Recitation 12 - Homework 5

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- Homework 5 Problems
- Reduction

Section 5.1 informal reductions. The book shows many problems are undecidable using the same method. If being able to decide some set A lets you decide A_{TM}. Then A must not be decidable, because A_{TM} is not.

Some undecidable problems:

$A_{TM}, HALT_{TM}, E_{TM}, REGULAR_{TM}, EQ_{TM}, E_{LBA}, ALL_{CFG}$.

Section 5.3 introduces a mathematical formalism for this concept.

Mapping reducibility: Given two languages A and B, A is mapping reducible to B ($A \leq_M B$) if there is a *computable* function f such that:

$$w \in A \iff f(w) \in B$$

Computable function f means some Turing machine can take in w and leave $f(w)$ on the tape for all w.

$$A = \{w \in \Sigma^* \mid w \text{ contains an even number of 0s}\}$$

$$B = \{w \in 0^* \mid w \text{ contains an even number of 0s}\}$$

Describe a mapping reduction.

$$A = \{w \in \Sigma^* \mid w \text{ contains an even number of 0s}\}$$

$$B = \{w \in 0^* \mid w \text{ contains an } \textit{odd} \text{ number of 0s}\}$$

Describe a mapping reduction.

Consider a reduction from ALL_{DFA} to E_{DFA}.

$ALL_{DFA} \leq_M E_{DFA}$ and E_{DFA} is Turing decidable, so ALL_{DFA} is Turing decidable. In general $A \leq_M B$ and B decidable implies A is decidable. What can we conclude if $A \leq_M B$:

- A is decidable $\implies$?
- B is undecidable $\implies$?
- A is undecidable $\implies$?

If $A \leq_M B$, then

- A is decidable $\implies$ nothing about B
- B is decidable $\implies$ A is decidable
- A is undecidable $\implies$ B is undecidable
- B is undecidable $\implies$ nothing about A

Describe a mapping reduction from A_{TM} to E_{TM}.

Need to describe a computable process, f, such that:

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E_{TM} expects as input a Turing machine, so we must describe how to construct a new Turing machine M' from (M, w) such that:

$$(M, w) \in A_{TM} \iff M' \in E_{TM}$$

The following Turing machine takes in (M, w) and converts it a suitable M'.

$T=$ On input (M, w).

1. Construct the following Turing machine:

   $S=$ On input x:

   1. If $x \neq w$, *reject*.
   2. Else, run M and w and accept if it does.

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If we could decide E_{TM}, then we could decide A_{TM} by taking its input and constructing S and then running our decider for E_{TM} on S. A_{TM} cannot be decided though, so a decider for E_{TM} cannot exist. E_{TM} is undecidable.

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or $A_{TM} \leq_M E_{TM}$ and A_{TM} is undecidable so E_{TM} is undecidable.