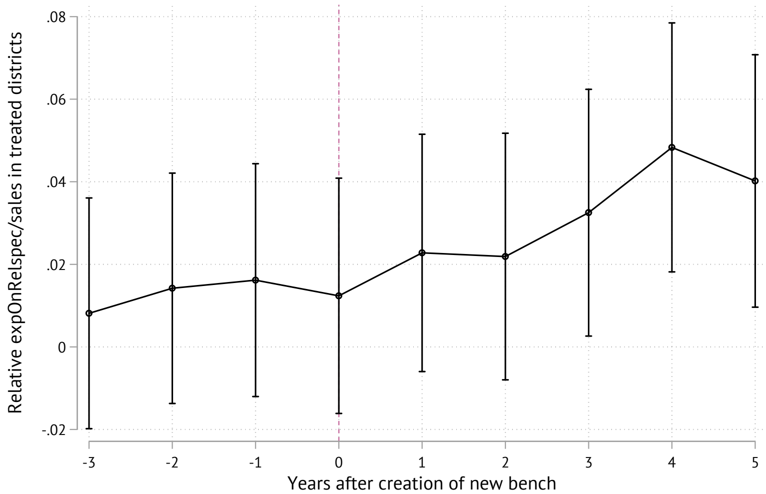
Our second set of regressions uses high court congestion ratios that vary by year, and that are published for a subset of High Courts in the 245th report of the Law Commission of India. For each year between 2002 and 2012, the report publishes the number of new and disposed cases during the year, and pending cases at the end of the year. We calculate the congestion ratio as

$$\text{Congestion Ratio}_{st} = \frac{(\text{Pending Cases})_{st}}{(\text{Disposed Cases})_{st}}.$$

This ratio can be interpreted as the number of years it takes to dispose of the backlog, if the number of disposed cases is constant over time.

[49]Source: Karnataka and Madras High Court websites: https://karnatakajudiciary.kar.nic.in/ and http://www.hcmadras.tn.nic.in/

Figure 6 Relative change in X^R/Sales after new court bench is set up



The figure shows the evolution of the share of expenditure on relationship-specific inputs in sales, in treated districts relative to non-treated districts. Treatment happens at the start of period 0. Regression includes firm × product fixed effects and year dummies.

We should stress that these congestion ratios are not a good way to measure the cross-sectional variation in court speed. The Law Commission report collects these data from surveys of the High Courts, and mentions explicitly that different courts measure cases (as well as institution and disposal of cases) very differently, making comparisons problematic. However, if the way measure cases and flows in a consistent way over time, we may still use them for regressions where we compare input use and court quality over time, within state-industry pairs. Tables XVIII to XX show these regressions. While these results cannot be taken as evidence for a causal channel (we do not know what drives changes in pendency ratios), it is worth noting that they are consistent with the results from Section 2.

Table XVIII Materials Shares and Court Quality – Time variation

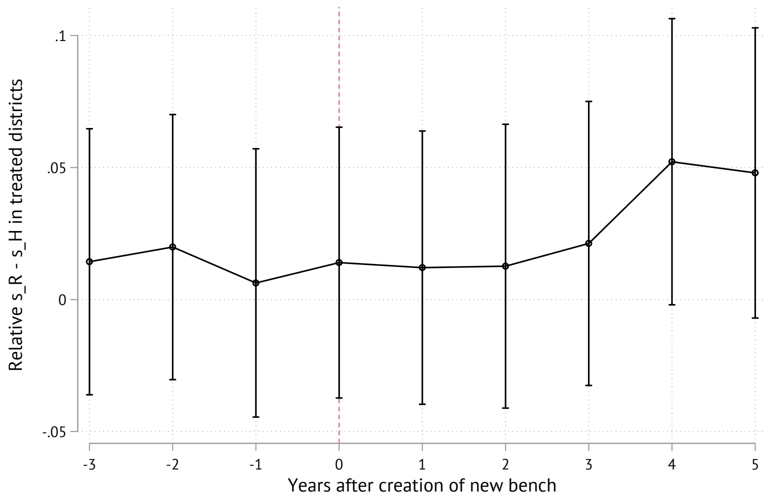
	Mat. Exp. / Total Cost
	(1)
Court Congestion Ratio * Rel. Spec.	-0.0573** (0.0066)
Rel. Spec. × State Controls	Yes
Industry × District FE	Yes
Estimator	OLS
R^2	0.718
Observations	86309

Standard errors in parentheses, clustered at the state × industry level.

$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

"Rel. Spec. × State Controls" are interactions of trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

Figure 7 Relative change in composition of input mix after new court bench is set up



The figure shows the evolution of $s^R - s^H$, in treated districts relative to non-treated districts. Treatment happens at the start of period 0. Regression includes firm × product fixed effects and year dummies.

Table XIX Input Mix and Court Quality – Time variation

	Dependent variable: $X_j^R/(X_j^R + X_j^H)$
	(1)
Court Congestion Ratio	-0.0118**
	(0.0041)
State Controls	Yes
Industry × District FE	Yes
Estimator	OLS
R^2	0.661
Observations	87936

Standard errors in parentheses, clustered at the state × industry level.

$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

"State Controls" are trust, language herfindahl, caste herfindahl, and corruption.

Table XX Vertical Distance and Court Quality – Time variation

	Dependent variable: Vertical Span
	(1)
Court Congestion Ratio * Rel. Spec.	0.0164
	(0.010)
Rel. Spec. × State Controls	Yes
Industry × District FE	Yes
Estimator	OLS
R^2	0.660
Observations	71667

Standard errors in parentheses, clustered at the state × industry level.

$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

"Rel. Spec. × State controls" are interactions of trust, language herfindahl, caste herfindahl, and corruption with relationship-specificity.

C.3 Distortions and International Sourcing

This section presents three tables describing how plants' usage of domestically- and foreign-sourced inputs varies with distortions. The fourth row of Table XXI shows that, relative to those in industries that tend to use standardized inputs, the expenditure shares on domestic inputs of plants in industries that use relationship-specific inputs declines as courts get more congested. The second row of Table XXI shows that this estimated relationship is stronger when we use our instrumental variables strategy with court age as an instrument for congestion.

Table XXI Domestic Materials Shares and Court Quality (Fact 3)

	Dependent variable: Dom. Mat. Exp. in Total Cost			
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	-0.0110^{+}	-0.0127*	-0.0349**	-0.0279**
	(0.0057)	(0.0063)	(0.0097)	(0.0085)
LogGDPC * Rel. Spec.		-0.0124		-0.0200^{+}
		(0.0098)		(0.010)
Trust * Rel. Spec.		-0.0415		-0.0331
		(0.043)		(0.043)
Language HHI * Rel. Spec.		0.00109		0.00139
		(0.056)		(0.056)
Caste HHI * Rel. Spec.		0.0466		0.0355
		(0.074)		(0.075)
Corruption * Rel. Spec.		0.0654		0.0439
		(0.12)		(0.12)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.432	0.439	0.431	0.439
Observations	208527	196748	208527	196748

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table XXII shows the same regressions but with the cost share of imported intermediates. Here, the OLS and IV specifications point to opposite results, and further investigation is required. The OLS specifications indicate that among those that rely more on more relationship-specific inputs, more congestion leads to lower shares of imported inputs relative to those that rely on standardized inputs. The IV specification indicates that more congestion leads to relatively higher imported input shares in those that rely on relationship-specific inputs, indicating that firms respond to distortions by substituting from domestic to foreign suppliers.

Table XXIII shows the results on the share of imports in the basket of relationship-specific (first three columns) and homogenous (last three columns) inputs. More congestion leads to a higher share of imports in both baskets, but the substitution is stronger in the basket of relationship-specific goods.

Table XXII Import Shares and Court Quality, OLS + IV

	Dependent variable: Imported Materials Expenditure in Total Cost				
	(1)	(2)	(3)	(4)	(5)
Avg age of Civil HC cases	-0.00105		0.000444		
	(0.0017)		(0.0032)		
Avg Age Of Civil Cases * Rel. Spec.	-0.00848^{+}	0.000870	0.0222**	0.0193**	0.00666
	(0.0045)	(0.0040)	(0.0068)	(0.0062)	(0.0042)
LogGDPC * Rel. Spec.		0.0227**			0.0255**
		(0.0054)			(0.0057)
Trust * Rel. Spec.		0.0716**			0.0683**
		(0.026)			(0.026)
Language HHI * Rel. Spec.		0.0599			0.0598
		(0.041)			(0.041)
Caste HHI * Rel. Spec.		0.0593			0.0635
		(0.051)			(0.052)
Corruption * Rel. Spec.		-0.00127			0.00691
		(0.073)			(0.074)
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes
District FE		Yes		Yes	Yes
Estimator	OLS	OLS	IV	IV	IV
R^2	0.276	0.342	0.265	0.329	0.342
Observations	223674	196748	223674	208527	196748

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table XXIII Substitution into Importing

	R-Imports in Total R		H-Imports in Total H	
	(1)	(2)	(3)	(4)
Avg age of Civil HC cases	0.0193**	0.00925**	0.0112**	0.00440**
	(0.0023)	(0.0018)	(0.0016)	(0.0013)
Log district GDP/capita		0.0224**		0.0180**
		(0.0027)		(0.0019)
Trust in other people (WVS)		0.110**		0.0564**
		(0.012)		(0.011)
Language Herfindahl		0.0162		-0.0292**
		(0.019)		(0.0093)
Caste Herfindahl		0.0584*		0.0171
		(0.028)		(0.013)
Corruption		0.0315		-0.0912**
		(0.028)		(0.022)
5-digit Industry FE	Yes	Yes	Yes	Yes
Estimator	IV	IV	IV	IV
R^2	0.227	0.251	0.180	0.197
Observations	168120	148165	168953	149623

Standard errors in parentheses, clustered at the state × industry level.
Dependent variable in columns (1) and (2) (resp. (3) and (4)) is the share of relationship-specific (homogeneous) imports in total relationship-specific (homogeneous) materials.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

C.4 Materials Shares with Size and Age

Table XXIV shows that materials cost shares do not correlate much with size and age of the plant. Table XXV shows correlations of input wedges with various plant-level characteristics.

Table XXIV Plant Age and Size

	Dependent variable: Mat. Exp in Total Cost		
	(1)	(2)	(3)
Plant Age	-0.000695**		-0.000679**
	(0.000065)		(0.000063)
Log Employment		-0.00257**	-0.00176*
		(0.00086)	(0.00083)
5-digit Industry FE	Yes	Yes	Yes
District FE	Yes	Yes	Yes
Estimator			
R^2	0.481	0.481	0.482
Observations	205109	208179	204767

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table XXV Wedges and Plant Characteristics

	Age	Size	Multiproduct	# Products
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	0.620+	-0.0253	-0.0121	-0.0580
	(0.32)	(0.040)	(0.0076)	(0.037)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
R^2	0.214	0.339	0.301	0.295
Observations	353392	359820	360316	360316

Note: Sample includes multiproduct plants. Industry dummies refer to the 5-digit industry with the plants' highest production value.

D Exploring the Nature of Contracting Frictions

D.1 Distortions and Revenue-Cost Margins

Table XXVI shows how measured markups, plants' ratio of sales to cost vary with the level of distortions. Court congestion should increase distortions for plants in industries that tend to rely on relationship-specific inputs relative to those in industries that tend to rely on homogeneous inputs. The first row of the table indicates that, across all measures, ratios of sales to cost decline with distortions. The second and third column shows that this appears to be the direct impact of the distortions rather than the indirect impact of distortions on size or age.

Table XXVI Sales over Total Cost

	Dependent variable: Sales/Total Cost					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	-0.0353**	-0.0347**	-0.0345**	-0.0494*	-0.0496*	-0.0508*
	(0.0097)	(0.0094)	(0.0093)	(0.022)	(0.022)	(0.022)
Plant Age		0.000574**	0.000258+		0.000575**	0.000259+
		(0.00014)	(0.00014)		(0.00014)	(0.00014)
Log Employment			0.0314**			0.0314**
			(0.0016)			(0.0016)
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.114	0.110	0.115	0.114	0.110	0.115
Observations	208527	205109	204767	208527	205109	204767

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

D.2 Distortions are costly

In our microfoundation, the reason a distortion is costly is that the resource the supplier saves by delivering an imperfect input is smaller than the resources used by the buyer correcting the input. An alternative possibility is that these costs are roughly equal, in which case there would be no resource cost of the distortion despite the fact that the buyer's reduced cost share of relationship-specific inputs. In the extreme, this would mean that despite the impact on cost shares, the loss in productivity would be smaller than suggested by Section 4.2. We argue here that distortions do indeed raise the buyer's cost. Table XXVII shows that when distortions are more likely to be severe—in industries that tend to rely on relationship-specific inputs in states with slower courts—industries tend to have fewer plants.

Table XXVII Extensive-margin regressions

	Dependent variable: $\log\lvert J^d_{\omega,t}\rvert$					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg Age Of Civil Cases * Rel. Spec.	-0.0413*	-0.0382*	-0.0259	-0.133**	-0.107**	-0.106**
	(0.017)	(0.017)	(0.018)	(0.035)	(0.035)	(0.032)
LogGDPC * Rel. Spec.		0.0505**	0.0518**		0.0313	0.0353*
		(0.016)	(0.016)		(0.019)	(0.018)
Trust * Rel. Spec.			-0.0482			0.00793
			(0.14)			(0.14)
Language HHI * Rel. Spec.			0.120			0.121
			(0.15)			(0.15)
Caste HHI * Rel. Spec.			0.225**			0.171^{+}
			(0.086)			(0.087)
Corruption * Rel. Spec.			0.537^{+}			0.381
			(0.32)			(0.33)
5-digit Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
State × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.410	0.417	0.423	0.410	0.417	0.423
Observations	191008	183214	177075	191008	183214	177075

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Note: Dependent variable is the log number of producers of ω in a state d at time t. Multi-product plants are counted once for each product. GDP per capita is the average district GDP per capita within each state.

D.3 Do distortions get paid in one particular component of primary inputs?

We have assumed that when a relationship-specific input is distorted, primary inputs get used up. Primary inputs includes labor, capital, services, and some other inputs. Table XXVIII shows regressions of the cost share of labor/capital/services/other inputs in total non-material inputs on the interaction of court quality and relationship-specificity. If one particular component of primary inputs was used to customize the distorted input, or if the distorted intermediate input were particularly complementary with one particular component, then we should find that the share of that input in non-materials costs should be higher whenever the contracting frictions are stronger. We find that this is not the case, suggesting that distortions are being paid in the overall bundle of primary inputs.

Table XXVIII Cost share of factors in primary inputs

	Labor	Capital	Services	Rest
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	-0.000504	0.00405	-0.000248	-0.00355
	(0.0054)	(0.0030)	(0.0046)	(0.0049)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
R^2	0.310	0.232	0.341	0.269
Observations	208527	208527	208527	208527

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$
The dependent variable is the cost share of the factor in the total cost of non-material inputs.

D.4 Plant-level substitutability of inputs

Fact 1 showed that when relationship-specific inputs are more severely distorted, the cost share of those inputs declines. In our model, this happens because the buyer must expend additional primary inputs in order to use the relationship-specific input. An alternative possibility is that the distortion raises the price paid to the supplier. If primary inputs and intermediate inputs were substitutes, the cost share of intermediates would fall. However, we believe the evidence does not support an elasticity of substitution between primary and intermediate inputs that is larger than unity. Indeed, in the model, the average cost share of intermediate inputs among plants that use any particular recipe is independent of the cost of intermediate inputs and the cost of labor.

We know of two estimates of long-run plant level elasticities of substitution between materials and primary inputs, Oberfield and Raval (2014) and Appendix B.3 of Atalay (2017). Each find elasticities slightly lower than unity. Further, we can investigate this elasticity in our context. Table XXIX uses the interaction of court quality with the average dependence on relationship-specific inputs among an industry's upstream industries to stand in for a shifter to the cost of intermediate inputs. The IV coefficient estimates are positive. Thus, we do not find support for an assertion that primary and intermediate inputs are substitutes at the plant-level.

Table XXIX Plant-level elasticity of substitution

	Dependent variable: Materials Expenditure in Total Cost			
	(1)	(2)	(3)	(4)
Avg Age Of Civil Cases * Rel. Spec.	-0.0147^{+}	-0.0135	-0.0397**	-0.0401**
	(0.0080)	(0.0089)	(0.013)	(0.013)
LogGDPC * Rel. Spec.		0.0112		0.00653
		(0.0090)		(0.0095)
Avg Age Of Civ. Cases * Rel. Spec. of Upstream Sector	-0.00360	0.00268	0.0450*	0.0349^{+}
	(0.011)	(0.012)	(0.019)	(0.020)
Trust * Rel. Spec.		0.0274		0.0342
		(0.038)		(0.038)
Language HHI * Rel. Spec.		0.0467		0.0501
		(0.032)		(0.032)
Caste HHI * Rel. Spec.		0.0980*		0.0897^{+}
		(0.050)		(0.050)
5-digit Industry FE	Yes	Yes	Yes	Yes
District FE	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV	IV
R^2	0.480	0.484	0.480	0.484
Observations	208527	196748	208527	196748

Standard errors in parentheses, clustered at the state $\times$ industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

E Imperfect Contract Enforcement

Suppliers can sell a good that is defective or imperfectly customized to the buyer. If this happens, the buyer must use labor correcting the defect or completing the customization. In principle, the supplier can save on production cost by producing defective/imperfectly customized inputs. To produce one unit of the intermediate input that is defective enough so that the buyer must use up ψ^x units of labor, the supplier's unit cost is $c_s(\psi)$, where $c_s(0) \equiv c_s$ is the supplier's unit cost of producing a defect-free unit of the input.

A contract between buyer and supplier is a triple (M^c, ψ^c, x^c), where $x^c \geq 0$ is the quantity of the good to be delivered, ψ^c is a desired customization level, M^c is a payment from the buyer to the supplier upon delivery. We assume that quantity x is costlessly enforceable, which ensures that the supplier chooses $x = x^c$.

Both the buyer and supplier anticipate equilibrium behavior. In line with the main text, we assume that the buyer has full bargaining power, in that she can make a take-it-or-leave-it offer to the supplier.

If the contract has been breached (either because the supplier chooses a $\psi > \psi^c$ or because the buyer chooses $M < M^c$), either party could enforce the contract in a court. The outcome of enforcement is deterministic, and enforcement is costly. The plaintiff has to pay enforcement costs, which amount to a proportion π of the value of the transaction (M^c), while the defendant expends δ of the value of the transaction. The value of the claim to the plaintiff is the net transfer to her that would arise under enforcement. Enforcement costs cannot be recovered in court. The enforcement cost for the plaintiff $\pi \in (0,1)$ is randomly drawn for each buyer-supplier pair.

In principle, the contracts can be written to get around this friction. However, the doctrine of *expectation damages* limits the damages the buyer can collect from the supplier. The damages cannot be more than what is needed so that the buyer's is as well off as she would have been had the contract been honored.[50]

Thus if the supplier breaches by choosing $\psi > \psi^c$ and the buyer enforces the contract in court, the buyer receives a gross transfer of $w\psi x - w\psi^c x$. Thus net of enforcement costs, the buyer receives $w\psi x - w\psi^c x - \pi M^c$, and the supplier pays $w\psi x - w\psi^c x + \delta M^c$. If the buyer chooses $M < M^c$, the court orders the buyer to pay $M^c - M$ to the buyer. Thus net of enforcement costs, the supplier receives $M^c - M - \pi M^c$, and the buyer pays $M^c - Mx + \delta M^c$.

Given the choice of supplier, the buyer's payoff from the contract (up to an additive normalizing constant) is $-w\psi x - M$ plus the value of any net transfers mandated by the court. Similarly, the payoff to the supplier (up to an additive normalizing constant) can be expressed as $-c(\psi)x + M$ plus the value of any net transfers mandated by the court.

E.1 Timing of events

1. The buyer makes a take-it-or-leave-it offer of a contract, (M^c, ψ^c, x^c)
2. The supplier decides whether to accept the offer. If the supplier accepts the contract, proceed to the next step.
3. The supplier produces x units at defectiveness level $\psi \geq 0$ for a unit cost of $c_s(\psi)$.
4. The buyer makes a transfer M.
5. If the contract has been breached, either party could enforce the contract in a court.
6. Production occurs.

E.2 Solving the game

Lemma 1 *$M^c > 0$, $M = (1-\pi) M^c$, and $\psi = \psi^c + \pi \frac{M^c}{xw}$, and neither party sues for breach of contract.*

Proof. First, M^c must be strictly positive because otherwise the supplier would not agree to the contract. The supplier would sue with positive probability if $(M^c - M) - \pi M^c \geq 0$. Thus it must be that $M \leq (1-\pi) M^c$ because otherwise the buyer could strictly improve her payoff by reducing M without risk of getting sued. On the other hand, if $M < (1-\pi) M^c$, the buyer could strictly increase her payoff by setting $M = (1-\pi) M^c + \epsilon$, for small enough ϵ, in which case the buyer would avoid getting sued and would avoid legal costs. Therefore it must be that $M \geq (1-\pi) M^c$. Together, these imply that $M = (1-\pi) M^c$. There cannot be an equilibrium in which $M = (1-\pi) M^c$ and the supplier sues with positive probability, because in that case, the buyer would have been strictly better off by setting $M = (1-\pi) M^c + \epsilon$, for small enough ϵ, and avoiding the legal costs of getting sued.

Similarly, the buyer would sue only if $\psi > \psi^c$ and $(\psi xw - \psi^c xw) - \pi M^c > 0$. It must be that $(\psi xw - \psi^c xw) - \pi M^c \geq 0$, because otherwise the supplier could strictly improve her payoff by raising ψ

[50] See Shavell (1980) and Boehm (2018).

without risk of getting sued. On the other hand, if $\psi xw > \psi^c xw + \pi M^c$, the buyer would sue. Thus the supplier would be better off setting $\tilde{\psi} = \psi^c + \frac{\pi M^c}{xw} - \epsilon$ for small enough ϵ and avoiding the legal costs of getting sued: $\frac{c'}{w} > -1$ implies that the change in payoff is positive for small enough ϵ:

$$
\begin{aligned}
-c\left(\tilde{\psi}\right)x - \left[-c\left(\psi\right)x - \left(\psi xw - \psi^c xw\right) - \delta M^c\right] &= xw\int_{\tilde{\psi}}^{\psi}\frac{c'\left(u\right)}{w}du + \left(\psi xw - \psi^c xw\right) + \delta M^c \\
&> xw\int_{\tilde{\psi}}^{\psi}\left(-1\right)du + \left(\psi xw - \psi^c xw\right) + \delta M^c \\
&= xw\left(\tilde{\psi} - \psi\right) + \left(\psi xw - \psi^c xw\right) + \delta M^c \\
&= xw\tilde{\psi} - \psi^c xw + \delta M^c \\
&= xw\left(\psi^c + \frac{\pi\left(T_0^c + T_1^c\right)}{xw} - \epsilon\right) - \psi^c xw + \delta M^c \\
&= \left(\pi + \delta\right)M^c - \epsilon xw
\end{aligned}
$$

Therefore it must be that $\psi xw = \psi^c xw + \pi M^c$. Finally, there cannot be an equilibrium in which $\psi xw = \psi^c xw + \pi M^c$ and the buyer sues with positive probability because in that case the supplier would have been strictly better off by setting $\tilde{\psi} = \psi^c + \frac{\pi M^c}{xw} - \epsilon$. ■

Given those, we can find the payoff to the buyer and supplier of an arbitrary contract (M^c, ψ^c, x^c):

$$
\begin{aligned}
\text{buyer} &: -\psi xw - M = -\psi^c xw - \pi M^c - \left(1-\pi\right)M^c \\
\text{supplier} &: -c\left(\psi\right)x + M = -c\left(\psi^c + \frac{\pi M^c}{xw}\right)x + \left(1-\pi\right)M^c
\end{aligned}
$$

It will be convenient to define $p^c \equiv \frac{M^c}{x^c}$ and $p \equiv \frac{M}{p}$ to be the average price as specified in the contract and in equilibrium. With this, we can express the contract and payoffs in per unit terms:

$$
\begin{aligned}
\text{buyer} &: -\psi^c w - \pi p^c - (1-\pi)p^c \\
\text{supplier} &: -c\left(\psi^c + \frac{\pi p^c}{w}\right) + (1-\pi)p^c
\end{aligned}
$$

The buyer makes a take-it-or-leave-it offer of

$$
\max_{p^c, \psi^c} -p^c - \psi^c w
$$

subject to

$$
-c\left(\psi^c + \frac{\pi p^c}{w}\right) + (1-\pi)p^c \geq 0
$$

Lemma 2 *Any contract for which the constraint does not bind can be improved upon.*

Proof. Suppose that (p^c, ψ^c) is a contract for which the constraint is not binding. Then consider the alternative contract $\left(\tilde{p}^c, \tilde{\psi}^c\right)$ in which $\tilde{p}^c = p^c - \epsilon$ and $\tilde{\psi}^c = \psi^c + \frac{\pi}{w}\epsilon$. In this alternative contract $\tilde{\psi}^c + \frac{\pi\tilde{p}^c}{w} = \psi^c + \frac{\pi p^c}{w}$. If ϵ is small enough, the constraint will not be violated, and the buyers payoff is strictly rises:

$$
-\tilde{p}^c - \tilde{\psi}^c w = -\left(p^c - \epsilon\right) - \left(\psi^c + \frac{\pi}{w}\epsilon\right)xw = -p^c - \psi^c w + (1-\pi)\epsilon > -p^c - \psi^c w
$$

■

Claim 1 $\psi^c = 0$ *and* p^c *is the unique solution to* $(1-\pi)p^c = c\left(\frac{\pi p^c}{w}\right)$.

Proof. First, note that $c'\left(\psi\right) > -w$ implies that that if $\psi > \tilde{\psi}$, $-c\left(\tilde{\psi}\right) > -c(\psi) + w\left(\tilde{\psi} - \psi\right)$. Consider a contract (p^c, ψ^c) with $\psi^c > 0$. The buyer's problem can be expressed as

$$\max_{p^c,\psi^c} -p^c - \psi^c w \;\text{ subject to }\; (1-\pi)p^c - c\left(\psi^c + \frac{\pi p^c}{w}\right) \geq 0.$$

Consider the alternative contract in which $\tilde{\psi}^c = \psi^c - \epsilon$ and $\tilde{p}^c = p^c + \epsilon w$. That alternative contract would leave the buyer with the same payoff. We next show that the constraint would not bind, which will imply that the contract can be improved upon. The left hand side of the constraint is

$$\begin{aligned}
LHS &= (1-\pi)\tilde{p}^c - c\left(\tilde{\psi}^c + \frac{\pi\tilde{p}^c}{w}\right) \\
&> (1-\pi)\,\tilde{p}^c + \left\{-c\left(\psi^c + \frac{\pi p^c}{w}\right) + \left[\left(\tilde{\psi}^c + \frac{\pi\tilde{p}^c}{w}\right) - \left(\psi^c + \frac{\pi p^c}{w}\right)\right] w\right\} \\
&\geq (1-\pi)\tilde{p}^c + \left\{-(1-\pi)p^c + \left[\left(\tilde{\psi}^c + \frac{\pi\tilde{p}^c}{w}\right) - \left(\psi^c + \frac{\pi p^c}{w}\right)\right] w\right\} \\
&= \tilde{p}^c - p^c + \left(\tilde{\psi}^c - \psi^c\right) w \\
&= 0
\end{aligned}$$

where the weak inequality imposes the fact that the constraint holds for the original contract.

Imposing that $\phi^c = 0$, the buyer's problem can then be expressed as

$$\max_{p^c} -p^c \;\text{ subject to }\; (1-\pi)p^c - c\left(\frac{\pi p^c}{w}\right) \geq 0$$

where the constraint binds. This means that p^c solves $(1-\pi)p^c = c\left(\frac{\pi p^c}{w}\right)$. Since the left hand side is increasing in p^c while the right hand side is decreasing in p^c, and since the two curves must cross at least once, there is a unique solution. ■

We study a limiting economy in which $c_s(\psi) \to c_s$, i.e., the supplier can customize the good at essentially no cost. For example, we could have $c(\psi) = \bar{c}e^{-\frac{bw\psi}{\bar{c}}}$, with $b < 1$, and study the limit as $b \to 0$. In this limit,

$$\begin{aligned}
p^c &= \frac{\bar{c}}{1-\pi} \\
\psi^c &= 0 \\
p &= \bar{c} \\
\psi &= \frac{\pi p^c}{w} = \frac{\pi}{1-\pi}\frac{\bar{c}}{w}
\end{aligned}$$

In this economy, π is drawn randomly. We define $t_x \equiv \frac{1}{1-\pi}$.

E.3 Production

Given prices and the defectiveness of each input, the buyer minimizes cost. Let l^{prod} be the mass of labor hired for production and let $l^x_{\hat{\omega}}$ be the mass of labor hired to customize the defective inputs. The firm's cost minimization problem can be described as:

$$\min w\left(l^{prod} + \sum_{\hat{\omega}\in\hat{\Omega}^\rho_R} l^x_{\hat{\omega}}\right) + \sum_{\hat{\omega}\in\Omega^\rho_R} p_{\hat{\omega}s}\left(\psi_{\hat{\omega}s}\right) x_{\hat{\omega}} + \sum_{\hat{\omega}\in\Omega^\rho_R} p_{\hat{\omega}s} x_{\hat{\omega}}$$

subject to

$$G\left(b_l l^{prod}, \left\{b_{\hat{\omega}} z_{\hat{\omega}} \min\left\{x_{\hat{\omega}}, \frac{l^x_{\hat{\omega}}}{\psi_{\hat{\omega}s}}\right\}\right\}_{\hat{\omega}\in\hat{\Omega}^\rho_R}, \left\{b_{\hat{\omega}} z_{\hat{\omega}s} x_{\hat{\omega}}\right\}_{\hat{\omega}\in\hat{\Omega}^\rho_H}\right) \geq y$$

This minimization problem can be rewritten as

$$\min wl^{prod} + \sum_{\hat{\omega}\in\Omega_R^\rho} \left(w\psi_{\hat{\omega}s}^x + p_{\hat{\omega}s}(\psi_{\hat{\omega}s})\right) x_{\hat{\omega}} + \sum_{\hat{\omega}\in\Omega_H^\rho} p_{\hat{\omega}s} x_{\hat{\omega}}$$

subject to

$$G\left(b_l l^{prod}, \{b_{\hat{\omega}} z_{\hat{\omega}s} x_{\hat{\omega}}\}_{\hat{\omega}\in\Omega^\rho}\right) \geq y$$

Thus if $\mathcal{C}$ is the unit cost function associated with G, the firms unit cost can be expressed as

$$\mathcal{C}\left(\frac{w}{b_l}, \left\{\frac{p_{\hat{\omega}s} + w\psi_{\hat{\omega}s}}{b_{\hat{\omega}} z_{\hat{\omega}s}}\right\}_{\hat{\omega}\in\hat{\Omega}_R^\rho}, \left\{\frac{p_{\hat{\omega}s}}{b_{\hat{\omega}} z_{\hat{\omega}s}}\right\}_{\hat{\omega}\in\hat{\Omega}_H^\rho}\right)$$

or, given the equilibrium actions of the buyers and suppliers,

$$\mathcal{C}\left(\frac{w}{b_l}, \left\{\frac{t_{\hat{\omega}s} p_{\hat{\omega}s}}{b_{\hat{\omega}} z_{\hat{\omega}s}}\right\}_{\hat{\omega}\in\hat{\Omega}_R^\rho}, \left\{\frac{p_{\hat{\omega}s}}{b_{\hat{\omega}} z_{\hat{\omega}s}}\right\}_{\hat{\omega}\in\hat{\Omega}_H^\rho}\right).$$

F Proofs

F.1 Proof of Proposition 1

Let $F_\omega(c)$ be the fraction of firms in industry ω with cost weakly less than c (when the wage is normalized to unity). Similarly, let $F_{\omega\rho}(c)$ be the fraction of firms in ω that have a technique of recipe ρ that delivers unit cost weakly less than c. These satisfy $1 - F_\omega(c) = \prod_{\rho\in\varrho(\omega)} [1 - F_{\omega\rho}(c)]$.

It will also be convenient to define, for recipe ρ that uses inputs $\hat{\Omega}^\rho = (\hat{\omega}_1, ..., \hat{\omega}_n)$, the functions $\mathcal{B}_{\omega\rho}(b)$ where $b = (b_l, b_1, ..., b_n)$ and $\mathcal{V}_{\omega\rho}(v)$ where $v = (v_l, v_1, ..., v_n)$. These functions are defined by $\mathcal{B}_{\omega\rho}(b) \equiv B_{\omega\rho} b_l^{-\beta_l^\rho} \prod_k b_k^{-\beta_{\hat{\omega}_k}^\rho}$ so that $\mathcal{B}_{\omega\rho}(db) = B_{\omega\rho}\beta_l^\rho b_l^{-\beta_l^\rho - 1} db_l \beta_{\hat{\omega}_1}^\rho b_1^{-\beta_{\hat{\omega}_1}^\rho - 1} db_1 ... \beta_{\hat{\omega}_n}^\rho b_n^{-\beta_{\hat{\omega}_n}^\rho - 1} db_n$. Similarly, let $\mathcal{V}_{\omega\rho}(v) \equiv v_l^{\beta_l^\rho} \prod_k v_k^{\beta_{\hat{\omega}_k}^\rho}$ so that $\mathcal{V}_{\omega\rho}(dv) = \beta_l^\rho v_l^{\beta_l^\rho - 1} dv_l \beta_{\hat{\omega}_1}^\rho v_1^{\beta_{\hat{\omega}_1}^\rho - 1} dv_1 ... \beta_{\hat{\omega}_n}^\rho v_n^{\beta_{\hat{\omega}_n}^\rho - 1} dv_n$.

Lemma 3 *Under Assumption 2, for a firm of type ω,*

$$\Pr(\lambda_{\hat{\omega}}(\phi) > \lambda | b_{\hat{\omega}}(\phi)) = e^{-(\lambda b_{\hat{\omega}}(\phi)/\Lambda_{\hat{\omega}})^{\zeta_{\hat{\omega}}}}$$

where

$$\Lambda_{\hat{\omega}} = \begin{cases} t_x^* \left[\int_0^\infty c^{-\zeta_R} dF_{\hat{\omega}}(c)\right]^{-1/\zeta_R}, & \hat{\omega} \in \Omega_R^\rho \\ \left[\int_0^\infty c^{-\zeta_H} dF_{\hat{\omega}}(q)\right]^{-1/\zeta_H}, & \hat{\omega} \in \Omega_H^\rho \end{cases}$$

and

$$t_x^* \equiv \left(\int_1^\infty t_x^{-\zeta_R} dT(t_x)\right)^{-1/\zeta_R}$$

.

Proof. Consider first a relationship-specific input $\hat{\omega} \in \hat{\Omega}_R^\rho$. Consider a technique with a common component of input-augmenting productivity $b_{\hat{\omega}}(\phi)$. The number of suppliers with match-specific component of input-augmenting productivity greater than z is Poisson with mean $z^{-\zeta_R}$. For a potential supplier with z and input wedge t_x, the probability that the supplier's cost is low enough so that the supplier delivers an effective cost weakly less than λ is $\Pr\left(\frac{p_s t_x}{b_{\hat{\omega}}(\phi) z} \leq \lambda\right) = \Pr\left(p_s \leq \lambda z b_{\hat{\omega}}(\phi)/t_x\right) = F_{\hat{\omega}}\left(\lambda z b_{\hat{\omega}}(\phi)/t_x\right)$, where $F_{\hat{\omega}}(c)$ is the fraction of firms in $\hat{\omega}$ with unit cost (and hence price) weakly less than c. Integrating over realizations of z and t_x, we have that the number of potential suppliers that deliver effective cost weakly less than λ follows a Poisson distribution with mean

$$\int_0^\infty \int_1^\infty F_{\hat{\omega}}\left(\lambda z b_{\hat{\omega}}(\phi)/t_x\right) dT(t_x) \zeta_R z^{-\zeta_R - 1} dz$$

Using the change of variables $v = \lambda z b_{\hat{\omega}}(\phi)/t_x$ and the definition of $\Lambda_{\hat{\omega}}$, this is

$$[\lambda b_{\hat{\omega}}(\phi)]^{\zeta_R} \int_0^\infty \int_1^\infty F_{\hat{\omega}}(v) t_x^{-\zeta_R} dT(t_x) \zeta_R v^{-\zeta_R - 1} dv = [\lambda b_{\hat{\omega}}(\phi)/\Lambda_{\hat{\omega}}]^{\zeta_R}$$

The probability that no such suppliers arrive is then simply

$$\Pr(\lambda_{\hat{\omega}}(\phi) > \lambda | b_{\hat{\omega}}(\phi)) = e^{-(\lambda b_{\hat{\omega}}(\phi)/\Lambda_{\hat{\omega}})^{\zeta_R}}$$

The logic for homogeneous inputs is the same. ■

Lemma 4 *Under Assumption 1,*

$$\int_0^\infty ... \int_0^\infty 1 \left\{ \mathcal{C}_{\omega\rho}\left(v_l, v_1, ..., v_n\right) \leq 1 \right\} \mathcal{V}_{\omega\rho}(dv) < \infty$$

Proof. Assumption 1 implies that for each $k \in \{l, 1, ..., n\}$ there is a $\bar{v}_k$ such that $\mathcal{C}_{\omega\rho}\left(0, ..., 0, \bar{v}_k, 0, ..., 0\right) = 1$. In other words, $\bar{v}_k$ is defined so that if the effective cost of the kth input were equal to $\bar{v}_k$ and the cost of all other inputs were equal to zero then the firm's cost would be 1. Thus if the firm's cost of the kth input were higher than $\bar{v}_k$, the firm's cost must be greater than 1. We therefore have

$$\begin{aligned}
\int_0^\infty ... \int_0^\infty 1 \left\{ \mathcal{C}_{\omega\rho}\left(v_l, v_1, ..., v_n\right) \leq 1 \right\} \mathcal{V}_{\omega\rho}(dv) &= \int_0^{\bar{v}_l} \int_0^{\bar{v}_1} ... \int_0^{\bar{v}_n} 1 \left\{ \mathcal{C}_{\omega\rho}\left(v_l, v_1, ..., v_n\right) \leq 1 \right\} \mathcal{V}_{\omega\rho}(dv) \\
&\leq \int_0^{\bar{v}_l} \int_0^{\bar{v}_1} ... \int_0^{\bar{v}_n} \mathcal{V}_{\omega\rho}(dv) \\
&= \mathcal{V}_{\omega\rho}(\bar{v}) \\
&< \infty
\end{aligned}$$

where $\bar{v} = \{\bar{v}_l, \bar{v}_1, ..., \bar{v}_n\}$. ■

Proposition 6 *Under Assumptions 1 and 2, the fraction of firms in industry ω with cost greater than c is*

$$e^{-(c/C_\omega)^\gamma}$$

where

$$\begin{aligned}
C_\omega &= \left\{ \sum_{\rho \in \varrho(\omega)} \kappa_{\omega\rho} B_{\omega\rho} \left((t_x^*)^{\alpha_R^\rho} (t_l)^{\alpha_L^\rho} \prod_{\hat{\omega} \in \hat{\Omega}^\rho} C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho} \right)^{-\gamma} \right\}^{-\frac{1}{\gamma}} && (6) \\
t_x^* &= \left(\int_1^\infty t_x^{-\zeta^R} dT(t_x) \right)^{-1/\zeta_R} \\
\kappa_{\omega\rho} &= \int_0^\infty ... \int_0^\infty 1 \left\{ \mathcal{C}_{\omega\rho}\left(v_l, v_1, ..., v_n\right) \leq 1 \right\} \mathcal{V}_{\omega\rho}(dv) \prod_{\hat{\omega} \in \hat{\Omega}^\rho} \Gamma\left(1 - \frac{\beta_{\hat{\omega}}^\rho}{\zeta_{\hat{\omega}}}\right) \Gamma\left(1 - \frac{\zeta_{\hat{\omega}}}{\gamma}\right)^{\beta_{\hat{\omega}}^\rho/\zeta_{\hat{\omega}}}
\end{aligned}$$

Proof. Consider recipe ρ that uses labor and intermediate inputs $\hat{\Omega}^\rho = \{\hat{\omega}_1, ..., \hat{\omega}_n\}$. Let $H_{\omega\rho}(c)$ be the arrival rate of a technique that delivers cost weakly less than c. Then $1 - F_{\omega\rho}(c)$ is the probability that no such techniques arrive, or $e^{-H_{\omega\rho}(c)}$. To find $H_{\omega\rho}(c)$, we consider first a technique of recipe ρ for which the common components of input-augmenting productivities are $b_l, b_1, ..., b_n$. To find the probability that the technique delivers unit cost weakly less than c we integrate over the effective cost of all inputs:

$$\int_0^\infty ... \int_0^\infty 1 \left\{ \mathcal{C}_{\omega\rho}\left(\lambda_l, \lambda_{\hat{\omega}_1}, ..., \lambda_{\hat{\omega}_n}\right) \leq c \right\} \prod_{k=1}^n e^{-\left(\lambda_k b_k / \Lambda_{\hat{\omega}_k}\right)^{\zeta_{\hat{\omega}_k}}} \frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}} \zeta_{\hat{\omega}_k} \lambda_k^{\zeta_{\hat{\omega}_k} - 1} d\lambda_k$$

To find $H_{\omega\rho}(c)$, we integrate over the arrival of such techniques:

$$H_{\omega\rho}(c) = \int_0^\infty ... \int_0^\infty 1\left\{\mathcal{C}_{\omega\rho}\left(\lambda_l, \lambda_{\hat{\omega}_1}, ..., \lambda_{\hat{\omega}_n}\right) \leq c\right\} \left(\prod_{k=1}^n e^{-\left(\lambda_k b_k/\Lambda_{\omega\hat{\omega}_k}\right)^{\zeta_{\hat{\omega}_k}}} \frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\omega\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}} \zeta_{\hat{\omega}_k} \lambda_k^{\zeta_{\hat{\omega}_k}-1} d\lambda_k\right) \mathcal{B}_{\omega\rho}(db)$$

Using the definition of $\lambda_l = \frac{t_l}{b_l}$ and the homogeneity of the cost function, this is

$$H_{\omega\rho}(c) = \int_0^\infty ... \int_0^\infty 1\left\{\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l c}, \frac{\lambda_{\hat{\omega}_1}}{c}, ..., \frac{\lambda_{\hat{\omega}_n}}{c}\right) \leq 1\right\} \left(\prod_{k=1}^n e^{-\left(\lambda_k b_k/\Lambda_{\hat{\omega}_k}\right)^{\zeta_{\hat{\omega}_k}}} \frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}} \zeta_{\hat{\omega}_k} \lambda_k^{\zeta_{\hat{\omega}_k}-1} d\lambda_k\right) \mathcal{B}_{\omega\rho}(db)$$

It will be useful to make the changes of variables $v_k = \lambda_k/c$, $v_l = \frac{t_l}{cb_l}$, and $m_j = \left(\lambda_l b_l/\Lambda_{\hat{\omega}_l}\right)^{\zeta_{\hat{\omega}_j}}$ to express $H_{\omega\rho}$ as

$$H_{\omega\rho}(c) = \int_0^\infty ... \int_0^\infty 1\left\{\mathcal{C}_{\omega\rho}\left(v_l, v_1, ..., v_n\right) \leq 1\right\} B_{\omega\rho}\left(\frac{t_l}{c}\right)^{-\beta_l^\rho} \left(\prod_{k=1}^n \left(\frac{m_k^{1/\zeta_{\hat{\omega}_k}} \Lambda_{\hat{\omega}_k}}{c}\right)^{-\beta_{\hat{\omega}_k}^\rho} e^{-m_k} dm_k\right) \mathcal{V}_{\omega\rho}(dv)$$

or more simply,

$$H_{\omega\rho}(c) = \tilde{\kappa}_{\omega\rho} B_{\omega\rho} t_l^{-\beta_l^\rho} \Lambda_{\hat{\omega}_1}^{-\beta_{\hat{\omega}_1}^\rho} ... \Lambda_{\hat{\omega}_n}^{-\beta_{\hat{\omega}_n}^\rho} c^\gamma$$

where

$$\tilde{\kappa}_{\omega\rho} \equiv \int_0^\infty ... \int_0^\infty 1\left\{\mathcal{C}_{\omega\rho}\left(v_l, v_1, ..., v_n\right) \leq 1\right\} \mathcal{V}_{\omega\rho}(dv) \prod_{k=1}^n \int_0^\infty m_k^{-\beta_{\hat{\omega}_k}^\rho/\zeta_{\hat{\omega}_k}} e^{-m_k} dm_k$$

Assumptions 1 and 2e guarantee that $\tilde{\kappa}_{\omega\rho}$ is finite: the first integral is finite because of Lemma 4, and the subsequent integrals can be written as

$$\int_0^\infty m_{\hat{\omega}_k}^{-\beta_{\hat{\omega}_k}^\rho/\zeta_{\hat{\omega}_k}} e^{-m_k} dm_k = \Gamma\left(1 - \frac{\beta_{\hat{\omega}_k}^\rho}{\zeta_{\hat{\omega}_k}}\right)$$

Using the fact that $1 - F_\omega(c) = \prod_{\rho \in \varrho(\omega)} \left[1 - F_{\omega\rho}(c)\right] = \prod_{\rho \in \varrho(\omega)} e^{-H_{\omega\rho}(c)}$, we have

$$1 - F_\omega(c) = e^{-(c/C_\omega)^\gamma}$$

where C_ω is defined as

$$C_\omega \equiv \left[\sum_{\rho \in \varrho(\omega)} \tilde{\kappa}_{\omega\rho} B_{\omega\rho} t_l^{-\beta_l^\rho} \Lambda_{\hat{\omega}_1}^{-\beta_{\hat{\omega}_1}^\rho} ... \Lambda_{\hat{\omega}_n}^{-\beta_{\hat{\omega}_n}^\rho}\right]^{-1/\gamma} \tag{7}$$

To complete the proof, we will derive expressions for $\Lambda_{\hat{\omega}}$ in terms of $C_{\hat{\omega}}$ and substitute into (7). Note first that for each $\hat{\omega} \in \hat{\Omega}^\rho$ we have

$$\begin{aligned}\int_0^\infty c^{-\zeta_\omega} dF_\omega(c) &= \int_0^\infty c^{-\zeta_\omega} C_\omega^{-\gamma} \gamma c^{\gamma-1} e^{-(c/C_\omega)^\gamma} dc = C_\omega^{-\zeta_\omega} \int_0^\infty v^{-\frac{\zeta_\omega}{\gamma}} e^{-v} dv \\ &= C_\omega^{-\zeta_\omega} \Gamma\left(1 - \frac{\zeta_\omega}{\gamma}\right)\end{aligned}$$

therefore Lemma 3 implies

$$\Lambda_{\hat{\omega}} = \begin{cases} t_x^* C_{\hat{\omega}} \Gamma\left(1 - \frac{\zeta_R}{\gamma}\right)^{-1/\zeta_R} & \hat{\omega} \in \hat{\Omega}_R \\ C_{\hat{\omega}} \Gamma\left(1 - \frac{\zeta_H}{\gamma}\right)^{-1/\zeta_H} & \hat{\omega} \in \hat{\Omega}_H \end{cases}$$

Plugging this into (7), defining $\kappa_{\omega\rho} \equiv \tilde{\kappa}_{\omega\rho} \prod_{\hat{\omega}\in\hat{\Omega}^\rho} \Gamma\left(1-\frac{\zeta_{\hat{\omega}}}{\gamma}\right)^{\beta^\rho_{\hat{\omega}}/\zeta_{\hat{\omega}}}$, and using $\alpha^\rho_{\hat{\omega}} = \frac{\beta^\rho_{\hat{\omega}}}{\gamma}$ gives the result. ■

F.2 Factor Shares

Consider a firm in industry ω. If, in equilibrium, the firm uses a technique of recipe ρ (that uses labor and intermediate inputs $\hat{\Omega}^\rho = \{\hat{\omega}_1, ..., \hat{\omega}_n\}$) with input-augmenting productivities $b = \{b_l, b_1, ..., b_n\}$, effective cost of intermediate inputs $\lambda = \{\lambda_1, ..., \lambda_n\}$, then its payment to supplier of a relationship-specific input $\hat{\omega}_i$ with wedge $t_{\hat{\omega}x}$ is

$$
\begin{aligned}
p_s x_s &= t^{-1}_{\hat{\omega}_i x} \lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda) y_j, \quad \omega_i \in \Omega^\rho_R \\
p_s x_s &= \lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda) y_j, \quad \omega_i \in \Omega^\rho_H \\
wl &= \lambda_l \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda_l, \lambda_1, ..., \lambda_n) y_j + \sum_{\hat{\omega}\in\hat{\Omega}^\rho_R} (1 - t^{-1}_{\hat{\omega}_i x}) \lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda) y_j
\end{aligned}
$$

where $\mathcal{C}_{\omega\rho l}$ and $\mathcal{C}_{\omega\rho\hat{\omega}}$ denote the partial derivatives of the cost function with respect to the cost of labor and to the cost of input $\hat{\omega}$ respectively.

We characterize average revenue shares of each input in several in several intermediate steps.

Effective Cost Shares

For any technique of recipe ρ that delivers cost c, let $D_{\omega\rho}(c)$ denote the probability that the technique is actually chosen by the firm.[51]

Lemma 5 *Under Assumptions 1 and 2, the average effective cost share of the ith intermediate input and of labor among firms that, in equilibrium, use recipe ρ and have unit cost weakly less than c_0 are, respectively,*

$$E\left[\frac{\lambda_{\hat{\omega}_i}\mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c \le c_0, \rho\right] = \frac{\int_v \frac{v_{\hat{\omega}_i}\mathcal{C}_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\{\mathcal{C}_{\omega\rho}(v) \le c_0\} D_{\omega\rho}(\mathcal{C}_{\omega\rho}(v)) \mathcal{V}_{\omega\rho}(dv)}{\int_v 1\{\mathcal{C}_{\omega\rho}(v) \le c_0\} D_{\omega\rho}(\mathcal{C}_{\omega\rho}(v)) \mathcal{V}_{\omega\rho}(dv)} \tag{8}$$

$$E\left[\frac{\lambda_l C_{\omega\rho l}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c \le c_0, \rho\right] = \frac{\int_v \frac{v_l C_{\omega\rho l}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\{\mathcal{C}_{\omega\rho}(v) \le c_0\} D_{\omega\rho}(\mathcal{C}_{\omega\rho}(v)) \mathcal{V}_{\omega\rho}(dv)}{\int_v 1\{\mathcal{C}_{\omega\rho}(v) \le c_0\} D_{\omega\rho}(\mathcal{C}_{\omega\rho}(v)) \mathcal{V}_{\omega\rho}(dv)} \tag{9}$$

Proof. For a firm in industry ω, the measure of the arrival rate of techniques of recipe ρ with $b = \{b_l, b_1, ..., b_n\}$ and $\lambda = \{\lambda_1, ..., \lambda_n\}$ that deliver cost weakly less than c_0 is

$$1\{\mathcal{C}_{\omega\rho}(\lambda) \le c_0\} \left(\frac{b_1\lambda_1}{\Lambda_{\hat{\omega}_1}}\right)^{\zeta_{\hat{\omega}_1}} \frac{\zeta_{\hat{\omega}_1}}{\lambda_1} e^{-\left(\lambda_1 b_1/\Lambda_{\hat{\omega}_1}\right)^{\zeta_{\hat{\omega}_1}}} d\lambda_1 ... \left(\frac{b_n\lambda_n}{\Lambda_{\hat{\omega}_n}}\right)^{\zeta_{\hat{\omega}_n}} \frac{\zeta_{\hat{\omega}_n}}{\lambda_n} e^{-(\lambda_n b_n/\Lambda_{\hat{\omega}_n})^{\zeta_{\hat{\omega}_n}}} d\lambda_n \mathcal{B}_{\omega\rho}(db) \tag{10}$$

Such a technique is actually used by the firm with probability $D_{\omega\rho}(\mathcal{C}_{\omega\rho}(\lambda))$. To find the density of b and λ among firms in industry ω that choose to use a technique of recipe ρ that delivers cost weakly less than c_0 we simply divide the product of (10) and $D_{\omega\rho}(\mathcal{C}_{\omega\rho}(\lambda))$ by the integral over all such combinations of input-augmenting productivities and effective cost, so that the conditional expectation of $\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}$ is

$$E\left[\frac{\lambda_i\mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c \le c_0, \rho\right] = \frac{\begin{array}{c}\int_b\int_\lambda \frac{\lambda_i C_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)} 1\{\mathcal{C}_{\omega\rho}(\lambda) \le c_0\} D_{\omega\rho}(\mathcal{C}_{\omega\rho}(\lambda)) \\ \times \left(\frac{b_1\lambda_1}{\Lambda_{\hat{\omega}_1}}\right)^{\zeta_{\hat{\omega}_1}} \frac{\zeta_{\hat{\omega}_1}}{\lambda_1} e^{-\left(\lambda_1 b_1/\Lambda_{\hat{\omega}_1}\right)} d\lambda_1 ... \left(\frac{b_n\lambda_n}{\Lambda_{\hat{\omega}_n}}\right)^{\zeta_{\hat{\omega}_n}} \frac{\zeta_{\hat{\omega}_n}}{\lambda_n} e^{-(\lambda_n b_n/\Lambda_{\hat{\omega}_n})} d\lambda_n \mathcal{B}_{\omega\rho}(db)\end{array}}{\begin{array}{c}\int_b\int_\lambda 1\{\mathcal{C}_{\omega\rho}(\lambda) \le c_0\} D_{\omega\rho}(\mathcal{C}_{\omega\rho}(\lambda)) \\ \times \left(\frac{b_1\lambda_1}{\Lambda_{\hat{\omega}_1}}\right)^{\zeta_{\hat{\omega}_1}} \frac{\zeta_{\hat{\omega}_1}}{\lambda_1} e^{-\left(\lambda_1 b_1/\Lambda_{\hat{\omega}_1}\right)} d\lambda_1 ... \left(\frac{b_n\lambda_n}{\Lambda_{\hat{\omega}_n}}\right)^{\zeta_{\hat{\omega}_n}} \frac{\zeta_{\hat{\omega}_n}}{\lambda_n} e^{-(\lambda_n b_n/\Lambda_{\hat{\omega}_n})} d\lambda_n \mathcal{B}_{\omega\rho}(db)\end{array}}$$

[51] While it is not relevant for the proof, it turns out that $D_{\omega\rho}(c) = 1 - F_{\tilde{\omega}}(c)$, the overall probability that a firm's best technique delivers cost c.

Making the change of variables of variables $m_k = (\lambda_k b_k / \Lambda_{\hat{\omega}_k})^{\zeta_{\hat{\omega}_k}}$ for each k and using the definition $\lambda_l = \frac{t_l}{b_l}$ gives

$$E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \le c_0, \rho\right] = \frac{\begin{array}{c}\int_b\int_m \frac{\frac{\Lambda_{\hat{\omega}_i}}{b_{\hat{\omega}_i}} m_i^{1/\zeta_{\hat{\omega}_i}} C_{\omega\rho\hat{\omega}_i}\left(\frac{t_l}{b_l},\left\{\frac{\Lambda}{b}m^{1/\zeta}\right\}\right)}{\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l},\left\{\frac{\Lambda}{b}m^{1/\zeta}\right\}\right)} 1\left\{\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l},\left\{\frac{\Lambda}{b}m^{1/\zeta}\right\}\right) \le c_0\right\} \\ \times D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l},\left\{\frac{\Lambda}{b}m^{1/\zeta}\right\}\right)\right) e^{-m_1}dm_1 ... e^{-m_n}dm_n \mathcal{B}_{\omega\rho}(db)\end{array}}{\begin{array}{c}\int_b\int_m 1\left\{\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l},\left\{\frac{\Lambda}{b}m^{1/\zeta}\right\}\right) \le c_0\right\} \\ \times D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l},\left\{\frac{\Lambda}{b}m^{1/\zeta}\right\}\right)\right) e^{-m_1}dm_1 ... e^{-m_n}dm_n \mathcal{B}_{\omega\rho}(db)\end{array}}$$

where $\mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l},\left\{\frac{\Lambda}{b}m^{1/\zeta}\right\}\right) = \mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l}, \frac{\Lambda_{\hat{\omega}_i}}{b_i} m_i^{1/\zeta_{\hat{\omega}_i}}, ..., \frac{\Lambda_{\hat{\omega}_n}}{b_n} m_n^{1/\zeta_{\hat{\omega}_n}}\right)$. A further change of variables $v_l = \frac{t_l}{b_l}$ and $v_k = \frac{\Lambda_{\hat{\omega}_k}}{b_k} m_k^{1/\zeta_{\hat{\omega}_k}}$ gives

$$E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \le c_0, \rho\right] = \frac{\begin{array}{c}\int_v\int_m \frac{v_{\hat{\omega}_i} C_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{C_{\omega\rho}(v) \le c_0\right\} D_{\omega\rho}\left(C_{\omega\rho}(v)\right) e^{-m_1}dm_1 ... e^{-m_n}dm_n \\ \times B_{\omega\rho} t_l^{-\beta_l^\rho}\left(\Lambda_{\hat{\omega}_1} m_1^{1/\zeta_{\hat{\omega}_1}}\right)^{\beta^\rho_{\hat{\omega}_1}} ... \left(\Lambda_{\hat{\omega}_n} m_n^{1/\zeta_{\hat{\omega}_n}}\right)^{\beta^\rho_{\hat{\omega}_n}} \mathcal{V}_{\omega\rho}(dv)\end{array}}{\begin{array}{c}\int_v\int_m 1\left\{\mathcal{C}_{\omega\rho}(v) \le c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) e^{-m_1}dm_1 ... e^{-m_n}dm_n \\ \times B_{\omega\rho} t_l^{-\beta_l^\rho}\left(\Lambda_{\hat{\omega}_1} m_1^{1/\zeta_{\hat{\omega}_1}}\right)^{\beta^\rho_{\hat{\omega}_1}} ... \left(\Lambda_{\hat{\omega}_n} m_n^{1/\zeta_{\hat{\omega}_n}}\right)^{\beta^\rho_{\hat{\omega}_n}} \mathcal{V}_{\omega\rho}(dv)\end{array}}$$

Canceling common terms from the numerator and denominator gives (8). (9) can be derived using identical logic. ■

Lemma 6

$$\int_v \frac{v_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \le c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right)\mathcal{V}_{\omega\rho}(dv) = \beta^\rho_{\hat{\omega}_i} A_{\omega\rho}(c_0) \tag{11}$$

$$\int_v \frac{v_l C_{\omega\rho l}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \le c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right)\mathcal{V}_{\omega\rho}(dv) = \beta^\rho_L A_{\omega\rho}(c_0) \tag{12}$$

where $A_{\omega\rho}(c_0)$ is a constant

Proof. For the proof of this lemma we need some additional notation. Define the function $K_{\omega\rho}(c; c_0) = \int_c^{c_0} \frac{1}{\tilde{c}} D_{\omega\rho}(\tilde{c})\, d\tilde{c}$. Define the function $\psi(v_{-i}, c)$ to be the solution to $C_{\omega\rho}(v_{-i}, \psi(v_{-i}, c)) = c$ if such a solution exists, or take the value of zero no such solution exists, i.e., if the v_{-i} are too large for a solution to exist. Finally, define $\mathcal{V}_{\omega\rho,-i}(v_{-i}) \equiv v_l^{\beta_l^\rho} \prod_{k \ne i} v_k^{\beta^\rho_{\hat{\omega}_k}}$ and $\mathcal{V}_{\omega\rho,l}(v_{-l}) \equiv \prod_k v_k^{\beta^\rho_{\hat{\omega}_k}}$

We can express the left hand side of (11) as

$$\begin{aligned} LHS &= \int_v \frac{v_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \le c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right)\mathcal{V}_{\omega\rho}(dv) \\ &= \int_{v_{-i}}\int_{v_i} \frac{v_i C_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{C_{\omega\rho}(v) \le c_0\right\} D_{\omega\rho}\left(C_{\omega\rho}(v)\right) \beta^\rho_{\hat{\omega}_i} v_i^{\beta^\rho_{\hat{\omega}_i}-1} dv_i \mathcal{V}_{\omega\rho,-i}(dv_{-i}) \end{aligned} \tag{13}$$

We can use the definition of $\psi(v_{-i}, c)$, integrate by parts, and then use the definition of $\psi(v_{-i}, c)$ again to express the innermost integral as

$$
\begin{aligned}
& \int_{v_i} \frac{v_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \beta^{\rho}_{\hat{\omega}_i} v_i^{\beta^{\rho}_{\hat{\omega}_i}-1} dv_i \\
= & \int_0^{\psi(v_{-i},c_0)} \frac{v_i C_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \beta^{\rho}_{\hat{\omega}_i} v_i^{\beta^{\rho}_{\hat{\omega}_i}-1} dv_i \\
= & \int_0^{\psi(v_{-i},c_0)} \frac{C_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \beta^{\rho}_{\hat{\omega}_i} v_i^{\beta^{\rho}_{\hat{\omega}_i}} dv_i \\
= & -K_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v); c_0\right) \beta^{\rho}_{\hat{\omega}_i} v_i^{\beta^{\rho}_{\hat{\omega}_i}} \Big|_0^{\psi(v_{-i},c_0)} + \int_0^{\psi(v_{-i},c_0)} K_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v); c_0\right) \left(\beta^{\rho}_{\hat{\omega}_i}\right)^2 v_i^{\beta^{\rho}_{\hat{\omega}_i}-1} dv_i \\
= & \int_0^{\psi(v_{-i},c_0)} K_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v); c_0\right) \left(\beta^{\rho}_{\hat{\omega}_i}\right)^2 v_i^{\beta^{\rho}_{\hat{\omega}_i}-1} dv_i \\
= & \beta^{\rho}_{\hat{\omega}_i} \int_{v_i} 1\left\{C_{\omega\rho}(v) \leq c_0\right\} K_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v); c_0\right) \beta^{\rho}_{\hat{\omega}_i} v_i^{\beta^{\rho}_{\hat{\omega}_i}-1} dv_i
\end{aligned}
$$

Plugging this back into (13) gives

$$
LHS = \beta^{\rho}_{\hat{\omega}_i} \int_v 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} K_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v), c_0\right) \mathcal{V}_{\omega\rho}(dv)
$$

The derivation of (12) follows identical logic. ■

Proposition 7 *Under Assumptions 1 and 2, the average effective cost share of the ith intermediate input and of labor among firms that, in equilibrium, use recipe ρ and have unit cost weakly less than c_0 are, respectively,*

$$
\begin{aligned}
E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] &= \alpha^{\rho}_{\hat{\omega}_i} \\
E\left[\left.\frac{\lambda_l C_{\omega\rho l}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] &= \alpha^{\rho}_{L}
\end{aligned}
$$

Proof. Lemma 5 gives

$$
E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] = \frac{\int_v \frac{v_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)} 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \mathcal{V}_{\omega\rho}(dv)}{\int_v 1\left\{\mathcal{C}_{\omega\rho}(v) \leq c_0\right\} D_{\omega\rho}\left(\mathcal{C}_{\omega\rho}(v)\right) \mathcal{V}_{\omega\rho}(dv)}
$$

Lemma 6 implies that the numerator equals $\beta^{\rho}_{\hat{\omega}_i} A_{\omega\rho}(c_0)$. The homogeneity of the cost function implies that $1 = \frac{v_l C_{\omega\rho l}(v)}{C_{\omega\rho}(v)} + \sum_i \frac{v_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(v)}{\mathcal{C}_{\omega\rho}(v)}$, so the denominator equals $\beta^{\rho}_{l} A_{\omega\rho}(c_0) + \sum_i \beta^{\rho}_{\hat{\omega}_i} A_{\omega\rho}(c_0) = \gamma A_{\omega\rho}(c_0)$. Together these imply that $E\left[\left.\frac{\lambda_i \mathcal{C}_{\omega\rho\hat{\omega}_i}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] = \alpha^{\rho}_{\hat{\omega}_i}$. Identical logic implies that $E\left[\left.\frac{\lambda_l C_{\omega\rho l}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\right| c \leq c_0, \rho\right] = \alpha^{\rho}_{L}$. ■

Actual Cost Shares

Lemma 7 *Among firms that produce ω that, in equilibrium, use a supplier for relationship-specific input $\hat{\omega} \in \Omega^{\rho}_{R}$ that delivers effective cost $\lambda_{\hat{\omega}}$, the harmonic average of the wedge is*

$$
\bar{t}_x = E\left[t^{-1}_{x\hat{\omega}} | \lambda_{\hat{\omega}}\right]^{-1} = \left(\int_1^{\infty} t_x^{-1} d\tilde{T}(t_x)\right)^{-1}
$$

where $\tilde{T}(t_x) \equiv \frac{\int_1^{t_x} t^{-\zeta_R} dT(t)}{\int_1^{\infty} t^{-\zeta_R} dT(t)}$.

Proof. Consider all suppliers drawn by j to supply input $\hat{\omega}$ for recipe ρ. The effective cost delivered by the supplier is $\lambda = \frac{p_s}{t_x z}$ where $p_s = c_s$ is the price charged by the supplier. Given the match-specific productivity z and wedge t_x, the probability that the supplier's efficiency is high enough to deliver an effective cost lower λ is the probability that c_s is small enough to so that $\frac{t_x c_s}{z} < \lambda$, i.e., $c_s < \lambda z/t_x$, or $1 - F_{\hat{\omega}}(\lambda z/t_x)$. Integrating over possible values of z and t_x, the probability that a supplier delivers effective cost lower than λ is $\int_0^\infty \int_1^\infty [1 - F_{\hat{\omega}}(\lambda z/t)]\, dT(t)\zeta_R z^{-\zeta_R - 1} dz$. Second, the probability that a supplier delivers effective cost less than λ and the wedge is less than t_x is $\int_0^\infty \int_1^{t_x} [1 - F_{\hat{\omega}}(\lambda z/t)]\, dT(t)\zeta_R z^{-\zeta_R - 1} dz$. Together, these imply that, among suppliers who deliver effective cost less than λ, the probability that the wedge is less than t_x is

$$\begin{aligned}
\Pr(t_{xs} < t_x | \lambda_s \leq \lambda) &= \frac{\int_0^\infty \int_1^{t_x} [1 - F_{\hat{\omega}}(\lambda z/t)]\, dT(t) z^{-\zeta_R - 1} dz}{\int_0^\infty \int_1^\infty [1 - F_{\hat{\omega}}(\lambda z/t)]\, dT(t) z^{-\zeta_R - 1} dz} \\
&= \frac{\int_0^\infty \int_1^{t_x} [1 - F_{\hat{\omega}}(u)]\, \lambda^{\zeta_R} t^{-\zeta_R} dT(t) u^{-\zeta_R - 1} du}{\int_0^\infty \int_1^\infty [1 - F_{\hat{\omega}}(u)]\, \lambda^{\zeta_R} t^{-\zeta_R} dT(t) u^{-\zeta_R - 1} du} \\
&= \frac{\int_1^{t_x} t^{-\zeta_R} dT(t)}{\int_1^\infty t^{-\zeta_R} dT(t)} \\
&= \tilde{T}(t_x)
\end{aligned}$$

where the second line uses the change of variables $u = \lambda t z$. With this, we have that among suppliers that deliver effective cost weakly greater than λ, the harmonic average of wedges is

$$\bar{t}_x \equiv E\left[t_{xs}^{-1} | \lambda_s \leq \lambda\right]^{-1}$$

Since $\bar{t}_x$ does not depend on λ, the expectation must be the same for each λ, i.e.,

$$\bar{t}_x = E\left[t_{xs}^{-1} | \lambda_s\right]^{-1}$$

Finally, this equation holds regardless of whether s is selected as a supplier. In other words,

$$E\left[t_{x\hat{\omega}}^{-1} | \lambda_{\hat{\omega}}\right] = E\left[t_{xs}^{-1} | \lambda_s, s \text{ selected as supplier}\right] = E\left[t_{xs}^{-1} | \lambda_s\right] = \bar{t}_x^{-1}$$

■

Proposition 8 *Among firms in industry ω that, in equilibrium, use recipe ρ and have unit cost c:*

- *the average share of expenditures spent on input $\hat{\omega} \in \Omega_R^\rho$ is $\frac{1}{\bar{t}_x}\alpha_{\hat{\omega}}^\rho$*
- *the average share of expenditures spent on input $\hat{\omega} \in \Omega_H^\rho$ is $\alpha_{\hat{\omega}}^\rho$*
- *the average share of expenditures spent on input $\hat{\omega} \in \Omega_H^\rho$ is $\alpha_L^\rho + (1 - \frac{1}{\bar{t}_x})\alpha_R^\rho$*

Proof. Consider a relationship specific-input $\hat{\omega}$. Note first that the share of j's expenditures spent on $\hat{\omega}$ is $\frac{1}{t_{\hat{\omega}x}} \frac{\lambda_{\hat{\omega}} \mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}$. Note that conditional on c, $\bar{p}_j$ is independent of any feature of the firm's sourcing decision, and conditional on λ_i, $t_{\hat{\omega}x}$ is independent of any other feature of the firm's sourcing decision. Putting the pieces together, we have, by iterated expectations,

$$\begin{aligned}
E\left[\frac{1}{t_{\hat{\omega}x}}\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right] &= E\left[\frac{1}{t_{x\hat{\omega}}}\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right] \\
&= E\left\{E\left[\frac{1}{t_{x\hat{\omega}}}\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho,b,\lambda\right]\middle| c,\rho\right\} \\
&= E\left\{E\left[\frac{1}{t_{x\hat{\omega}}}\middle| c,\rho,b,\lambda\right]\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right\} \\
&= E\left[\frac{1}{t_{x\hat{\omega}}}\middle| c,\rho,b,\lambda\right]E\left\{\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right\} \\
&= \frac{1}{t_x}\alpha^{\rho}_{\hat{\omega}}
\end{aligned}$$

The expression for homogeneous inputs follows directly from Proposition 7. The expression for labor follows from the fact that the cost shares sum to 1. ■

Corollary 2 *Among firms in industry ω that, in equilibrium, use recipe ρ:*

- *the average and aggregate shares of expenditures spent on input $\hat{\omega} \in \Omega^{\rho}_{R}$ are both $\frac{1}{t_x}\alpha^{\rho}_{\hat{\omega}}$*
- *the average and aggregate shares of expenditures spent on input $\hat{\omega} \in \Omega^{\rho}_{H}$ are $\alpha^{\rho}_{\hat{\omega}}$*
- *the average and aggregate shares of expenditures spent on input $\hat{\omega} \in \Omega^{\rho}_{H}$ are $\alpha^{\rho}_{L} + (1-\frac{1}{t_x})\alpha^{\rho}_{R}$*

Proof. This follows directly from the previous corollary by integrating over realizations of c. ■

We next turn to revenue shares. Let $s_{\omega j\hat{\omega}}$ be the ratio of j's expenditure on input $\hat{\omega}$ to j's revenue. Let $\bar{p}_j$ be j's average price across all buyers; j charges a price of c_j to other firms and $\frac{\varepsilon}{\varepsilon-1}c_j$ to the household.

Proposition 9 *Among firms in industry ω that, in equilibrium, use recipe ρ and have unit cost c:*

- *the average share of revenue spent on input $\hat{\omega} \in \Omega^{\rho}_{R}$ is $\frac{1}{t_x}\alpha^{\rho}_{\hat{\omega}}E[\frac{c}{\bar{p}}|c,\rho]$*
- *the average share of expenditures spent on input $\hat{\omega} \in \Omega^{\rho}_{H}$ is $\alpha^{\rho}_{\hat{\omega}}E[\frac{c}{\bar{p}}|c,\rho]$*
- *the average share of expenditures spent on input $\hat{\omega} \in \Omega^{\rho}_{H}$ is $\left(\alpha^{\rho}_{L} + (1-\frac{1}{t_x})\alpha^{\rho}_{R}\right)E[\frac{c}{\bar{p}}|c,\rho]$*

Proof. This proof closely follows the proof of cost shares. Consider a relationship-specific input $\hat{\omega}$ Note first that j's revenue share of input $\hat{\omega}$ is $\frac{c}{\bar{p}_j}\frac{1}{t_{\hat{\omega}x}}\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}$. Note that conditional on c, $\bar{p}_j$ is independent of any feature of the firm's sourcing decision, and conditional on λ_i, $t_{\hat{\omega}x}$ is independent of any other feature of the firm's sourcing decision. Putting the pieces together, we have

$$\begin{aligned}
E\left[\frac{c}{\bar{p}_j}\frac{1}{t_{\hat{\omega}x}}\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right] &= E\left[\frac{c}{\bar{p}}\frac{1}{t_{x\hat{\omega}}}\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right] \\
&= E\left\{E\left[\frac{c}{\bar{p}}\frac{1}{t_{x\hat{\omega}}}\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho,b,\lambda\right]\middle| c,\rho\right\} \\
&= E\left\{E\left[\frac{c}{\bar{p}}\middle| c,\rho,b,\lambda\right]E\left[\frac{1}{t_{x\hat{\omega}}}\middle| c,\rho,b,\lambda\right]\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right\} \\
&= E\left[\frac{c}{\bar{p}}\middle| c,\rho,b,\lambda\right]E\left[\frac{1}{t_{x\hat{\omega}}}\middle| c,\rho,b,\lambda\right]E\left\{\frac{\lambda_{\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle| c,\rho\right\} \\
&= E\left[\frac{c}{\bar{p}}\middle| c,\rho\right]\frac{1}{t_x}\alpha^{\rho}_{\hat{\omega}}
\end{aligned}$$

The derivation for homogeneous inputs is the same but without the wedge (i.e., setting the wedge to 1). The derivation for labor is similar. ■

Corollary 3 *Among firms in industry ω that, in equilibrium use recipe ρ, the following expressions hold:*

$$
\begin{aligned}
0 &= E\left[\frac{s_{jR}}{\frac{\alpha_R^\rho}{t_x}} - \frac{s_{jH}}{\alpha_H^\rho}\middle|\rho\right] \\
0 &= E\left[\frac{s_{jR}+s_{jL}}{\alpha_R^\rho+\alpha_L^\rho} - \frac{s_{jH}}{\alpha_H^\rho}\middle|\rho\right]
\end{aligned}
$$

Proof. This follows from rearranging the expressions in the previous proposition and integrating over c. ■

F.3 Counterfactuals

Given the household's preferences, let P_ω be the ideal price index for the consumption aggregate for industry ω and let P be the overall ideal price index. These satisfy $P_\omega \equiv \left(\int_0^{J_\omega} p_j^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$ and $P = \left(\sum_\omega \upsilon_\omega P_\omega^{1-\eta}\right)^{\frac{1}{1-\eta}}$.

Since each firm charges a fixed markup over marginal cost in sales to the household, firm j in industry ω would charge a price of $p_j = \frac{\varepsilon}{\varepsilon-1}c_j$. Thus the price index for ω satisfies

$$
P_\omega^{1-\varepsilon} = \int_0^{J_\omega}\left(\frac{\varepsilon}{\varepsilon-1}c_j\right)^{1-\varepsilon} dj = \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} J_\omega \int_0^\infty c^{1-\varepsilon} dF_\omega(c)
$$

Proposition 1 gives $F_\omega(c) = 1 - e^{-(c/C_\omega)^\gamma}$. Integrating yields $\int_0^\infty c^{1-\varepsilon} dF_\omega(c) = \Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right) C_\omega^{1-\varepsilon}$, which implies that the industry price index can be expressed as

$$
P_\omega = \frac{\varepsilon}{\varepsilon-1} J_\omega^{\frac{1}{1-\varepsilon}} \Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{1}{1-\varepsilon}} C_\omega
$$

The overall price index is therefore

$$
P = \frac{\varepsilon}{\varepsilon-1}\left(\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}\right)^{\frac{1}{1-\eta}}
$$

To find total profit, note that j's profit (which comes only from sales to the household because firm-to-firm sales are priced at marginal cost) is $\pi_{\omega j} = u_{\omega j}(p_{\omega j} - c_{\omega j})$. Using $u_{\omega j} = U\upsilon_\omega\left(\frac{P_\omega}{P}\right)^{-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{-\varepsilon} = UP\upsilon_\omega\left(\frac{P_\omega}{P}\right)^{1-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}\frac{1}{p_j}$ and $c_{\omega j} = \frac{\varepsilon-1}{\varepsilon}p_{\omega j}$, this is $\pi_{\omega j} = UP\upsilon_\omega\left(\frac{P_\omega}{P}\right)^{1-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}\frac{1}{\varepsilon}$. Total profit can be found by integrating over all firms

$$
\Pi = \sum_\omega \int_0^{J_\omega} \pi_{\omega j} dj = \sum_\omega \int_0^{J_\omega} UP\left(\frac{\upsilon_\omega P_\omega}{P}\right)^{1-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}\frac{1}{\varepsilon} dj = \frac{1}{\varepsilon}UP
$$

Total household income is profit plus wage income, while its expenditure is UP, so its budget is $UP = \Pi + wL = \frac{1}{\varepsilon}UP + wL$, or (normalizing the wage to unity)

$$
U = \frac{\varepsilon}{\varepsilon-1}\frac{w}{P}L = \left(\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}\right)^{\frac{1}{\eta-1}} L
$$

Proposition 10 *A change in the distribution of relationship-specific intermediate input wedges from T to T' leads to a change in household utility that can be summarized by*

$$
\frac{U'}{U} = \left(\sum_\omega HH_\omega\left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right)^{\frac{1}{\eta-1}} \tag{14}
$$

and the change in industry cost indexes satisfy the following system of equations

$$\frac{C'_\omega}{C_\omega} = \left[\sum_{\rho\in\varrho(\omega)} R_{\omega\rho} \left(\left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho} \prod_{\hat{\omega}\in\hat{\Omega}_\rho} \left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho} \right)^{-\gamma} \right]^{-\frac{1}{\gamma}} \tag{15}$$

Proof. The share of the household's spending on goods from ω is

$$HH_\omega = \frac{\upsilon_\omega P_\omega^{1-\eta}}{P^{1-\eta}} = \frac{\upsilon_\omega \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\eta} J_\omega^{\frac{1-\eta}{1-\varepsilon}} \Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{1-\eta}{1-\varepsilon}} C_\omega^{1-\eta}}{P^{1-\eta}} \tag{16}$$

Using $U = \frac{\varepsilon}{\varepsilon-1} L/P$ and rearranging gives

$$HH_\omega = \frac{\upsilon_\omega J_\omega^{\frac{1-\eta}{1-\varepsilon}} \Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{1-\eta}{1-\varepsilon}} C_\omega^{1-\eta}}{(U/L)^{\eta-1}}$$

Under the counterfactual, we have

$$\begin{aligned}
(U'/L)^{\eta-1} &= \sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{\eta-1}{\varepsilon-1}} (C'_\omega)^{1-\eta} \\
&= \sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} \Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta} \\
&= \sum_\omega HH_\omega (U/L)^{\eta-1} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}
\end{aligned}$$

Rearranging gives (14).

We next show that the share of revenue is $R_{\omega\rho} = \kappa_{\omega\rho} B_{\omega\rho} \left(\frac{t_l^{\alpha_L^\rho} (t_x^*)^{\alpha_R^\rho} \prod_{\hat{\omega}\in\hat{\Omega}^\rho} C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}}{C_\omega}\right)^{-\gamma}$, where we define $C_{\omega\rho} \equiv \kappa_{\omega\rho} \left(t_l^{\alpha_L^\rho} (t_x^*)^{\alpha_R^\rho} \prod_{\hat{\omega}\in\hat{\Omega}^\rho} (Q_{\hat{\omega}})^{\alpha_{\hat{\omega}}^\rho}\right)^\gamma$, so that $F_{\omega\rho}(q) = e^{-(q/Q_{\omega\rho})^{-\gamma}}$. This follows from two facts. First, among producers of ω that have efficiency q, the fraction that use recipe ρ is:

$$\begin{aligned}
\Pr(\rho|c,\omega) &= \frac{\left(\prod_{\tilde{\rho}\neq\rho}[1-F_{\omega\tilde{\rho}}(c)]\right) F'_{\omega\rho}(c)}{F'_\omega(c)} = \frac{F'_{\omega\rho}(c)/[1-F_{\omega\rho}(c)]}{F'_\omega(c)/[1-F_\omega(c)]} \\
&= \kappa_{\omega\rho} B_{\omega\rho} \left(\frac{t_l^{\alpha_L^\rho} (t_x^*)^{\alpha_R^\rho} \prod_{\hat{\omega}\in\hat{\Omega}^\rho} C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}}{C_\omega}\right)^{-\gamma}
\end{aligned}$$

where the last equality follows from $F_{\omega\rho}(c) = 1 - e^{-\kappa_{\omega\rho} B_{\omega\rho} \left(t_l^{\alpha_L^\rho} (t_x^*)^{\alpha_R^\rho} \prod_{\hat{\omega}\in\hat{\Omega}^\rho} C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}\right)^{-\gamma} c^\gamma}$ and $F_\omega(c) = 1 - e^{-(c/C_\omega)^\gamma}$. The second fact is that, conditional on c, revenue is independent of the recipe chosen by a firm or any other feature of the firm's sourcing decisions.

Finally, we have that the counterfactual industry cost indexes $\{C'_\omega\}$ satisfy

$$\begin{aligned} C'_\omega &= \left\{ \sum_{\rho \in \varrho(\omega)} \kappa_{\omega\rho} B_{\omega\rho} \left(t_l^{\alpha_L^\rho} (t_x^{*\prime})^{\alpha_R^\rho} \prod_{\hat{\omega} \in \hat{\Omega}^\rho} (C'_{\hat{\omega}})^{\alpha_{\hat{\omega}}^\rho} \right)^{-\gamma} \right\}^{-\frac{1}{\gamma}} \\ &= \left\{ \sum_{\rho \in \varrho(\omega)} \kappa_{\omega\rho} B_{\omega\rho} \left(t_l^{\alpha_L^\rho} (t_x^{*})^{\alpha_R^\rho} \prod_{\hat{\omega} \in \hat{\Omega}^\rho} (C_{\hat{\omega}})^{\alpha_{\hat{\omega}}^\rho} \right)^{-\gamma} \left(\left(\frac{t_x^{*\prime}}{t_x^*} \right)^{\alpha_R^\rho} \prod_{\hat{\omega} \in \hat{\Omega}_\rho} \left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}} \right)^{\alpha_{\hat{\omega}}^\rho} \right)^{-\gamma} \right\}^{-\frac{1}{\gamma}} \\ &= \left\{ \sum_{\rho \in \varrho(\omega)} R_{\omega\rho} C_\omega^{-\gamma} \left(\left(\frac{t_x^{*\prime}}{t_x^*} \right)^{\alpha_R^\rho} \prod_{\hat{\omega} \in \hat{\Omega}_\rho} \left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}} \right)^{\alpha_{\hat{\omega}}^\rho} \right)^{-\gamma} \right\}^{-\frac{1}{\gamma}} \end{aligned}$$

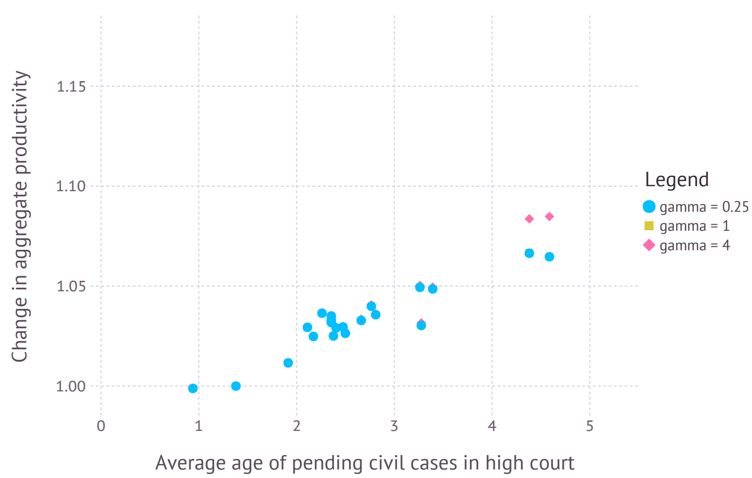
Rearranging yields (15). ■

G Additional Structural Results

G.1 Counterfactual: Robustness to different parameter values

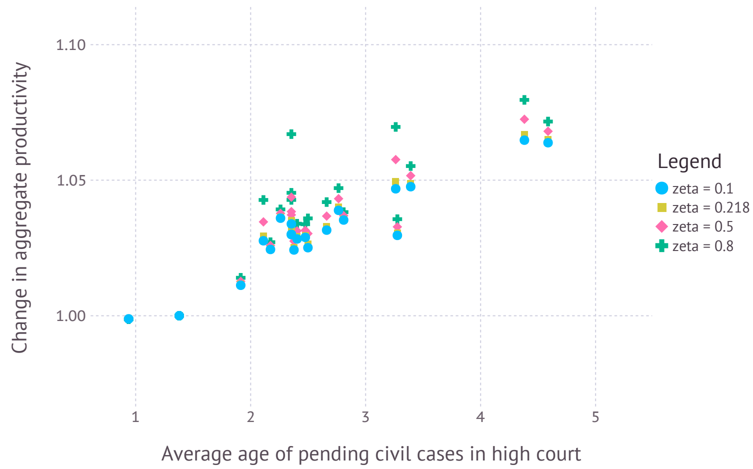
Figures 8 and 9 show the results from the welfare counterfactual for different values of γ and ζ.

Figure 8 Welfare counterfactual for different elasticities γ



The figure shows the counterfactual increase in U for each state for different values of γ. For $\Delta U < 5\%$ the differences are so small that the markers are overlapping.

Figure 9 Welfare counterfactual for different elasticities ζ



The figure shows the counterfactual increase in U for each state for different values of ζ.

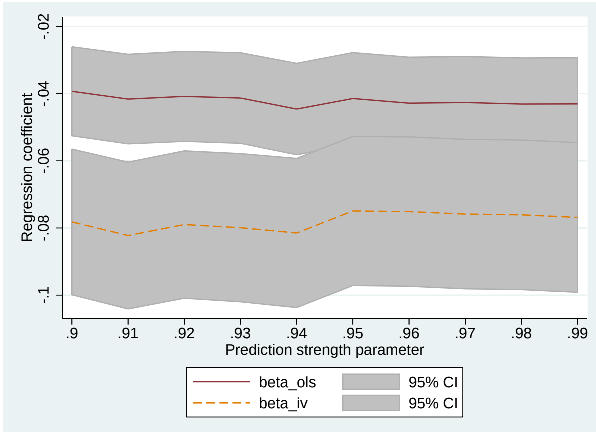
G.2 Fineness of recipes

This subsection explores the robustness of our estimates to the choice of how finely to define recipes. In our clustering procedure that defines recipes we use the prediction strength method of Tibshirani and Walther (2005) to find the number of clusters that we want to detect. Similar to cross-validation, the prediction strength method divides the sample into two subsamples (A and B) and assesses the predictive power of clusters obtained from each subsample. In our benchmark implementation that we use for the results in the paper, we choose a threshold parameter of 0.95. Here, we explore how much this choice matters. To do so, we run linear regressions that most closely mimic the structural regressions (cf. the GMM moment conditions in Proposition 5):

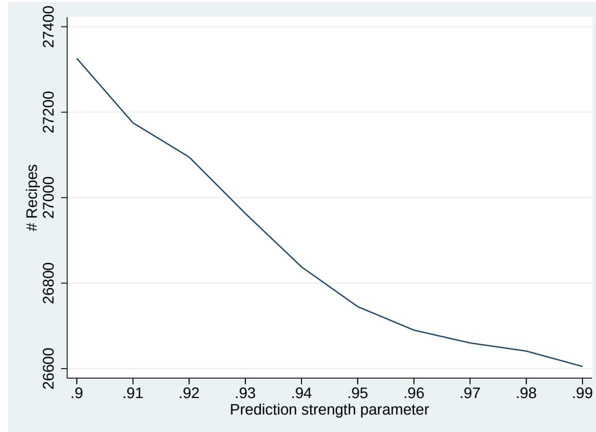
$$\log\left(\frac{\overline{s}_R^{\rho d}}{\overline{s}_H^{\rho d}}\right) = \beta \cdot (\text{Court quality})_d + \nu_\rho + \varepsilon_{\rho d}$$

where $\overline{s}_R^{\rho d}$ (and $\overline{s}_H^{\rho d}$) is the weighted average sales share of relationship-specific (homogeneous) inputs of plants that produce using recipe ρ in state d (weights are the probability weights as in the GMM procedure), and ν_ρ is a set of fixed effects. The estimate for β has a negative sign: Among plants that use a particular recipe, the sales shares of relationship-specific inputs is low compared to homogeneous inputs when courts are slow (i.e., when the average age of pending cases is high). The same holds when we instrument for court quality with the log age of court.

Figure G.2 shows that the point estimates for the regression coefficients do not change much with the threshold parameter. This is because the procedure identifies a similar number of recipes for a relatively broad range of threshold parameters.



(a) Regression Coefficients



(b) Number of Recipes

Figure 10 Regression coefficients & number of recipes for different levels of recipe fineness

G.3 Identifying Recipes and Distortions: Monte Carlo Exercises

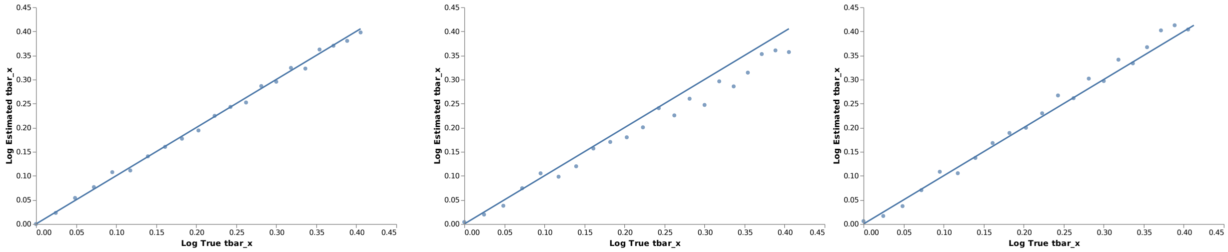
An important part of our exercise is to separate recipes from distortions. Fundamentally, identification is provided by the assumption that recipe production functions are invariant across states, whereas distortion parameters τ_d are state-specific. At the same time, our quantitative exercise also relies on a way to assign plants to recipes (which we do using the clustering algorithm), and on a way to determine the number of recipes in that procedure.

In this section, we explore the implications of the choice of the number of recipes, i.e. the number of clusters in the clustering procedure. We first discuss what could go wrong when choosing the "wrong" number of clusters, and illustrate these considerations with Monte Carlo simulations of small economies of our model. If the number of recipes we allow for is sufficiently large or the choice of recipes is uncorrelated with distortions, our estimator is likely to be consistent. We then perform a Monte Carlo study to assess the small-sample properties of our estimator (for the sample size we have).

G.3.1 In large samples

Our problem is closely related to the group fixed effect estimator of Bonhomme and Manresa (2015). Bonhomme and Manresa study the asymptotic properties (including consistency) of an estimator that estimates simultaneously group memberships, group fixed effects, and coefficients on observable characteristics in a linear panel model. In fact, our moment conditions can be mapped into a extension of their model (see equation 7 of their paper). Bonhomme and Manresa propose an iterative algorithm similar to ours, and show that if the number of groups is correct, the estimation of the covariates is consistent. In our context, this would mean that if we have the right number of recipes, our estimates of the distortions would be consistent. There are some small differences between our approach and that of Bonhomme and Manresa, so we use Monte Carlo simulations to confirm that this property does indeed seem to hold in our model (see discussion below).

To think about the properties of our estimator under the wrong number of clusters, we draw from another closely related paper, Moon and Weidner (2015), who study least squares estimators in panel models with a factor structure with an unknown number of factors, as well as the discussion in section S3 of the supplementary Appendix in Bonhomme and Manresa (2015), which also draws on Moon and Weidner (2015). In those models, allowing for too few groups/factors, estimates of the distortions may not be consistent, whereas if one allows for too many groups/factors, the estimates of the distortions will be consistent, although this comes at the cost of reduced power. Our model is very similar, but does not map exactly into that of Moon and Weidner, so we cannot apply their analytical results directly to our context. Nevertheless, Monte Carlo simulations of our model confirm that these properties give a good description of the performance of



(a) One cluster per product (b) Two clusters per product (c) Four clusters per product

Figure 11 MC results: Number of observations not skewed across states

The simulated economy has two products, one which is relationship-specific, the other one homogeneous. For each product there are two recipes; one intensive in the homogeneous good, the other one in the relationship-specific good. There are 21 states, each having the same number of producers using each recipe and producing each product, with distortions $\bar{t}$ as indicated on the horizontal axis. The average estimated $\bar{t}$ from large samples is shown on the vertical axis.

our estimator.

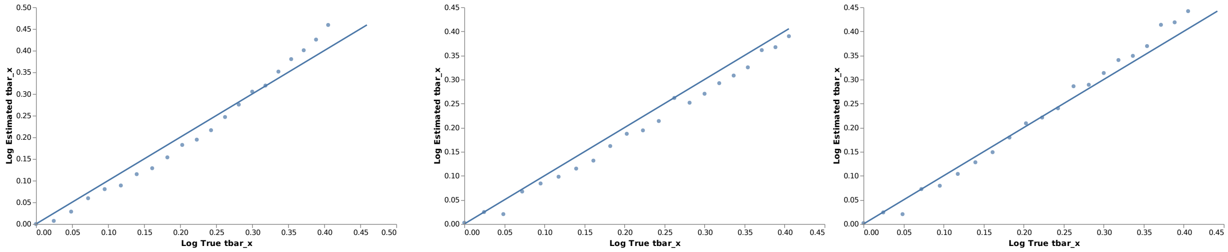
The bottom line is that if the number of recipes we allow for is sufficiently large (greater or equal than the actual number of recipes), our estimator is likely to be consistent. Allowing for too many recipes, however, comes at the cost of precision, as the $\bar{t}_x$ are estimated from within-recipe across-state variation in the shares of materials expenditure in sales, and with more recipes, there are on average fewer observations per recipe.

Here is the intuition: imagine a world with two recipes, one relying more on relationship-specific inputs ("R-inputs") than the other, and two states, one with no distortions and one for which R-inputs are distorted. Assume that we posited a single recipe, i.e., we assign all plants as belonging to the same recipe. If the number of producers using each recipe was the same across states, our estimator would still be consistent: the *average* plant in the distorted state would have a lower cost shares on R than the *average* plant in the undistorted state by exactly as much as the distortion (because the weights in the averages are the same).[52] On the other hand, if the more distorted state had relatively more firms using the low-R recipe (perhaps due to technology differences), then the assumption of a single recipe would lead us to overstate the distortions: some of the difference in cost shares on R is due to a different recipe mix, but the estimator would attribute it to larger distortions.

In contrast, if we allow for more recipes than present in the data, the "extra" identified recipe centers will be chosen in a way that is (asymptotically) orthogonal to the terms that identify the distortion parameters $\bar{t}_x$ (this is a formal result in Moon and Weidner (2015), which, based on our Monte Carlo's, seems to extend to our setup). We do, however, lose power, since we identify distortions from variation within recipes across states, and more recipes mean on average fewer observations in each recipe. In the extreme case in which there are more recipes than plants (or, a bit less extreme, if no recipe is used more than one state), distortions are obviously not identified anymore.

We illustrate this intuition using Monte Carlo experiments on large samples drawn from simulated small model economies. Each model economy consists of two products (one R, one H) and two recipes for each product (one intensive in the R good, the other one intensive in the H good). States vary in their distortions. In Figure 11 the number of plants is the same across states and recipes; in Figure 12 more strongly distorted states have fewer producers using the R-intensive recipe. We find that allowing for too few recipes (one, instead of two) leads to a bias in distortions when plants are skewed but not when they are the same across states. With the right number of recipes or with more, no such difference appears.

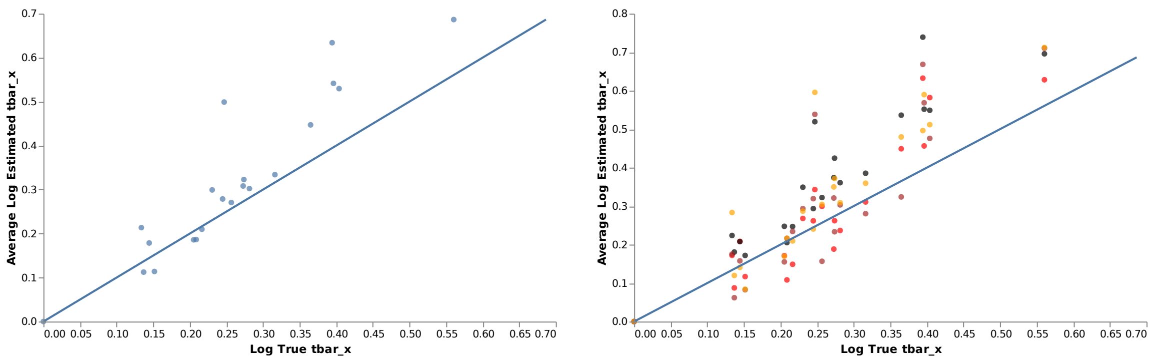
[52] Mathematically, let $s_{\rho_1}^{R,d_1} = \alpha_1/\bar{t}$, $s_{\rho_2}^{R,d_1} = \alpha_2/\bar{t}$, $s_{\rho_1}^{R,d_2} = \alpha_1$, $s_{\rho_2}^{R,d_2} = \alpha_2$, then $w_i^{d_1} = w_i^{d_2}$ implies $\sum_i w_i^{d_1} s_{\rho_i}^{R,d_1} / \sum_i w_i^{d_2} s_{\rho_i}^{R,d_2} = 1/\bar{t}$.



(a) One cluster per product (b) Two clusters per product (c) Four clusters per product

Figure 12 MC results: Number of observations skewed across states

Setup as above, but here states that are more distorted (higher $\bar{t}$) have relatively more producers using the H-intensive recipe for both products (the most distorted state has about 45% more producers using H-intensive recipes than the least distorted state) .



(a) Average of four MC simulations (b) All four MC simulation results

Figure 13 MC results using actual number of observations and estimated $\bar{t}$

The figure shows actual (horizontal axis) vs. estimated (vertical axis) distortions from a simulated model economy, where the parameters are the point estimates from our benchmark estimation and the number of simulated plants is the same as in our actual dataset. The left panel shows average estimated distortions across four runs, the right panel shows estimates from each individual run (coded in four different colors).

G.3.2 In small samples

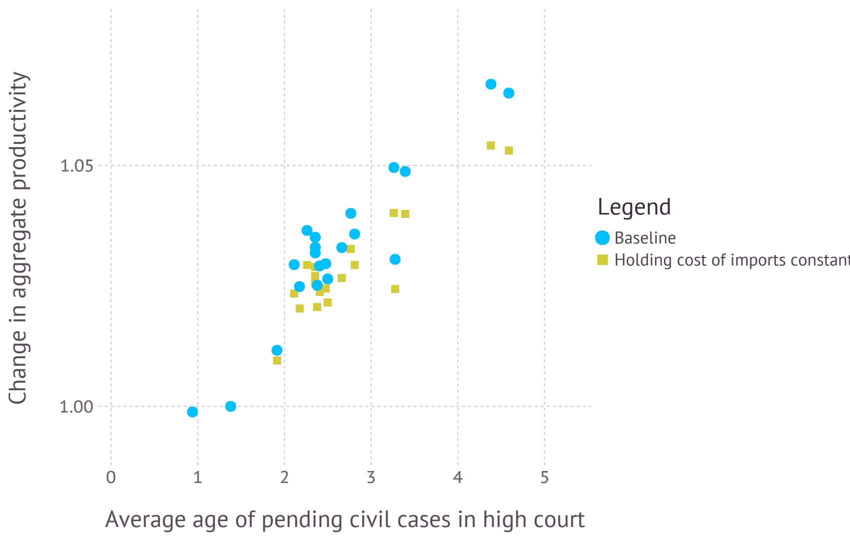
To assess the small-sample properties of our estimation procedure, we conduct a Monte Carlo simulation exercise where we use the actual number of observations in each state and recipe in our data. We simulate four datasets using the point estimates from our baseline results, and run the estimation and classification procedure on these simulated data. [53] Figure 13 shows the resulting estimates of $\bar{t}$. The left panel shows the average of the estimated $\bar{t}$ over the four runs. The small-sample bias of our estimation procedure seems to be relatively small, in particular for smaller distortions. Larger distortions may be upward biased to some extent. The right panel shows the estimates of each run coded in different colors. Estimates are relatively similar across the four runs, suggesting that the variance of our estimator is not very large.

G.4 Counterfactual with imports from other states

Our benchmark model economy is closed. In this subsection we explore departures from this assumption. Unfortunately, there is little publicly available data on trade across states, in particular for trade in interme-

[53] We do this only four times because each run takes about two days on our server.

Figure 14 Counterfactual with fixed cost distribution of imports



diate inputs. Van Leemput (2016) pieces together several datasets that cover interstate trade for a number of commodities and modes of transport (based on estimates from the Directorate General of Commercial Intelligence and Statistics). She estimates that on average (across states) imports of manufacturing goods are about 10% of domestic production (less for agriculture). Unfortunately, we do not know what fraction of that is trade in intermediate inputs.

In Figure 14, we report productivity gains from a counterfactual where we reduce court speed to the level enjoyed by the fastest court (as in the benchmark), but assume that $\frac{10\%}{10\%+100\%}$ of suppliers of each intermediate input is sourced from outside of the state. When reducing contracting frictions, we hold constant the cost distribution of those imports. As we would expect, the gains from reducing contracting frictions are a bit smaller than in our baseline counterfactual.

In our view, reality likely lies between the baseline counterfactual and this alternative. Producers from neighboring states may, in fact, experience cost reductions if contracting frictions are reduced for any producer in their supply chains. Hence, we believe this counterfactual provides a lower bound for the gains from a unilateral reduction in distortions in one state.

G.5 Entry

Suppose there is a representative entrepreneur that can choose the measure of firms in each industry according to a constant elasticity of transformation technology. The mass of firms in each industry $\{J_\omega\}_{\omega\in\Omega}$ must satisfy the constraint $\left(\sum_\omega (J_\omega/h_\omega)^{\frac{1+\beta}{\beta}}\right)^{\frac{\beta}{1+\beta}} \leq 1$, where h_ω indexes the ease of setting up firms in industry ω and β is an elasticity capturing diminishing returns to entering in any particular industry. This specification nests exogenous entry at the extreme of $\beta = 0$ and free entry at $\beta = \infty$. After entry, firms in each industry are ex-ante identical. Following entry, all firms draw techniques and then production occurs.

In equilibrium, let $\bar{\pi}_\omega$ be the average profit of firms in industry ω. The representative entrepreneur takes $\{\bar{\pi}_\omega\}_{\omega\in\Omega}$ as given when making entry decisions, and therefore maximizes expected profit

$$\max \sum_\omega J_\omega \bar{\pi}_\omega \text{ subject to } \left(\sum_\omega (J_\omega/h_\omega)^{\frac{1+\beta}{\beta}}\right)^{\frac{\beta}{1+\beta}} \leq 1$$

The entry choice is

$$J_\omega = h_\omega \left(\frac{(h_\omega \bar{\pi}_\omega)^{1+\beta}}{\sum_{\omega'} (h_{\omega'} \bar{\pi}_{\omega'})^{1+\beta}} \right)^{\frac{\beta}{1+\beta}}$$

We next find an expression for average profit in industry ω. $\bar{\pi}_\omega$. We have assumed that prices in buyer-supplier relationships are set at the supplier's marginal cost. This means that firms make profit only from sales to the final consumer. Since profit for a firm j in industry ω is price is $p_{\omega j} = \frac{\varepsilon}{\varepsilon-1} c_{\omega j}$, and final demand is $u_{\omega j} = U_\omega \left(\frac{p_{\omega j}}{P_\omega}\right)^{-\varepsilon} = v_\omega U \left(\frac{P_\omega}{P}\right)^{-\eta} \left(\frac{p_{\omega j}}{P_\omega}\right)^{-\varepsilon}$, its profit is $\pi_{\omega j} = (p_{\omega j} - c_{\omega j}) u_{\omega j} = \frac{1}{\varepsilon} p_{\omega j} u_{\omega j} = \frac{1}{\varepsilon} v_\omega PU \left(\frac{P_\omega}{P}\right)^{1-\eta} \left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}$. Total profit among firms in industry ω is then

$$\bar{\pi}_\omega J_\omega = \int \frac{1}{\varepsilon} v_\omega PU \left(\frac{P_\omega}{P}\right)^{1-\eta} \left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon} dj = \frac{1}{\varepsilon} v_\omega PU \left(\frac{P_\omega}{P}\right)^{1-\eta}$$

In equilibrium, the fraction of firms in industry ω with cost greater than c is $e^{-(c/C_\omega)^{-\theta}}$. Integrating over possible cost realizations gives

$$\begin{aligned} P_\omega^{1-\varepsilon} &= \int \left(\frac{\varepsilon}{\varepsilon-1} c_{\omega j}\right)^{1-\varepsilon} dj = J_\omega \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} \int c^{1-\varepsilon} \theta c^{\theta-1} C_\omega^{-\theta} e^{-(c/C_\omega)^\theta} dc \\ &= \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} \Gamma\left(1 - \frac{\varepsilon-1}{\theta}\right) J_\omega C_\omega^{1-\varepsilon} \end{aligned}$$

Putting these together, average profit is

$$\bar{\pi}_\omega = \frac{1}{\varepsilon} \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\eta} \Gamma\left(1 - \frac{\varepsilon-1}{\theta}\right)^{\frac{1-\eta}{1-\varepsilon}} P^\eta U v_\omega J_\omega^{\frac{1-\eta}{1-\varepsilon}-1} C_\omega^{1-\eta}$$

Claim 2 *The mass of firms in industry ω satisfies*

$$\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left(h_\omega^{\frac{1-\eta}{1-\varepsilon}} v_\omega C_\omega^{1-\eta}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} \left(h_{\omega'}^{\frac{1-\eta}{1-\varepsilon}} v_{\omega'} C_{\omega'}^{1-\eta}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

Proof. We first rearrange the expression for the mass of firms and then use the expression for average profit. The choice of entry in industry ω is $\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{(h_\omega \bar{\pi}_\omega)^{1+\beta}}{\sum_{\omega'} (h_{\omega'} \bar{\pi}_{\omega'})^{1+\beta}}$, which can be rearranged as

$$\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left(A_\omega (J_\omega/h_\omega)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)^{1+\beta}}{\sum_{\omega'} \left(A_{\omega'} (J_{\omega'}/h_{\omega'})^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)^{1+\beta}}$$

where $A_\omega \equiv h_\omega^{\frac{1-\eta}{1-\varepsilon}} \left(\bar{\pi}_\omega / J_\omega^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)$. We want to solve for the denominator. To do this, we can rearrange this further as

$$1 = \frac{A_\omega^{1+\beta} (J_\omega/h_\omega)^{\frac{\varepsilon-\eta}{1-\varepsilon}(1+\beta) - \frac{1+\beta}{\beta}}}{\sum_{\omega'} \left(A_{\omega'} (J_{\omega'}/h_{\omega'})^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)^{1+\beta}}$$

or

$$A_\omega^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}} = \frac{\left[A_\omega \left(J_\omega/h_\omega\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right]^{1+\beta}}{\left[\sum_{\omega'} \left(A_{\omega'} \left(J_{\omega'}/h_{\omega'}\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)^{1+\beta}\right]^{\frac{\frac{\varepsilon-\eta}{1-\varepsilon}(1+\beta)}{\frac{\varepsilon-\eta}{1-\varepsilon}(1+\beta)-\frac{1+\beta}{\beta}}}}$$

Summing across ω and then simplifying the right hand side gives

$$\sum_\omega A_\omega^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}} = \left[\sum_\omega A_\omega^{1+\beta} \left(J_\omega/h_\omega\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}(1+\beta)}\right]^{\frac{1}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}$$

or more simply

$$\sum_\omega \left(A_\omega \left(J_\omega/h_\omega\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)^{1+\beta} = \left(\sum_\omega A_\omega^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}\right)^{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}$$

We therefore have $\sum_{\omega'} \left(h_{\omega'}\bar{\pi}_{\omega'}\right)^{1+\beta} = \left(\sum_\omega A_\omega^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}\right)^{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}$ and $\left(h_\omega\bar{\pi}_\omega\right)^{1+\beta} = \left[A_\omega \left(J_\omega/h_\omega\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right]^{1+\beta}$. so that

$$\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left[A_\omega \left(J_\omega/h_\omega\right)^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right]^{1+\beta}}{\left(\sum_{\omega'} A_{\omega'}^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}\right)^{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}$$

So that solving for J_ω/h_ω gives

$$\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{A_\omega^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}}{\sum_{\omega'} A_{\omega'}^{\frac{1+\beta}{1-\frac{\varepsilon-\eta}{1-\varepsilon}\beta}}} = \frac{A_\omega^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} A_{\omega'}^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

Finally, The expression for profits gives $A_\omega \equiv h_\omega^{\frac{1-\eta}{1-\varepsilon}} \left(\bar{\pi}_\omega/J_\omega^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right) \propto h_\omega^{\frac{1-\eta}{1-\varepsilon}} v_\omega C_\omega^{1-\eta}$, yields the result. ■

We next show how we can solve for counterfactuals

Claim 3 *When costs change, the change in the mass of firms in industry ω is*

$$\left(\frac{J'_\omega}{J_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left(C'_\omega/C_\omega\right)^{\frac{1-\eta}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_\omega HH_\omega \left(C'_\omega/C_\omega\right)^{\frac{1-\eta}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

Proof. Again, using $A_\omega \equiv h_\omega^{\frac{1-\eta}{1-\varepsilon}} \left(\bar{\pi}_\omega/J_\omega^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right) \propto h_\omega^{\frac{1-\eta}{1-\varepsilon}} v_\omega C_\omega^{1-\eta}$, we have $\frac{A'_\omega}{A_\omega} = \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}$. We also have

$$\left(\frac{J'_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left(A'_\omega\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} \left(A'_{\omega'}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}} = \frac{A_\omega^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}} \left(\frac{A'_\omega}{A_\omega}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} A_{\omega'}^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}} \left(\frac{A'_{\omega'}}{A_{\omega'}}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

So dividing by $\left(\frac{J_\omega}{h_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{A_\omega^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} A_{\omega'}^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$ gives

$$\left(\frac{J'_\omega}{J_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{\left(\frac{A'_\omega}{A_\omega}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} \frac{A_{\omega'}^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega''} A_{\omega''}^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}} \left(\frac{A'_{\omega'}}{A_{\omega'}}\right)^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

Finally, using $J_\omega = h_\omega \left(\frac{(h_\omega \bar{\pi}_\omega)^{1+\beta}}{\sum_{\omega'} (h_{\omega'}\bar{\pi}_{\omega'})^{1+\beta}}\right)^{\frac{\beta}{1+\beta}}$ implies $h_\omega = \left(\sum_{\omega'} (h_{\omega'}\bar{\pi}_{\omega'})^{1+\beta}\right)^{\frac{\beta}{(1+\beta)^2}} J_\omega^{\frac{1}{\beta+1}} \bar{\pi}_\omega^{-\frac{\beta}{\beta+1}}$, we have

$$\begin{aligned}
A_\omega^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}} &= \left[h_\omega^{\frac{1-\eta}{1-\varepsilon}} \left(\bar{\pi}_\omega / J_\omega^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)\right]^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}} = \left[\left(\sum_{\omega'} (h_{\omega'}\bar{\pi}_{\omega'})^{1+\beta}\right)^{\frac{\beta}{(1+\beta)^2}\frac{1-\eta}{1-\varepsilon}} J_\omega^{\frac{1}{\beta+1}\frac{1-\eta}{1-\varepsilon}} \bar{\pi}_\omega^{-\frac{\beta}{\beta+1}\frac{1-\eta}{1-\varepsilon}} \left(\bar{\pi}_\omega / J_\omega^{\frac{\varepsilon-\eta}{1-\varepsilon}}\right)\right]^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}} \\
&= \left(\sum_{\omega'} (h_{\omega'}\bar{\pi}_{\omega'})^{1+\beta}\right)^{\frac{1}{1+\beta}\frac{\frac{\beta}{1+\beta}\frac{1-\eta}{1-\varepsilon}}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}} J_\omega \bar{\pi}_\omega
\end{aligned}$$

Thus we have

$$\left(\frac{J'_\omega}{J_\omega}\right)^{\frac{1+\beta}{\beta}} = \frac{(C'_\omega / C_\omega)^{\frac{1-\eta}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}{\sum_{\omega'} \frac{J_{\omega'}\bar{\pi}_{\omega'}}{\sum_{\omega''} J_{\omega''}\bar{\pi}_{\omega''}} (C'_{\omega'}/C_{\omega'})^{\frac{1}{1-\frac{1-\eta}{1-\varepsilon}\frac{\beta}{1+\beta}}}}$$

Finally, since markups are the same across sectors, we have that $\frac{J_{\omega'}\bar{\pi}_{\omega'}}{\sum_{\omega''} J_{\omega''}\bar{\pi}_{\omega''}} = HH_\omega$, giving the result. ■

Finally, we show how to use this to compute the aggregate counterfactual

Claim 4 *The change in aggregate productivity of the manufacturing sector is*

$$\frac{U'}{U} = \left\{\sum_{\omega\in\Omega} HH_\omega \left(\frac{J'_\omega}{J_\omega}\right)^{\frac{\eta-1}{\varepsilon-1}} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}}$$

Proof. Utility can be expressed as

$$\begin{aligned}
U &= \left\{\sum_{\omega\in\Omega} \Gamma\left(1 - \frac{\varepsilon_\omega - 1}{\gamma_\omega}\right) \nu_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}\right\}^{\frac{1}{\eta-1}} \\
U' &= \left\{\sum_{\omega\in\Omega} \Gamma\left(1 - \frac{\varepsilon_\omega - 1}{\gamma_\omega}\right) \nu_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta} \left(\frac{J'_\omega}{J_\omega}\right)^{\frac{\eta-1}{\varepsilon-1}} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}} \\
\frac{U'}{U} &= \left\{\sum_{\omega\in\Omega} \frac{\Gamma\left(1 - \frac{\varepsilon_\omega - 1}{\gamma_\omega}\right) \nu_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}}{\sum_{\omega\in\Omega} \Gamma\left(1 - \frac{\varepsilon_\omega - 1}{\gamma_\omega}\right) \nu_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}} \left(\frac{J'_\omega}{J_\omega}\right)^{\frac{\eta-1}{\varepsilon-1}} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}} \\
&= \left\{\sum_{\omega\in\Omega} HH_\omega \left(\frac{J'_\omega}{J_\omega}\right)^{\frac{\eta-1}{\varepsilon-1}} \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}}
\end{aligned}$$

■

This discussion leaves out a few channels. First: we allow for changes in the rates of entry across industries, but we do not allow for changes in the total entry rate to increase. While it would be easy to relax this, we think it is reasonable starting point because if entry costs are denominated in labor (as suggested by Bollard, Klenow and Li (2016)) then the total rate of entry would be invariant to changes in aggregate productivity, which raise both the payoff to and opportunity cost of starting a firm by the same proportion.

We also have ignored any feedback in the change in the number of firms on the number of potential suppliers drawn to provide a good for a technique. It is possible that the mass of such suppliers would increase with the measure of firms in the industry.

H Alternative Distortions

In the model presented in the main text, if firm uses a supplier that shirks on quality of the inputs because of contracting friction, the buyer uses labor to customize the good itself. This increased the total amount of labor used at all stages of production to produce the firm's good (by the firm, its suppliers, its suppliers' suppliers, etc). In this section, we take inspiration form Hsieh and Klenow (2009) and assume that when a firm draws a supplier of a relationship-specific input, she also draws a random wedge t_x from a distribution $T(t_x)$ that is independent of the supplier's cost. The buyer then behaves as if it must pay a tax at rate $t_x - 1$ on expenditures on that input. Like in the baseline, a firm's shadow cost of each input might differ from what it pays to the supplier. Here, however, the firm's shadow unit cost is larger than its actual expenditure on inputs.

Following the notation of the baseline model presented in Section 3, the effective shadow cost of using a supplier with input-augmenting productivity $z_s b_{\hat{\omega}}(\phi)$ that charges price p_s in the presences of the wedge t_{xs} would be $\frac{t_{xs} p_s}{b_{\hat{\omega}}(\phi) z_s}$. j's effective shadow cost of input $\hat{\omega}$ for technique ϕ is the minimum across all potential suppliers:

$$\lambda_{\hat{\omega}}(\phi) = \min_{s \in S_{\hat{\omega}}(\phi)} \frac{t_{xs}(\phi) p_s}{b_{\hat{\omega}}(\phi) z_s(\phi)}.$$

Similarly, the effective shadow cost of labor when using technique ϕ is $\lambda_l(\phi) = \frac{t_l w}{b_l(\phi)}$. For the remainder, we normalize the wage to unity, $w = 1$.

The shadow unit cost delivered by a technique depends on the effective shadow cost of each input. j's shadow unit cost of producing one unit of output using technique ϕ would be $\mathcal{C}_{\omega\rho}\left(\lambda_l(\phi), \{\lambda_{\hat{\omega}}(\phi)\}_{\hat{\omega} \in \hat{\Omega}^\rho}\right)$. Minimizing cost across all techniques, j's shadow unit cost is

$$\min_{\rho \in \varrho(\omega)} \min_{\phi \in \Phi_{\omega j \rho}} \mathcal{C}_{\omega\rho}\left(\lambda_l(\phi), \{\lambda_{\hat{\omega}}(\phi)\}_{\hat{\omega} \in \hat{\Omega}^\rho}\right)$$

One implication is that j's total shadow cost can be expressed as

$$c_j y_j = \frac{1}{t_l} w l_j + \sum_{\hat{\omega} \in \Omega_\rho} \frac{1}{t_{s_{\hat{\omega}}(\phi)}} p_{s_{\hat{\omega}}(\phi)} x_{s_{\hat{\omega}}(\phi)}$$

We assume that prices in firm-to-firm trade are set at the supplier's shadow unit cost, and that sales to final consumers are determined via monopolistic competition, so that prices are a fixed markup $\frac{\varepsilon}{\varepsilon - 1}$ over shadow unit cost.

While we have no microfoundation for why the wedges would take this particular form, it still may be interesting to explore how our results would differ under this alternative formulation.

A key difference from the baseline model is that the wedges themselves represent behavioral distortion but do not use up resources. To solve for the actual allocation, we need to solve for the resources that get used in producing each good. Towards this, define the "resource gap" for firm j, $a_j \in [0,1]$, to be the ratio of the labor used across all stages of production (by firm j, its suppliers, its suppliers' suppliers, etc.) to produce a unit of good j and the shadow unit cost of firm j. In other words, if c_j is j's shadow unit cost, $a_j c_j$ is the cumulative expenditure on primary inputs (i.e., labor) to produce a unit of good j. Suppose that j uses technique ϕ, and it produces y_j units of output using labor l_j and $x_{s_{\hat{\omega}}(\phi)}$ units of intermediate inputs used from respective suppliers $s_{\hat{\omega}}(\phi)$. If firm faces wedges t_l and $\left\{t_{s_{\hat{\omega}}(\phi)}\right\}$, then the resources gap satisfies the equation

$$a_j c_j y_j = w l_j + \sum_{\hat{\omega}\in\Omega_\rho} a_{s_{\hat{\omega}}(\phi)} c_{s_{\hat{\omega}}(\phi)} x_{s_{\hat{\omega}}(\phi)} \tag{17}$$

That is, cumulative expenditure on labor to make good j is equal to the sum of the labor directly used plus the cumulative labor used to make each input.

We briefly summarize the key equations that can be used to estimate the distortions and recipe technologies in this alternative environment, as well as to compute the counterfactual change in aggregate productivity that would result from changes in distortions in this alternative environment. Proofs are relegated to Appendix H.2 below.

Among firms in industry ω that, in equilibrium, use recipe ρ, the following moment conditions hold

$$\begin{aligned} E\left[\frac{s_{Rj}}{\frac{1}{\bar{t}_x}\alpha_R^\rho} - \frac{s_{Hj}}{\alpha_H^\rho}\right] &= 0 \\ E\left[\frac{s_{Lj}}{\frac{1}{t_l}\alpha_L^\rho} - \frac{s_{Hj}}{\alpha_H^\rho}\right] &= 0 \end{aligned} \tag{18}$$

where $\bar{t}_x$ is the harmonic average of the wedges that prevail in equilibrium, as in our baseline environment. The first moment condition is similar to the one that arises in our baseline environment: the expenditure on relationship-specific inputs is shaded down by distortions relative to what the expenditure would be in our baseline environment. The second moment condition differs because, in contrast to our baseline, here the distortions on relationship-specific intermediate inputs do not cause the firm to use extra labor. Following the algorithm outlined in Section 4, these two moment conditions can be used to estimate distortions for each state, $\bar{t}_x$, and technology parameters for each recipe.

We next turn to counterfactuals. As with Section 4, we are interested in the change in aggregate productivity that would come from a change in the distribution of distortions, $T(\cdot)$. Like in Proposition 4, we can solve for this in changes. The change in welfare is

$$\frac{U'}{U} = \frac{\left\{\sum_\omega HH_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\frac{\sum_\omega HH_\omega \bar{a}'_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}}{\sum_\omega HH_\omega \bar{a}_\omega}} \tag{19}$$

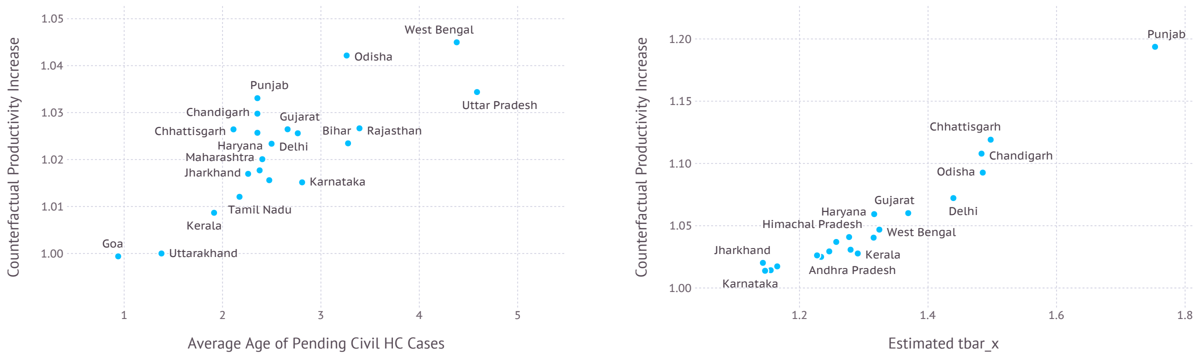
and the change in the shadow cost index for industry ω is

$$\left(\frac{C'_\omega}{C_\omega}\right)^{-\gamma} = \left\{\left(\frac{t'_l}{t_l}\right)^{\alpha_L^\rho}\left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\hat{\Omega}_\rho}\left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}\right\}^{-\gamma}$$

where HH_ω is the share of the household's expenditure spent on goods in industry ω in the current equilibrium, and $R_{\omega\rho}$ is the share of revenue in industry ω accounted for by firms that use recipe ρ in the current equilibrium. To find the change in aggregate productivity, we need to solve for two extra sets of variables, $\bar{a}_\omega$ and $\bar{a}'_\omega$, the average resource gap for each industry in the current equilibrium and in the counterfactual. These can be solved for recursively with the following two equations

$$\bar{a}_\omega = \sum_{\rho\in\varrho_\omega} R_{\omega\rho}\left\{\frac{1}{t_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\hat{\Omega}_\rho^R}\frac{1}{\bar{t}_x}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\hat{\Omega}_\rho^H}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho\right\}$$

$$\bar{a}'_\omega = \sum_{\rho\in\varrho_\omega} R_{\omega\rho}\left(\frac{\left(\frac{t'_l}{t_l}\right)^{\alpha_L^\rho}\left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\hat{\Omega}_\rho}\left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}}{C'_\omega/C_\omega}\right)^{-\gamma}\left\{\frac{1}{t'_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\hat{\Omega}_\rho^R}\frac{1}{\bar{t}'_x}\bar{a}'_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\hat{\Omega}_\rho^H}\bar{a}'_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho\right\}$$



(a) Improving court speed to level of best court

(b) Halving wedges

Figure 15 Counterfactual increases in aggregate productivity, alternative model

The figure shows the counterfactual increase in U when the wedges on relationship-specific inputs are reduced in the alternative model where distortions do not entail a resource cost. In the left panel we reduce $\bar{t}_x$ according to the fraction of $\bar{t}_x$ that is explained by court congestion in a linear IV regression; in the right panel we cut the $\bar{t}_x$ in half.

Provided that $\alpha_L^\rho > 0$ in each industry, each of these equations is a contraction. Note that relative to the baseline model in the main text of the paper, no extra information is required to estimate the model or compute counterfactuals.

H.1 Results for Alternative Formulation of the Distortion

Figure 15 shows the results from conducting the counterfactual (Equation 19) using the parameter estimates from the moment conditions (18). Counterfactual welfare changes are a bit smaller than in the benchmark model, where a reduction in the distortions also frees up labor that can be used in production.

H.2 Proofs for Alternative Formulation of the Distortion

Let $F_\omega(c)$ be the fraction of firms in industry ω with shadow unit cost weakly less than c. As in the baseline economy, Proposition 1 applies, so that $F_\omega(c) = 1 - e^{-(c/C_\omega)^\gamma}$ where the shadow cost indices for each industry $\{C_\omega\}$ satisfy

$$
\begin{aligned}
C_\omega &= \left\{ \sum_{\rho \in \varrho_\omega} \kappa_{\omega\rho} B_{\omega\rho} \left((t_x^*)^{\alpha_R^\rho} t_l^{\alpha_L^\rho} \prod_{\hat{\omega} \in \hat{\Omega}^\rho} C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho} \right)^{-\gamma} \right\}^{-\frac{1}{\gamma}} \qquad (20) \\
t_x^* &= \left(\int_1^\infty t_x^{-\zeta_R} dT(t_x) \right)^{-1/\zeta_R}
\end{aligned}
$$

and $\kappa_{\omega\rho}$ is a constant that depends on technological parameters.

We begin by deriving two results that will be helpful in characterizing the equilibrium. We also define $\tilde{F}_\omega(c, a)$ to be the fraction of firms in industry ω with shadow unit cost weakly less than c and resource gap weakly less than a.

Lemma 8 *Among suppliers of any input that are selected in equilibrium, the effective shadow cost λ delivered by the suppler, resource gap of the supplier, a, and the wedge facing the buyer of using that supplier, t_x, are mutually independent.*

Proof. Consider a technique of recipe ρ for which the common component of productivity is $b_{\hat{\omega}}(\phi)$. For any such technique, the arrival rate of suppliers with resource gap less than a, for which the wedge facing the buyer would be less that t_0 and that delivers effective shadow cost weakly less than λ (which means that if match-specific productivity is z, the supplier's shadow cost c is small enough so that $\lambda \leq \frac{tc}{zb_{\hat{\omega}}(\phi)}$, i.e., $\frac{\lambda z b_{\hat{\omega}}(\phi)}{t} \leq c$) is

$$\begin{aligned}
\int_0^\infty \int_1^{t_0} \tilde{F}_{\hat{\omega}}\left(\frac{\lambda z b_{\hat{\omega}}(\phi)}{t}, a\right) dT(t) \zeta_{\hat{\omega}} z^{-\zeta_{\hat{\omega}}-1} dz &= [\lambda b_{\hat{\omega}}(\phi)]^{\zeta_{\hat{\omega}}} \int_0^\infty \int_1^{t_0} \tilde{F}_{\hat{\omega}}(u,a) t^{-\zeta_{\hat{\omega}}} dT(t) \zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du \\
&= [\lambda b_{\hat{\omega}}(\phi)]^{\zeta_{\hat{\omega}}} \int_1^{t_0} t^{-\zeta_{\hat{\omega}}} dT(t) \int_0^\infty \tilde{F}_{\hat{\omega}}(u,a) \zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du \\
&= \left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}} \frac{\int_1^{t_0} t^{-\zeta_{\hat{\omega}}} dT(t)}{\int_1^\infty t^{-\zeta_{\hat{\omega}}} dT(t)} \frac{\int_0^\infty \tilde{F}_{\hat{\omega}}(u,a) \zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du}{\int_0^\infty F_{\hat{\omega}}(u) \zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du} \\
&= \left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}} \tilde{T}(t_0) A_{\hat{\omega}}(a)
\end{aligned}$$

where $\tilde{T}(t_0) \equiv \frac{\int_1^{t_0} t^{-\zeta_{\hat{\omega}}} dT(t)}{\int_1^\infty t^{-\zeta_{\hat{\omega}}} dT(t)}$ and $\Lambda_{\hat{\omega}} \equiv \begin{cases} t_x^* \left[\int_0^\infty c^{-\zeta_R} dF_{\hat{\omega}}(c)\right]^{-1/\zeta_R}, & \hat{\omega} \in \Omega_R^\rho \\ \left[\int_0^\infty c^{-\zeta_H} dF_{\hat{\omega}}(q)\right]^{-1/\zeta_H}, & \hat{\omega} \in \Omega_H^\rho \end{cases}$ are defined as in the baseline economy and $A_{\hat{\omega}}(a) \equiv \frac{\int_0^\infty \tilde{F}_{\hat{\omega}}(u,a) \zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du}{\int_0^\infty F_{\hat{\omega}}(u) \zeta_{\hat{\omega}} u^{-\zeta_{\hat{\omega}}-1} du}$. We can differentiate to find the arrival rate of suppliers that deliver effective cost λ and resource gap weakly less than v and with wedge weakly less than t_0,

$$\zeta_{\hat{\omega}} \lambda^{\zeta_{\hat{\omega}}-1} \left[\frac{b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}} \tilde{T}(t_0) A_{\hat{\omega}}(a)$$

Next note that arrival rate of suppliers that deliver effective shadow cost weakly less than λ is $\left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}}$, so the probability that no such techniques arrive is $e^{-\left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}}}$. Together, to find the probability that the best supplier of an input for a technique delivers effective cost weakly less than λ_0, has resource gap weakly less than a, and comes with a wedge for the buyer of t_0, we simply integrate over possible value of $\lambda \in [0, \lambda_0]$ the arrival rate of a supplier with these properties multiplied by the probability that there is no better supplier (these are indepenent events)

$$\begin{aligned}
\int_0^{\lambda_0} e^{-\left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}}} \zeta_{\hat{\omega}} \lambda^{\zeta_{\hat{\omega}}-1} \left[\frac{b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}} \tilde{T}(t_0) A_{\hat{\omega}}(a) d\lambda &= A_{\hat{\omega}}(a) \tilde{T}(t_0) \int_0^{\lambda_0} \zeta_{\hat{\omega}} \lambda^{\zeta_{\hat{\omega}}-1} \left[\frac{b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}} e^{-\left[\frac{\lambda b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}}} d\lambda \\
&= A_{\hat{\omega}}(a) \tilde{T}(t_0) e^{-\left[\frac{\lambda_0 b_{\hat{\omega}}(\phi)}{\Lambda_{\hat{\omega}}}\right]^{\zeta_{\hat{\omega}}}}
\end{aligned}$$

Since the resource gap, a, the wedge facing the buyer, t_0, and the effective cost λ_0 are multiplicatively separable, they are mutually independent. Finally, since only λ_0 enters the probability that the technique is actually used by the buyer, it must be that the three are mutually independent across suppliers of the input that are actually used in equilibrium. ■

Lemma 9 *Among firms in any industry, shadow cost c and resource gap a are independent.*

Proof. For a firm in industry ω, let $\tilde{H}_{\omega\rho}(c,a)$ be arrival rate of techniques with recipe ρ that delivers shadow cost weakly less than c and resource wedge weakly less than a for the buyer.

Consider a single technique of recipe ρ that uses labor and n intermediate inputs, for which the common components of input-augmenting productivities are $b = \{b_1, b_1, ..., b_n\}$. The probability that the technique delivers a shadow cost weakly less than c and resource gap weakly less than a_0 is

$$\int_0^\infty ... \int_0^\infty 1 \left\{ \begin{array}{c} \mathcal{C}_{\omega\rho}(\lambda_l, \lambda_1, ..., \lambda_n) \le c, \\ \frac{1}{t_l} \frac{\lambda_l \mathcal{C}_{\omega\rho l}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)} + \frac{a_1}{t_1} \frac{\lambda_1 \mathcal{C}_{\omega\rho\hat{\omega}_1}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)} + ... + \frac{a_n}{t_n} \frac{\lambda_n \mathcal{C}_{\omega\rho\hat{\omega}_n}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)} \le a_0 \end{array} \right\}$$
$$\times \prod_{k=1}^n A_{\hat{\omega}}(a_k) T_{\hat{\omega}}(t_k) e^{-\left(\frac{\lambda_k b_k}{\Lambda_{\hat{\omega}_k}}\right)^{\zeta_{\hat{\omega}}}} \frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}} \zeta_{\hat{\omega}_k} \lambda_k^{\zeta_{\hat{\omega}_k}-1} d\lambda_{\hat{\omega}_k}$$

where $\lambda_l = \frac{t_l}{b_l}$. To find $\tilde{H}_{\omega\rho}(c, a_0)$, we simply need to integrate over the arrival of such techniques across realizations of the vector b:

$$\begin{aligned} \tilde{H}(c, a_0) &= \int_0^\infty ... \int_0^\infty 1 \left\{ \begin{array}{c} \mathcal{C}_{\omega\rho}(\lambda_l, \lambda_1, ..., \lambda_n) \le c, \\ \frac{1}{t_l} \frac{\lambda_l \mathcal{C}_{\omega\rho l}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)} + \frac{a_1}{t_1} \frac{\lambda_1 \mathcal{C}_{\omega\rho\hat{\omega}_1}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)} + ... + \frac{a_n}{t_n} \frac{\lambda_n \mathcal{C}_{\omega\rho\hat{\omega}_n}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)} \le a_0 \end{array} \right\} \\ &\quad \times \prod_{k=1}^n A_{\hat{\omega}}(da_k) T_{\hat{\omega}}(dt_k) e^{-\left(\frac{\lambda_k b_k}{\Lambda_{\hat{\omega}_k}}\right)^{\zeta_{\hat{\omega}}}} \frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}} \zeta_{\hat{\omega}_k} \lambda_k^{\zeta_{\hat{\omega}_k}-1} d\lambda_{\hat{\omega}_k} \mathcal{B}_{\omega\rho}(db) \end{aligned}$$

Using the definition of $\lambda_l = \frac{t_l}{b_l}$ and the homogeneity of the cost function, this is

$$\begin{aligned} \tilde{H}(c, a_0) &= \int_0^\infty ... \int_0^\infty 1 \left\{ \begin{array}{c} \mathcal{C}_{\omega\rho}\left(\frac{t_l}{b_l c}, \frac{\lambda_1}{c}, ..., \frac{\lambda_n}{c}\right) \le 1, \\ \frac{1}{t_l} \frac{\lambda_l}{c} \mathcal{C}_{\omega\rho l}\left(\frac{\lambda}{c}\right) + \frac{a_1}{t_1} \frac{\lambda_1}{c} \mathcal{C}_{\omega\rho\hat{\omega}_1}\left(\frac{\lambda}{c}\right) + ... + \frac{a_n}{t_n} \frac{\lambda_n}{c} \mathcal{C}_{\omega\rho\hat{\omega}_n}\left(\frac{\lambda}{c}\right) \le a_0 \end{array} \right\} \\ &\quad \times \prod_{k=1}^n A_{\hat{\omega}}(da_k) T_{\hat{\omega}}(dt_k) e^{-\left(\frac{\lambda_k b_k}{\Lambda_{\hat{\omega}_k}}\right)^{\zeta_{\hat{\omega}}}} \frac{b_k^{\zeta_{\hat{\omega}_k}}}{\Lambda_{\hat{\omega}_k}^{\zeta_{\hat{\omega}_k}}} \zeta_{\hat{\omega}_k} \lambda_k^{\zeta_{\hat{\omega}_k}-1} d\lambda_{\hat{\omega}_k} \mathcal{B}_{\omega\rho}(db) \end{aligned}$$

It will be useful to make the change of variables $v_k = \frac{\lambda_k}{c}$, $v_l = \frac{t_l}{c b_l}$, and $m_j = \left(\frac{\lambda_l b_l}{\Lambda_{\hat{\omega}}}\right)^{\zeta_{\hat{\omega}_k}}$ to express $\tilde{H}(c, a)$ as

$$\begin{aligned} \tilde{H}(c, a_0) &= \int_0^\infty ... \int_0^\infty 1 \left\{ \begin{array}{c} \mathcal{C}_{\omega\rho}(v_l, v_1, ..., v_n) \le 1, \\ \frac{1}{t_l} v_l \mathcal{C}_{\omega\rho l}(v) + \frac{a_1}{t_1} v_1 \mathcal{C}_{\omega\rho\hat{\omega}_1}(v) + ... + \frac{a_n}{t_n} v_n \mathcal{C}_{\omega\rho\hat{\omega}_n}(v) \le a_0 \end{array} \right\} \\ &\quad \times B_{\omega\rho}\left(\frac{t_l}{c}\right)^{-\beta_l^\rho} \prod_{k=1}^n A_{\hat{\omega}}(da_k) T_{\hat{\omega}}(dt_k) \left(\frac{m_k^{1/\zeta_{\hat{\omega}_k}} \Lambda_\omega}{c}\right)^{-\beta_{\hat{\omega}_k}} e^{-m_k} dm_k \mathcal{V}(dv) \end{aligned}$$

or more simply

$$\tilde{H}(c, a_0) = \bar{A}_{\omega\rho}(a_0) c^\gamma$$

where

$$\begin{aligned} \bar{A}_{\omega\rho}(a_0) &\equiv B_{\omega\rho} t_l^{-\beta_l^\rho} \Lambda_{\hat{\omega}_1}^{-\beta_{\hat{\omega}_1}^\rho} ... \Lambda_{\hat{\omega}_n}^{-\beta_{\hat{\omega}_n}^\rho} \int_0^\infty ... \int_0^\infty 1 \left\{ \begin{array}{c} \mathcal{C}_{\omega\rho}(v_l, v_1, ..., v_n) \le 1, \\ \frac{1}{t_l} v_l \mathcal{C}_{\omega\rho l}(v) + \frac{a_1}{t_1} v_1 \mathcal{C}_{\omega\rho\hat{\omega}_1}(v) + ... + \frac{a_n}{t_n} v_n \mathcal{C}_{\omega\rho\hat{\omega}_n}(v) \le a_0 \end{array} \right\} \\ &\quad \times \prod_{k=1}^n A_{\hat{\omega}}(da_k) T_{\hat{\omega}}(dt_k) m_k^{-\beta_{\hat{\omega}_k}/\zeta_{\hat{\omega}_k}} e^{-m_k} dm_k \mathcal{V}(dv) \end{aligned}$$

We can differentiate to find the arrival rate of techniques that deliver shadow cost c and resource cost no greater than a

$$\bar{A}_{\omega\rho}(a) \gamma c^{\gamma-1}$$

Next note that arrival rate of techniques of recipe ρ that deliver shadow cost weakly less than c regardless of resource gap is $\bar{A}_{\omega\rho}(1)c^\gamma$, so that the probability of no such techniques across all recipes is $\prod_{\rho \in \varrho_\omega} e^{-\bar{A}_{\omega\rho}(1)c^\gamma} = e^{-c^\gamma \sum_{\rho \in \varrho_\omega} \bar{A}_{\omega\rho}(1)} = e^{-(c/C_\omega)^\gamma}$ (which follows from the definition of C_ω). Together, to find the probability that the best technique delivers shadow cost weakly less than c_0 and has resource gap weakly less than a, we simply integrate over possible values of $c \in [0, c_0]$ the arrival rate of a supplier with these properties multiplied by the probability that there is no better supplier (these are independent events)

$$
\begin{aligned}
\tilde{F}(c,a) &= \sum_{\rho\in\varrho_\omega}\int_0^{c_0} e^{-(c/C_\omega)^\gamma}\bar{A}_{\omega\rho}(a)\gamma c^{\gamma-1}dc \\
&= \sum_{\rho\in\varrho_\omega}\bar{A}_{\omega\rho}(a)\int_0^{c_0} e^{-(c/C_\omega)^\gamma}\gamma c^{\gamma-1}dc \\
&= \frac{\sum_{\rho\in\varrho_\omega}\bar{A}_{\omega\rho}(a)}{C_\omega^{-\gamma}}e^{-(c_0/C_\omega)^\gamma}
\end{aligned}
$$

The result follows from the fact that this is multiplicatively separable in c_0 and a. ■

Claim 5 *Among firms in industry ω that choose to use recipe ρ, the following two equations hold*

$$
\begin{aligned}
E\left[\frac{s_{Rj}}{\frac{1}{t_x}\alpha_R^\rho}-\frac{s_{Hj}}{\alpha_H^\rho}\middle|\rho\right] &= 0 \\
E\left[\frac{s_{Lj}}{\frac{1}{t_l}\alpha_L^\rho}-\frac{s_{Hj}}{\alpha_H^\rho}\middle|\rho\right] &= 0
\end{aligned}
$$

Proof. Let p_s be the actual price per unit paid to supplier so that $t_s p_s$ is the shadow cost per unit of the input to the buyer. The share of j's revenue paid to the supplier of input $\hat{\omega}$ can be expressed as

$$
s_{\hat{\omega}j}=\frac{p_{s_{\hat{\omega}}(\phi)}x_{s_{\hat{\omega}}(\phi)}}{\mathrm{Rev}_j}=\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\frac{t_{s_{\hat{\omega}}(\phi)}p_{s_{\hat{\omega}}(\phi)}x_{s_{\hat{\omega}}(\phi)}}{c_j y_j}\frac{c_j y_j}{\mathrm{Rev}_j}=\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{c_j y_j}{\mathrm{Rev}_j}
$$

where the last equality follows from Shephard's lemma. Similarly, the share of revenue spent on labor is

$$
s_{Lj}=\frac{wl_j}{\mathrm{Rev}_j}=\frac{1}{t_l}\frac{t_l w l_j}{c_j y_j}\frac{c_j y_j}{\mathrm{Rev}_j}=\frac{1}{t_l}\frac{\lambda_L\mathcal{C}_{\omega\rho L}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{c_j y_j}{\mathrm{Rev}_j}
$$

Among firms that produce using recipe ρ whose shadow cost is c, the average share of revenue spent on relationship-specific inputs is

$$
E\left[s_{Rj}|c_j,\rho\right]=\sum_{\hat{\omega}\in\Omega_\rho^R}E\left[s_{\hat{\omega}j}|c_j,\rho\right]=\sum_{\hat{\omega}\in\Omega_\rho^R}E\left[\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\mathrm{Rev}}\middle|c,\rho\right]
$$

Using the law of iterated expectations along with $E\left[\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\middle|\lambda\right]=\frac{1}{t_x}$, this is

$$
\begin{aligned}
E\left[s_{Rj}|c_j,\rho\right] &= \sum_{\hat{\omega}\in\Omega_\rho^R}E\left[E\left[\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\mathrm{Rev}}\middle|\lambda,c,\rho\right]\middle|c,\rho\right] \\
&= \sum_{\hat{\omega}\in\Omega_\rho^R}E\left[E\left[\frac{1}{t_{s_{\hat{\omega}}(\phi)}}\middle|\lambda,c,\rho\right]\frac{\lambda_{s_{\hat{\omega}}(\phi)}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\mathrm{Rev}}\middle|c,\rho\right] \\
&= \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{t_x}E\left[\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\mathrm{Rev}}\middle|c,\rho\right]
\end{aligned}
$$

Finally, note that, conditional on shadow cost c, downstream demand and prices do not depend on any of the determinants of c. This implies that for a firm in industry ω, $E\left[\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\mathrm{Rev}}\middle|c,\rho\right]=E\left[\frac{cy}{\mathrm{Rev}}\middle|c,\rho\right]E\left[\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\middle|c,\rho\right]$. Using Proposition 7, we have

$$
E\left[s_{Rj}|c_j,\rho\right]=\sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{t_x}\alpha_{\hat{\omega}}^\rho E\left[\frac{cy}{\mathrm{Rev}}|c,\rho\right]=\frac{1}{t_x}\alpha_R^\rho E\left[\frac{cy}{\mathrm{Rev}}|c,\rho\right]
$$

Using similar logic, we have

$$E\left[s_{Hj}|c_j\right] = \sum_{\hat{\omega}\in\Omega_\rho^H} E\left[s_{\hat{\omega}j}|c_j\right] = \sum_{\hat{\omega}\in\Omega_\rho^H} E\left[\frac{\lambda_{\hat{\omega}}(\phi)\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\text{Rev}}\bigg|c\right] = \sum_{\hat{\omega}\in\Omega_\rho^H}\alpha_{\hat{\omega}}^\rho E\left[\frac{cy}{\text{Rev}}\bigg|c\right] = \alpha_H^\rho E\left[\frac{cy}{\text{Rev}}\bigg|c\right]$$

$$E\left[s_{Lj}|c_j\right] = E\left[\frac{1}{t_l}\frac{\lambda_l\mathcal{C}_{\omega\rho L}(\lambda)}{\mathcal{C}_{\omega\rho}(\lambda)}\frac{cy}{\text{Rev}}\bigg|c\right] = \frac{1}{t_l}\alpha_L^\rho E\left[\frac{cy}{\text{Rev}}\bigg|c\right]$$

Elminating $E\left[\frac{cy}{\text{Rev}}\big|c\right]$, we have the two moment conditions

$$E\left[\frac{s_{Rj}}{\frac{1}{t_x}\alpha_R^\rho}-\frac{s_{Hj}}{\alpha_H^\rho}\right] = 0$$

$$E\left[\frac{s_{Lj}}{\frac{1}{t_l}\alpha_L^\rho}-\frac{s_{Hj}}{\alpha_H^\rho}\right] = 0$$

■

Lemma 10 *Let a_ω be the average resource gap among firms in industry ω. Then*

$$\bar{a}_\omega = \sum_{\rho\in\varrho_\omega} R_{\omega\rho}\left\{\frac{1}{t_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}_x}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho\right\}$$

Proof. We begin by dividing (17) by the total shadow cost $c_j y_j$ and rearranging.

$$\begin{aligned}
a_j &= \frac{wl_j}{c_j y_j} + \sum_{\hat{\omega}\in\Omega_\rho} a_{s_{\hat{\omega}}(\phi)}\frac{c_{s_{\hat{\omega}}(\phi)}x_{s_{\hat{\omega}}(\phi)}}{c_j y_j} \\
&= \frac{1}{t_l}\frac{t_l w l_j}{c_j y_j} + \sum_{\hat{\omega}\in\Omega_\rho}\frac{a_{s_{\hat{\omega}}(\phi)}}{t_{s_{\hat{\omega}}(\phi)}}\frac{t_{s_{\hat{\omega}}(\phi)}c_{s_{\hat{\omega}}(\phi)}x_{s_{\hat{\omega}}(\phi)}}{c_j y_j} \\
&= \frac{1}{t_l}\frac{\lambda_{jl}\mathcal{C}_{\omega\rho l}(\lambda_j)}{\mathcal{C}_{\omega\rho}(\lambda_j)} + \sum_{\hat{\omega}\in\Omega_\rho}\frac{a_{s_{\hat{\omega}}(\phi)}}{t_{s_{\hat{\omega}}(\phi)}}\frac{\lambda_{j\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda_j)}{\mathcal{C}_{\omega\rho}(\lambda_j)}
\end{aligned}$$

Among firms in industry ω that use recipe ρ and have a vector of effective shadow cost λ, the average resource gap is

$$\begin{aligned}
E\left[a_j|\lambda,\omega,\rho\right] &= \frac{1}{t_l}\frac{\lambda_{jl}\mathcal{C}_{\omega\rho l}(\lambda_j)}{\mathcal{C}_{\omega\rho}(\lambda_j)} + \sum_{\hat{\omega}\in\Omega_\rho} E\left[\frac{a_{s_{\hat{\omega}}(\phi)}}{t_{s_{\hat{\omega}}(\phi)}}|\omega,\rho\right]\frac{\lambda_{j\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda_j)}{\mathcal{C}_{\omega\rho}(\lambda_j)} \\
&= \frac{1}{t_l}\frac{\lambda_{jl}\mathcal{C}_{\omega\rho l}(\lambda_j)}{\mathcal{C}_{\omega\rho}(\lambda_j)} + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}_x}\bar{a}_{\hat{\omega}}\frac{\lambda_{j\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda_j)}{\mathcal{C}_{\omega\rho}(\lambda_j)} + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}_{\hat{\omega}}\frac{\lambda_{j\hat{\omega}}\mathcal{C}_{\omega\rho\hat{\omega}}(\lambda_j)}{\mathcal{C}_{\omega\rho}(\lambda_j)}
\end{aligned}$$

Then the average resource gap across all firms that use ρ is

$$E\left[a_j|\omega,\rho\right] = E\left[E\left[a_j|\lambda,\omega,\rho\right]|\omega,\rho\right] = \frac{1}{t_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}_x}\bar{v}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho$$

To find the average resource gap, we simply weight the previous equation by the probability that a firm chooses to use recipe ρ, which, in equilibrium, is equal to the share of revenue earned by firms that use recipe ρ.

$$\bar{a}_\omega = \sum_{\rho\in\varrho_\omega} R_{\omega\rho}\left\{\frac{1}{t_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}_x}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho\right\}$$

■

Claim 6 *Suppose that the distribution of wedges changes from T to T'. The following equations are sufficient to compute the counterfactual change in aggregate productivity:*

$$
\begin{aligned}
\frac{U'}{U} &= \frac{\left\{\sum_\omega HH_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\frac{\sum_\omega HH_\omega \bar{a}'_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}}{\sum_\omega HH_\omega \bar{a}_\omega}} \\
\left(\frac{C'_\omega}{C_\omega}\right)^{-\gamma} &= \left\{\left(\frac{t'_l}{t_l}\right)^{\alpha_L^\rho}\left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\Omega_\rho}\left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}\right\}^{-\gamma} \\
\bar{a}_\omega &= \sum_{\rho\in\varrho_\omega} R_{\omega\rho}\left\{\frac{1}{t_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}_x}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho\right\} \\
\bar{a}'_\omega &= \sum_{\rho\in\varrho_\omega} R_{\omega\rho}\left(\frac{\left(\frac{t'_l}{t_l}\right)^{\alpha_L^\rho}\left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho}\prod_{\hat{\omega}\in\Omega_\rho}\left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}}{C'_\omega/C_\omega}\right)^{-\gamma}\left\{\frac{1}{t'_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R}\frac{1}{\bar{t}'_x}\bar{a}'_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H}\bar{a}'_{\hat{\omega}}\alpha_{\hat{\omega}}^\rho\right\}
\end{aligned}
$$

Proof. As in our baseline, the price level in industry ω and in aggregate are respectively

$$
\begin{aligned}
P_\omega &= \frac{\varepsilon}{\varepsilon-1}J_\omega^{\frac{1}{1-\varepsilon}}\Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{1}{1-\varepsilon}}C_\omega \\
P &= \frac{\varepsilon}{\varepsilon-1}\left\{\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}}\Gamma\left(1-\frac{\varepsilon-1}{\gamma}\right)^{\frac{\eta-1}{\varepsilon-1}}C_\omega^{1-\eta}\right\}^{\frac{1}{1-\eta}}
\end{aligned}
$$

Total profit generated by the production of good j (revenue from final consumer less cumulative labor cost) is $(p_j - a_j c_j)\,u_j$. Using $c_j = \frac{\varepsilon-1}{\varepsilon}p_j$ and $u_j = U\upsilon_\omega\left(\frac{P_\omega}{P}\right)^{-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{-\varepsilon}$, gives

$$
\begin{aligned}
(p_j - a_j c_j)\,u_j &= \left(p_j - a_j\frac{\varepsilon-1}{\varepsilon}p_j\right)U\upsilon_\omega\left(\frac{P_\omega}{P}\right)^{-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{-\varepsilon} \\
&= \left(1 - a_j\frac{\varepsilon-1}{\varepsilon}\right)UP\upsilon_\omega\left(\frac{P_\omega}{P}\right)^{1-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}
\end{aligned}
$$

Summing across all firms in the economy and using the fact that υ_j is independent of p_j gives

$$
\begin{aligned}
\Pi &= \sum_\omega\int_0^{J_\omega}\left(1 - a_j\frac{\varepsilon-1}{\varepsilon}\right)UP\upsilon_\omega\left(\frac{P_\omega}{P}\right)^{1-\eta}\left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}dj \\
&= UP - \frac{\varepsilon-1}{\varepsilon}U\sum_\omega \upsilon_\omega P^\eta (P_\omega)^{1-\eta}\int_0^{J_\omega}a_j\left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}dj \\
&= UP - \frac{\varepsilon-1}{\varepsilon}U\sum_\omega \upsilon_\omega P^\eta (P_\omega)^{1-\eta}\bar{a}_\omega\int_0^{J_\omega}\left(\frac{p_{\omega j}}{P_\omega}\right)^{1-\varepsilon}dj \\
&= UP - \frac{\varepsilon-1}{\varepsilon}U\sum_\omega \upsilon_\omega P^\eta (P_\omega)^{1-\eta}\bar{a}_\omega
\end{aligned}
$$

Plugging this into the household's budget constraint gives

$$\begin{aligned} UP &= wL + \Pi \\ &= wL + UP - \frac{\varepsilon-1}{\varepsilon} U \sum_{\omega} \upsilon_\omega P^\eta (P_\omega)^{1-\eta} \bar{a}_\omega \end{aligned}$$

Rearranging and using the expression for and using the expressions for P_ω and P gives

$$\begin{aligned} U &= \frac{w}{\frac{\varepsilon-1}{\varepsilon}\sum_\omega \upsilon_\omega P^\eta (P_\omega)^{1-\eta} a_\omega} L \\ &= \Gamma \left(1 - \frac{\varepsilon-1}{\gamma}\right)^{\frac{1}{\varepsilon-1}} \frac{\left\{\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\sum_\omega \bar{a}_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} C_\omega^{1-\eta}} L \end{aligned}$$

To find a counterfactual, we thus have

$$\frac{U'}{U} = \frac{\frac{\left\{\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} (C'_\omega)^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\sum_\omega \bar{a}'_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} (C'_\omega)^{1-\eta}}}{\frac{\left\{\sum_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} (C_\omega)^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\sum_\omega \bar{a}_\omega \upsilon_\omega J_\omega^{\frac{\eta-1}{\varepsilon-1}} (C_\omega)^{1-\eta}}} = \frac{\left\{\sum_\omega HH_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}\right\}^{\frac{1}{\eta-1}+1}}{\frac{\sum_\omega HH_\omega \bar{a}'_\omega \left(\frac{C'_\omega}{C_\omega}\right)^{1-\eta}}{\sum_\omega HH_\omega \bar{a}_\omega}}$$

We thus need expressions for $\bar{a}_\omega$ and $\bar{a}'_\omega$. The expression for $\bar{a}_\omega$ comes directly from Lemma 10, which also delivers an expression for $\bar{a}'_\omega$:

$$\begin{aligned} \bar{a}'_\omega &= \sum_{\rho\in\varrho_\omega} R'_{\omega\rho} \left\{ \frac{1}{t'_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R} \frac{1}{t'_x} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho \right\} \\ &= \sum_{\rho\in\varrho_\omega} \frac{R'_{\omega\rho}}{R_{\omega\rho}} R_{\omega\rho} \left\{ \frac{1}{t'_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R} \frac{1}{t'_x} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho \right\} \\ &= \sum_{\rho\in\varrho_\omega} \frac{R'_{\omega\rho}}{R_{\omega\rho}} R_{\omega\rho} \left\{ \frac{1}{t'_l}\alpha_L^\rho + \sum_{\hat{\omega}\in\Omega_\rho^R} \frac{1}{t'_x} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho + \sum_{\hat{\omega}\in\Omega_\rho^H} \bar{a}'_{\hat{\omega}} \alpha_{\hat{\omega}}^\rho \right\} \end{aligned}$$

Then, noting that among firms in industry ω, the share of revenue of those that use recipe ρ is $R_{\omega\rho} = \kappa_{\omega\rho} B_{\omega\rho} \left(\frac{t_l^{\alpha_L^\rho} (t_x^*)^{\alpha_R^\rho} \prod_{\hat{\omega}\in\Omega_\rho} C_{\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}}{C_\omega} \right)^{-\gamma}$, the change in revenue share in the counterfactual is

$$\frac{R'_{\omega\rho}}{R_{\omega\rho}} = \left(\frac{\left(\frac{t'_l}{t_l}\right)^{\alpha_L^\rho} \left(\frac{t_x^{*\prime}}{t_x^*}\right)^{\alpha_R^\rho} \prod_{\hat{\omega}\in\Omega_\rho} \left(\frac{C'_{\hat{\omega}}}{C_{\hat{\omega}}}\right)^{\alpha_{\hat{\omega}}^\rho}}{C'_\omega / C_\omega} \right)^{-\gamma}$$

The results follows from combining these last two equations. ■

I A Hsieh-Klenow exercise

In the following, we try to do a quantification exercise that is as close as possible to Hsieh and Klenow (2009), and point out some of the issues we would face.

Suppose that each plant in industry ω that uses recipe ρ uses the production function

$$y_{\omega\rho j} = A_{\omega\rho j} L_{\omega\rho j}^{\alpha_L^\rho} \prod_{\hat{\omega}\in\hat{\Omega}^\rho} x_{\omega\rho j\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}$$

The planner maximizes output as

$$u^* = \max_{u_\omega, y_{\omega j}, L_{\omega j}, x_{\omega j\hat{\omega}}} \left(\sum_{\omega\in\Omega} \beta_\omega^{\frac{1}{\eta}} u_\omega^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

subject to

$$\lambda_{\omega\rho j}^* : y_{\omega\rho j} \leq A_{\omega\rho j} L_{\omega\rho j}^{\alpha_L^\rho} \prod_{\hat{\omega}\in\hat{\Omega}^\rho} x_{\omega\rho j\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}$$

$$\lambda_\omega^* : u_\omega + \sum_{\omega'} \sum_{\rho\in\varrho_{\omega'}} \sum_{j\in J_{\omega'\rho}} x_{\omega'\rho j\omega} \leq \left(\sum_{\rho\in\varrho_\omega} \sum_{j\in J_{\omega\rho}} y_{\omega\rho j}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$w^* : \sum_\omega \sum_{\rho\in\varrho_\omega} \sum_{j\in J_{\omega\rho}} L_{\omega\rho j} \leq L$$

If u is output in the current equilibrium, allocational efficiency is $\frac{u}{u^*}$. We use the following result:

Proposition 11 *Define $M_\omega \equiv \frac{\lambda_\omega^*/w^*}{p_\omega/w}$. Allocational efficiency is*

$$\frac{u}{u^*} = \frac{pu}{wL} \left(\sum \frac{p_\omega u_\omega}{pu} M_\omega^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

and the $\{M_\omega\}$ solve the system of equations

$$M_\omega = \left\{ \sum_{\rho\in\varrho_\omega} \sum_{j\in J_{\omega\rho}} \frac{p_{\omega\rho j} y_{\omega\rho j}}{p_\omega y_\omega} \left[\left(\frac{w L_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}} \right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho} \left(M_{\hat{\omega}} \frac{p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}} \right)^{\alpha_{\hat{\omega}}^\rho} \right]^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}.$$

Proof. We begin by showing that $u^* = w^* L$. To see this, note that the FOCs for all firms' inputs imply

$$w^* L_\omega^* + \sum_{\rho\in\varrho_\omega} \sum_{\hat{\omega}\in\Omega^\rho} \lambda_{\hat{\omega}}^* x_{\omega\hat{\omega}}^* = \lambda_\omega^* y_\omega^* = \lambda_\omega^* u_\omega^* + \sum_{\omega'} \sum_{\rho\in\varrho_{\omega'}} \lambda_\omega^* x_{\omega'\omega}^*$$

Summing across ω gives and noticing that the terms for intermediate inputs drop gives

$$w^* L = \sum_\omega w^* L_\omega^* = \sum_\omega \lambda_\omega^* u_\omega^*$$

The planner's FOC for u_ω^* is $u_\omega^* = (\lambda_\omega^*)^{-\eta} u^*$, which implies $\left[\sum_\omega (\lambda_\omega^*)^{1-\eta} \right]^{\frac{1}{1-\eta}} = 1$ and hence $\sum_\omega \lambda_\omega^* u_\omega^* = u^*$. The latter implies $w^* L = u^*$

Next, we derive the expression for $\frac{u}{u^*}$, which can be rearranged as $\frac{u}{u^*} = \frac{u}{w^* L} = \frac{u\left(\sum_\omega (\lambda_\omega^*)^{1-\eta}\right)^{\frac{1}{1-\eta}}}{w^* L}$. Using

$\lambda_\omega^* = M_\omega p_\omega \frac{w^*}{w}$, this can be expressed as

$$
\begin{aligned}
\frac{u}{u^*} &= \frac{u}{w^* L}\left(\sum_\omega (\lambda_\omega^*)^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
&= \frac{u}{w^* L}\left(\sum_\omega \left(M_\omega p_\omega \frac{w^*}{w}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
&= \frac{pu}{wL}\left(\sum_\omega \left(\frac{p_\omega}{p}\right)^{1-\eta} M_\omega^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
&= \frac{pu}{wL}\left(\sum_\omega \frac{p_\omega u_\omega}{pu} M_\omega^{1-\eta}\right)^{\frac{1}{1-\eta}}
\end{aligned}
$$

where the last line uses $\left(\frac{p_\omega}{p}\right)^{1-\eta} = \frac{p_\omega u_\omega}{pu}$.

Lastly we derive the system of equations for $\{M_\omega\}$. The first order conditions for $L_{\omega\rho j}$ and $\{x_{\omega\rho j\hat{\omega}}\}$ imply

$$
\begin{aligned}
\lambda_{\omega\rho j}^* &= \frac{\left(\frac{w^*}{\alpha_L^\rho}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{\lambda_{\hat{\omega}}^*}{\alpha_{\hat{\omega}}^\rho}\right)^{\alpha_{\hat{\omega}}^\rho}}{A_{\omega\rho j}} \\
&= \frac{1}{y_{\omega\rho j}}\left(\frac{w^* L_{\omega\rho j}}{\alpha_L^\rho}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{\lambda_{\hat{\omega}}^* x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho}\right)^{\alpha_{\hat{\omega}}^\rho} \\
&= p_{\omega\rho j}\left(\frac{w^* L_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{\lambda_{\hat{\omega}}^* x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_{\hat{\omega}}^\rho}
\end{aligned}
$$

where the second line uses $A_{\omega\rho j} = y_{\omega\rho j}/\left(L_{\omega\rho j}^{\alpha_L^\rho}\prod_{\hat{\omega}\in\Omega^\rho} x_{\omega\rho j\hat{\omega}}^{\alpha_{\hat{\omega}}^\rho}\right)$. Using $\lambda_{\hat{\omega}}^* = M_{\hat{\omega}} p_{\hat{\omega}} \frac{w^*}{w}$, this is

$$
\begin{aligned}
\lambda_{\omega\rho j}^* &= p_{\omega\rho j}\left(\frac{w^* L_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{M_{\hat{\omega}}\frac{w^*}{w} p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_{\hat{\omega}}^\rho} \\
&= p_{\omega\rho j}\frac{w^*}{w}\left(\frac{w L_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{M_{\hat{\omega}}\frac{w^*}{w} p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_{\hat{\omega}}^\rho}
\end{aligned}
$$

Household demand satisfies $y_{\omega\rho j} = y_\omega\left(\frac{p_{\omega\rho j}}{p_\omega}\right)^{-\varepsilon}$ which implies $p_{\omega\rho j} = p_\omega\left(\frac{p_{\omega\rho j} y_{\omega\rho j}}{p_\omega y_\omega}\right)^{\frac{1}{1-\varepsilon}}$. Plugging this in and rearranging gives

$$
\frac{\lambda_{\omega\rho j}^*/w^*}{p_\omega/w} = \left(\frac{p_{\omega\rho j} y_{\omega\rho j}}{p_\omega y_\omega}\right)^{\frac{1}{1-\varepsilon}}\left(\frac{w L_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_L^\rho} \prod_{\hat{\omega}\in\Omega^\rho}\left(\frac{M_{\hat{\omega}}\frac{w^*}{w} p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_{\hat{\omega}}^\rho}
$$

Finally, we have

$$
\begin{aligned}
M_\omega &= \frac{\lambda_\omega^*/w^*}{p_\omega/w} = \left\{\sum_{\rho\in\varrho_\omega}\sum_{j\in J_{\omega\rho}}\left(\frac{\lambda_{\omega\rho j}^*/w^*}{p_\omega/w}\right)^{1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}} \\
&= \left\{\sum_{\rho\in\varrho_\omega}\sum_{j\in J_{\omega\rho}}\frac{p_{\omega\rho j} y_{\omega\rho j}}{p_\omega y_\omega}\left[\left(\frac{w L_{\omega\rho j}}{\alpha_L^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_L^\rho}\prod_{\hat{\omega}\in\Omega^\rho}\left(M_{\hat{\omega}}\frac{p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha_{\hat{\omega}}^\rho p_{\omega\rho j} y_{\omega\rho j}}\right)^{\alpha_{\hat{\omega}}^\rho}\right]^{1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}}
\end{aligned}
$$

■

Empirically implementing Theorem 11 to get robust measures of allocational efficiency is hard. We face the following problems:

- It is not clear how to include multi-product plants in this calculation. Here, we would need to know how much of an input $\hat{\omega}$ is being used in the plant's use of recipe ρ. In contrast, to do the counterfactual in the main text of our paper, we need to know only the recipe sales shares of each plant.
- One decision we have to take is how to interpret firms having a mix of intermediate inputs that is not exactly the same (along the extensive margin) as that of the recipe.
- Most papers on misallocation (in particular Hsieh and Klenow (2007)) winsorize the cost shares before doing the exercise. Since the dispersion of cost shares is crucial to the magnitude of the result, where the winsorizing threshold is set matters a lot for the outcomes (Rotemberg and White demonstrate this problem very nicely). See below for more on this.
- How exactly to do the winsorizing in a way that is model-consistent is not clear: should it be done by changing expenditure (but then it should also change sales of other firms! which firms?), or sales (but then we would have to change either final consumer purchases or expenditures of other firms?).

In the following, we implement one version of this exercise. We pretend that the economy of each state consists only of firms that correspond to the single-product plants in our data. We choose $\varepsilon = 4$. We winsorize cost shares such that $\frac{p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}}{\alpha^{\rho}_{\hat{\omega}} p_{\omega\rho j} y_{\omega\rho j}}$ is above a particular threshold (and the inverse is above the inverse of the threshold). We do this by adjusting expenditures $p_{\hat{\omega}} x_{\omega\rho j\hat{\omega}}$ of j, but not sales of other firms, or final demand.

Figure 16 shows the distribution of resulting (inverses of) M_{ω} for the state of Himachal Pradesh, for winsorizing thresholds of 2%, 5%, and 10%. The resulting counterfactual increases in the final consumer's utility aggregate u are 600%, 330%, and 170%. Hence, results depend crucially on the winsorizing thresholds, which are completely arbitrary. In Figure 17 we show this across states. The figure shows the consumer utility aggregate relative to its counterfactual undistorted one (u/u^* in Theorem 11) by state and winsorizing threshold, and separately for whether we use variation within recipes or within 5-digit industries. Again the results depend heavily on the winsorizing threshold. Interestingly, u/u^* are relatively similar across states. The welfare gains accrue to a large extent from the extreme cost shares, so that it does not matter much whether one looks at within-recipe or within-industry variation.

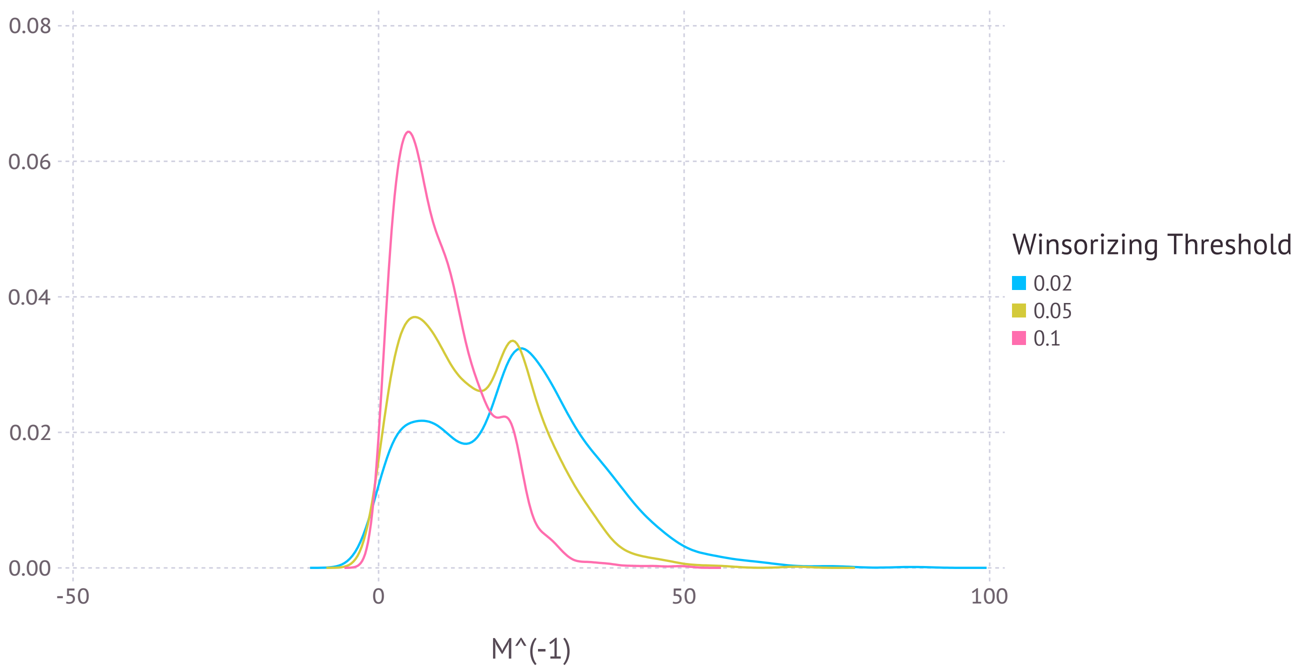
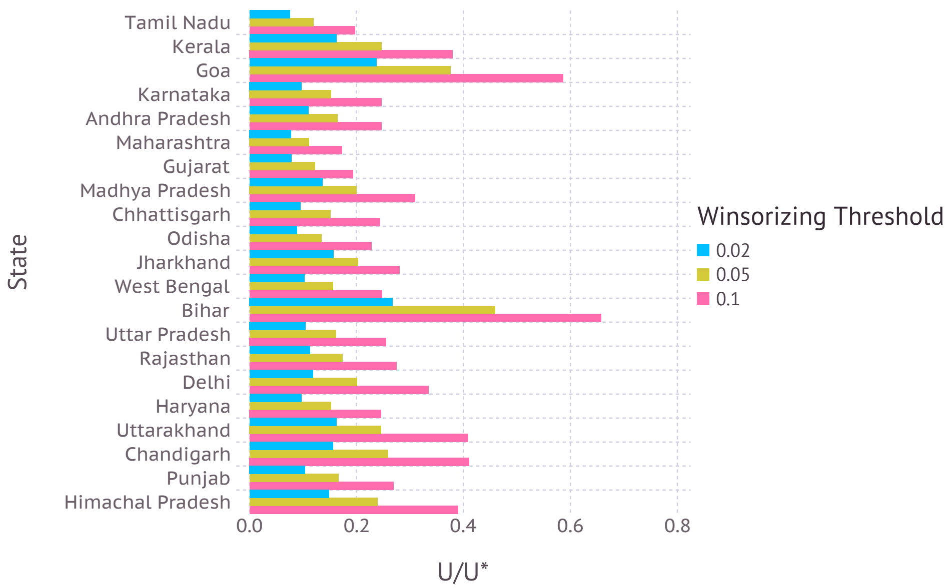
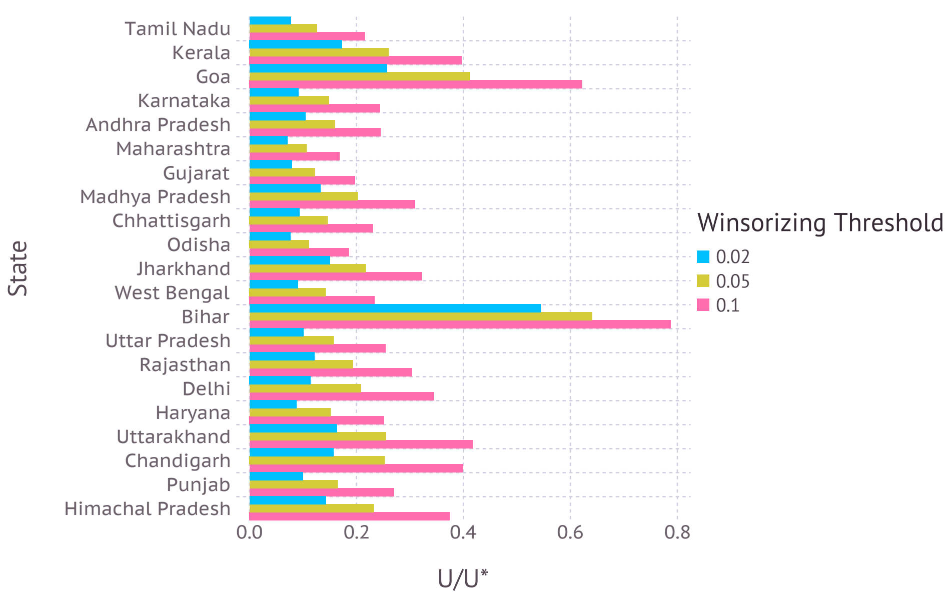


Figure 16 Counterfactual increases in M_ω, by industry



(a) Within 5-digit Products



(b) Within Recipes

Figure 17 Hsieh-Klenow Exercise Results, By State

In panel (a), we assume that each product has only one recipe; recipes are therefore equal to products. In panel (b), we use the recipes that emerge from the benchmark results in the main text of the paper.