Heat Conduction in Sand

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Lab Report – PY3107

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February 2023

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1. Introduction

This report details the first practical attended on campus for PY3107. The experiment detailed throughout is (one of five campus-based experiments) 'Heat Conduction of Sand'.

1.1 Overview

Conduction is one of three common fundamental mechanisms of heat transfer (*Convection*, *Radiation*, and *Conduction*). Heat conduction is a process in which thermal energy is transferred from one point to another within a solid material, without any net movement of the material itself. The objective of this experiment is to determine the rate of heat transfer through a sand sample by experimentally determining the *heat conductivity coefficient* of the sand sample, k_h.

1.2 Theory

"Heat has dimensions of energy [J], but is not a state variable"[1] Heat is unlike temperature in the fact that it has a dependence on the past history of the system. Classical Thermodynamics states:

$$dQ = mc_p dT$$

where dQ is the increase of heat, dT is the increase of temperature, m is the affected mass of specific heat capacity c_p. Heat transfer by means of conductivity is caused by local temperature differences in a material (*"Temperature Gradient, ∇T"*), where heat flows from areas of higher to lower temperature. According to Fourier's Law, heat flux q'' is proportional to the temperature difference by a factor k_h, the conductivity coefficient.

$$q'' = -k_h \nabla T$$

For this experiment, it was a necessity to ensure that the environment from which data would be collected was in *thermal equilibrium*, i.e. the system is receiving heat at the same rate at which it is expelling it. This was to ensure that variables within the environment remained consistent throughout measurement (aptly, the temperature at any given point within the experimental setup).

$$\frac{dT}{dt} = 0$$

A method of achieving this condition is by simply heating the material for an extended period of time until thermal equilibrium is achieved. In this case, this was done using an electrical heating element placed in the centre of the sand sample, which was contained within an insulated cylindrical bucket. Assuming that the entire electrical energy is converted to heat, the amount of energy transformed into heat (per unit time) is characterised by

$$P = UI$$

where U is the driving voltage and I is the current through the heating element. Under these conditions we can briefly conclude:

$$dQ = mc_p dT \rightarrow \frac{dQ}{dt} = mc_p \frac{dT}{dt} = 0, \quad as \ \frac{dT}{dt} = 0$$

where $\frac{dQ}{dt}$ is the heat flux of the system. The system should theoretically be in steady state. The geometry of the system has a major impact on the temperature gradient. Fig 1.2.1 visualises the temperature gradient of a battery during use. This is a highly complicated geometrical configuration that would not be suitable for this experiment, but demonstrates well the concept and issue at hand.

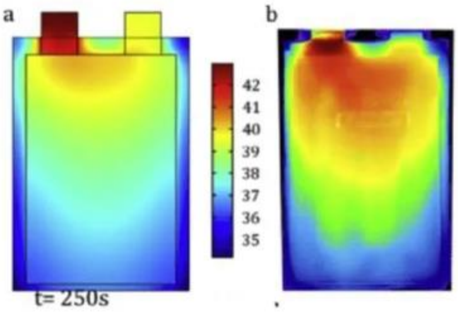


Fig 1.2.1 [2] - Surface temperature evolution of a pouch cell during 5C constant current discharge obtained by a) simulation and b) measurement. See Reference [2] for source.

The heating elements in the lab are cylindrical in design and should in theory expel heat into the environment radially. The obvious solution is to use a cylindrical environment (i.e. a bucket) meaning the spatial dependence of the temperature in cylindrical co-ordinates is given by

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

Which has a solution

$$T(r) = c_1 \ln(r) + c_2$$

with boundary conditions:

$T = T_i$ at $r = r_i$ $\quad$ $T = T_0$ at $r = r_0$

Since we are working with cylindrical geometry we can assume that $\nabla T = \frac{\partial T}{\partial r}$ and conclude that

$$P = q = Aq'' = (2\pi r L) k_h \frac{\partial T}{\partial r} = 2\pi L k_h c_1$$

$Proof$: $\frac{dT}{dr} = \frac{c_1}{r}$

$$\frac{d^2T}{dr^2} = \frac{d}{dr}\frac{c_1}{r} = -\frac{c_1}{r^2}$$

$$\therefore \frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = -\frac{c_1}{r^2} + \frac{1}{r}\frac{c_1}{r}$$

$$= -\frac{c_1}{r^2} + \frac{c_1}{r^2} = 0$$

Using the above we can finally determine the heat conductivity coefficient as

$$k_h = -\frac{P}{2\pi L c_1}$$

where c_1 is the slope of *T(r) Vs. ln(r)* to be determined experimentally.

This cylindrical setup is excellent because heat, like potentials from a point source, expel/attract or repel radially. We should not have issues of the like shown in Fig 1.2.1.

2. Experimental Methods

Section 2, *Experimental Methods,* details procedure and experimental data produced from the experiment while providing brief discussion of reported data and fulfilling specified criteria of the lab brief.

The experiment was set up as shown in Figure 2.0.1 for both Part 1 and Part 2.

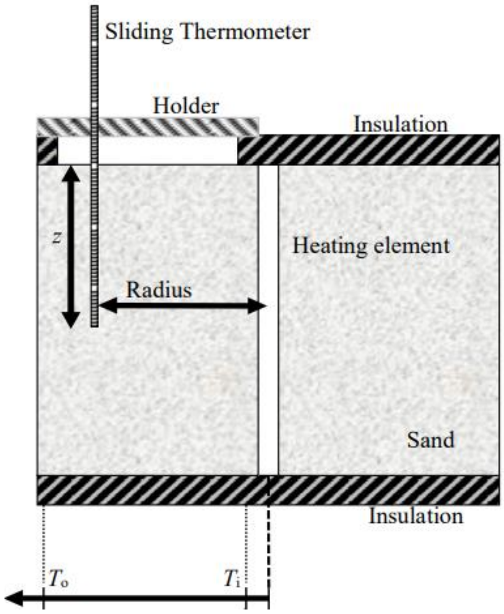


Fig 2.0.1 [1] – Experimental setup – sand contained in a standard bucket

2.1 Part I – Axial Temperature Profile Measurement

In Part I, a number of axial temperature profiles were recorded in preparation for Part II. This is because the bucket and heating element are not infinitely long, and as such a suitable depth for radial profiling must be determined. This was done by: Inserting the thermometer into the bucket at an arbitrary distance from the centre, *r*, measuring the temperature at various points on the axial z-coordinate by pulling the thermometer out of the sand in steps of ~2/3cm (waiting at least 2 minutes before taking readings in any new position) and repeating for a number of radial co-ordinates.

By plotting *z* Vs. *T(r)*, or by qualitative analysis, a region of suitable cylindrical approximation can be determined.

2.1.1 Results

r m	z m	T °C	
0.05	0.235	27.2	±0.05
	0.205	33.4	±0.05
	0.175	38	±0.05
	0.145	42.6	±0.05
	0.115	45.7	±0.05
	0.085	46.8	±0.05

r m	z m	T °C	
0.08	0.235	23.6	±0.05
	0.205	24.5	±0.05
	0.175	28.5	±0.05
	0.145	29.2	±0.05
	0.115	29.7	±0.05
	0.085	30.8	±0.05

r m	z m	T °C	
0.11	0.235	21.6	±0.05
	0.205	21.5	±0.05
	0.175	22	±0.05
	0.145	22.8	±0.05
	0.115	23.5	±0.05
	0.085	23.4	±0.05

r m	z m	T °C	
0.13	0.145	21.3	±0.05
	0.115	21.4	±0.05
	0.085	21.6	±0.05

By brief qualitative analysis of the data, it is very evident that a suitable depth where the cylindrical approximation is evident is in the region of 0.115m to 0.085m.

For completeness sake, the data has been plotted in Fig 2.1.1, where identical conclusions can be drawn.

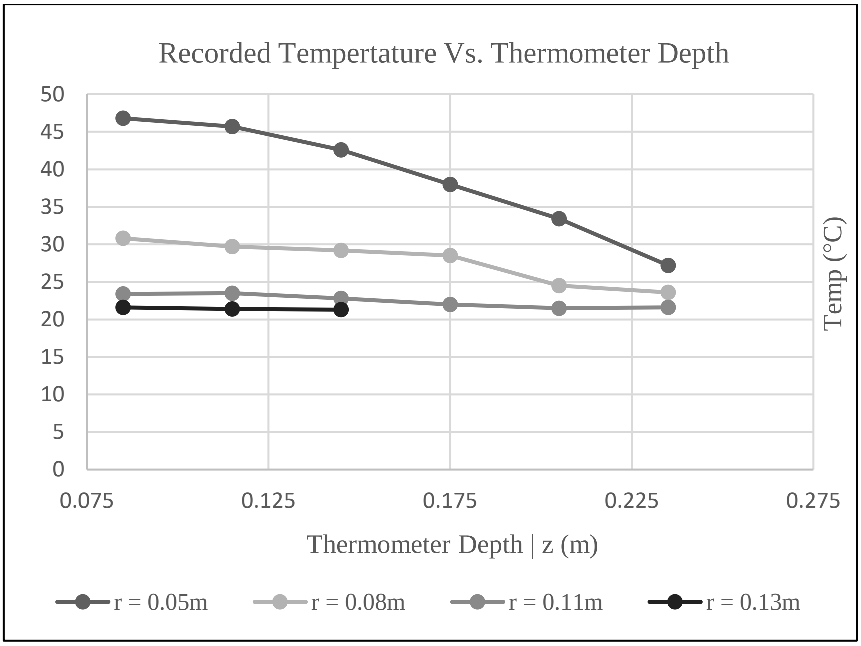


Fig 2.1.1 – Identical conclusions to that of the qualitative analysis can be drawn here. The plotted data has a subtle tendency to plateau at approximately z = 0.1m.

From brief analysis of the data, the thermometer depth chosen to complete Part II with was

$$z = 0.1m \ (10cm)$$

Considering that there are no major analytical conclusions to be drawn, it does not seem fitting to do an analysis of any error that may have impacted the final choice given the nature of the method and conclusion.

2.2 Part II – Radial Temperature Profile Measurement

In Part II, we determine the heat conductivity of the sand. This was done by: performing temperature readings of the sand at different radial distances from the centre rod at a single depth, *z*. In Part I, it was determined that, for this experimental setup, an approximate depth of *z* = 0.1m had no plausible deniability to be the optimal depth for measurement. As discussed in Section 1.2, the conductivity coefficient can be determined by plotting the results in the form *T(r) Vs. ln(r)* by recognising that

$$k_h = -\frac{P}{2\pi L c_1}$$

where c_1 is the slope of the produced data.

(Please turn over)

2.2.1 Results

z m	r cm	r m		T °C		ln(r) cm	ln(r) m
0.1	1.6	0.016	±0.0005	153.6	±0.5	0.470004	-4.1352
	2	0.02	±0.0005	108.4	±0.05	0.693147	-3.912
	5	0.05	±0.0005	59.7	±0.05	1.609438	-2.9957
	6	0.06	±0.0005	50	±0.05	1.791759	-2.8134
	7	0.07	±0.0005	41.9	±0.05	1.94591	-2.6593
	8	0.08	±0.0005	35.3	±0.05	2.079442	-2.5257
	9	0.09	±0.0005	31.5	±0.05	2.197225	-2.4079
	10	0.1	±0.0005	28.3	±0.05	2.302585	-2.3026
	11	0.11	±0.0005	25.9	±0.05	2.397895	-2.2073
	12	0.12	±0.0005	24.4	±0.05	2.484907	-2.1203
	13	0.13	±0.0005	23.4	±0.05	2.564949	-2.0402

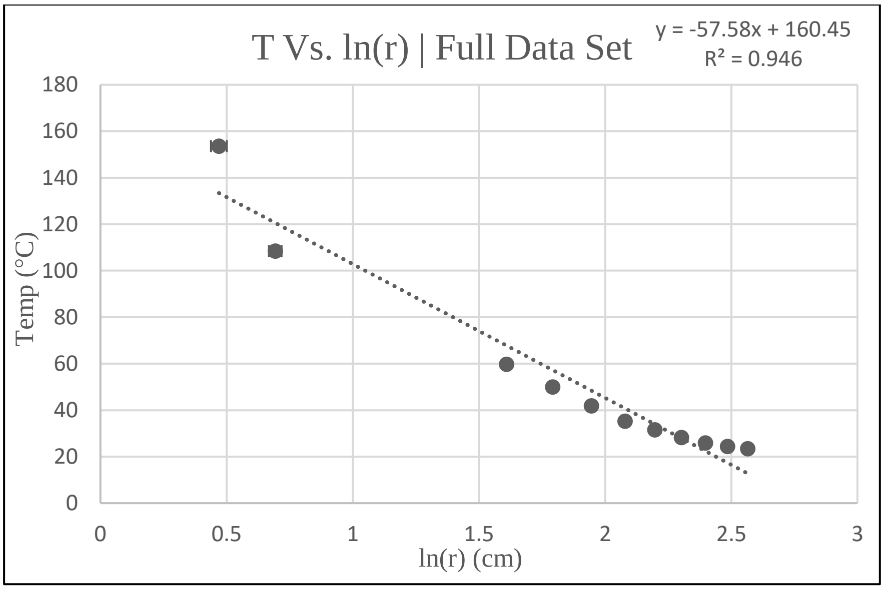


Fig 2.2.1 – Full Plotted Data Set (Inc. Error Bars + Trendline) - c_1 = -57.58

It is quite clear that the relationship is not quite linear, contrary to theory.

There is, however, evidently a number of data points that appear to be very well correlated approximately about the radius centre (i.e approximately half way between the bucket edge and heating element)

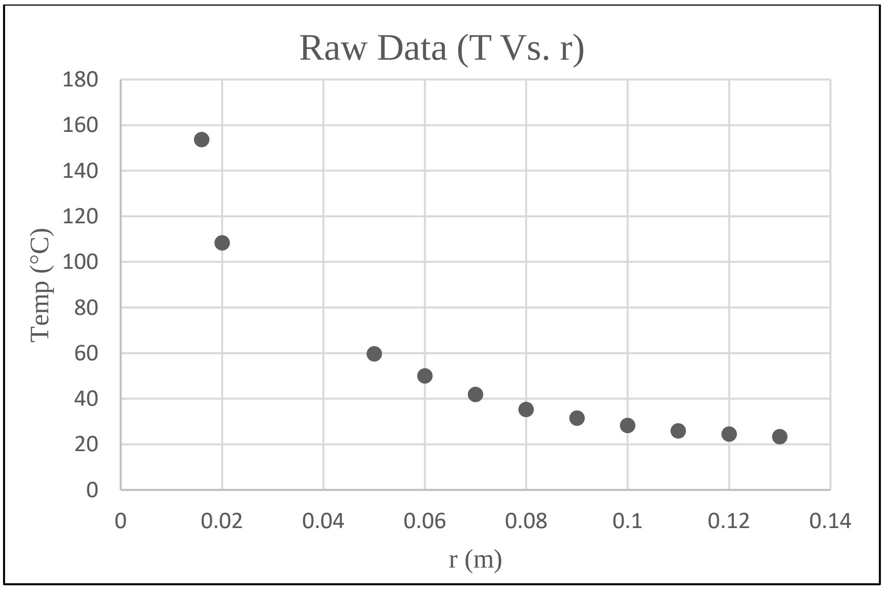
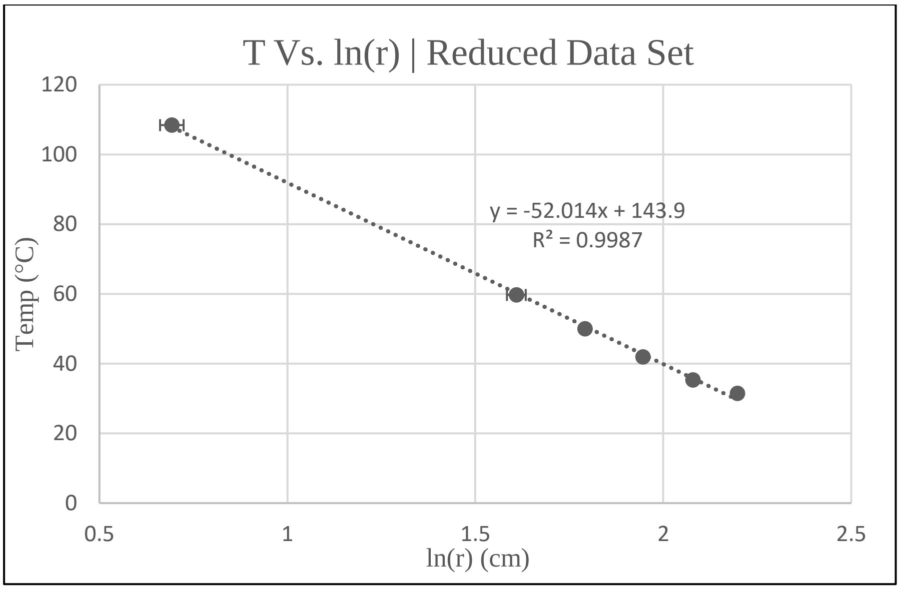


Fig 2.2.2 – Raw Plotted Data Set

This is better demonstrated in the raw experimental data shown Fig 2.2.2, and a reduced data set is shown in Fig 2.2.3.



There is plausible reason to believe that this subset of data showcases an area of the bucket that was in a true steady state, producing extremely well correlated data.

This is discussed in more detail in Section 3.

Fig 2.2.3 – Reduced Data set – c_1 = -52.014

Power Source: Voltage = 31.8V, Current = 0.88A

Heating Coil: Length (L) = 0.2m ± 0.0005m

2.2.2 Calculation/Error Analysis

Considering the result from the reduced data set:

$$k_h = -\frac{P}{2\pi L c_1} = -\frac{(31.8\,V)(0.88\,A)}{2\pi(0.2\,m)(-57.014\,Km^{-1})} = 0.428\,Wm^{-1}K^{-1}$$

$$\frac{\Delta k_h}{k_h} = \frac{\Delta c_1}{c_1} + \frac{\Delta L}{L} + \frac{\Delta U}{U} + \frac{\Delta I}{I} = \frac{0.95575}{52.014} + \frac{0.0005}{0.2} + \frac{0.05}{31.8} + \frac{0.005}{0.88} = 0.028129$$

$$\therefore k_h = 0.428 \ \pm 0.012\,Wm^{-1}K^{-1}$$

Considering the result from the full data set:

$$k_h = -\frac{P}{2\pi L c_1} = -\frac{(31.8\,V)(0.88\,A)}{2\pi(0.2\,m)(-57.54\,Km^{-1})} = 0.387\,Wm^{-1}K^{-1}$$

$$\frac{\Delta k_h}{k_h} = \frac{\Delta c_1}{c_1} + \frac{\Delta L}{L} + \frac{\Delta U}{U} + \frac{\Delta I}{I} = \frac{4.5835}{57.54} + \frac{0.0005}{0.2} + \frac{0.05}{31.8} + \frac{0.005}{0.88} = 0.08851$$

$$\therefore k_h = 0.387 \ \pm 0.03788\,Wm^{-1}K^{-1}$$

In both cases the uncertainty in the slope has been determined via an Excel algorithm.

3. Discussion

This was a very interesting experiment to complete. There are several points I would like to bring focus to for this discussion. Firstly, the results seem very realistic. The typical Thermal Conductivity of arbitrary dry sand can be found to be from 0.25 to 0.6 W/mK depending on where you look online. So overall, the result(s) is quite reflective of reality. I am more compelled to believe that the result of the reduced data set is a better reflection of the actual value. There a number of reasons for this. Firstly, upon arrival to the lab, the sand that was supplied was not sufficiently heated to a steady state. The lab brief states that the sand should be heated for upwards of 6 hours, but on this occasion the heating coil had been active for less than 1.5 hours. This meant that we had to wait additional period before beginning the experiment, with less time overall for completion. I feel that this has shown in the results negatively, particularly in the brief discussion in section 2.2.1, where there is clearly not a linear relationship seen between T and ln(r). There were several data points in the centre of the conduction region that correlated excellently. My theory is that this region was in fact in a true steady state, while the bucket edges and region around the heating element were not, possibly due to the insufficient heating time before arrival to the lab or due to poor insulation or inconsistencies in the sand. I more inclined to assume the former, as Fig 2.2.1 can be interpreted as having separated regions of the bucket. The data recorded closer to the heating does not match up with the expected steady state data, as with the data points from the edge of the bucket, where heat may have been continuously lost to the environment's surroundings. An additional possible cause of this could have been due to a drop in room temperature while conducting the experiment, disrupting the temperature gradient of the bucket, however this is less likely than the alternative possibilities given the discussion points previous.

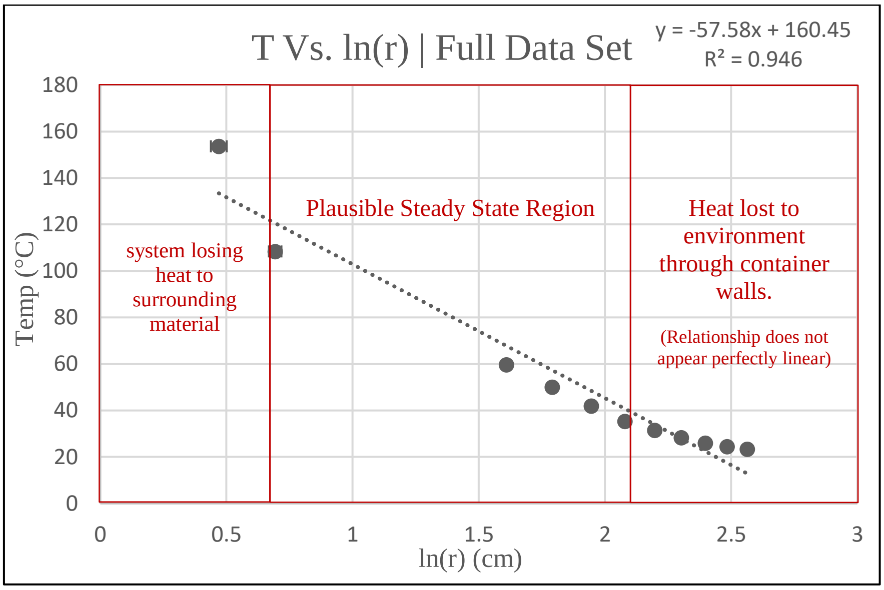


Fig 2.2.1 – Full Plotted Data Set (Annotated) - c_1 = -57.58

For this reason, 2 results are shown in section 2.2.2 for this final write up (one for each data set).

I am satisfied over all with the results and completion of the practical. I also feel that, even though the error analysis feels scarce, especially so for section 2.1.1 (where an analysis of error would have been a stretch for an approximate value of depth), there is quite little to draw any additional conclusions or analysis from, especially considering the seemingly positive final answer produced and the overlap in error margin between the two.

References

[1] : UCC Department of Physics PY3107 Lab Manual.

[2] : Figure 1.2.1 source: https://www.batterydesign.net/cell-temperature-gradient/