# Homework #3

John Devivo

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## Due: Tuesday, September 23

Collaborators: None

## E-1.20

For each of the following languages, give two strings that are members and two strings that are *not* members- a total of four strings for each part. Assume the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ in all parts.

a. $\mathtt{a}^*\mathtt{b}^*$
**Answer:** Members: [ $aab, abbbb$] NonMembers: [$aba, baa$]

b. $\mathtt{a(ba)}^*\mathtt{b}$
**Answer:** Members: [ $abab, ababab$] NonMembers: [$aaaa, bbbbbb$]

c. $\mathtt{a}^* \cup \mathtt{b}^*$
**Answer:** Members: [$aabb, aaaa$] NonMembers: [$ba, bababa$]

d. $(\mathtt{aaa})^*$
**Answer:** Members: [$aaa, aaaaaaaaa$] NonMembers: [$a, abababbb$]

## E-1.46

Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection, and complement.

a. $\{0^n 1^m 0^n | m, n \geq 0\}$
**Answer:** Let $B$ be the language $\{0^n 1^n | n \geq 0\}$. We use the pumping lemma to prove that $B$ is not regular. The proof is by contradiction.
Assume to the contrary that $B$ is regular. Let $p$ be the pumping length given by the pumping lemma. Choose $s$ to be the string $0^p 1 0^p$. Because $s$ is a member of $B$ and $s$ has a length more than $p$, the pumping lemma states that $s$ can be split into three pieces, $s = xyz$, where for any $i \geq 0$ the string $xy^iz$ is in $B$.
When dividing $s$ into $xyz$ we know the pumping lemma that $|xy| \leq p$. As such, since the first $p$ elements in the string $s$ are 0's, we know that both $x$ and $y$ must be composed entirely of 0's. Knowing this we can show the string $s$ cannot be pumped. Since $x$ and $y$ must be composed entirely of 0's, we can express $x = 0^{p-a}$, $y = 0^a$, and $z = 10^p$, making $xyz = 0^{p-a}0^a10^p$. Pumping up, the string $xyyz = 0^{p+a}10^p$ has more 0's on the left hand side of the 1, than the right making $xyyz \notin B$. If $B$ were a regular language then $xyyz$ would be an element of $B$, but since it isn't we reach a contradiction, $B$ is not a regular language.

c. $\{w | w \in \{0,1\}^* \text{ is not a palindrome}\}$
**Answer:** Let $A$ be the language $\{w | w \in \{0,1\}^* \text{ is not a palindrome}\}$. We use closure of regular languages under compliment and the pumping lemma to prove that $A$ is not regular. The proof is by contradiction.
Assume for sake of contradiction that $A$ is regular. Since regular languages are closed under compliment, we know that the compliment of the set $A$, $B = \{w | w \in \{0,1\}^* \text{ is a palindrome}\}$ is a regular language.
Let $p$ be the pumping length of $B$. Choose $s$ to be the string $0^p 1 0^p$. Because $s$ is a member of $B$, and $|s|$ is greater than $p$, the pumping lemma states that $s$ can be split into three pieces $s = xyz$ where for any $i \geq 0$, the string $xy^iz \in B$. Since the pumping lemma states $xy \leq p$, we know that $y$ is composed entirely of 0's. Knowing this we can express $xyz = 0^{p-a}0^a10^p$

with $x = 0^p - a$, $y = 0^a$, and $z = 10^p$. According to the first condition of the pumping lemma, $xyyz \in B$, however we know that $xyyz = 0^{p+a}10^p$ and $0^{p+a}10^p \notin B$ because it is not a palindrome. Here is where we find our contradiction that shows our assumption that $A$ is regular is false. $A$ is not a regular language.

d. $\{wtw|w, t \in \{0,1\}^+\}$
**Answer:** Let $A$ be the language $\{wtw|w, t \in \{0,1\}^+\}$. We will use the pumping lemma to prove that $A$ is not regular. To do this we will assume for sake of contradiction that $A$ is regular.
Let $p$ be the pumping length of $A$. Choose $s$ to be the string $0^p110^p1 \in A$. By the pumping lemma we know that $s$ can be split into three parts $xyz$. We also know from the pumping lemma's second condition that $|xy| \leq p|$ and knowing this we can say that $y = 0^a$ for some $a > 0$. By the pumping lemma we know that $xy^iz \in A$ for some $i \geq 0$. However when we pump up $xyyz = 0^{p+a}110^p1 \notin A$. This provides our contradiction. $A$ is not a regular language.

# E-1.49

a. Let $B = \{1^k y|y \in \{0,1\}^*$ and $y$ contains at least $k$ 1s, for $k \geq 1\}$. Show that $B$ is a regular language.
**Answer:** We can prove that $B$ is a regular language by constructing a regular expression that describes the language. The regular expression $1 \circ 0^\star \circ 1 \circ (0 \cup 1)^\star$

b. Let $C = \{1^k y|y \in \{0,1\}^*$ and $y$ contains at most $k$ 1s, for $k \geq 1\}$. Show that $C$ isn't a regular language.
**Answer:** Assume for sake of contradiction that $C$ is a regular language.
Let $p$ be the pumping length of $C$. Let $s$ be the string $1^p0^p1^p \in C$. The pumping lemma states that $s$ can be split into three sections $xyz$ where $xy \leq p$. With this we know that $x = 1^{p-k}$, $y = 1^k$, and $z = 0^p1^p$ so we see $xyz = 1^p0^p1^p \in C$. The pumping lemma also states that $xy^iz \in C$ for all $i \geq 0$. However if we pump down we see that $xz = 1^{p-k}0^p1^p \notin C$ providing our contradiction. $C$ is not a regular language.

# E-1.53

Let $\Sigma = \{\mathtt{0}, \mathtt{1}, +, =\}$ and

$$ADD = \{x{=}y{+}z|x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$$

Show that $ADD$ is not regular.
**Answer:** Assume for sake of contradiction that ADD is regular.
Let $p$ be the pumping length of ADD. Let $s$ be as string such that $s = 1^p = 1^{p-1}0 + 1 \in$ ADD. The pumping lemma states that $s$ can be broken down into three portions $xyz = 1^p = 1^{p-1}0 + 1$ and since, from the pumping lemma, we know $|xy| \leq p$ and $|y| \geq 1$ we can say $x = 1^{p-k}$, $y = p^k$, and $z == 1^{p-1}0 + 1$. We know from the pumping lemma that $xy^iz \in$ ADD for all $i \geq 0$. However $xy^2z = p^{p+k} = 1^{p-1}0 + 1 \notin$ ADD. This provides our contradiction. ADD is not regular.

# E-1.57 (Extra Credit)

If $A$ is any language, let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in $A$ so that

$$A_{\frac{1}{2}-} = \{x| \text{ for some } y, |x| = |y| \text{ and } xy \in A\}.$$

Show that if $A$ is regular, then so is $A_{\frac{1}{2}-}$.
**Answer:**