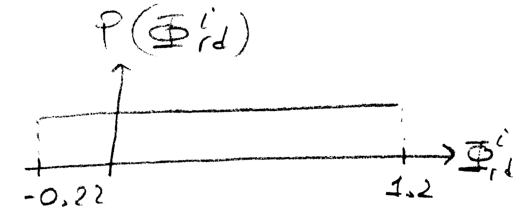
• Given $\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (\Phi^i_{th} - \Phi^i_{exp})^2$, what is the probability of having a set of $\Phi^{\{i\}}_{random}$ such that $\frac{1}{N} \sum_{i=1}^{N} (\Phi^i_{rd} - \Phi^i_{exp})^2 < \sigma^2$??



• Assume Φ^i_{rd} is a random variable with a flat probability distribution in $[-0.22, 1.20]$, which is the range of experimental Φ-values.

$$\mathcal{P}(\Phi^i_{rd}) = \begin{cases} \frac{1}{1.42} & -0.22 < \Phi^i_{rd} < 1.2 \\ 0 & \text{elsewhere} \end{cases}$$

Then we have:

$$\langle (\Phi^i_{th} - \Phi^i_{exp})^2 \rangle = \frac{1}{1.42} \int_{-0.22}^{1.2} d\Phi^i_{rd} \, (\Phi^i_{rd} - \Phi^i_{exp})^2 = \frac{(1.2 - \Phi^i_{exp})^3 + (0.22 + \Phi^i_{exp})^3}{(1.42) \cdot 3}$$

$$\langle (\Phi^i_{th} - \Phi^i_{exp})^4 \rangle = \frac{(1.2 - \Phi^i_{exp})^5 + (0.22 + \Phi^i_{exp})^5}{(1.42) \cdot 5}$$

• Assume all the Φ^i_{rd} are uncorrelated. Then by the central limit theorem, the probability distribution of σ^2 is

$$\mathcal{P}(\sigma^2) = \frac{1}{\sqrt{2\pi b^2}} e^{-(\sigma^2 - a)^2 / 2b^2}, \text{ where}$$

$$a = \frac{1}{N} \sum_{i=1}^{N} \langle (\Phi^i_{rd} - \Phi^i_{exp})^2 \rangle \quad \text{and} \quad b^2 = \frac{1}{N} \sum_{i=1}^{N} \langle (\Phi^i_{rd} - \Phi^i_{exp})^4 \rangle - a^2$$

• Given a particular σ^2, its associated p-value is $\int_{-\infty}^{\sigma^2} d\sigma^2 \mathcal{P}(\sigma^2)$

• Change of variables: $x = \frac{\sigma^2 - a}{b} \Rightarrow \mathcal{P}(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$. The p-value of x is the output of integral.exe x