Matrix Background: $\lambda_{min}(A)x^Tx \leq x^TAx \leq \lambda_{max}x^Tx$. (Holds for some x)
$\max_{x\neq 0} \|Ax\|/\|X\| = \sqrt{\lambda_{max}}(A^TA)$ (same for min)
Eigenvalue Decomposition on S^n, $X = Q^T\Lambda Q = \sum_i \lambda_i q_i q_i^T$, with $Q^TQ = I$.
$A > B \Rightarrow \forall x \neq 0\ x^TAx > x^TBx$ — $A^TA \geq 0$.
$X = [AB; B^TC]$ and $\det A \neq 0 \Rightarrow S = C - B^TAB$ and $\det X = \det S \det A$.
$X > 0 \iff A, S > 0$, $A > 0 \Rightarrow (X \geq 0 \iff S \geq 0)$
$min_u u^TAu + 2v^TB^Tu + v^TC^v = v^TSv$ if $\det A \neq 0$
Else if $A \geq 0$ and $Bv \in R(A), v^T(C - B^TA^\dagger B)v$ ($-$inf without B cond.)
$\nabla(x^TPx + q^Tx + r) = Px + q$ — $\nabla(\log\det X \in S_+^n+) = X^{-1}$
$(a^Ta)(b^Tb) \leq (a^Tb)^2$
$\|X\|_1$ (Max col abs sum, ∞ is row) — $\|X\|_2 = sqrt(\lambda_{max}(X^TX))$
$\sup\{a_i^Tu|\|u\|_2 \leq r\} = r\|a_i\|_2$
$[x, y; y, z] \in S_+^2 \iff x, z \geq 0, xz \geq y^2$
$\|A\|_2 \leq s \iff A^TA \leq s^2I(s \geq 0)$ — $A^TA \leq t^2I \iff [tI, A; A^T, tI] \geq 0$

Hyperplane: $\{x|a^Tx = b\} = \{x|a^T(x - x_0) = 0\}$ — Halfspace: $\{x|a^Tx \leq b\}$.
Norm Cone: $\{(x,t)|\|x\| \leq t\}$
Polyhedra: $\{x|a_j^Tx \leq b_j, ...c_j^Tx = d_j\}$
Affine Independence: $\{v_1 - v_0, ..., v_k - v_0\}$ indep $\Rightarrow \{v0, ..., v_k\}$ affinely independent.
Simplex : Convex hull of an affinely independent set.
Operations that preserve convexity of sets: Arbitrary intersection, Affine Transformation, Inverse of affine image, projection on subset of coordinates, set sumation, cartesian product, $\{(x, y_1 + y_2)|(x, y_1) \in S_1, (x, y_2) \in S_2\}$. Perspective projection. K proper cone if convex, closed, non empty interior, and $x, -x \in K \Rightarrow x = 0$.
$x \leq_K y \Rightarrow y - x \in K$ and $x <_K y \Rightarrow y - x \in intK$
x minimum element if $\forall y \in S\ x \leq_K y$. X minimal element if $\forall y \in S\ y \leq_K x \Rightarrow y = x$.
Below is seperating hyperplane theorem, converse holds if at least one set is open
C, D convex, $C \cap D = \phi \Rightarrow \exists a \neq 0$ s.t. $\forall x \in C\ a^Tx \leq b$ and $\forall x \in D\ a^Tx \geq b$
Supported HP Thm: C convex $\Rightarrow \forall x_0 \in bdC\ \exists a \neq 0$ s.t. $\forall x \in C\ a^Tx \leq a^Tx_0$.
Every closed convex set S is the intersection of halfspaces.

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f differentable with convex domain:
f convex (concave) $\iff \forall x, y\ f(y) \geq (\leq) f(x) + \nabla f(x)^T(y - x)$
f strict convex $\iff \forall x \neq y\ f(y) > f(x) + \nabla f(x)^T(y - x)$
Convex: e^{ax}, x^a : $a \leq 0$ or ≥ 1 (cc. else), $|x|^p$ for $p \geq 1$, $x\log x$, any norm, x^2/y for $y > 0$.
Convex: $\log(\sum e^{x_i})$, $x^TYx\ Y \in S_{++}^n$, λ_{max} on S^n, $1/x$
Concave: $\log x$, $\Pi x_i^{1/n}$, $\log\det X$, $\log\int_0^x exp(-t^2)$
Sublevel (superlevel) sets of convex (concave) functions are convex
Ops preserve convexity: Arb. suming or maxing, $g(x,t) = t * f(x/t) s.t. t > 0$, $\inf_y f(x,y)$
g convex $\Rightarrow$: $e^{g(x)}, g(x)^p (p \geq 1), -log(-g(x))$ convex.
g concave $\Rightarrow$: $\log g(x)$ is cc and $1/g(x)$ is cx.
$h(g_i(x))$ cx if h cx, nd (ni) $\forall$ arg, g_i cx (cc) — $h(g_i(x))$ cc if h cc, nd (ni) $\forall$ arg, g_i cc (cx(scalar))
Rank X on S_+^n is quasiconcave - LF is quasilinear, $\|x - a\|_2/\|x - b\|_2$ (QCX when ≤ 0)
On R, f quasiconvex if noninc., nondec, or noninc then nondec. QCX $\iff$ QCX on all lines.
QCX preserved under max, LF transform of domain, composition with monotonic func, $\inf_x f(x,y)$, sums of QCX not QCX.
f diff w/cx dom $\Rightarrow f$ quasiconvex $\iff \forall x, y f(y) \leq f(x) \Rightarrow \nabla f(x)^T(y - x) \leq 0$.
f quasiconvex $\Rightarrow y^T\nabla f(x) = 0 \Rightarrow y^T\nabla^2 f(x) y \geq 0$. ($\Leftarrow$ if > 0 when $y \neq 0$)
Log convexity: $\log f$ cvx or $f(\theta x + (1 - \theta)y) \leq f(x)^\theta f(y)^{1-\theta}$.
lcx $\Rightarrow$ cx, positive cc $\Rightarrow$ lcc, lcc $\Rightarrow$ quasicc, lcx $\Rightarrow$ qcx.
sum lcc not lcc, sum lcx is lcx, f,g lcc $\rightarrow$ fg lcc, $\int f(x,y)dy$ lcc if f is

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LP Stand. Form: $\min c^Tx$ s.t. $Ax = b, x \geq 0$. Unconstrained vars ($x_i = x_i^+ - x_i^-$, both pos)
$\min\max_i f_i \ \rightarrow\ \min t$ s.t. $f_i \leq t \quad i = 1, ..., m$
$\min(c^Tx + d)/(e^T + f) s.t. Gx \leq h, Ax = B \rightarrow \min c^Ty + dz s.t. Gy - hz \leq 0, Ay - bz = 0, e^Ty + fz = 1, z \geq 0$ because $y = x/(e^Tx + f)$ and $z = 1/(e^T + f)$.
SOCP: $\min f^tx$ s.t. $\|A_ix + b_i\|_2 \leq c_i^Tx + d_i, Fx = g$

Monimial: $cx_1^{a_1}\cdots x_n^{a_n}$ for $c > 0$. Posynomial are sum of monimials.
GP: $f_0, ...f_m$ are posynomials and h_i are monomials. Convex when take $x_i = e^{y_i}$ and log transform problem.
SPD: $\min c^Tx$ s.t. $x_1F_1 + \cdots + x_nF_n + G \leq 0, Ax = b$
SDP(SF): $\min tr(CX)$ s.t. $tr(A_iX) = b_i, X \geq 0$
$\min\|Ax - b\|_\infty \Rightarrow \min t$ s.t. $Ax - b \leq t1, Ax - b \geq -1t$
$\min\|Ax - b\|_1 \Rightarrow \min 1^Ty$ s.t. $Ax - b \leq y, Ax - b \geq y$.
$\min\|Ax - b\|^2$ If A Full Rank: $x_{ls} = (A^TA)^{-1}A^Tb$ In general $A^\dagger b + v(\in N(A))$.
$\min\|x\|^2$ s.t. $Ax = b$. Full Rank A: $A^T(AA^T)^{-1}b$ In genral $A^\dagger b$.
$\min c^Tx$ s.t. $x^TA(> 0)x \leq 1\ x_{opt} = -A^{-1}c/sqrt(c^TA^{-1}c)$ (Proof $y = A^{1/2}x, \tilde{c} = A^{-1/2}c$)
$\min c^Tx$ s.t. $G + x_1F_F1 + ... \leq 0 \rightarrow \max tr(GZ)$ s.t. $tr(F_iZ) + c_i = 0, Z \geq 0$
RLP: $\min c^Tx$ s.t. $a_i^Tx \leq b_i \forall a_i \in \{\bar{a}_i + F_iu|\|u\| \leq 1\} \rightarrow \min c^Tx$ s.t. $a_i^Tx + \|F_ix\| \leq b_i$
$\min\|A(x) = A_0 + x_1A_1 + ...\| = \min t$ s.t. $[tI, A(x); A^T(x), tI] \geq 0$.
$\min(c^Tx)^2/d^Tx$ s.t. $Ax \leq b = \min t$ s.t. $[diag(b - Ax), 0, 0; 0, t, c^Tx; 0, c^Tx, d^Tx] \geq 0$ (Schur).
$x^Tx \leq yz, y, z \geq 0 \iff |[2x; y - z]\|_2 \leq y + z, y, z \geq 0$

$\min c^Tx$ s.t. $Ax = b, x \geq 0 \Rightarrow g(\lambda, v) = -b^Tv$ if $A^Tv - \lambda(\geq 0) + c = 0 = -b^Tv$ if$A^Tv + c \geq 0$
$\min c^Tx$ s.t. $Ax \leq b \rightarrow \max -b^T\lambda$ s.t. $A^T\lambda + c = 0, \lambda \geq 0$
$\min f_0(x)$ s.t. $Ax \leq b, Cx = d\ g(\lambda, v) = -b^Tv - d^Tv - f_0^*(Y) \quad Y = -A^T\lambda - C^Tv \in domf_0^*$
$\min c^Tx$ s.t. $Ax = b, x \geq_K 0 \rightarrow maxb^Ty$ s.t. $A^Ty \leq_K *c$
$\min x^TP(> 0)x$ s.t. $Ax \leq b \rightarrow \max -(1/4)\lambda^TAP^{-1}A^T\lambda - b^\lambda$ s.t. $\lambda \geq 0$.
$\min x^Tx$ s.t. $Ax = b \rightarrow \max -1/4v^TAA^Tv - b^Tv \quad v* = -2(AA^T)^{-1}b$
$\min\sum x_i\log x_i$ s.t. $Ax < b, 1^Tx = 1 \rightarrow \max -b^T\lambda - \log\sum e^{-a_i^T\lambda}$ s.t. $\lambda \geq 0$.
$\min\log\det X^{-1}$ s.t. $a_i^TXa_i \leq 1 \rightarrow \max\log\det(\sum^m \lambda_ia_ia_i^T) - 1^T\lambda + n$
$\min x^TA_0x + 2b_0^Tx + c_0$ s.t. $x^TA_1x + 2b_1^Tx + c_1 \leq 0$
$g(\lambda) = c_0 + \lambda c_1 - (b_0 + \lambda b_1)^T(A_0 + \lambda A_1)^\dagger(b_0 + \lambda b_1)$ s.t.
$A_0 + \lambda A_1 \geq 0, b_0 + \lambda b_1 \in R(A_0 + \lambda A_1) \rightarrow$
$max\gamma$ s.t. $\lambda \geq 0, [A_0 + \lambda A_1, b_0 + \lambda b_1; (b_0 + \lambda b_1)^T, c_0 + \lambda c_1 - \gamma] \geq 0$
$KKT: f_i \leq 0, h_i = 0, \lambda_i \geq 0, \lambda_if_i = 0, \nabla_x(L(x^*, \lambda^*, v^*) = 0$
All optimals must satifsy KKT if SD holds, for convex converse is true.
Feasibility and ($\lambda \geq 0, g(\lambda, v) > 0$) always at most one true, exactly for convex
$K^* = \{y|x^Ty \geq 0 \forall x \in K\}$. $f^*(y) = sup_{x\in dom}(y^Tx - f(x))$. Always convex.
$\log x \rightarrow -1 - log(-y)(y < 0)$, $x^TP(> 0)x \rightarrow 1/4 * y^TP^{-1}y$

Self-concordance: $|f'''(t)| \leq 2f''(t)^{3/2}$ (Affinely Invariant).
$f, gSC \Rightarrow f + gSC, \alpha(\geq 1)fSC$. SC: $-logx, -logdetX, -log(t^2 - x^Tx)$ (on Cone).