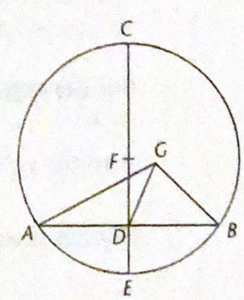
Book III

Definitions

1. *Equal circles* are those the diameters of which are equal, or the radii of which are equal.
2. A straight line is said to *touch a circle* which, meeting the circle and being produced, does not cut the circle.
3. *Circles* are said to *touch one another* which, meeting one another, do not cut one another.
4. In a circle straight lines are said *to be equally distant from the centre* when the perpendiculars drawn to them from the centre are equal.
5. And that straight line is said to be *at a greater distance* on which the greater perpendicular falls.
6. A *segment of a circle* is the figure contained by a straight line and a circumference of a circle.
7. An *angle of a segment* is that contained by a straight line and a circumference of a circle.
8. An *angle in a segment* is the angle which, when a point is taken on the circumference of the segment and straight lines are joined from it to the extremities of the straight line which is the *base of the segment*, is contained by the straight lines so joined.
9. And, when the straight lines containing the angle cut off a circumference, the angle is said to *stand upon* that circumference.
10. A *sector of a circle* is the figure which, when an angle is constructed at the centre of the circle, is contained by the straight lines containing the angle and the circumference cut off by them.
11. *Similar segments of circles* are those which admit equal angles, or in which the angles are equal to one another.

Proposition 1

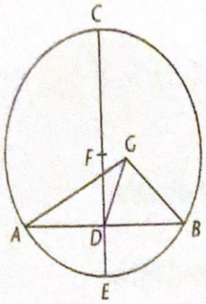
To find the centre of a given circle.



Let *ABC* be the given circle;
thus it is required to find the centre of the circle *ABC*.

Let a straight line *AB* be drawn through it at random, and let it be bisected at the point *D*;
from *D* let *DC* be drawn at right angles to *AB* and let it be drawn through to *E*; let *CE* be bisected at *F*;
I say that *F* is the centre of the circle *ABC*.

For suppose it is not, but, if possible, let *G* be the centre,



and let GA, GD, GB be joined.

Then, since AD is equal to DB, and DG is common,
the two sides AD, DG are equal to the two sides BD, DG respectively;
and the base GA is equal to the base GB, for they are radii;
therefore the angle ADG is equal to the angle GDB. [I. 8]

But, when a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right; [I. Def. 10]
therefore the angle GDB is right.

But the angle FDB is also right;
therefore the angle FDB is equal to the angle GDB,
the greater to the less: which is impossible.

Therefore G is not the centre of the circle ABC.

Similarly we can prove that neither is any other point except F.

Therefore the point F is the centre of the circle ABC.

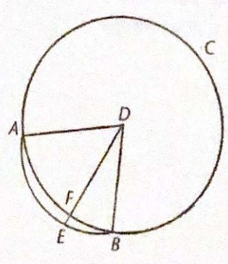
Q.E.F.

PORISM. From this it is manifest that, if in a circle a straight line cut a straight line into two equal parts and at right angles, the centre of the circle is on the cutting straight line.

Proposition 2

If on the circumference of a circle two points be taken at random, the straight line joining the points will fall within the circle.

Let ABC be a circle, and let two points A, B be taken at random on its circumference;
I say that the straight line joined from A to B will fall within the circle.



For suppose it does not, but, if possible, let it fall outside, as AEB;
let the centre of the circle ABC be taken [III. 1], and let it be D;
let DA, DB be joined, and let DFE be drawn through.

Then, since DA is equal to DB,
the angle DAE is also equal to the angle DBE. [I. 5]

And, since one side AEB of the triangle DAE is produced,
the angle DEB is greater than the angle DAE. [I. 16]

But the angle DAE is equal to the angle DBE;
therefore the angle DEB is greater than the angle DBE.

And the greater angle is subtended by the greater side;
therefore DB is greater than DE. [I. 19]

But DB is equal to DF;

therefore DF is greater than DE,
the less than the greater: which is impossible.

Therefore the straight line joined from A to B will not fall outside the circle.

Similarly we can prove that neither will it fall on the circumference itself;
therefore it will fall within.

Therefore etc.

Q.E.D.

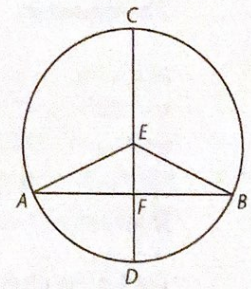
Proposition 3

If in a circle a straight line through the centre bisect a straight line not through the centre, it also cuts it at right angles; and if it cut it at right angles, it also bisects it.

Let ABC be a circle, and in it let a straight line CD through the centre bisect a straight line AB not through the centre at the point F;
I say that it also cuts it at right angles.

For let the centre of the circle ABC be taken, and let it be E; let EA, EB be joined.

Then, since AF is equal to FB, and FE is common,
two sides are equal to two sides;
and the base EA is equal to the base EB;
therefore the angle AFE is equal to the angle BFE. [I. 8]



But, when a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right; [I. Def. 10]
therefore each of the angles AFE, BFE is right.

Therefore CD, which is through the centre, and bisects AB which is not through the centre, also cuts it at right angles.

Again, let CD cut AB at right angles;
I say that it also bisects it, that is, that AF is equal to FB.

For, with the same construction,
since EA is equal to EB,
the angle EAF is also equal to the angle EBF. [I. 5]

But the right angle AFE is equal to the right angle BFE,
therefore EAF, EBF are two triangles having two angles equal to two angles and one side equal to one side, namely EF, which is common to them, and subtends one of the equal angles;
therefore they will also have the remaining sides equal to the remaining sides; [I. 26]
therefore AF is equal to FB.

Therefore etc.

Q.E.D.