Recitation 15 - Homework 6 and 7 Answers and Questions

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December 6, 2006

- Extra Monday (12/18) Office Hours
  - 12:30-1:30pm
  - 5:00-7:00pm
- Homework 6 Answers
- Homework 7 Answers
- Wrap-Up

Problem 1. Show EQ_{CFG} is undecidable.

Problem 2. If $A \leq_M B$ and B is regular, does that imply A is regular? Why or why not?

$$T = \{M \mid M \text{ accepts } w^R \text{ whenever } M \text{ accepts } w\}$$

Problem 3. Show that T is undecidable in two ways.

Problem 4. Consider

$$A = \{(M, w) \mid M \text{ moves left on left most tape pos. on } w\}$$

Show A is undecidable.

Problem 5. Consider

$$A = \{(M, w) \mid M's \text{ head ever moves left on } w\}$$

Show A is decidable.

Problem 6.

$J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$

Big-O Problems

- $n^2 = O(n)$
- $3^n = 2^{O(n)}$
- $2^{2^n} = O(2^{2^n})$

Little-o Problems

- $n = o(2n)$
- $1 = o(1/n)$

Two graphs are isomorphic if the nodes of one graph can be relabelled in such a way that the resulting graph is equal to the second graph.

$$ISO = \{(G, H) \mid G \text{ is isomorphic to } H\}$$

Show $ISO \in NP$.

$$MODEXP = \{(a, b, c, p) \mid (a^b \mod p) = (c \mod p)\}$$

Demonstrate $MODEXP \in P$. That is some deterministic TM decides $MODEXP$ in polynomial time.

Problem 5. Part a.

$$SPATH = \{(G, a, b, k) \mid G \text{ contains a simple path from } a \text{ to } b \text{ of length at most } k\}$$

Show $SPATH \in P$.

Problem 5. Part b.

$$LPATH = \{(G, a, b, k) \mid G \text{ contains a simple (loopless) path from } a \text{ to } b \text{ of length at least } k\}$$

Show $LPATH$ is NP-Complete.

Problem 6. Problem 7.28.

$$SETSPLITTING = \{(S, C = \{C_i\}) \mid C_i \subseteq S \;\&\; (S, C) \text{ colorable}\}$$

S can be colored if each element can be chosen to be *red* or *blue* such that no C_i's elements are all the same color. Show $SETSPLITTING$ is NP-complete.

$$Variables = \{x_1, x_2, x_3\}$$

$$\phi = \{x_1 \vee x_2 \vee x_3\} \wedge \{\bar{x_2} \vee x_2 \vee x_3\} \wedge \{\bar{x_1} \vee x_3 \vee \bar{x_3}\}$$

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$$\phi = \{x_1 \vee x_2 \vee x_3\} \wedge \{\bar{x_2} \vee x_2 \vee x_3\} \wedge \{\bar{x_1} \vee x_3 \vee \bar{x_3}\}$$

$$\Downarrow\Downarrow\Downarrow$$

$$S = \{x_1, \bar{x_1}, x_2, \bar{x_2}, x_3, \bar{x_3}\}$$

$$C = \{\{x_1, x_2, x_3\}, \{\bar{x_2}, \bar{x_2}, \bar{x_3}\}, \{\bar{x_1}, x_3, \bar{x_3}\}\}$$

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$$\Downarrow\Downarrow\Downarrow$$

$$S = \{{\color{red}x_1}, {\color{blue}\bar{x_1}}, {\color{blue}x_2}, {\color{red}\bar{x_2}}, {\color{red}x_3}, {\color{blue}\bar{x_3}}\}$$

$$C = \{\{{\color{red}x_1}, {\color{blue}x_2}, {\color{red}x_3}\}, \{{\color{red}\bar{x_2}}, {\color{red}\bar{x_2}}, {\color{blue}\bar{x_3}}\}, \{{\color{blue}\bar{x_1}}, {\color{red}x_3}, {\color{blue}\bar{x_3}}\}\}$$

$\{x_1, x_2\}$

$$\{x_1 \vee x_1 \vee x_2\} \wedge \{x_1 \vee x_1 \vee \bar{x_2}\} \wedge \{\bar{x_1} \vee \bar{x_1} \vee x_2\} \wedge \{\bar{x_1} \vee \bar{x_1} \vee \bar{x_2}\}\}$$

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$S = \{x_1, \bar{x_1}, x_2, \bar{x_2}\}$

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$$\Downarrow\Downarrow\Downarrow$$

$$S = \{\color{red}{x_1}, \color{red}{\bar{x_1}}, \color{blue}{x_2}, \color{blue}{\bar{x_2}}\}$$

$$\{\{\color{red}{x_1}, \color{red}{x_1}, \color{blue}{x_2}\}, \{\color{red}{x_1}, \color{red}{x_1}, \color{blue}{\bar{x_2}}\color{blue}{\}}, \{\color{red}{\bar{x_1}}, \color{red}{\bar{x_1}}, \color{blue}{x_2}\}, \{\color{red}{\bar{x_1}}, \color{red}{\bar{x_1}}, \color{blue}{\bar{x_2}}\}\}$$

$\{x_1, x_2\}$

$$\{x_1 \vee x_1 \vee x_2\} \wedge \{x_1 \vee x_1 \vee \bar{x_2}\} \wedge \{\bar{x_1} \vee \bar{x_1} \vee x_2\}\}$$

$$\Downarrow\Downarrow\Downarrow$$

$S = \{x_1, \bar{x_1}, x_2, \bar{x_2}\}$

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- Content of recitations?
- Pace of recitations?
- Accessible?
- Grading?
- Obtaining help?