Recitation 2 - Deterministic Finite Automata

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We will be reviewing how to construct DFAs. We will start with some simple ones and use regular operations to construct more complicated ones.

- Motivation and relevancy
- Use of Whiteboard/Chalkboard
- Slides

- For alphabet $\Sigma = \{a, b\}$, $R_1 = \{w \mid w$ has an odd number of b's$\}$. Find a DFA, $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, that accepts R_1
- How many and which states are required?
- What should happen when a is read?
- What should happen when b is read?
- Label start and accept states.

$$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1) = (\{q_{odd}, q_{even}\}, \Sigma, \delta_1, q_{even}, \{q_{odd}\})$$

with δ_1 as follows

δ_1	a	b
q_{odd}	q_{odd}	q_{even}
q_{even}	q_{even}	q_{odd}

- For alphabet $\Sigma = \{a, b\}$, $R_3 = \{w \mid w$ contains at least three a's$\}$. Find a DFA, $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, that accepts R_2
- How many and which states are required?
- What should happen when a is read?
- What should happen when b is read?
- Label start and accept states.

$$\begin{aligned} M_2 =& (Q_2, \Sigma, \delta_2, q_2, F_2) \\ =& (\{q_{20}, q_{21}, q_{22}, q_{23}\}, \Sigma, \delta_2, q_{20}, \{q_{23}\}) \end{aligned}$$

with δ_2 as follows

δ_2	a	b
q_{20}	q_{21}	q_{20}
q_{21}	q_{22}	q_{21}
q_{22}	q_{23}	q_{22}
q_{23}	q_{23}	q_{23}

- For alphabet $\Sigma = \{a, b\}$, $R_3 = \{w \mid w$ starts with a b and ends with an $a\}$. Find a DFA, $M_3 = (Q_3, \Sigma, \delta_3, q_3, F_3)$, that accepts R_3

$$
\begin{aligned}
M_3 =& (Q_3, \Sigma, \delta_3, q_3, F_3) \\
=& (\{q_{3start}, q_{3dead}, q_{3a}, q_{3b}\}, \Sigma, \delta_3, q_{3start}, \{q_{3a}\})
\end{aligned}
$$

with δ_3 as follows

δ_3	a	b
q_{3start}	q_{3dead}	q_{3b}
q_{3dead}	q_{3dead}	q_{3dead}
q_{3a}	q_{3a}	q_{3b}
q_{3b}	q_{3a}	q_{3b}

Given

$$\begin{aligned} M_1 =& (Q_1, \Sigma, \delta_1, q_1, F_1) \\ =& (\{q_{odd}, q_{even}\}, \Sigma, \delta_1, q_{even}, \{q_{odd}\}) \\ M_2 =& (Q_2, \Sigma, \delta_2, q_2, F_2) \\ =& (\{q_{20}, q_{21}, q_{22}, q_{23}\}, \Sigma, \delta_2, q_{20}, \{q_{23}\}) \end{aligned}$$

Construct DFA which accepts $R_1 \cup R_2 = (Q_\cup, \Sigma, \delta_\cup, q_0, F_\cup)$.

$Q_\cup$

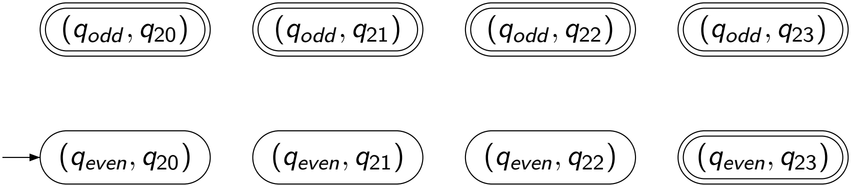
$$\begin{aligned} Q_\cup =& Q_1 \times Q_2 \\ =& \{(q_{odd}, q_{20}), (q_{odd}, q_{21}), (q_{odd}, q_{22}), (q_{odd}, q_{23}), \\ & (q_{even}, q_{20}), (q_{even}, q_{21}), (q_{even}, q_{22}), (q_{even}, q_{23})\} \end{aligned}$$

(q_{odd}, q_{20})	(q_{odd}, q_{21})	(q_{odd}, q_{22})	(q_{odd}, q_{23})
(q_{even}, q_{20})	(q_{even}, q_{21})	(q_{even}, q_{22})	(q_{even}, q_{23})

q_0 and $F_\cup$

$$\begin{aligned} q_0 =& (q_1, q_2) \\ =& (q_{even}, q_{20}) \end{aligned}$$

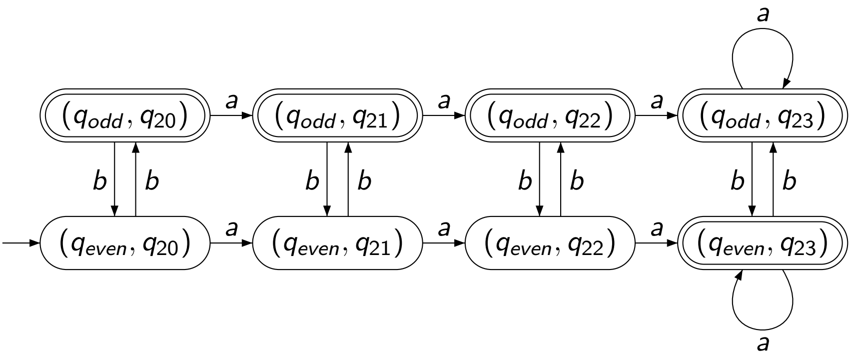
$$\begin{aligned} F_\cup =& \{(r_1, r_2) \mid r_1 \in F_1 \text{ or } r_2 \in F_2\} \\ =& \{(q_{odd}, q_{20}), (q_{odd}, q_{21}), (q_{odd}, q_{22}), (q_{odd}, q_{23}), (q_{even}, q_{23})\} \end{aligned}$$


(q_{odd}, q_{20})
(q_{odd}, q_{21})
(q_{odd}, q_{22})
(q_{odd}, q_{23})
(q_{even}, q_{20})
(q_{even}, q_{21})
(q_{even}, q_{22})
(q_{even}, q_{23})

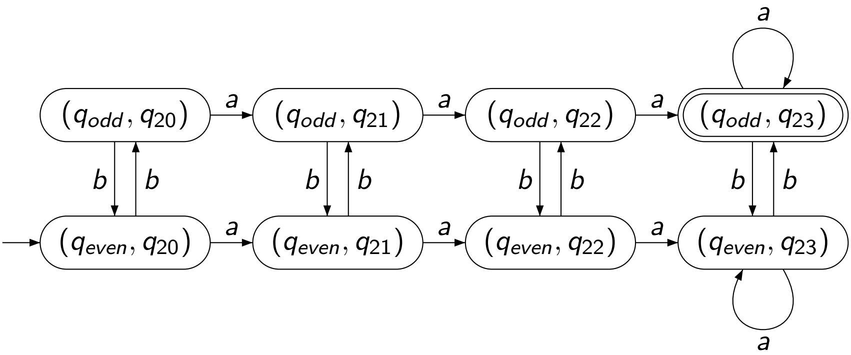
δ_1	a	b
q_{odd}	q_{odd}	q_{even}
q_{even}	q_{even}	q_{odd}

δ_2	a	b
q_{20}	q_{21}	q_{20}
q_{21}	q_{22}	q_{21}
q_{22}	q_{23}	q_{22}
q_{23}	q_{23}	q_{23}

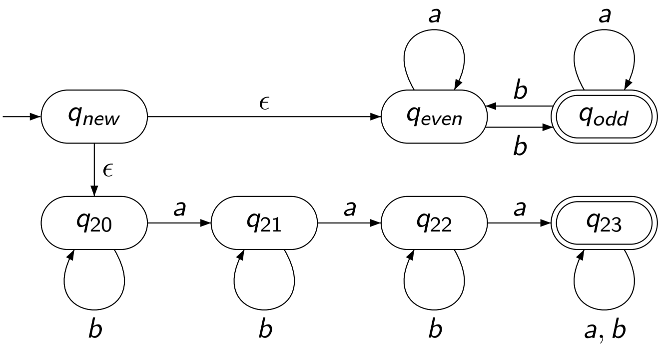
$\delta_\cup$	a	b
(q_{odd}, q_{20})	(q_{odd}, q_{21})	(q_{even}, q_{20})
(q_{odd}, q_{21})	(q_{odd}, q_{22})	(q_{even}, q_{21})
(q_{odd}, q_{22})	(q_{odd}, q_{23})	(q_{even}, q_{22})
(q_{odd}, q_{23})	(q_{odd}, q_{23})	(q_{even}, q_{23})
(q_{even}, q_{20})	(q_{even}, q_{21})	(q_{odd}, q_{20})
(q_{even}, q_{21})	(q_{even}, q_{22})	(q_{odd}, q_{21})
(q_{even}, q_{22})	(q_{even}, q_{23})	(q_{odd}, q_{22})
(q_{even}, q_{23})	(q_{even}, q_{23})	(q_{odd}, q_{23})


(q_{odd}, q_{20})
(q_{odd}, q_{21})
(q_{odd}, q_{22})
(q_{odd}, q_{23})
(q_{even}, q_{20})
(q_{even}, q_{21})
(q_{even}, q_{22})
(q_{even}, q_{23})
a
b

Intersection. $F_\cap = \{(r_1, r_2) \mid r_1 \in F_1 \text{ and } r_2 \in F_2\}$



Producing an NFA that is that accepts the union of two regular languages given by DFAs (or NFAs) is even easier.



How about intersection?

- Reminder: $R_3 = \{w \mid w \text{ starts with a } b \text{ and ends with an } a\}$
- What is the complement of R_3
- Design a DFA which accepts $\bar{R_3}$.
- Does the same trick work for NFAs?

- Easy, straightforward methods given to compute union, intersection, and complement of regular languages
- Star and concatenation are are slightly more complicated general...
- $R_4 = \{w \mid w \text{ contains exactly 2 } a\text{'s}\}$
- $R_5 = \{w \mid w \text{ contains exactly 2 } b\text{'s}\}$
- $R_4 \circ R_5$ - consider *baabbab* and *baabab*
- $(R_4 \cup R_5)^*$ - difficult for same reason

- Consider $R_6 = baa^*$.
- Design a DFA which accepts R_6^*
- Design a DFA which accepts $R_6 \circ R_1$