Sheet 7

Solutions to be handed in before class on Wednesday May 22

There is the following remark about last week's lecture:

In the argument that the coroot is unique one needs that in the last step the field has characteristic zero, since n should not be zero.

Problem 34. Let V be a finite-dimensional vector space over a field of characteristic 0. Let $R \subseteq V$ be an irreducible root system. Let W be the associated Weyl group. Show that V is an irreducible representation of W.   (3 points)

Problem 35.

1. Show that up to isomorphism there are 4 rank 2 root systems, and draw their pictures.   (4 points)

2. Compute their Cartan matrices, and draw their Dynkin diagrams. (4 points)

3. Which of these are Langlands dual or self-dual?   (2 points)

Problem 36. Compute the order of the Weyl group of type B_2, and show which familiar group it is isomorphic to.   (3 points)