

HOMework 1

ADAM C. KNAPP

1. PROBLEMS

Friday, February 1

For the questions below, create a document which includes the Matlab code that you create, and any data or graphs that you generate.

Based on the code we created in class, run the Gambler's ruin simulation with a variety of starting wallets. (Ex: 3\$, 7\$, 12\$, 25\$, 50\$, 100\$, 150\$, 200\$, etc.)

Question 1 (Gambler's Ruin). For each, compute the mean and standard deviation of the number of rounds until bankruptcy.

Question 2 (Gambler's Ruin). How does the mean number of rounds until bankruptcy depend on the size of the starting wallet?

The mean, \bar{X} , of N numbers x_1, \dots, x_N is computed in the usual way:

$$\bar{X} = \frac{1}{N} \sum_{i=1}^N x_i$$

The sample standard deviation, s_X , is computed by the formula

$$s_X = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{X})^2}$$

and is a measure of how much variability is in the collection of numbers.

Consider the family of ODEs

$$x' = f(x, a, b) = -x^3 + ax - b$$

parameterized by a, b .

Question 3 (Family of ODEs). Fix $a = 1$ and experiment with various values of b . Find the value of b for which the qualitative behavior of the ODE changes.

Question 4 (Family of ODEs). Fix $b = 0$ and experiment with various values of a . What kind of behavior do we have when $a < 0$ and $a > 0$? How does this relate to the graph of $f(x, a, 0) = -x^3 + ax$?

Question 5 (Family of ODEs). **Bonus:** Formulate and test a conjecture about the (a, b) pairs at which qualitative behavior changes.