Recitation 4 - Proving Regularity and Nonregularity

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- Proving Regularity
  - A^R
  - $NOPREFIX(A)$
- Proving Nonregularity - Pumping Lemma
  - 1.47
  - $\{w \mid |w| \text{ is prime}\}$

Give a or describe a construction for a DFA, NFA, or regular expression that recognizes the required language.

- Alto of people came up with a correct construction, but many failed to prove it or argue why it works.

Given $w = w_1 w_2 \ldots w_n \in A$. $w \in LANG(M) \implies \exists r_0 r_1 r_2 \ldots r_n$ such that (P 40):

- $r_0 = q_0$
- $r_n \in F$
- $r_{i+1} = \delta(r_i, w_{i+1})$ for $i = 0, 1, \ldots, n-1$

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To show $w^R \in LANG(M')$ need to express w^R as $y = y_1 y_2 \dots y_m$ and find sequence of states $s_0 s_1 \dots s_m$ such that (Page 54):

- $s_0 = q'_0$
- $s_m \in F'$
- $s_{i+1} \in \delta'(s_i, y_{i+1})$ for $i = 0, 1, \dots, m-1$

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- Example: 10 is a prefix and proper prefix of 1011.
- Example: 1011 is a prefix of 1011, but not a proper prefix.

Problem 1.40 part a.

$$NOPREFIX(A) = \{w \in A \mid \text{no proper prefix of } w \text{ is a member of } A\}$$

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If $A = \{100, 101, 1011\}$, then $NOPREFIX(A) = \{100, 101\}$.

"Solution" from the book. Given NFA $M = (Q, \Sigma, \delta, q_0, F)$ which recognizes A, construct the following NFA, $M' = (Q, \Sigma, \delta', q_0, F)$ with:

$$\delta'(r, a) = \begin{cases} \delta(r, a) & \text{if } r \notin F \\ \phi & \text{if } r \in F \end{cases}$$

IDEA: If after reading some string, x, M is in an accept state and then M continues to read in characters and ends in the same or another accept state then we shouldn't accept that string because x is a proper prefix of the whole string and is in A. So drop all outgoing transitions on accept states.

Proof:

- $w \in NOPREFIX(A)$:

- $w \notin NOPREFIX(A)$ and $w \in A$:

- $w \notin NOPREFIX(A)$ and $w \notin A$:

For every regular language A, there exists an integer p (the *pumping length*) such that any string $w \in A$ on length at least p can be expressed as $w = xyz$, with

- $|xy| \leq p$
- $|y| \geq 1$ (y isn't empty)
- $xz \in A$, $xyz \in A$, $xy^2z = xyyz \in A$, $xy^3z \in A$

To show a language is nonregular, pick a string in A of length at least p, then show no possible way of assigning x, y, and z results in an assignment that can be pumped.

$$\Sigma = \{1, \#\}$$

$$Y = \{w \mid w = x_1 \# x_2 \# \ldots \# x_k \text{ with } x_i = 1^* \text{ and } x_i \neq x_j\}$$

- $11111 \in Y$

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- $1\#11\#111 \in Y$
- $11\#11 \notin Y$
- $11\#1\#11 \notin Y$

- One Idea: $1^p\#1^{p+1}$

For $\Sigma = \{0, 1\}$, consider $A = \{w \mid |w| \text{ is prime}\}$.

- $111 \in A$
- $1111 \notin A$

- Consider $w = 1^q$. Where q is the smallest prime number larger than the p.
- Assume A is regular and consider some decomposition of w as promised by the pumping lemma:

$$w = 1^q = xyz = 1^k 1^l 1^m$$

- For this decomposition we know:
  - $q = k + l + m$
  - $|xy| \leq p \implies k + l \leq p$
  - $|y| \geq 1 \implies l \geq 1$
  - If A is regular, then $xy^n z$ should be of prime length for any n
  - Now we need to derive a contradiction, which will show A isn't regular.