Recitation 5

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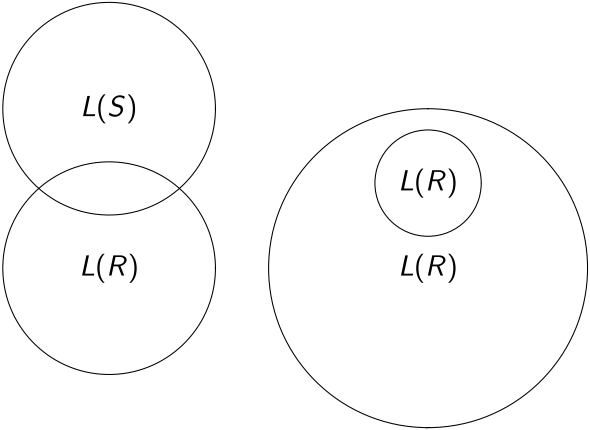
Assume some f exists that is a one-to-one mapping between $\mathcal{B}$ and $\mathcal{N}$.

n	$f(n)$
1	$\underline{\mathbf{0}}01010101...$
2	$1\underline{\mathbf{0}}0101001...$
3	$10\underline{\mathbf{1}}111011...$
4	$001\underline{\mathbf{0}}01010...$
5	$1010\underline{\mathbf{1}}1101...$
6	$11100\underline{\mathbf{1}}010...$
$\vdots$	$\vdots$

Then construct a new infinite sequence in $\mathcal{B}$ that is not mapped to by any natural number.



$$L(R) \subseteq L(S) \iff L(R) \cap L(S)^C = \phi$$

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$S =$ On input $< R, S >$

1. Convert regular expressions R & S to DFAs D_R & D_S
2. Create a new DFA D_{SC} recognizing the complement of D_S
3. Create a new DFA D_{new} recognizing $L(D_R) \cap L(D_{SC})$
4. Run D_{new} as input on a decider of E_{DFA},
   accept if it accepts, otherwise *reject*

S is a decider for A, so A is a decidable language.

$ST = \{< M, w > \mid M$ is a two-tape TM and M writes to its second tape during the computation of input $w\}$

Attempt 1

On input $< M, w >$:

1. Construct the following TM U.

   $U =$ "On input x

   1. Run M on x, if M writes to the 2^{nd} tape, *accept*.
   2. If M halts without writing to the second tape, *reject*."

// U will accept w iff M writes to second tape on w

2. Run T (the decider for A_{TM}) on $< U, w >$.
3. *Accept* if T accepts, otherwise *reject*.

Attempt 2

On input $< M, w >$:

1. Construct the following two-tape Turing machine.

   $U =$ "On input x:

   1. Simulate M on input x using only the first tape.
   2. If M accepts x, write a symbol to the second tape.
   3. Upon completion, *reject*."

// U writes to the second tape iff M accepts its input

2. Run S (the decider for ST) on input $< U, w >$.
3. *Accept* if S accepts, otherwise *reject*."

Some Undecidable languages.

A_{TM}	TM M accepts string w
$HALT_{TM}$	TM M halts on input w
E_{TM}	TM M accepts no strings
EQ_{TM}	TM M accepts same language as TM N
ALL_{CFG}	CFG G generates Σ^*
PCP	"Dominos" have a match

$$DIFF_{CFG} = \{< G, H > \mid G, H \text{ are CFGs and } L(G) \neq L(H)\}$$

Show $DIFF_{CFG}$ is undecidable.

Show $DIFF_{CFG}$ is Turing recognizable.

$$SEVEN_{TM} = \{< M > \ \mid M \text{ accepts the input } 7\}$$

$$SEVEN2_{TM} = \{< M > \ \mid M \text{ accepts exactly } 7 \text{ strings}\}$$

Show $SEVEN_{TM}$ and $SEVEN2_{TM}$ are undecidable.

Are $SEVEN_{CFG}$ and/or $SEVEN2_{CFG}$ decidable.

$INFINITE_{TM} = \{< M > \mid M \text{ is a TM with an infinite language. } \}$

Show $INFINITE_{TM}$ is undecidable.

Based on your intuition is $INFINITE_{TM}$ Turing recognizable or not?

Show $INFINITE_{DFA}$ is decidable. (HINT: Think pumping lemma.)

$\{< M, w > \mid M$ doesn't move its heads left on computation of $w\}$

Is this language decidable? Prove your answer.