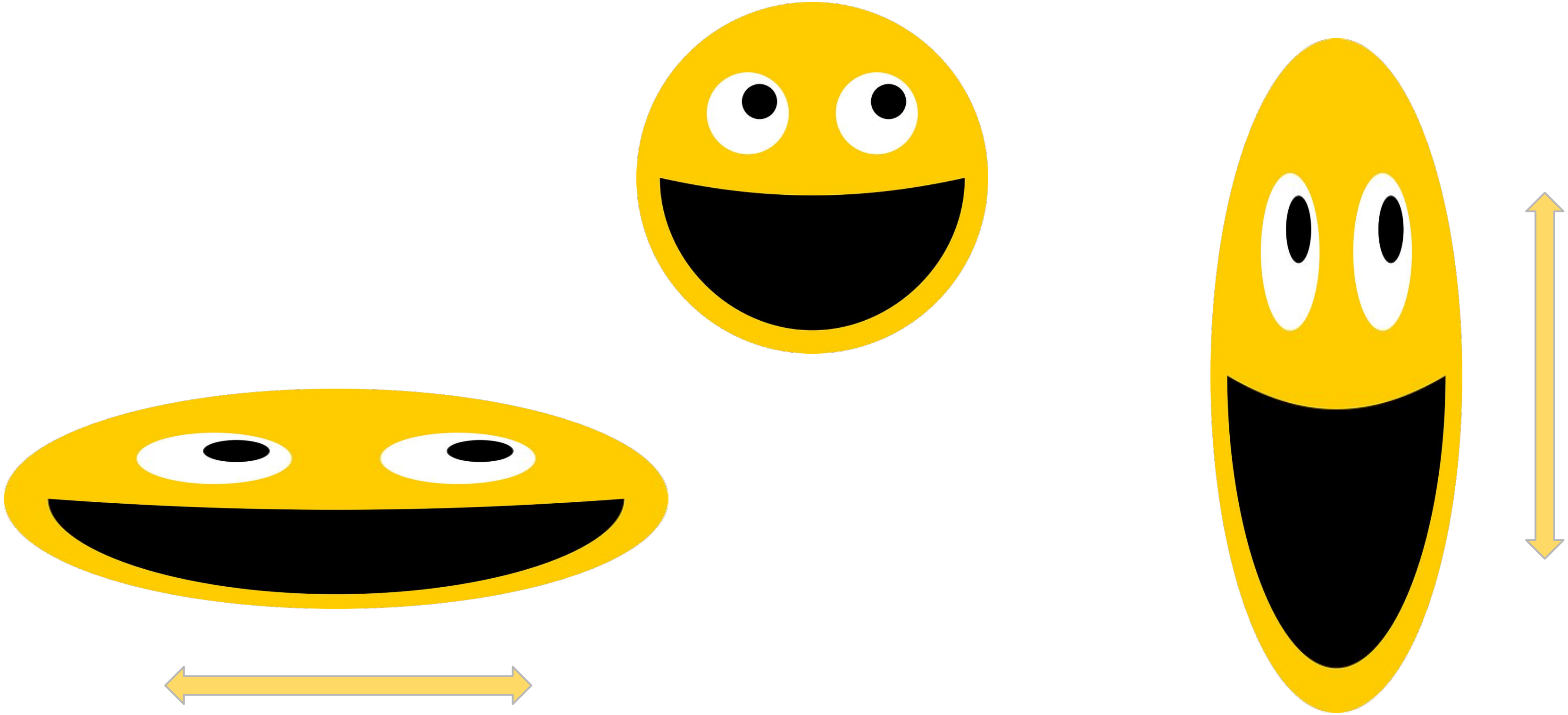
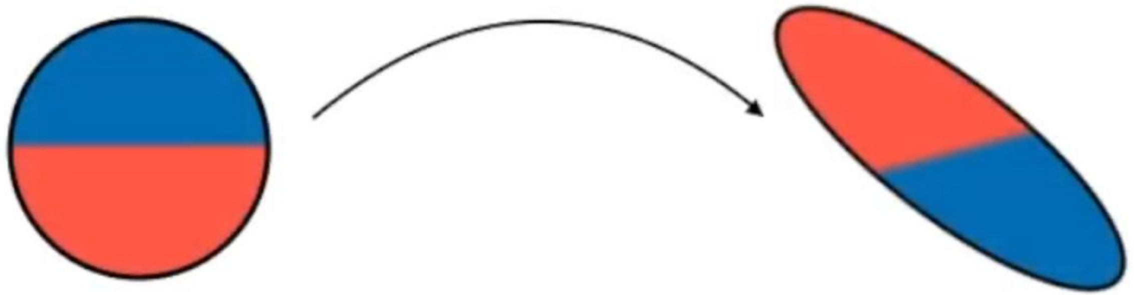
Entendamos las transformaciones en el plano

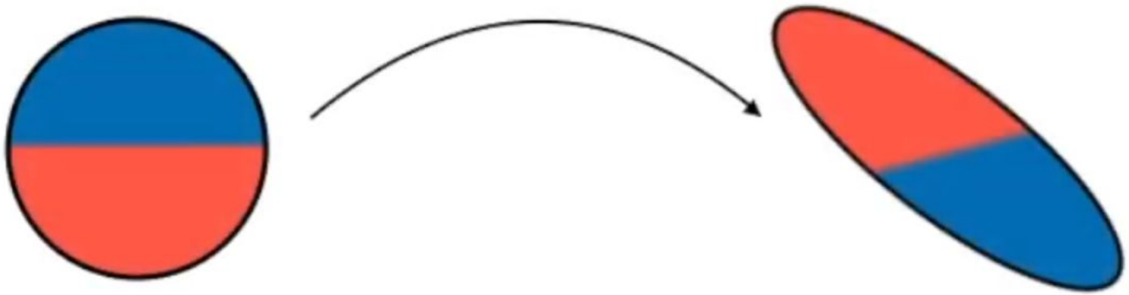


Rotar

Problema (dificil)



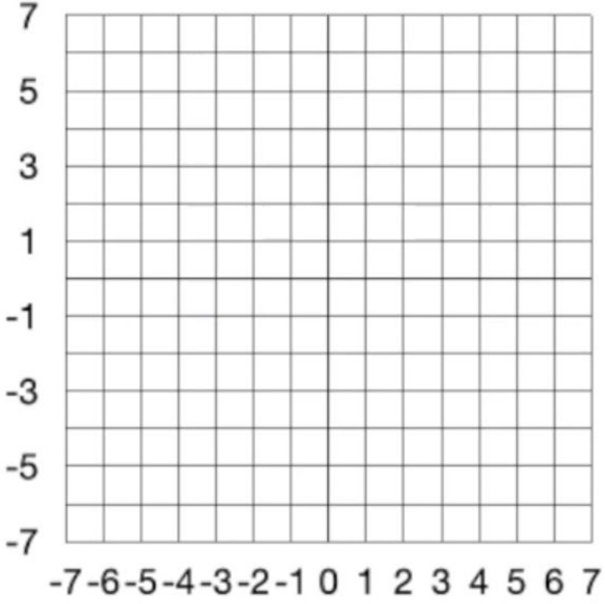
Problema (dificil)

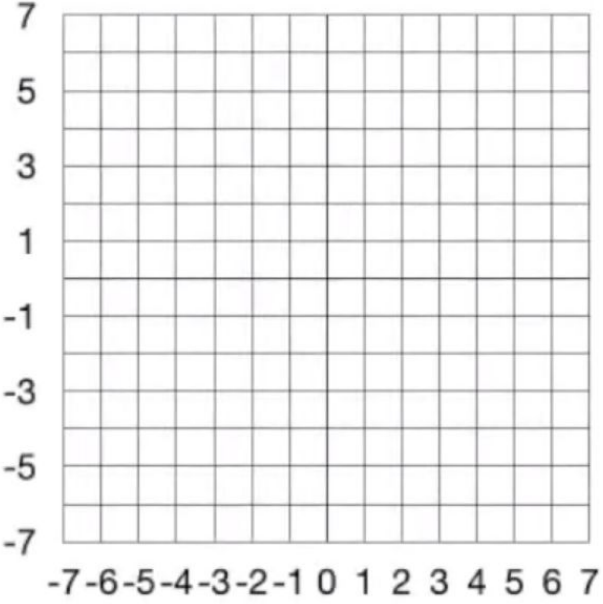


Solución: Rotación + estiramiento + compresión + Rotación

Qué tiene que ver esto con matrices?

$$A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$$



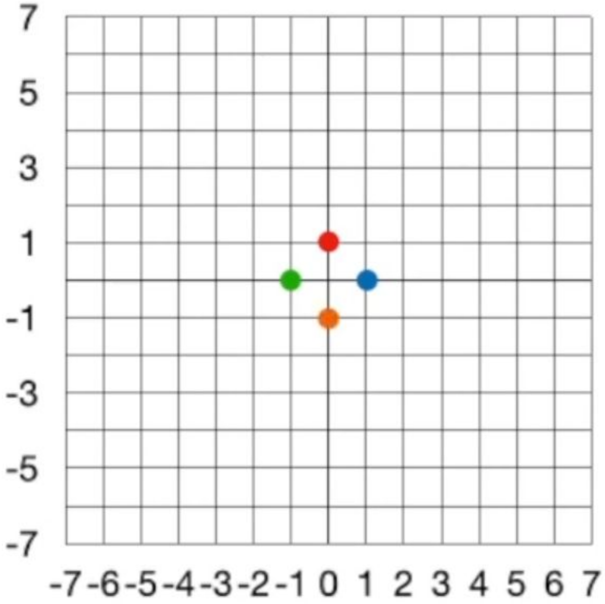


Recordemos el producto de matrices

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 5 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -1 & 0 \\ 5 & 1 & -1 \\ -2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -2 & -1 & 0 \\ 11 & -2 & -1 \\ 1 & -6 & 1 \end{bmatrix}$$

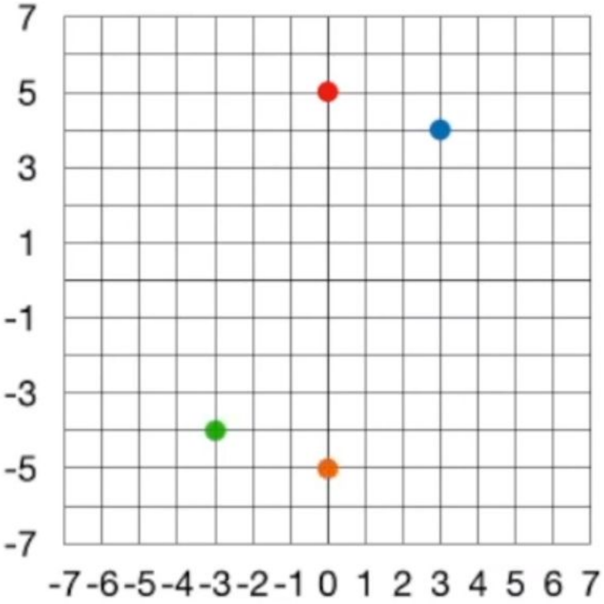
$$3 \times 2 + 1 \times 5 + 0 \times -2 = 11$$

Qué tiene que ver esto con matrices?



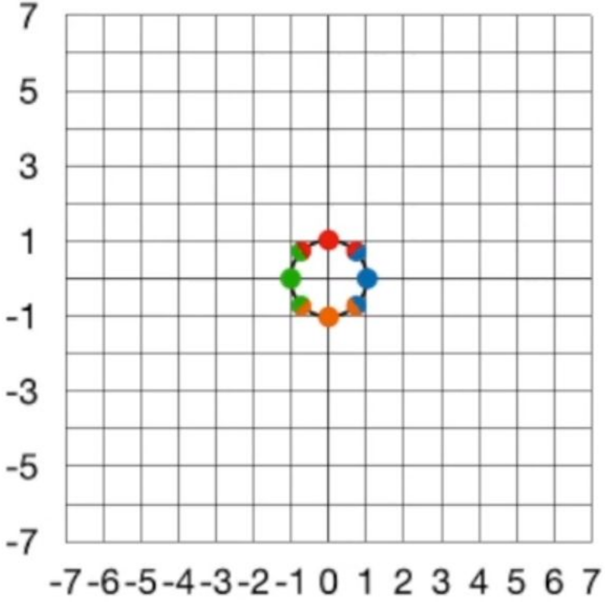
$$A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$$

(p,q)	(3p+0q, 4p+5q)
(1,0)	(3, 4)
(0,1)	(0, 5)
(-1,0)	(-3, -4)
(0,-1)	(0, -5)

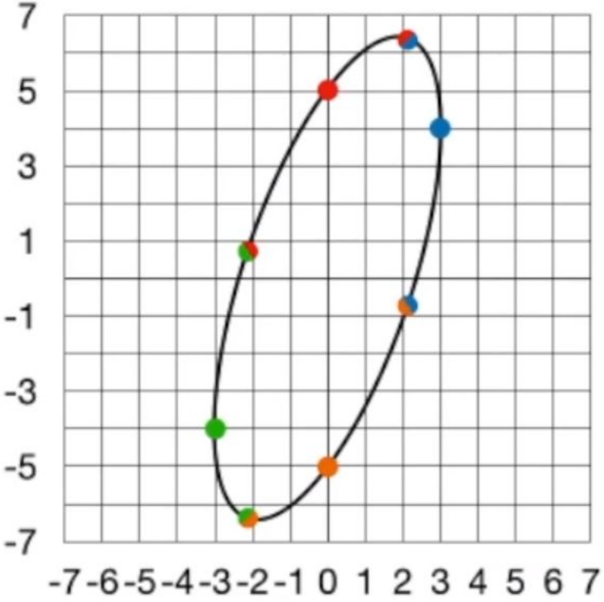


Qué tiene que ver esto con matrices?

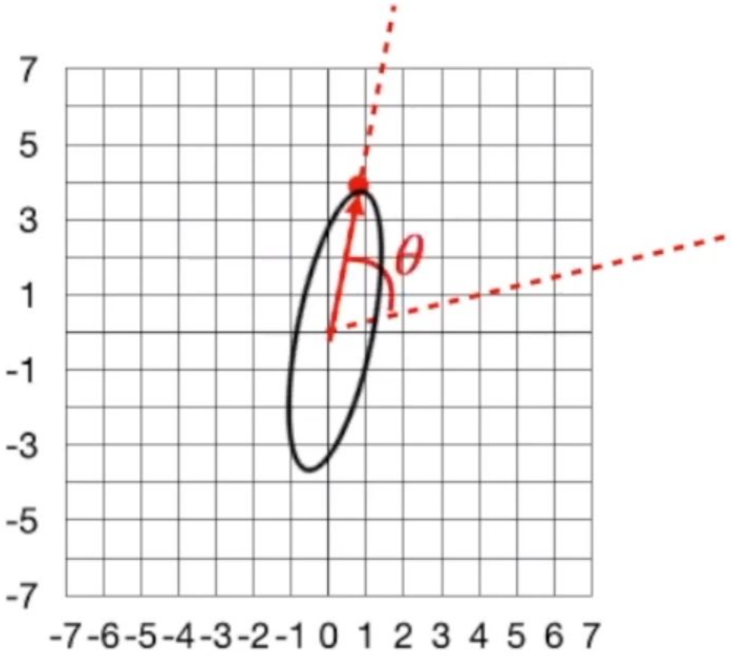
$$A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$$



(p,q)	(3p+0q, 4p+5q)
(1,0)	(3, 4)
(0,1)	(0, 5)
(-1,0)	(-3, -4)
(0,-1)	(0, -5)

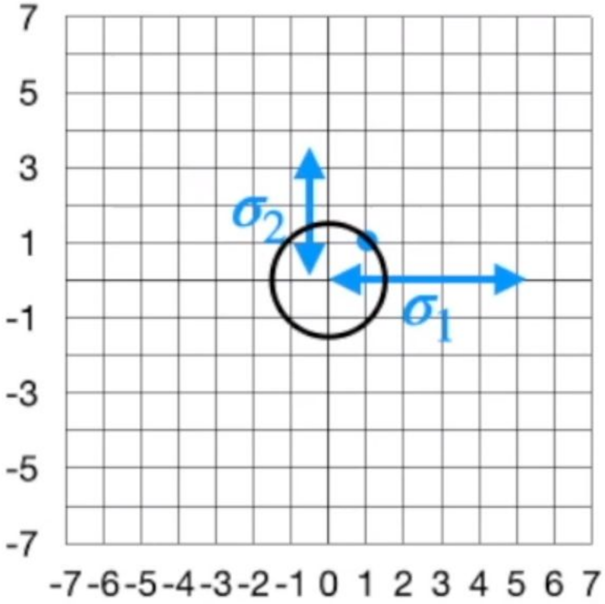


Matrices de rotación

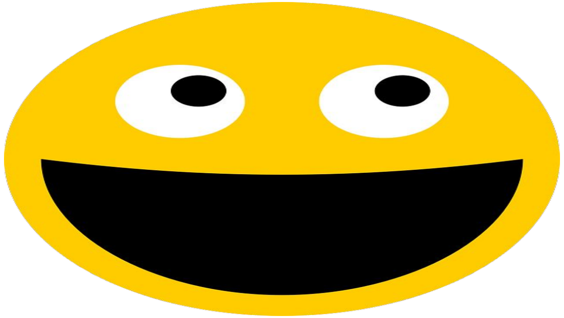


$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

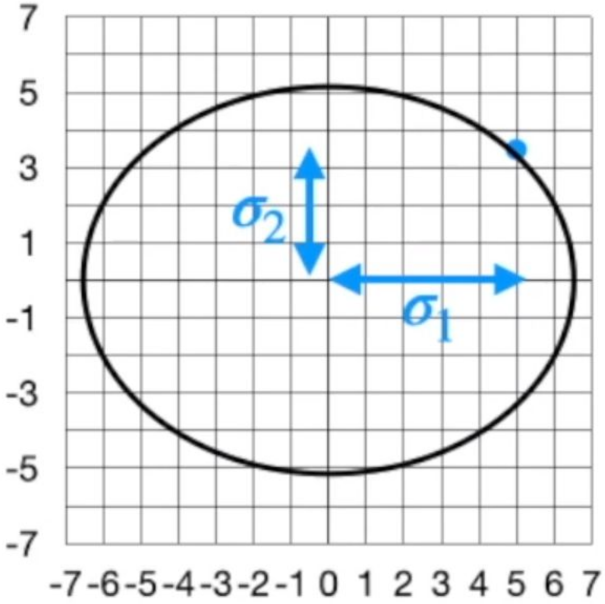
Matrices de estiramiento



$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$



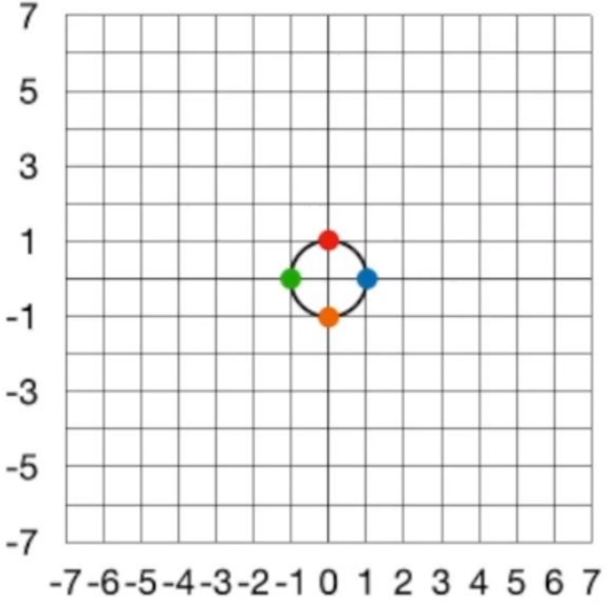
Matrices de estiramiento



$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

Qué tiene que ver esto con matrices?

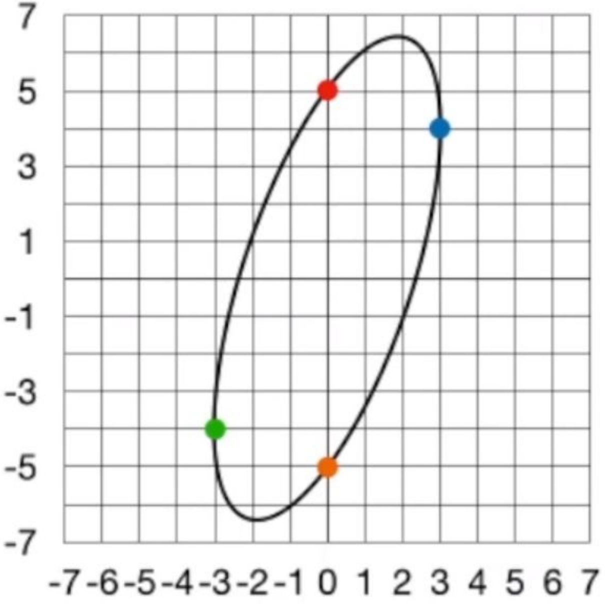
$$A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$$

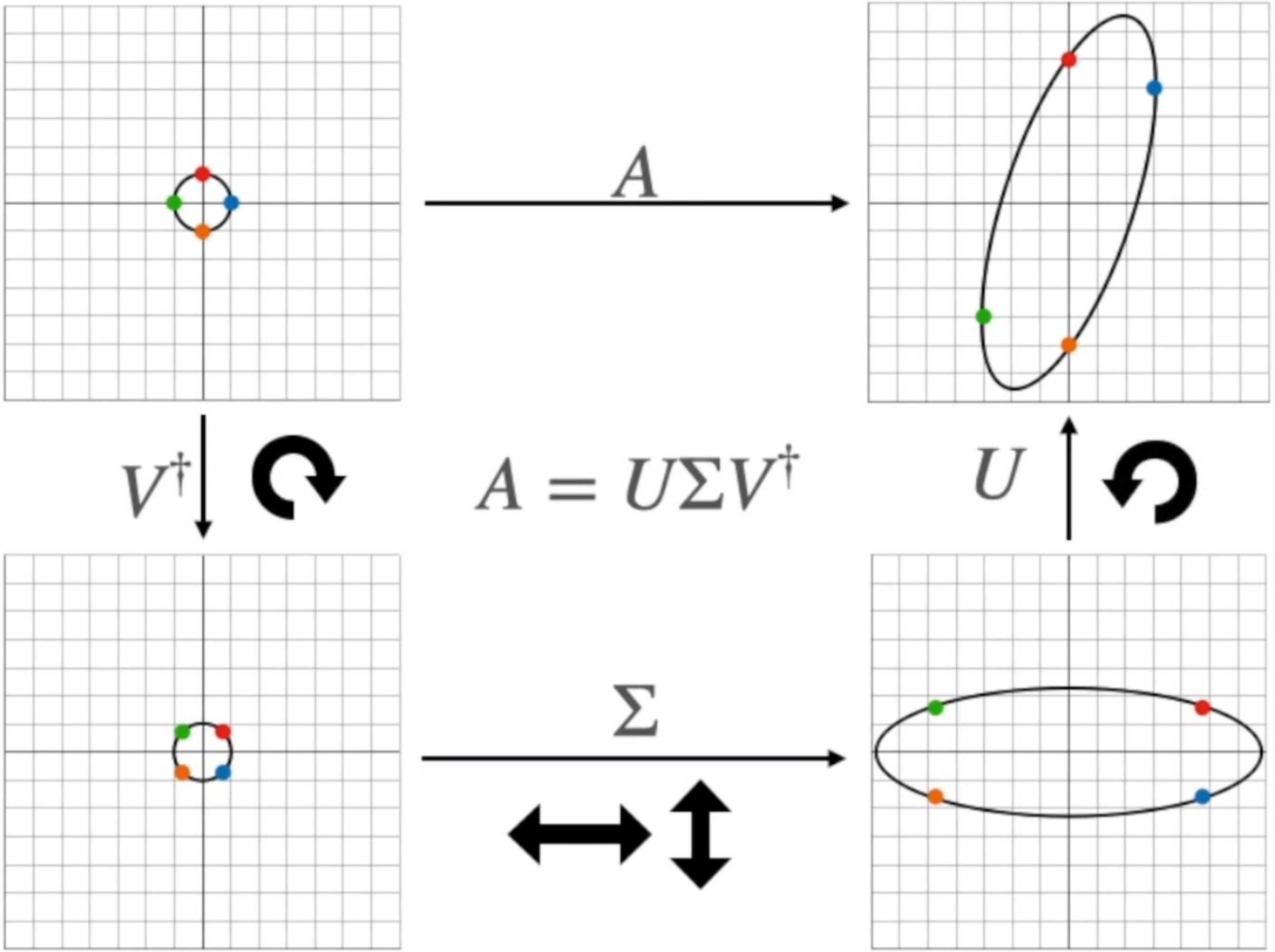


$$\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

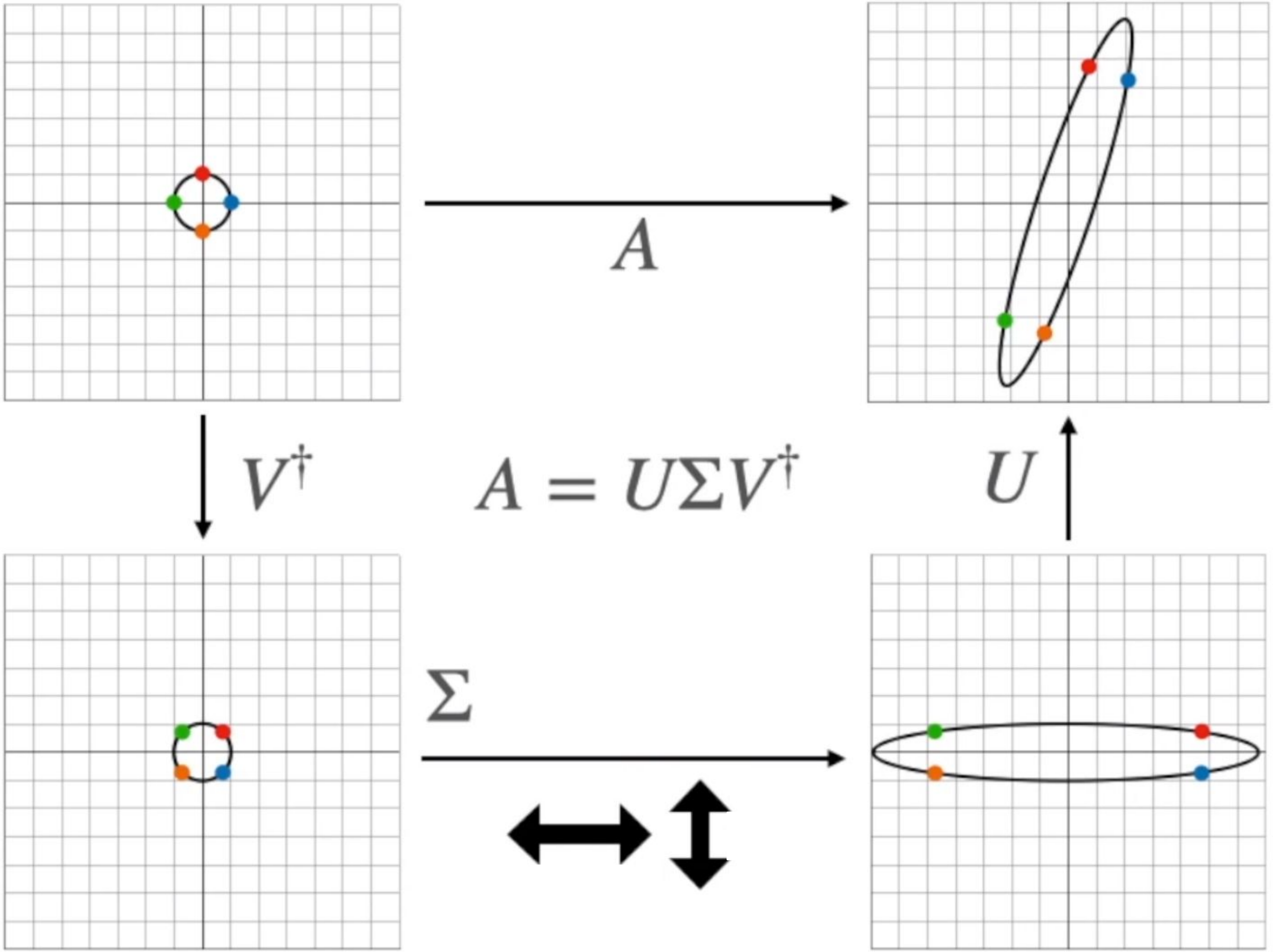
$$\begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix}$$

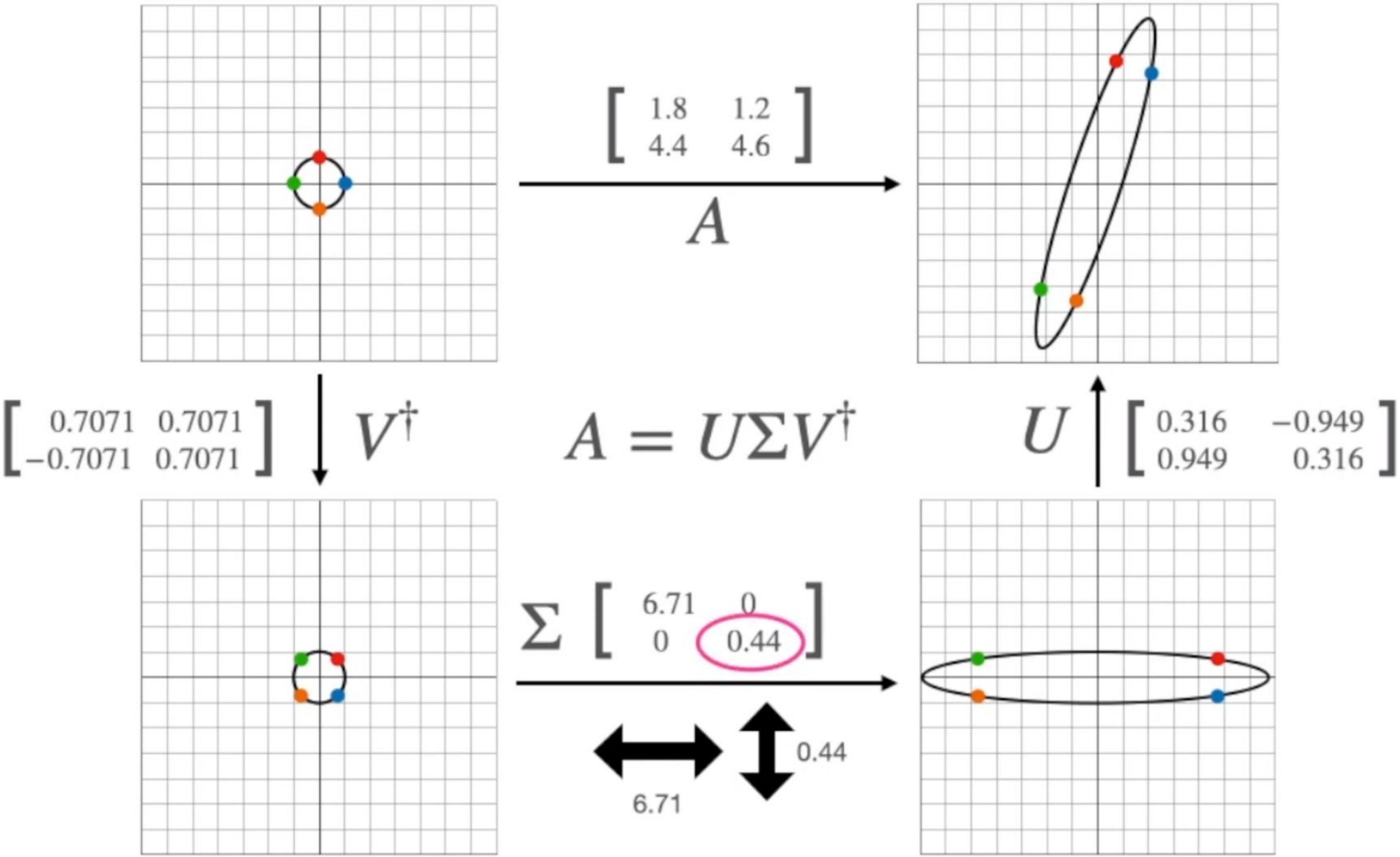




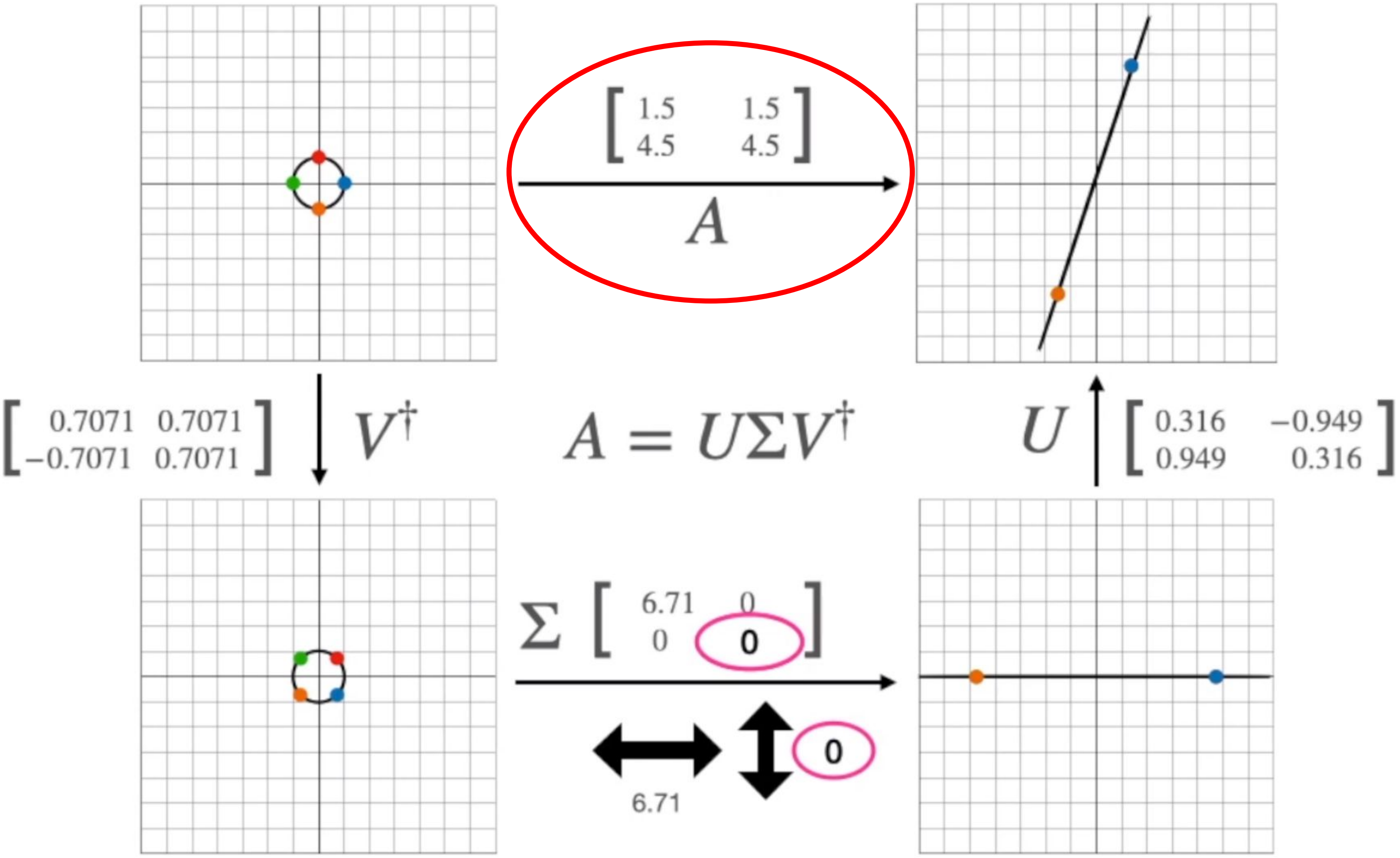
A
$V^\dagger$
$A = U\Sigma V^\dagger$
U
Σ

Simplificación de matrices
(Reducción de dimensionalidad)

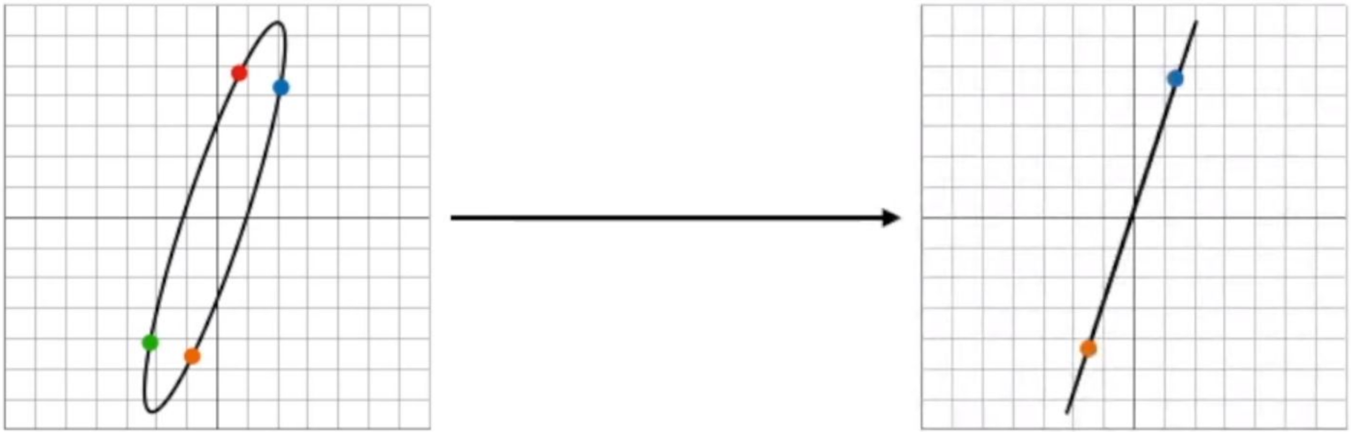

A
$V^\dagger$
$A = U\Sigma V^\dagger$
U
Σ



$\begin{bmatrix} 1.8 & 1.2 \\ 4.4 & 4.6 \end{bmatrix}$
A
$\begin{bmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}$
$V^\dagger$
$A = U\Sigma V^\dagger$
U
$\begin{bmatrix} 0.316 & -0.949 \\ 0.949 & 0.316 \end{bmatrix}$
Σ
$\begin{bmatrix} 6.71 & 0 \\ 0 & 0.44 \end{bmatrix}$
0.44
6.71



$\begin{bmatrix} 1.5 & 1.5 \\ 4.5 & 4.5 \end{bmatrix}$
A
$\begin{bmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}$
$V^\dagger$
$A = U\Sigma V^\dagger$
U
$\begin{bmatrix} 0.316 & -0.949 \\ 0.949 & 0.316 \end{bmatrix}$
Σ
$\begin{bmatrix} 6.71 & 0 \\ 0 & 0 \end{bmatrix}$
6.71
0



$$\begin{bmatrix} 1.8 & 1.2 \\ 4.4 & 4.6 \end{bmatrix} \qquad \begin{bmatrix} 1.5 & 1.5 \\ 4.5 & 4.5 \end{bmatrix}$$

Rango 2 | Rango 1

Cual de estas dos matrices es más simple?

1	2	3	4
-1	-2	-3	-4
2	4	6	8
10	20	30	40

3	1	4	1
5	9	2	6
5	3	5	8
9	7	9	3

Matrices de rango 1

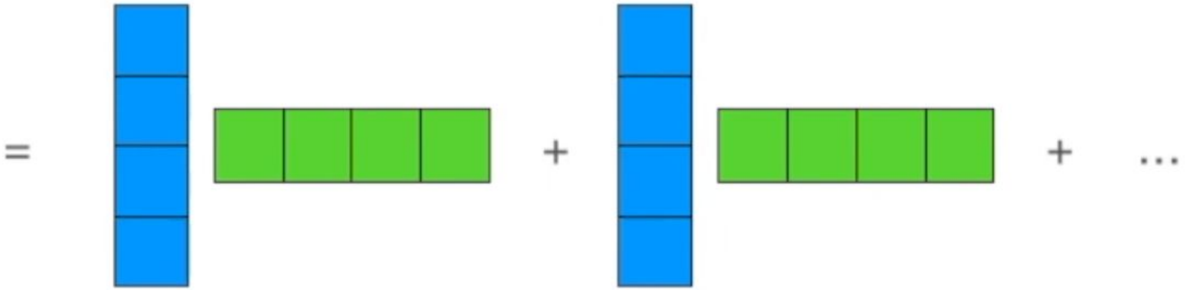
$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 2 & 3 & 4 \\ -1 & -1 & -2 & -3 & -4 \\ 2 & 2 & 4 & 6 & 8 \\ 10 & 10 & 20 & 30 & 40 \end{array} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 10 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

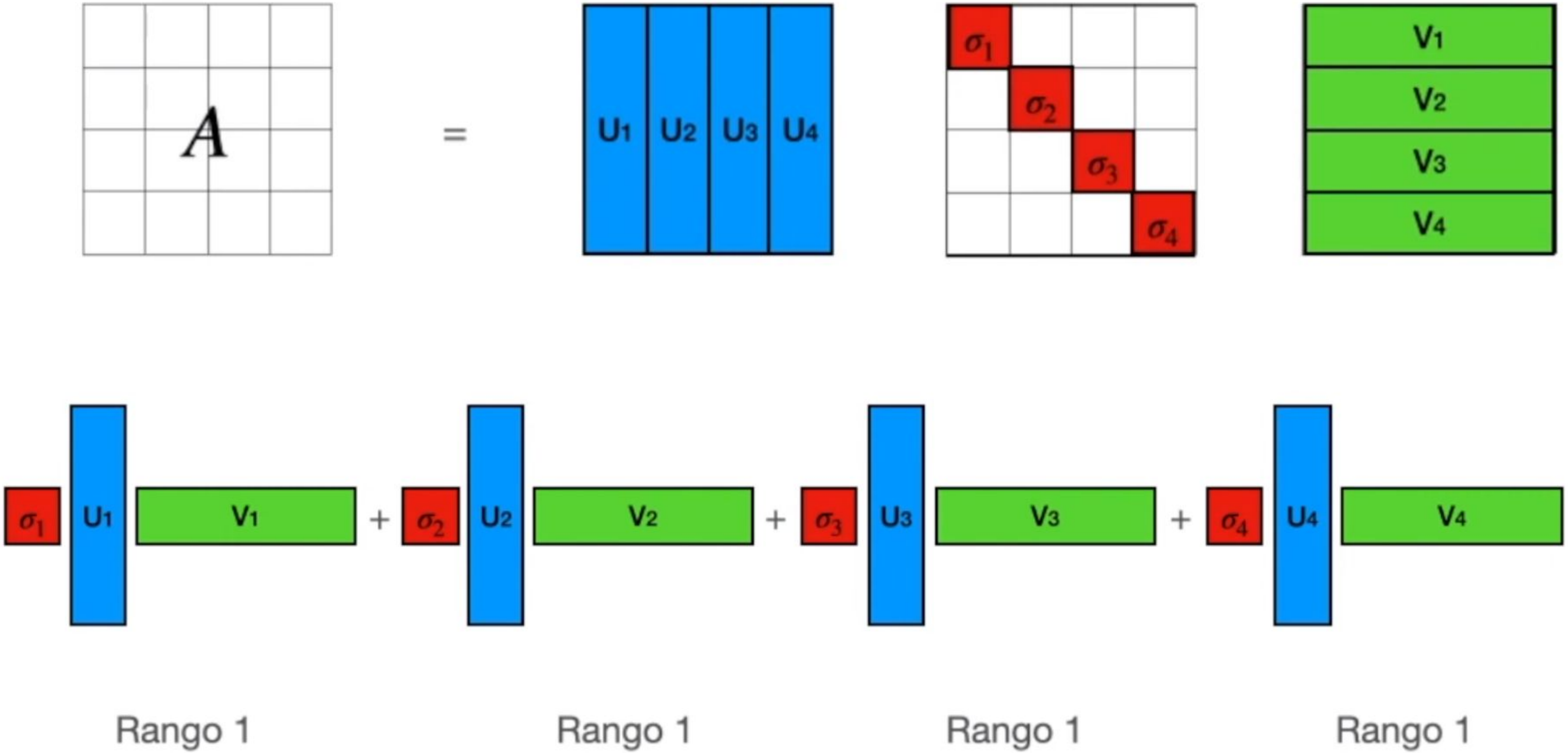
16 números

8 números

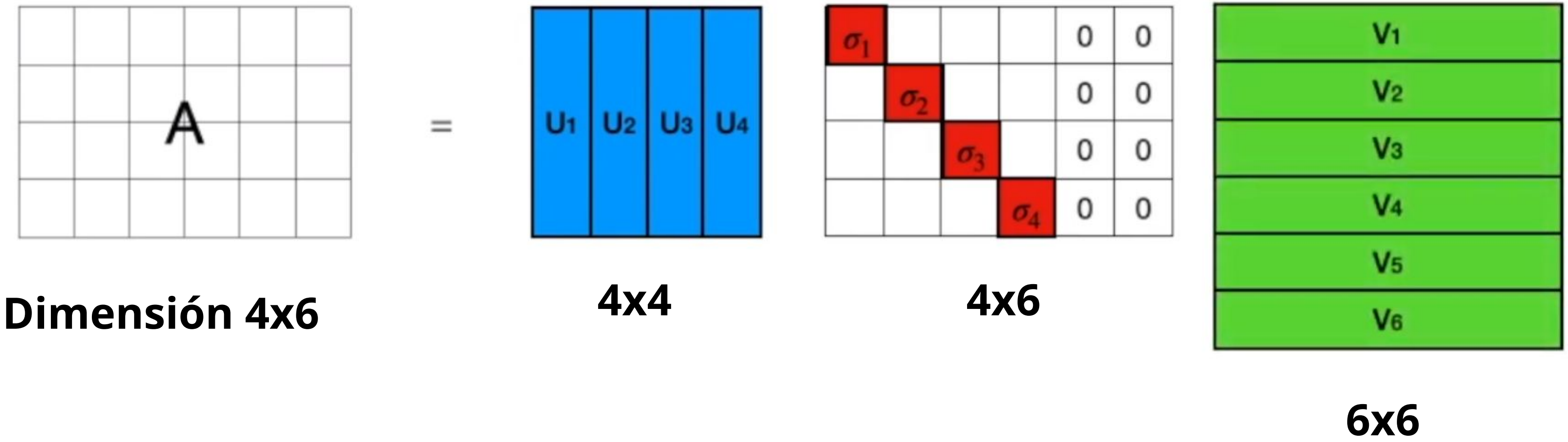
Aproximación usando matrices de rango 1

3	1	4	1
5	9	2	6
5	3	5	8
9	7	9	3

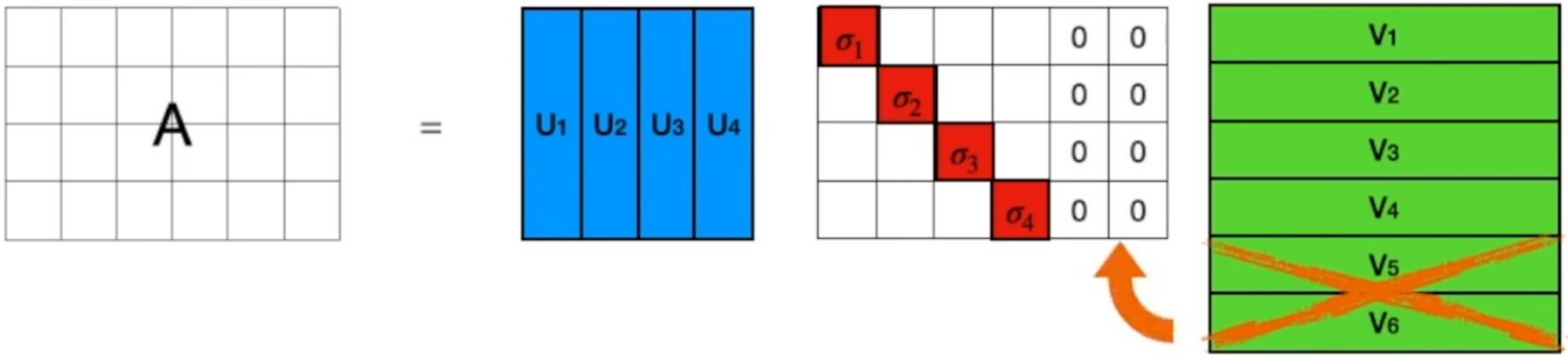



A
=
U1 U2 U3 U4
σ1 σ2 σ3 σ4
V1
V2
V3
V4
σ1 U1 V1 + σ2 U2 V2 + σ3 U3 V3 + σ4 U4 V4
Rango 1
Rango 1
Rango 1
Rango 1

La matriz no es cuadrada? No hay problema!



La matriz no es cuadrada? No hay problema!



Ejemplo Real

(Imagen en Python)