Elephant Cross Correlation Test

There may be an issue with the scaling of cross correlation at least for single pairs using `elephant.signal_processing.cross_correlation_function()` tested using the example below

In [1]:

```
%load_ext watermark
%matplotlib notebook

import numpy
import numpy as np
import matplotlib.pyplot as plt

import elephant
from elephant import signal_processing

import neo
from quantities import Hz, mV, sec, ms

%watermark -v -m -p numpy,scipy,neo,elephant
```

```
CPython 3.5.2
IPython 7.8.0

numpy 1.17.3
scipy 1.3.1
neo 0.7.2
elephant 0.6.3

compiler   : GCC 5.4.0 20160609
system     : Linux
release    : 4.4.0-165-generic
machine    : x86_64
processor  : x86_64
CPU cores  : 8
interpreter: 64bit
```

Example

The example problem converted from matlab (see http://www.mechanicalvibration.com/Using_cross_correlation_lin.html (http://www.mechanicalvibration.com/Using_cross_correlation_lin.html)).

In [2]:

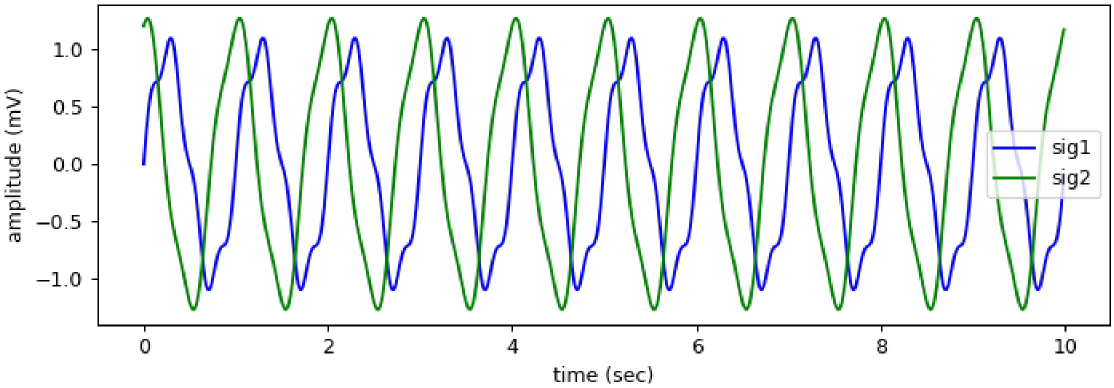
```
fs = 100.0
t = np.arange(0.0, 10.0, 1/fs)
f = 1
```

In [3]:

```
sig1 = 1.0 * np.sin(2.0*np.pi*f*t) + 0.15 * np.sin(2.0*np.pi*4.0*f*t)
sig2 = 1.2 * np.sin(2.0*np.pi*f*t + np.pi/2.0) +  0.15 * np.sin(2.0*np.pi*3.0*f*
t)
```

In [4]:

```
plt.figure(figsize=(8,3))
plt.plot(t, sig1,'b-', label='sig1')
plt.plot(t, sig2, 'g-', label='sig2')
plt.xlabel('time (sec)')
plt.ylabel('amplitude (mV)')
plt.legend()
plt.tight_layout()
```



Numpy Version

Performed cross correlation of these two signals using `numpy.correlate()` and, to match matlab used same normalization used in `matlab.xcorr()` ([https://www.mathworks.com/matlabcentral/answers/5275-algorithm-for-coeff-scaling-of-xcorr (https://www.mathworks.com/matlabcentral/answers/5275-algorithm-for-coeff-scaling-of-xcorr)](https://www.mathworks.com/matlabcentral/answers/5275-algorithm-for-coeff-scaling-of-xcorr)):

In [5]:

```
cxx = np.correlate(sig1, sig2, "full")/np.sqrt(np.sum(np.abs(sig1)**2)*np.sum(np
.abs(sig2)**2))
```

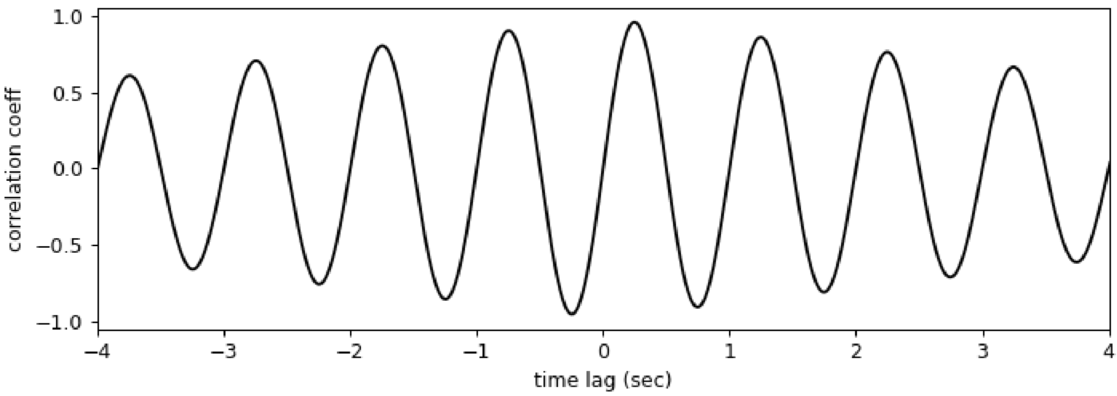
In [6]:

```
tx = np.arange(-t[-1],t[-1]+(0.5/fs),1/fs)
```

The cross correlation plot looks similar to original matlab example:

In [7]:

```
plt.figure(figsize=(8,3))
plt.plot(tx, cxx, 'k-')
plt.xlim(-4,4)
plt.xlabel('time lag (sec)')
plt.ylabel('correlation coeff')
plt.tight_layout()
```



Use peak of cross-correlation to find time shift between sig1 and sig2, which was 0.25 in original matlab example:

In [8]:

```
tshift = tx[np.argmax(cxx)]
tshift
```

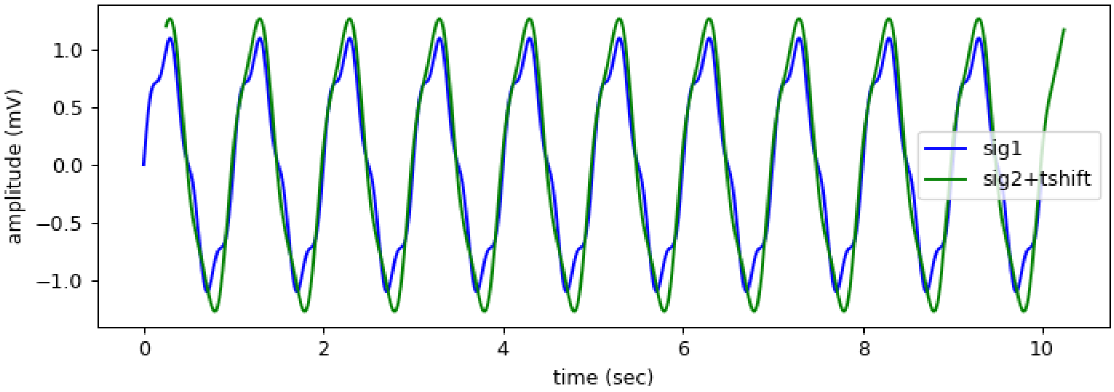
Out[8]:

```
0.2499999999997815
```

Now replot sig2 using this time shift to check signals align:

In [9]:

```
plt.figure(figsize=(8,3))
plt.plot(t, sig1,'b-', label='sig1')
plt.plot(t+tshift, sig2, 'g-', label='sig2+tshift')
plt.xlabel('time (sec)')
plt.ylabel('amplitude (mV)')
plt.legend()
plt.tight_layout()
```



Elephant Version

`elephant.signal_processing.cross_correlation_function()` does not give quite the same results as `numpy.correlate()` function

In [10]:

```
x = np.vstack([sig1,sig2]).T
x.shape
```

Out[10]:

```
(1000, 2)
```

In [11]:

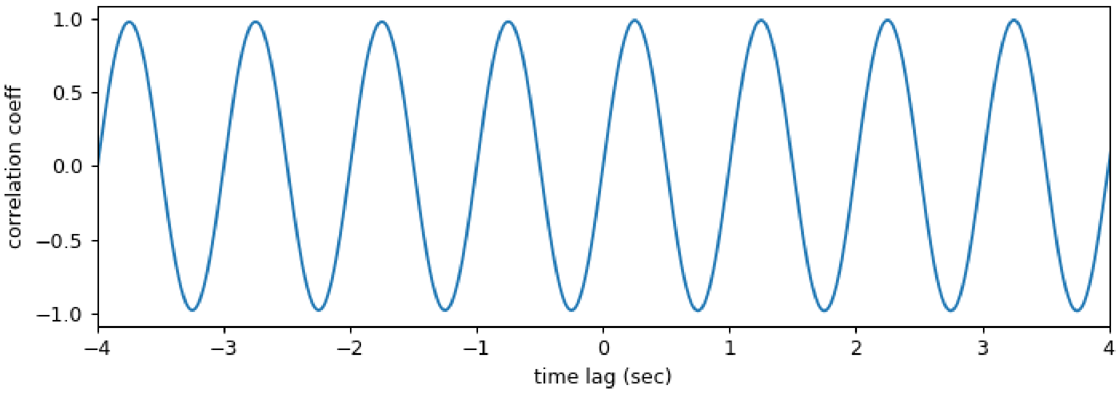
```
signal = neo.AnalogSignal(x, units='mV', t_start=0.*sec, sampling_rate=fs*Hz, dt
ype=float)
```

In [12]:

```
rho = elephant.signal_processing.cross_correlation_function(signal, [0,1])
```

In [13]:

```
plt.figure(figsize=(8,3))
plt.plot(rho.times, rho)
plt.xlabel('time lag (sec)')
plt.ylabel('correlation coeff')
plt.xlim(-4,4)
plt.tight_layout()
```



Differences...

Examined `cross_correlation_function()` code (https://github.com/NeuralEnsemble/elephant/blob/master/elephant/signal_processing.py (https://github.com/NeuralEnsemble/elephant/blob/master/elephant/signal_processing.py)). Essentially it performs an FFT convolution of normalized signals, where tau is the time lags and xcorr is correlation coefficient:

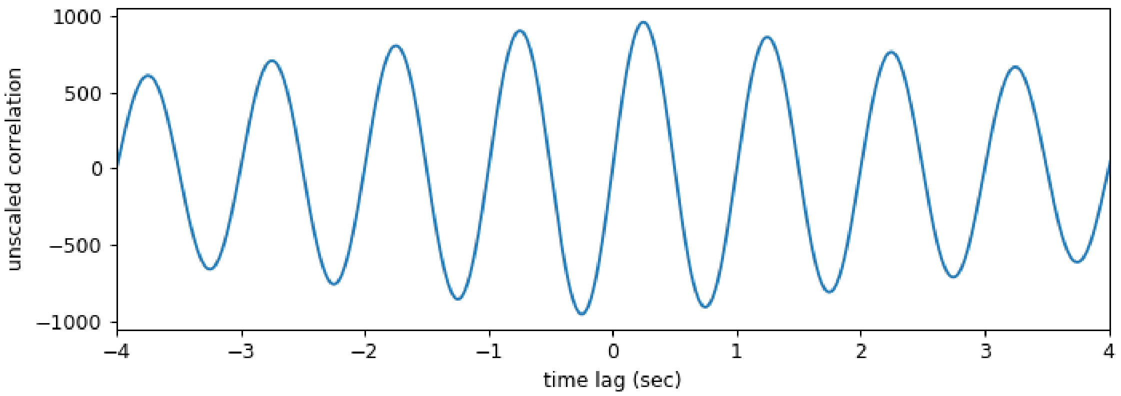
In [14]:

```
from scipy.signal import fftconvolve
from scipy.stats import zscore

zsig1 = zscore(sig1)
zsig2 = zscore(sig2)

nt = np.shape(zsig1)[0]

tau = (np.arange(nt) - nt//2)

xcorr = fftconvolve(zsig1, zsig2[::-1], mode='same')

# did not perform scaling of xcorr:
# xcorr = xcorr / npm.repmat((nt-abs(tau)), nch, 1).T
```

The unscaled "correlation coefficient" appears same as numpy correlate():

In [15]:

```
plt.figure(figsize=(8,3))
plt.plot(tau/fs, xcorr)
plt.xlabel('time lag (sec)')
plt.ylabel('unscaled correlation')
plt.xlim(-4,4)
plt.tight_layout()
```



The time shift matches numpy result too:

In [16]:

```
tshift = tau[np.argmax(xcorr)]/fs
tshift
```

Out[16]:

```
0.25
```

Conclusion

This suggests at least for single pairs there may be a problem in the scaling of xcorr. This is scaling step for a single channel case (one pair):

In [17]:

```
nch = 1
R = np.matlib.repmat((nt-abs(tau)), nch, 1).T
```

In [18]:

```
R.shape
```

Out[18]:

```
(1000, 1)
```

In [19]:

```
np.squeeze(R)
```

Out[19]:

```
array([ 500,  501,  502,  503,  504,  505,  506,  507,  508,  509,
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        511,  512,  513,  514,  515,  516,  517,  518,  519,  520,
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        742,  743,  744,  745,  746,  747,  748,  749,  750,  751,
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        753,  754,  755,  756,  757,  758,  759,  760,  761,  762,
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        764,  765,  766,  767,  768,  769,  770,  771,  772,  773,
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        775,  776,  777,  778,  779,  780,  781,  782,  783,  784,
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        786,  787,  788,  789,  790,  791,  792,  793,  794,  795,
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            510,  509,  508,  507,  506,  505,  504,  503,  502,  501])
```
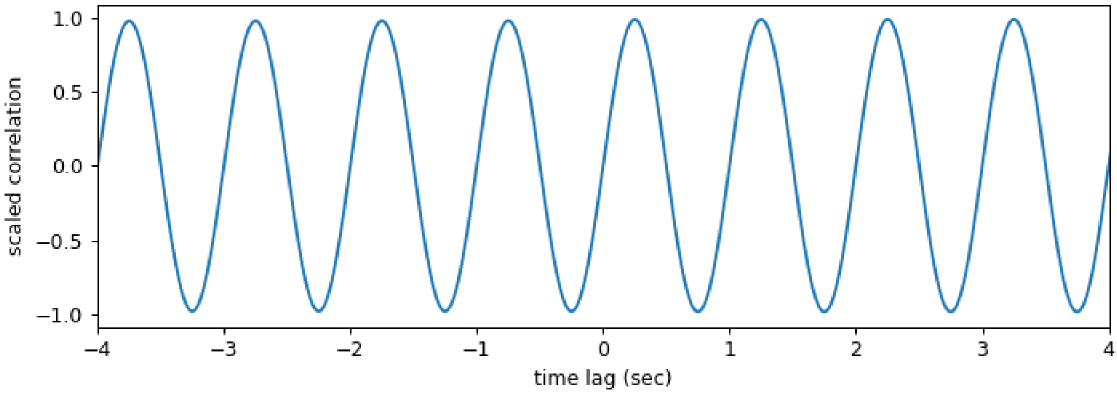

So it appears that xcorr is scaled by abs(lag) which compensates for decay from peak correlation:

In [20]:

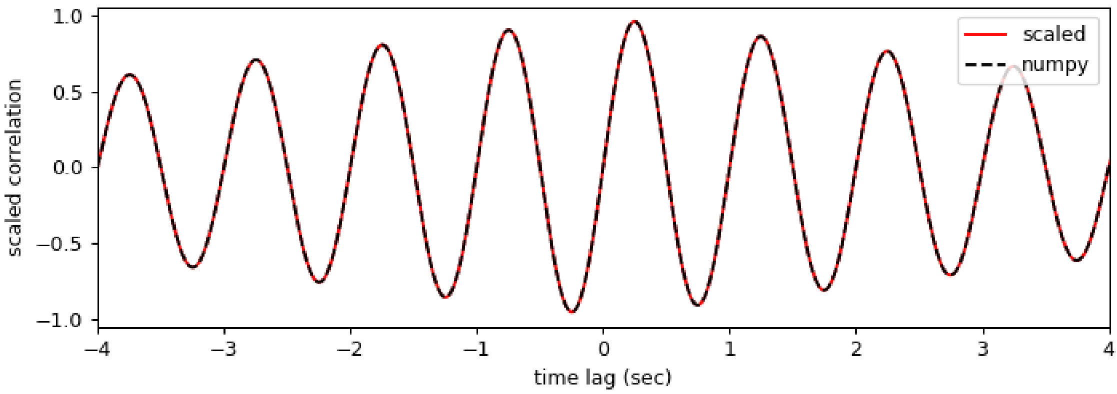
```
plt.figure(figsize=(8,3))
plt.plot(tau/fs, xcorr/np.squeeze(R))
plt.xlabel('time lag (sec)')
plt.ylabel('scaled correlation')
plt.xlim(-4,4)
plt.tight_layout()
```



Whereas dividing by sample length appears to scale correlation identically to numpy version:

In [21]:

```
plt.figure(figsize=(8,3))
plt.plot(tau/fs, xcorr/nt,'r-',label='scaled')
plt.plot(tx, cxx, 'k--', label='numpy')
plt.xlabel('time lag (sec)')
plt.ylabel('scaled correlation')
plt.xlim(-4,4)
plt.legend()
plt.tight_layout()
```



In [ ]: