20

CHAPTER

Understanding Options

Pop quiz: What do the following events have in common?

- Hershey buys options that put a ceiling on the price that it will pay for its future purchases of cocoa.
- Flatiron offers its president a bonus if the company's stock price exceeds $120.
- Blitzen Computer dips a toe in the water and enters a new market.
- Malted Herring postpones investment in a positive-NPV plant.
- Hewlett-Packard exports partially assembled printers even though it would be cheaper to ship the finished product.
- Dominion installs a dual-fired unit at its Possum Point power station that can use either fuel oil or natural gas.
- in 2017, vTv Therapeutics issues 38,006 warrants. Each warrant entitles its owner to buy an additional Class A share for $5.92.
- Twitter issues $1.8 billion of convertible bonds. Each bond can be exchanged for 12.9 shares.

Answers: (1) Each of these events involves an option, and (2) they illustrate why the financial manager of an industrial company needs to understand options.

Companies regularly use commodity, currency, and interest-rate options to reduce risk. For example, a meat-packing company that wishes to put a ceiling on the cost of beef might take out an option to buy live cattle at a fixed price. A company that wishes to limit its future borrowing costs might take out an option to sell long-term bonds at a fixed price. And so on. In Chapter 26, we explain how firms employ options to limit their risk.

Many capital investments include an embedded option to expand in the future. For instance, the company may invest in a patent that allows it to exploit a new technology, or it may purchase adjoining land that gives it the option in the future to increase capacity. In each case, the company is paying money today for the opportunity to make a further investment. To put it another way, the company is acquiring *growth opportunities.*

Here is another disguised option to invest: You are considering the purchase of a tract of desert land that is known to contain gold deposits. Unfortunately, the cost of extraction is higher than the current price of gold. Does this mean the land is almost worthless? Not at all. You are not obliged to mine the gold, but ownership of the land gives you the option to do so. Of course, if you know that the gold price will remain below the extraction cost, then the option is worthless. But if there is uncertainty about future gold prices, you could be lucky and make a killing.[1]

If the option to expand has value, what about the option to bail out? Projects don't usually go on until the equipment disintegrates. The decision to terminate a project is usually taken by management, not by nature. Once the project is no longer profitable, the company will cut its losses and exercise its option to abandon the project. Some projects have higher abandonment value than others. Those that use standardized

[1]In Chapter 11, we valued Kingsley Solomon's gold mine by calculating the value of the gold in the ground and then subtracting the value of the extraction costs. That is correct only if we know that the gold will be mined. Otherwise, the value of the mine is increased by the value of the option to leave the gold in the ground if its price is less than the extraction cost.

TABLE 20.1 Selected prices of put and call options on Amazon.com stock in April 2017, when the closing stock price was about $900

Maturity Date	Exercise Price	Price of Call Option	Price of Put Option
July 2017	$820	$95.58	$14.40
	860	66.03	24.73
	900	42.80	41.15
	940	25.35	63.63
	980	14.08	92.43
October 2017	$820	$113.30	$28.75
	860	86.65	42.15
	900	**64.30**	**59.55**
	940	45.95	81.18
	980	31.75	110.30
January 2018[a]	$820	$128.23	$40.68
	860	103.08	55.60
	900	81.23	73.15
	940	62.58	94.75
	980	46.83	119.58

[a] Long-term options are called "LEAPS."

Source: Yahoo! Finance, finance.yahoo.com.

Now look at the quotes for options maturing in October 2017 and January 2018. Notice how the option price increases as option maturity is extended. For example, at an exercise price of $900, the July 2017 call option costs $42.80, the October 2017 option costs $64.30, and the January 2018 option costs $81.23. The longer you have to decide whether you want to exercise, the more valuable is the option.

Option analysts often draw a *position diagram* to illustrate the possible payoffs from an option. For example, the position diagram in Figure 20.1*a* shows the possible consequences of investing in Amazon October 2017 call options with an exercise price of $900 (boldfaced in Table 20.1). The outcome from investing in Amazon calls depends on what happens to

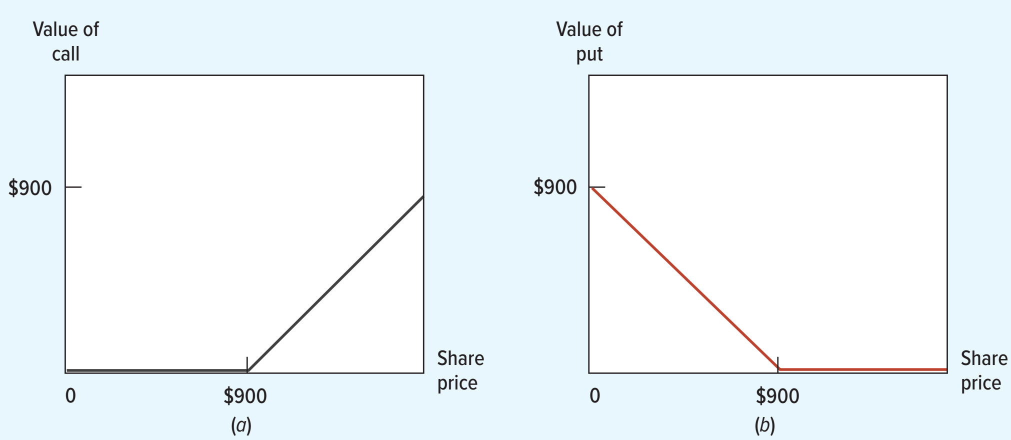


FIGURE 20.1 Position diagrams show how payoffs to owners of Amazon calls and puts (shown by the colored lines) depend on the share price. (*a*) Result of buying Amazon call exercisable at $900. (*b*) Result of buying Amazon put exercisable at $900.

the stock price. If the stock price at the end of this six-month period turns out to be less than the \$900 exercise price, it will not make sense to pay the exercise price to obtain the share. Your call will, in that case, be worthless. On the other hand, if the stock price turns out to be greater than \$900, it will pay to exercise your option to buy the share. In this case, when the call expires, it will be worth the market price of the share minus the \$900 that you must pay to exercise the option. For example, suppose that the price of Amazon stock rises to \$980. Your call will then be worth \$980 – \$900 = \$80. That is your payoff, but of course, it is not all profit. Table 20.1 shows that you had to pay \$64.30 to buy the call.

Put Options

Now let us look at the Amazon **put options** in the right-hand column of Table 20.1. Whereas a call option gives you the right to *buy* a share for a specified exercise price, a put gives you the right to *sell* the share. For example, the boldfaced entry in the right-hand column of Table 20.1 shows that for \$59.55, you could acquire an option to sell Amazon stock for a price of \$900 any time before October 2017. The circumstances in which the put turns out to be valuable are just the opposite of those in which the call is profitable. You can see this from the position diagram in Figure 20.1*b*. If Amazon's share price immediately before expiration turns out to be *greater* than \$900, you won't want to sell stock at that price. You would do better to sell the share in the market, and your put option will be worthless. Conversely, if the share price turns out to be *less* than \$900, it will pay to buy stock at the low price and then take advantage of the option to sell it for \$900. In this case, the value of the put option on the exercise date is the difference between the \$900 proceeds of the sale and the market price of the share. For example, if the share is worth \$800, the put is worth \$100:

$$\text{Value of put option at expiration} = \text{exercise price} - \text{market price of the share}$$
$$= \$900 - \$800 = \$100$$

Selling Calls and Puts

Let us now look at the position of an investor who *sells* these investments. If you sell, or "write," a call, you promise to deliver shares if asked to do so by the call buyer. In other words, the buyer's asset is the seller's liability. If the share price is below the exercise price when the option matures, the buyer will not exercise the call and the seller's liability will be zero. If it rises above the exercise price, the buyer will exercise and the seller must give up the shares. The seller loses the difference between the share price and the exercise price received from the buyer. Notice that it is the buyer who always has the option to exercise; option sellers simply do as they are told.

Suppose that the price of Amazon stock turns out to be \$980, which is above the option's exercise price of \$900. In this case, the buyer will exercise the call. The seller is forced to sell stock worth \$980 for only \$900 and so has a payoff of – \$80.[4] Of course, that \$80 loss is the buyer's gain. Figure 20.2*a* shows how the payoffs to the seller of the Amazon call option vary with the stock price. Notice that for every dollar the buyer makes, the seller loses a dollar. Figure 20.2*a* is just Figure 20.1*a* drawn upside down.

In just the same way, we can depict the position of an investor who sells, or writes, a put by standing Figure 20.1*b* on its head. The seller of the put has agreed to pay \$900 for the share if the buyer of the put should request it. Clearly the seller will be safe as long as the share price remains above \$900 but will lose money if the share price falls below this figure. The worst thing that can happen is that the stock becomes worthless. The seller would then be obliged to pay \$900 for a stock worth \$0. The payoff to the seller would be –\$900.

[4]The seller has some consolation, for he or she was paid \$64.30 in April for selling the call.

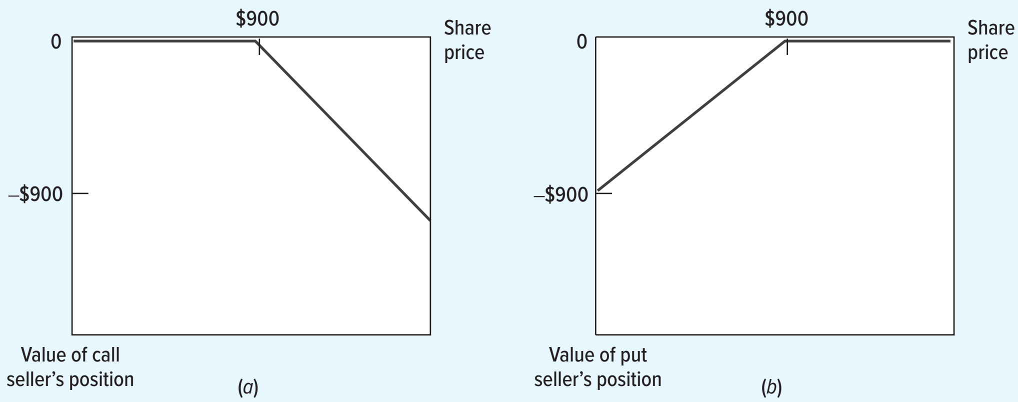


FIGURE 20.2 Payoffs to *sellers* of Amazon calls and puts (shown by the colored lines) depend on the share price. (*a*) Result of selling Amazon call exercisable at $900. (*b*) Result of selling Amazon put exercisable at $900.

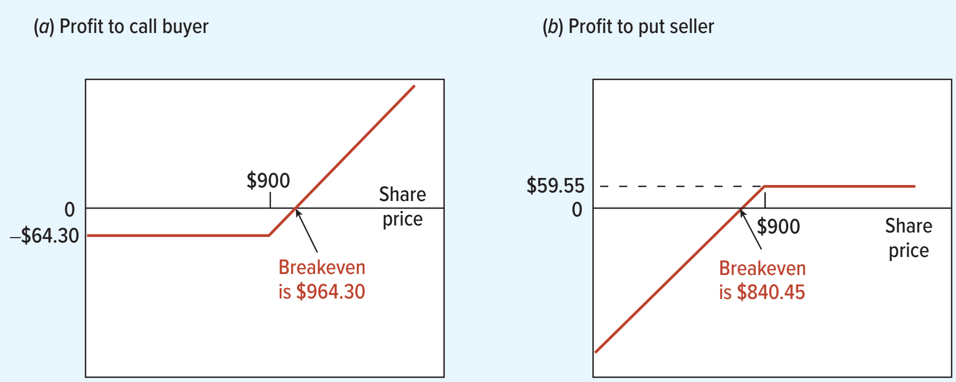
Position Diagrams Are Not Profit Diagrams

Position diagrams show *only* the payoffs at option exercise; they do not account for the initial cost of buying the option or the initial proceeds from selling it.

This is a common point of confusion. For example, the position diagram in Figure 20.1*a* makes purchase of a call *look* like a sure thing—the payoff is at worst zero, with plenty of upside if Amazon's stock price goes above $900 by October 2017. But compare the *profit diagram* in Figure 20.3*a*, which subtracts the $64.30 *cost* of the call in April 2017 from the payoff at maturity. The call buyer loses money at all share prices less than $900 + 64.30 = $964.30. Take another example: The position diagram in Figure 20.2*b* makes selling a put *look* like a sure loss—the *best* payoff is zero. But the profit diagram in Figure 20.3*b*, which recognizes the $59.55 received by the seller, shows that the seller gains at all prices above $900 – 59.55 = $840.45.[5]

FIGURE 20.3

Profit diagrams incorporate the costs of buying an option or the proceeds from selling one. In panel (*a*), we subtract the $64.30 cost of the Amazon call from the payoffs plotted in Figure 20.1*a*. In panel (*b*), we add the $59.55 proceeds from selling the Amazon put to the payoffs in Figure 20.2*b*.



[5]The fact that you have made a profit on your position is not necessarily a cause for rejoicing. The profit needs to compensate you for the risk that you took.

Profit diagrams like those in Figure 20.3 may be helpful to the options beginner, but options experts rarely draw them.[6] Now that you've graduated from the first options class we won't draw them either. We stick to position diagrams, because you have to focus on payoffs at exercise to understand options and to value them properly.

20-2 Financial Alchemy with Options

BEYOND THE PAGE

Option payoffs

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Look now at Figure 20.4*a*, which shows the payoff if you buy Amazon stock at $900. You gain dollar-for-dollar if the stock price goes up and you lose dollar-for-dollar if it falls. That's trite; it doesn't take a genius to draw a 45-degree line.

Look now at panel (b), which shows the payoffs from an investment strategy that retains the upside potential of Amazon stock but gives complete downside protection. In this case, your payoff stays at $900 even if the Amazon stock price falls to $800, $500, or zero. Panel (b)'s payoffs are clearly better than panel (a)'s. If a financial alchemist could turn panel (a) into panel (b), you'd be willing to pay for the service.

Now, as you have probably suspected, this financial alchemy is for real. You can do the transmutation shown in Figure 20.4. You do it with options, and we will show you how. Look at row 1 of Figure 20.5. The first diagram again shows the payoff from buying a share of Amazon stock, while the next diagram in row 1 shows the payoffs from buying an Amazon put option with an exercise price of $900. The third diagram shows the effect of combining these two positions. You can see that if Amazon's stock price rises above $900, your put option is valueless, so you simply receive the gains from your investment in the share. However, if the stock price falls below $900, you can exercise your put option and sell your stock for $900. Thus, by adding a put option to your investment in the stock, you have protected yourself against loss.[7] This is the strategy that we depicted in Figure 20.4. Of course, there is no gain without pain. The cost of insuring yourself against loss is the amount that you pay for a put option on Amazon stock with an exercise price of $900. In April 2017, the price of this put was $59.55. This was the going rate for financial alchemists.

We have just seen how put options can be used to provide downside protection. We now show you how call options can be used to get the same result. This is illustrated in row 2 of Figure 20.5. The first diagram shows the payoff from placing the present value of $900 in a

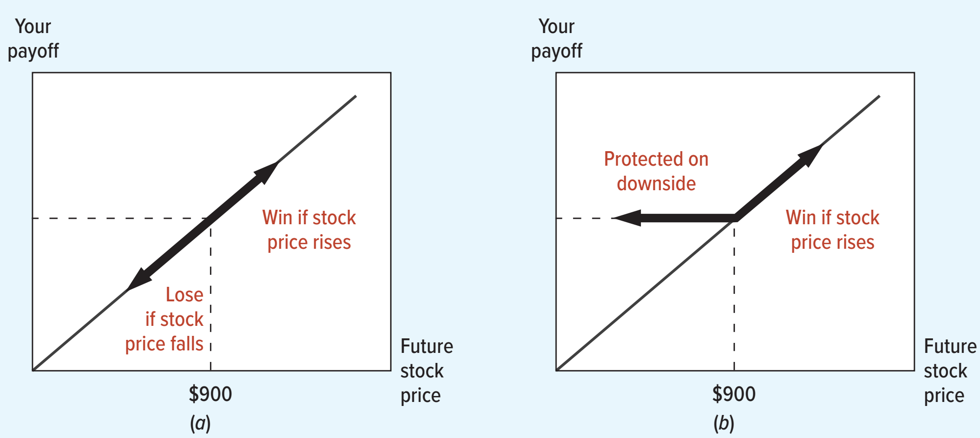


FIGURE 20.4 Payoffs at the end of six months to two investment strategies for Amazon stock. (*a*) You buy one share for $900. (*b*) No downside. If stock price falls, your payoff stays at $900.

[6]Profit diagrams such as Figure 20.3 deduct the initial cost of the option from the final payoff. They therefore ignore the first lesson of finance—"A dollar today is worth more than a dollar in the future."

[7]This combination of a stock and a put option is known as a *protective put*.

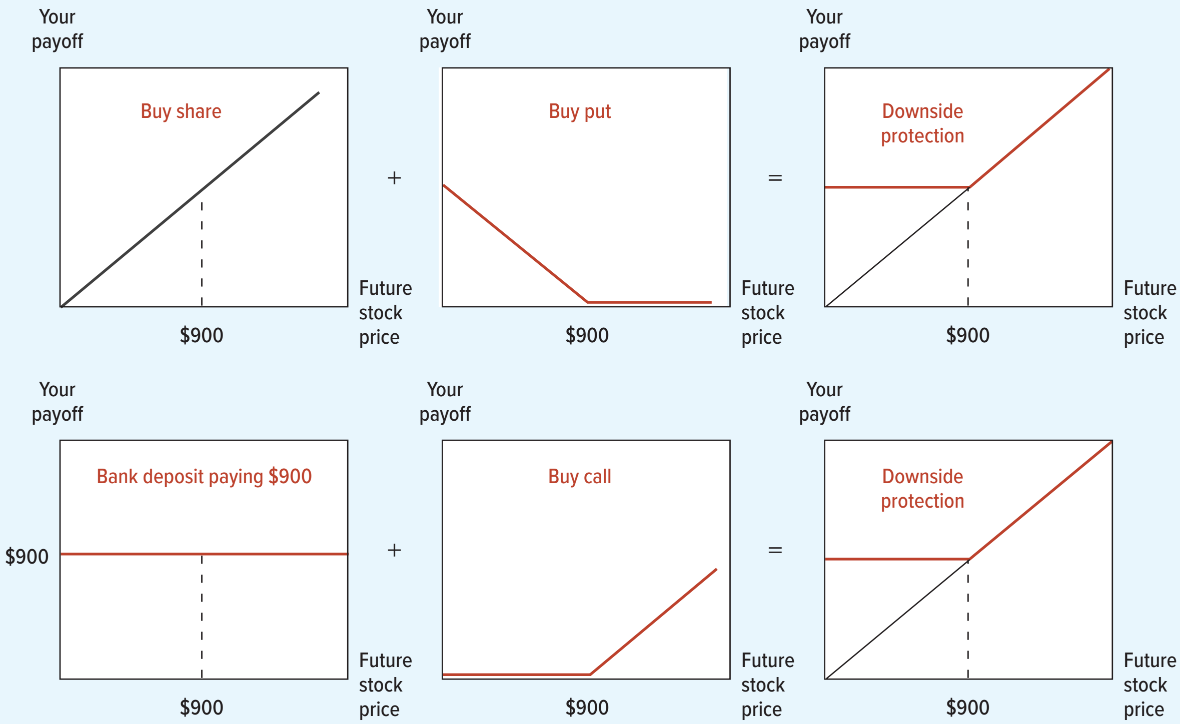


FIGURE 20.5 Each row in the figure shows a different way to create a strategy where you gain if the stock price rises but are protected on the downside (strategy [*b*] in Figure 20.4)

bank deposit. Regardless of what happens to the price of Amazon stock, your bank deposit will pay off $900. The second diagram in row 2 shows the payoff from a call option on Amazon stock with an exercise price of $900, and the third diagram shows the effect of combining these two positions. Notice that if the price of Amazon stock falls, your call is worthless, but you still have your $900 in the bank. For every dollar that Amazon stock price rises above $900, your investment in the call option pays off an extra dollar. For example, if the stock price rises to $980, you will have $900 in the bank and a call worth $80. Thus you participate fully in any rise in the price of the stock, while being fully protected against any fall. So we have just found another way to provide the downside protection depicted in panel (b) of Figure 20.4.

These two rows of Figure 20.5 tell us something about the relationship between a call option and a put option. Regardless of the future stock price, both investment strategies provide identical payoffs. In other words, if you buy the share and a put option to sell it for $900, you receive the same payoff as from buying a call option and setting enough money aside to pay the $900 exercise price. Therefore, if you are committed to holding the two packages until the options expire, the two packages should sell for the same price today. This gives us a fundamental relationship for European options:

Value of call + present value of exercise price = value of put + share price

To repeat, this relationship holds because the payoff of

buy call, invest present value of exercise price in safe asset[8]

[8]The present value is calculated at the *risk-free* rate of interest. It is the amount that you would have to invest today in a bank deposit or Treasury bills to realize the exercise price on the option's expiration date.

is identical to the payoff from

buy put, buy share.

This basic relationship among share price, call and put values, and the present value of the exercise price is called **put–call parity**.[9]

Put–call parity can be expressed in several ways. Each expression implies two investment strategies that give identical results. For example, suppose that you want to solve for the value of a put. You simply need to twist the put–call parity formula around to give

Value of put = value of call + present value of exercise price – share price

From this expression you can deduce that

buy put

is identical to

buy call, invest present value of exercise price in safe asset, sell share.

In other words, if puts are not available, you can get exactly the same payoff by buying calls, putting cash in the bank, and selling shares.

If you find this difficult to believe, look at Figure 20.6, which shows the possible payoffs from each position. The diagram on the left shows the payoffs from a call option on Amazon stock with an exercise price of $900. The second diagram shows the payoffs from placing the present value of $900 in the bank. Regardless of what happens to the share price, this

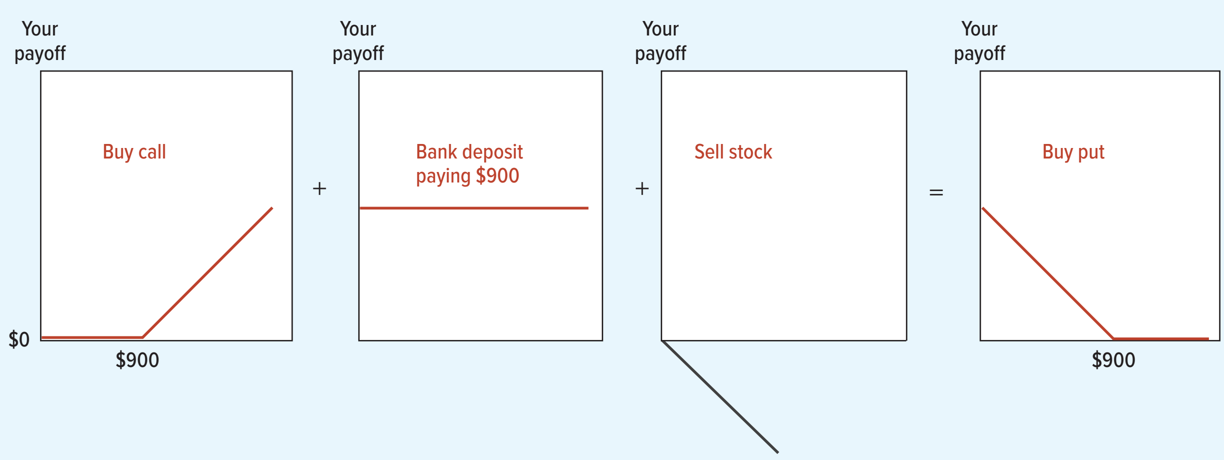


FIGURE 20.6 A strategy of buying a call, depositing the present value of the exercise price in the bank, and selling the stock is equivalent to buying a put

[9]Put–call parity holds only if you are committed to holding the options until the final exercise date. It therefore does not hold for American options, which you can exercise *before* the final date. We discuss possible reasons for early exercise in Chapter 21. Also if the stock makes a dividend payment before the final exercise date, you need to recognize that the investor who buys the call misses out on this dividend. In this case the relationship is

Value of call + present value of exercise price = value of put + share price – present value of dividend investment will pay off \$900. The third diagram shows the payoffs from selling Amazon stock. When you sell a share that you don't own, you have a liability—you must sometime buy it back. As they say on Wall Street:

He who sells what isn't his'n
Buys it back or goes to pris'n

Therefore, the best that can happen to you is that the share price falls to zero. In that case, it costs you nothing to buy the share back. But for every extra dollar on the future share price, you will need to spend an extra dollar to buy the share. The final diagram in Figure 20.6 shows that the *total* payoff from these three positions is the same as if you had bought a put option. For example, suppose that when the option matures, the stock price is \$800. Your call will be worthless, your bank deposit will be worth \$900, and it will cost you \$800 to repurchase the share. Your total payoff is 0 + 900 – 800 = \$100, exactly the same as the payoff from the put.

If two investments offer identical payoffs, then they should sell for the same price today. If the law of one price is violated, you have a potential arbitrage opportunity. So let's check whether there are any arbitrage profits to be made from our Amazon calls and puts. In April 2017, the price of a six-month call with a \$900 exercise price was \$64.30, the interest rate was about .5% for 6 months, and the price of Amazon stock was \$900. Therefore the cost of a homemade put was

Buy call	+	present value of exercise price	−	share price	=	cost of homemade put	
64.30	+	900/1.005	−	900	=	\$59.82	

This is almost exactly the same as it would have cost you to buy a put directly.

Spotting the Option

Options rarely come with a large label attached. Often, the trickiest part of the problem is to identify the option. When you are not sure whether you are dealing with a put or a call or a complicated blend of the two, it is a good precaution to draw a position diagram. Here is an example.

The Flatiron and Mangle Corporation has offered its president, Ms. Higden, the following incentive scheme: At the end of the year Ms. Higden will be paid a bonus of \$50,000 for every dollar that the price of Flatiron stock exceeds its current figure of \$120. However, the maximum bonus that she can receive is set at \$2 million.[10]

You can think of Ms. Higden as owning 50,000 tickets, each of which pays nothing if the stock price fails to beat \$120. The value of each ticket then rises by \$1 for each dollar rise in the stock price up to the maximum of \$2,000,000/50,000 = \$40. Figure 20.7 shows the payoffs from just one of these tickets. The payoffs are not the same as those of the simple put and call options that we drew in Figure 20.1, but it is possible to find a combination of options that exactly replicates Figure 20.7. Before going on to read the answer, see if you can spot it yourself. (If you are someone who enjoys puzzles of the make-a-triangle-from-just-two-matchsticks type, this one should be a walkover.)

The answer is in Figure 20.8. The solid black line represents the purchase of a call option with an exercise price of \$120, and the dotted line shows the sale of another call option with an exercise price of \$160. The colored line shows the payoffs from a combination of the purchase and the sale—exactly the same as the payoffs from one of Ms. Higden's tickets.

[10]Bonus schemes in many companies follow a pattern similar to Ms. Higden's scheme. See, for example, A. Edmans, X. Gabaix, and D. Jenter, "Executive Compensation: A Survey of Theory and Evidence," European Corporate Governance Institute, June 26, 2017.

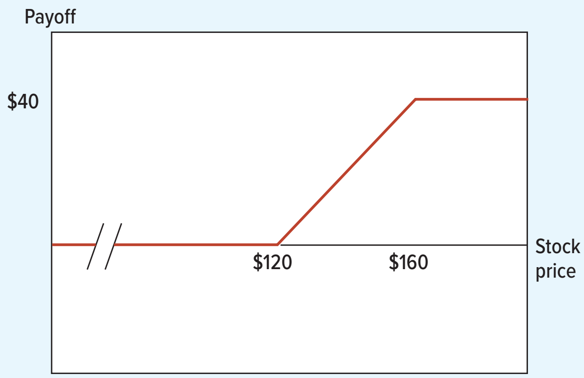


FIGURE 20.7

The payoff from one of Ms. Higden's "tickets" depends on Flatiron's stock price

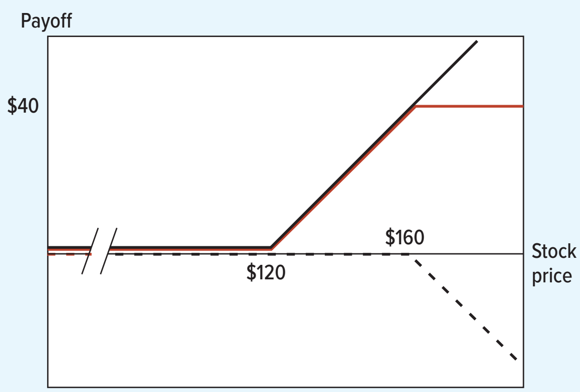


FIGURE 20.8

The solid black line shows the payoff from buying a call with an exercise price of $120. The dotted line shows the sale of a call with an exercise price of $160. The combined purchase and sale (shown by the colored line) is identical to one of Ms. Higden's "tickets."

Thus, if we wish to know how much the incentive scheme is costing the company, we need to calculate the difference between the value of 50,000 call options with an exercise price of $120 and the value of 50,000 calls with an exercise price of $160.

We could have made the incentive scheme depend in a much more complicated way on the stock price. For example, the bonus could peak at $2 million and then fall steadily back to zero as the stock price climbs above $160.[11] You could still have represented this scheme as a combination of options. In fact, we can state a general theorem:

> Any set of contingent payoffs—that is, payoffs that depend on the value of some other asset—can be constructed with a mixture of simple options on that asset.

In other words, you can create any position diagram—with as many ups and downs or peaks and valleys as your imagination allows—by buying or selling the right combinations of puts and calls with different exercise prices.[12]

Finance pros often talk about **financial engineering,** which is the practice of packaging different investments to create new tailor-made instruments. Perhaps a German company

[11]This is not as nutty a bonus scheme as it may sound. Maybe Ms. Higden's hard work can lift the value of the stock by so much and the only way she can hope to increase it further is by taking on extra risk. You can deter her from doing this by making her bonus start to decline beyond some point. Too bad that before the financial crisis the bonus schemes for some bank CEOs did not contain this feature.

[12]In some cases, you may also have to borrow or lend money to generate a position diagram with your desired pattern. Lending raises the payoff line in position diagrams, as in the bottom row of Figure 20.5. Borrowing lowers the payoff line.

would like to set a minimum and maximum cost at which it can buy dollars in six-months' time. Or perhaps an oil company would like to pay a lower rate of interest on its debt if the price of oil falls. Options provide the building blocks that financial engineers use to create these interesting payoff structures.

20-3 What Determines Option Values?

So far we have said nothing about how the market value of an option is determined. We do know what an option is worth when it matures, however. Consider, for instance, our earlier example of an option to buy Amazon stock at $900. If Amazon's stock price is below $900 on the exercise date, the call will be worthless; if the stock price is above $900, the call will be worth $900 less than the value of the stock. This relationship is depicted by the heavy, lower line in Figure 20.9.

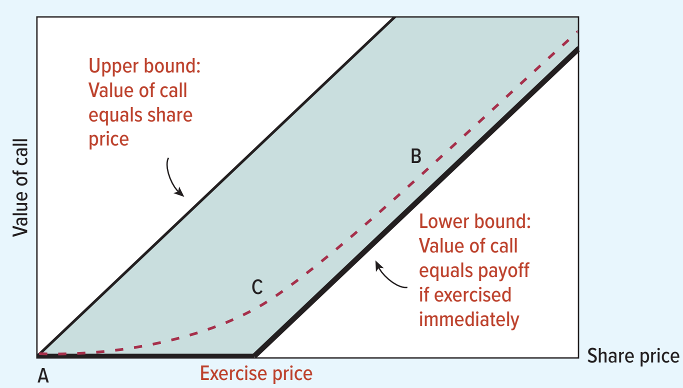
Even before maturity the price of the option can never remain *below* the heavy, lower-bound line in Figure 20.9. For example, if our option were priced at $20 and the stock were priced at $980, it would pay any investor to sell the stock and then buy it back by purchasing the option and exercising it for an additional $900. That would give an arbitrage opportunity with a profit of $60. The demand for options from investors seeking to exploit this opportunity would quickly force the option price up, at least to the heavy line in the figure. For options that still have some time to run, the heavy line is therefore a *lower bound* on the market price of the option. Option geeks express the same idea more concisely when they write Lower bound = max(stock price – exercise price, 0).

The diagonal line in Figure 20.9 is the *upper bound* to the option price. Why? Because the option cannot give a higher ultimate payoff than the stock. If at the option's expiration the stock price ends up *above* the exercise price, the option is worth the stock price *less* the exercise price. If the stock price ends up *below* the exercise price, the option is worthless, but the stock's owner still has a valuable security. For example, if the option's exercise price is $900, then the extra dollar returns realized by stockholders are shown in the following table:

	Stock Payoff	Option Payoff	Extra Payoff from Holding Stock Instead of Option
Option exercised (stock price greater than $900)	Stock price	Stock price – $900	$900
Option expires unexercised (stock price less than or equal to $900)	Stock price	0	Stock price

FIGURE 20.9

Value of a call before its expiration date (dashed line). The value depends on the stock price. It is always worth more than its value if exercised now (heavy line). It is never worth more than the stock price itself.



If the stock and the option have the same price, everyone will rush to sell the option and buy the stock. Therefore, the option price must be somewhere in the shaded region of Figure 20.9. In fact, it will lie on a curved, upward-sloping line like the dashed curve shown in the figure. This line begins its travels where the upper and lower bounds meet (at zero). Then it rises, gradually becoming parallel to the upward-sloping part of the lower bound.

But let us look more carefully at the shape and location of the dashed line. Three points, *A, B,* and *C,* are marked on the dashed line. As we explain each point you will see why the option price has to behave as the dashed line predicts.

Point *A* *When the stock is worthless, the option is worthless.* A stock price of zero means that there is no possibility the stock will ever have any future value.[13] If so, the option is sure to expire unexercised and worthless, and it is worthless today.

That brings us to our first important point about option value:

The value of an option increases as stock price increases, if the exercise price is held constant.

That should be no surprise. Owners of call options clearly hope for the stock price to rise and are happy when it does.

Point *B* *As the stock price increases, the option price approaches the stock price less the present value of the exercise price.* Notice that the dashed line representing the option price in Figure 20.9 eventually becomes parallel to the ascending heavy line representing the lower bound on the option price. The reason is as follows: The higher the stock price, the higher is the probability that the option will eventually be exercised. If the stock price is high enough, exercise becomes a virtual certainty; the probability that the stock price will fall below the exercise price before the option expires becomes trivially small.

If you own an option that you *know* will be exchanged for a share of stock, you effectively own the stock now. The only difference is that you don't have to pay for the stock (by handing over the exercise price) until later, when formal exercise occurs. In these circumstances, buying the call is equivalent to buying the stock but financing part of the purchase by borrowing. The amount implicitly borrowed is the present value of the exercise price. The value of the call is therefore equal to the stock price less the present value of the exercise price.

This brings us to another important point about options. Investors who acquire stock by way of a call option are buying on credit. They pay the purchase price of the option today, but they do not pay the exercise price until they actually take up the option. The delay in payment is particularly valuable if interest rates are high and the option has a long maturity.

Thus, the value of an option increases with both the rate of interest and the time to maturity.

Point *C* *The option price always exceeds its minimum value* (except when stock price is zero). We have seen that the dashed and heavy lines in Figure 20.9 coincide when stock price is zero (point *A*), but elsewhere the lines diverge; that is, the option price must exceed the minimum value given by the heavy line. The reason for this can be understood by examining point *C.*

At point *C,* the stock price exactly equals the exercise price. The option is therefore worthless if exercised today. However, suppose that the option will not expire until three months hence. Of course, we do not know what the stock price will be at the expiration date. There is roughly a 50% chance that it will be higher than the exercise price and a 50% chance that it will be lower. The possible payoffs to the option are therefore

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[13]If a stock *can* be worth something in the future, then investors will pay *something* for it today, although possibly a very small amount.

Outcome	Payoff
Stock price rises (50% probability)	Stock price less exercise price (option is exercised)
Stock price falls (50% probability)	Zero (option expires worthless)

If there is a positive probability of a positive payoff, and if the worst payoff is zero, then the option must be valuable. That means the option price at point *C* exceeds its lower bound, which at point *C* is zero. In general, the option prices will exceed their lower-bound values as long as there is time left before expiration.

One of the most important determinants of the *height* of the dashed curve (i.e., of the difference between actual and lower-bound value) is the likelihood of substantial movements in the stock price. An option on a stock whose price is unlikely to change by more than 1% or 2% is not worth much; an option on a stock whose price may halve or double is very valuable.

As an option holder, you gain from volatility because the payoffs are not symmetric. If the stock price falls *below* the exercise price, your call option will be worthless, regardless of whether the shortfall is a few cents or many dollars. On the other hand, for every dollar that the stock price rises *above* the exercise price, your call will be worth an extra dollar. Therefore, the option holder gains from the increased volatility on the upside, but does not lose on the downside.

A simple example may help to illustrate the point. Consider two stocks, X and Y, each of which is priced at $100. The only difference is that the outlook for Y is much less easy to predict. There is a 50% chance that the price of Y will rise to $150 and a similar chance that it will fall to $70. By contrast, there is a 50–50 chance that the price of X will either rise to $130 or fall to $90.

Suppose that you are offered a call option on each of these stocks with an exercise price of $100. The following table compares the possible payoffs from these options:

	Stock Price Falls	Stock Price Rises
Payoff from option on X	$0	$130 – $100 = $30
Payoff from option on Y	$0	$150 – $100 = $50

In both cases, there is a 50% chance that the stock price will decline and make the option worthless but, if the stock price rises, the option on Y will give the larger payoff. Because the chance of a zero payoff is the same, the option on Y is worth more than the option on X.

Of course, in practice future stock prices may take on a range of different values. We have recognized this in Figure 20.10, where the uncertain outlook for Y's stock price shows up in the wider probability distribution of future prices.[14] The greater spread of outcomes for stock Y again provides more upside potential and, therefore, increases the chance of a large payoff on the option.

Figure 20.11 shows how volatility affects the value of an option. The upper curved line depicts the value of the Amazon call option assuming that Amazon's stock price, like that of stock Y, is highly variable. The lower curved line assumes a lower (and more realistic) degree of volatility.[15]

[14] Figure 20.11 continues to assume that the exercise price on both options is equal to the current stock price. This is not a necessary assumption. Also, in drawing Figure 20.11, we have assumed that the distribution of stock prices is symmetric. This also is not a necessary assumption, and we will look more carefully at the distribution of stock prices in the next chapter.

[15] The option values shown in Figure 20.12 were calculated by using the Black-Scholes option-valuation model. We explain this model in Chapter 21 and use it to value the Amazon option.

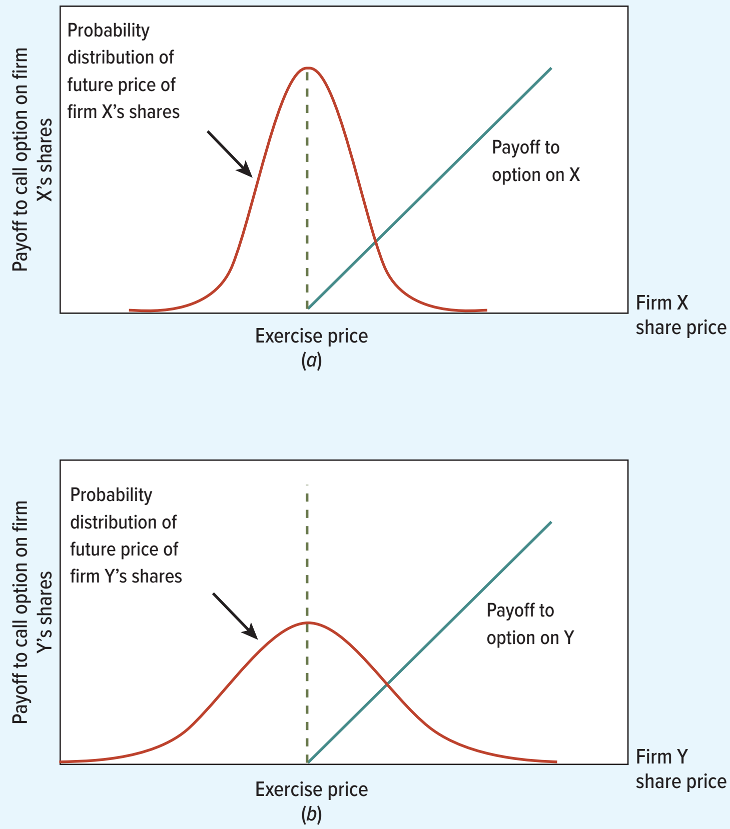


FIGURE 20.10

Call options on the shares of (*a*) firm X and (*b*) firm Y. In each case, the current share price equals the exercise price, so each option has a 50% chance of ending up worthless (if the share price falls) and a 50% chance of ending up "in the money" (if the share price rises). However, the chance of a large payoff is greater for the option on firm Y's shares because Y's stock price is more volatile and therefore has more upside potential.

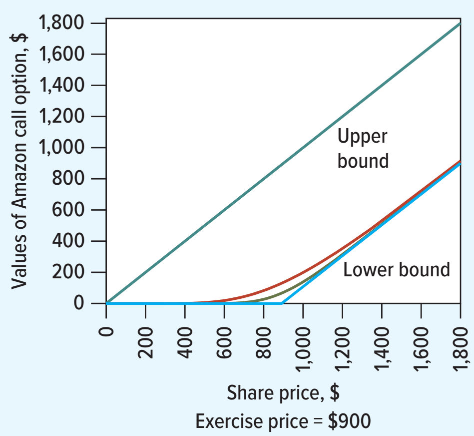


FIGURE 20.11

How the value of the Amazon call option increases with the volatility of the stock price. Each of the curved lines shows the value of the option for different initial stock prices. The only difference is that the upper line assumes a much higher level of uncertainty about Amazon's future stock price.

The probability of large stock price changes during the remaining life of an option depends on two things: (1) the variance (i.e., volatility) of the stock price *per period* and (2) the number of periods until the option expires. If there are t remaining periods, and the variance per period is σ^2, the value of the option should depend on cumulative variability $\sigma^2 t$.[16]

[16]Here is an intuitive explanation: If the stock price follows a random walk (Section 13-2), successive price changes are statistically independent. The cumulative price change before expiration is the sum of t random variables. The variance of a sum of independent random variables is the sum of the variances of those variables. Thus, if σ^2 is the variance of the daily price change, and there are t days until expiration, the variance of the cumulative price change is $\sigma^2 t$

TABLE 20.2
What the price of a call option depends on

*The direct effect of increases in r_f or σ on option price, given the stock price. There may also be indirect effects. For example, an increase in r_f could reduce stock price P. This in turn could affect option price.

1. If There Is an *Increase* in:	The Change in the Call Option Price Is:
Stock price (P)	Positive
Exercise price (EX)	Negative
Interest rate (r_f)	Positive*
Time to expiration (t)	Positive
Volatility of stock price (σ)	Positive*

2. Other Properties of Call Options:

a. *Upper bound.* The option price is always less than the stock price.
b. *Lower bound.* The call price never falls below the payoff to immediate exercise ($P - EX$ or zero, whichever is larger).
c. If the stock is worthless, the call is worthless.
d. As the stock price becomes very large, the call price approaches the stock price less the present value of the exercise price.

Other things equal, you would like to hold an option on a volatile stock (high σ^2). Given volatility, you would like to hold an option with a long life ahead of it (large t).

Thus the value of an option increases with both the volatility of the share price and the time to maturity.

It's a rare person who can keep all these properties straight at first reading. Therefore, we have summed them up in Table 20.2.

Risk and Option Values

In most financial settings, risk is a bad thing; you have to be paid to bear it. Investors in risky (high-beta) stocks demand higher expected rates of return. High-risk capital investment projects have correspondingly high costs of capital and have to beat higher hurdle rates to achieve positive NPV.

For options it's the other way around. As we have just seen, options written on volatile assets are worth *more* than options written on safe assets.[17] If you can understand and remember that one fact about options, you've come a long way.

EXAMPLE 20.1 • Volatility and Executive Stock Options

Suppose you have to choose between two job offers, as CFO of either Establishment Industries or Digital Organics. Establishment Industries' compensation package includes a grant of the stock options described on the left side of Table 20.3. You demand a similar package from Digital Organics, and they comply. In fact, they match the Establishment Industries options in every respect, as you can see on the right side of Table 20.3. (The two companies' current stock prices just happen to be the same.) The only difference is that Digital Organics' stock is 50% more volatile than Establishment Industries' stock (36% annual standard deviation versus 24% for Establishment Industries).

[17]This is not as crazy as it may at first sound. *Given the price of the stock,* the option is more valuable when the stock is volatile. However, that same volatility may have reduced the amount that investors are prepared to pay for the stock.

	Establishment Industries	Digital Organics
Number of options	100,000	100,000
Exercise price	$25	$25
Maturity	5 years	5 years
Current stock price	$22	$22
Stock price volatility (standard deviation of return)	24%	36%

TABLE 20.3 Which package of executive stock options would you choose? The package offered by Digital Organics is more valuable, because the volatility of that company's stock is higher.

If your job choice hinges on the value of the executive stock options, you should take the Digital Organics offer. The Digital Organics options are written on the more volatile asset and, therefore, are worth more.

We value the two stock-option packages in the next chapter.

SUMMARY

If you have managed to reach this point, you are probably in need of some fresh air and a run round the block. So we will summarize what we have learned so far and take up the subject of options again in the next chapter when you are refreshed.

There are two types of option. An American call is an option to buy an asset at a specified exercise price on or before a specified maturity date. Similarly, an American put is an option to sell the asset at a specified price on or before a specified date. European calls and puts are exactly the same except that they cannot be exercised before the specified maturity date. Calls and puts are the basic building blocks that can be combined to give any pattern of payoffs.

What determines the value of a call option? Common sense tells us that it ought to depend on three things:

1. To exercise an option you have to pay the exercise price. Other things being equal, the less you are obliged to pay, the better. Therefore, the value of a call option increases with the ratio of the asset price to the exercise price.
2. You do not have to pay the exercise price until you decide to exercise the option. Therefore, a call option gives you a free loan. The higher the rate of interest and the longer the time to maturity, the more this free loan is worth. So the value of a call option increases with the interest rate and time to maturity.
3. If the price of the asset falls short of the exercise price, you won't exercise the call option. You will, therefore, lose 100% of your investment in the option no matter how far the asset depreciates below the exercise price. On the other hand, the more the price rises *above* the exercise price, the more profit you will make. Therefore, the option holder does not lose from increased volatility if things go wrong, but gains if they go right. The value of an option increases with the variance per period of the stock return multiplied by the number of periods to maturity.

Always remember that an option written on a risky (high-variance) asset is worth more than an option on a safe asset. It's easy to forget, because in most other financial contexts, increases in risk reduce present value.

FURTHER READING

See Further Readings for Chapter 21.

PROBLEM SETS

Select problems are available in McGraw-Hill's *Connect*. Please see the preface for more information.

1. **Vocabulary*** Complete the following passage:

   A _____ option gives its owner the opportunity to buy a stock at a specified price that is generally called the _____ price. A _____ option gives its owner the opportunity to sell stock at a specified price. Options that can be exercised only at maturity are called _____ options.

2. **Option payoffs*** Note Figure 20.12 below. Match each diagram, (*a*) and (*b*), with one of the following positions:
   - Call buyer
   - Call seller
   - Put buyer
   - Put seller

3. **Option payoffs** Look again at Figure 20.12. It appears that the investor in panel (*b*) can't lose and the investor in panel (*a*) can't win. Is that correct? Explain. (*Hint:* Draw a profit diagram for each panel.)

4. **Option payoffs** What is a call option worth at maturity if (a) the stock price is zero? (b) the stock price is extremely high relative to the exercise price?

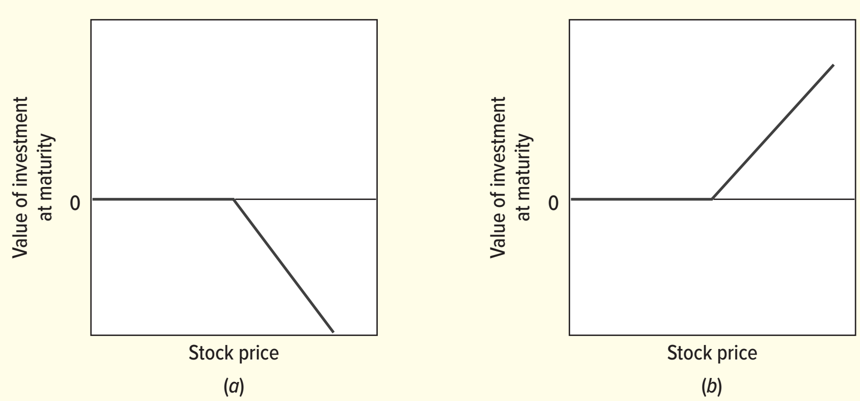
5. **Option payoffs** "The buyer of the call and the seller of the put both hope that the stock price will rise. Therefore the two positions are identical." Is the speaker correct? Illustrate with a position diagram.

6. **Option combinations*** Suppose that you hold a share of stock and a put option on that share. What is the payoff when the option expires if (a) the stock price is below the exercise price? (b) the stock price is above the exercise price?

7. **Option combinations** Dr. Livingstone I. Presume holds £600,000 in East African gold stocks. Bullish as he is on gold mining, he requires absolute assurance that at least £500,000 will be available in six months to fund an expedition. Describe two ways for Dr. Presume to achieve this goal. There is an active market for puts and calls on East African gold stocks, and the rate of interest is 6% per year.

FIGURE 20.12

See Problem 2



8. **Option combinations*** Suppose you buy a one-year European call option on Wombat stock with an exercise price of $100 and sell a one-year European put option with the same exercise price. The current stock price is $100, and the interest rate is 10%.
   a. Draw a position diagram showing the payoffs from your investments.
   b. How much will the combined position cost you? Explain.
9. **Option combinations** Suppose that Mr. Colleoni borrows the present value of $100, buys a six-month put option on stock Y with an exercise price of $150, and sells a six-month put option on Y with an exercise price of $50.
   a. Draw a position diagram showing the payoffs when the options expire.
   b. Suggest two other combinations of loans, options, and the underlying stock that would give Mr. Colleoni the same payoffs.
10. **Option combinations** Option traders often refer to "straddles" and "butterflies." Here is an example of each:
    - *Straddle:* Buy one call with exercise price of $100 and simultaneously buy one put with exercise price of $100.
    - *Butterfly:* Simultaneously buy one call with exercise price of $100, sell two calls with exercise price of $110, and buy one call with exercise price of $120.

    Draw position diagrams for the straddle and butterfly, showing the payoffs from the investor's net position. Each strategy is a bet on variability. Explain briefly the nature of each bet.
11. **Option combinations** Ms. Higden has been offered yet another incentive scheme (see Section 20-2). She will receive a bonus of $500,000 if the stock price at the end of the year is $120 or more; otherwise she will receive nothing. (Don't ask why anyone should want to offer such an arrangement. Maybe there's some tax angle.)
    a. Draw a position diagram illustrating the payoffs from such a scheme.
    b. What combination of options would provide these payoffs? (*Hint:* You need to buy a large number of options with one exercise price and sell a similar number with a different exercise price.)
12. **Option combinations** Discuss briefly the risks and payoffs of the following positions:
    a. Buy stock and a put option on the stock.
    b. Buy stock.
    c. Buy call.
    d. Buy stock and sell call option on the stock.
    e. Buy bond.
    f. Buy stock, buy put, and sell call.
    g. Sell put.
13. **Put–call parity** Which *one* of the following statements is correct?
    a. Value of put + present value of exercise price = value of call + share price
    b. Value of put + share price = value of call + present value of exercise price
    c. Value of put – share price = present value of exercise price – value of call
    d. Value of put + value of call = share price – present value of exercise price

    The correct statement equates the value of two investment strategies. Plot the payoffs to each strategy as a function of the stock price. Show that the two strategies give identical payoffs.
14. **Put-call parity** A European call and put option have the same maturity. Both are at-the-money, so that the stock price equals the exercise price. The stock does not pay a dividend. Which option should sell for the higher price? Explain.

15. Put–call parity

a. If you can't sell a share short, you can achieve exactly the same final payoff by a combination of options and borrowing or lending. What is this combination?

b. Now work out the mixture of stock and options that gives the same final payoff as investment in a risk-free loan.

16. Put–call parity The common stock of Triangular File Company is selling at $90. A 26-week call option written on Triangular File's stock is selling for $8. The call's exercise price is $100. The risk-free interest rate is 10% per year.

a. Suppose that puts on Triangular stock are not traded, but you want to buy one. How would you do it?

b. Suppose that puts *are* traded. What should a 26-week put with an exercise price of $100 sell for?

17. Put–call parity What is put–call parity and why does it hold? Could you apply the parity formula to a call and put with different exercise prices?

18. Put–call parity There is another strategy involving calls and borrowing or lending that gives the same payoffs as the strategy described in Problem 6. What is the alternative strategy?

19. Put–call parity It is possible to buy three-month call options and three-month puts on stock Q. Both options have an exercise price of $60 and both are worth $10. If the interest rate is 5% a year, what is the stock price? (*Hint:* Use put–call parity.)

20. Put–call parity* In April 2017, Facebook's stock price was about $145. An eight-month call on the stock, with an exercise price of $145, sold for $10.18. The risk-free interest rate was 1% a year. How much would you be willing to pay for a put on Facebook stock with the same maturity and exercise price? Assume that the Facebook options are European options. (*Note:* Facebook does not pay a dividend.)

21. Option bounds Pintail's stock price is currently $200. A one-year *American* call option has an exercise price of $50 and is priced at $75. How would you take advantage of this great opportunity?

22. Option values* How does the price of a call option respond to the following changes, other things equal? Does the call price go up or down?

a. Stock price increases.

b. Exercise price is increased.

c. Risk-free rate increases.

d. Expiration date of the option is extended.

e. Volatility of the stock price falls.

f. Time passes, so the option's expiration date comes closer.

23. Option values Respond to the following statements.

a. "I'm a conservative investor. I'd much rather hold a call option on a safe stock like Exxon Mobil than a volatile stock like Amazon."

b. "I bought an American call option on Fava Farms stock, with an exercise price of $45 per share and three more months to maturity. Fava Farms' stock has skyrocketed from $35 to $55 per share, but I'm afraid it will fall back below $45. I'm going to lock in my gain and exercise my call right now."

24. Option values FX Bank has succeeded in hiring ace foreign exchange trader Lucinda Cable. Her remuneration package reportedly includes an annual bonus of 20% of the profits that she generates in excess of $100 million. Does Ms. Cable have an option? Does it provide her with the appropriate incentives?

25. **Option values** Look at actual trading prices of call options on stocks to check whether they behave as the theory presented in this chapter predicts. For example,
    a. Follow several options as they approach maturity. How would you expect their prices to behave? Do they actually behave that way?
    b. Compare two call options written on the same stock with the same maturity but different exercise prices.
    c. Compare two call options written on the same stock with the same exercise price but different maturities.
26. **Option values** Is it more valuable to own an option to buy a portfolio of stocks or to own a portfolio of options to buy each of the individual stocks? Say briefly why.
27. **Option values** You've just completed a month-long study of energy markets and conclude that energy prices will be *much* more volatile in the next year than historically. Assuming you're right, what types of option strategies should you undertake? (*Note:* You can buy or sell options on oil-company stocks or on the price of future deliveries of crude oil, natural gas, fuel oil, etc.)
28. **Option values*** Table 20.4 lists some prices of options on common stocks (prices are quoted to the nearest dollar). The interest rate is 10% a year. Can you spot any mispricing? What would you do to take advantage of it?

CHALLENGE

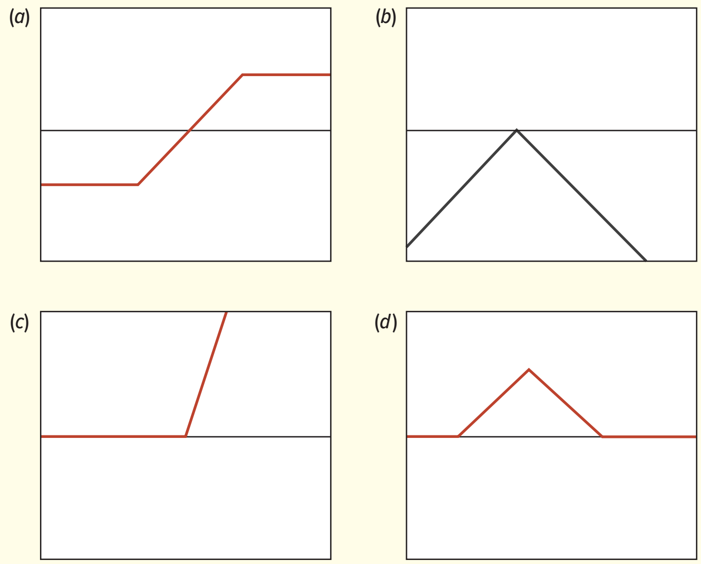
29. **Option bounds** Problem 21 considered an arbitrage opportunity involving an American option. Suppose that this option was a European call. Show that there is a similar possible arbitrage profit.
30. **Option payoffs** Figure 20.13 shows some complicated position diagrams. Work out the combination of stocks, bonds, and options that produces each of these positions.
31. **Option payoffs** Some years ago the Australian firm Bond Corporation sold a share in some land that it owned near Rome for A$110 million and as a result boosted its annual earnings by A$74 million. A television program subsequently revealed that the buyer was given a put option to sell its share in the land back to Bond for A$110 million and that Bond had paid A$20 million for a call option to repurchase the share in the land for the same price.
    a. What happens if the land is worth more than A$110 million when the options expire? What if it is worth less than A$110 million?
    b. Use position diagrams to show the net effect of the land sale and the option transactions.
    c. Assume a one-year maturity on the options. Can you deduce the interest rate?
    d. The television program argued that it was misleading to record a profit on the sale of land. What do you think?

Stock	Time to Exercise (months)	Exercise Price	Stock Price	Put Price	Call Price
Drongo Corp.	6	$ 50	$80	$20	$52
Ragwort, Inc.	6	100	80	10	15
Wombat Corp.	3	40	50	7	18
Wombat Corp.	6	40	50	5	17
Wombat Corp.	6	50	50	8	10

TABLE 20.4 Prices of options on common stocks (in dollars). See Problem 28.

FIGURE 20.13

Some complicated position diagrams. See Problem 30.



32. Option values Three six-month call options are traded on Hogswill stock:

Exercise Price	Call Option Price
$ 90	$ 5
100	11
110	15

How would you make money by trading in Hogswill options? (*Hint:* Draw a graph with the option price on the vertical axis and the ratio of stock price to exercise price on the horizontal axis. Plot the three Hogswill options on your graph. Does this fit with what you know about how option prices should vary with the ratio of stock price to exercise price?) Now look in the newspaper at options with the same maturity but different exercise prices. Can you find any money-making opportunities?

FINANCE ON THE WEB

Go to **finance.yahoo.com.** Check out the delayed quotes for Amazon options for different exercise prices and maturities. Take the mean of the bid and ask prices.

a. Confirm that higher exercise prices mean lower call prices and higher put prices.

b. Confirm that longer maturity means higher prices for both puts and calls.

c. Choose an Amazon put and call with the same exercise price and maturity. Confirm that put–call parity holds (approximately). (*Note:* You will have to use an up-to-date risk-free interest rate.)

CHAPTER 21

Valuing Options

In Chapter 20, we introduced you to call and put options. Call options give the owner the right to buy an asset at a specified exercise price; put options give the right to sell. We also took the first step toward understanding how options are valued. The value of a call option depends on five variables:

1. The higher the price of the asset, the more valuable an option to buy it.
2. The lower the price that you must pay to exercise the call, the more valuable the option.
3. You do not need to pay the exercise price until the option expires. This delay is most valuable when the interest rate is high.
4. If the stock price is below the exercise price at maturity, the call is valueless regardless of whether the price is \$1 below or \$100 below. However, for every dollar that the stock price rises above the exercise price, the option holder gains an additional dollar. Thus, the value of the call option increases with the volatility of the stock price.
5. Finally, a long-term option is more valuable than a short-term option. A distant maturity delays the point at which the holder needs to pay the exercise price and increases the chance of a large jump in the stock price before the option matures.

In this chapter, we show how these variables can be combined into an exact option-valuation model—a formula we can plug numbers into to get a definite answer. We first describe a simple way to value options, known as the binomial model. We then introduce the Black–Scholes formula for valuing options. Finally, we provide a checklist showing how these two methods can be used to solve a number of practical option problems.

The most efficient way to value most options is to use a computer. But in this chapter, we will work through some simple examples by hand. We do so because unless you understand the basic principles behind option valuation, you are likely to make mistakes in setting up an option problem, and you won't know how to interpret the computer's answer and explain it to others.

In Chapter 20, we looked at the put and call options on Amazon stock. In this chapter, we stick with that example and show you how to value the Amazon options. But remember *why* you need to understand option valuation. It is not to make a quick buck trading on an options exchange. It is because many capital budgeting and financing decisions have options embedded in them. We discuss a variety of these options in subsequent chapters.

21-1 A Simple Option-Valuation Model

Why Discounted Cash Flow Won't Work for Options

For many years, economists searched for a practical formula to value options until Fischer Black and Myron Scholes finally hit upon the solution. Later we will show you what they found, but first we should explain why the search was so difficult.

Our standard procedure for valuing an asset is to (1) figure out expected cash flows and (2) discount them at the opportunity cost of capital. Unfortunately, this is not practical for options. The first step is messy but feasible, but finding *the* opportunity cost of capital is impossible because the risk of an option changes every time the stock price moves.

When you buy a call, you are taking a position in the stock but putting up less of your own money than if you had bought the stock directly. Thus, an option is always riskier than the underlying stock. It has a higher beta and a higher standard deviation of returns.

How much riskier the option is depends on the stock price relative to the exercise price. A call option that is in the money (stock price greater than exercise price) is safer than one that is out of the money (stock price less than exercise price). Thus a stock price increase raises the option's expected payoff *and* reduces its risk. When the stock price falls, the option's payoff falls *and* its risk increases. That is why the expected rate of return investors demand from an option changes day by day, or hour by hour, every time the stock price moves.

We repeat the general rule: The higher the stock price is relative to the exercise price, the safer is the call option, although the option is always riskier than the stock. The option's risk changes every time the stock price changes.

Constructing Option Equivalents from Common Stocks and Borrowing

If you've digested what we've said so far, you can appreciate why options are hard to value by standard discounted-cash-flow formulas and why a rigorous option-valuation technique eluded economists for many years. The breakthrough came when Black and Scholes exclaimed, "Eureka! We have found it![1] The trick is to set up an *option equivalent* by combining common stock investment and borrowing. The net cost of buying the option equivalent must equal the value of the option."

We'll show you how this works with a simple numerical example. We'll travel back to April 2017 and consider a six-month call option on Amazon stock with an exercise price of $900. We'll pick a day when Amazon stock was also trading at $900, so that this option is *at the money*. The short-term, risk-free interest rate was $r = 0.5\%$ for 6 months, or about 1% a year.

To keep the example as simple as possible, we assume that Amazon stock can do only two things over the option's six-month life. The price could rise by 20% to $900 \times 1.2 = \$1{,}080$. Alternatively, it could fall by the same proportion to $900 \div 1.2 = \$750$, which is equivalent to a decline of $(900 - 750)/900 = 16.667\%$. The upward move is sometimes written as $u = 1.2$, and the downward move as $d = 1/u = 1/1.2$.

Warning: We will round some of our calculations slightly. So, if you are following along with your calculator, don't worry if your answers differ in the last decimal place.

If Amazon's stock price falls to $750, the call option will be worthless, but if the price rises to $1,080, the option will be worth $1,080 – 900 = $180. The possible payoffs to the option are therefore as follows:

	Stock Price = $750	Stock Price = $1,080
1 call option	$0	$180

[1] We do not know whether Black and Scholes, like Archimedes, were sitting in bathtubs at the time.

Now compare these payoffs with what you would get if you bought .54545 Amazon share and borrowed the present value of \$409.09 from the bank:[2]

	Stock Price = \$750	Stock Price = \$1,080
0.54545 share	\$409.09	\$589.09
Repayment of loan + interest	−409.09	−409.09
Total payoff	\$ 0	\$180.00

Notice that the payoffs from the levered investment in the stock are identical to the payoffs from the call option. Therefore, the law of one price tells us that both investments must have the same value:

$$\begin{aligned}\text{Value of call} &= \text{value of .54545 shares} - \text{value of bank loan}\\ &= .54545 \times \$900 - 409.09/1.005 = \$83.85\end{aligned}$$

Presto! You've valued a call option.

To value the Amazon option, we borrowed money and bought stock in such a way that we exactly replicated the payoff from a call option. This is called a **replicating portfolio.** The number of shares needed to replicate one call is called the **hedge ratio or option delta.** In our Amazon example, one call is replicated by a levered position in .54545 share. The option delta is, therefore, .54545.

How did we know that Amazon's call option was equivalent to a levered position in .54545 share? We used a simple formula that says:

$$\text{Option delta} = \frac{\text{spread of possible option prices}}{\text{spread of possible share prices}} = \frac{180 - 0}{1{,}080 - 750} = .54545$$

You have learned not only to value a simple option but also learned that you can replicate an investment in the option by a levered investment in the underlying asset. Thus, if you can't buy or sell a call option on an asset, you can create a homemade option by a replicating strategy—that is, you buy or sell delta shares and borrow or lend the balance.

Risk-Neutral Valuation Notice why the Amazon call option should sell for \$83.85. If the option price is higher than \$83.85, you could make a certain profit by buying .54545 share of stock, selling a call option, and borrowing the present value of \$409.09. Similarly, if the option price is less than \$83.85, you could make an equally certain profit by selling .54545 share, buying a call, and lending the balance. In either case, there would be an arbitrage opportunity.[3]

If there's a possible arbitrage profit, everyone scurries to take advantage of it. So when we said that the option price had to be \$83.85 or there would be an arbitrage opportunity, we did not need to know anything about investor attitudes to risk. High-rolling speculators and total wimps would all jostle each other in the rush to realize a possible arbitrage profit. Thus, the option price cannot depend on whether investors detest risk or do not care a jot.

[2]The exact number of shares to buy is 180/330 = .54545. . . as explained below.

[3]Of course, you don't get seriously rich by dealing in .54545 share. But if you multiply each of our transactions by a million, it begins to look like real money.

This suggests an alternative way to value the option. We can *pretend* that all investors are *indifferent* about risk, work out the expected future value of the option in such a world, and discount it back at the risk-free interest rate to give the current value. Let us check that this method gives the same answer.

If investors are indifferent to risk, the expected return on the stock must be equal to the risk-free rate of interest:

$$\text{Expected return on Amazon stock} = 0.5\% \text{ per six months}$$

We know that Amazon stock can either rise by 20% to $1,080 or fall by 16.667% to $750. We can, therefore, calculate the probability of a price rise in our hypothetical risk-neutral world:

$$\begin{aligned}\text{Expected return} &= [\text{probability of rise} \times 20\%] \\ &\quad + [(1 - \text{probability of rise}) \times (-6.667\%)] \\ &= 0.5\%\end{aligned}$$

Therefore,

$$\text{Probability of rise} = .46815 \text{ or } 46.815\%$$

Notice that this is *not* the *true* probability that Amazon stock will rise. Since investors dislike risk, they will almost surely require a higher expected return than the risk-free interest rate from Amazon stock. Therefore, the true probability is greater than .46815.

The general formula for calculating the risk-neutral probability of a rise in value is

$$p = \frac{\text{interest rate} - \text{downside change}}{\text{upside change} - \text{downside change}} = \frac{r - d}{u - d}$$

In the case of Amazon:

$$p = \frac{.005 - (-.16667)}{.20 - (-.16667)} = .46815$$

We know that if the stock price rises, the call option will be worth $180; if it falls, the call will be worth nothing. Therefore, if investors are risk-neutral, the expected value of the call option is

$$\begin{aligned}&[\text{Probability of rise} \times 180] + [(1 - \text{probability of rise}) \times 0] \\ &= (.46815 \times 180) + (.53185 \times 0) \\ &= \$84.27\end{aligned}$$

And the *current* value of the call is

$$\frac{\text{Expected future value}}{1 + \text{interest rate}} = \frac{84.27}{1.005} = \$83.85$$

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Exactly the same answer that we got earlier!

We now have two ways to calculate the value of an option:

1. Find the combination of stock and loan that replicates an investment in the option. Since the two strategies give identical payoffs in the future, they must sell for the same price today.
2. Pretend that investors do not care about risk so that the expected return on the stock is equal to the interest rate. Calculate the expected future value of the option in this

hypothetical *risk-neutral* world, and discount it at the risk-free interest rate. This idea may seem familiar to you. In Chapter 9, we showed how you can value an investment either by discounting the expected cash flows at a risk-adjusted discount rate or by adjusting the expected cash flows for risk and then discounting these *certainty-equivalent* flows at the risk-free interest rate. We have just used this second method to value the Amazon option. The certainty-equivalent cash flows on the stock and option are the cash flows that would be expected in a risk-neutral world.

Valuing the Amazon Put Option

Valuing the Amazon call option may well have seemed like pulling a rabbit out of a hat. To give you a second chance to watch how it is done, we will use the same method to value another option—this time, the six-month Amazon put option with a $900 exercise price.[4] We continue to assume that the stock price will either rise to $1,080 or fall to $750.

If Amazon's stock price rises to $1,080, the option to sell for $900 will be worthless. If the price falls to $750, the put option will be worth $900 – 750 = $150. Thus, the payoffs to the put are

	Stock Price = $750	Stock Price = $1,080
1 put option	$150	$0

We start by calculating the option delta using the formula that we presented previously:[5]

$$\text{Option delta} = \frac{\text{spread of possible option prices}}{\text{spread of possible stock prices}} = \frac{0 - 150}{1{,}080 - 750}$$
$$= -.45455$$

Notice that the delta of a put option is always negative; that is, you need to *sell* delta shares of stock to replicate the put. In the case of the Amazon put you can replicate the option payoffs by *selling* .45455 Amazon share and *lending* the present value of $490.91. Since you have sold the share short, you will need to lay out money at the end of six months to buy it back, but you will have money coming in from the loan. Your net payoffs are exactly the same as the payoffs you would get if you bought the put option:

	Stock Price = $750	Stock Price = $1,080
Sale of 0.45455 share	–$340.91	–$490.91
Repayment of loan + interest	+ 490.91	+ 490.91
Total payoff	$150.00	$ 0

Since the two investments have the same payoffs, they must have the same value:

$$\text{Value of put} = -(.45455) \text{ shares} + \text{value of bank loan}$$
$$= -(.45455) \times 900 + 490.91/1.005 = \$79.37$$

[4]When valuing *American* put options, you need to recognize the possibility that it will pay to exercise early. We discuss this complication later in the chapter, but it is unimportant for valuing the Amazon put and we ignore it here.

[5]The delta of a put option is always equal to the delta of a call option with the same exercise price minus one. In our example, delta of put = .54545 – 1 = –.45455.

Valuing the Put Option by the Risk-Neutral Method Valuing the Amazon put option with the risk-neutral method is a cinch. We already know that the probability of a rise in the stock price is .46815. Therefore, the expected value of the put option in a risk-neutral world is

$$\begin{aligned}&[\text{Probability of rise} \times 0] + [(1 - \text{probability of rise}) \times 150]\\ &\quad = (.46815 \times 0) + (.53185 \times \$150)\\ &\quad = \$79.78\end{aligned}$$

And therefore the *current* value of the put is

$$\frac{\text{Expected future value}}{1 + \text{interest rate}} = \frac{79.78}{1.005} = \$79.38$$

Apart from a minor rounding error, the two methods give the same answer.

The Relationship between Call and Put Prices We pointed out earlier that for European options there is a simple relationship between the values of the call and the put.[6]

$$\text{Value of put} = \text{value of call} + \text{present value of exercise price} - \text{share price}$$

Since we had already calculated the value of the Amazon call, we could also have used this relationship to find the value of the put:

$$\text{Value of put} = 83.85 + \frac{900}{1.005} - 900 = \$79.37$$

Everything checks.

21-2 The Binomial Method for Valuing Options

The essential trick in pricing any option is to set up a package of investments in the stock and the loan that will exactly replicate the payoffs from the option. If we can price the stock and the loan, then we can also price the option. Equivalently, we can pretend that investors are risk-neutral, calculate the expected payoff on the option in this fictitious risk-neutral world, and discount by the rate of interest to find the option's present value.

These concepts are completely general, but the example in the last section used a simplified version of what is known as the **binomial method.** The method starts by reducing the possible changes in the next period's stock price to two, an "up" move and a "down" move. This assumption that there are just two possible prices for Amazon stock at the end of six months is clearly fanciful.

We could make the Amazon problem a trifle more realistic by assuming that there are two possible price changes in each three-month period. This would give a wider variety of six-month prices. And there is no reason to stop at three-month periods. We could go on to take shorter and shorter intervals, with each interval showing two possible changes in Amazon's stock price and giving an even wider selection of six-month prices.

We illustrate this in Figure 21.1. The top diagram shows our starting assumption: just two possible prices at the end of six months. Moving down, you can see what happens when there are two possible price changes every three months. This gives three possible stock prices when the option matures. In Figure 21.1*c*, we have gone on to divide the six-month period into 26 weekly periods, in each of which the price can make one of two small moves. The distribution of prices at the end of six months is now looking much more realistic.

[6]*Reminder:* This formula applies only when the two options have the same exercise price and exercise date.

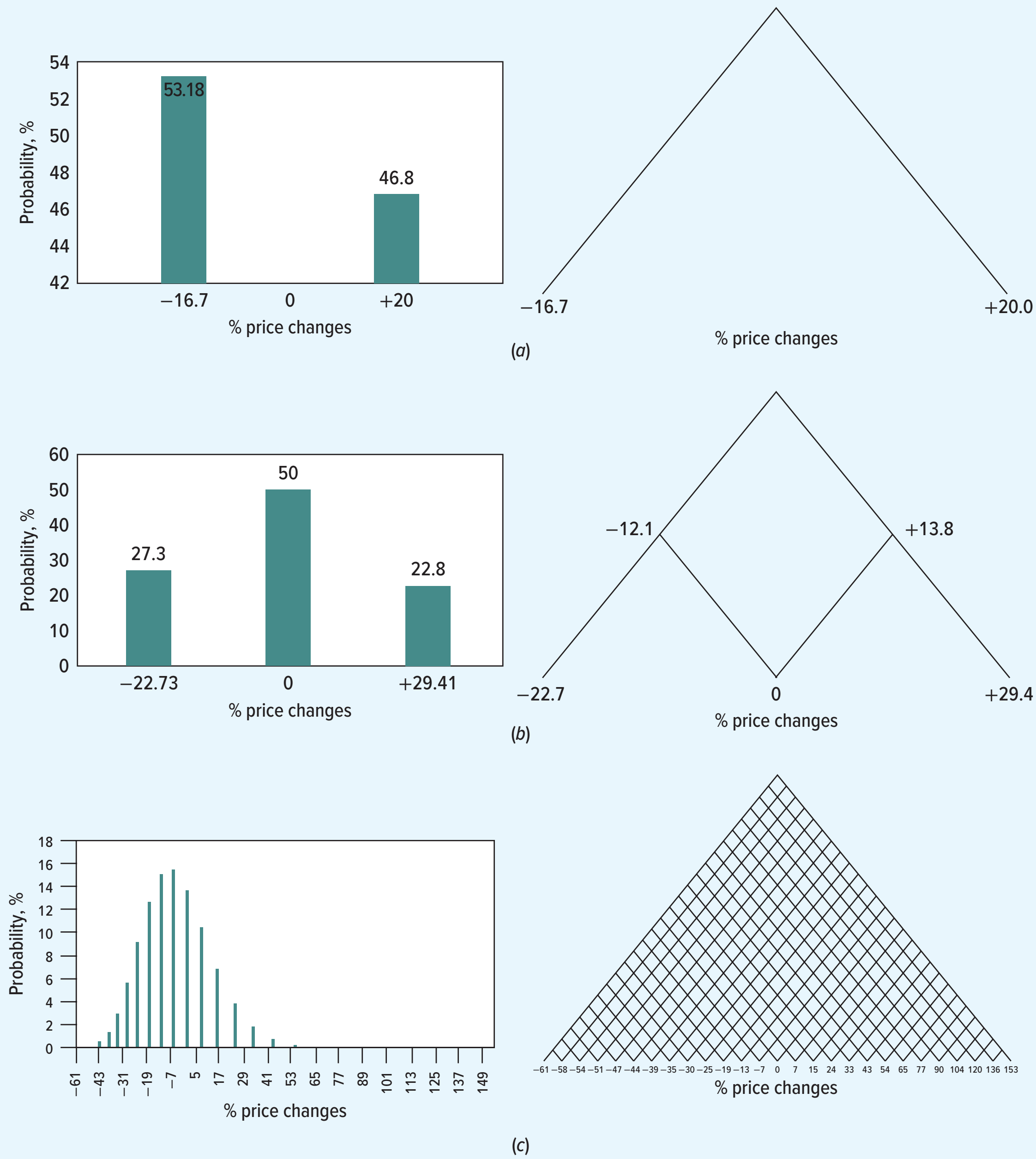

FIGURE 21.1 This figure shows the possible six-month price changes for Amazon stock assuming that the stock makes a single up or down move each six months (Fig. 21.1a); 2 moves, one every three months (Fig. 21.1b); or 26 moves, one every week (Fig. 21.1c). Beside each tree we show a histogram of the possible six-month price changes, assuming investors are risk-neutral.

We could continue in this way to chop the period into shorter and shorter intervals, until eventually we would reach a situation in which the stock price is changing continuously and there is a continuum of possible future stock prices. We demonstrate first with our simple two-step case in Figure 21.1*b*. Then we work up to the situation where the stock price is changing continuously. Don't panic; that won't be as bad as it sounds.

Example: The Two-Step Binomial Method

Dividing the period into shorter intervals doesn't alter the basic approach for valuing a call option. We can still find at each point a levered investment in the stock that gives exactly the same payoffs as the option. The value of the option must therefore be equal to the value of this replicating portfolio. Alternatively, we can pretend that investors are risk-neutral and expect to earn the interest rate on all their investments. We then calculate at each point the expected future value of the option and discount it at the risk-free interest rate. Both methods give the same answer.

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If we use the replicating-portfolio method, we must recalculate the investment in the stock at each point, using the formula for the option delta:

$$\text{Option delta} = \frac{\text{spread of possible option prices}}{\text{spread of possible stock prices}}$$

Recalculating the option delta is not difficult, but it can become a bit of a chore. It is simpler in this case to use the risk-neutral method, and that is what we will do.

Figure 21.2 is taken from Figure 21.1 and shows the possible prices of Amazon stock, assuming that in each three-month period the price will either rise by 13.76% or fall by 12.10%.[7] We show in parentheses the possible values at maturity of a six-month call option with an exercise price of \$900. For example, if Amazon's stock price turns out to be \$695.45 in month 6, the call option will be worthless; at the other extreme, if the stock value is \$1,164.72, the call will be worth \$1,164.72 – \$900 = \$264.72. We haven't worked out yet what the option will be worth before maturity, so we will just put question marks there for now.

We continue to assume an interest rate of .5% for 6 months, which is equivalent to about .25% a quarter. We now ask: If investors demand a return of .25% a quarter, what is the probability (p) at each stage that the stock price will rise? The answer is given by our simple formula:

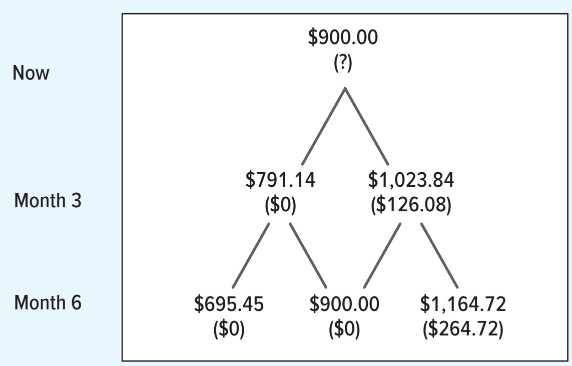
$$p = \frac{\text{interest rate} - \text{downside change}}{\text{upside change} - \text{downside change}} = \frac{.0025 - (-.1210)}{.1376 - (-.1210)} = .4774$$

We can check that if there is a 47.74% chance of a rise of 13.76% and a 52.26% chance of a fall of 12.10%, then the expected return must be equal to the .25% risk-free rate:

$$(.4774 \times 13.76) + (.5226 \times -12.10) = .25\%$$

FIGURE 21.2

Present and possible future prices of Amazon stock assuming that in each three-month period the price will either rise by 13.76% or fall by 12.10%. Figures in parentheses show the corresponding values of a six-month call option with an exercise price of \$900. The interest rate is .25% a quarter.



[7] We explain shortly why we picked these figures.

Option Value in Month 3 Now we can find the possible option values in month 3. Suppose that by the end of three months, the stock price has risen to $1,023.84. In that case, investors know that when the option finally matures in month 6, the option value will be either $0 or $264.72. We can therefore use our risk-neutral probabilities to calculate the expected option value at month 6:

$$\begin{aligned}\text{Expected value of call in month 6} &= (\text{probability of rise} \times 264.72) + (\text{probability of fall} \times 0)\\ &= (.4774 \times 264.72) + (.5226 \times 0) = \$126.39\end{aligned}$$

And the value in month 3 is 126.39/1.0025 = $126.08.

What if the stock price falls to $791.14 by month 3? In that case the option is bound to be worthless at maturity. Its expected value is zero, and its value at month 3 is also zero.

Option Value Today We can now get rid of two of the question marks in Figure 21.2. Figure 21.3 shows that if the stock price in month 3 is $1,023.84, the option value is $126.08, and if the stock price is $791.14, the option value is zero. It only remains to work back to the option value today.

There is a 47.74% chance that the option will be worth $126.08 and a 52.26% chance that it will be valueless. So the expected value in month 3 is

$$(.4774 \times 126.08) + (.5626 \times 0) = \$60.19$$

And the value today is 60.19/1.0025 = $60.05.

The General Binomial Method

Moving to two steps when valuing the Amazon call probably added extra realism. But there is no reason to stop there. We could go on, as in Figure 21.1, to chop the period into smaller and smaller intervals. We could still use the binomial method to work back from the final date to the present. Of course, it would be tedious to do the calculations by hand but simple to do so with a computer.

Since a stock can usually take on an almost limitless number of future values, the binomial method gives a more realistic and accurate measure of the option's value if we work with a large number of subperiods. But that raises an important question. How do we pick sensible figures for the up and down changes in value? For example, why did we pick figures of +13.76% and –12.1% when we revalued Amazon's option with two subperiods? Fortunately,

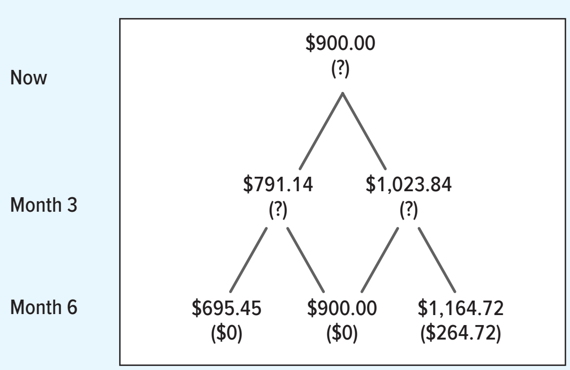


FIGURE 21.3

Present and possible future prices of Amazon stock. Figures in parentheses show the corresponding values of a six-month call option with an exercise price of $900.

there is a neat little formula that relates the up and down changes to the standard deviation of stock returns:

$$1 + \text{upside change} = u = e^{\sigma\sqrt{h}}$$
$$1 + \text{downside change} = d = 1/u$$

where

e = base for natural logarithms = 2.718
σ = standard deviation of (continuously compounded) stock returns
h = interval as fraction of a year

When we said that Amazon's stock price could either rise by 20% or fall by 16.667% over six months ($h = .5$), our figures were consistent with a figure of 25.784% for the standard deviation of annual returns:[8]

$$1 + \text{upside change(6-month interval)} = u = e^{.25784\sqrt{.5}} = 1.2$$
$$1 + \text{downside change} = d = 1/u = 1/1.2 = .833$$

To work out the equivalent upside and downside changes when we divide the period into two three-month intervals ($h = .25$), we use the same formula:

$$1 + \text{upside change(3-month interval)} = u = e^{.25784\sqrt{.25}} = 1.1376$$
$$1 + \text{downside change} = d = 1/u = 1/1.1376 = .879$$

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The center columns in Table 21.1 show the equivalent up and down moves in the value of the firm if we chop the period into six monthly or 26 weekly periods, and the final column shows the effect on the estimated option value. (We explain the Black–Scholes value shortly.)

The Binomial Method and Decision Trees

Calculating option values by the binomial method is basically a process of solving decision trees. You start at some future date and work back through the tree to the present. Eventually, the possible cash flows generated by future events and actions are folded back to a present value.

TABLE 21.1 As the number of steps is increased, you must adjust the range of possible changes in the value of the asset to keep the same standard deviation. But you will get increasingly close to the Black–Scholes value of the Amazon call option.

Note: The standard deviation is $\sigma = .25784$.

	Change per Interval (%)		
Number of Steps	Upside	Downside	Estimated Option Value
1	+20.00	−16.67	$83.85
2	+13.76	−12.10	60.05
6	+7.73	−7.17	64.82
26	+3.64	−3.51	66.84
		Black–Scholes value =	67.47

[8] To find the standard deviation given u, we turn the formula around:

$$\sigma = \log(u)/\sqrt{h}$$

where log = natural logarithm. In our example,

$$\sigma = \text{Log}(1.20)/\sqrt{(0.5)} = 0.1823/\sqrt{(0.5)} = 0.25784.$$

Is the binomial method *merely* another application of decision trees, a tool of analysis that you learned about in Chapter 10? The answer is no, for at least two reasons. First, option pricing theory is absolutely essential for discounting within decision trees. Discounting expected cash flows doesn't work within decision trees for the same reason that it doesn't work for puts and calls. As we pointed out in Section 21-1, there is no single, constant discount rate for options because the risk of the option changes as time and the price of the underlying asset change. There is no single discount rate inside a decision tree because, if the tree contains meaningful future decisions, it also contains options. The market value of the future cash flows described by the decision tree has to be calculated by option pricing methods.

Second, option theory gives a simple, powerful framework for describing complex decision trees. For example, suppose that you have the option to abandon an investment. The complete decision tree would overflow the largest classroom whiteboard. But now that you know about options, the opportunity to abandon can be summarized as "an American put." Of course, not all real problems have such easy option analogies, but we can often approximate complex decision trees by some simple package of assets and options. A custom decision tree may get closer to reality, but the time and expense may not be worth it. Most men buy their suits off the rack even though a custom-made Armani suit would fit better and look nicer.

21-3 The Black–Scholes Formula

Look back at Figure 21.1, which showed what happens to the distribution of possible Amazon stock price changes as we divide the option's life into a larger and larger number of increasingly small subperiods. You can see that the distribution of price changes becomes increasingly smooth.

If we continued to chop up the option's life in this way, we would eventually reach the situation shown in Figure 21.4, where there is a continuum of possible stock price changes at maturity. Figure 21.4 is an example of a lognormal distribution. The lognormal distribution is often used to summarize the probability of different stock price changes.[9] It has a number of good commonsense features. For example, it recognizes the fact that the stock price can

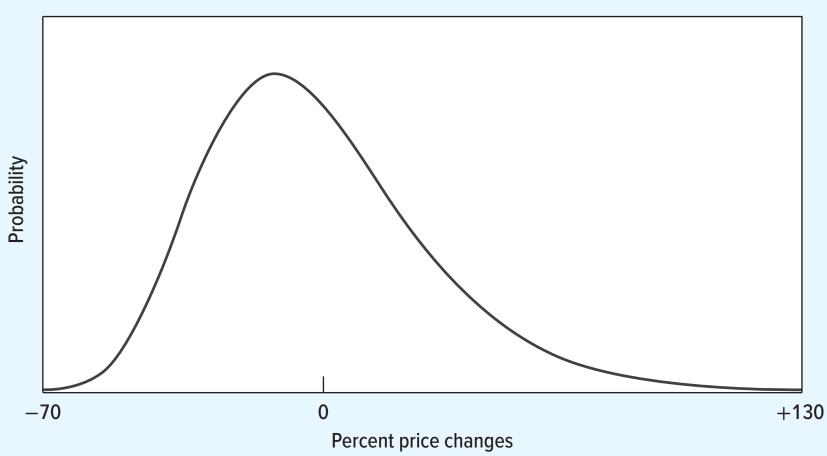

FIGURE 21.4

As the option's life is divided into more and more subperiods, the distribution of possible stock price changes approaches a lognormal distribution

[9]When we first looked at the distribution of stock price changes in Chapter 8, we depicted these changes as normally distributed. We pointed out at the time that this is an acceptable approximation for very short intervals, but the distribution of changes over longer intervals is better approximated by the lognormal.

never fall by more than 100% but that there is some, perhaps small, chance that it could rise by much more than 100%.

Subdividing the option life into indefinitely small slices does not affect the principle of option valuation. We could still replicate the call option by a levered investment in the stock, but we would need to adjust the degree of leverage continuously as time went by. Calculating option value when there is an infinite number of subperiods may sound a hopeless task. Fortunately, Black and Scholes derived a formula that does the trick.[10] It is an unpleasant-looking formula, but on closer acquaintance you will find it exceptionally elegant and useful. The formula is

$$\begin{array}{rcccccc} \text{Value of call option} = & [\text{delta} & \times & \text{share price}] & - & [\text{bank loan}] \\ & \uparrow & & \uparrow & & \uparrow \\ & [N(d_1) & \times & P] & - & [N(d_2) \times \text{PV(EX)}] \end{array}$$

where

$$d_1 = \frac{\log[P/\text{PV(EX)}]}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

$N(d)$ = cumulative normal probability density function[11]

EX = exercise price of option; PV(EX) is calculated by discounting at the risk-free interest rate r_f

t = number of periods to exercise date

P = price of stock now

σ = standard deviation per period of (continuously compounded) rate of return on stock

Notice that the value of the call in the Black–Scholes formula has the same properties that we identified earlier. It increases with the level of the stock price P and decreases with the present value of the exercise price PV(EX), which in turn depends on the interest rate and time to maturity. It also increases with the time to maturity and the stock's variability ($\sigma\sqrt{t}$).

To derive their formula, Black and Scholes assumed that there is a continuum of stock prices, and therefore to replicate an option, investors must continuously adjust their holding in the stock.[12] Of course, this is not literally possible, but even so, the formula performs remarkably well in the real world, where stocks trade only intermittently and prices jump from one level to another. The Black–Scholes model has also proved very flexible; it can be adapted to value options on a variety of assets such as foreign currencies, bonds, and commodities. It is not surprising, therefore, that it has been extremely influential and has become the standard model for valuing options. Every day, dealers on the options exchanges use this formula to make huge trades. These dealers are not for the most part trained in the formula's mathematical derivation; they just use a computer or a specially programmed calculator to find the value of the option.

Using the Black–Scholes Formula

The Black–Scholes formula may look difficult, but it is very straightforward to apply. Let us practice using it to value the Amazon call.

---

[10] The pioneering articles on options are F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81 (May–June 1973), pp. 637–654; and R. C. Merton, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science* 4 (Spring 1973), pp. 141–183.

[11] That is, $N(d)$ is the probability that a normally distributed random variable $\tilde{x}$ will be less than or equal to d. $N(d_1)$ in the Black–Scholes formula is the option delta. Thus the formula tells us that the value of a call is equal to an investment of $N(d_1)$ in the common stock less borrowing of $N(d_2) \times \text{PV(EX)}$.

[12] The important assumptions of the Black–Scholes formula are that (1) the price of the underlying asset follows a lognormal random walk, (2) investors can adjust their hedge continuously and costlessly, (3) the risk-free rate is known, and (4) the underlying asset does not pay dividends.

Here are the data that you need:

- Price of stock now = $P = 900$
- Exercise price = EX = 900
- Standard deviation of continuously compounded annual returns = $\sigma = .25784$
- Years to maturity = $t = .5$
- Interest rate per annum = $r_f = .5\%$ for 6 months or about 1% per annum[13]

Remember that the Black–Scholes formula for the value of a call is

$$[N(d_1) \times P] - [N(d_2) \times \text{PV(EX)}]$$

where

$$d_1 = \log[P/\text{PV(EX)}]/\sigma\sqrt{t} + \sigma\sqrt{t}/2$$
$$d_2 = d_1 - \sigma\sqrt{t}$$
$$N(d) = \text{cumulative normal probability function}$$

There are three steps to using the formula to value the Amazon call:

Step 1 Calculate d_1 and d_2. This is just a matter of plugging numbers into the formula (noting that "log" means *natural* log):

$$\begin{aligned} d_1 &= \log[P/\text{PV(EX)}]/\sigma\sqrt{t} + \sigma\sqrt{t}/2 \\ &= \log[900/(900/1.005)]/(.25784 \times \sqrt{.5}) + .25784 \times \sqrt{.5}/2 \\ &= .1184 \\ d_2 &= d_1 - \sigma\sqrt{t} = .1184 - .25784 \times \sqrt{.5} = -.0639 \end{aligned}$$

Step 2 Find $N(d_1)$ and $N(d_2)$. $N(d_1)$ is the probability that a normally distributed variable will be less than d_1 standard deviations above the mean. If d_1 is large, $N(d_1)$ is close to 1.0 (i.e., you can be almost certain that the variable will be less than d_1 standard deviations above the mean). If d_1 is zero, $N(d_1)$ is .5 (i.e., there is a 50% chance that a normally distributed variable will be below the average).

The simplest way to find $N(d_1)$ is to use the Excel function NORMSDIST. For example, if you enter NORMSDIST(.1184) into an Excel spreadsheet, you will see that there is a .5471 probability that a normally distributed variable will be less than .1184 standard deviations above the mean.

Again you can use the Excel function to find $N(d_2)$. If you enter NORMSDIST (–.0639) into an Excel spreadsheet, you should get the answer .4745. In other words, there is a probability of .4745 that a normally distributed variable will be less than .0639 standard deviations *below* the mean.

[13]When valuing options, it is more common to use continuously compounded rates to calculate PV(EX) (see Section 2-4). As long as the two rates are equivalent, both methods give the same answer, so why do we bother to mention the subject here? It is simply because most computer programs for valuing options call for a continuously compounded rate. If you enter an annually compounded rate by mistake, the error will usually be small, but you can waste a lot of time trying to trace it.

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Step 3 Plug these numbers into the Black–Scholes formula. You can now calculate the value of the Amazon call:

$$[\text{Delta} \times \text{price}] - [\text{bank loan}]$$
$$= [N(d_1) \times P] - [N(d_2) \times \text{PV(EX)}]$$
$$= [.5471 \times 900] - [.4745 \times (900/1.005)] = 492.43 - 424.96 = \$67.47$$

In other words, you can replicate the Amazon call option by investing $492.43 in the company's stock and borrowing $424.96. Subsequently, as time passes and the stock price changes, you may need to borrow a little more to invest in the stock or you may need to sell some of your stock to reduce your borrowing.

Some More Practice Suppose you repeat the calculations for the Amazon call for a wide range of stock prices. The result is shown in Figure 21.5. You can see that the option values lie along an upward-sloping curve that starts its travels in the bottom left-hand corner of the diagram. As the stock price increases, the option value rises and gradually becomes parallel to the lower bound for the option value. This is exactly the shape we deduced in Chapter 20 (see Figure 20.9).

The height of this curve of course depends on risk and time to maturity. For example, if the risk of Amazon stock had suddenly increased, the curve shown in Figure 21.5 would rise at every possible stock price. For example, Figure 20.11 shows what would happen to the curve if the risk of Amazon stock doubled.

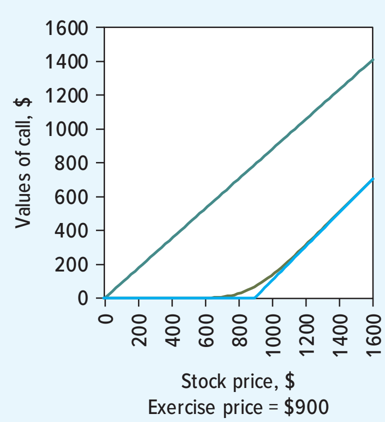
The Risk of an Option

How risky is the Amazon call option? We have seen that you can exactly replicate a call by a combination of risk-free borrowing and an investment in the stock. So the risk of the option must be the same as the risk of this replicating portfolio. We know that the beta of any portfolio is simply a weighted average of the betas of the separate holdings. So the risk of the option is just a weighted average of the betas of the investments in the loan and the stock.

On past evidence, the beta of Amazon stock is $\beta_{\text{stock}} = 1.5$; the beta of a risk-free loan is $\beta_{\text{loan}} = 0$. You are investing $492.43 in the stock and –$424.96 in the loan. (Notice that the investment in the loan is negative—you are *borrowing* money.) Therefore the beta of the option is $\beta_{\text{option}} = (-424.96 \times 0 + 492.43 \times 1.5)/(-424.96 + 492.43) = 10.95$. Notice that because a call option is equivalent to a levered position in the stock, it is always riskier than the stock itself. In Amazon's case, the option is about 7 times as risky as the stock and 11 times as risky as the market. As time passes and the price of Amazon stock changes, the risk of the option will also change.

FIGURE 21.5

The curved line shows how the value of the Amazon call option changes as the price of Amazon stock changes



The Black–Scholes Formula and the Binomial Method

Look back at Table 21.1, where we used the binomial method to calculate the value of the Amazon call. Notice that, as the number of intervals is increased, the values that you obtain from the binomial method begin to snuggle up to the Black–Scholes value of $67.47.

The Black–Scholes formula recognizes a continuum of possible outcomes. This is usually more realistic than the limited number of outcomes assumed in the binomial method. The formula is also more accurate and quicker to use than the binomial method. So why use the binomial method at all? The answer is that there are many circumstances in which you cannot use the Black–Scholes formula, but the binomial method will still give you a good measure of the option's value. We will look at several such cases in Section 21-5.

21-4 Black–Scholes in Action

To illustrate the principles of option valuation, we focused on the example of Amazon's options. But financial managers turn to the Black–Scholes model to estimate the value of a variety of different options. Here are four examples.

Executive Stock Options

In fiscal year 2017, Larry Ellison, the CEO of Oracle Corporation, received a salary of $1, but he also pocketed $21 million in the form of options and other stock-related awards.

Executive stock options are often an important part of compensation. For many years, companies were able to avoid reporting the cost of these options in their annual statements. However, they must now treat options as an expense just like salaries and wages, so they need to estimate the value of all new options that they have granted. For example, Oracle's financial statements show that in fiscal 2017, the company issued a total of 18 million options with an average life of 4.8 years. Oracle calculated that the average value of these options was $8.18. How did it come up with this figure? It just used the Black–Scholes model assuming a standard deviation of 23%.[14]

Some companies have disguised how much their management is paid by backdating the grant of an option. Suppose, for example, that a firm's stock price has risen from $20 to $40. At that point, the firm awards its CEO options exercisable at $20. That is generous but not illegal. However, if the firm pretends that the options were *actually* awarded when the stock price was $20 and values them on that basis, it will substantially understate the CEO's compensation.[15] The Beyond the Page app discusses the backdating scandal.

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The Perfect Payday

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Speaking of executive stock options, we can now use the Black–Scholes formula to value the option packages you were offered in Section 20-3 (see Table 20.3). Table 21.2 calculates the value of the options from the safe-and-stodgy Establishment Industries at $5.26 each. The options from risky-and-glamorous Digital Organics are worth $7.40 each. Congratulations.

Warrants

When Owens Corning emerged from bankruptcy in 2006, the debtholders became the sole owners of the company. But the old stockholders were not left entirely empty-handed. They

[14]Many of the recipients of these options may not have agreed with Oracle's valuation. First, the options were less valuable to their owners if they created substantial undiversifiable risk. Second, if the holders planned to quit the company in the next few years, they were liable to forfeit the options. For a discussion of these issues see J. I. Bulow and J. B. Shoven, "Accounting for Stock Options," *Journal of Economic Perspectives* 19 (Fall 2005), pp. 115–135.

[15]Until 2005, companies were obliged to record as an expense any difference between the stock price when the options were granted and the exercise price. Thus, as long as the options were granted at-the-money (exercise price equals stock price), the company was not obliged to show any expense.

TABLE 21.2 Using the Black–Scholes formula to value the executive stock options for Establishment Industries and Digital Organics (see Table 20.3)

	Establishment Industries	Digital Organics
Stock price (P)	\$22	\$22
Exercise price (EX)	\$25	\$25
Interest rate (r_f)	0.04	0.04
Maturity in years (t)	5	5
Standard deviation (σ)	0.24	0.36
$d_1 = \log[P/\text{PV(EX)}]/\sigma\sqrt{t} + \sigma\sqrt{t}/2$	0.3955	0.4873
$d_2 = d_1 - \sigma\sqrt{t}$	−0.1411	−0.3177
Call value $= [N(d_1) \times P] - [N(d_2) \times \text{PV(EX)}]$	\$5.26	\$7.40

were given warrants to buy the new common stock at any point in the next seven years for \$45.25 a share. Because the stock in the restructured firm was worth about \$30 a share, the stock needed to appreciate by 50% before the warrants would be worth exercising. However, this option to buy Owens Corning stock was clearly valuable, and shortly after the warrants started trading, they were selling for \$6 each. You can be sure that before shareholders were handed this bone, all the parties calculated the value of the warrants under different assumptions about the stock's volatility. The Black–Scholes model is tailor-made for this purpose.[16]

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The Chinese Warrants Bubble
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Portfolio Insurance

BEYOND THE PAGE
VIX
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Your company's pension fund owns an \$800 million diversified portfolio of common stocks that moves closely in line with the market index. The pension fund is currently fully funded, but you are concerned that if it falls by more than 20%, it will start to be underfunded. Suppose that your bank offers to insure you for one year against this possibility. What would you be prepared to pay for this insurance? Think back to Section 20-2 (Figure 20.5), where we showed that you can shield against a fall in asset prices by buying a protective put option. In the present case, the bank would be selling you a one-year put option on U.S. stock prices with an exercise price 20% below their current level. You can get the value of that option in two steps. First use the Black–Scholes formula to value a call with the same exercise price and maturity. Then back out the put value from put–call parity. (You may have to adjust for dividends, but we'll leave that to the next section.)

Calculating Implied Volatilities

So far, we have used our option pricing model to calculate the value of an option given the standard deviation of the asset's returns. Sometimes it is useful to turn the problem around and ask what the option price is telling us about the asset's volatility. For example, the

[16]Postscript: Unfortunately, Owens Corning's stock price never reached \$45 and the warrants expired worthless.

Chicago Board Options Exchange trades options on several market indexes. As we write this, the Standard and Poor's 500 Index is about 2375, while a seven-month at-the-money call on the index is priced at 89. If the Black–Scholes formula is correct, then an option value of 89 makes sense only if investors believe that the standard deviation of index returns is about 11.4% a year.[17]

The Chicago Board Options Exchange regularly publishes the implied volatility on the Standard and Poor's index, which it terms the VIX (see the nearby box on the "fear index"). There is an active market in the VIX. For example, suppose you feel that the implied volatility is implausibly low. Then you can "buy" the VIX at the current low price and hope to "sell" it at a profit when implied volatility has increased.

You may be interested to compare the current implied volatility that we calculated earlier with Figure 21.6, which shows past measures of implied volatility for the Standard and Poor's index and for the Nasdaq index (VXN). Notice the sharp increase in investor uncertainty at the height of the credit crunch in 2008. This uncertainty showed up in the price that investors were prepared to pay for options.

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The option smile

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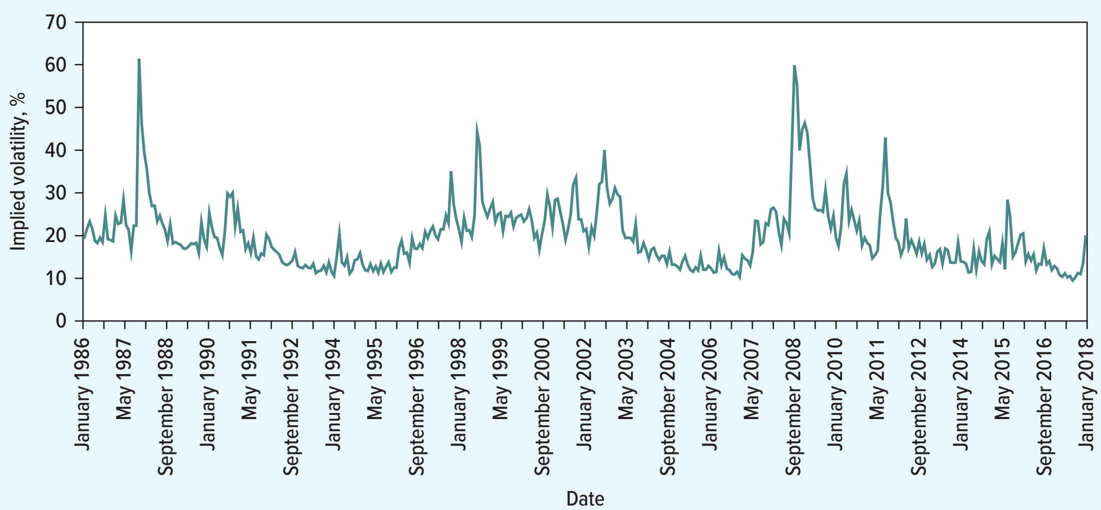

FIGURE 21.6 VIX measures standard deviation of market returns implied by option prices on the S&P 500 Index

Source: finance.yahoo.com.

[17]In calculating the implied volatility we need to allow for the dividends paid on the shares. We explain how to take these into account in the next section.

FINANCE IN PRACTICE

The Fear Index*

The Market Volatility Index, or VIX, measures the volatility that is implied by near-term Standard & Poor's 500 Index options and is therefore an estimate of expected *future* market volatility over the next 30 calendar days. Implied market volatilities have been calculated by the Chicago Board Options Exchange (CBOE) since January 1986, though in its current form, the VIX dates back only to 2003.

Investors regularly trade volatility. They do so by buying or selling VIX futures and options contracts. Since these were introduced by the Chicago Board Options Exchange (CBOE), combined trading activity in the two contracts has grown to more than 100,000 contracts per day, making them two of the most successful innovations ever introduced by the exchange.

Because VIX measures investor uncertainty, it has been dubbed the "fear index." The market for index options tends to be dominated by equity investors who buy index puts when they are concerned about a potential drop in the stock market. Any subsequent decline in the value of their portfolio is then offset by the increase in the value of the put option. The more that investors demand such insurance, the higher the price of index put options. Thus VIX is an indicator that reflects the cost of portfolio insurance.

Between January 1986 and February 2018, the VIX has averaged 20.0%, almost identical to the long-term level of market volatility that we cited in Chapter 7. The high point for the index was in October 1987 when the VIX closed the month at 61%,** but there have been several other short-lived spikes, for example, at the time of Iraq's invasion of Kuwait and the subsequent response by UN forces.

Although the VIX is the most widely quoted measure of volatility, volatility measures are also available for several other U.S. and overseas stock market indexes (such as the FTSE 100 Index in the U.K. and the CAC 40 in France), as well as for gold, oil, and the euro.

*For a review of the VIX index, see R. E. Whaley, "Understanding the VIX," *Journal of Portfolio Management 35* (Spring 2009), pp. 98–105.

**On October 19, 1987 (Black Monday), the VIX closed at 150. Fortunately, the market volatility returned fairly rapidly to less exciting levels.

21-5 Option Values at a Glance

BEYOND THE PAGE

Dilution

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So far, our discussion of option values has assumed that investors hold the option until maturity. That is certainly the case with European options that *cannot* be exercised before maturity but may not be the case with American options that can be exercised at any time. Also, when we valued the Amazon call, we could ignore dividends, because Amazon did not pay any. Can the same valuation methods be extended to American options and to stocks that pay dividends?

Another question concerns dilution. When investors buy and then exercise traded options, there is no effect on the number of shares issued by the company. But sometimes the company itself may give options to key employees or sell them to investors. When these options are exercised, the number of outstanding shares *does* increase, and therefore the stake of existing stockholders is diluted. Option valuation models need to be able to cope with the effect of dilution. The Beyond the Page feature shows how to do this.

In this section, we look at how the possibility of early exercise and dividends affect option value.

American Calls—No Dividends Unlike European options, American options can be exercised any time. However, we know that in the absence of dividends, the value of a call option increases with time to maturity. So, if you exercised an American call option early, you would needlessly reduce its value. Because an American call should not be exercised before maturity, its value is the same as that of a European call, and the Black–Scholes model applies to both options.

European Puts—No Dividends If we wish to value a European put, we can use the put–call parity formula from Chapter 20:

$$\text{Value of put} = \text{value of call} - \text{value of stock} + \text{PV(exercise price)}$$

American Puts—No Dividends It can sometimes pay to exercise an American put before maturity in order to reinvest the exercise price. For example, suppose that immediately after you buy an American put, the stock price falls to zero. In this case, there is no advantage to holding onto the option because it *cannot* become more valuable. It is better to exercise the put and invest the exercise money. Thus, an American put is always more valuable than a European put. In our extreme example, the difference is equal to the present value of the interest that you could earn on the exercise price. In all other cases, the difference is less.

Because the Black–Scholes formula does not allow for early exercise, it cannot be used to value an American put exactly. But you can use the step-by-step binomial method as long as you check at each point whether the option is worth more dead than alive and then use the higher of the two values.

European Calls and Puts on Dividend-Paying Stocks Part of the share value comprises the present value of dividends. The option holder is not entitled to dividends. Therefore, when using the Black–Scholes model to value a European option on a dividend-paying stock, you should reduce the price of the stock by the present value of the dividends to be paid before the option's maturity.

Dividends don't always come with a big label attached, so look out for instances where the asset holder gets a benefit and the option holder does not. For example, when you buy foreign currency, you can invest it to earn interest; but if you own an option to buy foreign currency, you miss out on this income. Therefore, when valuing an option to buy foreign currency, you need to deduct the present value of this foreign interest from the current price of the currency.[18]

American Calls on Dividend-Paying Stocks We have seen that when the stock does not pay dividends, an American call option is *always* worth more alive than dead. By holding on to the option, you not only keep your option open, but also earn interest on the exercise money. Even when there are dividends, you should never exercise early if the dividend you gain is less than the interest you lose by having to pay the exercise price early. However, if the dividend

[18]For example, just suppose that it costs \$2 to buy £1 and that this pound can be invested to earn interest of 5%. The option holder misses out on interest of $.05 \times \$2 = \$.10$. So, before using the Black–Scholes formula to value an option to buy sterling, you must adjust the current price of sterling: Adjusted price of sterling = current price − PV(interest) = $\$2 - .10/1.05 = \$1.905$

is sufficiently large, you might want to capture it by exercising the option just before the ex-dividend date.

The only general method for valuing an American call on a dividend-paying stock is to use the step-by-step binomial method. In this case, you must check at each stage to see whether the option is more valuable if exercised just before the ex-dividend date than if held for at least one more period.

21-6 The Option Menagerie

Our focus in the past two chapters has been on plain-vanilla puts and calls or combinations of them. An understanding of these options and how they are valued will allow you to handle most of the option problems that you are likely to encounter in corporate finance. However, you may occasionally encounter some more unusual options. We are not going to be looking at them in this book, but just for fun and to help you hold your own in conversations with your investment banker friends, here is a crib sheet that summarizes a few of these exotic options:

Asian (or average) option	The exercise price is equal to the *average* of the asset's price during the life of the option.
Barrier option	Option where the payoff depends on whether the asset price reaches a specified level. A knock-in option (up-and-in call or down-and-in put) comes into existence only when the underlying asset reaches the barrier. Knock-out options (down-and-out call or up-and-out put) *cease* to exist if the asset price reaches the barrier.
Bermuda option	The option is exercisable on discrete dates before maturity.
Caput option	Call option on a put option.
Chooser (as-you-like-it) option	The holder must decide before maturity whether the option is a call or a put.
Compound option	An option on an option.
Digital (binary or cash-or-nothing) option	The option payoff is zero if the asset price is the wrong side of the exercise price and otherwise is a fixed sum.
Lookback option	The option holder chooses as the exercise price any of the asset prices that occurred before the final date.
Rainbow option	Call (put) option on the best (worst) of a basket of assets.

SUMMARY

In this chapter, we introduced the basic principles of option valuation by considering a call option on a stock that could take on one of two possible values at the option's maturity. We showed that it is possible to construct a package of the stock and a loan that would provide exactly the same payoff as the option *regardless* of whether the stock price rises or falls. Therefore, the value of the option must be the same as the value of this replicating portfolio.

We arrived at the same answer by pretending that investors are risk-neutral, so that the expected return on every asset is equal to the interest rate. We calculated the expected future value of the option in this imaginary risk-neutral world and then discounted this figure at the interest rate to find the option's present value.

The general binomial method adds realism by dividing the option's life into a number of subperiods in each of which the stock price can make one of two possible moves. Chopping the period into these shorter intervals doesn't alter the basic method for valuing a call option. We can still replicate the call by a package of the stock and a loan, but the package changes at each stage.

Finally, we introduced the Black–Scholes formula. This calculates the option's value when the stock price is constantly changing and takes on a continuum of possible future values.

An option can be replicated by a package of the underlying asset and a risk-free loan. Therefore, we can measure the risk of any option by calculating the risk of this portfolio. Naked options are often substantially more risky than the asset itself.

When valuing options in practical situations there are a number of features to look out for. For example, you may need to recognize that the option value is reduced by the fact that the holder is not entitled to any dividends.

FURTHER READING

Three readable articles about the Black–Scholes model are:

F. Black, "How We Came up with the Option Formula," *Journal of Portfolio Management* 15 (1989), pp. 4–8.

F. Black, "The Holes in Black–Scholes," *RISK Magazine* 1 (1988), pp. 27–29.

F. Black, "How to Use the Holes in Black–Scholes," *Journal of Applied Corporate Finance* 1 (Winter 1989), pp. 67–73.

There are a number of good books on option valuation. They include:

J. Hull, *Options, Futures and Other Derivatives,* 10th ed. (Cambridge, UK: Pearson, 2017).

R. L. McDonald, *Derivatives Markets,* 3rd ed. (Cambridge, UK: Pearson, 2012).

P. Wilmott, *Paul Wilmott on Quantitative Finance,* 2nd ed. (New York: John Wiley & Sons, 2006).

PROBLEM SETS

Mc Graw Hill Education connect®

Select problems are available in McGraw-Hill's *Connect.* Please see the preface for more information.

1. **Binomial model*** Over the coming year, Ragwort's stock price will halve to \$50 from its current level of \$100 or it will rise to \$200. The one-year interest rate is 10%.
   a. What is the delta of a one-year call option on Ragwort stock with an exercise price of \$100?
   b. Use the replicating-portfolio method to value this call.
   c. In a risk-neutral world, what is the probability that Ragwort stock will rise in price?
   d. Use the risk-neutral method to check your valuation of the Ragwort option.
   e. If someone told you that in reality there is a 60% chance that Ragwort's stock price will rise to \$200, would you change your view about the value of the option? Explain.

2. **Binomial model** Imagine that Amazon's stock price will either rise by 33.3% or fall by 25% over the next six months (see Section 21-1). Recalculate the value of the call option (exercise price = \$900) using (a) the replicating portfolio method and (b) the risk-neutral method. Explain intuitively why the option value rises from the value computed in Section 21-1.
3. **Binomial model** The stock price of Heavy Metal (HM) changes only once a month: Either it goes up by 20% or it falls by 16.7%. Its price now is \$40. The interest rate is 1% per month.
   a. What is the value of a one-month call option with an exercise price of \$40?
   b. What is the option delta?
   c. Show how the payoffs of this call option can be replicated by buying HM's stock and borrowing.
   d. What is the value of a two-month call option with an exercise price of \$40?
   e. What is the option delta of the two-month call over the first one-month period?
4. **Binomial model** Suppose a stock price can go up by 15% or down by 13% over the next year. You own a one-year put on the stock. The interest rate is 10%, and the current stock price is \$60.
   a. What exercise price leaves you indifferent between holding the put or exercising it now?
   b. How does this break-even exercise price change if the interest rate is increased?
5. **Two-step binomial model*** Take another look at our two-step binomial trees for Amazon in Figure 21.2. Use the risk-neutral method to value six-month call and put options with an exercise price of \$750. Assume the Amazon stock price is \$900.
6. **Two-step binomial model** Buffelhead's stock price is \$220 and could halve or double in each six-month period (equivalent to a standard deviation of 98%). A one-year call option on Buffelhead has an exercise price of \$165. The interest rate is 21% a year.
   a. What is the value of the Buffelhead call?
   b. Now calculate the option delta for the second six months if (1) the stock price rises to \$440 and (2) the stock price falls to \$110.
   c. How does the call option delta vary with the level of the stock price? Explain intuitively why.
   d. Suppose that in month 6, the Buffelhead stock price is \$110. How, at that point, could you replicate an investment in the stock by a combination of call options and risk-free lending? Show that your strategy does indeed produce the same returns as those from an investment in the stock.
7. **Two-step binomial model** Suppose that you have an option that allows you to sell Buffelhead stock (see Problem 6) in month 6 for \$165 *or* to buy it in month 12 for \$165. What is the value of this unusual option?
8. **Two-step binomial model** Johnny Jones's high school derivatives homework asks for a binomial valuation of a 12-month call option on the common stock of the Overland Railroad. The stock is now selling for \$45 per share and has an annual standard deviation of 24%. Johnny first constructs a binomial tree like Figure 21.2, in which stock price moves up or down every six months. Then he constructs a more realistic tree, assuming that the stock price moves up or down once every three months, or four times per year.
   a. Construct these two binomial trees.
   b. How would these trees change if Overland's standard deviation were 30%? (*Hint:* Make sure to specify the right up and down percentage changes.)
9. **Option delta***
   a. Can the delta of a call option be greater than 1.0? Explain.
   b. Can it be less than zero?

c. How does the delta of a call change if the stock price rises?

d. How does it change if the risk of the stock increases?

10. **Option delta** Suppose you construct an option hedge by buying a levered position in delta shares of stock and selling one call option. As the share price changes, the option delta changes, and you will need to adjust your hedge. You can minimize the cost of adjustments if changes in the stock price have only a small effect on the option delta. Construct an example to show whether the option delta is likely to vary more if you hedge with an in-the-money option, an at-the-money option, or an out-of-the-money option.

11. **Black–Scholes model***

a. Use the Black-Scholes formula to find the value of the following call option.

i. Time to expiration 1 year.

ii. Standard deviation 40% per year.

iii. Exercise price $50.

iv. Stock price $50.

v. Interest rate 4% (effective annual yield).

b. Now recalculate the value of this call option, but use the following parameter values. Each change should be considered independently.

i. Time to expiration 2 years.

ii. Standard deviation 50% per year.

iii. Exercise price $60.

iv. Stock price $60.

v. Interest rate 6%.

c. In which case did increasing the value of the input *not* increase your calculation of option value?

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Try it!
The Black-Scholes model

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12. **Black–Scholes model** Use the Black–Scholes formula to value the following options:

a. A call option written on a stock selling for $60 per share with a $60 exercise price. The stock's standard deviation is 6% per month. The option matures in three months. The risk-free interest rate is 1% per month.

b. A put option written on the same stock at the same time, with the same exercise price and expiration date.

Now for each of these options, find the combination of stock and risk-free asset that would replicate the option.

13. **Binomial and Black–Scholes models** The current price of United Carbon (UC) stock is $200. The standard deviation is 22.3% a year, and the interest rate is 21% a year. A one-year call option on UC has an exercise price of $180.

a. Use the Black–Scholes model to value the call option on UC. You may find it helpful to use the spreadsheet version of Table 21.2, accessible through the Beyond the Page feature.

b. Use the formula given in Section 21-2 to calculate the up-and-down moves that you would use if you valued the UC option with the one-period binomial method. Now value the option by using that method.

c. Recalculate the up-and-down moves and revalue the option by using the two-period binomial method.

d. Use your answer to part (c) to calculate the option delta (1) today, (2) next period if the stock price rises, and (3) next period if the stock price falls. Show at each point how you would replicate a call option with a levered investment in the company's stock.

14. **Option risk** "A call option is always riskier than the stock it is written on." True or false? How does the risk of an option change when the stock price changes?

15. **Option risk***
    a. In Section 21-3, we calculated the risk (beta) of a six-month call option on Amazon stock with an exercise price of $900. Now repeat the exercise for a similar option with an exercise price of $750. Does the risk rise or fall as the exercise price is reduced?
    b. Now calculate the risk of a one-year call on Amazon stock with an exercise price of $750. Does the risk rise or fall as the maturity of the option lengthens?

16. **Warrants** Use the Black–Scholes program from the Beyond the Page feature to value the Owens Corning warrants described in Section 21-4. The standard deviation of Owens Corning stock was 41% a year and the interest rate when the warrants were issued was 5%. Owens Corning did not pay a dividend. Ignore the problem of dilution.

17. **Pension fund insurance** Use the Black–Scholes program to estimate how much you should be prepared to pay to insure the value of your pension fund portfolio for the coming year. Make reasonable assumptions about the volatility of the market and use current interest rates. Remember to subtract the present value of likely dividend payments from the current level of the market index.

18. **American options** For which of the following options *might* it be rational to exercise before maturity? Explain briefly why or why not.
    a. American put on a non-dividend-paying stock.
    b. American call—the dividend payment is $5 per annum, the exercise price is $100, and the interest rate is 10%.
    c. American call—the interest rate is 10%, and the dividend payment is 5% of future stock price. (*Hint:* The dividend depends on the stock price, which could either rise or fall.)

19. **American options** The price of Moria Mining stock is $100. During each of the next two six-month periods the price may either rise by 25% or fall by 20% (equivalent to a standard deviation of 31.5% a year). At month 6, the company will pay a dividend of $20. The interest rate is 10% per six-month period. What is the value of a one-year American call option with an exercise price of $80? Now recalculate the option value, assuming that the dividend is equal to 20% of the with-dividend stock price.

20. **American options** Suppose that you own an American put option on Buffelhead stock (see Problem 6) with an exercise price of $220.
    a. Would you ever want to exercise the put early?
    b. Calculate the value of the put.
    c. Now compare the value with that of an equivalent European put option.

21. **American options** Recalculate the value of the Buffelhead call option (see Problem 6), assuming that the option is American and that at the end of the first six months the company pays a dividend of $25. (Thus, the price at the end of the year is either double or half the *ex*-dividend price in month 6.) How would your answer change if the option were European?

22. **American options** The current price of the stock of Mont Tremblant Air is C$100. During each six-month period it will either rise by 11.1% or fall by 10% (equivalent to an annual standard deviation of 14.9%). The interest rate is 5% per six-month period.
    a. Calculate the value of a one-year European put option on Mont Tremblant's stock with an exercise price of C$102.
    b. Recalculate the value of the Mont Tremblant put option, assuming that it is an American option.

23. **American options** Other things equal, which of these American options are you most likely to want to exercise early?
    a. A put option on a stock with a large dividend or a call on the same stock.
    b. A put option on a stock that is selling below exercise price or a call on the same stock.
    c. A put option when the interest rate is high or the same put option when the interest rate is low.

    Illustrate your answer with examples.

24. **Option exercise** Is it better to exercise a call option on the with-dividend date or on the ex-dividend date? How about a put option? Explain.

CHALLENGE

25. **Option delta** Use the put-call parity formula (see Section 20-2) and the one-period binomial model to show that the option delta for a put option is equal to the option delta for a call option minus 1.

26. **Option delta** Show how the option delta changes as the stock price rises relative to the exercise price. Explain intuitively why this is the case. (What happens to the option delta if the exercise price of an option is zero? What happens if the exercise price becomes indefinitely large?)

27. **Dividends** Your company has just awarded you a generous stock option scheme. You suspect that the board will either decide to increase the dividend or announce a stock repurchase program. Which do you secretly hope they will decide? Explain. (You may find it helpful to refer back to Chapter 16.)

28. **Option risk** Calculate and compare the risk (betas) of the following investments: (a) a share of Amazon stock; (b) a one-year call option on Amazon; (c) a one-year put option; (d) a portfolio consisting of a share of Amazon stock and a one-year put option; (e) a portfolio consisting of a share of Amazon stock, a one-year put option, and the sale of a one-year call. In each case, assume that the exercise price of the option is $900, which is also the current price of Amazon stock.

29. **Option risk** In Section 21-1, we used a simple one-step model to value two Amazon options each with an exercise price of $900. We showed that the call option could be replicated by borrowing $407.06 and investing $490.91 in .54545 share of Amazon stock. The put option could be replicated by selling short $409.10 of Amazon stock and lending $488.46.
    a. If the beta of Amazon stock is 1.5, what is the beta of the call according to the one-step model?
    b. What is the beta of the put?
    c. Suppose that you were to buy one call and invest the present value of the exercise price in a bank loan. What would be the beta of your portfolio?
    d. Suppose instead that you were to buy one share and one put option of Amazon. What would be the beta of your portfolio now?
    e. Your answers to parts (c) and (d) should be the same. Explain.

30. **Option maturity** Some corporations have issued *perpetual* warrants. Warrants are call options issued by a firm, allowing the warrant holder to buy the firm's stock.
    a. What does the Black–Scholes formula predict for the value of an infinite-lived call option on a non-dividend-paying stock? Explain the value you obtain. (*Hint:* What happens to the present value of the exercise price of a long-maturity option?)
    b. Do you think this prediction is realistic? If not, explain carefully why. (*Hints:* What about dividends? What about bankruptcy?)

FINANCE ON THE WEB

Look at the stocks listed in Table 7.3. Pick at least three stocks, and find call option prices for each of them on **finance.yahoo.com.** Now find monthly adjusted prices and calculate the standard deviation from the monthly returns using the Excel function STDEV.P. Convert the standard deviation from monthly to annual units by multiplying by the square root of 12.

a. For each stock, pick a traded option with a maturity of about six months and an exercise price equal to the current stock price. Use the Black–Scholes formula and your estimate of standard deviation to value each option. If the stock pays dividends, remember to subtract from the stock price the present value of any dividends that the option holder will miss out on. How close is your calculated value to the traded price of the option?

b. Your answer to part (a) will not exactly match the traded price. Experiment with different values for the standard deviation until your calculated values match the prices of the traded options as closely as possible. What are these implied volatilities? What do the implied volatilities say about investors' forecasts of future volatility?

MINI-CASE

Bruce Honiball's Invention

It was another disappointing year for Bruce Honiball, the manager of retail services at the Gibb River Bank. Sure, the retail side of Gibb River was making money, but it didn't grow at all in 2017. Gibb River had plenty of loyal depositors but few new ones. Bruce had to figure out some new product or financial service—something that would generate some excitement and attention.

Bruce had been musing on one idea for some time. How about making it easy *and safe* for Gibb River's customers to put money in the stock market? How about giving them the upside of investing in equities—at least *some* of the upside—but none of the downside?

Bruce could see the advertisements now:

> How would you like to invest in Australian stocks completely risk-free? You can with the new Gibb River Bank *Equity-Linked Deposit.* You share in the good years; we take care of the bad ones.
>
> Here's how it works. Deposit A\$100 with us for one year. At the end of that period, you get back your A\$100 *plus* A\$5 for every 10% rise in the value of the Australian All Ordinaries stock index. But, if the market index falls during this period, the Bank will still refund your A\$100 deposit in full.
>
> There's no risk of loss. Gibb River Bank is your safety net.

Bruce had floated the idea before and encountered immediate skepticism, even derision: "Heads they win, tails we lose—is that what you're proposing, Mr. Honiball?" Bruce had no ready answer. Could the bank really afford to make such an attractive offer? How should it invest the money that would come in from customers? The bank had no appetite for major new risks.

Bruce has puzzled over these questions for the past two weeks but has been unable to come up with a satisfactory answer. He believes that the Australian equity market is currently fully valued, but he realizes that some of his colleagues are more bullish than he is about equity prices.

Fortunately, the bank had just recruited a smart new MBA graduate, Sheila Liu. Sheila was sure that she could find the answers to Bruce Honiball's questions. First she collected data on the Australian market to get a preliminary idea of whether equity-linked deposits could work. These data are shown in Table 21.3. She was just about to undertake some quick calculations when she received the following further memo from Bruce:

Sheila, I've got another idea. A lot of our customers probably share my view that the market is overvalued. Why don't we also give them a chance to make some money by offering a "bear market deposit"? If the market goes up, they would just get back their A$100 deposit. If it goes down, they get their A$100 back plus $5 for each 10% that the market falls. Can you figure out whether we could do something like this? Bruce.

QUESTION

1. What kinds of options is Bruce proposing? How much would the options be worth? Would the equity-linked and bear-market deposits generate positive NPV for Gibb River Bank?

Year	Interest Rate	Market Return	Dividend Yield	Year	Interest Rate	Market Return	Dividend Yield
1995	8.0%	20.2%	4.0	2007	6.6	18.0	4.3
1996	7.4	14.6	4.1	2008	7.3	–40.4	6.8
1997	5.5	12.2	3.7	2009	3.2	39.6	5.3
1998	5.0	11.6	3.6	2010	4.3	3.3	4.2
1999	4.9	19.3	3.3	2011	4.8	–11.4	4.4
2000	5.9	5.0	3.3	2012	3.7	18.8	5.1
2001	5.2	10.1	3.3	2013	2.8	19.7	4.5
2002	4.6	–8.1	3.5	2014	2.5	5.0	4.5
2003	4.8	15.9	4.2	2015	2.1	3.8	4.7
2004	5.4	27.6	3.7	2016	1.8	11.6	4.8
2005	5.6	21.1	3.8	2017	1.5	12.5	4.4
2006	5.9	25.0	3.8				

TABLE 21.3 Australian interest rates and equity returns, 1995–2017