

Homework #4

John Devivo

October 9, 2014

Due: Thursday, October 2

Collaborators: None

E-2.1

Recall the CFG G_4 that we gave in Example 2.4. For convenience, let's rename the variables with single letters as follows.

$$E \rightarrow E + T \mid T$$

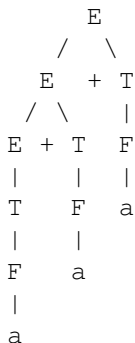
$$T \rightarrow T \times F \mid F$$

$$F \rightarrow (E) \mid a$$

Give parse trees and derivations for each of the following (just c):

c. $a+a+a$

Answer:



Here is a partial derivation:

$E \rightarrow$
 $E + T \rightarrow$
 $E + F \rightarrow$
 $E + a \rightarrow$
 $E + T + a \rightarrow$
 $E + F + a \rightarrow$
 $E + a + a \rightarrow$
 $T + a + a \rightarrow$
 $F + a + a \rightarrow$
 $a + a + a$

E-2.2

- a. Use the languages $A = \{a^m b^n c^n \mid m, n \geq 0\}$ and $B = \{a^n b^n c^m \mid m, n \geq 0\}$ together with Example 2.36 to show that the class of context-free languages is not closed under intersection.

Answer: We begin by assuming that Context Free Languages are closed under the intersection operation for the sake of contradiction.

The first step in using languages A and B to show that CFLs are closed under intersection is to prove that both languages are CFLs. We can do this by constructing context free grammars to represent A and B .

A context free grammar that represents A can be shown as

$$\begin{aligned} S &\rightarrow FG|F|G|\epsilon \\ G &\rightarrow bc|bGc|\epsilon \\ F &\rightarrow a|aF|\epsilon \end{aligned}$$

thus A is a context free language.

A context free grammar that represents B can be shown as

$$\begin{aligned} S &\rightarrow FG|F|G|\epsilon \\ F &\rightarrow ab|aFb|\epsilon \\ G &\rightarrow c|cG|\epsilon \end{aligned}$$

thus B is a context free language.

If the class of context free languages were closed under intersection, the intersection of A and B , $A \cap B = C = \{a^n b^n c^n \mid n \geq 0\}$ would also be a context free language. However, example 2.36 in the book uses the pumping for context free languages to prove that C is not a context free language. We have a contradiction, context free languages are not closed under intersection.

- b. Use part (a) and DeMorgan's law (Theorem 0.20) to show that the class of context-free languages is not closed under complementation.

Answer: In order to prove that the set of context free languages is not closed under compliment, we first assume that they are closed under compliment. This would imply that $D = \bar{A}$ is a context free grammar and $E = \bar{B}$ is a context free language. Since Context free languages are closed under the union operation, we know that $F = D \cup E$ is a context free language.

If CFLs are closed under compliment, we know that \bar{F} is a context free language. Using DeMorgan's law we know that $F = C$, however we showed in part a that C is not a context free language, thus we have our contradiction. Context Free Languages are not closed under compliment.

E-2.4

Give context-free languages that generate the following languages. In all parts, the alphabet is $\Sigma = \{0, 1\}$.

- e. $\{w = w^R, \text{ that is, } w \text{ is a palindrome.}\}$

Answer: $S \rightarrow 0S0|1S1| =$

E-2.6

Give context-free grammars generating the following languages.

- b. The complement of the language $\{a^n b^n \mid n \geq 0\}$.

Answer: $S \rightarrow A|B$
 $A \rightarrow 1A0|1A|1$
 $B \rightarrow 1B0|B0|0$

E-2.19

Let CFG G be the following grammar.

$$\begin{aligned} S &\rightarrow aSb \mid bY \mid Ya \\ Y &\rightarrow bY \mid aY \mid \epsilon \end{aligned}$$

Give a simple description of $L(G)$ in English. Use that description to give a CFG for $\overline{L(G)}$, the complement of $L(G)$.

Answer: The CFG G describes the language of all permutations of $n \geq 0a$ characters and $m \geq 0b$ characters but also excludes the empty set. The complement of $L(G)$ would be a language that only describes the empty string $\{\epsilon\}$.