Sodium Light Diffraction from a Reflection Grating

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Lab Report – PY3107

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February 2023

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1. Introduction

This report details the second practical attended on campus for PY3107. The experiment detailed throughout is (one of five campus-based experiments) 'Sodium Light Diffraction from a Reflection Grating'.

1.1 Overview

The use of diffraction gratings to study the properties of light has been a fundamental technique in the field of optics for over a century. In this lab report, we investigate the separation between the two components of the yellow doublet in the optical spectrum of sodium using a reflection diffraction grating. Sodium, a common element in many everyday materials such as table salt, possesses a unique spectral signature that appears as a yellow doublet in its optical emission spectrum. The doublet is a result of the two closely spaced emission lines from the excited state of sodium (*Also called "D-lines" – this doublet makes up the fourth line (when observed together) of the sun's absorption spectrum, hence the letter D* [4]).

1.2 Theory

The reflection grating works by diffracting the incident light into several orders. By adjusting the grating angle and measuring the position of the spectral orders, we can determine the separation between the two lines in the sodium doublet.

The resolving power, R, of a diffraction grating is a measure of its ability to separate adjacent spectral lines. It is determined by the number of lines per unit length (i.e. the grating density) and the order of the spectral lines.

$$R = Nm$$

where m is the order of diffraction and N is the number of rulings in the grating illuminated by the collimated beam. In this experiment, the resolving power can be estimated by measuring the angular width $\delta\theta$ of a peak of the horizontal light intensity profile that corresponds to a single diffraction line. If $\delta\theta$ corresponds directly to a wavelength separation, the resolving power is therefore

$$R = \frac{\lambda}{\delta\lambda}$$

where λ is the wavelength corresponding to the maximum peak and

$$\delta\lambda = \Delta\lambda \frac{\delta\theta}{\Delta\theta}$$

See *Appendix 1* for proof that $R = \lambda / \delta\lambda$. As such,

$$\delta\lambda = \frac{\lambda}{Nm}$$

and from this we can evaluate and compare the separation according to theory and experimental results (as determined in Section 2).

2. Experimental Methods

Section 2, *Experimental Methods*, details procedure and experimental data produced from the experiment while providing brief discussion of reported data and fulfilling specified criteria of the lab brief.

2.1 Experimental Setup

The experimental setup consists of a spectrometer, sodium lamp and a diffraction grating, which is shown, as formally setup in the lab, in Figure 2.1.1. A camera was in-place of the eyepiece of the telescope for ease of accuracy, of which displayed its output on a specially curated program installed on the laboratory computers. After calibration (as discussed in section 2.2), the reflection grating was seated appropriately on the spectrometer table leading to the geometry of the setup as shown in Figure 2.1.2.

Fig 2.1.1 [1] – Spectrometer and Sodium Light

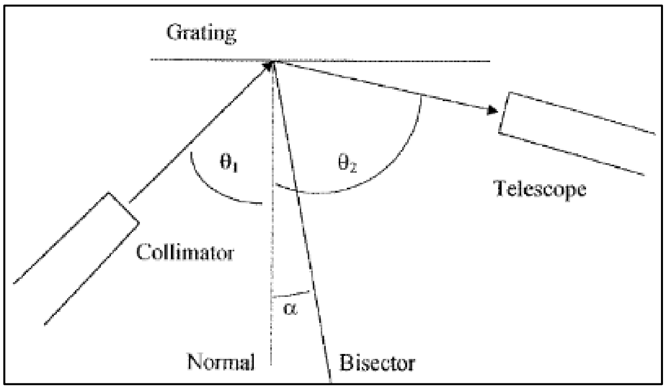

Fig 2.1.2 [2] – Geometry of the experiment

The relationship between θ_1 and θ_2 is given by

$$d\, sin\theta_1 - d\, sin\theta_2 = m\lambda$$

Where d is the line separation, m is the order of diffraction and λ is the wavelength of incident light. The left-hand side of the equation is the difference of the optical path between two parallel rays emerging from two adjacent grating lines. When the difference of optical path is equal to an integer number of wavelengths, the interference between these two rays is constructive. The combined emerging rays will produce a visible image of the source at the wavelength. When $m = 0$, the grating acts as a mirror and no doublets are visible to the observer.

By using the equation derived in the lab brief,

$$-2d\cos\alpha\ \Delta\alpha\ \approx m\Delta\lambda,$$

a graph of $-2d\cos\alpha\ \Delta\alpha$ Vs. m can be plotted, from which the slope, $\Delta\lambda$, can be extracted.

2.2 Procedure

Before any meaningful data could be collected, the setup had to be calibrated. The sodium lamp was required to be on and active for a number of minutes before the lamp was warm enough to emit visible light suitable for competent doublet detection. The spectrometer was calibrated and aligned using 2 half-meter sticks and recording any zero-point error that would affect results negatively. A brass slit was placed approximately 2cm from the collimator slit of the spectrometer. This was adjusted to a suitable width and the observing arm was rotated so that the sodium line was aligned with the blue mark in the camera viewing software. The angular span of the camera was documented by adjusting the apparatus and recording the angle on the vernier scale such that the visible sodium light was at either side of the screen for two measurements respectively (and as such, taking their difference to obtain the angular resolution). The telescope was then set at 40° from the collimator and the refection grating was placed on the table such that it was aligned with visible white guide markings. The reflection (not diffracted image) of the sodium light was then located by rotating the table.

The experimental data (as shown in section 2.3) was obtained by slowly rotating the spectrometer table in one direction until the first order (*m = 1*) diffracted images of the D-lines could be seen. The angle α, as shown in Figure 2.1.1, could then be measured and the intensity graphs and images exported from the camera software. This process was repeated/continued for higher orders of *m*.

The data recorded using this procedure (as shown in section 2.3) was sufficient to determine $\Delta\lambda$ (via the method described in section 2.1).

The lab brief states: *"Finding the grating resolving power by (a) estimating the number N of grating lines illuminated by the beam of incident light and (b) measuring the angular width of peaks in the horizontal light intensity profile of the camera image". (b)* is satisfied by the method described above, however to satisfy criteria *(a)* there is extra calculation required. The geometry required to calculate N is shown in Figure 2.4.2 (Section 2.4). This is a simple trigonometry problem which will be dealt with in the analysis section.

2.3 Experimental Data

Img. Left deg°	Img. Right deg°	ΔImg deg°	ΔImg rads	#px N	rad px
153.01	153.3	0.29	0.005061455	640	7.91E-06

Tab 2.3.1 – Camera Angular Resolution/Span Data for image analysis

m au	α deg°	$\alpha_{Vernier}$ cdeg°	α rad
1	96.5	1.7	0.26197
2	108	4.5	0.46318
3	120	1	0.672
4	134.5	5	0.92577
			±0.00035

Zero Point		
zero deg°	$zero_{vernier}$ cdeg°	zero rad
153.5	4.4	2.67984835
		±0.00035

Tab 2.3.2 – Compiled experimental data for corresponding measurements shown in Fig 2.3.1 – 2.3.4

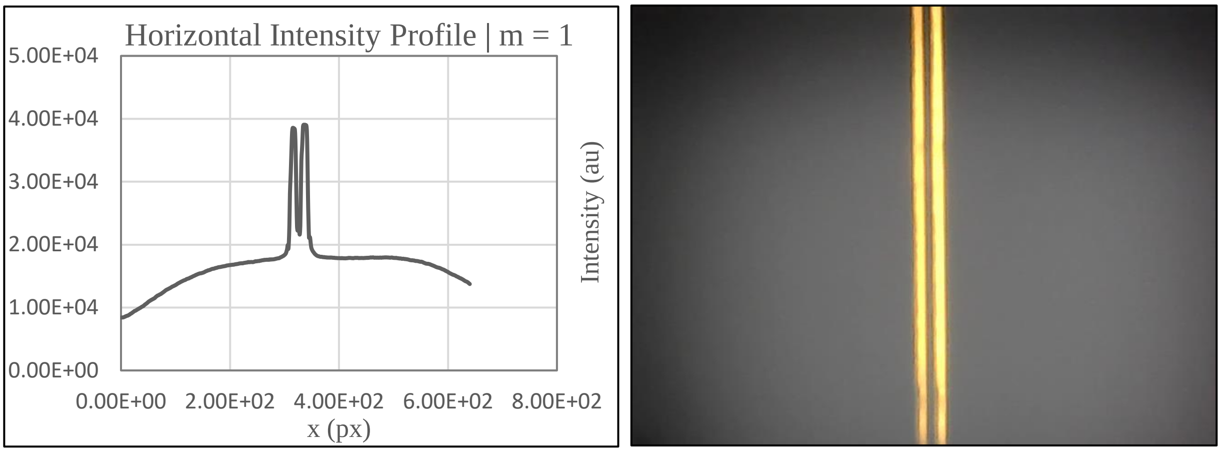

Fig 2.3.1 | m = 1 | | α = 0.26197

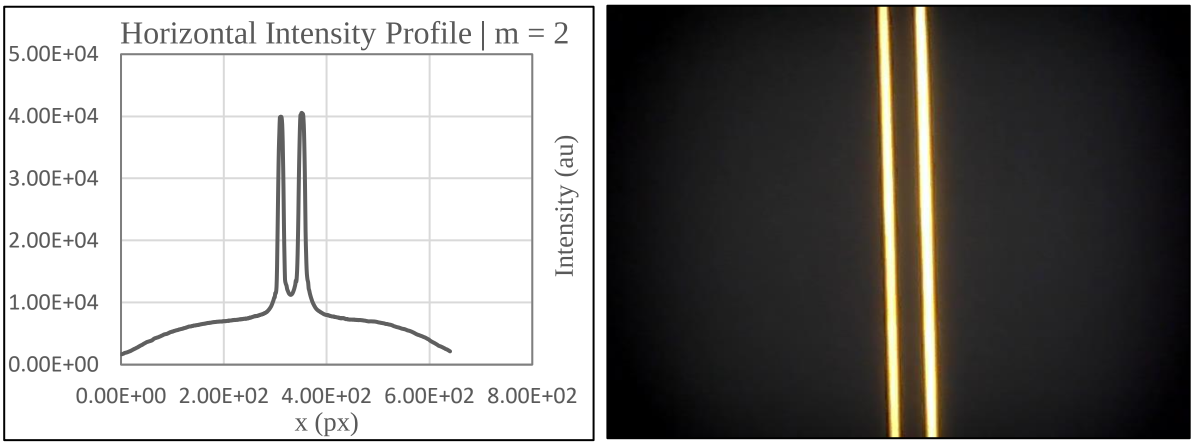

Fig 2.3.2 | m = 2 | α = 0.46318

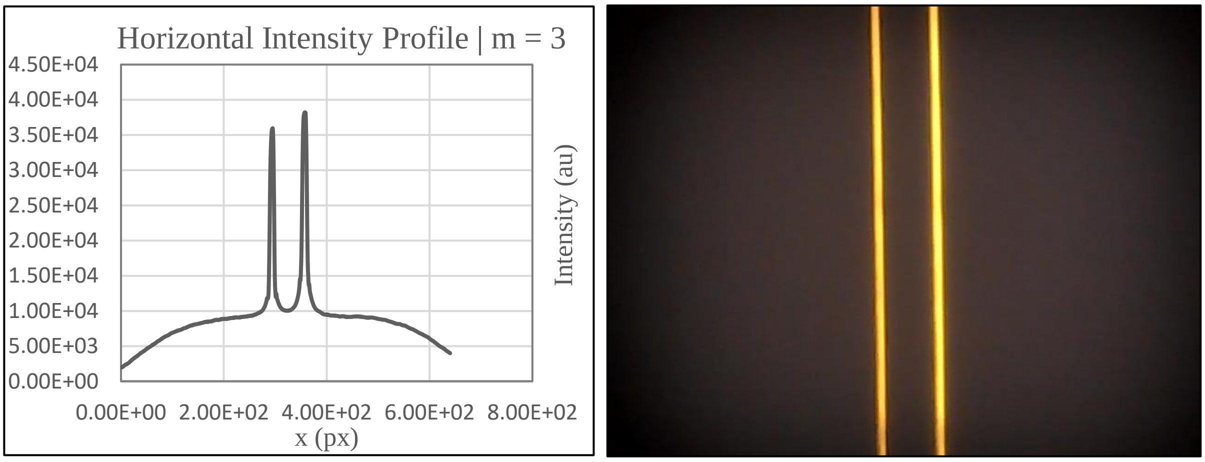

Fig 2.3.3 | m = 3 | α = 0.672

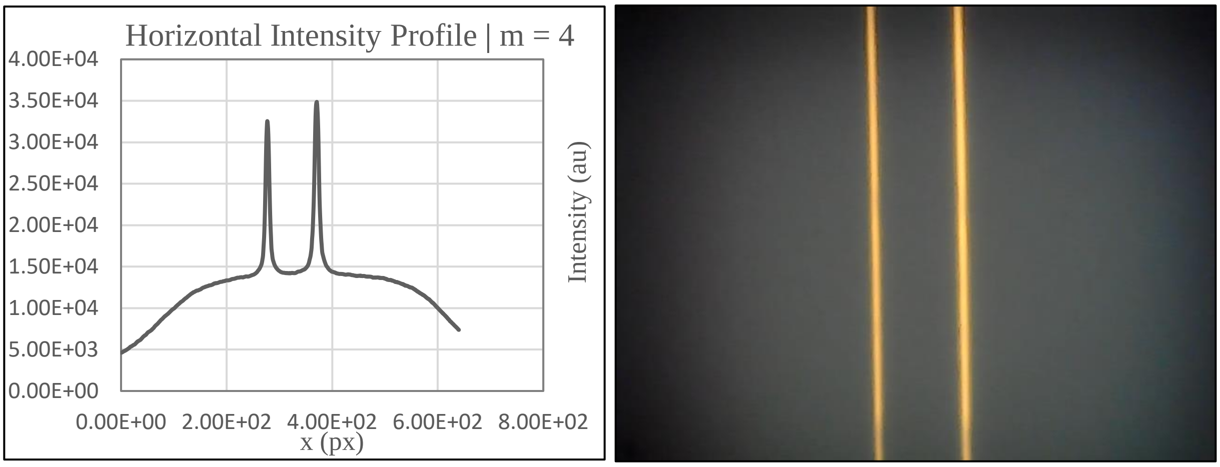

Fig 2.3.4 | m = 4 | α = 0.92577

2.4 Analysis / Calculation

m au	Peak $_{left}$ px	Peak $_{right}$ px	ΔPeak px	Δα rad
1	319	339	20	0.00015817
2	312	354	42	0.000332158
3	295	359	64	0.000506145
4	371	277	94	0.000743401
	±0.25	±0.25		

Tab 2.4.1 – Calculation of Δα from data in Fig 2.3.1 – 2.3.4

m au	Δα rad	α rad	2dCos(α)Δα m
1	0.00015817	0.26197	5.09246E-10
2	0.000332158	0.46318	9.90535E-10
3	0.000506145	0.672	1.32033E-09
4	0.000743401	0.92577	1.48982E-09
		±0.00035	

Tab 2.4.2 – Compiled Analytical and Experimental data for Δλ relationship.

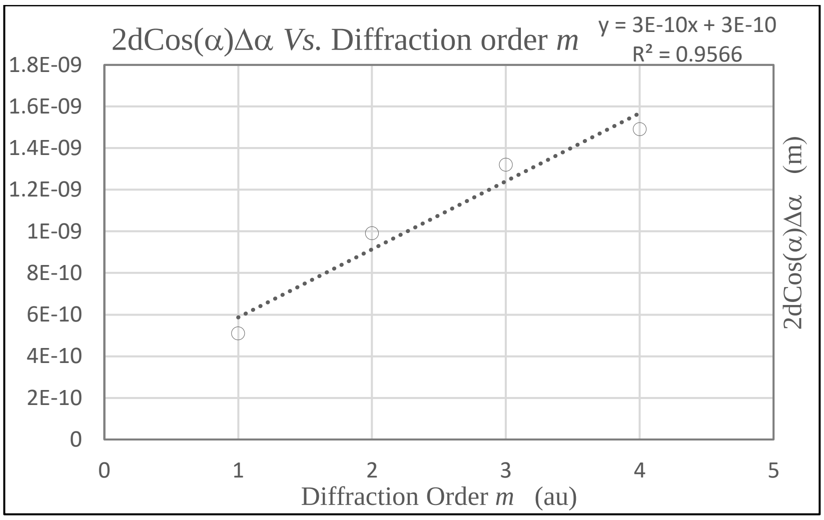

Fig 2.4.1 – LINEST: *s* = Δλ = 3.2715e-10 m | Δs = 4.9303e-11 m

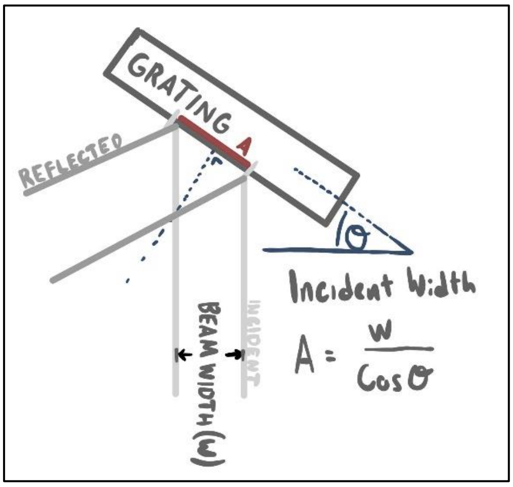

Fig 2.4.2 – Geometry for calculating N

Calculation of Incident N via trigonometry (Fig 2.4.2)

$$\theta_0 \sim 65.02° = (1.13481 \pm 0.00035)\, rad$$

$$W \approx 0.002 \pm 0.001 \text{ m}$$

$$\rightarrow \theta_m = |\theta_0 - \alpha_m|$$

$$A_m = W \frac{1}{\cos(\theta_m)}$$

(See tables below)

θ_0 deg°	θ_0 rad	W m
65.02	1.13481	0.002
	±0.00035	±0.0005

Tab 2.4.3 – Zero Point data For Table rotation

m au	θ deg°	θ rad	A m	$N_{incident}$ au	RP_N au
1	31.48	0.54943	0.0023452	1407.092	1407.092334
2	42.98	0.75014	0.0027338	1640.259	3280.518268
3	54.98	0.95958	0.0034852	2091.094	6273.281505
4	69.48	1.21265	0.0057056	3423.345	13693.381

Tab 2.4.4 – Resolving Power based on approximate number of lines hit by incident beam due to θ

P_1 px	P_2 px	ΔP px	FWHM rad	δλ m	RP_λ au
311	331	20	0.0001582	3.27E-10	1800.383088
305	316.5	24	0.0001898	1.87E-10	3150.670404
289	298	20	0.0001582	1.02E-10	5761.225882
273	282	20	0.0001582	6.96E-11	8461.800514
±0.25	±0.25				

Tab 2.4.5 – Resolving Power based on extracted intensity data and δλ - P1 and P2 represent pixels where the half full intensity horizontal bisector intersects the data. FWHM determined by taking their difference and using the rad/px ratio determined in Table 2.3.1.

2.5 Error Analysis

Δλ | From Fig 2.4.1:

$$s = \Delta\lambda = 3.2715 \times 10^{-10}\, m, \quad \Delta s = \Delta(\Delta\lambda) = 4.3026 \times 10^{-11}\, m$$

$$\therefore \Delta\lambda = (3.3 \pm 0.4) \times 10^{-10}\, m \equiv (0.33 \pm 0.04) nm$$

Δλ | From Tab 2.4.1 and Tab 2.4.2:

$$\Delta\lambda = \frac{2d\cos\alpha\ \Delta\alpha}{m}$$

$$m = 1 \mid \Delta\lambda = 5.09246\mathrm{E}-10\ \mathrm{m}$$

$$m = 2 \mid \Delta\lambda = \frac{9.90535E-10}{2} = 4.9526E-10\ \mathrm{m}$$

$$m = 3 \mid \Delta\lambda = \frac{1.32033\mathrm{E}-09}{3} = 4.4011E-10\ \mathrm{m}$$

$$m = 4 \mid \Delta\lambda = \frac{1.48982\mathrm{E}-09}{4} = 3.72455E-10\ \mathrm{m}$$

$$\overline{\Delta\lambda} = \frac{0.509 + 0.495 + 0.44 + 0.372}{4} nm$$

$$\therefore \overline{\Delta\lambda} = 0.454nm$$

Standard Deviation:

$$\Delta\left(\overline{\Delta\lambda}\right) = \sqrt{\frac{\sum_{m=1}^{4}\left(\Delta\lambda_m - \overline{\Delta\lambda}\right)^2}{4}}$$

$$\Delta\left(\overline{\Delta\lambda}\right) = \sqrt{\frac{(0.509-0.454)^2 + (0.495-0.454)^2 + (0.44-0.454)^2 + (0.372-0.454)^2}{4}} nm$$

$$= 0.0539nm$$

$$\therefore \Delta\lambda = (0.45 \pm 0.05)nm$$

RP_N | From Table 2.4.4:

$$\Delta\theta = 0.00035\ rad,\quad \Delta W = 0.0005m$$

$$RP = Nm \ \rightarrow \ \frac{\Delta RP_m}{RP_m} = \frac{\Delta N_m}{N_m} = \frac{\Delta\cos\theta}{\cos\theta} + \frac{\Delta W}{W} \equiv \frac{\sin\theta\,\Delta\theta}{\cos\theta} + \frac{\Delta W}{W}$$

$$\left(\frac{\Delta\cos\theta}{\cos\theta} \equiv \frac{\sin\theta\,\Delta\theta}{\cos\theta}\right)$$

$$m = 1 \mid \frac{\Delta RP_1}{RP_1} = \frac{\sin\theta\,\Delta\theta}{\cos\theta} + \frac{\Delta W}{W} = \frac{\sin 0.549\,(0.00035)}{\cos 0.549} + \frac{0.0005\,m}{0.002\,m} = 0.2502$$

$$m = 2 \mid \frac{\Delta RP_2}{RP_2} = \frac{\sin\theta\,\Delta\theta}{\cos\theta} + \frac{\Delta W}{W} = \frac{\sin 0.75\,(0.00035)}{\cos 0.75} + \frac{0.0005\,m}{0.002\,m} = 0.2503$$

$$m = 3 \mid \frac{\Delta RP_3}{RP_3} = \frac{\sin\theta\,\Delta\theta}{\cos\theta} + \frac{\Delta W}{W} = \frac{\sin 0.96\,(0.00035)}{\cos 0.96} + \frac{0.0005\,m}{0.002\,m} = 0.2504$$

$$m = 4 \mid \frac{\Delta RP_4}{RP_4} = \frac{\sin\theta\,\Delta\theta}{\cos\theta} + \frac{\Delta W}{W} = \frac{\sin 1.21\,(0.00035)}{\cos 1.21} + \frac{0.0005\,m}{0.002\,m} = 0.2509$$

$$\frac{\Delta RP}{RP} \approx 0.25$$

RP_λ | Using data from Table 2.4.5 and Table 2.4.1:

$$\Delta(\Delta P) \equiv \Delta(\Delta Peak) = 0.5\,px, \quad \frac{\Delta\left(\frac{rad}{px}\right)}{\frac{rad}{px}} \equiv \frac{\Delta(\Delta IMG)}{\Delta IMG} = \frac{0.00035}{0.0051} = 0.069$$

$$RP = \frac{\lambda}{\delta\lambda} \rightarrow \frac{\Delta RP}{RP} = \frac{\Delta\delta\lambda}{\delta\lambda} = \frac{\Delta FWHM}{FWHM} + \frac{\Delta(\Delta\alpha)}{\Delta\alpha} = \frac{\Delta(\Delta P)}{\Delta P} + 2\frac{\Delta(\Delta IMG)}{\Delta IMG} + \frac{\Delta(\Delta Peak)}{\Delta Peak}$$

$$m = 1 \mid \frac{\Delta RP_1}{RP_1} = \frac{0.5}{20} + 2(0.069) + \frac{0.5}{20} = 0.188$$

$$m = 2 \mid \frac{\Delta RP_2}{RP_2} = \frac{0.5}{24} + 2(0.069) + \frac{0.5}{42} = 0.171$$

$$m = 3 \mid \frac{\Delta RP_3}{RP_3} = \frac{0.5}{20} + 2(0.069) + \frac{0.5}{64} = 0.171$$

$$m = 4 \mid \frac{\Delta RP_4}{RP_4} = \frac{0.5}{20} + 2(0.069) + \frac{0.5}{94} = 0.168$$

$$\frac{\Delta RP}{RP} \approx 0.17$$

2.6 Results

From the analysis of section 2.4 and error analysis of section 2.5, the following results are to be concluded:

$$\Delta\lambda = (0.45 \pm 0.05)nm \quad or \quad \Delta\lambda = (0.33 \pm 0.04)nm$$

(depending on preferential method of analysis)

$$m = 1 \mid \quad RP_N \approx 1400 \pm 350 \quad \mid \; RP_\lambda \approx 1800 \pm 300$$

$$m = 2 \mid \quad RP_N \approx 3300 \pm 800 \quad \mid \; RP_\lambda \approx 3150 \pm 500$$

$$m = 3 \mid \quad RP_N \approx 5800 \pm 1000 \quad \mid \; RP_\lambda \approx 6300 \pm 1500$$

$$m = 4 \mid \quad RP_N \approx 8500 \pm 1400 \quad \mid \; RP_\lambda \approx 13700 \pm 3400$$

3 Discussion

It is of my opinion that the experiment was a success under the circumstances in which it was performed. **This was a *very* difficult experiment to complete**, and there are several points I would like to bring to focus for this discussion.

Firstly, it is staggering how sensitive the equipment is to its setup. I am of the belief that *blu-tac*, the material provided to keep the diffraction grating in place, is not fit for purpose in the setup of this experiment – *this is a strong opinion shared among my peers*. There was many points throughout data collection in which my lab partner and I could visibly see (on the camera output) diffracted sodium doublets displacing themselves and slowly *'crawling'*, one may say, off the visible output due to the blu-tac assumedly expanding or contracting. One of the biggest issues faced during completing the experiment was successfully locating images of diffraction orders of *m = 2* and onwards due to the unpredictable nature of the blu-tac and the difficulty of setting the grating direction perpendicular to the spectrometer imaging plane (in fact, this is even evident in the data as shown in Figures 2.3.1 to 2.3.4 – it was a consistent battle to keep the diffracted images upright).

Once the grating had been optimally aligned, it was very satisfying to see the doublets and corresponding infrared images + emission spectra. Figure 3.0.1 shows an image captured of the infrared doublet of order m = 5 (unfortunately, we had lost the position of the m = 5 doublet due to the experimental issues previously discussed), which according to lab demonstrators is a very difficult image to obtain due to its faintness and angular position on the diffracted image plane.

Fig 3.0.1 – m = 5 | infrared

I consider the experiment overall to be a success due mainly to the results shown in section 2.6. The resolving powers (RP) obtained via the methods specified within the lab brief are quite convincingly within the determined margins of error for all orders of diffraction except m = 4. This may have been due to a rounding error when determining the FWHM. However, in the experimental data shown in Figure 2.4.1, the corresponding y-value for $m = 4$ seems qualitatively less than what would have been expected from the trend set by the other orders of diffraction, but this also could have been due to a vernier rounding error in measurement.

The measurements of $\Delta\lambda$ were, as expected, greater than the minimum resolvable separation as determined in the lab brief ($\delta\lambda$ = 0.17nm). This shows that there is likely no errors in measurement or calculation, and the corresponding minimum resolvable separations as shown in Tab 2.4.5 for increasing incident grating lines, N, behaves as expected (decreases).

According to various sources (on the internet) the wavelength difference between the doublets should be approximately *0.6nm*. This indicates that our results have fallen short of their expected value. The preferred method of analysis, which seems to me to be the more rigorous of the two, has produced a difference of $(0.45 \pm 0.05)nm$, which I would deem satisfactory considering the experimental complications that were so frequently evident. Plausible sources of error that may have caused this include the blu-tac setup (as previously discussed), accidental movement of the camera and/or spectrometer, and potential error in determining the angular span of the camera.

The issue of decreasing $\Delta\lambda$ in the calculated results (This is shown comprehensively on page 7) indicates another systematic error with the equipment used, as this was evident in my peers' attempts who have also completed the experiment – $\Delta\lambda\ should\ be\ constant.$

To conclude, it is obvious from the data shown in this report the strongest emission line of sodium is a doublet and that the resolving power of a diffraction/reflection grating increases with order of diffraction.

References

[1] : University College Cork, Department of Physics website, Image Source: https://www.ucc.ie/en/physics/study/undergraduate/thelabratories/thirdyearphysicslab/determinationoftheseparationbetweentheyellowdoubletofthesodiumspectrum/

[2] : "The plane reflection grating revisited", by U. M. T. Buckley and F. A. Deeney, Eur. J. Phys. 19 231, (1998)

[3] : http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/gratres.html#c4

[4] : Adapted from Britannica definition, Source: https://www.britannica.com/science/D-lines