



MACS 30111



The fundamental package for scientific computing with Python

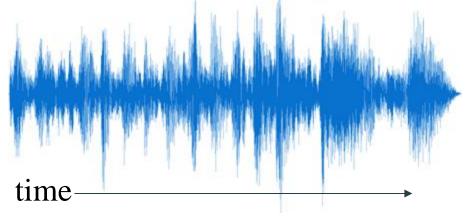
Chapter: 4.2

Documentation: https://numpy.org/devdocs/use

Topics:

- Introduction and Motivation for array programming
- Creating Numpy arrays
- Indexing into Numpy arrays
- Working with arrays and array operations
- Advanced array manipulations
- An extended example of standardizing features

Audio data (1-dimensional)

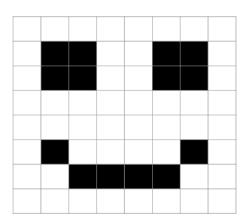


```
array([-0.00270751, -0.00303302, -0.00159557, ..., -0.0012889 , -0.00184731, -0.00210062], dtype=float32)
```

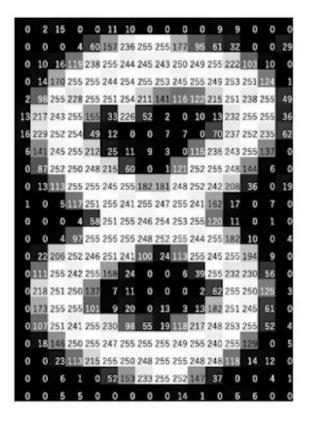
Kaggle/UrbanSound8K

- Machine learning
- Deep learning

Image data



0	0	0	0	0	0	0	0
0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	0	0	0	0	1	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0





		165	187	209	58	7
	14	125	233	201	98	159
253	144	120	251	41	147	204
67	100	32	241	23	165	30
209	118	124	27	59	201	79
210	236	105	169	19	218	156
35	178	199	197	4	14	218
115	104	34	111	19	196	
32	69	231	203	74		

- Machine learning
- Deep learning

Kaggle/Animals 151

Video data



		165	187	209	58	7
	14	125	233	201	98	159
253	144	120	251	41	147	204
67	100	32	241	23	165	30
209	118	124		59	201	79
210	236	105	169	19	218	156
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35	178	199	197		14	218
115	104	34	111	19	196	
32	69	231	203	74		

Kaggle/youtube-video-dataset

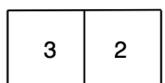
- Machine learning
- Deep learning

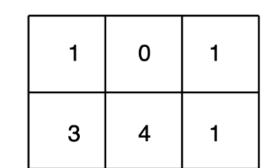
1D Array

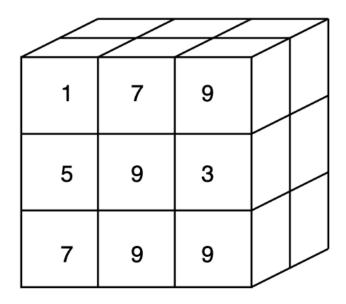
2D Array

3D Array

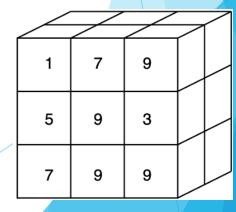
4D Array







\angle		$\overline{}$	$\overline{}$
1	7	9	
5	9	3	
7	9	9	



Nested Lists

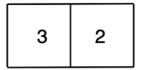
```
[["M", "M", "F", "M", "M"],
["F", "B", "B", "B", "F"]
["M", "M", "M", "M", "B"]
["B", "B", "B", "F", "B"]
["B", "M", "M", "M", "F"]]
```

Benefits: Easy to code up (you already know how!)

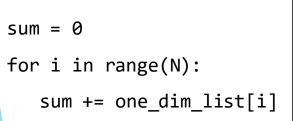
(0,0)	(0,1)	(0,2)	(0,3)	(0,4)
0	0	1	1	0
(1,0)	(1,1)	(1,2)	(1,3)	(1,4)
0	1	1		0
(2,0) 0	(2,1) 0	(2,2)	(2,3)	(2,4) 1
(3,0) 1	(3,1)	(3,2)	(3,3)	(3,4)
(4,0)	(4,1)	(4,2)	(4,3)	(4,4)
0	1	1	0	

Drawbacks

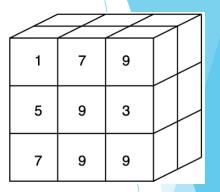
Sum the elements in 1, 2, and 3-dimensional arrays:



1	0	1
3	4	1



```
sum = 0
for i in range(N):
  for j in range(M):
    sum += two_dim_list[i][j]
```



Writing dimension-independent generic code is difficult when using lists-of-lists.

Drawbacks: for loop VS numpy operation

```
X = make_big_list(10000000)

total_sum = 0
for i in range(10000000):
   total_sum += X[i]
```

 $X = make_big_array(10000000)$

total_sum = np.sum(X)

1.17 seconds

0.007 seconds

Numpy operation is faster

Writing your own for loops can be slow. Performance matters when working with large datasets!

Numpy: scientific computing with n-d array



NumPy (pronounced "numb-pie") is an open-source library in Python. It supports:

- 1. Multidimensional data structure (the ndarray)
- 2. Fast mathematical operations
- 3. An interface that reuses much of the Python list interface

Tradeoff: NumPy arrays can only hold values of the same *type*

Python list can hold different data types.

A NumPy array can only hold the same type: integers, floats, Boolean.

 Most numpy operations are performed on all elements together, different data types have different operations.

```
1 a = [1, 2.0, 'cat', False]
2 type(a)
```

```
list
```

```
import numpy as np
b = np.array([True, False, False])
type(b)
```

numpy.ndarray

Reference:

- https://numpy.org/devdocs/user/absolute_beginners.html
- https://numpy.org/doc/stable/user/basics.types.html

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Creating new NumPy arrays

- From existing lists
- Creating arrays with the same value
- Creating arrays with a range of values (e.g., *arange()*, *linspace*)
- Joining arrays (e.g. np.array([array1, array2]))
- The data type, or "dtype", of an array
- Array Shape

Coding practice: 4.2.2

Creating an array from an existing list

First, we usually **import** the NumPy library using a shorter name, "np"

Then we can **create** a new array from a list

$$x = np.array([1, 2, 3])$$

We can then **index** into the array like a list:

```
1 print(x[2])
```

3

Creating multi-dimensional arrays from nested lists

To create a two-dimensional array, we can use two nested lists:

$$y = np.array([[1, 2, 3], [4, 5, 6]])$$

To print this array:

```
1 print(y)
```

To access elements in this array: $\begin{bmatrix}
[1 & 2 & 3] \\
[4 & 5 & 6]
\end{bmatrix}$

```
1 print(y[1,2])
```

6

Coding practice: 4.2.2

Creating arrays: constant values

If you need to create an array of all zeros, you can use *np.zeros*

```
1 np.zeros(10)
array([0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

You can create multidimensional arrays like this as well, by passing a shape-like tuple

Numpy also supports creating arrays of ones like this

Coding practice: 4.2.2

Creating arrays: ranges

Numpy supports creating arrays of numerical ranges with the arange and linspace functions.

```
np.arange(start, stop, step)
                                      np.linspace(start, stop, numsteps)
                                       first value
        exclusive, just like
                            step size
                                                  last value in
                                                                   total number of
                                        in array
        Python's range()
                                                   array, inclusive
                                                                   steps in array
        np.arange(1,5)
                                          np.linspace(0, 1, 5)
    array([1, 2, 3, 4])
                                     array([0. , 0.25, 0.5 , 0.75, 1.
                                                        Coding practice: 4.2.2
Reference:
```

https://numpy.org/doc/stable/reference/generated/numpy.arange.html

https://numpy.org/doc/stable/reference/generated/numpy.linspace.html#r

Array types

Remember that the contents of NumPy arrays must all be of the **same type**.

.dtype: the array's data type

Integers (64-bit)

```
1 x = np.array([1, 2, 3])
2 print(x)

[1 2 3]

1 x.dtype

dtype('int64')
```

```
x = np.array([1.2, 2.5, 3.7])
    print(x)
[1.2 2.5 3.7]
    x.dtype
dtype('float64')
Floating-point (64-
bit)
            Coding practice: 4.2.2
```

The dimension of an arrays: shape

The .shape attribute: a tuple describing the length in each dimension

```
1 x = np.array([1, 2, 3])
2 x.shape
(3,)
```

- The length of the shape attribute: the number of dimensions
- One-dimension: row/column vectors

Coding practice: 4.2.2

Numpy: methods and routines

zeros: pass tuple (shape, dtype)

ones: pass tuple (shape, dtype)

random: pass tuple (size)

arange: pass ([start,]stop, [step],]dtype)

linspace: pass (start, stop, num=50, e ndpoint=True, dtype=N one, axis=0)

loadtxt: loads data into an array (needs file name)

Task 1

- Create an array of 3 x 5 filled with zeros
- Create an array of 3 x 5 filled with ones
- \checkmark Create an array of 3 x 5 that has random values from 0 to 1 in it
- Then, combine all these arrays into one array.
 - Find its shape and data type

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One-dimensional array indexing

Same as a Python list:

```
1 x = np.array([1, 2, 3])
2 print(x[2])
```

3

Multi-dimensional array indexing

We can create a two dimensional array

```
1 y = np.array([[1, 2, 3], [4, 5, 6]])
2 y
```

```
array([[1, 2, 3], [4, 5, 6]])
```

What if we only use one dimension? *y[0]*

```
[[<mark>1 2 3</mark>]
[4 5 6]]
```

And index with y[0, 1]

```
[[1 <mark>2</mark> 3]
[4 5 6]]
```

two indices separated by a comma Coding practice: 4.2.3

Array slicing

```
The same Python slicing operations
                     y = np.array([[1, 2, 3, 4]])
                       [5, 6, 7, 8]
                       [9, 10, 11, 12])
                                                    y[1:,:3
 y[0,:3]
\lceil \lceil 1 \mid 2 \mid 3 \mid 4 \rceil
                                                [[1 2 3
                                                  [<mark>5 6 7 8]</mark>
 [5 6 7 8]
 [9 10 11 12]
                                                  9 10 11 12]
                                             Coding practice: 4.2.3
```

Task 2

- Set your seed to zero (why? How?)
- Create a matrix of random integers from 0 to 10 (inclusive) that is 5×5
- Slice your matrix so that you have the following:
 - Rows: odd numbered rows
 - Columns: numbered one through three

Quiz

X is an array with X.shape = (5, 10). How many elements are returned by X[:3, :2]?

- a. 3
- b. 5
- c. 6

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Mathematical operations with arrays

- List operation: whole list as a unit
- Array operation: element-by-element
- +, -, *, /, %
- >, <, ==
- np.sin, np.cos

Array*array: not matrix operation

Coding practice: 4.2.4

Reference:

- https://numpy.org/doc/stable/reference/routines.statistics.html
- https://numpy.org/doc/stable/reference/routines.linalg.html
- https://numpy.org/doc/stable/reference/routines.logic.html

Reshaping arrays

.reshape: change the shape of an array:

- one-dimensional to two-dimensional
- two dimensional back into one-dimensional

Note:

- the new array must have the same number of elements as the original array
- element updates on reshaped array mirrors back to the original array

Coding practice: 4.2.5

Task 2.1

Create a matrix that goes from 0 to 10 that has the dimensions of 3x5. Divide the range evenly across the 15 values.

Quiz

x = np.array([1, 2, 3, 4]). What is the value of x * 2?

- ► A. np.array([1, 2, 3, 4, 1, 2, 3, 4])
- ► B. np.array([3, 4, 5, 6])
- C. np.array([2, 4, 6, 8])

Arrays and axes

Reductions along an axis

Axis = 0: goes along ROWS

Axis = 1: goes along COLUMNS

Reductions on arrays

Reduce arrays to lower dimensions:

operation across all elements or along specific dimensions

```
array([[1, 2, 3], [4, 5, 6]])
```

```
array([[1, 2, 3], [4, 5, 6]])
```

- · slicing in the axis dimension
- · every index in the non-axis dimension

```
np.sum(x, axis=0)
x[:,0], x[:,1], x[:2]
array([5, 7, 9])
np.sum(x, axis=1)
x[0,:], x[1,:]
array([ 6, 15])
```

Coding practice: 4.2.6

Vectors, Matrices, Tensors

- Vector (akin to list): one-d arrays (only one axis)
 - MATH WORKS WEIRD HERE IN NUMPY!!
 - Can reshape into a 'matrix' (meaning you give it a second dimension)
- Matrix (like regular 2d matrix): two axes (0 and 1)
 - Math is OK
- Tensor (multiple / higher dimensions) (AAHHH!)
 - Harder to visualize but possible (good for dimension reduction!)

Fun: Images

```
Import the image (Pillow library)
```

```
Can get image info: pixels
WEBP
(2048, 2048)
RGB
```

Dimensions:

```
<class 'numpy.ndarray'>
(2048, 2048, 3)
```

Red dimension: numpydata[:,:,0]

```
[[65 51 64 ... 53 61 62]

[54 36 44 ... 47 48 57]

[57 48 50 ... 49 44 44]

...

[48 44 47 ... 40 51 52]

[54 57 53 ... 44 58 46]

[56 63 55 ... 71 66 53]]
```



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Advanced array manipulations

- Fancy indexing:
 - Indexing with a list/an arrays of indexes (array indexing)
 - Indexing with an array of Booleans to select elements (masked indexing)
- Performing operations between arrays of different dimensions ("broadcasting")

Coding practice: 4.2.7

Index a subset of elements

```
a = np.array([1, 4, 9, 16, 25, 36, 49])
```

Array indexing with:

• one index (e.g., *a*[3])

```
a = np.array([1, 4, 9, 16, 25, 36, 49])
• a list of indexes (e.g., a[3, 1, 6])
```

```
a = np.array([1, 4, 9, 16, 25, 36, 49])

• a multi-dimensional arrays of indexes (e.g. alnn array([[1,3], [5,2]])])

array([16, 4, 49])
```

```
a = np.array([1, 4, 9, 16, 25, 36, 49]) array([[ 4, 16], [36, 9]]
```

Coding practice: 4.2.7

Masked indexing

Select a subset of elements via a boolean array:

```
c = np.array([100, 200, 300])
mask = np.array([True, False, True])
```

Relational operations:

```
1 c>100

array([False, True, True])

1 c[(c>100) & (c/100==2)]

array([200])
```

```
1 c[c>100]

array([200, 300])

1 c[(c>100) | (c/100==2)]

array([200, 300])
```

Coding practice: 4.2.7

Task 3: bringing it all together

- Create a 10 x 10 matrix of random numbers between 0 and 100 (inclusive) with a seed of 10.
- Create a sub-matrix of this where you select even-numbered rows and odd-numbered columns. If they are greater than 50, keep the values make them zero otherwise
- Add the columns

Quiz

```
Consider the NumPy arrays x = np.array([1, 3, 5, 7, 9]) and y = np.array([1, 2, 3, 2, 1]).
```

What is the value of the expression x [y < 2]?

- a np.array([True, False, False, False, True])
- np.array([True, 3, 5, 7, True])
- np.array([1, 9])
- d. np.array([1, 3, 7, 9])



Advanced topics

- Broadcasting
- Standardizing

It's possible to perform operations on arrays that have compatible but not identical shapes. In these cases, NumPy *logically* constructs intermediate values that have the same shape using broadcasting before performing the element-by-element operations.

You can perform operations (e.g., +, *) on arrays with compatible shapes.

```
1 x = np.array([1, 2, 3])
2 y = np.array([7, 8])
3 x.shape, y.shape

((3,), (2,))
```

- same sizes for all dimensions
- one of the dimension is

```
Traceback (most recent call las
t)
<ipython-input-99-e32109319f52> in <module>
----> 1 x * y

ValueError: operands could not be broadcast together with shapes (3,)
(2,)
```

Reference: https://numpy.org/doc/stable/user/basics.broadcasting.html

Multiply an array by a scalar

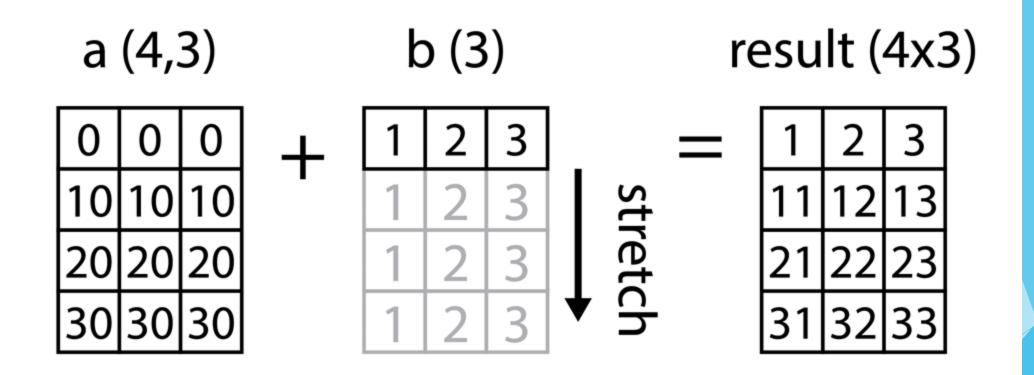
a (3) b (1) result (3)

1 2 3 *
$$444 = 4812$$

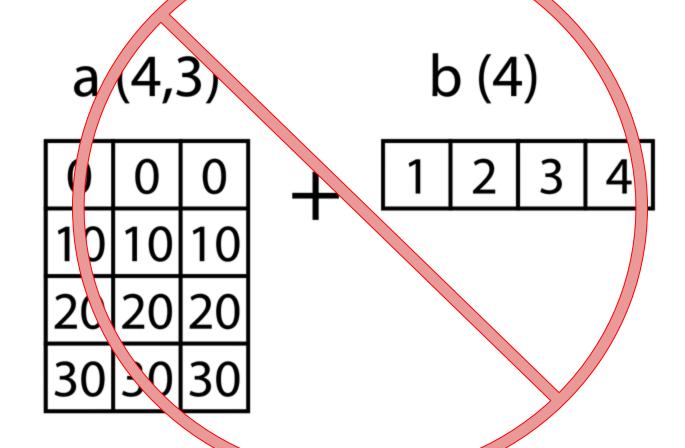
stretch

Numpy handles dimensions when two arrays' shapes don't agree.

E.g., if a dimension is one, numpy magically stretches array b to match the shape of array a.



Reference: https://numpy.org/doc/stable/user/basics.broadcasting.html



Reference:

https://pumpy.org/doc/stable/user/basics.broadcasting.htm

Only valid for compatible arrays

Compare array shape from right to left by each dimension:

- equal
- or one of them is 1

Reference: https://numpy.org/doc/stable/user/basics.broadcas/

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Standardizing

Allows you to compare items with others, even when they have different scales

The need for standardized features

Patient number	Height (inches)	Weight (lbs)
11471	72	190
9412	60	124
12911	65	170
42311	75	225
32141	50	85

Patient number	Height (mi)	Weight (g)
11471	0.001136	86182.6
9412	0.0009470	56245.5
12911	0.0010259	77110.7
42311	0.0011837	102058
32141	0.0007891	38555.4

The need for standardized features

We can take an unknown feature and "standardize" it to have a mean of zero and a standard deviation of one. This way we can have some intuition about how far away a value is from "average" (zero mean).

This is a very common operation for most machine learning and statistics tasks, where often you have more than just two features -- sometimes you have two million! How would you ever make sense of all those units?

How to standardize data

```
data = [[89.0, 66.0, 23.0, 94.0],
        [137.0, 40.0, 35.0, 168.0],
        [78.0, 50.0, 32.0, 88.0],
        [197.0, 70.0, 45.0, 543.0],
        [189.0, 60.0, 23.0, 846.0],
        [166.0, 72.0, 19.0, 175.0],
        [118.0, 84.0, 47.0, 230.0],
        [103.0, 30.0, 38.0, 83.0],
        [115.0, 70.0, 30.0, 96.0],
        [126.0, 88.0, 41.0, 235.0]]
```

$$\mu_j = 1/N * \sum_{i=0}^{N-1} m_{i,j}$$

$$\sigma_j = \sqrt{1/N * \sum_{i=0}^{N-1} (m_{i,j} - \mu_j)^2}$$

$$m_{i,j}' = (m_{i,j} - \mu_j)/\sigma_j$$

The regular python way of standardizing features

```
def standardize_features(data):
    N = len(data)
    M = len(data[0])
    # initialize the result w/ NxM list of lists of zeros.
    rv = []
    for i in range(N):
        rv.append([0] * M)
    # for each feature
    for j in range(M):
        mu = compute_feature_mean(data, j)
        sigma = compute_feature_stdev(data, j)
       # standardized feature
       for i in range(N):
           rv[i][j] = (data[i][j] - mu)/sigma
    return rv
```

$$\mu_j = 1/N * \sum_{i=0}^{N-1} m_{i,j}$$

```
def compute_feature_mean(data, j):
    111
    Compute the mean of feature (column) j
    Inputs:
      data (list of list of floats)
    Returns (float): mean of feature j
    111
    N = len(data)
    total = 0
    for i in range(N):
        total += data[i][j]
    return total/N
```

```
\sigma_j = \sqrt{1/N * \sum_{i=0}^{N-1} (m_{i,j} - \mu_j)^2}
```

```
def compute_feature_stdev(data, j):
    111
    Compute the standard deviation of feature (column) j
    Inputs:
      data (list of lists of floats)
    Returns (float): standard deviation of feature j
    111
   N = len(data)
    mu = compute_feature_mean(data, j)
    total = 0
    for i in range(N):
        total = total + (data[i][j] - mu) ** 2
    return math.sqrt(1 / N * total)
```

Feature standardization with NumPy

```
def standardize_features(data):
   1 1 1
    Standardize features to have mean 0.0 and standard deviation 1.0.
     Inputs:
       data (2D NumPy array): data to be standardized
      Returns (2D NumPy array): standardized data
    1 1 1
    mu vec = data.mean(axis=0)
    sigma vec = data.std(axis=0)
    return (data - mu_vec)/ sigma_vec
```

Simply feature standardization with Numpy

We significantly simplify the above code with NumPy:

- 1. The NumPy code is less work to write
- 2. The functions in NumPy are already thoroughly debugged, reducing the likelihood of a mistake
- The NumPy code will be faster, and will therefore let us handle more data.

Quiz

```
Assume x = np.array([1.0, 2.0, 3.0]).
Which is closest to the output of x / x.sum()
```

- A. 2
- np.array([0.166, 0.333, 0.5])
- np.array([1.0, 2.0, 3.0])

Additional fun!

- np.where:
 - Can use as within a matrix to select indices based on conditions
 - ► Can use as a standalone for conditional replacement

```
m = np.array([[3,2,1],[3,2,1],[1,2,3]])
m[np.where(m>1)]
np.where(m>1, m, m+2)
```

List VS array

- Different data types allowed
- +: concatenation
- Pop(), append(), extend()
- List as a whole unit

- Efficiency
- ► Index[], [], []

- Same data type required
- Mathematical
- Element-wise
- Broadcasting
- ► Fast, generic
- ▶ Index, [, , ,]
- reshape

NumPy: methods and routines

- zeros: pass tuple (*shape, dtype*)
- ones: pass tuple (shape, dtype)
- random: pass tuple (size)
- arange: pass ([start,]stop, [step,]dtype)
- linspace: pass (start, stop, num=50, endpoint=True, dtype=None, axis=0)
- loadtxt: loads data into an array (needs file name)
- savetxt: saves data into an array (needs file name)
- where: can return indices or conditional replacement

(Time permitting) Task 4

- Create a 20 x 20 array filled with random numbers drawn from a standard normal distribution
- Export it
- Sum along rows
- Sum along columns
- Calc the average