Determination of the Refractive Index of Glass (and other materials) using a Prism Spectrometer

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Lab Report – PY3107

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Determination of Refractive Indices using a Prism Spectrometer

1. Introduction

This report details the fourth practical attended on campus for PY3107. The experiment detailed throughout is ‘Sodium Light Diffraction from a Reflection Grating’.

1.1 Overview

The refractive index of a material is a measure of how light propagates through it, and generally it can be calculated by the ratio of the speed of light in a vacuum, *c*, to its speed in the propagating material, *v*.

$$n = \frac{c}{v}$$

An ordinary triangular prism (like the ones used throughout this report) can separate white light into its constituent colours, called a spectrum. Each colour, or wavelength, making up the light is refracted a different amount; the shorter wavelengths (those toward the violet end of the spectrum) are bent the most, and the longer wavelengths (those toward the red end of the spectrum) are bent the least.

In this experiment a prism is used in conjunction with a spectrometer to analyse the spectral components in sodium light and determine the refractive index of the prism(s’) material.

1.2 Theory

The refractive index of a material, as determined from the setup shown in Section 2.1, can be calculated using

$$n(\lambda) = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where *A* is the angle of the prism, and *D* is the minimum angle of deviation, as shown graphically in Figure 1.2.1. The angle between the beam emerging from the collimator of a spectrometer and the position of the telescope is referred to as the angle of deviation; it is a property of a prism under incident light (in this situation; an emergent beam from a collimator) that this angle goes through a minimum, hence, its apt name the ‘minimum angle of deviation’ (*D*).

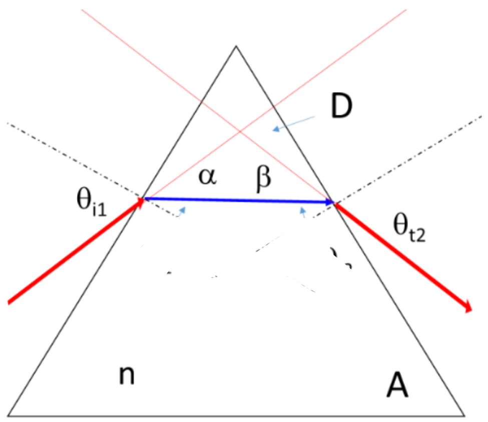

Fig 1.2.1 [1] | Modified diagram from lab manual | Geometrical setup as described above

Given the sprawling nature of the requirements as laid out by the lab manual, other details of theory will be addressed as they are encountered throughout the report (or in appendices) in order to keep this document a coherent read. The formula shown above is derived *in the lab manual*.

2. Experimental Methods

Section 2, *Experimental Methods*, details the procedure and experimental data produced from the experiment while providing brief discussion of reported data and fulfilling specified criteria of the lab brief.

2.1 Experimental Setup

The setup of the experiment consists of a spectrometer, various prisms depending on the measurement in question, a sodium lamp and a PC. The PC was wired up to a camera which was positioned at the end of the viewing end of the telescope, allowing pictures to be obtained of the refracted/reflected components of the sodium lamp. An image of the setup, identical to that used to collect the data shown in this report, is shown in Figure 2.0.1. A spectrometer consists mainly of a collimator and a telescope. The collimator produces a parallel beam of light, which passes through the prism and is refracted. The telescope is then used to observe the refracted light, and the angle of deviation is measured using the spectrometers vernier scale.

Fig 2.0.1 [2] | Experimental Setup

2.2 Procedure

As a preliminary, before any measurements are taken the sodium lamp must be given sufficient time to heat up and any zero-point errors on the spectrometers vernier must be accounted for. Firstly, we measure the angle of the edges of the prism. This is achieved by shining a slit of the sodium light directly onto the edge of the prism. The beam will be reflected off both sides of the prism (each acting like a plane mirror) so two images of the slit can be observed. By measuring the angle between each reflection and dividing it in two we can get the angle, *A*, of the prism edge (Figure 2.2.1 shows the geometry of the situation described). In other words, with reference to Figure 2.2.1, we can say

$$A = \frac{\theta_{position\ 1} + \theta_{position\ 2}}{2}.$$

This is proved in *Appendix 1*, as per the lab manual.

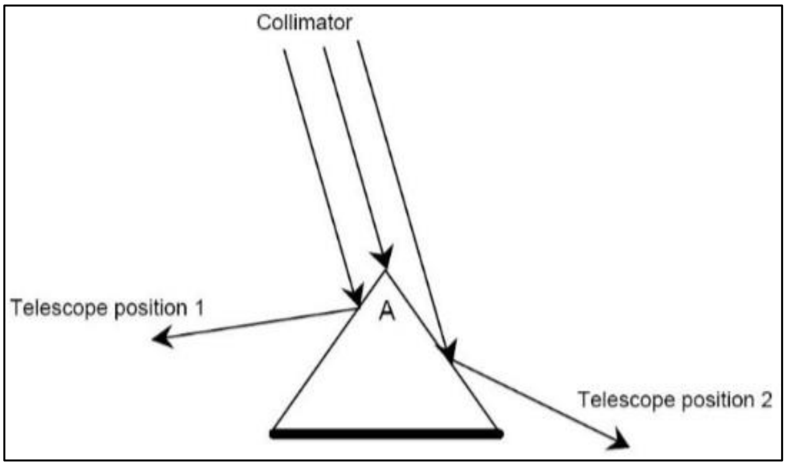

Fig 2.2.1 [1] | Beam split by prism edge

The minimum angle of deviation, *D*, is required also to obtain the refractive index of the material in question as per the formula shown at the beginning of Section 1.2. The angle of deviation is at its minimum when, on viewing the refracted rays through the camera and rotating both the prism and telescope (keeping the refracted rays in view), the images as viewed suddenly slow to a halt and change their direction of projected motion. That is, the projected beams emergent from the prism begin to move opposite in direction relative to the table's continuous rotation. The minimum angle of deviation is precisely the angle at which the image is stationary. This occurs for all incident sides of the prism; so by rotating the prism 130° a minimum angle of deviation can be found at which the telescope is positioned on the opposite side of the spectrometer (with respect to the collimators incident path - See Figure 2.2.2).

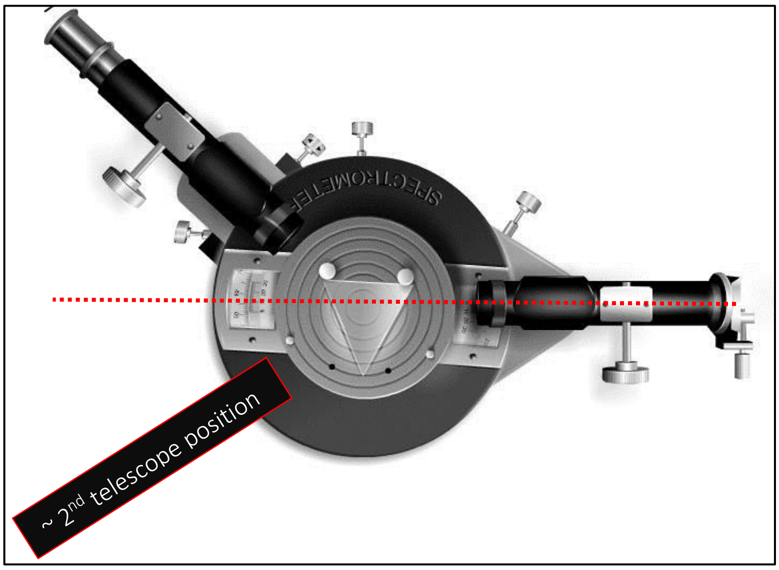

Fig 2.2.2 [3] | Telescope position (post-rotation) | Annotated Image, See reference for source.

This gives us two angles from the vernier which can be corrected (zero-point error) to obtain D_{LHS} and D_{RHS}. From these we obtain the relevant D_{min} from realising that

$$D_{min} = \frac{D_{LHS} + D_{RHS}}{2}^{[1]} \quad or \quad \frac{D_{ver\,[1]} + D_{ver\,[2]}}{2} \quad \textit{(As presented in Tab 2.3.3)}$$

which provides us with all of the information required to determine the refractive index of the given material (this relation is derived in detail in the PY3107 Lab Manual – copying the derivation step by step is of no benefit).

2.3 Experimental Data

θ_0 deg	θ_0 rad
353.03	6.1615359
±0.005	±0.00009

Tab 2.3.1 | Zero point measurements for spectrometer

Glass		θ_{ver} deg	$\theta_{corrected}$ deg	$\theta_{corrected}$ rad	$\Delta\theta_{corrected}$ rad	A rad	A deg
1	**reflected (right)**	290.012	63.018	1.0998716	2.1034359	1.051718	60.259
	reflected (left)	50.53	57.5	1.0035643			
2	**reflected (right)**	285.073	67.957	1.1860734	2.0992297	1.0496148	60.1385
	reflected (left)	45.35	52.32	0.9131563			
3	**reflected (right)**	289.042	63.988	1.1168013	2.1130352	1.0565176	60.534
	reflected (left)	50.11	57.08	0.9962339			
		±0.005	±0.005	±0.00009	±0.00009	±0.00009	

Tab 2.3.2 | Determination of angle A under various incident angles (See Figure 2.2.1)

Material		D_{ver} deg	ΔD_{ver} deg	D_{min} deg	D_{min} rad	n au
Glass	*1*	309.04	88.48	44.24	0.772133661	1.575205901
	2	37.52				
Water	*1*	17.04	37.54	18.77	0.327598301	1.267587864
	2	339.53				
Ethanol	*1*	327.04	51.48	25.74	0.449247749	1.358668537
	2	18.52				
		±0.005	±0.005	±0.005	±0.00009	

Tab 2.3.3 | Raw readings and initial calculations from spreadsheet model for refractive indices of materials

NOTE: The angle 'A' was not calculated for the liquid filled prisms in Tab 2.3.3 (the same prism was used for measurements of ethanol and water). This task, with the setup provided, is likely impossible for the allotted time of 3 hours. The value used was as calculated in Tab 2.3.2 (mean value). Please see Section 3 for further discussion.

2.4 Calculation / Error Analysis

A (glass prism) | From Tab 2.3.2:

$$\Delta A = 0.005° = 0.00009 \text{ rad } \textit{(known)}$$

$$\bar{A} = \frac{60.259° + 60.1385° + 60.534°}{3} = 60.3105° = 1.05262 \text{ rad}$$

$$\therefore A = (60.3105 \pm 0.005)° \equiv (1.05262 \pm 0.00009) \text{ rad}$$

$$n(\lambda) = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad \frac{\Delta n}{n} = \Delta A\left(\frac{\cos\left(\frac{A}{2}\right)}{2\sin\left(\frac{A}{2}\right)} + \frac{\cos\left(\frac{A+D}{2}\right)\left(1+\frac{D}{A}\right)}{\sin\left(\frac{A+D}{2}\right)}\right)$$

(See Appendix 3 for derivation)

$$\Delta A \equiv \Delta D = 0.005° = 0.00009\, rad$$

$n(\lambda)_{glass}$| From Tab 2.3.3:

$$n(\lambda)_{glass} = \frac{sin\left(\frac{44.24° + 60.3105°}{2}\right)}{\sin\left(\frac{18.77°}{2}\right)} = 1.575205901$$

$$\frac{\Delta n(\lambda)_{glass}}{n(\lambda)_{glass}} = \Delta A\left(\frac{\cos\left(\frac{44.24°}{2}\right)}{2\sin\left(\frac{44.24°}{2}\right)} + \frac{\cos\left(\frac{44.24° + 60.3105°}{2}\right)\left(1+\frac{44.24°}{60.3105°}\right)}{\sin\left(\frac{44.24° + 60.3105°}{2}\right)}\right)$$
$$= 0.000342114$$

$$\rightarrow \Delta n(\lambda)_{glass} = 0.0005389$$

$$\therefore n(\lambda)_{glass} = 1.575 \pm 0.001$$

$n(\lambda)_{water}$| From Tab 2.3.3:

$$n(\lambda)_{water} = \frac{sin\left(\frac{18.77° + 60.3105°}{2}\right)}{\sin\left(\frac{44.24°}{2}\right)} = 1.267587864$$

$$\frac{\Delta n(\lambda)_{water}}{n(\lambda)_{water}} = \Delta A\left(\frac{\cos\left(\frac{18.77°}{2}\right)}{2\sin\left(\frac{18.77°}{2}\right)} + \frac{\cos\left(\frac{18.77° + 60.3105°}{2}\right)\left(1+\frac{18.77°}{60.3105°}\right)}{\sin\left(\frac{18.77° + 60.3105°}{2}\right)}\right)$$
$$= 0.000687484$$

$$\rightarrow \Delta n(\lambda)_{water} = 0.000871446$$

$$\therefore n(\lambda)_{water} = 1.268 \pm 0.001$$

$n(\lambda)_{ethanol}|$ From Tab 2.3.3:

$$n(\lambda)_{ethanol} = \frac{sin\left(\frac{25.74° + 60.3105°}{2}\right)}{\sin\left(\frac{25.74°}{2}\right)} = 1.358668537$$

$$\frac{\Delta n(\lambda)_{ethanol}}{n(\lambda)_{ethanol}} = \Delta A\left(\frac{\cos\left(\frac{25.74°}{2}\right)}{2\sin\left(\frac{25.74°}{2}\right)} + \frac{\cos\left(\frac{25.74° + 60.3105°}{2}\right)\left(1 + \frac{25.74°}{60.3105°}\right)}{\sin\left(\frac{25.74° + 60.3105°}{2}\right)}\right)$$
$$= 0.000531491$$

$$\rightarrow \Delta n(\lambda)_{ethanol} = 0.00072212$$

$$\therefore n(\lambda)_{ethanol} = 1.359 \pm 0.001$$

2.5 Results

From the data of Section 2.3 and analysis of Section 2.4, we can conclude the following results:

$$A_{glass} = (60.3105 \pm 0.005)° \equiv (1.05262 \pm 0.00009)\text{ rad}$$

$$n(\lambda)_{glass} = 1.575 \pm 0.001$$

$$n(\lambda)_{water} = 1.2676 \pm 0.0004$$

$$n(\lambda)_{ethanol} = 1.359 \pm 0.001$$

2.6 Additional Experimental Work

The lab brief states; "*By plotting the refractive index calculated versus the corresponding wavelengths, it is possible to determine the dispersion curve n(λ) of the specific glass the given prism is made of.*" The dispersion curve should be proportional to the square of the inverse of wavelength, according to Cauchy's transmission equation

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots$$

when approximated to

$$n(\lambda) \approx A + \frac{B}{\lambda^2}.$$

This is a reasonable approximation to make because λ is typically on the scale of 10^{-9} (nano scale), rendering C onwards to be effectively just constants (with very little effect on the curves shape) and can be coupled into the determined value of *A*.

Doing this additional optional task produced the data and figures, complete with a fit Cauchy relation, as shown in Tables 2.7.1, 2.7.2 and Figures 2.7.1, 2.7.2.

Glass	θ_{ver} deg	$\theta_{corrected}$ deg	$\theta_{corrected}$ rad	n au	λ nm	Cauchy Fit au
blue 1	308.08	44.95	0.784525499	1.582732635	498	1.58E+00
blue 2	308.26	44.77	0.781383906	1.580830187	515	1.58E+00
green	308.73	44.3	0.773180859	1.575844306	568	1.58E+00
yellow	308.9	44.13	0.770213799	1.57403437	589.3	1.57E+00
red	309.06	43.97	0.767421272	1.572327735	615	1.57E+00
infrared 1	309.67	43.36	0.756774764	1.5657931	768	1.57E+00
infrared 2	309.77	43.26	0.755029434	1.564717611	819	1.56E+00

Tab 2.7.1 | Additional Experimental Work |

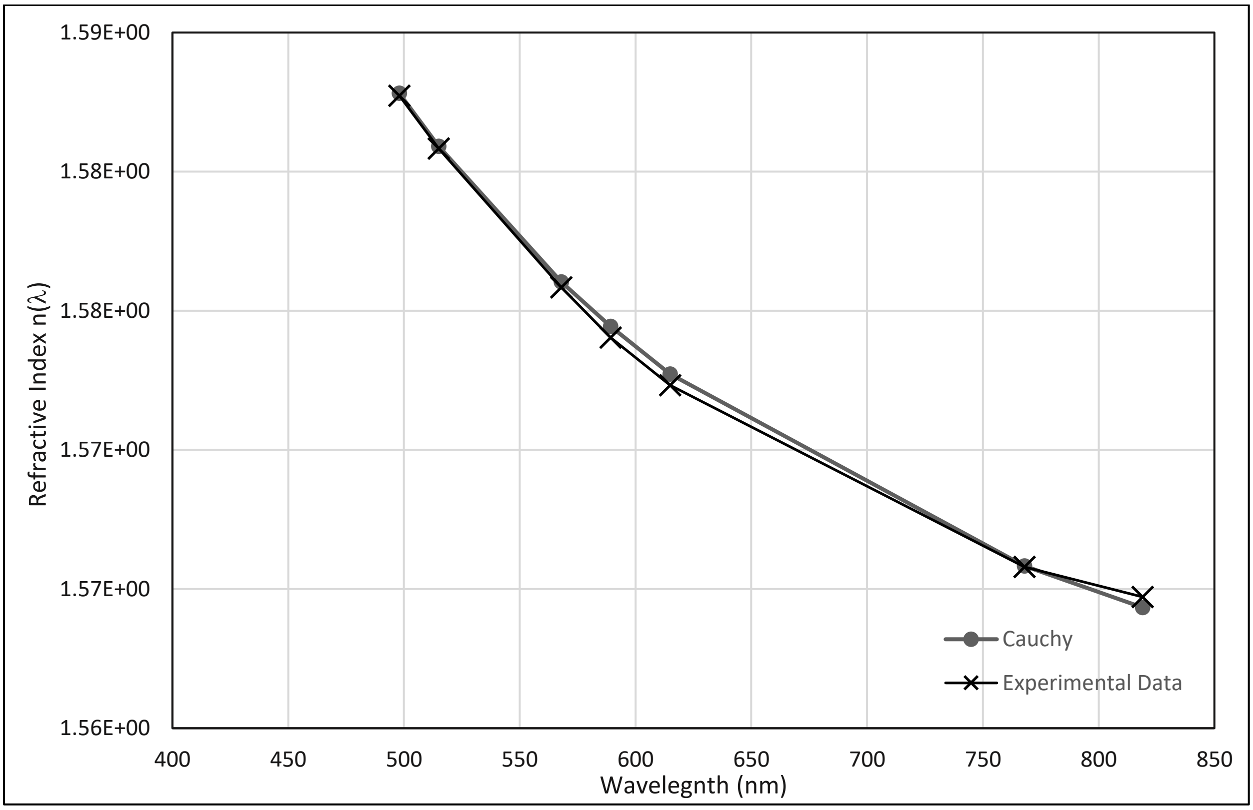

Fig 2.7.1 | Additional Experimental Work | n(λ) Vs. Wavelength (nm) | Includes Cauchy fit of data with corresponding parameters given in Table 2.7.2 and experimentally determined data of which the function was fitted to.

A au	B au
1.5535	7.27E+03

Tab 2.7.2 | Cauchy fit parameters used for data shown in Fig 2.7.1

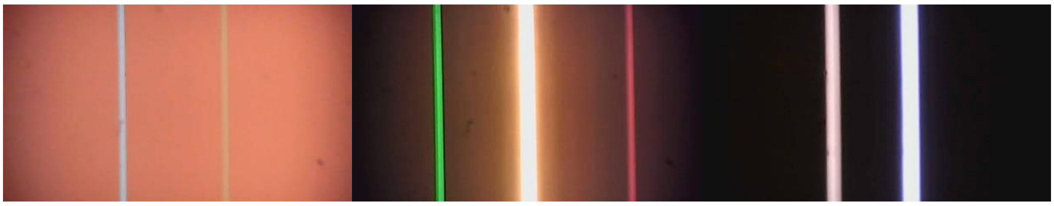

Fig 2.7.2 | Reconstructed spectrum as captured using the camera

3 Discussion

I personally would consider this experiment to be a success. The experiment was stressful and, at times, exhausting.

My first point of discussion will be the calculation of angle *A* for the liquid filled prisms. This was, in my opinion, simply not possible in the allotted time. This may not be the case at other stations with different prisms, however for the prisms that were provided to us on the day I do believe this to be impossible. The prisms for the liquids were large and their glass shell/walls thick and weathered, so much so that it did not seem that the joined edges (corners of the prism) could reflect the light due to other optical effects. From what I could tell, unless the prism were aligned precisely head on with an optimal beam width, the light from the sodium lamp would just disperse into the glass casing and be lost. Only mere 'smudges' of interference could be located. After measuring the prism's corner angles with a protractor, it was obvious that the triangular plan was equilateral (all corners read ~60° to less than 1°). As such, it was well more than plausible to assume *A* to be what was determined for the initial glass prism throughout the remainder of the experiment.

From literature (see references), the following results for refractive indices can be found:

$$n(\lambda)_{glass} = 1.52$$ [5]

$$n(\lambda)_{water} = 1.333 \quad (20°\text{C})$$ [4]

$$n(\lambda)_{ethanol} = 1.361 \quad (20°\text{C})$$ [4]

In comparison to the results determined in this report, I am satisfied with the values obtained. The only discrepancy between our results and the results above is the determined index of water. Having completed other spectroscopy experiments for PY3107, I am likely to conclude that this down to error reading the vernier scales of the spectrometer (systematic human error), as both of our other results are accurate to a relative error of less than 0.01%. This also could have stemmed from the fact that we did not experimentally calculate the prism angle *A*, however, I have already discussed the difficulties faced there.

I find it difficult to conclude much else from these results and the main experiment overall, other than that the method detailed and exemplified throughout this document for determining the refractive indices of various materials can be accurate to a very high degree.

In terms of the additional activity, judging by the comparison between the Cauchy Fit and the experimental data in Figure 2.7.1, the dispersion curve of sodium light is in fact proportional to the square of the inverse of wavelength. I am very satisfied with this.

I am uncertain only in the error analysis, as I was acutely unsure of the approach to take for this experiment. I was surprised by the calculated errors and how small they seemed, however, the results *(excluding the refractive index of water)* are identical to those seen in literature, so the analysis may in fact be valid.

References

[1] : UCC Department of Physics PY3107 Laboratory Manual

[2] : UCC Department of Physics website, Source: https://www.ucc.ie/en/physics/study/undergraduate/thelaboratories/thirdyearphysicslab/determinationofrefractiveindexusingaprism/

[3] : Image Source: https://vlab.amrita.edu/?sub=1&brch=281&sim=1513&cnt=2

[4] : Zajac, Alfred; Hecht, Eugene (18 March 2003). Optics, Fourth Edit. Pearson Higher Education.

[5] : "High temperature glass melt property database for process modeling"; Eds.: Thomas P. Seward III and Terese Vascott; The American Ceramic Society, Westerville, Ohio, 2005

Appendices

1. **Proof that $A = \frac{\theta_{position\ 1} + \theta_{position\ 2}}{2}$**

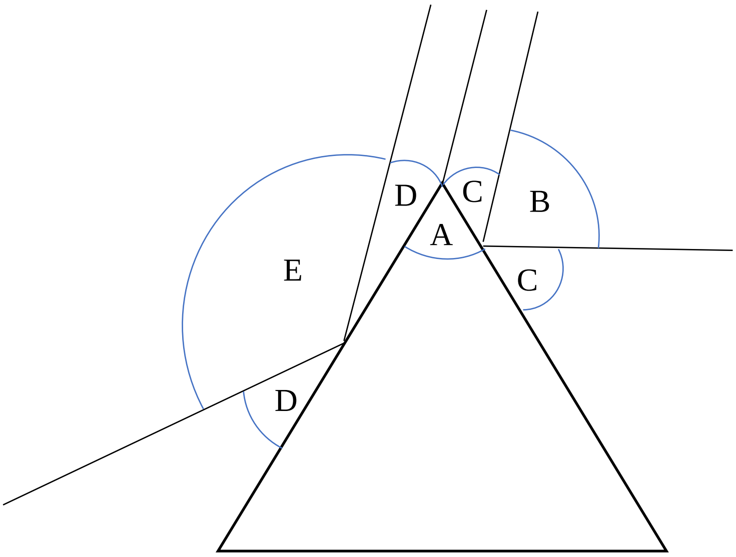

Fig A.1

Consider Figure A.1

From Figure A.1 we can qualitatively conclude the following:

$$2C + B = 180°$$

$$2D + E = 180°$$

$$\therefore 2D + 2C + B + E = 360°$$

$$and\ also;\ A + B + C + D + E = 360°$$

$$\rightarrow A + B + C + D + E = 2D + 2C + B + E$$

$$A = C + D$$

The angle between the telescope positions in terms of Fig A.1 would be

$$A + C + D$$

which from the previous term we can evaluate to be

$$(C + D) + C + D = 2C + 2D = 2A$$

QED

2. Derivation of $\frac{\Delta n}{n}$

$$\frac{\Delta n}{n} = \frac{\Delta sin\left(\frac{A+D}{2}\right)}{sin\left(\frac{A+D}{2}\right)} + \frac{\Delta \sin\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where

$$\frac{\Delta \sin\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\cos\left(\frac{A}{2}\right)}{2\sin\left(\frac{A}{2}\right)}\Delta A$$

and

$$\frac{\Delta \sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A+D}{2}\right)} = \frac{\cos\left(\frac{A+D}{2}\right)\Delta\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A+D}{2}\right)} \equiv \frac{\cos\left(\frac{A+D}{2}\right)\left(\Delta A + \frac{(\Delta A)D}{A}\right)}{\sin\left(\frac{A+D}{2}\right)} \; since\ \Delta A \equiv \Delta D$$

where

$$\frac{\Delta\left(\frac{A+D}{2}\right)}{\left(\frac{A+D}{2}\right)} = \frac{\Delta A}{A} + \frac{\Delta D}{D} = 2\frac{\Delta A}{A} \quad since\ \Delta A \equiv \Delta D$$

$$\therefore \Delta\left(\frac{A+D}{2}\right) = 2\frac{\Delta A}{A}\left(\frac{A+D}{2}\right) = \Delta A + \frac{(\Delta A)D}{A}$$

therefore, allowing us to conclude that

$$\frac{\Delta n}{n} = \Delta A\left(\frac{\cos\left(\frac{A}{2}\right)}{2\sin\left(\frac{A}{2}\right)} + \frac{\cos\left(\frac{A+D}{2}\right)\left(1+\frac{D}{A}\right)}{\sin\left(\frac{A+D}{2}\right)}\right)$$