For Age and Want save while you may;
No morning Sun lasts a whole Day.

Ben Franklin

NOTE: Compute all dollar-denominated answers to the nearest dollar unless otherwise specified, and compute all other answers to four decimal places (*i.e.,* .0001). Unless otherwise stated, all interest rates or rates of return are annual rates. Excel function hints are given in brackets.

ANNUITIES AND PERPETUITIES

1. Mick Jagger (age 77), lead singer of the Rolling Stones, welcomed at the end of 2016 his 8th child, Devereaux. The mother, Melanie Hamrick (then age 32!), is a recently retired ballerina with the American Ballet Theatre and told the DailyMail two years ago that she doesn't think of herself as "[Mick's] baby mama." Being the responsible man he is, Jagger has reportedly agreed to provide support of $15,000 per month for the newborn until he reaches the age of 18. (There apparently are other undisclosed financial payments for additional schooling and a humble abode for Ms. Hamrick, but we'll ignore those.) Thus, Mick will have no further financial obligations the year that young Devereaux turns 18.

   Let's assume that Mick's newest scion was born on January 1, 2020, will live a long and fruitful life, and the payments are very low risk, given Mick's wealth. The 20-year U.S. treasury bond is currently yielding 1.18%, so let's use 3.00% as the appropriate discount rate. A sample of the Rolling Stones' wonderful *oeuvre* that provides an artistic interpretation of such relationships can be found here: Rolling Stones. For this problem, *assume that each year's total monthly payments are paid annually).*

   (a) What is the PV (as of Jan. 1, 2020) of the payments if the total payments for each year ($180,000) are paid on December 31 of each year? *[PV]*

   (b) What is the PV (as of Jan. 1, 2020) of the payments if each year's total payments are paid at the beginning of each year, i.e., each January 1? *[PV]*

   (c) Suppose that Melanie negotiates an escalator clause whereby each year's payment, *after* the first payment, will be increased by the year's inflation (let's assume 2% per year). What is the PV (as of Jan. 1, 2020) of the payments if they are paid *at the end of each year*, i.e., each December 31?

   (d) Suppose that Melanie wants a lump sum payment on the day of the birth of her child (January 1, 2020) instead of the installment payments, but Mick insists on the annuity payout. Melanie turns to J.G. Wentworth and sells the promised payments for a lump sum in a structured settlement agreement. If J.G. Wentworth discounts the payments at 6% (assume they are made at year end and

there is no escalator clause), how much would they offer her? *[PV]*

(e) Suppose that Melanie is a very responsible mom and decides to invest the payments instead of spending them. If the payments are received at year end and there is no escalator clause, what will be the account balance when young Jagger turns 18 if Melanie is able to earn 4% per year? 6% per year?*[FV]*

2. You want to have $3 million when you retire in 35 years. Assume that you begin saving one year from today and make 35 equal, annual additions at year end to your retirement account. How much do you have to save each year to reach this goal if your annual rate of return is 4%? 6%? *[PMT]*

3. To get an idea of the importance of time in compounding (i.e., starting early), answer question 2 but now assume that you make the 35 equal additions at the beginning of each year instead at year end. *[PMT]*

4. Now that you have accumulated all of that wealth, you want to endow a chair in corporate finance at the FLS that will pay $250,000 per year beginning one year from today–remember, be generous when you decide to give back. How much do you have to donate *today* to the FLS to fund the chair if you want the chair to be endowed for 30 years, after which there will be no funds left, and the market rate of interest is 5%? *[PV]*

5. Same facts as the previous question, except that you want the chair to last indefinitely (a perpetuity). How big is the check that you will happily write now?

6. Read the WSJ Cryonaut article on the class web site. The Reanimation Foundation states that a $10,000 investment could grow to $8,677,163 in 100 years. What is the annual rate of return they are assuming? An interesting book that explores a variation on this theme is Michel Houellebecq's, *The Possibility of an Island. [Rate]*

7. On Immediate Annuities, for a payment today of $1 million, I can receive a guaranteed *monthly* payment of $8,901 for ten years. If each payment is received at month end, what's the *effective annual interest rate* the insurance company is offering me on the investment? *[Rate]*