Beware of little Expenses: a small Leak will sink a great Ship.

Ben Franklin

NOTE: Compute all dollar-denominated answers to the nearest dollar unless otherwise specified, and compute all other answers to four decimal places (*i.e.,* .0001). Unless otherwise stated, all interest rates or rates of return are annual (effective) rates. Excel hints are given in brackets. For the multiple choice questions, give the correct choice and very briefly explain your answer.

1. Starting at end of 2024, big law lawyer Luiza saves $10,000 per year for 10 years. She then discovers that corporate law practice isn't making her happy and opts to work for one of those charities that raises money to battle some fashionably fatal disease. For the rest of her working life, she doesn't earn enough money to add to her retirement account, but she leaves the entire 2033 ending balance invested for the next 30 years.

   Lazy Larry comes late in life to a knowledge of the time value of money and at the end of 2034 begins to save $10,000 per year for the next 30 years. Both Luiza and Lazy earn 7% per year.

   Who finishes the race with the most money at the end of 2064? In sum, Luiza invests $10,000 for 10 years and allows that balance to grow for 30 years, and Lazy, who starts 10 years later, invests $10,000 for 30 years. *[FV]*

2. You borrow $500,000 at 7% per year (APR, not EAR) for 30 years to purchase an apartment (in some city that is not New York).

   (a) What is your monthly payment? *[PMT]*

   (b) If you make an additional monthly payment of $250, in how many years will the loan be repaid? *[NPER or N]*

   (c) Using the original facts, if you want to repay the loan in 15 years instead of 30, how much more do you need to pay each month? *[PMT] and [NPER or N]*

   (d) Using the original facts, immediately after the 72nd payment, you receive a $50,000 bonus, which you use to pay down the principal. By how many years and months do you reduce the mortgage? *Note: When you prepay a mortgage, your monthly payment does not change but the interest that accrues each month is less (because the principal is less), and you will make fewer payments. [PV and NPER*]

3. You've found your dream house/apartment/room, and you apply for a $500,000 mortgage from your local bank after seeing an ad on your Instagram feed for a 7.50%,

30-year mortgage. After you show your W-2 (wage statement) to the loan officer, the bank concludes that you may not be able to afford the monthly payment. They therefore offer you a rate of 7.00% (APR) but you must also pay an upfront fee of 3 "points." A point, in loan parlance, means 1% of the loan amount. Thus, if you borrow $100,000 and pay 3 points, you must pay $3,000. Even though you receive only the loan proceeds *less* the points paid, you must repay the *entire* amount using the amortization schedule for that amount.

Points paid are economically equivalent to additional interest on the loan, except the interest is paid upfront. (You should convince yourself that the result is the same whether the bank deducts the points from the loan proceeds or you write a separate check for them.)

(a) What is your true interest rate (use APR) as a result of paying the 3 points assuming that you pay off the loan over the original 30 year term? *[Rate]*. **Hint:** *The PMT is calculated using the loan amount, but the amount you receive is the loan amount* less *the points.*

(b) Suppose that your income qualifies you for either mortgage. In helping you to decide between the two mortgages, one back-of-the-envelope calculation that often appears online, is the breakeven point, that is, how long before you recover the points paid with the lower monthly payment. If you plan to stay in your home for a period exceeding the breakeven point, then the breakeven approach suggests that paying the points is superior. What is the breakeven point for these two mortgages? Using some of the principles we covered this semester, very, very briefly critique the breakeven methodology. What is other information that could be relevant to the analysis?

(c) Assume that you choose the 7.00% plus 3 points mortgage and that you pay off the remaining loan balance at the end of three years instead of over 30 years. What is the true interest rate (use APR) that you have paid over the three years?

Hint: Assume you repay the loan after one day. You will have borrowed the net proceeds (after points) but must repay the entire $500,000 balance and one day's interest. You can see that's indeed a hefty interest rate. If, however, the loan is repaid over 30 years as in (a) above, the extra interest is spread out over the 30 years. Your true interest rate over 3 years is going to fall between the rate in (a) and the rate you pay when you borrow the net proceeds and repay $500,000 one day later.

There are a couple of ways to tackle this problem. The easiest is to use *IRR*. Input the cash flows—the points, the loan proceeds, the 36 payments, and the loan repayment (the loan balance at the end of the 3 years)—using the correct

sign convention, positive for inflows and negative for outflows. *IRR* will give you the correct answer. To solve it with Excel using the *NPER, RATE, PV, PMT, and FV* formulas, remember that your PV is the net loan proceeds (after points), your PMT remains the same, but the FV is the amount that you owe to the bank after 36 payments–you must calculate that.