Recitation 6

John Chilton

July 18, 2007

At the bottom of the proof for 5.13, write down one specific thing that you were the most unclear about while doing homework 5. Try to be as specific as possible.

$LEFT_{TM} = \{< M, w > \mid M$ moves its head left while on leftmost tape position during computation of $w\}$

$LEFT_{TM} = \{< M, w > \mid M$ moves its head left while on leftmost tape position during computation of $w\}$

Assume $LEFT_{TM}$ is decidable. Let L be some decider of it. Now consider the following decider of A_{TM}.

$S =$ On input $< M, w >$

1. Create a new TM, M' based on M but
   a) M' adds @ at the beginning of input and moves all input to the right
   b) Adjust transitions so that on @ M' stays in the same state and moves to the right
   c) Replace q_{accept} with a state that moves left indefinitely
2. Run decider L on this new TM
3. *Accept* if L accepts, else *reject*.

$J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$

Show J and $\overline{J}$ are not Turing-*recognizable*.

$J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$

Method 1: Normal Reduction. Assume J is recognizable by some TM S. Then the following would recognize $\overline{A_{TM}}$, a contradiction.

$T =$ On input $< M, w >$

1. Run S on input $1 < M, w >$.
2. If at some point S accepts, *accept*.

$J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$

Method 1: Normal Reduction. Assume J is recognizable by some TM S. Then the following would recognize $\overline{A_{TM}}$, a contradiction.

$T =$ On input $< M, w >$

1. Run S on input $1 < M, w >$.
2. If at some point S accepts, *accept*.

Method 2: Mapping Reduction. The following proves $\overline{A_{TM}} \leq_M J$

$T =$ On input $< M, w >$

1. Output $1 < M, w >$.

Give an example of an undecidable language B such than $B \leq_m \overline{B}$

Give an example of an undecidable language B such that $B \leq_m \overline{B}$

$J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$

$\overline{J} = \{w \mid w = 0x \text{ for some } x \in \overline{A_{TM}} \text{ or } w = 1y \text{ for some } y \in A_{TM}\}$

Give an example of an undecidable language B such that $B \leq_m \overline{B}$

$J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$

$\overline{J} = \{w \mid w = 0x \text{ for some } x \in \overline{A_{TM}} \text{ or } w = 1y \text{ for some } y \in A_{TM}\}$

$T =$ On input $y < M, w >$
1. If $y == 0$, output $1 < M, w >$.
2. If $y == 1$, output $0 < M, w >$.

In the SPCP, in each pair the top string has the same length as the bottom string. Show that SPCP is decidable.

Show that the PCP is decidable over $\Sigma = \{1\}$.

Show that the PCP is decidable over $\Sigma = \{1\}$.
Three cases:

- For each tile, top always longer than bottom, or vise versa
- There exists at least one tile with equal length top and bottom
- There exists at least one tile with longer top and one with longer bottom

$T =$ On input T (a set of tiles)

1. If each tile has more 1s on top than bottom, *reject*
2. If each tile has fewer 1s on top than bottom, *reject*
3. Else, *accept*.

T decides PCP over $\Sigma = \{1\}$..

Show PCP over $\Sigma = \{0, 1\}$ is undecidable, call this language $2PCP$.

The following proves $PCP \leq_m 2PCP$, hence $2PCP$ is undecidable.

$T =$ "On input T (a set of Tiles)

1. Find a fixed-length binary encoding capable of uniquely encoding each symbol in language of T.
2. Create a new set of tiles T' based on T, but with each symbol replaced by its binary encoding.
3. Output T' "

Show A is Turing-recognizable iff $A \leq_m A_{TM}$.

Show A is Turing-recognizable iff $A \leq_m A_{TM}$.

$A \leq_m A_{TM} \implies A$ is Turing-recognizable.

Show A is Turing-recognizable iff $A \leq_m A_{TM}$.

$A \leq_m A_{TM} \implies A$ is Turing-recognizable.

Proof: Follows right from Theorem 5.28 and the fact A_{TM} is Turing recognizable. $\square$

Show A is Turing-recognizable iff $A \leq_m A_{TM}$.

$A \leq_m A_{TM} \implies A$ is Turing-recognizable.

Proof: Follows right from Theorem 5.28 and the fact A_{TM} is Turing recognizable. $\square$

A is Turing-recognizable $\implies A \leq_m A_{TM}$

Show A is Turing-recognizable iff $A \leq_m A_{TM}$.

$A \leq_m A_{TM} \implies A$ is Turing-recognizable.

Proof: Follows right from Theorem 5.28 and the fact A_{TM} is Turing recognizable. $\square$

A is Turing-recognizable $\implies A \leq_m A_{TM}$

Proof: If A is Turing-recognizable, some TM, say M, recognizes it. We can use this TM to create a function that maps instances of A to A_{TM}.

Show A is Turing-recognizable iff $A \leq_m A_{TM}$.

$A \leq_m A_{TM} \implies A$ is Turing-recognizable.

Proof: Follows right from Theorem 5.28 and the fact A_{TM} is Turing recognizable. $\square$

A is Turing-recognizable $\implies A \leq_m A_{TM}$

Proof: If A is Turing-recognizable, some TM, say M, recognizes it. We can use this TM to create a function that maps instances of A to A_{TM}.

$T =$ "On input w

1. Output $< M, w >$

Show A is Turing-recognizable iff $A \leq_m A_{TM}$.

$A \leq_m A_{TM} \implies A$ is Turing-recognizable.

Proof: Follows right from Theorem 5.28 and the fact A_{TM} is Turing recognizable. $\square$

A is Turing-recognizable $\implies A \leq_m A_{TM}$

Proof: If A is Turing-recognizable, some TM, say M, recognizes it. We can use this TM to create a function that maps instances of A to A_{TM}.

$T =$ "On input w

1. Output $< M, w >$

$w \in A \Leftrightarrow < M, w > \in A_{TM}$, hence $A \leq_m A_{TM}$.

Show that there is an undecidable subset of $\{1\}^*$.

Show that there is an undecidable subset of $\{1\}^*$.

Key: Set of all Turing machines over the language $\{1\}$ can be enumerated and hence is countably infinite. The deciders are a subset of these and so they too must be countably infinite.

Show that there is an undecidable subset of $\{1\}^*$.

Assume that a bijective mapping exists between the deciding TMs and subsets of $\{1\}$. This table displays one such possible mapping.

	ϵ	1	11	111	1111	...
M_1	$\underline{1}$	1	0	1	1	...
M_2	1	$\underline{0}$	1	0	0	...
M_3	0	1	$\underline{0}$	0	0	...
M_4	1	0	0	$\underline{1}$	1	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

The contradiciton is derived by noting that for any such mapping you can construct a subset of $\{1\}^*$ that is not decided by any of the deciders. For instance

$$\{1^i \mid 1^i \text{ is rejected by } M_i\}$$

$PRIMESET = \{s \mid s \subset \mathcal{Z}$ with a subset adding to a prime number$\}$

Show $PRIMESET \in NP$. Do you think $PRIMESET \in P$?

$7SPANTREE = \{< G > \mid G$ is a weighted graph with a spanning tree of weight at most $7\}$

Show $7SPANTREE \in NP$. Do you think $7SPANTREE \in P$?

$$7MOD = \{n \mid n \text{ is a binary number evenly divisible by 7}\}$$

What is wrong with the following argument that $7MOD \in P$?

Proof: $7MOD \in P$ because the following TM, T, decides $7MOD$ in polynomial time.

$T =$ On input n (a binary number)

1. While number on tape > 7:
   Subtract 7 from number
2. If number is zero, *accept*, else *reject*

$\square$

What can you conclude about $7MOD$s membership in P? Solve the easier problem of showing $7MOD \in$NP.

Consider problems about lists of binary numbers. Assume that the lists are represented by simply seperating each number by a pound sign, i.e. the following would be a sample input list:

$$1010\#111\#010$$

Do you think the subset of P that is problems of this form is closed under $*$? Prove that the subset of NP that is problems of this form is closed under $*$?

Answer the same two questions but assume the sample list above actually looks like:

$$\#1010\#111\#010\#$$