

An investment in knowledge pays the best interest.

Ben Franklin

Finance TVM Formulas

1. How is the *net return* of an investment determined?
2. What's the formula to calculate the rate of return (r) of an investment?
3. What's the formula for the future value (FV) of a single CF today that earns an annual rate of return of $r\%$ for t years?
4. Same as previous question. What if r is compounded semi-annually? Monthly?
5. What's the formula for the present value (PV) of a CF to be received t years from today if the discount rate is $r\%$, compounded annually?
6. What's the formula to calculate the *discount factor* of a CF to be received in t years if the discount rate is $r\%$?
7. What's the formula for the PV of a perpetuity?
8. What's the formula for a PV of a growing perpetuity?
9. What's the formula for the PV of an annuity? [*Excel: PV*]
10. What's the formula for the PV of a growing annuity?
11. What's the formula for the FV of an annuity? [*Excel: FV*]
12. What is the formula for NPV? What is the relevance of NPV in deciding whether to undertake a project?

Routine Applications of Finance TVM Formulas

13. You invest \$100 and sometime later the investment is worth \$120. What's the rate of return, return, and net return?
14. Give an example of cash flows where you could use the formula for the PV of an annuity, the PV of a growing annuity, and FV of an annuity.
15. What is an APR? What is the EAR?
16. Your student loans have an APR of 6%. What's the EAR? [*Excel: EFFECT*]
17. You are an attorney drafting a loan document for the *lender*. Which of the following interest rates should you put in the document? [*Excel: EFFECT*]
 - (a) 9.90% compounded annually.

- (b) 9.73% compounded semiannually.
 - (c) 9.67% compounded quarterly.
18. You buy a bond yielding 6.25%. A few days later, it's yielding 6.35%. By how many bps did the yield increase?
19. A bond issued by Intel promises to pay \$1,000 in principal and \$50 in interest one year from today. What's the price of the bond today (PV) if the yield (discount rate) is 6% p.a.?
20. You will receive \$29,375 ten years from today. To calculate the PV of \$29,375, what's the *discount factor* you would use?
21. You invest \$10,000 today. How many years until you have \$1,000,000 if you earn 8% p.a.?
22. What's the relationship between bond prices and bond yields (interest rates)? Explain why.
23. What's the rule of 72?
- (a) You have \$10,000 today. If you can earn 7% per annum, how many years will it take for you to have \$20,000?
 - (b) Same facts as above, if you plan to work for the next 40 years, how much will you have when you retire?
24. You have just won the Lucky Charms—they are magically delicious—cereal box-top lottery and have been offered the choice of the following prizes:
- (a) \$250,000 today
 - (b) \$360,000 at the end of five years
 - (c) \$22,800 a year forever (assume that your heirs can receive the payments) with the first payment one year from today
 - (d) \$40,000 for the next ten years with the first payment starting *today*
 - (e) \$13,000 next year and each annual payment increasing 3% per year forever
- Which prize is the most valuable if the appropriate discount rate is 8%? [*Excel: PV*]
25. A stock is expected to pay a \$1 dividend next year, and the dividend is expected to grow by 5% p.a. forever. What's the value of the stock if the interest rate is 10%?
26. A share of preferred stock is expected to pay a dividend of \$5/year forever. What's the value of the stock if the discount rate is 10%?

27. A stock is expected to pay a dividend of \$5 next year, and the dividend is expected to grow by 5%/year forever. What's the value of the stock if the discount rate is 10%?

Real-World Applications of Finance TVM Formulas

28. After a very successful law career, you want to endow a chair in corporate finance at the FLS that will pay \$300,000 per year beginning one year from today—remember, be generous when you decide to give back. How much do you have to donate *today* to the FLS to fund the chair if you want the chair to be endowed for 25 years, after which there will be no funds left, and the market rate of interest is 7%? *[PV]*
29. Upon his death on April 17, 1790, the great American, Ben Franklin, bequeathed in trust approximately \$4,500 (it was actually £1,000) to each Boston and Philadelphia. (As a point of comparison, this is roughly equivalent in purchasing power to about \$159,00 in 2025.) Both trusts were required to loan the money at 5% p.a. to married tradesmen who were seeking to establish their own businesses. The cities could not spend the money for 100 years, at which time they could spend £100,000; the remaining balance could not be spent for another 100 years (until 1990). The final balance in the Boston trust was about \$5,000,000, but Philadelphia didn't fare nearly as well as its final balance was only \$2,300,000—apparently the default rate was a bit higher in Philadelphia than in Boston. This should come as no surprise if you have ever been to Philadelphia or met anyone from there. For the questions below, *disregard* the disbursements of funds in 1890 and use the dollar figures. An excellent bibliography of Franklin is Walter Isaacson's [Benjamin Franklin: An American Life](#).
- What was the 200-year holding period return ($r_{0,200}$) of the Boston trust? Assume that the funds were invested for exactly 200 years. Note, it's a pretty big return.
 - What was the Boston trust's annual rate of return? *[Excel: Rate]*
 - What would have been the Boston trust's final balance if its investments had actually earned 5% p.a. over the 200 years, as envisioned by Ben Franklin? *[Excel: FV]*
30. *This question highlights two of the most important concepts that we cover this semester: (1) the importance of time and returns in compounding wealth, and (2) a very keen appreciation of the utterly devastating effect of fees. You can't control future returns, but you can control amounts invested, time invested, and fees. Your goal should be to maximize amounts and time invested and minimize fees.*

You decide to be generous to your first unborn great-great grandchild: you establish a trust on 1 Jan 2025 and fund it with \$1,000. Under the terms of the trust, when your great-great grandchild turns 18, the trust will distribute all of the trust assets

to him or her or them. Assume that you make no other contributions to the trust, and your great–great grandchild turns 18 on 1 Jan 2105.

- (a) What will be the amount distributed to the trust beneficiary if the trust earns 5% per annum? [*Excel: FV*]
- (b) What will be the amount distributed to the trust beneficiary if instead the trust earns 10% per annum? Although the annual return in this question is twice that in the previous question, notice how many times greater the ending balance is. [*Excel: FV*]
- (c) Assume that the trust earns 10% per annum on its investments but pays an *administrative fee to helpers* of 2% per year *at year end*. These *helpers* are known as brokers, financial advisers, or financial planners, but you should think of them as basically used car salesmen. Note, the fee of 2% is roughly the average annual administrative costs of an actively managed mutual fund. What’s the final balance? [*Excel: FV*]

Hint: To get the correct “r” in the compounding factor—the $(1 + r)^T$ —it may be useful to write out an algebraic formula for the first year accumulation and reduction for the administrative fee. Remember that in this problem the 2% fee is levied on the year-end balance, after the annual return. Alternatively you can think of the administrative fee as being an annual negative return.

31. In 2023, it was announced that the Japanese baseball player, Shohei Ohtani, had signed a ten-year, \$700 million contract with the Los Angeles Dodgers. The contract calls for ten annual payments of \$2 million followed by ten annual deferred payments of \$68 million for a total of \$700 million.
- (a) Although the Dodgers are obligated to pay Ohtani a total of \$700 million over the next twenty years, why may it be inaccurate to label the contract a \$700 million contract?
 - (b) Given the cash flows of Ohtani’s contract, briefly describe two approaches to calculate the PV of the cash flows? No calculations are necessary.
 - (c) Assuming a discount rate of 4.43%, what is the PV of the 20 contractual payments assuming the payments are received at year end?
 - (d) In calculating CFs for revenues and expenses, such as salaries, it is often assumed that the payments are made or received in the middle of the year. Would such an adjustment increase or decrease the PV calculated in 2.c.? Describe at least one method for making the adjustment.
 - (e) One question is why the contract was structured with such large deferred payments. Apparently it has to do with the way a team’s annual payroll is deter-

mined for purposes of the *Competitive Balance Tax* (CBT), which is a penalty that applies if a team's payroll exceeds a certain threshold specified in the 442-page Collective Bargaining Agreement (CBA) between the league and the MLB Players Association. The CBA also specifies the discount rate for deferred contract payments as being the *applicable federal mid-term rate*, which was 4.43% when the contract was signed.

In determining the annual cost of a player's contract (referred to as the *Annual Average Value* (AAV)) the non-deferred annual amounts (\$2 million/year) are *not* discounted, but the deferred amounts (\$68 million/year) are discounted at the above rate to the season to which they are attributed. (An amount is deferred compensation if it is received after services are required to be rendered.)

Thus, if there were no deferred amounts, Ohtani's AAV would be \$70 million per year, but under the CBA, each year's AAV would be the sum of \$2 million plus the PV of the deferred \$68 million— $\frac{68}{(1+.0443)^{10}}$ —or \$44.08 million, for a total of \$46.08 million. The lower AAV therefore allows the Dodgers to sign better players while reducing any potential CBT liability.

Is the PV of Ohtani's contract calculated under the CBT greater or lesser than the value you calculated in 2.c.? Why?

- (f) Why may the *applicable federal mid-term rate* not be the appropriate discount rate?
32. This question addresses financial issues that may arise in certain *discrete* family law matters.

Mick Jagger (then age 78), lead singer of the Rolling Stones, welcomed at the end of 2016 his 8th child, Devereaux. The mother, Melanie Hamrick (then age 32!), is a retired ballerina with the American Ballet Theatre. Some recent updates for the truly curious: [Mick's recent 82nd birthday party](#). Ms. Hamrick apparently is a woman of many talents: in addition to her ballet achievements, she is a twice published [author](#).

Being the responsible frontman he is, Jagger reportedly agreed to provide support of \$15,000 per month for the newborn until he reaches the age of 18. (There apparently are other undisclosed financial payments for additional schooling and a humble abode for Ms. Hamrick, but we'll ignore those.) Thus, Mick will have no further financial obligations the year that young Devereaux turns 18.

Let's assume that Mick's newest scion was born on January 1, 2025, will live a long and fruitful life, and the payments are very low risk, given Mick's wealth. The 20-year U.S. treasury bond is currently yielding 4.83%, so let's use 8.80% as the appropriate discount rate.

For this problem, *assume that each year's total monthly payments are paid annually*, and remember, the year that he turns 18 he will not be entitled to any further payments.

- (a) What is the PV (as of Jan. 1, 2025) of the payments if the total payments for each year (\$180,000) are paid on December 31 of each year? *[PV]*
 - (b) What is the PV (as of Jan. 1, 2025) of the payments if each year's total payments are paid at the beginning of each year, i.e., each January 1? *[PV]*
 - (c) Suppose that Melanie negotiates an escalator clause whereby each year's payment, *after* the first payment, will be increased by the year's inflation (let's assume 2% per year). What is the PV (as of Jan. 1, 2025) of the payments if they are paid *at the end of each year*, i.e., each December 31?
 - (d) Suppose that Melanie wants a lump sum payment on the day of the birth of her child (January 1, 2025) instead of the installment payments, but Mick insists on the annuity payout. Melanie turns to [J.G. Wentworth](#) and sells the promised payments for a lump sum in a structured settlement agreement. If J.G. Wentworth discounts the payments at 10% (assume they are made at year end and there is no escalator clause), how much would they offer her? *[PV]*
 - (e) Suppose that Melanie is a very responsible mommie and decides to invest the payments instead of spending them. If the payments are received at year end and there is no escalator clause, what will be the account balance when young Jagger turns 18 if Melanie is able to earn 5% per year? 7% per year? *[FV]*
33. You made the very wise choice to have borrowed \$200,000 to finance your FLS education. What is your monthly payment if the APR on the loan is 7.20% and the term of the loan 10 years? *[PMT]*
34. Same facts as previous question. How long will it take to pay off 50% of the principal amount?
35. Your goal is to have \$5 million when you retire in 35 years—it may seem like a lot, but remember the dreaded inflation tax. Assume that you begin saving one year from today and make 35 equal, annual additions at year end to your retirement account. How much do you have to save each year to reach this goal if your annual rate of return is 4%? 6%? *[PMT]*
36. On [Immediate Annuities](#), for a payment today of \$1 million, I can receive a guaranteed *monthly* payment of \$10,200 for ten years (10-year period certain). (Note, when interest rates were very low over the last 10 years, the monthly payment has been as low as \$4,000. Interest rates matter!) If each payment is received at month end, what's the *effective annual interest rate* the insurance company is offering me on the

annuity investment? *[Rate]*

37. Read the WSJ Cryonaut article on the class web site. The Reanimation Foundation states that a \$10,000 investment could grow to \$8,677,163 in 100 years. What is the annual rate of return they are assuming? Some interesting books that explore variations on this theme are Michel Houellebecq's [The Possibility of an Island](#), and Dani and Eytan Kollin's [The Unicorporated Man](#). *[Rate]*