Recitation 2 - Regular Languages

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Things I am planning for us to cover

- Proving Things About Machine Models
- Converting NFAs to Regular Expressions
- Using the Pumping Lemma to show a language isn't regular

Things I am not planning to cover

- Converting NFAs to DFAs
- Converting Regular Expressions to NFAs

- If s can be written as xy for strings x and y, then x is a *prefix* of s.
- If $y \neq \epsilon$, then x is a *proper prefix* of s.
- Example: 10 is a prefix and proper prefix of 1011.
- Example: 1011 is a prefix of 1011, but not a proper prefix.
- Question: Is ϵ a proper prefix of 1011?

$NOPREFIX(A) = \{w \in A \mid \text{no proper prefix of } w \text{ is a member of } A\}$

If $A = \{100, 101, 1011\}$, then $NOPREFIX(A) = \{100, 101\}$.

$NOPREFIX(A) = \{w \in A \mid \text{no proper prefix of } w \text{ is a member of } A\}$

$\{100, 101, 110, 111, 11\}$

$111^* \cup 00$

1^*

$$NOPREFIX(A) = \{w \in A \mid \text{no proper prefix of } w \text{ is a member of } A\}$$

$\{100, 101, 110, 111, 11\}$
$\quad \rightarrow \{100, 101, 11\}$

$111^* \cup 00$

1^*

$NOPREFIX(A) = \{w \in A \mid \text{no proper prefix of } w \text{ is a member of } A\}$

$\{100, 101, 110, 111, 11\}$
$\quad \rightarrow \{100, 101, 11\}$

$111^* \cup 00$
$\quad \rightarrow \{11, 00\}$

1^*

$$NOPREFIX(A) = \{w \in A \mid \text{no proper prefix of } w \text{ is a member of } A\}$$

$\{100, 101, 110, 111, 11\}$
$\quad \rightarrow \{100, 101, 11\}$

$111^* \cup 00$
$\quad \rightarrow \{11, 00\}$

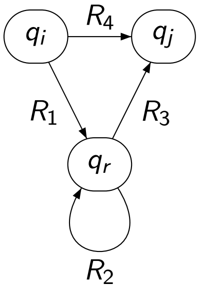
1^*
$\quad \rightarrow \{\epsilon\}$

$$NOPREFIX(A) = \{w \in A \mid \text{no proper prefix of } w \text{ is a member of } A\}$$

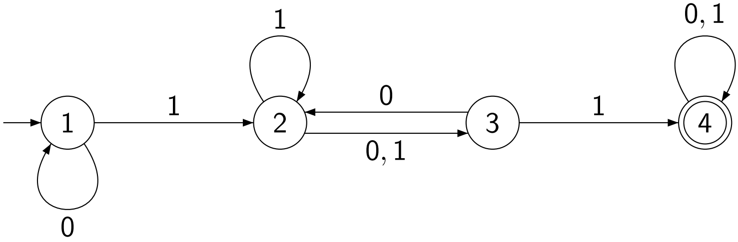
Show that the set of regular languages is closed under the *NOPREFIX* operation.

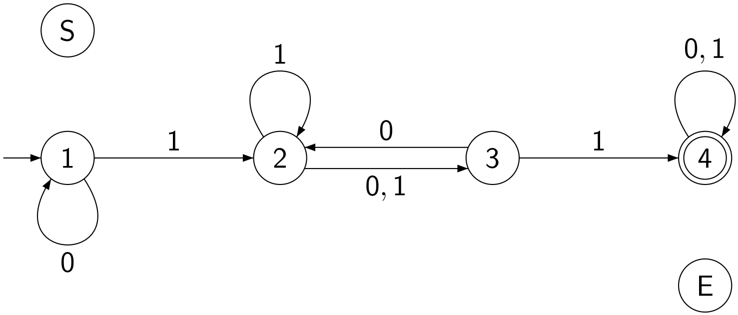
Place marker for Jigsaw Activity.

1. Convert DFA to a GNFA with 2 additional states.
2. Recursively remove original states



Remove R_1 and R_3 and replace R_4 with $R_1 R_2^* R_3 \cup R_4$.


1
0, 1
0
1
2
3
4
1
1
0, 1
0


S
1
2
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4
E
0
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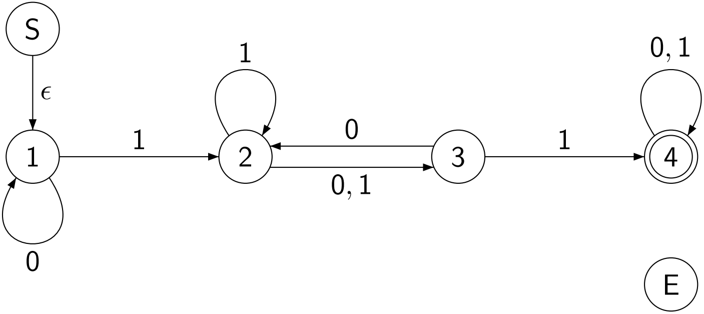
0, 1
1
0, 1


S
ϵ
1
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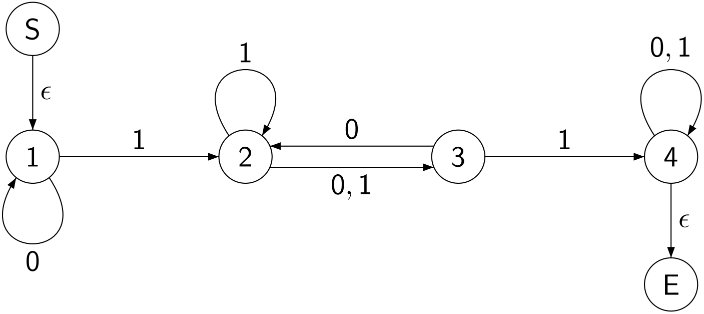
1
0
0, 1
3
1
4
0, 1
E


S
ϵ
1
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0, 1
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ϵ
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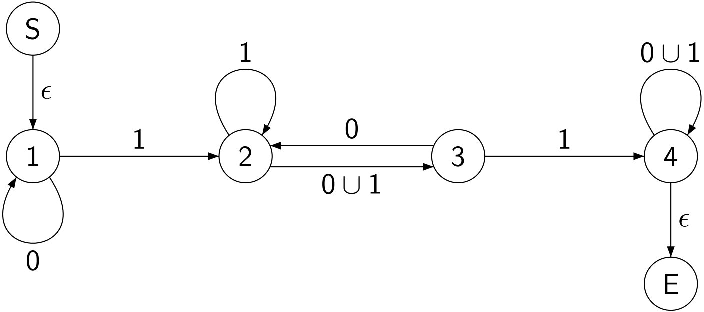
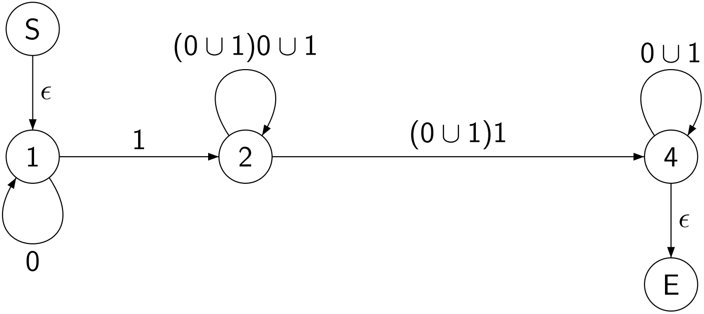
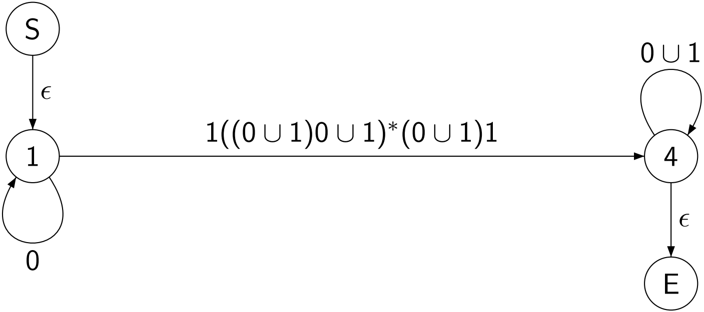
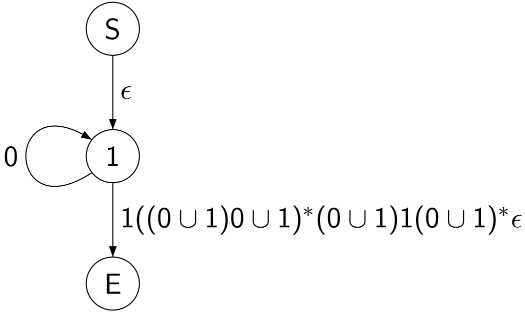
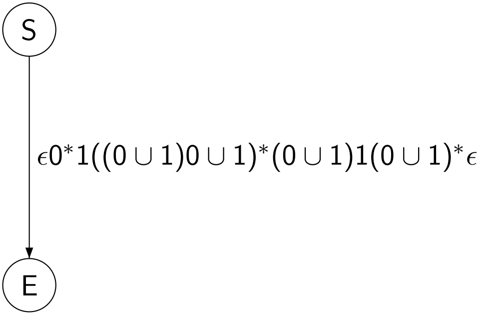

S
ϵ
1
0
1
2
1
0
$0 \cup 1$
3
1
4
$0 \cup 1$
ϵ
E


S
ϵ
(0 ∪ 1)0 ∪ 1
0 ∪ 1
1
1
2
(0 ∪ 1)1
4
0
ϵ
E


S
ϵ
1
0
$1((0 \cup 1)0 \cup 1)^*(0 \cup 1)1$
$0 \cup 1$
4
ϵ
E


S
ϵ
0
1
$1((0 \cup 1)0 \cup 1)^*(0 \cup 1)1(0 \cup 1)^*\epsilon$
E



So regular expression is: $\epsilon 0^* 1((0 \cup 1)0 \cup 1)^*(0 \cup 1)1(0 \cup 1)^* \epsilon$

Your Turn.

For every regular language A, there exists an integer p (the *pumping length*) such that any string $w \in A$ on length at least p can be expressed as $w = xyz$, with

- $|xy| \leq p$
- $|y| \geq 1$ (y isn't empty)
- $xz \in A$, $xyz \in A$, $xy^2z = xyyz \in A$, $xy^3z \in A, \ldots$

To show a language is not regular, pick a string in A of length at least p, then show no possible way of assigning x, y, and z results in an assignment that can be pumped.

$\Sigma = \{0, \#\}$

$$A = 0^n \# 0^n$$

- $0000\#0000, \#, 0\#0 \in A$
- Why are these poor choices for pumping lemma strings: 000, #, 000#000?
- What are some better choices?

$$A = 0^n \# 0^n$$

Assume A is regular with pumping length p. Let $0^p \# 0^p = xyz$ be a decomposition given by the pumping lemma.

What does $|xy| < p$ imply?

xy substring contains only zeros from first group

What does $|y| > 0$ imply?

y contains at least 1 zero

Is $xyyz \in A$?

No! Contains more zeros before $\#$ than after.

Use a simillar arguments to show the following languages are not regular

$$\{0^n \# 0^m \mid n \leq m\}$$

$$\{0^n \# 0^m \mid n \geq m\}$$

Following these arguments, what can you conclude about $0^* \# 0^*$?

Show the following language is not regular

$$\{wtw \mid w, t \in \{0,1\}^* \text{ and } |t| > 0\}$$