Growth and the Fragmentation of Production

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Motivation

Since Adam Smith, economists have been postulating a link between specialization and productivity:

> *"The greatest improvement in the productive powers of labour [...] seem to have been the effects of the division of labour." (Wealth of Nations, Chapter 1, 1776)*

In the context of supply chains: how is value chain broken down into work done by different plants?

Motivation

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In the context of supply chains: how is value chain broken down into work done by different plants?

This paper: specialization in value chain *among plants* and growth. E.g. do plants produce shirts from cloth or from yarn?

1. Empirical facts about organization and performance using manufacturing data from India
   - macro: vertical specialization $\Leftrightarrow$ income per capita
   - micro: vertical specialization $\Leftrightarrow$ plant size
2. Model: positive predictions, implications for growth

Literature

Specialization and productivity:

- **Theory:** Young (1928), Stigler (1951), Rosen (1978), Baumgardner (1988), Becker and Murphy (1992), Rodriguez-Clare (1996), Akerman and Py (2010), Chaney and Ossa (2013), Limao and Xu (2021)
- **Empirical evidence:** Baumgardner (1988), Brown (1992), Garicano and Hubbard (2009), Duranton and Jayet (2011), Tian (2018), Hansman et al. (2020), Chor et al. (2020), Bergeaud et al. (2021), Bartelme et al. (2021), Atalay, Sotelo, and Tannenbaum (2021)

Smithian Growth:

- Boreland and Yang (1991), Kelly (1997), Legros, Newman, Proto (2014), Menzio (2020)

Indian Trade Liberalization:

- Panagariya (2004), Sivadasan (2009), Khandelwal and Topalova (2010), Goldberg et al. (2010), Peter and Ruane (2020)

Input-Output Structure and Growth:

- Ciccone (2002), Boehm (2022), Boehm and Oberfield (2020), Acemoglu and Azar (2020), Chenery et al (1986), Jones (2013), Fadinger, Ghiglino, and Teteryatnikova (2021), Bartelme and Gorodnichenko (2015)

Endogenous Production Networks:

- Oberfield (2018), Eaton, Kortum, Kramarz (2022), Lim (2018), Chaney (2014), Dhyne et al (2021), Startz (2021), Grant and Startz (2021), Taschereau-Dumochel (2017), Huneeus (2018), Miyauchi (2018), Panigrahi (2021)

Manufacturing Plants in India

Data: **Indian Annual Survey of Industries**, 1989/90-2014/15 (with gaps)

- Plant-level panel survey of formal manufacturing plants
  - All plants that have 100+ employees
  - 1/5 of all plants between 20 (10 if using power) and 100 employees

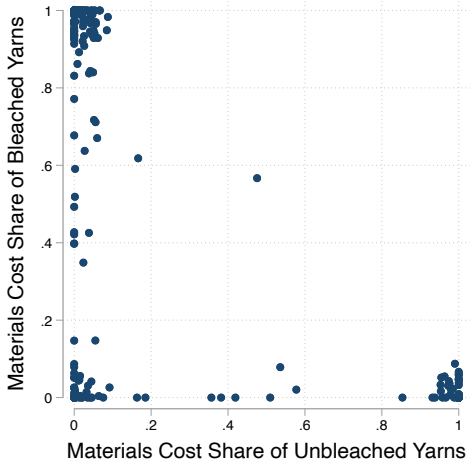
Most important part of the survey:

- Quantities, unit values & 5-digit product codes for all manufacturing output and intermediate inputs (domestic and imported)

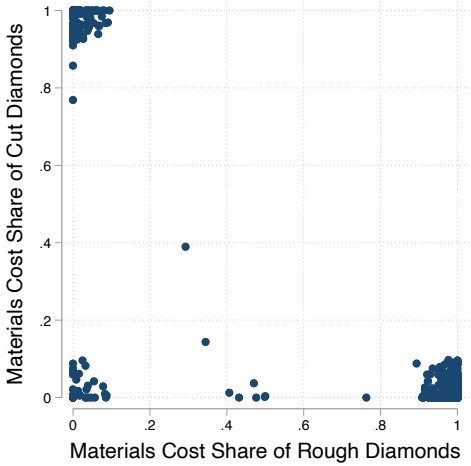
Example product codes: Silk yarn, bleached (61222), beryllium copper wire (72246), aluminium ingots (73107)

	min	p25	p50	p75	max	count
Multi-product	1	1	3	5	117	235,566
Single-product	1	1	2	4	69	359,894
Total	1	1	3	5	117	595,460

Within narrow industries, firms use different inputs



(a) Input mixes for Bleached Cotton Cloth (63303)

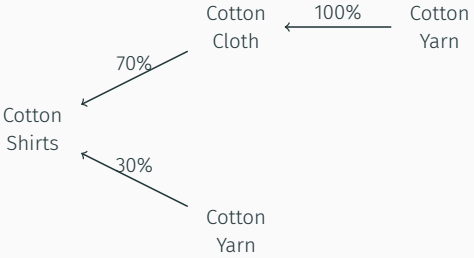


(b) Input mixes for Polished Diamonds (92104)

Measuring the vertical span of production (Boehm & Oberfield, 2020)

Fix an output industry.

1. **Vertical Distance of Input from Output** in supply chain: is input close or far from output?
   - Similar to upstreamness of Alfaro et al. (2019)



- Cloth: Close (1 plant away)
- Yarn: Farther (1.7 plants away)

2. **Vertical Span of Plant**: Does shirt producer use inputs that are distant or close?

$$\text{span}_{jt} = \sum_{\hat{\omega}} \frac{X_{j\hat{\omega}}}{\sum_{\tilde{\omega}} X_{j\tilde{\omega}}} d_{\omega\hat{\omega}}$$

Vertical Distance of inputs from output – Examples

Table 1: Vertical distance examples for 63428: *Cotton Shirts*

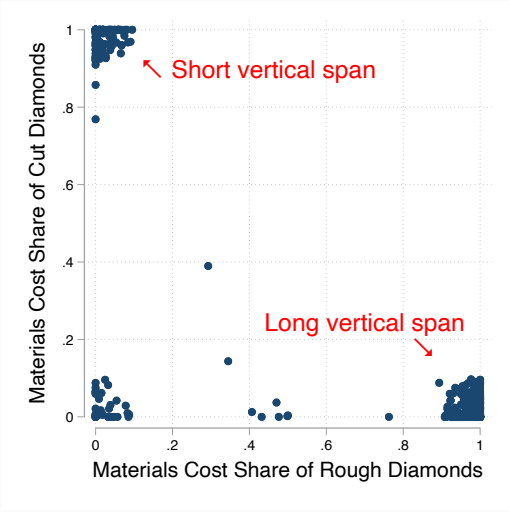
	Mean Vertical Distance
Fabrics/Cloths	1.66
Yarns	2.58
Ginned & pressed cotton	3.44
Raw cotton	4.09

Table 2: Vertical distance examples for 73107: *Aluminium Ingots*

	Vertical Distance
Anodes, copper	1.00
Aluminium scrap	1.19
Aluminium oxide	1.25
Bauxite, calcined	2.18
Caustic soda (sodium hydroxide)	2.39
Bauxite, raw	3.03
Coal	3.43

Long and short vertical span

Figure 1: Input mixes for Polished Diamonds (92104)



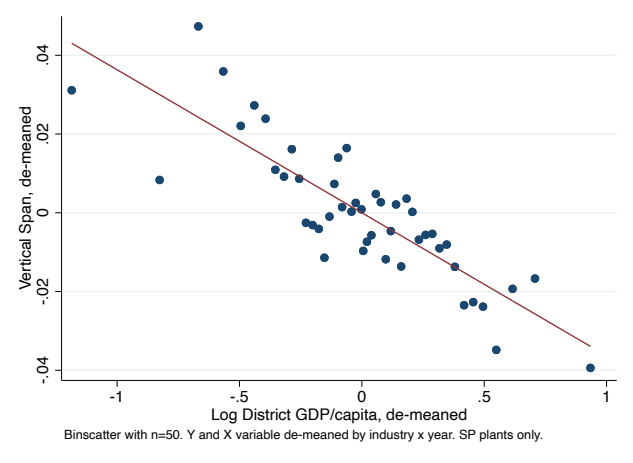
Motivational Facts about Vertical Specialization

Macro facts: Vertical specialization is positively correlated with development

Within industry × year:

Plants in **richer districts** are on average **more vertically specialized**

Holds in the **time dimension**:
In Indian states that grew faster, plants vert. specializing more ▸ show

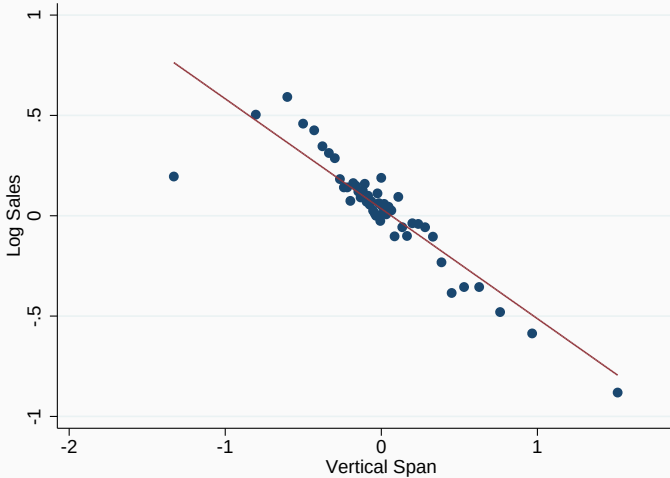


Micro facts: Vertical specialization is positively correlated with plant size

Within industry × year:

Plants with **higher sales** tend to have **shorter vertical span**

Holds also in the **time dimension**: Plants that v. specialize more grow faster show



SP plants only. Regression includes industry x year FE's.

Single-plant

Indian Trade Liberalization to get at causality

- **Until end of 80s:** India in near-autarky
  - Import licensing system
  - Very high tariffs. Large variation (up to 355%), average ∼80%. Was set in the 1950s.
- **July 1991:** Balance of Payments crisis. Removal of import licensing system, starts cutting tariffs.
- **1992-1997:** Tariffs come down to average of 35%, ending up fairly uniform. ▸ Tariffs
- ⇒ **tariff change was determined in the 50's**
- ⇒ **tariff changes are uncorrelated with 1992 industry characteristics** (Khandelwal and Topalova, 2010: "as exogenous to the state of the industries as a researcher might hope for").

See also Panagariya (2004), Sivadasan (2009), Khandelwal and Topalova (2010), Goldberg et al. (2010).

▸ Tariff details

Tariff changes act as demand & supply shocks

	Dependent variable: Log Sales			
	(1)	(2)	(3)	(4)
$\log(1+\tau_{it}^{\text{output}})$	0.315**	0.385**		0.726**
	(0.10)	(0.11)		(0.16)
$\log(1+\bar{\tau}_{it}^{\text{input}})$		-0.192^{+}		-0.607**
		(0.12)		(0.20)
$\text{ExDemand}_{\omega t}$			0.0467**	0.0507**
			(0.0080)	(0.010)
Year FE	Yes	Yes	Yes	Yes
Plant × Industry FE	Yes	Yes	Yes	Yes
R^2	0.943	0.943	0.947	0.949
Observations	160254	160233	162421	124098

Standard errors in parentheses, clustered at the state × industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

⇒ Use Δ **import tariffs** in the output good or **Hummels instrument** as a **demand shifter**

Demand ↗ ⇒ firms reduce vertical span

	Dependent variable: Vertical Span					
	(1)	(2)	(3)	(4)	(5)	(6)
Log Sales	-0.0191**	-0.0197**	-0.512^{+}	-0.253*	-0.164*	-0.268**
	(0.0020)	(0.0024)	(0.28)	(0.13)	(0.073)	(0.094)
$\log(1+\bar{\tau}^{\text{input}}_{j\omega t})$		-0.0211		0.0198		0.0552
		(0.050)		(0.065)		(0.10)
$\sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})\overline{\text{span}}_j$		-0.0905		-0.219*		-0.372**
		(0.056)		(0.099)		(0.13)
$\sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})(\text{distance}_{\omega i} - \overline{\text{span}}_j)$		-0.121		-0.337*		-0.498*
		(0.095)		(0.15)		(0.21)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Plant × Product FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV 1	IV 1	IV 2	IV 2
R^2	0.765	0.731	-1.049	-0.232	-0.0816	-0.227
First stage w. id. robust F			5.566	10.91	26.53	15.70
Observations	186628	145165	138204	137059	143428	110578

Standard errors in parentheses, clustered at the state-industry level. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Instrument: Col. 3, 4: log output tariff: Col 5, 6: Export demand shifter

Smith (1776): "The division of labour is limited by the extent of the market"

▸ Changes ▸ 90s ▸ GIV

Demand ↗ ⇒ firms reduce the actual number of inputs

	Dependent variable: # Inputs					
	(1)	(2)	(3)	(4)	(5)	(6)
Log Sales	0.0359**	0.0374**	-1.136*	-0.468+	-0.383+	-0.542*
	(0.0030)	(0.0035)	(0.54)	(0.26)	(0.20)	(0.25)
$\log(1+\bar{\tau}^{\text{input}}_{j\omega t})$		-0.441**		-0.366*		-0.207
		(0.16)		(0.14)		(0.19)
$\sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})\overline{\text{span}}_j$		0.125		-0.126		-0.337
		(0.10)		(0.14)		(0.21)
$\sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})(\text{distance}_{\omega i} - \overline{\text{span}}_j)$		0.162		-0.202		-0.722*
		(0.10)		(0.25)		(0.30)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Plant × Product FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV 1	IV 1	IV 2	IV 2
R^2	0.856	0.847	-4.788	-0.874	-0.574	-1.059
First stage w. id. robust F			5.566	10.91	26.53	15.70
Observations	186628	145165	138204	137059	143428	110578

Changes within plant-products
Standard errors in parentheses, clustered at the state-industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Young (1928): Economies of scale? Increasing returns? Network economies?

Plant j in industry ω

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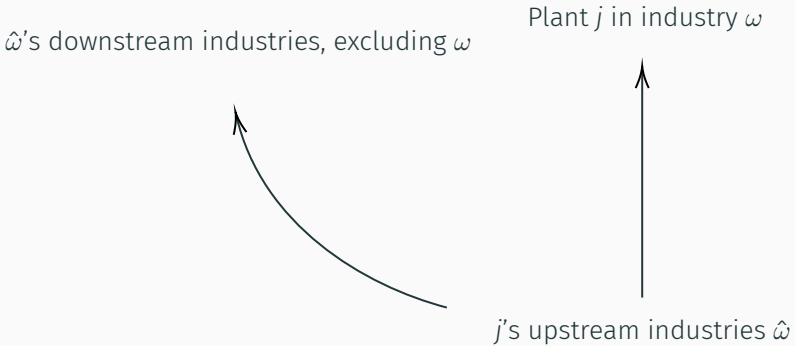


Young (1928): Economies of scale? Increasing returns? Network economies?

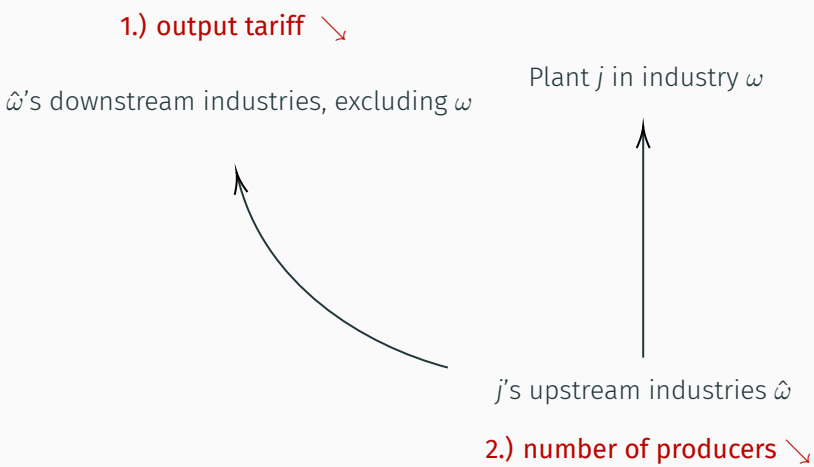
1.) output tariff ↘

$\hat{\omega}$'s downstream industries, excluding ω

Plant j in industry ω

j's upstream industries $\hat{\omega}$

Young (1928): Economies of scale? Increasing returns? Network economies?



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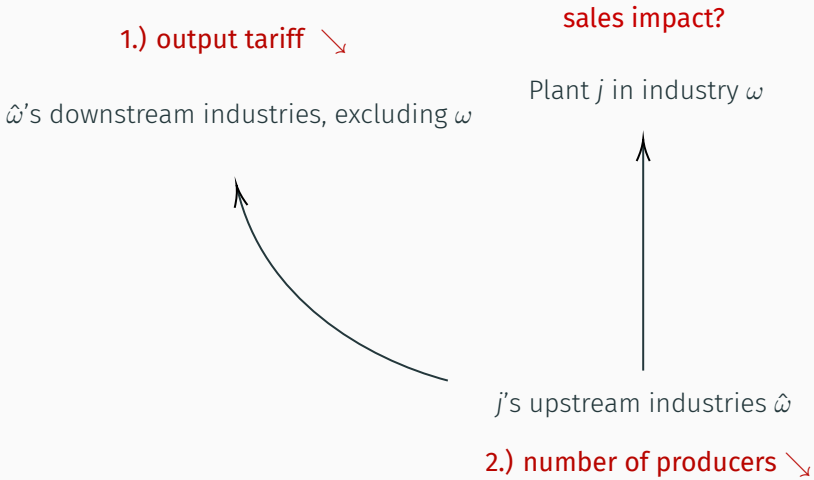


Upstream entry and sales

	Dependent variable: log Sales					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg. log #Producers in Upstream Ind.	0.0383**	0.0375**	0.0611**	0.0735**	0.254**	0.121**
	(0.0050)	(0.0060)	(0.017)	(0.018)	(0.036)	(0.018)
$\log(1+\bar{\tau}^{\text{input}}_{j\omega t})$		-0.0242		-0.0207		-0.0162
		(0.096)		(0.097)		(0.097)
$\sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})(\text{distance}_{\omega i} - \overline{\text{span}}_j)$		-0.331**		-0.334**		-0.337**
		(0.11)		(0.11)		(0.11)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Plant × Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV 1	IV 1	IV 2	IV 2
R^2	0.952	0.954	0.000636	0.000292	-0.0301	-0.00391
First-stage w. id. F			615.6	517.1	88.48	437.5
Observations	199039	142006	199039	142006	199039	142006

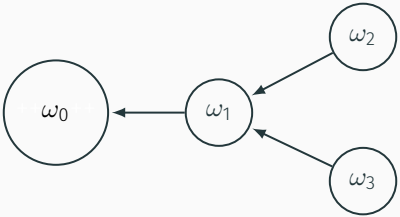
Standard errors in parentheses, clustered at the industry-year level. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

▸ 90s
▸ Sales

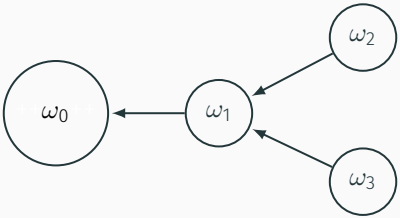
Other empirical results

- **Tariff supply & demand shocks affect entry.**
  Lower output tariffs decreases the number of plants
  Lower input tariffs increases the number of plants.
  ▸ Table
- **Input tariffs affect input adoption.**
  Lower input tariffs lead to an increased probability of plants using that input.
  ▸ Table

Simple Model



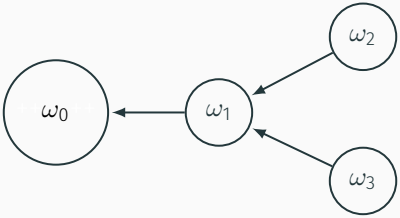
Firm produces good ω_0. Two (perf. substitutable) ways of producing:



Firm produces good ω_0. Two (perf. substitutable) ways of producing:

· Buying ω_1 from a supplier ('shirts from cloth')

· Buying ω_2 and ω_3 from suppliers ('shirts from yarn & dye')



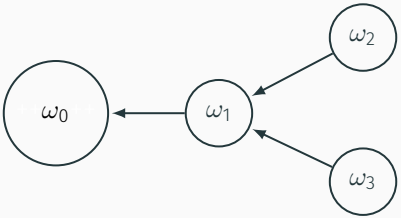
Firm produces good ω_0. Two (perf. substitutable) ways of producing:

1. Buying ω_1 from a supplier ('shirts from cloth')

Firms search for ω_1 suppliers. Search effort h_1.

Cost of production: $c_{j\omega_0} = \frac{1}{q_j} w^{\alpha_l^0} \left(\tilde{c}_j^1\right)^{1-\alpha_l^0}$ $\qquad$ $\tilde{c}_j^1 = \min_{s \in S_j^1} \frac{\text{price}_s}{\text{match-specific prod}_{js}}$

Arrival rate of supplier matches + match-specific productivity so that $\tilde{c}_j^1 \sim EV(h_1 v_1, \zeta)$



Firm produces good ω_0. Two (perf. substitutable) ways of producing:

2. **Buying ω_2 and ω_3 from suppliers ('shirts from yarn & dye')**

Firms search for ω_2, ω_3 suppliers. Search efforts h_2, h_3

Cost of production: $c_{j\omega_0} = \frac{1}{q_j} w^{\alpha_l^0} \Big(\underbrace{\frac{1}{b_j} w^{\alpha_l^1} (\tilde{c}_j^2)^{\alpha_2^1} (\tilde{c}_j^3)^{\alpha_3^1}}_{\sim EV\left((h_2 v_2)^{\alpha_2^1} (h_3 v_3)^{\alpha_3^1}, \zeta\right)} \Big)^{1-\alpha_l^0}$

$\tilde{c}_j^2 \sim EV(h_2 v_2, \zeta), \qquad \tilde{c}_j^3 \sim EV(h_3 v_3, \zeta), \qquad \chi(\log b_j) = \frac{\Gamma(1-\zeta it)}{\Gamma(1-\alpha_2^1 \zeta it)\Gamma(1-\alpha_3^1 \zeta it)}$

Search problem

- Firm born with productivity q_j, make search choice based only on that.
- Profits from sales to households, isoelastic demand, isoelastic search costs:

$$\max_{\{h\}_i} \quad \mathbb{E}\left(\pi_j | q_j, \{h\}_i\right) - \sum_{i=1,2,3} \frac{k}{1+\gamma} h_i^{1+\gamma}$$

$$A_{\omega_0} q^{\varepsilon-1} \mathbb{E}\left(c_j | q_j, \{h\}_i\right)^{1-\varepsilon} - \sum_{i=1,2,3} \frac{k}{1+\gamma} h_i^{1+\gamma}$$

$$A_{\omega_0} q^{\varepsilon-1} \left[h_1 v_1 + (h_2 v_2)^{\alpha_2^1} (h_3 v_3)^{\alpha_3^1}\right]^{(1-\alpha_i^0)\frac{\varepsilon-1}{\zeta}} - \sum_{i=1,2,3} \frac{k}{1+\gamma} h_i^{1+\gamma}$$

- **Nonhomotheticity:** return from searching in upstream industries (i.e. 2, 3) is more concave than in downstream industry (1).

  $\Rightarrow$ Plants born with high q will be more likely to be vertically specialized (use ω_1 rather than ω_2, ω_3). **Size $\leftrightarrow$ Span relationship** in the data

Nonhomotheticity

- A firm with a higher Hicks-neutral productivity q_j will search more in all markets
- But if the elasticity of substitution *across nests* is higher than *within nests* then $\log h_1 \nearrow$ more than $\log h_2 \nearrow$ (or $\log h_3 \nearrow$).
  - Why? When organizational forms are substitutable, x_{ω_1} is more elastic than $(x_{\omega_2}, x_{\omega_3})$ bundle
  - Searching more in upstream industries would increase $(x_{\omega_2}, x_{\omega_3})$ by less, since extra labor also needs to get hired (compared to increase in x_1 from searching more in $\omega - 1$)

Proposition

Under the optimal search effort, the probability of using ω_1 is

- *increasing in Hicks-neutral productivity q,*
- *increasing in the final consumer's demand for ω_0*

Generalizations

Extreme value math extends to any finite "production tree"

- Any (finite) number of inputs in each stage
- Any (finite) depth of the tree

Conditional on search effort choices, the distributions of input unit costs are EV

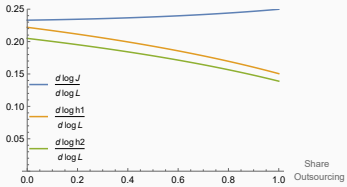
Search choices depend on Hicks-neutral productivity and upstream cost distributions

$\Rightarrow$ solve search problem recursively starting with most upstream (leaf) nodes

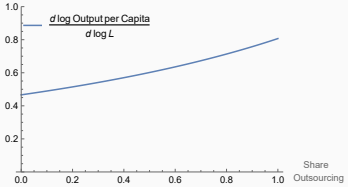
Full Model:

- (Imperfectly) elastic entry into industries ω on a large "production tree"
- Positive profits from sales to households, marginal cost pricing to firms further downstream
- Firms born with Hicks-neutral q. Increasing returns to scale through input search.
- Potentially network economies through arrival rate of draws also depending on upstream sector characteristics.

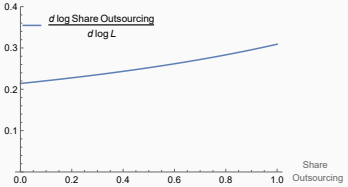
Economy-wide increase in scale: $L \uparrow$



(a) Response of Search Effort, Entry



(b) Response of Income/capita

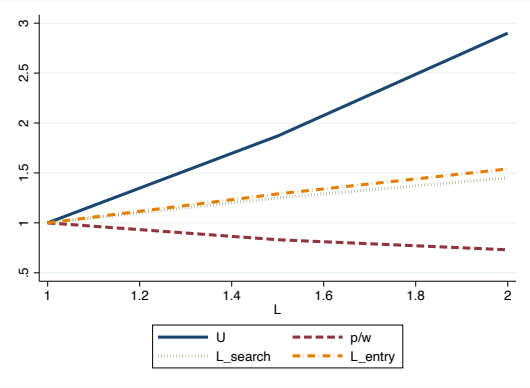


(c) Response of Share Outsourcing

- $L \uparrow \implies$ Search effort, entry $\uparrow$
- Aggregate increasing returns to scale: $L \uparrow \implies u \uparrow$
- Like IO multiplier: Response is larger if firms are more specialized
- Feedback: $L \uparrow \implies$ more specialization $\implies$ Larger response

An Economy-wide increase in scale

Figure 2: Counterfactual increase in size of the economy *L*



Conclusion

How do firms organize their vertical activities?

What are the implications for relationship between specialization and growth?

- Facts:
  - Macro: specialization correlated with development
  - Micro: plants with larger scales have shorter vertical spans
- Model
  - Firms that are ex-ante more productive search disproportionately more for inputs further downstream $\implies$ shorter vertical span
  - Economies of scale in production
  - Strong link between specialization and growth

Full Model

Demand & entry

Large number of industries ω, each with continuum of firms producing differentiated varieties

Consumption: Representative household has standard nested CES preferences

$$u = \left(\sum_{\omega} \delta_{\omega}^{\frac{1}{\eta}} u_{\omega}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \qquad u_{\omega} = \left(\int_{J_{\omega}} u_{\omega j}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \varepsilon > \eta > 1$$

Market Structure: Firms sell to firms further downstream, and to final consumers.

- Firms price at marginal cost when selling to other firms*
- Firms are monopolistically competitive when selling to final consumers.

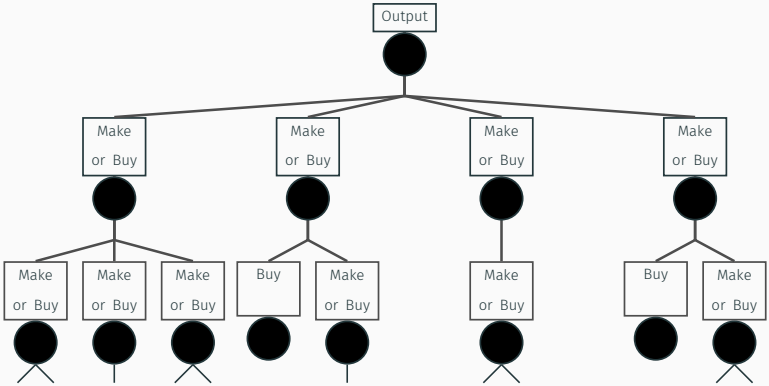
Entry: Representative entrepreneur chooses

$$\max \sum_{\omega} J_{\omega} \bar{\pi}_{\omega} \qquad \text{subject to} \qquad \left(\sum_{\omega} J_{\omega}^{\frac{1+\beta}{\beta}} \right)^{\frac{\beta}{1+\beta}} \leq 1$$

This nests free entry ($\beta \to \infty$) and inelastic entry ($\beta = 0$) as special cases. Assume $\beta < \infty$.

Production: technology menu

Each firm produces using **production modules** that make up a **production tree** (of finite depth):



The firm faces a **make-or-buy decision** for each non-leaf production module.

Production modules: make-or-buy decision

A firm's **effective unit cost of input** $\tilde{\omega}$ (in production tree) is

$$c_{j\tilde{\omega}} = \min\{c^o_{j\tilde{\omega}}, c^i_{j\tilde{\omega}}\}$$

1. **Buy input from supplier**
   - Search effort yields set of potential suppliers, $S_{j\tilde{\omega}}$
   - For each $s \in S_{j\tilde{\omega}}$: price p_s and match-specific productivity z_{js}

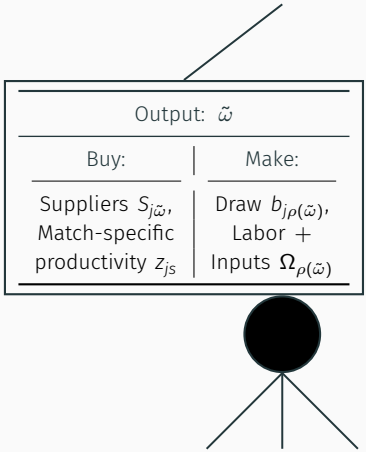
$$c^o_{j\tilde{\omega}} = \min_{s \in S_{j\tilde{\omega}}} \frac{p_j}{z_{js}}$$

2. **Produce in-house** using a production module, $\rho(\tilde{\omega})$
   - Module-specific productivity draw, $b_{j\rho(\tilde{\omega})}$
   - Module prod. fct. is Cobb-Douglas in labor and inputs $\hat{\Omega}_{\rho(\tilde{\omega})}$

$$c^i_{j\tilde{\omega}} = \frac{1}{b_{j\rho(\tilde{\omega})}} w^{\alpha_\ell^{\rho(\tilde{\omega})}} \prod_{\hat{\omega} \in \hat{\Omega}_{\rho(\tilde{\omega})}} c_{j\hat{\omega}}^{\alpha_{\hat{\omega}}^{\rho(\tilde{\omega})}}$$

Firm's effective unit cost for its output ω is

$$c_{j\omega} = \frac{1}{q_j} w^{\alpha_\ell^{\rho(\omega)}} \prod_{\hat{\omega} \in \hat{\Omega}_{\rho(\omega)}} c_{j\hat{\omega}}^{\alpha_{\hat{\omega}}^{\rho(\omega)}}$$



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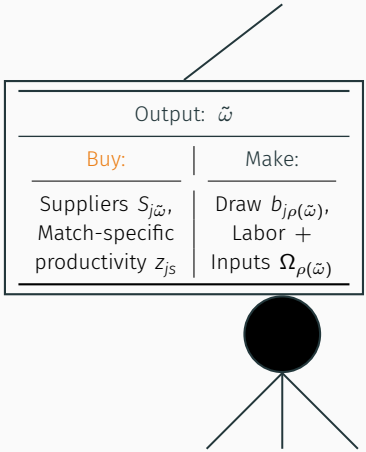

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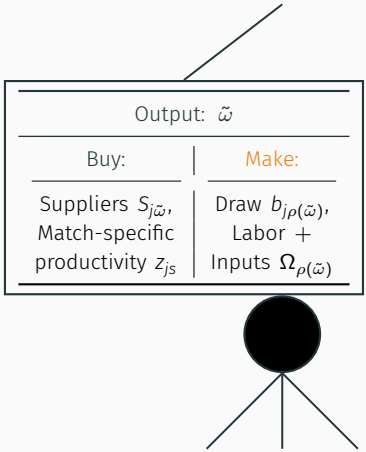
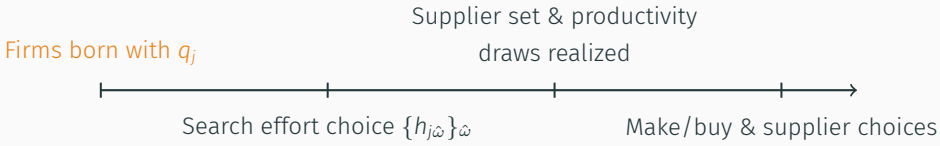


Timing & functional form assumptions

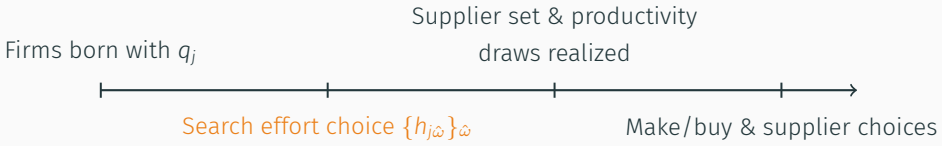


1. **Firms born** with q_j.
   Assume distribution of birth productivities has sufficiently thin tail

2. **Search.** Assume isoelastic and additive search cost, then the firm chooses search efforts $\{h_{j\hat{\omega}}\}_{\hat{\omega}}$ to maximize

$$\max_{\{h_{j\hat{\omega}}\}_{\hat{\omega}\in\hat{\Omega}^{\infty}_{\rho(\omega)}}} E\left[\pi_j | q_j, \{h_{j\hat{\omega}}\}_{\hat{\omega}}\right] - \sum_{\hat{\omega}} \frac{k}{1+\gamma} h_{j\hat{\omega}}^{1+\gamma}$$

Timing & functional form assumptions



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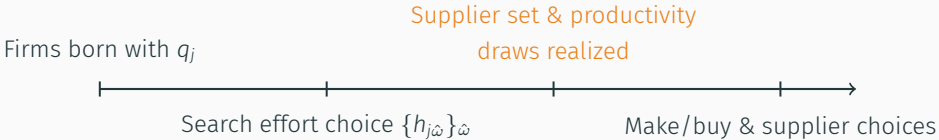

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$$h_{j\hat{\omega}} m(J_{\hat{\omega}}) z^{-\zeta}$$

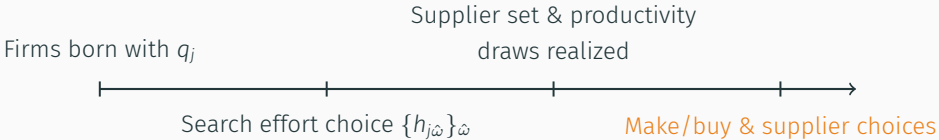
Log of module/task productivity $b_{j\rho}$ drawn from dist with characteristic function

$$\frac{\Gamma(1-\zeta it)}{\prod_{\hat{\omega}\in\hat{\Omega}_\rho}\Gamma(1-\alpha_{\hat{\omega}}^{\rho}\zeta it)}$$

Distribution is backward engineered to help with discrete choice.

4. **Make/buy & supplier choice** to minimize ex-post cost ($\Leftrightarrow$ maximize profit)

Timing & functional form assumptions



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With functional form assumptions

Normalize $w = 1$. Then the **distribution of unit cost of an input** $\tilde{\omega}$ conditional on $\{h_{j\hat{\omega}}\}$ is Weibull:

$$P\left(c_{j\tilde{\omega}} > c | \{h_{j\hat{\omega}}\}\right) = e^{-T_{j\rho(\tilde{\omega})} c^{\zeta}}$$

where

$$T_{j\rho(\tilde{\omega})} = \begin{cases} \prod_{\hat{\omega} \in \hat{\Omega}_{\rho(\tilde{\omega})}} \left(h_{j\hat{\omega}} v_{\hat{\omega}} + T_{j\rho(\hat{\omega})}\right)^{\alpha^{\rho}_{\hat{\omega}}}, & \text{input nodes} \\ \prod_{\hat{\omega} \in \hat{\Omega}_{\rho(\tilde{\omega})}} \left(h_{j\hat{\omega}} v_{\hat{\omega}}\right)^{\alpha^{\rho}_{\hat{\omega}}}, & \text{terminal production modules (leaves)} \end{cases}$$

where $v_{\hat{\omega}} \equiv m(J_{\hat{\omega}}) \int_0^{\infty} c^{-\zeta} dF_{\hat{\omega}}(c)$ indexes the cost distribution of suppliers.

$\Rightarrow$ Conditional on requiring input $\tilde{\omega}$, the probability that the firm uses a supplier for it is

$$\frac{h_{j\tilde{\omega}} v_{\tilde{\omega}}}{h_{j\tilde{\omega}} v_{\tilde{\omega}} + T_{j\rho(\tilde{\omega})}}.$$

Demand shocks in ω

Proposition

If δ_ω increases (= positive demand shock),

- *more entry in industry ω: $J_\omega \nearrow$*
- *the price level in industry ω falls: $p_\omega \searrow$ (and $v_\omega \nearrow$)*
- *For each input $\hat{\omega}$, the probability of buying it (rather than making it) increases.*

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Intuition:

(1) $\delta_\omega \nearrow \Rightarrow \bar{\pi}_\omega \nearrow \Rightarrow J_\omega$

(2) $\delta_\omega \nearrow \Rightarrow \bar{\pi}_\omega \nearrow \Rightarrow$ search efforts $\nearrow \Rightarrow p_\omega \searrow$

(3) Firms shift search effort toward more downstream suppliers

Demand shocks in upstream industry $\hat{\omega} \in \hat{\Omega}_{\rho(\omega)}$

Proposition

If $\delta_{\hat{\omega}}$ increases (= positive demand shock in the upstream industry), then if γ is sufficiently large (search effort not too elastic):

- *more entry in industry $\hat{\omega}$: $J_{\hat{\omega}} \nearrow$, $v_{\hat{\omega}} \nearrow$*
- *the fraction of ω firms buying $\hat{\omega}$ increases*
- *total sales in industry ω increase*

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- *total sales in industry ω increase*

Intuition:

(1) As before
   - More matches $m(J_{\hat{\omega}}) \nearrow$
   - firms in $\hat{\omega}$ search more $\Rightarrow$ lower cost

(2) $v_{\hat{\omega}} \nearrow$ but $p_{\omega} \searrow$. If γ sufficiently large, $v_{\hat{\omega}} \nearrow$ dominates for all q.

(3) $v_{\omega} \nearrow$ and $p_{\omega} \searrow$, and demand elastic

Going forward

- **Differentiated vs Standardized Inputs** (preliminary) empirical patterns driven by use of differentiated inputs
- **Profits from firm-to-firm trade**
  - Account explicitly for demand shocks from downstream sectors
  - What is internalized?
- **Identification of scale economies** through h and m

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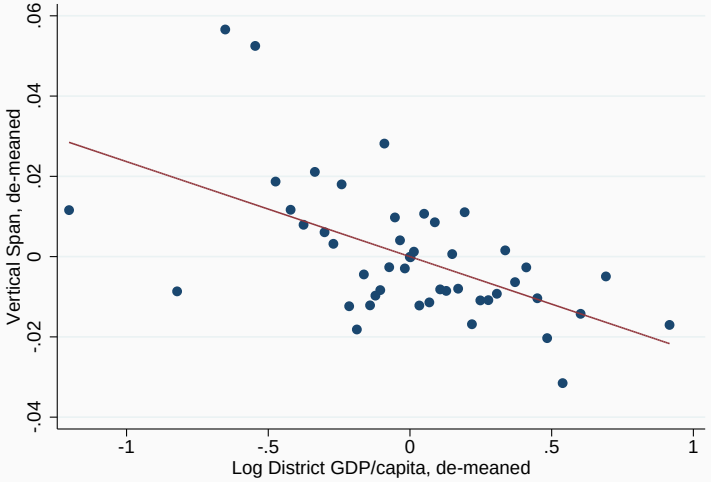
Conclusion:
Indian Microdata suggests

- Internal economies of scale from search
- Possibly external economies of scale through matching process

Overall, try to make progress on quantitative models of growth. How important is "Smithian" growth?

Fact 1: In richer districts, plants are more specialized (shorter vertical span)

Within industry × year:



Binscatter with n=50. Y and X variable de-meaned by industry x year. SP plants only.

Fact 2: Increased vertical specialization positively correlated with state growth

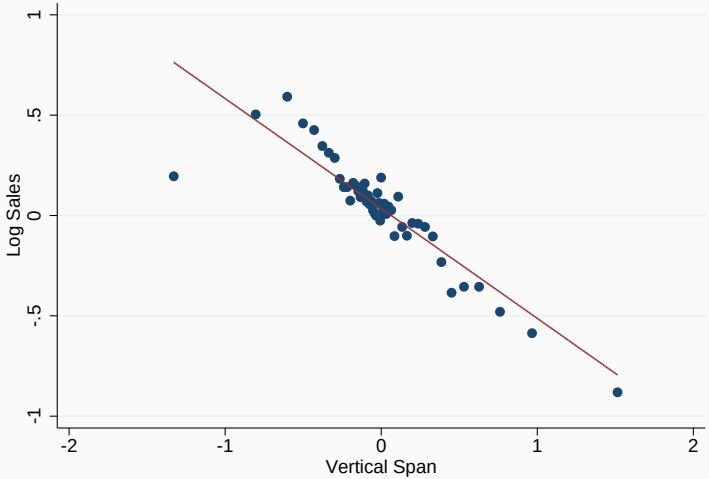
Within plant, over time:

	Dependent variable: Vertical Span		
	(1)	(2)	(3)
Log GDP/capita$_{st}$	-0.0552	-0.0647	-0.0741^{+}
	(0.048)	(0.045)	(0.043)
Year FE	Yes	Yes	Yes
Plant FE	Yes	Yes	
5-digit Industry FE		Yes	
Plant $\times$ 5-digit Industry FE			Yes
R^2	0.644	0.720	0.780
Observations	95727	94754	61073

Standard errors in parentheses, clustered at the state $\times$ 5-dgt industry level. SP plants only.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Fact 3: More specialized plants (shorter span) are larger

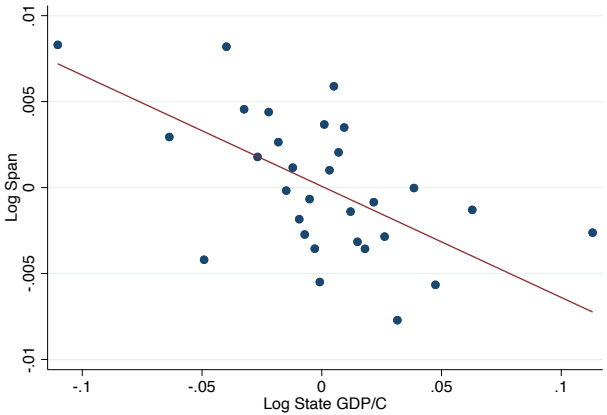
Plants with higher sales tend to have shorter vertical span (within industry × year)



SP plants only. Regression includes industry x year FE's.

Fact 2: Increased vertical specialization is positively correlated with state growth

Within plant×industry, year:



Binscatter with n=30. SP plants only. Variables de-meaned by plant-industry and year.

Fact 4: Sales growth is correlated with increased vertical specialization

	Dependent variable: Δ log Sales			
	(1)	(2)	(3)	(4)
Δ Vertical Span	-0.0655**	-0.0445**	-0.0284*	-0.0259*
	(0.0082)	(0.0087)	(0.013)	(0.011)
Year FE	Yes			
Product × Year FE		Yes	Yes	Yes
Plant FE			Yes	
Plant × Product FE				Yes
R^2	0.00819	0.149	0.432	0.431
Observations	120436	111244	83026	74707

Changes within plant-products

Standard errors in parentheses, clustered at the state-industry level.

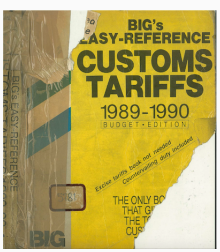
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

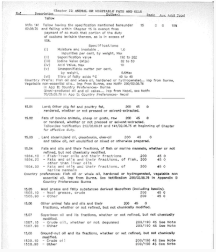
back

Details on tariff construction

We use UNCTAD tariffs, complemented by hand-digitized effective tariff rates for early years of the liberalization (1990-1996).

- Exclude agricultural tariffs (which changed in response to domestic shocks)
- Exclude mechanical & electrical machinery (HS headings 84, 85): long list of exceptions



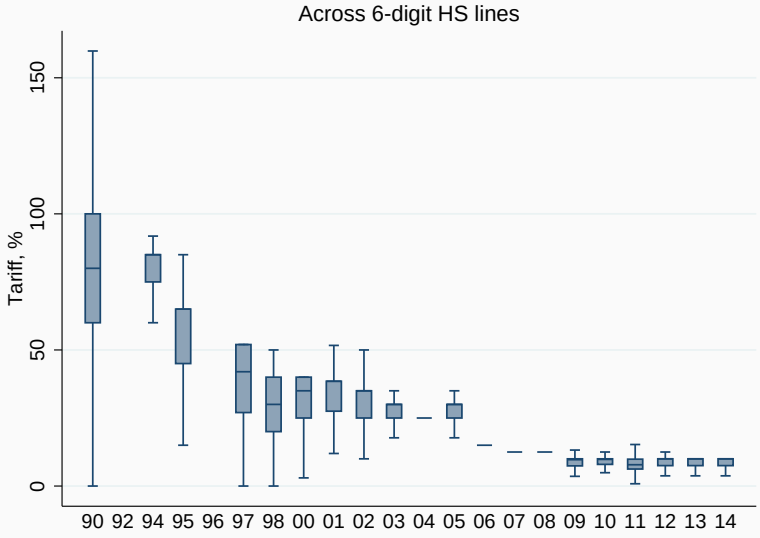


Plants with shorter span are larger: Details

	Dependent variable: Log Sales				
	(1)	(2)	(3)	(4)	(5)
Vertical Span	-0.719**	-0.670**	-0.431**	-0.432**	-0.193**
	(0.024)	(0.023)	(0.034)	(0.034)	(0.015)
Age				0.00616**	-0.00314**
				(0.0012)	(0.00068)
Log Employment					1.067**
					(0.015)
Year FE	Yes	Yes	Yes	Yes	Yes
5-digit Industry FE	Yes	Yes			
District FE		Yes			
Industry × District × Year FE			Yes	Yes	Yes
R^2	0.394	0.440	0.700	0.701	0.859
Observations	353659	295094	140610	136831	136608

Standard errors in parentheses, clustered at the 5-dgt industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Import Tariffs, India, 1990-2014



Changes since 1990: tariffs and sales

	Dep. var.: Δ^t_{1990} log Sales	
	(1)	(2)
$\Delta^t_{1990} \log(1+\tau^{\text{output}}_{\omega t})$	1.302$^+$	1.533$^+$
	(0.75)	(0.79)
$\Delta^t_{1990} \log(1+\bar{\tau}^{\text{input}}_{\omega t})$		-1.188
		(0.77)
Year FE	Yes	Yes
R^2	0.0852	0.0903
Observations	2376	2376

Standard errors in parentheses, clustered at the state $\times$ industry level.
$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Changes since 1990: vertical span and demand

	Dependent variable: Δ^t_{1990} Vertical Span		
	(1)	(2)	(3)
Δ^t_{1990} log Sales	-0.147$^+$	-0.166$^+$	-0.237$^+$
	(0.084)	(0.086)	(0.12)
$\Delta^t_{1990} \log(1+\bar{\tau}^{\text{input}}_{it})$		0.194	1.421$^+$
		(0.24)	(0.77)
$\Delta \sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})(\text{distance}_{\omega i} - \overline{\text{span}}_j)$			-0.747
			(0.75)
$\Delta \sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})\overline{\text{span}}_j$			-1.031$^+$
			(0.62)
Year FE	Yes	Yes	Yes
R^2	-0.194	-0.255	-0.498
Observations	2179	2179	2128

Standard errors in parentheses, clustered at the state $\times$ industry level.
$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Δ^t_{1990} log sales is instrumented by the change in the log output tariff since 1990.

Demand shocks affect vertical specialization

	Dependent variable: Δ Vertical Span					
	(1)	(2)	(3)	(4)	(5)	(6)
Δ log Sales	-0.0158**	-0.0160**	-0.0165**	-0.0301*	-0.0457+	-0.0652
	(0.0020)	(0.0020)	(0.0024)	(0.013)	(0.025)	(0.052)
$\Delta \log(1+\bar{\tau}^{\text{input}}_{j\omega t})$		-0.0173	0.00538		-0.0354	-0.0188
		(0.020)	(0.047)		(0.044)	(0.051)
$\Delta \sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})(\text{distance}_{\omega i} - \overline{\text{span}}_j)$			-0.00198			-0.0731
			(0.087)			(0.14)
$\Delta \sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})\overline{\text{span}}_j$			-0.00455			-0.0478
			(0.041)			(0.096)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.00207	0.00208	0.00229	0.000325	-0.00220	-0.00774
Observations	123666	123666	94795	90115	90115	89301

Standard errors in parentheses, clustered at the state-industry level. $^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Columns (3), (4) instrument Δ log sales by the change in the log output tariff.

Vertical span and demand, generated instruments à la Wooldridge (2002, Ch 6)

IV for log sales: fitted values of a Poisson regression of sales on exogenous variables.

	Dependent variable: Vertical Span		
	(1)	(2)	(3)
Log Sales	-0.0909**	-0.0841**	-0.0933**
	(0.021)	(0.020)	(0.021)
$\log(1+\bar{\tau}^{\text{input}}_{j\omega t})$		-0.0931*	-0.0265
		(0.039)	(0.057)
$\sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})\overline{\text{span}}_j$			-0.133*
			(0.064)
$\sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})(\text{distance}_{\omega i} - \overline{\text{span}}_j)$			-0.203$^+$
			(0.11)
Year FE	Yes	Yes	Yes
Plant × Product FE	Yes	Yes	Yes
Weak-Id robust 1st stage F	221.3	230.4	215.6
Estimator	G-IV	G-IV	G-IV
R^2	-0.0212	-0.0169	-0.0218
Observations	138204	138204	137059

Standard errors in parentheses, clustered at the state-industry level.
$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Demand $\nearrow \Rightarrow$ firms reduce the actual number of inputs

	Dependent variable: # Inputs					
	(1)	(2)	(3)	(4)	(5)	(6)
Log Sales	0.0359**	0.0374**	-1.136*	-0.468^{+}	-0.383^{+}	-0.542*
	(0.0030)	(0.0035)	(0.54)	(0.26)	(0.20)	(0.25)
$\log(1+\bar{\tau}_{j\omega t}^{\text{input}})$		-0.441**		-0.366*		-0.207
		(0.16)		(0.14)		(0.19)
$\sum_i \alpha_i \log(1+\bar{\tau}_{it}^{\text{input}})\overline{\text{span}}_j$		0.125		-0.126		-0.337
		(0.10)		(0.14)		(0.21)
$\sum_i \alpha_i \log(1+\bar{\tau}_{it}^{\text{input}})(\text{distance}_{\omega i} - \overline{\text{span}}_j)$		0.162		-0.202		-0.722*
		(0.10)		(0.25)		(0.30)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Plant × Product FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV 1	IV 1	IV 2	IV 2
R^2	0.856	0.847	-4.788	-0.874	-0.574	-1.059
First stage w. id. robust F			5.566	10.91	26.53	15.70
Observations	186628	145165	138204	137059	143428	110578

Changes within plant-products
Standard errors in parentheses, clustered at the state-industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Firms are more likely to adopt inputs when faced with a cost decrease

	Dependent variable: Input Used Dummy $\mathbf{1}(X_{j\hat{\omega}t} > 0)$	
	(1)	(2)
$\log(1+\tau_{it})$	-0.0506**	-0.0373**
	(0.0067)	(0.0071)
Year FE	Yes	Yes
Plant × Input FE	Yes	Yes
Plant × Product FE		Yes
R^2	0.337	0.361
Observations	2460831	2454899

Standard errors in parentheses.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Supply and demand shifters determine entry

	Dependent variable: log Producers $\lvert J \rvert_{d\omega t}$	
	(1)	(2)
$\log(1+\bar{\tau}_{it}^{\text{input}})$	-0.108**	-0.0496**
	(0.025)	(0.015)
$\log(1+\tau_{it}^{\text{output}})$	0.186**	0.251**
	(0.021)	(0.013)
Year FE	Yes	
State FE	Yes	
Industry FE	Yes	
State × Year FE		Yes
State × Industry FE		Yes
R^2	0.481	0.844
Observations	548180	537013

Standard errors in parentheses.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

The left-hand side is the log number of producers of a good ω at time t in state d.

Firms reduce the effective number of inputs when demand ↗

	Dependent variable: Inverse Input HHI					
	(1)	(2)	(3)	(4)	(5)	(6)
Log Sales	0.0101	0.00930	-1.804^{+}	-1.100^{+}	-1.705^{+}	-2.135^{+}
	(0.0072)	(0.0082)	(1.08)	(0.63)	(0.99)	(1.18)
$\log(1+\bar{\tau}^{\text{input}}_{j\omega t})$		-0.745^{+}		-0.574^{+}		-0.354
		(0.42)		(0.30)		(0.50)
$\sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})\overline{\text{span}}_j$		0.428		-0.0487		-1.362^{+}
		(0.34)		(0.29)		(0.78)
$\sum_i \alpha_i \log(1+\bar{\tau}^{\text{input}}_{it})(\text{distance}_{\omega i} - \overline{\text{span}}_j)$		0.630		-0.138		-1.906^{+}
		(0.39)		(0.55)		(1.16)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Plant × Product FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	IV 1	IV 1	IV 2	IV 2
R^2	0.807	0.799	-3.137	-1.132	-2.464	-3.560
First stage w. id. robust F			4.732	10.81	28.71	15.73
Observations	192795	145189	142265	137084	147328	110594

Changes within plant-products
Standard errors in parentheses, clustered at the state-industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Effective # of inputs measured by the inverse of the HHI of cost shares. Results similar for actual (unweighted) number of inputs.

Sample of 1990 plants: upstream industry size and sales

	Dependent variable: log Sales					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg. log #Producers in Upstream Ind.	0.0655**	0.0560**	0.0551**	0.0201	0.119**	0.115**
	(0.013)	(0.018)	(0.018)	(0.043)	(0.044)	(0.044)
$\log(1+\bar{\tau}^{\text{input}}_{j\omega t})$			0.540*			0.519*
			(0.26)			(0.26)
Year FE	Yes			Yes		
Industry × Year FE		Yes	Yes		Yes	Yes
Plant × Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.916	0.943	0.943	0.00262	-0.000638	0.000690
Observations	13683	9768	9757	13683	9768	9757

Standard errors in parentheses, clustered at the industry-year level.
$^+$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Sample: all SP plants observed in 1990

(except (3) and (6) which further condition on $t \leq 2000$)

Upstream industry size and sales

	Dependent variable: log Sales					
	(1)	(2)	(3)	(4)	(5)	(6)
Avg. log Sales in Upstream Ind.	0.00367**	0.00251**	0.00251**	0.00642*	0.00930**	0.00936**
	(0.00034)	(0.00038)	(0.00038)	(0.0029)	(0.0026)	(0.0026)
$\log(1+\bar{\tau}^{\text{input}}_{j\omega t})$			0.0241			0.0193
			(0.085)			(0.086)
Year FE	Yes			Yes		
Industry × Year FE		Yes	Yes		Yes	Yes
Plant × Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	OLS	OLS	IV	IV	IV
R^2	0.942	0.952	0.952	0.000572	-0.00304	-0.00311
Observations	215805	199039	198727	215805	199039	198727

Standard errors in parentheses, clustered at the industry-year level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Increased vertical specialization is positively correlated with state growth

Within plant, over time:

	Dependent variable: Vertical Span		
	(1)	(2)	(3)
Log GDP/capita$_{st}$	-0.0716*	-0.0601*	-0.0551*
	(0.028)	(0.026)	(0.026)
Year FE	Yes	Yes	
Plant FE	Yes	Yes	
5-digit Industry FE		Yes	
5-digit Industry × Year FE			Yes
Plant × 5-digit Industry FE			Yes
R^2	0.592	0.656	0.808
Observations	270003	269399	163668

Standard errors in parentheses, clustered at the state × 5-dgt industry level. SP plants only.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Standard errors in parentheses, clustered at the state × 5-dgt industry level. SP plants only.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Sales growth is correlated with increased vertical specialization

	Dependent variable: Δ log Sales			
	(1)	(2)	(3)	(4)
Δ Vertical Span	-0.0655**	-0.0445**	-0.0284*	-0.0259*
	(0.0082)	(0.0087)	(0.013)	(0.011)
Year FE	Yes			
Product × Year FE		Yes	Yes	Yes
Plant FE			Yes	
Plant × Product FE				Yes
R^2	0.00819	0.149	0.432	0.431
Observations	120436	111244	83026	74707

Changes within plant-products

Standard errors in parentheses, clustered at the state-industry level.

$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

SP plants only.

Fact 4: Sales growth is correlated with increased vertical specialization

	Dependent variable: Δ log Sales			
	(1)	(2)	(3)	(4)
Δ Vertical Span	-0.0693**	-0.0668**	-0.0577**	-0.0546**
	(0.0085)	(0.0085)	(0.011)	(0.011)
Δ R-Share in Materials	-0.0242*	-0.0247*	-0.0270^{+}	-0.0346*
	(0.012)	(0.012)	(0.015)	(0.014)
Δ Vertical Span × Δ R-Share in Materials	-0.0359*	-0.0408*	-0.0549*	-0.0544*
	(0.016)	(0.016)	(0.025)	(0.023)
Constant	0.194**	0.194**	0.181**	0.171**
	(0.0046)	(0.0025)	(0.0015)	(0.00030)
Year FE	Yes	Yes	Yes	Yes
Product FE		Yes		
Plant FE			Yes	
Plant × Product FE				Yes
R^2	0.00825	0.0409	0.305	0.314
Observations	116199	115643	89440	80377

Unit Costs and Tariff changes

	Dependent variable: Δ^{t}_{1990} log Unit Cost	
	(1)	(2)
$\Delta^{t}_{1990} \log(1+\tau^{\text{output}}_{it})$	-0.789**	-0.949**
	(0.10)	(0.17)
$\Delta^{t}_{1990} \log(1+\bar{\tau}^{\text{input}}_{j\omega t})$		0.226
		(0.17)
Year FE	Yes	Yes
R^2	0.0566	0.0583
Observations	920	916

Standard errors in parentheses, clustered at the state $\times$ industry level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Generalizations

Extreme value math extends to any finite "production tree"

- Any (finite) number of inputs in each stage
- Any (finite) depth of the tree

Conditional on search effort choices, the distributions of input unit costs are EV

Search choices depend on Hicks-neutral productivity and upstream cost distributions

$\Rightarrow$ solve search problem recursively starting with most upstream (leaf) nodes

Full Model:

- (Imperfectly) elastic entry into industries ω on a large "production tree"
- Positive profits from sales to households, marginal cost pricing to firms further downstream
- Firms born with Hicks-neutral q. Increasing returns to scale through input search.
- Potentially network economies through arrival rate of draws also depending on upstream sector characteristics.

Discrete Choice Math

- Lowest cost way of acquiring good $\omega - 1$

$$\min \left\{ \min_{s \in S_1} \frac{p_s}{z_s} \, , \; \frac{1}{b_2} \min_{s \in S_2} w^{1-\alpha} \left(\frac{p_s}{z_s} \right)^{\alpha} \right\}$$

- Arrival of suppliers with $z_s > z$ is Poisson with arrival rate $\propto z^{-\zeta}$

$$\min_{s \in S_1} \frac{p_s}{z_s} \sim Weibull(scale_1, \zeta) \tag{1}$$

$$\min_{s \in S_2} w^{1-\alpha} \left(\frac{p_s}{z_s} \right)^{\alpha} \sim Weibull \left(scale_2, \frac{\zeta}{\alpha} \right) \tag{3}$$

Back

- Lowest cost way of acquiring good $\omega - 1$

$$\min \left\{ \min_{s \in S_1} \frac{p_s}{z_s} \ , \ \frac{1}{b_2} \min_{s \in S_2} w^{1-\alpha} \left(\frac{p_s}{z_s} \right)^{\alpha} \right\}$$

- Arrival of suppliers with $z_s > z$ is Poisson with arrival rate $\propto z^{-\zeta}$

$$\min_{s \in S_1} \frac{p_s}{z_s} \sim Weibull(scale_1, \zeta) \tag{1}$$

$$\min_{s \in S_2} w^{1-\alpha} \left(\frac{p_s}{z_s} \right)^{\alpha} \sim Weibull \left(scale_2, \frac{\zeta}{\alpha} \right) \tag{2}$$

$$\frac{1}{b_2} \min_{s \in S_2} w^{1-\alpha} \left(\frac{p_s}{z_s} \right)^{\alpha} \sim Weibull \left(scale_3, \zeta \right) \tag{3}$$

- Follows from:

$$Z \sim \text{standard exponential}, \ Y \sim \alpha\text{-Stable} \qquad \Rightarrow \qquad (Z/Y)^{\alpha} \sim Z$$

Back

Nested CES Example

Imagine the production function was a Nested CES:

$$y_j = q\left\{(A_1 h_1 x_1)^{\frac{\eta-1}{\eta}} + \left[(A_0 l)^{\frac{\phi-1}{\phi}} + (A_2 h_2 x_2)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}}$$

Proposition

If $\gamma \geq \eta - 2$ *and* $\gamma \geq \phi - 2$, *then*

$$\frac{d \ln h_1}{d \ln q} > \frac{d \ln h_2}{d \ln q} \qquad \textit{iff} \qquad \eta > \phi$$

Our setting is a special case with $\eta \to \infty$ and $\phi \to 1$.

Where does the nonhomotheticity come from?

- Imagine a production function where search effort is factor-augmenting.

$$\max_{h_1,h_2} \delta g\left\{C\left(w, \frac{p_1}{h_1}, \frac{p_2}{h_2}\right)\right\} - \frac{h_1^{1+\gamma}}{1+\gamma} - \frac{h_2^{1+\gamma}}{1+\gamma}$$

- *Levels* of optimal search effort are determined by cost shares:

$$0 = -\delta g' \frac{p_i}{h_i^2} C_i\left(w, \frac{p_1}{h_1}, \frac{p_2}{h_2}\right) - h_i^{\gamma}$$

- Relative elasticity of h_1 vs h_2 is therefore determined by relative *elasticity* of cost shares … and these are encoded in the Morishima elasticities of substitution σ_{21}, σ_{12}
- If γ sufficiently large, $d \log h_1 / d \log q > d \log h_2 / d \log q$ iff $\sigma_{21} > \sigma_{12}$.
- In particular that's satisfied when there is perfect substitutability between a nested and non-nested production function:

$$y_j = \begin{cases} q_j f(l_{j0}, x_{j1}) & \text{or} \\ q_j f(l_{j0}, g(l_{j1}, x_{j2})) \end{cases}$$

(assuming g is imperfectly substitutable…)