Recitation 1 - Review and the Cardinality of Sets

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- Come with questions and ask questions!

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- Fairly straight forward for finite sets, try one $\{1, 5, 9\}$

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- Some intresting functions:
  - *One-to-one* : No two elements in the domain map to the same element in the range
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  - *Bijection* : A function that is both one-to-one and onto

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An equivalence relation that describes intuitive concept of two sets being the same "size". Works for finite and infinite sets.

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- *Uncountably Infinite* if A is infinite and not equinumerous with $\mathcal{N}$, all such sets are "larger" than $\mathcal{N}$

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$$f(n) = \begin{cases} 2|n| & n \geq 0 \\ 2|n| - 1 & n < 0 \end{cases}$$

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- Proof: Show f is a bijection mapping $\mathcal{N} \times \mathcal{N}$ to $\mathcal{N}$. Show it is one-to-one and onto.

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- Consider $P(\mathcal{N})$
- Assume a bijection $f : P(\mathcal{N}) \rightarrow \mathcal{N}$ exists and show a contradiction.

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- S is a member of $P(\mathcal{N})$ but does not map to any element in $\mathcal{N}$ - a contradiction.