Homework #2

John Devivo

September 17, 2014

Due: Tuesday, September 16
Collaborators: Ryan Schwarz

E-1.12

Let $D = \{w|w$ contains an even number of a's and an odd number of b's and does not contain the substring ab$\}$. Give a DFA with five states that recognizes D and a regular expression that generates D. (Suggestion: Describe D more simply.)

Answer: The DFA should be saved as the file `1_12.jff`. The regular expression is $b(bb)^*(aa)^*$

E-1.17

a. Give an NFA recognizing the language $(01 \cup 001 \cup 010)^*$.
Answer: The NFA should be saved as the file `1_17_a.jff`.

b. Convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state.
Answer: The DFA should be saved as the file `1_17_b.jff`.

E-1.19

Use the procedure described in Lemma 1.55 to convert the following regular expressions to nondeterministic finite automata.

a. $(0 \cup 1)^*000(0 \cup 1)^*$
Answer: The NFA should be saved as the file `1_19_a.jff`.

b. $(((00)^*(11)) \cup 01)^*$
Answer: The NFA should be saved as the file `1_19_b.jff`.

E-1.36

Let $B_n = \{a^k | k$ is a multiple of $n\}$. Show that for each $n \geq 1$, the language B_n is regular.

Answer: For every multiple of n, k, where $n \geq 1$, k is an integer where $k \geq 1$. We also know that every a, raised to a positive integer is a regular language. Knowing this we can say that all a^k are regular languages. We also know that the union of any number of regular languages will represent a regular language. B_n is the union of all a^k and each a^k represents a regular language. Knowing this we can say that B_n is a regular language.

E-1.38

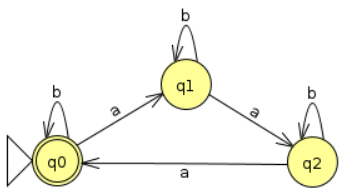
An *all*-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if *every* possible state that M coud be in after reading input x is a state from F. Note, in contrast, that an ordinary NFA accepts a string if some state among these possible states is an accept state. Prove that all-NFAs recognize the class of regular languages.

Answer: In an all-NFA, the set of accept states F can be described as $F = \Sigma^*$, where Σ is the alphabet. We can prove that an all-NFA represents a regular language in two ways, proving it can be converted to a regular expression, or proving that all all-NFAs can be converted to regular NFAs. Showing that an all-NFA can be converted to a regular expression is easiest. Using the same alphabet, every all-NFA can be represented as the regular expression Σ^* since the set of accept string for and all-NFA is the set Σ^*. Since we can convert every all-NFA of any alphabet to an equivalent regular expression, all-NFAs are regular.

E-1.43

Let A be any language. Define DROP-OUT(A) to be the language containing all strings that can be obtained by removing one symbol from a string in A. Thus, DROP-OUT(A) = $\{xz | xyz \in A$ where $x, z \in \Sigma^*, y \in \Sigma\}$. Show that the class of regular languages is closed under the DROP-OUT operation. Give both a proof by picture and a more formal proof by construction as in Theorem 1.47.

Answer: See the following figure for a proof by picture.



Formally, Every language, A, represents a set of strings that are accepted. We define the DROP-OUT operation to take a string $xyz \in A$, and remove a symbol. We are also able to represent A as a regular expression that is the union of every string xyz where $xyz \in A$. We also know that every string of symbols,$xyz \in A$ is a regular expression. Since we know that all $xyz \in A$ are merely strings of characters that are elements of Σ, and we can safely remove one symbol from any string (besides the empty string which has no symbols to remove but is still a valid regular language) and still have that string, xy, be a regular language. Since this is what the DROP-OUT operator does, we know that even when DROP-OUT is called on A, A can still be represented as the union of a number of regular languages, thus making A a regular language. Since DROP-OUT(A), where A is a regular language, yields a regular language as its output, we know that regular languages are closed under the DROP-OUT operation.