AM2060 Assignment 4

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November 30, 2020

This assignment is worth 25% of your final grade in this module

1 Introduction

You are to solve the epidemiological SIR model using Euler's method.

Specific requirements include:

1. Create a class called EulerSIR which implements Euler's method to solve the problem (see below).
   (a) Allow the user of the object to set the step size.
   (b) Allow the user to modify the initial conditions.
   (c) Allow the user to output the result of the computation to a csv file.
   (d) Provide a solve method which performs the computation.
   (e) Store the results in a suitably sized 2D array.
2. Use exception handling where necessary and deal with all potential errors.
3. Keep your code tidy and readable. Put in a reasonable amount of comments but no essays.
4. Provide code in main which tests your implementation. e.g.
   EulerSIR e = new EulerSIR();
   e.Stepsize = 0.001;
   e.SetInit(0.999,0,0.001);
   e.Solve();
   e.WriteToFile("c:\myfile.csv");
5. Submit your work in a single .cs file (combine all files into one file).

The following notes may be of help.

2 The SIR Model

The SIR model comprise three compartments containing subsets of the total population:

1. The susceptible (S) have not been exposed to infection,
2. The infected (I) who are infected and infectious,
3. The recovered (R) who were infected and have acquired immmunity i.e. cannot be reinfected.

For ease lets use proportions of the population in each set and the evolution equations are:

$$\begin{aligned}\dot{S} &= -\beta IS \\ \dot{I} &= \beta IS - \gamma I \\ \dot{R} &= \gamma I\end{aligned}$$

At all times $S + I + R = 1$ and $\dot{S} + \dot{I} + \dot{R} = 0$.

The infectious rate, β, represents the probability of transmitting disease between a susceptible and an infectious individual. The recovery rate, $\gamma = \frac{1}{D}$, is determined by the average time a person stays in the infected compartment, D, and is typically measured in days. Usually $D \approx 14$.

Often people in the media talk in terms of the basic reproduction number (R_0)or the R number. An epidemic occurs if $\dot{I} > 0$

$$\begin{aligned}\dot{I} = \beta IS - \gamma I &> 0 \\ \frac{\beta S}{\gamma} &> 1\end{aligned}$$

At the beginning of an epidemic then $S \approx 1$ thus the condition is $R_0 = \frac{\beta}{\gamma} > 1$ for a disease to spread. Because we know γ and R_0, i.e. for COVID $R_0 \approx 2.4$, we can estimate values for β. Typical initial conditions are $S(0) = 1 - \epsilon$,$R(0) = 0$ and $I(0) = \epsilon$, where $\epsilon \ll 1$ is a small number e.g. 0.001, which in Ireland would correspond to 4500 initial infections.

3 Euler's method

Consider the equation $\frac{dy}{dx} = f(x, y)$ along with some initial condition $y(0) = y_0$, and we wish to numerically solve it so that we know the solution $y_0, y_1, \cdots, y_n$ at times $x_0, x_1, \cdots, x_n$ where the fixed step size $h = x_{i+1} - x_i$. Then the scheme is

$$\begin{aligned}y_{j+1} &= y_j + hf(x_j, y_j) \\ x_{j+1} &= x_j + h\end{aligned}$$

In order to adapt this scheme to a system of equations let's suppose we have a system of 3 equations:

$$
\begin{aligned}
\dot{y} &= f(x, y, z, p) \\
\dot{z} &= g(x, y, z, p) \\
\dot{p} &= k(x, y, z, p).
\end{aligned}
$$

Then the corresponding Euler scheme is:

$$
\begin{aligned}
y_{j+1} &= y_j + hf(x_j, y_j, z_j, p_j) \\
z_{j+1} &= z_j + hg(x_j, y_j, z_j, p_j) \\
p_{j+1} &= p_j + hk(x_j, y_j, z_j, p_j) \\
x_{j+1} &= x_j + h
\end{aligned}
$$

For example for the SIR model above this becomes:

$$
\begin{aligned}
S_{j+1} &= S_j + h(-\beta S_j I_j) \\
I_{j+1} &= I_j + h(\beta S_j I_j - \gamma I_j) \\
R_{j+1} &= R_j + h(\gamma I_j) \\
x_{j+1} &= x_j + h
\end{aligned}
$$