Homework #4

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Collaborators: None

E-2.1

Recall the CFG G_4 that we gave in Example 2.4. For convenience, let's rename the variables with single letters as follows.

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T \times F \mid F \\
F &\rightarrow (E) \mid \mathtt{a}
\end{aligned}
$$

Give parse trees and derivations for each of the following (just c):

c. a+a+a

Answer:

```
          E
        /   \
       E   + T
      / \    |
     E + T   F
     |   |   |
     T   F   a
     |   |
     F   a
     |
     a
```

Here is a partial derivation:

$E \rightarrow$
$E + T \rightarrow$
$E + F \rightarrow$
$E + a \rightarrow$
$E + T + a \rightarrow$
$E + F + a \rightarrow$
$E + a + a \rightarrow$
$T + a + a \rightarrow$
$F + a + a \rightarrow$
$a + a + a$

E-2.2

a. Use the languages $A = \{a^m b^n c^n |\ m, n \geq 0\}$ and $B = \{a^n b^n c^m |\ m, n \geq 0\}$ together with Example 2.36 to show that the class of context-free languages is not closed under intersection.
Answer: We begin by assuming that Context Free Languages are closed under the intersection operation for the sake of contradiction.
The first step in using languages A and B to show that CFLs are closed under intersection is to prove that both languages are CFLs. We can do this by constructing context free grammars to represent A and B.
A context free grammar that represents A can be shown as

$S \rightarrow FG|F|G|\epsilon$
$G \rightarrow bc|bGc|\epsilon$
$F \rightarrow a|aF|\epsilon$

thus A is a context free language.

A context free grammar that represents B can be shown as

$S \rightarrow FG|F|G|\epsilon$
$F \rightarrow ab|aFb|\epsilon$
$G \rightarrow c|cG|\epsilon$

thus B is a context free language.

If the class of context free languages were closed under intersection, the intersection of A and B, $A \cap B = C = \{a^n b^n c^n | n \geq 0\}$ would also be a context free language. However, example 2.36 in the book uses the pumping for context free languages to prove that C is not a context free language. We have a contradiction, context free languages are not closed under intersection.

b. Use part (a) and DeMorgan's law (Theorem 0.20) to show that the class of context-free languages is not closed under complementation.
Answer: In order to prove that the set of context free languages is not closed under compliment, we first assume that they are closed under compliment. This would imply that $D = \bar{A}$ is a context free grammar and $E = \bar{B}$ is a context free language. Since Context free languages are closed under the union operation, we know that $F = D \cup E$ is a context free language.

If CFLs are closed under compliment, we know that $\bar{F}$ is a context free language. Using DeMorgan's law we know that $F = C$, however we showed in part a that C is not a context free language, thus we have our contradiction. Context Free Languages are not closed under compliment.

E-2.4

Give context-free languages that generate the following languages. In all parts, the alphabet is $\Sigma = \{0, 1\}$.

e. $\{w = w^R,$ that is, w is a palindrome.$\}$
Answer: $S \rightarrow 0S0|1S1| =$

E-2.6

Give context-free grammars generating the following languages.

b. The complement of the language $\{a^n b^n | n \geq 0\}$.
Answer: $S \rightarrow A|B$
$A \rightarrow 1A0|1A|1$
$B \rightarrow 1B0|B0|0$

E-2.19

Let CFG G be the following grammar.

$$\begin{array}{rcl} S & \rightarrow & aSb \mid bY \mid Ya \\ Y & \rightarrow & bY \mid aY \mid \epsilon \end{array}$$

Give a simple description of $L(G)$ in English. Use that description to give a CFG for $\overline{L(G)}$, the complement of $L(G)$.
Answer: The CFG G describes the language of all permutations of $n \geq 0a$ characters and $m \geq 0b$ characters but also excludes the empty set. The compliment of $L(G)$ would be a language that only describes the empty string $\{\epsilon\}$.