Exercise 1

Part (a).

As the social planner wants to maximize social surplus, there will not be any monopolistic distortions like they are present in the pricing decision of monopolists. Hence, he will set each varieties' price equal to its (common) marginal costs ψ. Using this, we get from the consumer's optimality condition

$$\left(\frac{c_i}{c_{i'}}\right)^{-\frac{1}{\varepsilon}} = \frac{p_i}{p_{i'}} = \frac{\psi}{\psi} = 1,$$

i.e. all varieties will be consumed in the same amount $c_i = c_{i'} = c$. Note that this is also true in the equilibrium. For a given number of varieties N the social planner will therefore chose a consumption aggregator

$$C \equiv \left(\sum_{i=1}^{N} c_i^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(Nc^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = cN^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where c is the consumption level of each variety. To allocate resources between the consumption goods c and the $y-$good and to decide about the number of varieties N, the social planner solves the problem

$$\max_{y,c,N} cN^{\frac{\varepsilon}{\varepsilon-1}} + y^{1-\alpha}(1-\alpha) \tag{2}$$

$$\text{s.t.} \quad m = Nc\psi + y + N\mu, \tag{3}$$

where the resource constraint stems from the fact that each good is produced in quantity c and costs ψ. Another way to see this (which is more in line with the exposition in the book) is that the ideal price index is equal to $P \equiv \left(\sum_{i=1}^{N} \psi^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} = \psi N^{1/(1-\varepsilon)}$. Note that P also denotes the unit costs of producing the aggregate good C. As c units of each variety are bought, the social planner buys $N^{\frac{\varepsilon}{\varepsilon-1}}c = C$ units of differentiated varieties (see (1)) so that we can also express (2) as

$$\max_{y,C,N} C + y^{1-\alpha}/(1-\alpha)$$

$$\text{s.t.} \quad m = CP + y + N\mu,$$

where the constraint follows from the fact that

$$Nc\psi = N^{\frac{\varepsilon}{\varepsilon-1}} N^{\frac{-1}{\varepsilon-1}} c\psi = N^{\frac{\varepsilon}{\varepsilon-1}} cN^{\frac{1}{1-\varepsilon}}\psi = CP.$$

This is exactly the form given in the book (using the specific utility function given here). Solving the constraint in (3) for the consumption level $c = \frac{m-y-N\mu}{N\psi}$ and substituting this into (2), we arrive at the unconstrained maximization problem

$$\max_{y,N} \frac{m-y-N\mu}{\psi} N^{\frac{1}{\varepsilon-1}} + y^{1-\alpha}/(1-\alpha).$$

The corresponding first-order conditions are

$$y^{-\alpha} = \frac{1}{\psi} N^{\frac{1}{\varepsilon-1}} \tag{4}$$

$$m - y = \mu\varepsilon N. \tag{5}$$

Using that from (4) we get that $y = \psi^{1/\alpha} N^{-\frac{1}{\alpha(\varepsilon-1)}}$, (5) determines the optimal number of varieties of the social planner N^{SP} by

$$m - \psi^{1/\alpha}\left(N^{SP}\right)^{-\frac{1}{\alpha(\varepsilon-1)}} = \mu\varepsilon N^{SP}. \tag{6}$$

Part (b).

Let us now suppose that the social planner is not able to control prices, i.e. he has to take the monopolistic prices $p = \frac{\varepsilon}{\varepsilon-1}\psi$ as given. The ideal price index in this case is given by $P = \frac{\varepsilon}{\varepsilon-1}\psi N^{1/(1-\varepsilon)}$. Hence, the only difference from the problem solved in Part (a) is, that the consumption good is now more expensive (relative to the $y-$good), as the monopolistic pricing decision involves the mark-up $\frac{\varepsilon}{\varepsilon-1}$. Hence the social planer solves the problem

$$\max_{y,N} \frac{m-y-N\mu}{\frac{\varepsilon}{\varepsilon-1}\psi} N^{\frac{1}{\varepsilon-1}} + y^{1-\alpha}/\left(1-\alpha\right),$$

which has the first-order conditions

$$\begin{aligned} y^{-\alpha} &= \frac{1}{\frac{\varepsilon}{\varepsilon-1}\psi} N^{\frac{1}{\varepsilon-1}} \\ m-y &= \mu\varepsilon N. \end{aligned}$$

Note especially that the second condition $m-y=\mu\varepsilon N$ is not affected by the different pricing. Similarly to (6), the optimal number of varieties N^C (with the constraint that prices cannot be changed) solves the equation

$$m - \left(\frac{\varepsilon}{\varepsilon-1}\psi\right)^{1/\alpha} \left(N^C\right)^{\frac{-1}{\alpha(\varepsilon-1)}} = \mu\varepsilon N^C. \tag{7}$$

Part (c).

Consider finally the equilibrium number of varieties, which is determined by free entry. To do so we have to find the expression for monopolistic profits. From the consumers' first-order condition we get that for each variety i

$$C^{1/\varepsilon} c_i^{-1/\varepsilon} = p_i y^{-\alpha}. \tag{8}$$

Since the monopolist of variety i faces an isoelastic demand function, the monopolistic price is given by

$$p_i = p_j = \frac{\varepsilon}{\varepsilon-1}\psi, \tag{9}$$

which immediately implies that $c_i = c_j = c$, i.e. all varieties are consumed by the same amount. From the definition of C we therefore get that

$$C = \left(\sum_{i=1}^{N} c_i^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = cN^{\frac{\varepsilon}{\varepsilon-1}}. \tag{10}$$

Substituting (9) and (10) into (8), we arrive at

$$C^{1/\varepsilon} c^{-1/\varepsilon} = N^{1/(\varepsilon-1)} = \frac{\varepsilon}{\varepsilon-1}\psi y^{-\alpha}. \tag{11}$$

Together with the budget constraint[1]

$$m = y + \sum_{i=1}^{N} p_i c_i = y + \frac{\varepsilon}{\varepsilon-1}\psi N c = y + \frac{\varepsilon}{\varepsilon-1}\psi N^{\frac{-1}{\varepsilon-1}} C \tag{12}$$

we get two equations in two unknowns (C and y) which we can solve. Substituting y from (11) into (12) yields

$$C = \frac{\varepsilon-1}{\varepsilon\psi} N^{\frac{1}{\varepsilon-1}} \left(m - \left(\frac{\varepsilon}{\varepsilon-1}\psi\right)^{1/\alpha} N^{\frac{-1}{\alpha(\varepsilon-1)}}\right) \tag{13}$$

[1]Note that $m = y + \sum_{i=1}^{N} p_i c_i$ is the correct budget constraint for the respresentative consumer. Even though the consumer is the owner of the N firms in the market and will therefore receive the profits $\sum_{i=1}^{N} \pi_i = N\pi$, those profits are exactly spent on the entry costs $N\mu$. Hence, the consumer has only his initial income m, which he can spend on the two consumption goods C and y.

as a function of N and parameters.

To solve for the equilibrium number of firms N^{EQ}, we have to derive the monopolistic profits in this economy. These are given by

$$\begin{aligned}\pi &= (p_i - \psi)c_i = \frac{1}{\varepsilon - 1}\psi C N^{\frac{-\varepsilon}{\varepsilon-1}} \\ &= \frac{1}{\varepsilon}N^{-1}\left(m - \left(\frac{\varepsilon}{\varepsilon-1}\psi\right)^{1/\alpha} N^{\frac{-1}{\alpha(\varepsilon-1)}}\right),\end{aligned}$$

where the second line followed upon substituting (13). Hence, the equilibrium number of firms N^{EQ} is given by the zero profit condition $\pi = \mu$, which in this example is given by

$$m - \left(\frac{\varepsilon}{\varepsilon-1}\psi\right)^{1/\alpha}\left(N^{EQ}\right)^{\frac{-1}{\alpha(\varepsilon-1)}} = \mu\varepsilon N^{EQ}. \tag{14}$$

When we compare the respective conditions (6), (7) and (14) we see that the structure is really similar and that we can learn about the sources of the differences between those allocations. Consider first the equilibrium number of varieties N^{EQ} determined in (14). This condition is exactly the same as for the number of varieties N^C the social planner would choose if he would have to take monopolistic prices as given (determined in (7)). In fact this result is relatively general in this kind of model. Dixit and Stiglitz (1977) work with the more general utility function

$$U\left(y, \left(\sum_{i=1}^{N} c_i^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right) \tag{15}$$

and show that a social planner who is choosing p_i, N and c_i subject to the constraint that each monopolist has to break even will in fact set $p_i = \frac{\varepsilon}{\varepsilon-1}\psi$. Furthermore they show that even in this more general case the social planner will choose the same number of firms as in the equilibrium allocation.

When we compare (7) or (14) to (6), we see that the difference between the optimal and the equilibrium number of varieties comes from the fact that the social planner internalizes that the marginal rate of transformation between a new variety and the y-good is equal to the marginal costs ψ and not equal to $\frac{\varepsilon}{\varepsilon-1}\psi$ as in either (7) or (14). Hence, the only source of distortions in the equilibrium number of varieties comes from the fact that prices are set monopolistically. Conditional on equilibrium prices, the zero-profit condition determines the number of varieties at exactly the number the social planner would also have chosen.

To see that the unconstrained social planner will in fact provide strictly more varieties, i.e. $N^{SP} > N^{EQ}$, consider the following argument. Although this could also be shown from the first-order conditions, we think the proof below is instructive as it illustrates various important properties of the Dixit-Stiglitz model. It is also closely related to the original argument provided in Dixit and Stiglitz (1977). We showed above that the consumers' problem can be thought of as choosing the two goods C and y with prices $p_C = P$ and $p_y = 1$. Hence, in both the equilibrium and the social planners solution the marginal condition $\frac{\partial U/\partial C}{\partial U/\partial y} = \frac{p_C}{p_y} = P$ will hold true. With the utility function assumed above this yields

$$P = \frac{\partial U/\partial C}{\partial U/\partial y} = \frac{1}{y^{-\alpha}} = y^{\alpha}. \tag{16}$$

Hence, y is increasing in P. Above we showed that $P^{SP} = \psi N^{1/(1-\varepsilon)} < \frac{\varepsilon}{\varepsilon-1}\psi N^{1/(1-\varepsilon)} = P^{EQ}$, i.e. due to the monopolistic distortions, the equilibrium price index will be higher. (16) then implies that $y^{EQ} > y^{SP}$, i.e. in equilibrium a higher quantity of the y-good will be consumed. But now note that we will have $U(C^{SP}, y^{SP}) > U(C^{EQ}, y^{EQ})$. This follows simply from the fact that the social planner could have chosen to set the monopolistic prices $p = \frac{\varepsilon}{\varepsilon-1}\psi$ but decided not to. As U is increasing in both arguments and $y^{EQ} > y^{SP}$, it will necessarily be the case that $C^{SP} > C^{EQ}$, i.e. given that less of the y-good will be consumed, the social planner will provide more of the consumption aggregate C. Intuitively, this could either be achieved by $c^{SP} > c^{EQ}$ or $N^{SP} > N^{EQ}$. Economically speaking, the social planner could either increase the scale of each firm and save the fixed costs expenses or he could exploit the aggregate demand externality and chose a higher number of firms.

To see that he will decide to use the latter channel, we are going to show that the social planner will in fact choose the same consumption level of each variety as the equilibrium allocation, i.e. $c^{SP} = c^{EQ}$. To see this, note that from the budget constraint we have that

$$m = p^{SP} c^{SP} N^{SP} + y^{SP} + N^{SP} \mu.$$

Hence,

$$c^{SP} = \frac{m - y^{SP} - N^{SP}\mu}{\psi N^{SP}}, \tag{17}$$

where we substituted $p^{SP} = \psi$. Now note that the first-order condition of the social planner (see (5)) is given by $m - y^{SP} = \mu\varepsilon N^{SP}$, so that (17) implies that

$$c^{SP} = \frac{\mu\varepsilon N^{SP} + N^{SP}\mu}{\psi N^{SP}} = \frac{\mu\varepsilon - \mu}{\psi} = \frac{\varepsilon - 1}{\psi}\mu. \tag{18}$$

This determines the variety-specific consumption level in the social planner's allocation as a function of parameters only.

Now consider the equilibrium. The profit of each firm producing one variety is given by

$$\pi = \left(p^{EQ} - \psi\right) c^{EQ} = \frac{1}{\varepsilon - 1}\psi c^{EQ}$$

where the second equality uses (9). In equilibrium, firms make zero profits, i.e. we will have $\pi = \mu$. This however implies that

$$c^{EQ} = \frac{\varepsilon - 1}{\psi}\mu. \tag{19}$$

Hence, (18) and (19) show that $c^{EQ} = c^{SP}$, i.e. in both the equilibrium and the optimal allocation the consumption level of each variety is exactly the same. Using this and the definition of the consumption aggregate C (see (1)), we therefore get that

$$C^{SP} = c^{SP}\left(N^{SP}\right)^{\frac{\varepsilon}{\varepsilon-1}} = c^{EQ}\left(N^{EQ}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(\frac{N^{SP}}{N^{EQ}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = C^{EQ}\left(\frac{N^{SP}}{N^{EQ}}\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

which from $C^{SP} > C^{EQ}$ directly implies that $N^{SP} > N^{EQ}$. Hence, the social planner will provide the same amount of each variety as in the equilibrium but will provide a larger number of varieties. Again, this result is not a consequence of the special structure of the preferences assumed in this exercise. Dixit and Stiglitz (1977) show that the same result is true for general preferences of the form given in (15).

Exercise 2

Part (a)

An equilibrium in this economy are consumption levels, machine expenditures and research expenses $[C(t), X(t), Z(t)]_{t=0}^{\infty}$, wages, prices for intermediary products and value functions $[w(t), [p^x(\nu,t)]_{\nu=1}^{N(t)}, [V(\nu,t)]_{\nu=1}^{N(t)}]_{t=0}^{\infty}$ and interest rates $[r(t)]_{t=0}^{\infty}$ such that markets clear, the allocation is consistent with utility maximization of the representative household, firms maximize profits, the evolution of $N(t)$ is consistent with the innovation possibilities frontier

$$\dot{N}(t) = \eta N(t)^{-\phi} Z(t), \tag{20}$$

and the value function is consistent with free entry.

Note that there are negative externalities in innovation, that is, the greater the number of machines, the more costly it is to innovate a new machine. This specification for the R&D technology corresponds to a view where innovation ideas are driven from a common pool and innovation today creates a fishing out effect and makes future innovations more difficult. Except for the innovation possibilities frontier the structure of this economy is entirely analogous to the baseline model of endogenous growth. As machine demand will be isoelastic, the monopolistic price of intermediaries is again given by

$$p^x(\nu,t) = \frac{\psi}{1-\beta} = 1,$$

where the last equality follows from our normalization $\psi = 1-\beta$. The labor market is competitive so that wages are given by the marginal product of labor which is just

$$\frac{\partial Y(t)}{\partial L} = \frac{\beta}{1-\beta}\int_0^{N(t)} x(\nu,t)^{1-\beta} d\nu L^{\beta-1} = \frac{\beta}{1-\beta}N(t),$$

where we used that $x(\nu,t) = L(t)$ for all ν. To derive the free entry condition in this economy we again have to derive the value function. As per period profits of research firms are still given by

$$\pi(\nu,t) = \beta L(t) \tag{21}$$

and the value function solves the Hamilton-Jacobi-Bellman equation

$$r(t)V(\nu,t) - \dot{V}(\nu,t) = \pi(\nu,t), \tag{22}$$

we get that along the BGP (where interest rates are constant and equal to r^*) the value function is given by

$$V(\nu,t) = \frac{\pi(t)}{r^* - g_\pi} = \frac{\beta L\,(t)}{r^* - n},$$

where the last equality follows since profits grow at the same rate n as $L\,(t)$ (see (21)).

Using (20) the free entry condition in this economy is given by

$$\eta N(t)^{-\phi} V(\nu,t) = \eta N(t)^{-\phi}\frac{\beta L(t)}{r^* - n} \leq 1 \text{ with equality if } Z\,(t) > 0. \tag{23}$$

To understand (23), note that one unit of the final good invested in research yields a flow rate of innovation equal to $\eta N(t)^{-\phi}$ and each innovation has a value of $V(\nu,t)$.

Part (b)

Now consider the case where population is constant, i.e. $n = 0$ and $L(t) = L$. In that case, the free entry condition (23) requires that on the BGP we have

$$\eta N(t)^{-\phi}\frac{\beta L}{r^*} \leq 1 \text{ with equality if } Z\,(t) > 0. \tag{24}$$

If this condition is satisfied with strict inequality, then we have $Z\,(t) = 0$ and $N\,(t)$ remains constant. If it is satisfied with equality, then $N\,(t)$ is also constant at the level $(\eta\beta L/r^*)^{1/\phi}$. This shows that along the BGP we have

$$N(t) = N^* \geq \left(\eta\frac{\beta L}{r^*}\right)^{1/\phi}. \tag{25}$$

Hence, there will be no growth and total output is constant. From the consumer's Euler equation we then get that $r^* = \rho$ as consumption has to be constant too. That consumption is constant follows from the fact that (20) implies that $Z(t) = 0$ once $N(t)$ reaches its long-run level determined by (25), so that from the resource constraint, consumption is given by

$$Y(t) - X(t) = \frac{\beta(2-\beta)}{1-\beta}N(t)L = C(t).$$

Hence, as long as there is no population growth, the economy will not be able to generate sustained growth. The reason is the following: with population being constant, the profits from intermediary producers are constant over time. However, R&D gets more and more expensive as the flow rate of innovation is decreasing in the current level of varieties. Hence, there is no endogenous growth in this model as long as the population is constant.

Note that when $N(0) < N^*$, $N(t)$ will *converge to* N^*, as in contrast to the baseline model of the lab equipment formulation, this economy will have transitional dynamics. Along the transition path, $N(t)$ will gradually increase to N^*, while the interest rate will gradually decline to $r^* = \rho$. Note also that when $N(0) > N^*$, the free entry condition in (24) will be slack. However, since there is no depreciation of machines, $N(t)$ will remain at the higher level and thus this economy has a continuum of steady states.

Part (c)

Consider now the case where the population grows over time at rate n. Again we can use the free entry condition to determine the joint evolution of $N(t)$ and $L(t)$. On an equilibrium with positive R&D, the free entry condition (23) will be satisfied with equality, so that

$$1 = \eta N(t)^{-\phi} V(\nu, t). \tag{26}$$

Differentiating this condition with respect to time yields

$$\frac{\dot{V}(\nu,t)}{V(\nu,t)} = \phi \frac{\dot{N}(t)}{N(t)} = \phi g_N(t). \tag{27}$$

From the HJB equation (22) we therefore get that

$$V(\nu,t) = \frac{\pi(\nu,t)}{r(t) - \frac{\dot{V}(\nu,t)}{V(\nu,t)}} = \frac{\beta L(t)}{r(t) - \phi g_N(t)}.$$

Along the BGP interest rates are constant and $N(t)$ grows at a constant rate g_N. Hence, $V(\nu,t)$ is given by $V(\nu,t) = \frac{\beta L(t)}{r - \phi g_N}$, so that

$$\frac{\dot{V}(\nu,t)}{V(\nu,t)} = \frac{\dot{L}(t)}{L(t)} = n.$$

Hence, (27) implies that

$$\frac{\dot{N}(t)}{N(t)} = g_N(t) = \frac{n}{\phi} > 0. \tag{28}$$

The reason why the economy now generates sustained growth is precisely that research becomes more valuable over time as population growth increases per period profits (by the usual market size effect). This counteracts the fact that research becomes more costly due to the congestion effects $N(t)^{-\phi}$. It is still the case that total output is given by

$$Y(t) = \frac{1}{1-\beta} \int_0^{N(t)} x(\nu,t)^{1-\beta} d\nu L(t)^{\beta} = \frac{1}{1-\beta} N(t) L(t),$$

so that

$$g_Y = \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{N}(t)}{N(t)} + \frac{\dot{L}(t)}{L(t)} = \frac{1+\phi}{\phi} n.$$

Similarly we can show that research expenditures $Z(t)$ and total consumption expenditures $C(t) = c(t)L(t)$ grow at the same rate. To see this, note that from from the innovation possibilities frontier (20) and Eq. (28) we have

$$\frac{\dot{N}(t)}{N(t)} = \frac{n}{\phi} = \eta \frac{Z(t)}{N(t)^{1+\phi}}.$$

Hence, $\frac{Z(t)}{N(t)^{1+\phi}}$ has to be constant, so that

$$\frac{\dot{Z}(t)}{Z(t)} = (1+\phi)\frac{\dot{N}(t)}{N(t)} = (1+\phi)\frac{n}{\phi} = g_Y,$$

i.e. $Z(t)$ is proportional to $Y(t)$. Therefore we can write $Z(t) = zY(t) = z\frac{1}{1-\beta}N(t)L(t)$, so that the resource constraint implies

$$\begin{aligned} C(t) &= Y(t) - X(t) - Z(t) \\ &= \left[\frac{\beta(2-\beta) - z}{1-\beta}\right] N(t)L(t). \end{aligned}$$

This shows that aggregate consumption is also proportional to $N(t)L(t)$, i.e. grows at rate

$$g_C = \frac{\dot{N}(t)}{N(t)} + \frac{\dot{L}(t)}{L(t)} = \frac{n}{\phi} + n = \frac{1+\phi}{\phi}n,$$

and per capita consumption grows at the same rate as the number of varieties $N(t)$, that is

$$g_c = \frac{n}{\phi} = \frac{1}{\theta}(r^* - \rho), \tag{29}$$

where the last equality is simply the Euler equation. Note that the described path will correspond to a BGP equilibrium with positive growth if

$$0 < \frac{n\,(1-\theta)}{\phi} < \rho - n,$$

where the second inequality ensures that the transversality condition holds.

Note that there are transitional dynamics in this economy. In particular, (26) and (29) imply that on a BGP, we have

$$\frac{N\,(t)^{\phi}}{L\,(t)} = \frac{\eta\beta}{r^* - n} = \frac{\eta\beta}{\theta g_c + \rho - n} = \frac{\eta\beta}{n\left(\frac{\theta}{\phi} - 1\right) + \rho - n} = \left(\frac{N^{\phi}}{L}\right)^{BGP}. \tag{30}$$

Hence, if $N\,(0)^{\phi}/L\,(0)$ ratio is below this level, that is, the economy starts with a low level of technology relative to its population, then $N\,(t)$ will initially grow faster than n/ϕ and $N\,(t)^{\phi}/L\,(t)$ will gradually increase towards its BGP value given in (30). Intuitively, the economy initially has higher incentives to innovate (since the diminishing returns to innovation, $N\,(t)^{-\phi}$, have not kicked in yet) and grows faster along the transition path.

Finally, note that the equilibrium is not Pareto optimal, but the socially planned economy does not always feature higher growth than the equilibrium allocation. In this model, there are both monopoly distortions and negative technological externalities in innovation. Without the technology externalities, monopoly distortions would make the equilibrium grow at a slower rate, because entrants do not capture the entire surplus of the innovation. However, the technological externalities create an opposing force, since each innovating firm fails to take into account the fact that it is making innovation for future firms more difficult. A social planner will internalize this effect and thus may want to slow down innovation and growth.[2]

Exercise 3

Part (a)

We first characterize the static equilibrium for a given number of machines $N\,(t)$ and $L_E\,(t)$ employed in the production sector. In particular, we calculate the aggregate variables $Y\,(t)\,, w\,(t)$ and firm profits $\pi\,(\nu, t)$ for a given level of $N\,(t)$ and $L_E\,(t)$, which we then use to consider the dynamic trade-offs in this economy.

Characterization of the Static Equilibrium for given $N\,(t)$ and $L_E\,(t)$. We denote by $p\,(\nu, t)$ the price of the monopolist. We normalize the price of the final good to 1, i.e. $p^Y\,(t) = 1$ for each period. Final good firms are competitive hence they solve

$$\max_{\{y(\nu,t)\}_{\nu=0}^{N(t)}} \left[\int_0^{N(t)} y\,(\nu, t)^{\beta}\, d\nu\right]^{1/\beta} - \int_0^{N(t)} y\,(\nu, t)\, p\,(\nu, t)\, d\nu.$$

[2] Analyzing the social planner's problem shows that the number of varieties in the social planner's allocation asymptotically grows at the same constant rate n/ϕ, but it may grow slower than the equilibrium allocation along the transition path to the BGP.

The first-order condition for $y(\nu,t)$ gives the isoelastic demand for intermediate goods

$$y(\nu,t) = p(\nu,t)^{-1/(1-\beta)}\, Y(t)\,. \tag{31}$$

We assume $\beta \in (0,1)$ so that the demand elasticity for each monopolist, $1/(1-\beta)$, is between $(1,\infty)$, since otherwise the monopolist either charges an infinite price or shuts down production. Note also that the ideal price index (the unit cost of producing the final good) is equal to the price of the final good, which is normalized to 1, that is

$$\int_0^{N(t)} p(\nu,t)^{-\beta/(1-\beta)}\, d\nu = 1. \tag{32}$$

Note that the intermediate monopolists maximize profits, i.e. they solve the problem

$$\max_{p(\nu,t)} y(\nu,t)\,(p(\nu,t) - w(t))\,,$$

where $y(\nu,t)$ is given by the isoelastic demand in Eq. (31). The optimal monopoly price is

$$p(\nu,t) = \frac{1}{\beta} w(t)\,, \tag{33}$$

i.e. each monopolist charges a constant markup over its marginal cost. Plugging in the prices from Eq. (33) in the ideal price index equation (32), we have

$$w(t) = \beta N(t)^{(1-\beta)/\beta}\,. \tag{34}$$

In other words, wages in this economy are uniquely pinned down by the number of varieties.

Given that each firm charges the same price (cf. Eq. (33)), the demand for each firm is also the same (cf. Eq. (31)), consequently labor employed by each firm is also the same. Hence, if the total labor employed is $L_E(t)$, we have

$$y(\nu,t) = l(\nu,t) = \frac{L_E(t)}{N(t)}. \tag{35}$$

Each firm's per period profits are then given by

$$\begin{aligned} \pi(\nu,t) &= y(\nu,t)\,(p(\nu,t) - w(t)) \\ &= \frac{L_E(t)}{N(t)} \frac{1-\beta}{\beta} w(t) \\ &= (1-\beta)\, L_E(t)\, N(t)^{(1-2\beta)/\beta}\,, \end{aligned} \tag{36}$$

where the last line substitutes from Eq. (34). Substituting Eq. (35) also gives an expression for the final output,

$$Y(t) = \left[\int_0^{N(t)} y(\nu,t)^\beta\, d\nu\right]^{1/\beta} = L_E(t)\, N(t)^{(1-\beta)/\beta}\,. \tag{37}$$

Note that the output is linearly increasing in labor employed, and is also increasing (nonlinearly) in the number of varieties. This completes our characterization of the static equilibrium.

Characterization of the Dynamic Equilibrium. We conjecture a BGP equilibrium on which $L_E(t) = L - L_R^*$ is constant, $r(t) = r^*$ is constant and $N(t)$ grows at a constant rate g_N. Note that this already implies by Eqs. (34) and (37) that wages and output also grow at the same rate at our conjectured BGP (albeit at a different constant rate). Recall that the value function $V(\nu,t)$ for firm ν at time t can be expressed as the discounted sum of future profits

$$V(\nu,t) = \int_t^\infty \exp(-r^*(t'-t))\, \pi(\nu,t')\, dt'. \tag{38}$$

Note also that by Eq. (36), profits grow (or shrink) at the constant rate $g_N (1-2\beta)/\beta$. Hence the value function can be solved from the previous integral as

$$V(\nu,t) = \frac{\pi(\nu,t)}{r^* - g_N(1-2\beta)/\beta} \tag{39}$$

$$= \frac{(1-\beta) L_E^* N(t)^{(1-2\beta)/\beta}}{r^* - g_N(1-2\beta)/\beta}, \tag{40}$$

where the second line substitutes the expression for profits from Eq. (36).[3] In particular, the value function in this economy also grows at the same rate as profits $g_N(1-2\beta)/\beta$.

We next consider the free entry condition, which, in this economy takes the form

$$\eta N(t)^{\phi} V(\nu,t) = w(t). \tag{41}$$

Using the fact that $\phi = 1$ for this part and substituting the expressions for $V(\nu,t)$ from (40) and $w(t)$ from Eq. (34), we have

$$\begin{aligned} \eta N(t) \frac{(1-\beta) L_E^* N(t)^{(1-2\beta)/\beta}}{r^* - g_N(1-2\beta)/\beta} &= \beta N(t)^{(1-\beta)/\beta} \\ \eta \frac{(1-\beta)(L-L_R^*)}{r^* - g_N(1-2\beta)/\beta} &= \beta, \end{aligned} \tag{42}$$

which shows that a BGP equilibrium is possible. Eq. (42) provides a relation between the growth rate g_N, the interest rate r^* and the share of labor employed in production L_E^*. Note how the growing terms canceled out of the innovation trade-off. In this economy, the value of new blueprints grow (or shrink) at rate $w(t)/N(t)$ and new blueprints are created by R&D labor. The externalities in the R&D technology are chosen at exactly the right level (proportional to $N(t)$) so that the cost of R&D also grows at rate $w(t)/N(t)$, hence the innovation incentives are balanced.

Next, note that the R&D technology $\dot{N} = \eta N L_R$ provides another expression for g_N

$$g_N = \eta L_R^*. \tag{43}$$

Plugging this value of g_N in Eq. (42), we get

$$\eta \frac{(1-\beta)(L-L_R^*)}{r^* - \eta L_R^*(1-2\beta)/\beta} = \beta. \tag{44}$$

Note also that that in this economy all output is consumed (output is not an input to any production process) hence market clearing for the final good implies $C(t) = Y(t)$. From Eq. (37), $Y(t)$ grows at rate $g_N(1-\beta)/\beta$, hence $C(t)$ also grows at this rate. Then, the Euler equation implies

$$g_N \frac{1-\beta}{\beta} = \frac{1}{\theta}(r^* - \rho) = \eta L_R^* \frac{1-\beta}{\beta}, \tag{45}$$

where the second equality uses Eq. (43). Note that Eqs. (44) and (45) constitute 2 equations in two unknowns L_R^*, r^*. These equations characterize the dynamic trade-off in this economy, that is, they characterize how labor is allocated between production and research so as to balance consumer's preferences [cf. Eq. (45)] and the value from further innovation [cf. Eq. (44)]. The equations have a unique solution given by

$$L_R^* = \frac{(1-\beta) L - \rho\beta/\eta}{\theta(1-\beta)+\beta} \tag{46}$$

[3] This expression could also be derived using the Hamilton–Jacobi-Bellman equation

$$r(t) V(\nu,t) = \pi(\nu,t) + \dot{V}(\nu,t),$$

which is essentially a convenient way to represent the integral in Eq. (38).

and the growth rate of varieties is

$$g_N = \frac{\eta(1-\beta)L - \rho\beta}{\theta(1-\beta)+\beta}. \tag{47}$$

The growth rate of output (and consumption) is given by $g_C \equiv g_N(1-\beta)/\beta$. The interest rate can also be solved as

$$r^* = \frac{\eta\theta\frac{1-\beta}{\beta}L + \frac{\beta}{1-\beta}\rho}{\theta + \frac{\beta}{1-\beta}}.$$

Finally, we have to make assumptions on parameters such that growth is positive and the transversality condition holds. For positive growth, we assume

$$(1-\beta)L > \rho\beta/\eta$$

and to satisfy the transversality condition $\lim_{t\to\infty}\exp(-r^*t)N(t)V(\nu,t) = 0$, we assume $r^* > g_N + g_V$ (or equivalently, $g_C(1-\theta) < \rho$), which gives

$$\eta L_R^* \frac{1-\beta}{\beta}(1-\theta) < \rho.$$

These assumptions jointly also ensure that L_R^* in Eq. (46) lies in $(0, L)$ so that the equilibrium path above is well defined. It can then be verified that the path we have described is an equilibrium. In equilibrium, starting at any $N(0)$, a constant share of labor L_R^* is employed in R&D, $N(t)$ grows at a constant rate g_N, and $C(t) = Y(t)$ and $w(t)$ grows at the constant rate $g_C = g_N(1-\beta)/\beta$, where L_R^* and g_N are given in terms of parameters as before. Moreover, this path is an equilibrium starting with any $N(0)$ hence there are no transitional dynamics.

Note from Eq. (47) that the long run growth rate in this economy increases with L and decreases with θ. The growth rate increases in L due to two effects that work in the same direction. The first effect is a standard market size effect: the larger L, the larger the population employed in production, the larger profits (cf. Eq. (36)) and the larger the incentives for innovation, leading to a higher growth rate g_N. The second effect is that since the R&D sector also uses labor, with a larger L, a larger population can be employed in research without increasing wages (the cost of R&D), which leads to a higher growth rate. The growth rate decreases in θ, the inverse of the elasticity of intertemporal substitution. When consumption is less substitutable between today and tomorrow, consumers prefer a flatter consumption profile for a given interest rate, which reduces savings and the investment in R&D.

Part (b)

We claim that there cannot be a BGP equilibrium in which the interest rate is constant and the number of varieties grow at a constant rate. Suppose, to reach a contradiction, that there is such a BGP. The R&D technology equation, $L_R(t) = \left(\dot{N}(t)/N(t)\right)/\eta = L_R^*$, implies that the labor employed in R&D is also constant. Then, the analogue of Eq. (42) applies to this economy, that is, we have

$$\eta\frac{(1-\beta)(L(t) - L_R^*)}{r^* - g_N(1-2\beta)/\beta} = \beta. \tag{48}$$

This equation cannot be satisfied for all t when population grows, which yields a contradiction and proves that there does not exist a BGP. Intuitively, the value of a machine grows faster than the cost of producing a machine since the monopolists' profits are increasing in population through the market size effect. Consequently, the free entry condition will be violated on a BGP allocation of this kind. The only way to restore the free entry condition is to employ more and more labor in the R&D sector, which increases the growth rate (and hence the interest rate) and which reduces employment in production and hence profits. Both of these effects will reduce the value of the firm and will help restore the free entry condition Eq. (48). Therefore, in equilibrium, we expect to have more and more of labor employed in R&D and we expect the growth rate to be increasing, which can also be seen from the R&D technology equation $\dot{N}(t) = \eta N(t) L_R(t)$.

Part (c)

The static equilibrium characterization of Part (a) continues to apply, that is, for a given $L_E(t)$ and $N(t)$, wages, profits and output are still given by Eqs. (34), (36) and (37).

For the dynamic analysis, we conjecture a BGP equilibrium in which $r(t) = r^*$, $L_R(t) = l_R^* L(t)$ for some $l_R^* \in (0,1)$ and the growth rate of $N(t)$ is constant. The calculation of the value function is now slightly changed from Eq. (39) to

$$V(\nu,t) = \frac{\pi(\nu,t)}{r^* - n - g_N(1-2\beta)/\beta}, \tag{49}$$

where the denominator now also features n since population growth leads to further growth in profits. After plugging in the static equilibrium values for $\pi(\nu,t)$ in the value function (49) and using the static equilibrium value of wages $w(t)$, the free entry condition Eq. (41) can be written as

$$\begin{aligned} N(t)^{\phi} \frac{(1-\beta) l_E^* L(t) N(t)^{(1-2\beta)/\beta}}{r^* - n - g_N(1-2\beta)/\beta} &= \beta N(t)^{(1-\beta)/\beta}. \\ N(t)^{\phi-1} L(t) \frac{(1-\beta)(1-l_R^*)}{r^* - n - g_N(1-2\beta)/\beta} &= \beta \end{aligned} \tag{50}$$

Differentiating both sides with respect to t, the growth rate of N is uniquely solved as

$$g_N = \frac{n}{1-\phi}. \tag{51}$$

In other words, this is the only growth rate for N that is consistent with the free entry condition.

Next note that we have another expression for the growth rate that comes from the R&D technology $\dot{N} = N(t)^{\phi} L(t) l_R^*$, which, after combining with Eq. (51), implies

$$\frac{n}{1-\phi} = \left[N(t)^{\phi-1} L(t)\right] l_R^*, \tag{52}$$

Note also that the representative consumer's problem gives the Euler equation

$$g_c = \frac{1}{\theta}\left((r^* - n) - (\rho - n)\right) = \frac{1}{\theta}(r^* - \rho).$$

Note that this time we have $c(t) L(t) = Y(t)$ from the final good resource constraint, hence g_c (growth rate of consumption per capita) is equal to $g_Y - n$. The expression for the final output $Y(t)$ in Eq. (37) implies $g_Y = n + \frac{1-\beta}{\beta} g_N$ and thus $g_c = \frac{1-\beta}{\beta} g_N$, that is, consumption per capita grows at the same rate as wages. Using this in the Euler equation, we have

$$\frac{n}{1-\phi}\frac{1-\beta}{\beta} = \frac{1}{\theta}(r^* - \rho). \tag{53}$$

Note that Eqs. $(50), (52)$ and (53) are three equations in three unknowns r^*, l_R^* and $N(t)^{\phi-1} L(t)$, which can uniquely be solved for. Under the parametric restrictions

$$(1-\theta) g_c < \rho - n,$$

the path we have constructed is indeed an equilibrium. Note that there are transitional dynamics in this case. In particular, our solution shows that $N(t)^{\phi-1} L(t) = \chi(t)$ must be at a specific value χ^* on the BGP. Starting from any other level, this ratio adjusts to its steady state level along the transitional path. For example, if $N(0)^{\phi-1} L(0) > \chi^*$, the economy starts with too few machines relative to labor and the economy would invest more in R&D early on, that is, $l_R(t)$ would decrease towards its steady state level l_R^*.

Note that in this case the growth rate of machines is given by $g_N = n/(1-\phi)$ and the growth rate of output and consumption per labor by $g_c = g_N(1-\beta)/\beta$. As opposed to Part (a), the growth rate in this economy does not depend on L but it depends on n. Intuitively, the knowledge externalities in this economy are not sufficient to generate growth and the engine of growth is the increase in the population. Ultimately, the same economic force (the market size effect) is present in both this economy and the economy analyzed in Part (a), but the effect is weaker here (due to diminishing externalities) and does not generate a scale effect, that is, the level of population does not increase the growth rate. The scale effect is a disputed aspect of the growth models, hence the fact that the present model does not feature a scale effect may be viewed as a success. Note, however, that the growth rate still depends on population growth, if not population level, hence scale effects are present but in a different guise. To see this, note that $N(t)^{\phi-1} L(t) = \chi^*$ is a constant independent of population as Eqs. $(50), (52)$ and (53) do not depend on $L(t)$. Therefore, a greater level of population translates into a higher level of $N(t)$ and $Y(t)/L(t)$ (cf. Eq. (37)), hence scale effects are still present if we compare, for example, cross country levels of output per capita.

Note also that, at first glance, the configuration $\phi < 1$ and $n > 0$ seems more plausible than the knife-edge case $\phi = 1$ and $n = 0$ of Part (a). On the other hand, the model with $\phi < 1$ and $n > 0$ is unappealing in the sense that the growth rate does not respond to variables that we think are important determinants of growth. For example, as opposed to the economy analyzed in Part (a), the growth rate does not respond to θ, moreover, if we added tax policy, the growth rate would not respond to that either. These models are sometimes called semi-endogenous growth models, and they are probably not a good representation of reality despite the fact that they apply for many more sets of parameters than models along the lines of the one analyzed in Part (a). There are other (perhaps more realistic) models we can write down which eliminates the scale effect but which retains the desirable properties of the model in Part (a) (see, for example, Howitt (1999)).

Exercise 4

Part (a)

An equilibrium is a collection of time paths of aggregate resource allocations, the set of machine varieties whose patents haven't expired (denoted by $N_1(t)$), the set of machine varieties whose patents expired (denoted by $N_2(t)$), quantities, prices and the value function for each machine, and interest rates and wages $\left[\begin{array}{c} Y(t), C(t), Z(t), X(t), \\ (p^x(\nu,t), x(\nu,t), V(\nu,t))|_{\nu \in N_1(t)}, [p^x(\nu, \end{array} \right.$ such that consumers choose consumption and asset holdings optimally, the evolution of patented machines is determined by free entry in R&D and the expiration of patents, machine producers with patents set prices to maximize profits, machines with expired patents are produced competitively, the final good is produced competitively, and asset and the final good markets clear.

We first characterize the static equilibrium allocations for given $N_1(t)$ and $N_2(t)$. The demand for machines from the final good producers is given by $x(\nu,t) = p^x(\nu,t)^{-1/\beta} L$. The machine producers with patents set the monopoly prices. Thus given the isoelastic demand, we have[4]

$$p^x(\nu,t) = \psi/(1-\beta) \text{ and } x(\nu,t) = \left(\frac{\psi}{1-\beta}\right)^{-1/\beta} L \text{ for } \nu \in N_1(t).$$

The monopolists' per period profits are

$$\pi(\nu,t) = \left(\frac{\psi}{1-\beta}\right)^{-(1-\beta)/\beta} \beta L. \tag{54}$$

The machines with expired patents are priced at marginal cost, hence we have

$$p^x(\nu,t) = \psi \text{ and } x(\nu,t) = \psi^{-1/\beta} L \text{ for } \nu \in N_2(t).$$

Total output is therefore given by

$$Y(t) = \frac{1}{1-\beta} L \psi^{-(1-\beta)/\beta} \left(N_1(t)(1-\beta)^{(1-\beta)/\beta} + N_2(t) \right), \tag{55}$$

[4]In this exercise, we do not impose the normalization assumption $\psi = 1-\beta$ to provide a slightly more general solution.

and equilibrium wages by

$$w(t) = \frac{\beta}{1-\beta}\psi^{-(1-\beta)/\beta}\left(N_1(t)(1-\beta)^{(1-\beta)/\beta} + N_2(t)\right).$$

Note also that the aggregate machine expenditure is given by

$$X(t) = L\psi^{-(1-\beta)/\beta}\left(N_1(t)(1-\beta)^{1/\beta} + N_2(t)\right). \tag{56}$$

We next turn to the dynamic trade-offs in this economy. The value function $V(\nu, t)$ for machine producers with patents satisfies the HJB equation

$$r(t)V(\nu,t) = \pi(\nu,t) + \dot{V}(\nu,t) - \iota V(\nu,t), \tag{57}$$

where the last term captures the fact that with a flow rate of ι, the firm loses the patent and its monopoly power at which point the value drops to 0. We are interested in equilibria in which $Z(t) > 0$ for all t, which implies that the value function is uniquely pinned down from free entry in R&D as

$$\eta V(\nu,t) = 1.$$

Using this and the expression for $\pi(\nu,t)$ in Eq. (54) to solve Eq. (57), we have that $r(t)$ is constant at all t and given by

$$r(t) = \left(\frac{\psi}{1-\beta}\right)^{-(1-\beta)/\beta}\eta\beta L - \iota.$$

Consumer optimization gives the Euler equation

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho),$$

hence the growth rate of consumption is also constant and given by

$$g = \frac{1}{\theta}\left[\left(\frac{\psi}{1-\beta}\right)^{-(1-\beta)/\beta}\eta\beta L - \iota - \rho\right]. \tag{58}$$

Since consumption grows at a constant rate, we have

$$C(t) = C(0)\exp(gt). \tag{59}$$

Next note that the evolution of $N_1(t)$ and $N_2(t)$ are given by

$$\begin{aligned} \dot{N}_1(t) &= \eta Z(t) - \iota N_1(t), \text{ with } N_1(0) \text{ given} \\ \dot{N}_2(t) &= \iota N_1(t), \text{ with } N_2(0) \text{ given,} \end{aligned} \tag{60}$$

where the expression $\iota N_1(t)$ in both equations capture the fact that the patent for each machine expires at a flow rate of ι. Now, using Eqs. (55), (56), (59), and market clearing in the final good, we have

$$Z(t) = \frac{1}{1-\beta}L\psi^{-(1-\beta)/\beta}\left(N_1(t)(1-\beta)^{1/\beta}\left[\frac{1}{1-\beta} - (1-\beta)\right] + \beta N_2(t)\right) - C(0)\exp(gt). \tag{61}$$

Plugging this in (60) gives us a set of differential equations with two variables $N_1(t)$ and $N_2(t)$ and two initial conditions, which can be solved for a given $C(0)$. Among the possible choices for $C(0)$, only one gives a stable solution for $N_1(t)$ and $N_2(t)$ where N_1 and N_2 asymptotically grow at rate g, and this solution satisfies all equilibrium requirements (the unstable solutions either violate the transversality condition or the resource constraints). Hence, the equilibrium is saddle path stable and is uniquely characterized by the two differential equations for N_1 and N_2.

We are interested in the BGP equilibrium, so we conjecture that N_1 and N_2 grow at the same constant rate as g. From the differential equation system in (60), we have that the BGP values of N_1 and N_2 must satisfy

$$\frac{N_1}{N_2} = \frac{g}{\iota} \tag{62}$$

Note also that from Eq. (60), we have

$$\begin{aligned} Z(t) &= \left(\dot{N}_1(t) + \iota N_1(t)\right)/\eta \\ &= N_1(t)(g+\iota)/\eta, \end{aligned}$$

where the second line uses our BGP conjecture that $N_1(t)$ grows at the constant rate g. Then, on our conjectured BGP, Eq. (61) can be rewritten

$$\begin{aligned} & N_1(0)\exp(gt)\frac{g+\iota}{\eta} \\ = & \left\{\frac{1}{1-\beta}L\psi^{-(1-\beta)/\beta}\left(N_1(0)(1-\beta)^{1/\beta}\left[\frac{1}{1-\beta}-(1-\beta)\right]+\beta N_1(0)\frac{\iota}{g}\right)-C(0)\right\}\exp(gt). \end{aligned}$$

Canceling the growing terms $\exp(gt)$ from each side and collecting the $N_1(0)$ terms, we have

$$C(0) = N_1(0)\left\{\frac{1}{1-\beta}L\psi^{-(1-\beta)/\beta}\left((1-\beta)^{1/\beta}\left[\frac{1}{1-\beta}-(1-\beta)\right]+\beta\frac{\iota}{g}\right)-\frac{g+\iota}{\eta}\right\},$$

which characterizes the initial level of consumption. We assume

$$\left(\left(\frac{\psi}{1-\beta}\right)^{-(1-\beta)/\beta}\eta\beta L-\iota\right)(1-\theta)<\rho, \tag{63}$$

so that the described path also satisfies the transversality condition, and

$$\left(\frac{\psi}{1-\beta}\right)^{-(1-\beta)/\beta}\eta\beta L-\iota>\rho, \tag{64}$$

so that there is positive growth (which we need to verify our assumption that there is positive R&D investment in equilibrium).

It follows that when the parametric restrictions in Eqs. (63) and (64) are satisfied and the initial values of the technology, $N_1(0)$ and $N_2(0)$, satisfy Condition (62), there exists a BGP equilibrium in which $N_1(t), N_2(t), C(t), Y(t), w(t)$ all grow at the constant rate g given by Eq. (58). Note also that if the initial levels of $N_1(0), N_2(0)$ do not satisfy Condition (62), then there will be transitional dynamics in this economy: $N_1(0)/N_2(0)$ ratio will monotonically converge to g/ι and the aggregate variables will asymptotically grow at rate g.

Part (b)

We have shown that the BGP growth rate is given by the expression in (58) hence the value of ι that maximizes the growth rate is $\iota^* = 0$. When patents expire faster, incentives for innovation are lower, that is, firms' expected profits are lower for a given interest rate. To have entry in the R&D sector, the interest rates will have to decline. With lower interest rates, consumers demand a flatter consumption profile and reduce their savings, which leads to lower investment in R&D and lower growth.

Part (c)

We first make a couple of observations about the nature of the distortions in this economy. Note that there are static monopoly distortions in this economy which reduce net output for a given level of machines $N(t)$. Note also that, as in the baseline expanding varieties model analyzed in Section **??**, there are dynamic distortions since the marginal value of a new technology is higher for the social planner for two reasons. First, the social planner takes into account the effect of new technologies on both wages and profits while the equilibrium firms only care about profits, and second, the social planner produces a higher net output for a given level of machines (since it avoids the monopoly distortions). Since the marginal value of a new technology is higher for the planner, the growth rate in the socially planned economy is also higher than the equilibrium growth rate.

Next, in view of these observations, we note that the effect of patents are two-fold. On the one hand, increasing ι increases the rate at which products become competitive and increases the static output for a given level of machines. This is best seen in Eq. (55): there is a coefficient $(1-\beta)^{1/\beta} < 1$ in front of $N_1(t)$, so for a given level of $N(t) = N_1(t) + N_2(t)$, total output is increasing in $N_2(t)$. The effect through this channel is welfare improving since it alleviates some of the static monopoly distortions. On the other hand, as we have seen in Part (b), increasing ι decreases the growth rate in this economy. Since the growth rate in the economy is less than optimal to begin with (as we have noted in the previous paragraph), increasing ι reduces welfare through this channel.

Depending on consumer preferences one or the other effect may dominate and increasing ι may be welfare improving or welfare reducing. The less patient the consumers are (the higher the discount rate ρ) and the lower the intertemporal substitution (the higher θ), the more likely it is that the first effect will dominate and increasing ι will be welfare enhancing. In this case, consumers care relatively more about consumption today and they dislike a growing consumption profile, hence they may prefer immediate benefits of a more competitive market to delayed benefits of the monopolistic market.

Viewed differently, increasing ι is not the best policy to cure the inefficiencies in this economy. This argument is also forcefully made by Romer (1990). To achieve efficiency, we need to reduce the distortions through the monopolistic mark-ups but we also need to give sufficient surpluses to the monopolists so they have the right incentives to innovate. When $\iota = 0$, a linear subsidy on the monopolist output (just enough to get the production to competitive levels) financed by a lump-sum tax on the consumers can decentralize the social planner's solution.[5] However, increasing ι is only an imperfect solution and may or may not be welfare improving. For a discussion along those lines, see also Romer (1987).

[5] Even though this policy is Pareto optimal in the model, in reality it would be difficult to implement and it may also be undesirable. If we add heterogeneity to the model and assume that the firms' shares are held by a small fraction of the population, this policy would most likely increase wealth inequality and may therefore be undesirable if the social planner has a preference for lower inequality.