Recitation 1 - Review and the Cardinality of Sets

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- A few words about slides...
- Note cards - at the end of recitation write down one thing that presented clearly and one thing that wasn't. Feel free to not write anything.
- Come with questions and ask questions!

- Cardinality of set A, represented as $|A|$, is the "number" of elements in A.
- Fairly straight forward for finite sets, try one $\{1, 5, 9\}$

- A *function* or *mapping* is an object that specifies an input-output relationship.
- Notation: $f : D \rightarrow R$
- Some intresting functions:
  - *One-to-one* : No two elements in the domain map to the same element in the range
  - *Onto* : Each element in the range set has an element from the domain mapped to it
  - *Bijection* : A function that is both one-to-one and onto

Equinumerosity : Sets A and B are equinumerous if there exists a bijection

$$f : A \rightarrow B$$

An equivalence relation that describes intuitive concept of two sets being the same "size". Works for finite and infinite sets.

The cardinality of set A is

- n if A is equinumerous with $\{1, 2, \ldots, n\}$
- *Countably Infinite* if A is equinumerous with $\mathcal{N}$, we represent this symbolically as $|A| = |\mathcal{N}| = \aleph_0$.
- *Uncountably Infinite* if A is infinite and not equinumerous with $\mathcal{N}$, all such sets are "larger" than $\mathcal{N}$

- $A = \{2, 3, 4, \ldots\}$

$$f(n) = n - 2$$

- Set of even natural numbers

$$f(n) = n/2$$

- Set of integers $\mathcal{Z}$

$$f(n) = \begin{cases} 2|n| & n \geq 0 \\ 2|n| - 1 & n < 0 \end{cases}$$

- Question: Is the set $\mathcal{N} \times \mathcal{N}$ $(\{(i,j)|i,j \in \mathcal{N}\})$ countable?
- Answer: Yes, a suitable bijection can be shown to be

$$f(i,j) = \frac{(i+j)*(i+j+1)}{2} + i$$

- Proof: Show f is a bijection mapping $\mathcal{N} \times \mathcal{N}$ to $\mathcal{N}$. Show it is one-to-one and onto.

- Cantor's Theorem: The power set of any set is "larger" than the set itself.
- Consider $P(\mathcal{N})$
- Assume a bijection $f : P(\mathcal{N}) \to \mathcal{N}$ exists and show a contradiction.

- For $i = 0, 1, 2, \ldots$ let A_i be the element of $P(\mathcal{N})$ that maps to i.
- Now consider the set $S = \{i \in \mathcal{N} \mid i \notin A_i\}$.
- S is a member of $P(\mathcal{N})$ but does not map to any element in $\mathcal{N}$ - a contradiction.