THE UNIVERSITY OF CHICAGO

MASTERS IN COMPUTATIONAL SOCIAL SCIENCE
THE UNIVERSITY OF CHICAGO

MACS 30111

NumPy

The fundamental package for scientific computing with Python

Chapter: 4.2
Documentation: https://numpy.org/devdocs/user/

Topics:

- Function-based API VS OOP
- Class construction, attributes, and methods
- **Object-oriented modeling**
- Class composition
- Private attributes
- Dunder methods

Modeling students

We could represent students with a dictionary of student data.

```
d1 = {"name": "Chloe Student",
      "majors": ["Computer Science"],
      "year": 2}

d2 = {"name": "Sam Student",
      "majors": ["Computer Science", "Political Science"],
      "year": 1}

d3 = {"name": "Sophia Student",
      "majors": ["Mathematics"],
      "year": 4}

d4 = {"name": "Marcus Student",
      "majors": ["Sociology"],
      "year": 4}

dict_students = [d1, d2, d3, d4]
```

Modeling students

A better way to represent students is to create a Student class.

```
class Student:

    def __init__(self, name, majors, year):
        self.name = name
        self.majors = majors
        self.year = year

s1 = Student("Chloe Student", ["Computer Science"], 2)
s2 = Student("Sam Student",
             ["Computer Science", "Political Science"], 1)
s3 = Student("Sophia Student", ["Mathematics"], 4)

s4 = Student("Marcus Student", ["Sociology"], 4)

students = [s1, s2, s3, s4]
```

What is a 'student'?
What 'properties' do they have?

Modeling students

In particular, we can add methods that associate operations with the Student class.

```
class Student:

    def __init__(self, name, majors, year):
        self.name = name
        self.majors = majors
        self.year = year

    def num_majors(self):
        return len(self.majors)

    def __repr__(self):
        return "Student: {}".format(self.name)

# Create Student objects
```

Valid input

The assert statement can be used in a class constructor to check input.

```
class Student:

    def __init__(self, name, majors, year):
        assert isinstance(name, str), "name must be a string"
        assert isinstance(majors, list) and \
               all([isinstance(major, str) for major in majors]),\
               "majors must be a list of strings"
        assert 1 <= year <= 4, "year must be between 1 and 4"

        self.name = name
        self.majors = majors
        self.year = year

    # Student methods
```

Learning check

Try:

```
for s in students:
    print(s)
```

```
class Student:

    def __init__(self, name, majors, year):
        assert isinstance(name, str), "name must be a string"
        assert isinstance(majors, list) and \
               all([isinstance(major, str) for major in majors]),
         "majors must be a list of strings"
        assert 1 <= year <= 4, "year must be between 1 and 4"

        self.name = name
        self.majors = majors
        self.year = year

s1 = Student("Chloe Student", ["Computer Science"], 2)
s2 = Student("Sam Student",
             ["Computer Science", "Political Science"], 1)
s3 = Student("Sophia Student", ["Mathematics"], 4)

s4 = Student("Marcus Student", ["Sociology"], 4)

students = [s1, s2, s3, s4]
```

Internal representation

The Student class with three attributes: name, major, and year.

```
class Student:

    def __init__(self, name, majors, year):
        self.name = name
        self.majors = majors
        self.year = year

    def num_majors(self):
        return len(self.majors)

    def __repr__(self):
        return "Student: {}".format(self.name)
```

```
# Create Student objects
students = [s1, s2, s3]

# Print number of majors
for s in students:
    print(s)
    print("Number of majors:",
          s.num_majors())
    print()
```

Internal representation

We can change the internals of Student without affecting the users of our class when they use it **moving forward**

```
class Student:

    def __init__(self, name, majors, year):
        self.name = name
        self.primary_major = majors[0]
        self.secondary_majors = majors[1:]
        self.year = year

    def num_majors(self):
        return 1 + len(self.secondary_majors)

    def __repr__(self):
        return "Student: {}".format(self.name)
```

```
# Create Student objects
students = [s1, s2, s3]

# Print number of majors
for s in students:
    print(s)
    print("Number of majors:",
          s.num_majors())
    print()
```

Real-world example

The course registration website defines these classes:

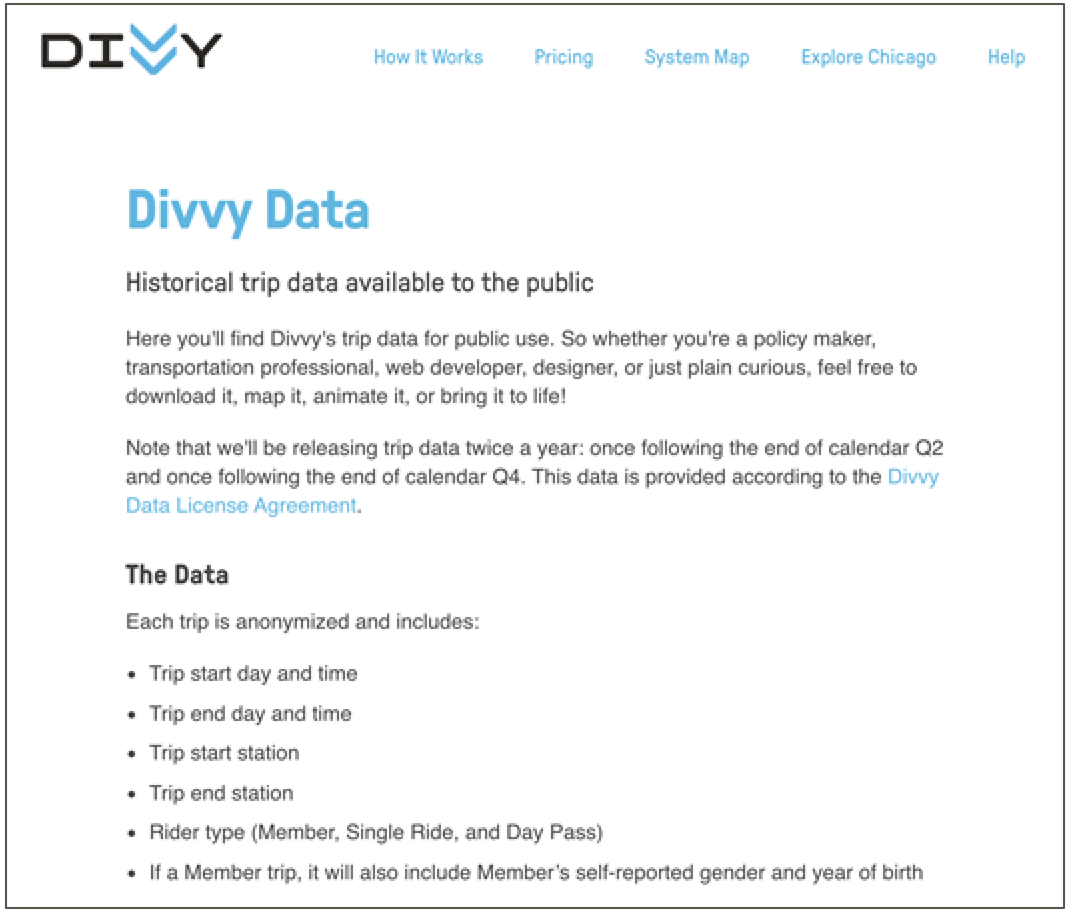
- Student
- Major
- Instructor
- Quarter
- Course
- LectureSlot
- LectureSection
- WaitlistRequest
- …

Topics:

- Function-based API VS OOP
- Class construction, attributes, and methods
- Object-oriented modeling
- **Class composition**
- Private attributes
- Dunder methods

Divvy Data Challenge

What is the total duration and total distance of all the Divvy trips taken in 2013?


DIVVY

How It Works | Pricing | System Map | Explore Chicago | Help

Divvy Data

Historical trip data available to the public

Here you'll find Divvy's trip data for public use. So whether you're a policy maker, transportation professional, web developer, designer, or just plain curious, feel free to download it, map it, animate it, or bring it to life!

Note that we'll be releasing trip data twice a year: once following the end of calendar Q2 and once following the end of calendar Q4. This data is provided according to the Divvy Data License Agreement.

The Data

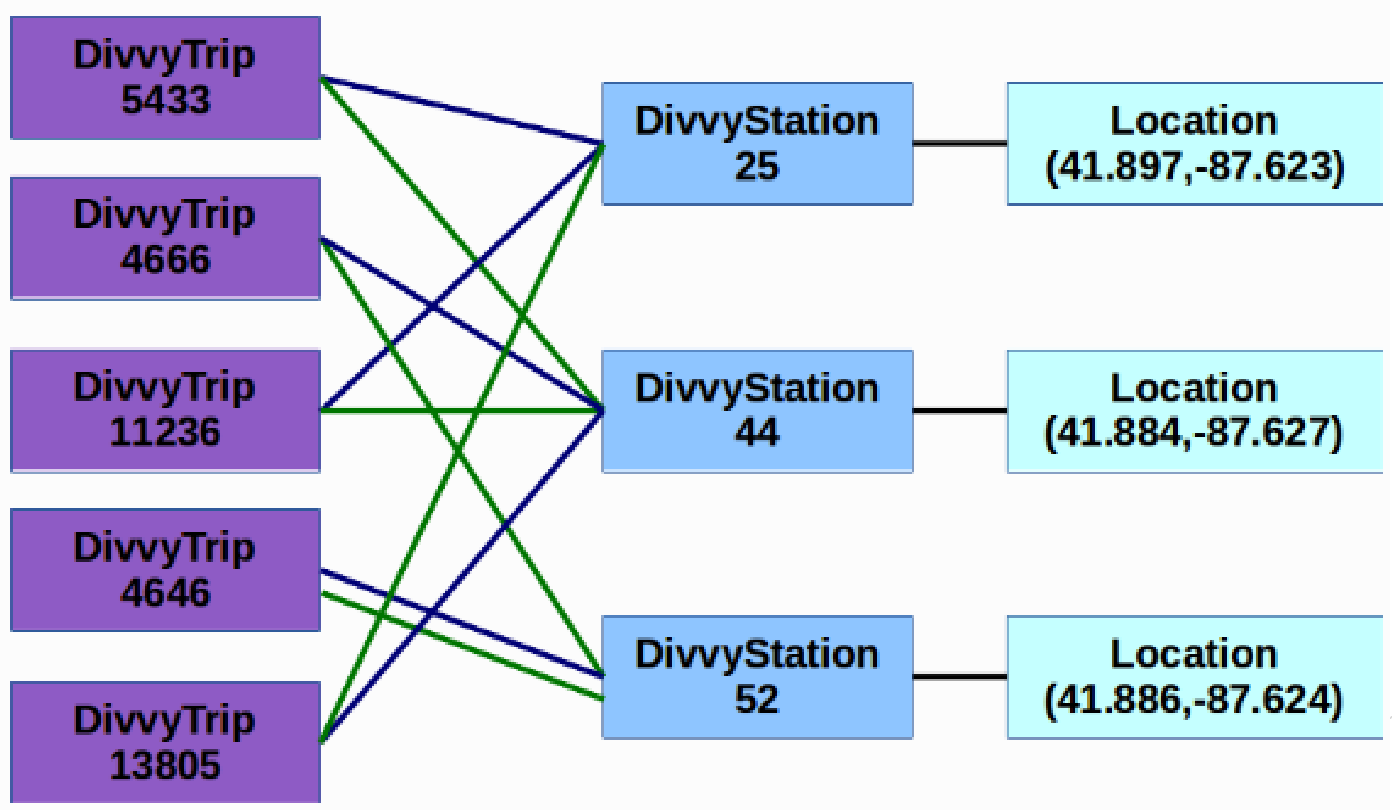
Each trip is anonymized and includes:

- Trip start day and time
- Trip end day and time
- Trip start station
- Trip end station
- Rider type (Member, Single Ride, and Day Pass)
- If a Member trip, it will also include Member's self-reported gender and year of birth

https://www.divvybikes.com/system-data

Class composition

Using DivvyStation **objects as attributes** of the DivvyTrip class is an example of *class composition*.



Coding practice: 2.4.2

Recap

- Classes are ways to deal with objects / formalize making copies of a “THING” with rules
- Super handy in agent based modeling
- They have properties or attributes you can access

NUMPY

Topics:

- Introduction and Motivation for array programming
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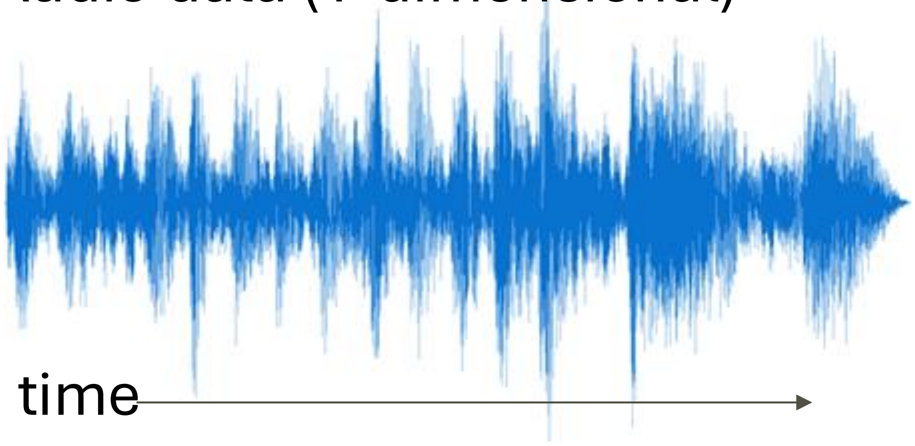
- Indexing into Numpy arrays
- Working with arrays and array operations
- Advanced array manipulations
- An extended example of standardizing features

The need for high-dimensional array

Audio data (1-dimensional)



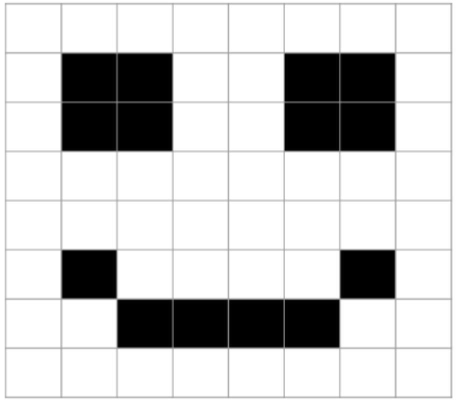

```
array([-0.00270751, -0.00303302, -0.00159557, ..., -0.0012889 ,
       -0.00184731, -0.00210062], dtype=float32)
```

Kaggle/UrbanSound8K

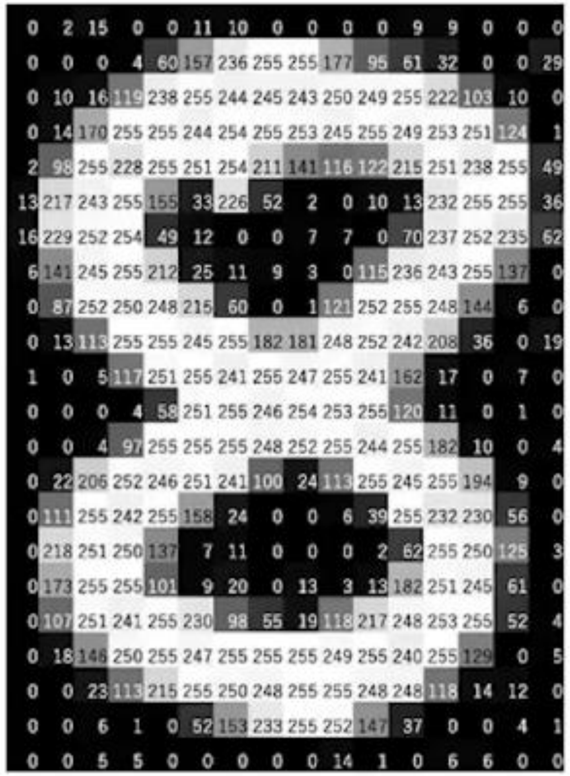
- Machine learning
- Deep learning

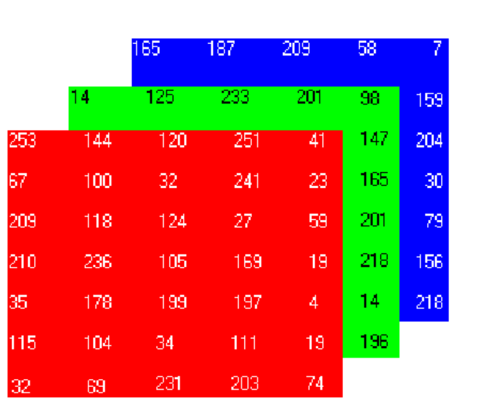
The need for high-dimensional array

Image data



0	0	0	0	0	0	0	0
0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	0	0	0	0	1	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0



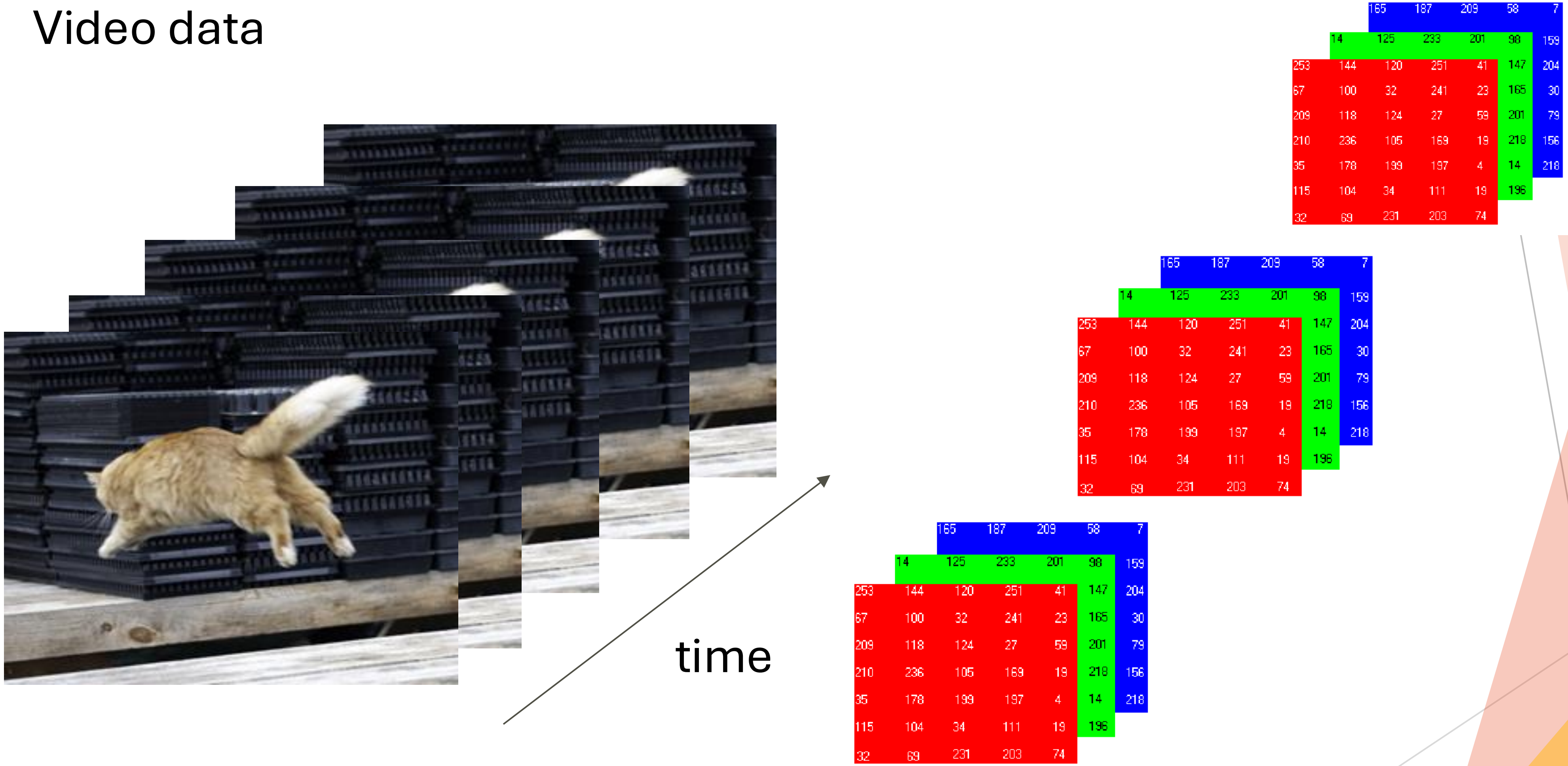


- Machine learning
- Deep learning

[Kaggle/Animals 151]

The need for high-dimensional array

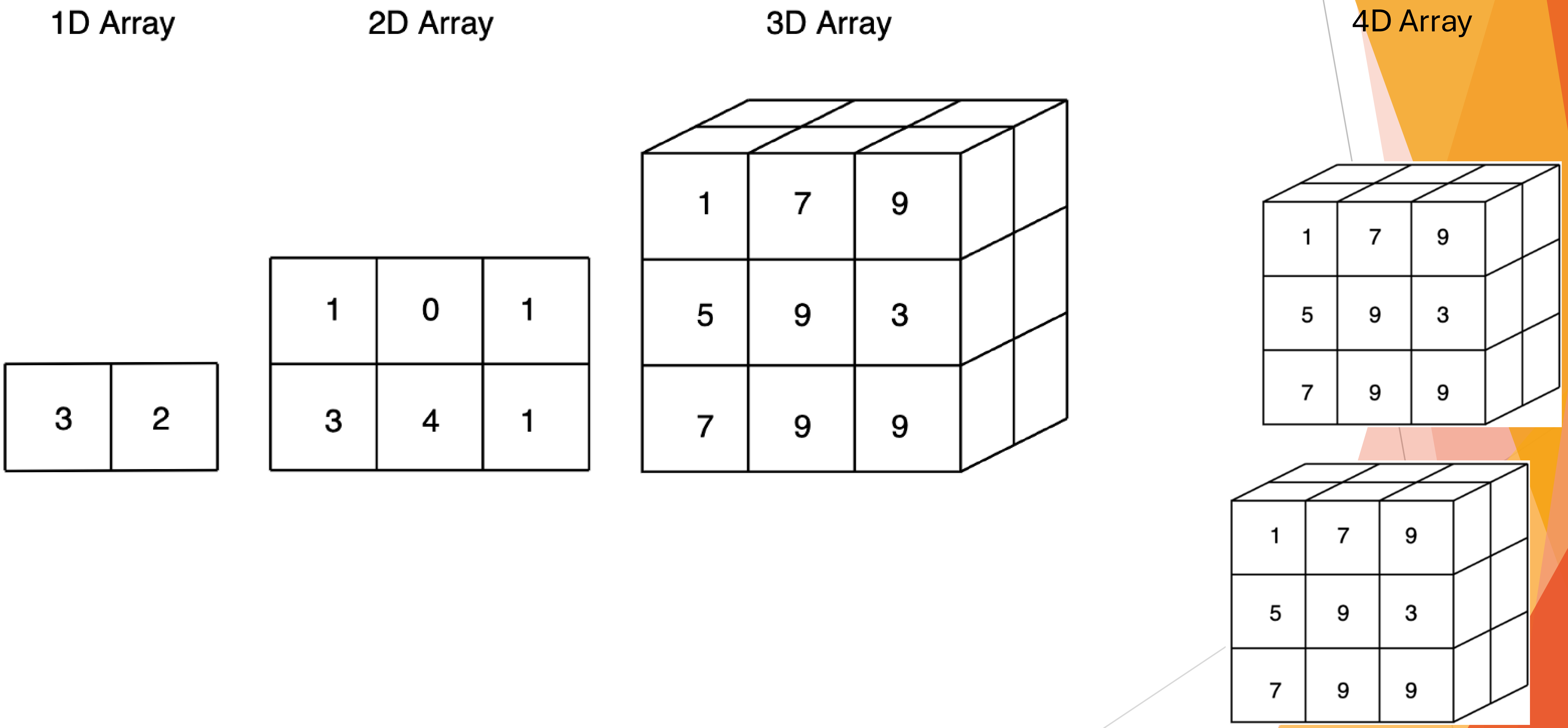
Video data



Kaggle/youtube-video-dataset

- Machine learning
- Deep learning

The need for high-dimensional array



Nested Lists

```
[["M", "M", "F", "M", "M"],
 ["F", "B", "B", "B", "F"]
 ["M", "M", "M", "M", "B"]
 ["B", "B", "B", "F", "B"]
 ["B", "M", "M", "M", "F"]]
```

(0,0) 0	(0,1) 0	(0,2) 1	(0,3) 1	(0,4) 0
(1,0) 0	(1,1) 1	(1,2) 1	(1,3) 2	(1,4) 0
(2,0) 0	(2,1) 0	(2,2) 2	(2,3) 2	(2,4) 1
(3,0) 1	(3,1) 2	(3,2) 1	(3,3) 1	(3,4) 2
(4,0) 0	(4,1) 1	(4,2) 1	(4,3) 0	(4,4) 2

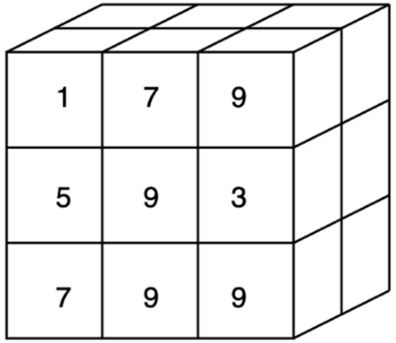
Benefits: Easy to code up
(you already know how!)

Drawbacks

Sum the elements in 1, 2, and 3-dimensional arrays:

3	2

1	0	1
3	4	1




```
sum = 0
for i in range(N):
    sum += one_dim_list[i]
    for j in range(M):
     sum += two_dim_list[i][j]
```

```
sum = 0
for i in range(N):
    for j in range(M):
        for k in range(N):
            sum +=
three_dim_list[i][j][k]
```

- Writing dimension-independent generic code is difficult when using lists-of-lists.

Drawbacks: for loop VS numpy operation

```
X = make_big_list(10000000)

total_sum = 0
for i in range(10000000):
    total_sum += X[i]
```

1.17 seconds

```
X = make_big_array(10000000)


total_sum = np.sum(X)
```

0.007 seconds

Numpy operation is faster

- Writing your own for loops can be slow. Performance matters when working with large datasets!

Numpy: scientific computing with n-d array

NumPy

NumPy (pronounced "numb-pie") is an open-source library in Python. It supports:

1. Multidimensional data structure (the ndarray)
2. Fast mathematical operations
3. An interface that reuses much of the Python list interface

Tradeoff: NumPy arrays can only hold values of the same type

Python list can hold different data types.
A NumPy array can only hold the same type: integers, floats, Boolean.

- Most numpy operations are performed on all elements together, different data types have different operations.

```python
a = [1, 2.0, 'cat', False]
type(a)
```

```
list
```

```python
import numpy as np
b = np.array([True, False, False])
type(b)
```

```
numpy.ndarray
```

Reference:

- https://numpy.org/devdocs/user/absolute_beginners.html
- https://numpy.org/doc/stable/user/basics.types.html

Topics:

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Creating new NumPy arrays

- From existing lists
- Creating arrays with the same value
- Creating arrays with a range of values (e.g., arange(), linspace)
- Joining arrays (e.g. np.array([array1, array2]))
- The data type, or “dtype”, of an array
- Array Shape

Coding practice: 4.2.2

Creating an array from an existing list

First, we usually import the NumPy library using a shorter name, “np”

```
import numpy as np
```

Then we can create a new array from a list

```
x = np.array([1, 2, 3])
```

We can then index into the array like a list:

```
print(x[2])
```

```
3
```

å practice: 4.2.2

Creating multi-dimensional arrays from nested lists

To create a two-dimensional array, we can use two nested lists:

```
y = np.array([[1, 2, 3], [4, 5, 6]])
```

To print this array:

```
1 print(y)
```

```
[[1 2 3]
 [4 5 6]]
```

To access elements in this arra

```
1 print(y[1,2])
```

```
6
```

Coding practice: 4.2.2

Creating arrays: constant values

If you need to create an array of all zeros, you can use np.zeros

```
np.zeros(10)
```

```
array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

You can create multidimensional arrays like this as well, by passing a shape-like tuple

```
np.zeros((3, 5))
```

```
array([[0., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0.]])
```

Num … of ones like this

```
np.ones((5, 8))
```

```
array([[1., 1., 1., 1., 1., 1., 1., 1.],
       [1., 1., 1., 1., 1., 1., 1., 1.],
       [1., 1., 1., 1., 1., 1., 1., 1.],
       [1., 1., 1., 1., 1., 1., 1., 1.],
       [1., 1., 1., 1., 1., 1., 1., 1.]])
```

Coding practice: 4.2.2

Creating arrays: ranges

Numpy supports creating arrays of numerical ranges with the arange and linspace functions.

`np.arange(start, stop, step)`

- stop: exclusive, just like Python's range()
- step: step size

`np.linspace(start, stop, numsteps)`

- start: first value in array
- stop: last value in array, inclusive
- numsteps: total number of steps in array

```
1  np.arange(1,5)
```

```
array([1, 2, 3, 4])
```

```
1  np.linspace(0, 1, 5)
```

```
array([0.  , 0.25, 0.5 , 0.75, 1.  ])
```

Reference:

- https://numpy.org/doc/stable/reference/generated/numpy.arange.html
- https://numpy.org/doc/stable/reference/generated/numpy.linspace.html#numpy.linspace

Coding practice: 4.2.2

Array types

Remember that the contents of NumPy arrays must all be of the same type.

.dtype: the array's data type

```
x = np.array([1, 2, 3])
print(x)
```

```
[1 2 3]
```

```
x.dtype
```

```
dtype('int64')
```

Integers (64-bit)

```
x = np.array([1.2, 2.5, 3.7])
print(x)
```

```
[1.2 2.5 3.7]
```

```
x.dtype
```

```
dtype('float64')
```

Floating-point (64-bit)

Coding practice: 4.2.2

The dimension of an arrays: shape

The .shape attribute: a tuple describing the length in each dimension

```
x = np.array([1, 2, 3])
x.shape
```

```
(3,)
```

```
y = np.array([[1, 2, 3],
              [4, 5, 6]])
y.shape
```

```
(2, 3)
```

- The length of the shape attribute : the number of dimensions
- One-dimension: row/column vectors

Coding practice: 4.2.2

Numpy: methods and routines

https://numpy.org/doc/stable/reference/

zeros: pass tuple (***shape***, ***dtype***)

ones: pass tuple (***shape***, ***dtype***)

random: pass tuple (**size**)

arange: pass (**[*start*,]*stop*, [*step*,]*dtype***)

linspace: pass
(***start***, ***stop***, ***num=50***, ***endpoint=True***, ***dtype=None***, ***axis=0***)

loadtxt: loads data into an array (needs file name)

Task 1

- Create an array of 3 x 5 filled with zeros
- Create an array of 3 x 5 filled with ones
- Create an array of 3 x 5 that has random values from 0 to 1 in it

- Then, combine all these arrays into one array.
  - Find its shape and data type

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One-dimensional array indexing

Same as a Python list:

```
In [89]: x = np.array([1,2,3])


In [90]: print(x[2])
3
```

Coding practice: 4.2.3

Multi-dimensional array indexing

We can create a two dimensional array

```
In [92]: y = np.array([[1,2,3],[4,5,6]])


In [93]: y
Out[93]:
array([[1, 2, 3],
       [4, 5, 6]])
```

What if we only use one dimension? *y[0]*

```
[[1  2  3]
 [4  5  6]]
```

And index with *y[0, 1]*

```
[[1  2  3]
 [4  5  6]]
```

two indices separated by a comma

Coding practice: 4.2.3

Array slicing

The same Python slicing operations

```
In [99]: y = np.array([[0,1,2,3],[4,5,6,7],[8,9,10,11]])
```

```
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])
```

```
y[0,:3]
```

```
[[0, 1, 2, 3]
 [4, 5, 6, 7]
 [8, 9, 10, 11]]
```

```
y[1:,:3]
```

```
[[0, 1, 2, 3]
 [4, 5, 6, 7]
 [8, 9, 10, 11]]
```

Coding practice: 4.2.3

Task: Ed post

- Set your seed to 13 (why? How?)
- Create a matrix of random integers from 0 to 10 (inclusive) that is 5 x 5
- Slice your matrix so that you have the following:
  - Rows: odd numbered rows
  - Columns: numbered one through three

Quiz

X is an array with X.shape = (5, 10). How many elements are returned by X[:3, :2]?

1. 3
2. 5
3. 6

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Mathematical operations with arrays

- List operation: whole list as a unit
- Array operation: element-by-element

- +, -, *, /, %
- >, <, ==
- np.sin, np.cos

Array*array: not matrix operation

Coding practice: 4.2.4

Reference:

- https://numpy.org/doc/stable/reference/routines.statistics.html
- https://numpy.org/doc/stable/reference/routines.linalg.html
- https://numpy.org/doc/stable/reference/routines.logic.html

Reshaping arrays

.reshape: change the shape of an array:

- one-dimensional to two-dimensional
- two dimensional back into one-dimensional

Note:

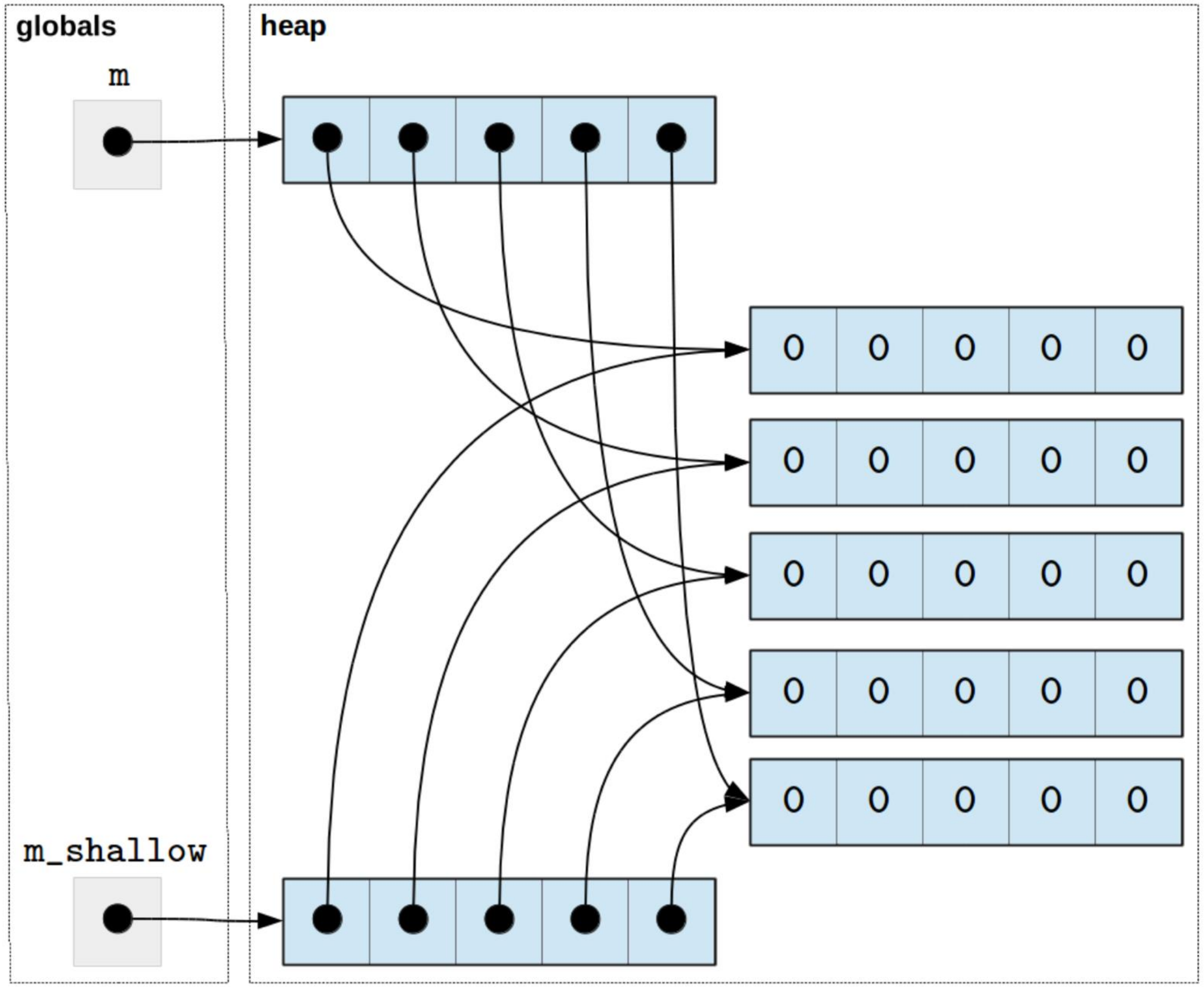
- the new array must have the same number of elements as the original array
- element updates on reshaped array mirrors back to the original array

Coding practice: 4.2.5

Task 2.1

- Create a matrix that goes from 0 to 10 that has the dimensions of 3x5. Divide the range evenly across the 15 values.

Reshaping arrays



Coding practice: 2.1.11

Reshape along an axis (2d (matrix) example)

Axis = 0: goes along ROWS

Axis = 1: goes along COLUMNS

Reductions on arrays

Reduce arrays to lower dimensions:

- operation across all elements or along specific dimensions

```
array([[1, 2, 3],
       [4, 5, 6]])
```

```
np.sum(x, axis=0)
x[:,0], x[:,1], x[:2]
array([5, 7, 9])
```

```
array([[1, 2, 3],
       [4, 5, 6]])
```

```
np.sum(x, axis=1)
x[0,:], x[1,:]
array([ 6, 15])
```

- slicing in the axis dimension
- every index in the non-axis dimension

Coding practice: 4.2.6

Vectors, Matrices, Tensors

- **Vector** (akin to list) : one-d arrays (only one axis)
  - MATH WORKS WEIRD HERE IN NUMPY!!
  - Can reshape into a ‘matrix’ (meaning you give it a second dimension)
- **Matrix** (like regular 2d matrix): two axes (0 and 1)
  - Math is OK
- **Tensor** (multiple / higher dimensions) (AAHHH!)
  - Harder to visualize but possible (good for dimension reduction!)

Quiz

```
x = np.array([1, 2, 3, 4])
```

What is the value of x * 2?

- A. np.array([1, 2, 3, 4, 1, 2, 3, 4])
- B. np.array([3, 4, 5, 6])
- C. np.array([2, 4, 6, 8])

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Advanced array manipulations

1. **Fancy indexing:**
   - Indexing with a list/an arrays of indexes (array indexing)
   - Indexing with an array of Booleans to select elements (masked indexing)
2. Performing operations between arrays of different dimensions ("**broadcasting**")

Coding practice: 4.2.7

Index a subset of elements

```
a = np.array([1, 4, 9, 16, 25, 36, 49])
```

Array indexing with:

- one index (e.g., a[3])

```
a = np.array([1, 4, 9, 16, 25, 36, 49])
```

- a list of indexes (e.g., a[3, 1, 6])

```
a = np.array([1, 4, 9, 16, 25, 36, 49])
```

- a multi-dimensional arrays of indexes (e.g., a[np.array([[1,3], [5,2]])])

```
array([16,  4, 49])
```

```
a = np.array([1, 4, 9, 16, 25, 36, 49])
```

```
array([[ 4, 16],
       [36,  9]])
```

Coding practice: 4.2.7

Masked indexing

Coding practice: 4.2.7

Select a subset of elements via a boolean array:

```
c = np.array([100, 200, 300])
```

```
mask = np.array([True, False, True])
```

Relational operations:

```
c>100
```

```
array([False,  True,  True])
```

```
c[c>100]
```

```
array([200, 300])
```

```
c[(c>100) & (c/100==2)]
```

```
array([200])
```

```
c[(c>100) | (c/100==2)]
```

```
array([200, 300])
```

Task 3: bringing it all together

- **Create a 10 x 10 matrix** of random numbers between 0 and 100 (inclusive) with a seed of 10.
- **Create a sub-matrix** of this where you select even-numbered rows and odd-numbered columns. If they are greater than 50, keep the values – make them zero otherwise

Quiz

Consider the NumPy arrays `x = np.array([1, 3, 5, 7, 9])` and `y = np.array([1, 2, 3, 2, 1])`

What is the value of the expression x [y < 2] ?

a. np.array([True, False, False, False, True])
b. np.array([True, 3, 5, 7, True])
c. np.array([1, 9])
d. np.array([1, 3, 7, 9])

Advanced topics

- Broadcasting
- Standardizing

Broadcasting

- It's possible to perform operations on arrays that have compatible but not identical shapes. In these cases, NumPy *logically* **constructs intermediate values that have the same shape using *broadcasting*** before performing the element-by-element operations.

Broadcasting

You can perform operations (e.g., +, *) on arrays with compatible shapes.

```
x = np.array([1, 2, 3])
y = np.array([7, 8])
x.shape, y.shape
```

```
((3,), (2,))
```

- same sizes for all dimensions
- one of the dimension is 1

```
x * y
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
<ipython-input-99-e32109319f52> in <module>
----> 1 x * y

ValueError: operands could not be broadcast together with shapes (3,) (2,)
```

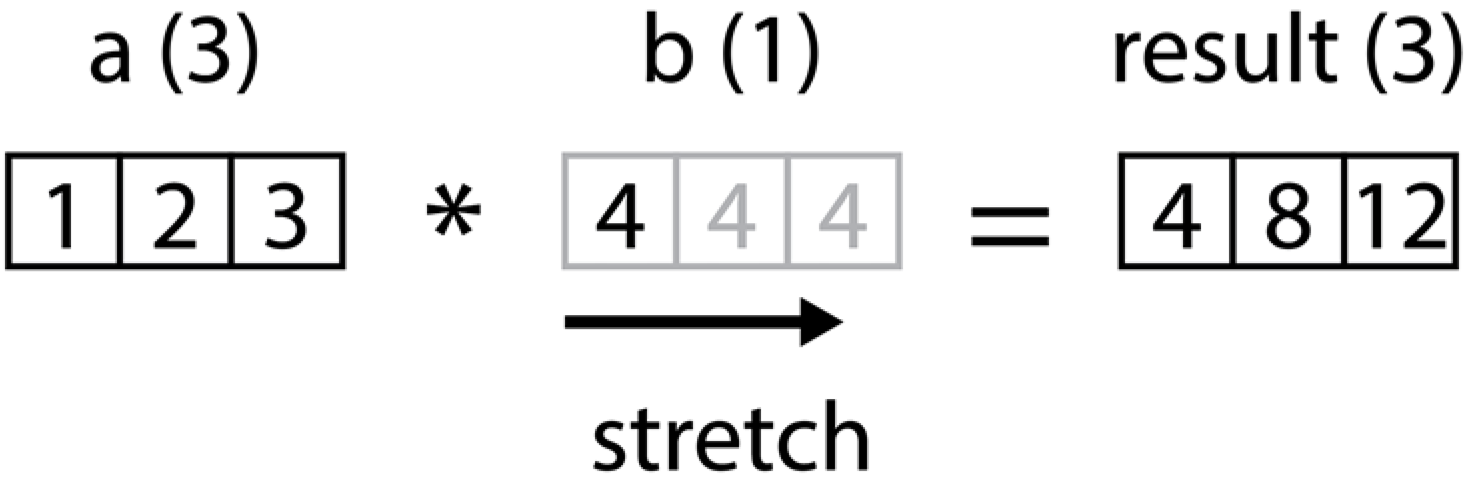
Reference: https://numpy.org/doc/stable/user/basics.broadcasting.html

Broadcasting

Multiply an array by a scalar

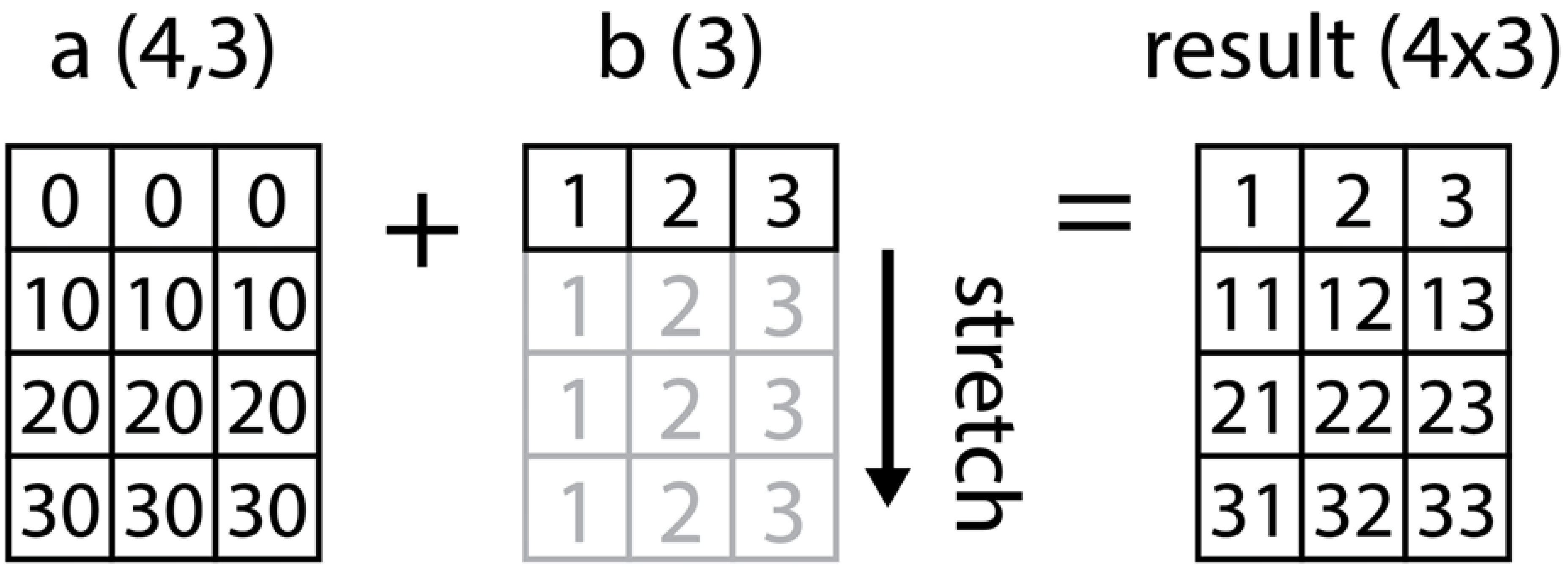
```
x = np.array([1, 2, 3])
x * 4
```

```
array([ 4,  8, 12])
```



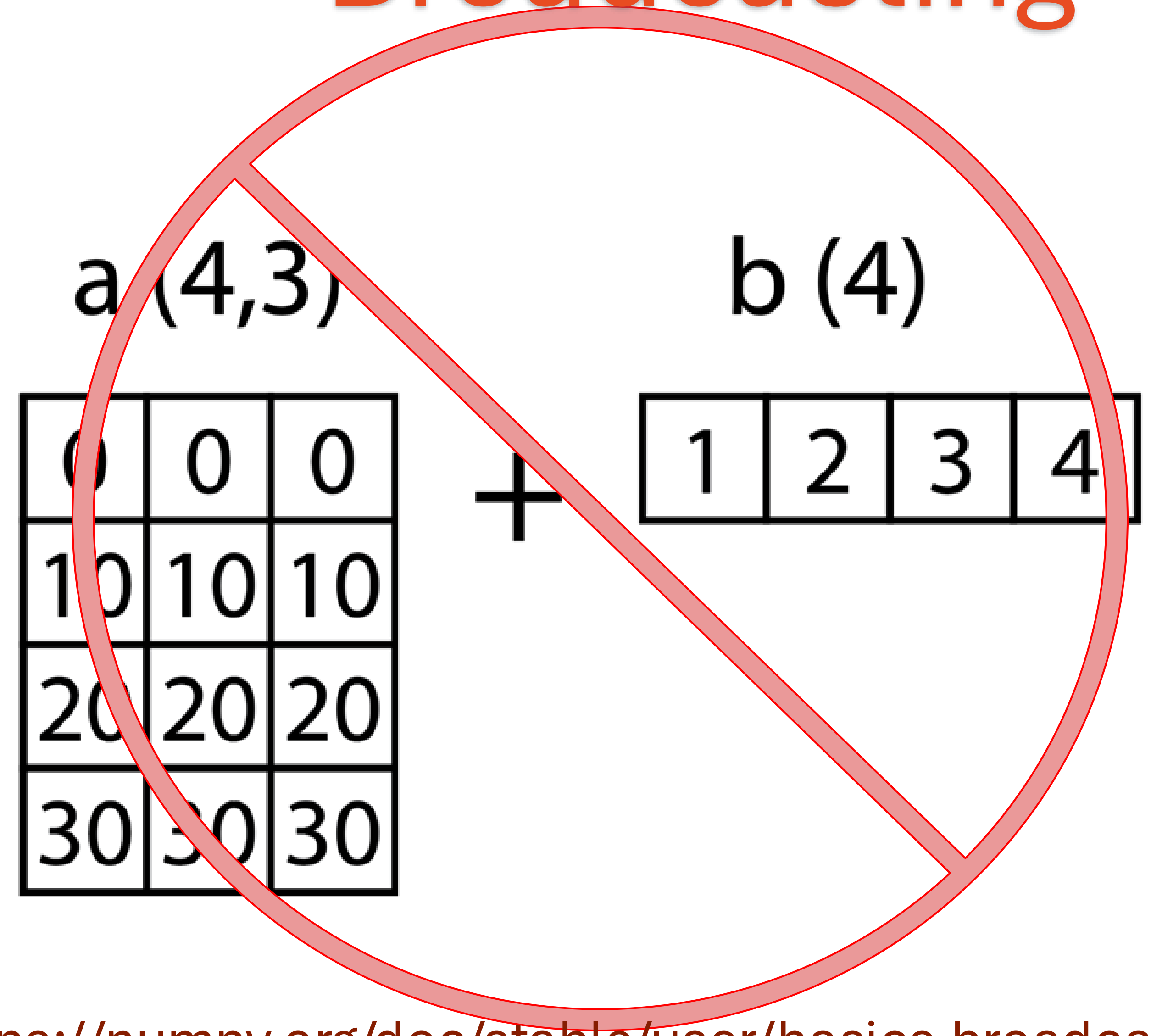
Numpy handles dimensions when two arrays' shapes don't agree.
E.g., if a dimension is one, numpy magically stretches array b to match the shape of array a.

Broadcasting



Reference: https://numpy.org/doc/stable/user/basics.broadcasting.html

Broadcasting



Reference: https://numpy.org/doc/stable/user/basics.broadcasting.html

Broadcasting

Only valid for compatible arrays

Compare array shape from right to left by each dimension:

- equal
- or one of them is 1

```
Image  (3d array): 256 x 256 x 3
Scale  (1d array):             3
Result (3d array): 256 x 256 x 3
```

```
A      (4d array):  8 x 1 x 6 x 1
B      (3d array):      7 x 1 x 5
Result (4d array):  8 x 7 x 6 x 5
```

```
A      (3d array):  15 x 3 x 5
B      (3d array):  15 x 1 x 5
Result (3d array):  15 x 3 x 5
```

```
A      (1d array):  3
B      (1d array):  4
```

```
A      (2d array):      2 x 1
B      (3d array):  8 x 4 x 3
```

Reference: https://numpy.org/doc/stable/user/basics.broadcasting.html

Topics:

- Introduction and Motivation for array programming
- Creating Numpy arrays
- Indexing into Numpy arrays
- Working with arrays and array operations
- Advanced array manipulations
- **An extended example of standardizing features**

Standardizing

- Allows you to compare items with others, even when they have different scales

The need for standardized features

Patient number	Height (inches)	Weight (lbs)
11471	72	190
9412	60	124
12911	65	170
42311	75	225
32141	50	85

Patient number	Height (mi)	Weight (g)
11471	0.001136	86182.6
9412	0.0009470	56245.5
12911	0.0010259	77110.7
42311	0.0011837	102058
32141	0.0007891	38555.4

The need for standardized features

We can take an unknown feature and “standardize” it to have a mean of zero and a standard deviation of one. This way we can have some intuition about how far away a value is from “average” (zero mean).

This is a very common operation for most machine learning and statistics tasks, where often you have more than just two features -- sometimes you have two million! How would you ever make sense of all those units?

How to standardize data

```
data = [[89.0, 66.0, 23.0, 94.0],
        [137.0, 40.0, 35.0, 168.0],
        [78.0, 50.0, 32.0, 88.0],
        [197.0, 70.0, 45.0, 543.0],
        [189.0, 60.0, 23.0, 846.0],
        [166.0, 72.0, 19.0, 175.0],
        [118.0, 84.0, 47.0, 230.0],
        [103.0, 30.0, 38.0, 83.0],
        [115.0, 70.0, 30.0, 96.0],
        [126.0, 88.0, 41.0, 235.0]]
```

$$\mu_j = 1/N * \sum_{i=0}^{N-1} m_{i,j}$$

$$\sigma_j = \sqrt{1/N * \sum_{i=0}^{N-1} (m_{i,j} - \mu_j)^2}$$

$$m'_{i,j} = (m_{i,j} - \mu_j)/\sigma_j$$

The regular python way of standardizing features

```
def standardize_features(data):
    N = len(data)
    M = len(data[0])

    # initialize the result w/ NxM list of lists of zeros.
    rv = []
    for i in range(N):
        rv.append([0] * M)

    # for each feature
    for j in range(M):
        mu = compute_feature_mean(data, j)
        sigma = compute_feature_stdev(data, j)

        # standardized feature
        for i in range(N):
            rv[i][j] = (data[i][j] - mu)/sigma
    return rv
```

$$\mu_j = 1/N * \sum_{i=0}^{N-1} m_{i,j}$$

```
def compute_feature_mean(data, j):
    '''
    Compute the mean of feature (column) j

    Inputs:
      data (list of list of floats)

    Returns (float): mean of feature j
    '''

    N = len(data)
    total = 0
    for i in range(N):
        total += data[i][j]
    return total/N
```

$$\sigma_j = \sqrt{1/N * \sum_{i=0}^{N-1} (m_{i,j} - \mu_j)^2}$$

```python
def compute_feature_stdev(data, j):
    '''
    Compute the standard deviation of feature (column) j

    Inputs:
      data (list of lists of floats)

    Returns (float): standard deviation of feature j
    '''

    N = len(data)
    mu = compute_feature_mean(data, j)
    total = 0
    for i in range(N):
        total = total + (data[i][j] - mu) ** 2
    return math.sqrt(1 / N * total)
```

Feature standardization with Numpy

```
def standardize_features(data):
    '''
    Standardize features to have mean 0.0 and standard deviation 1.0.
     Inputs:
       data (2D NumPy array): data to be standardized
      Returns (2D NumPy array): standardized data
     '''

    mu_vec = data.mean(axis=0)
    sigma_vec = data.std(axis=0)

    return (data - mu_vec)/ sigma_vec
```

Simply feature standardization with Numpy

We significantly simplify the above code with NumPy:

1. The NumPy code is less work to write
2. The functions in NumPy are already thoroughly debugged, reducing the likelihood of a mistake
3. The NumPy code will be faster, and will therefore let us handle more data.

Quiz

Assume `x = np.array([1.0, 2.0, 3.0])`.

Which is closest to the output of x / x.sum()

A. 2
B. np.array([0.166, 0.333, 0.5])
C. np.array([1.0, 2.0, 3.0])
D. 1

Additional fun!

- np.where:
  - Can use as within a matrix to select indices based on conditions
  - Can use as a standalone for conditional replacement

```
m = np.array([[3,2,1],[3,2,1],[1,2,3]])
m[np.where(m>1)]
np.where(m>1, m, m+2)
```

Translation: if m>1, replace it with m. Otherwise, replace it with m+2

List VS array

- Different data types allowed
- +: concatenation
- Pop(), append(), extend()
- List as a whole unit

- Efficiency
- Index[], [], []

- Same data type required
- Mathematical
- Element-wise
- Broadcasting
- Fast, generic
- Index, [, , ,]
- reshape

Numpy: methods and routines

https://numpy.org/doc/stable/reference/

- **zeros**: pass tuple (***shape***, ***dtype***)
- **ones**: pass tuple (***shape***, ***dtype***)
- **random**: pass tuple (**size**)
- **arange**: pass ([***start***,]***stop***, [***step***,]***dtype***)
- **linspace**: pass (***start***, ***stop***, ***num=50***, ***endpoint=True***, ***dtype=None***, ***axis=0***)
- **loadtxt**: loads data into an array (needs file name)
- **savetxt**: saves data into an array (needs file name)
- **where**: can return indices or conditional replacement

Time permitting exercise

- Create a 20 x 20 array filled with random numbers drawn from a standard normal distribution with seed at 89
- Export it
- Sum along rows
- Sum along columns
- Calc the average

Recap

- **Classes**: a great time – objects with properties
- **Numpy**: new way to deal with arrays and can move into matrix land better than with lists
  - Lots of new vocabulary