Homework #5

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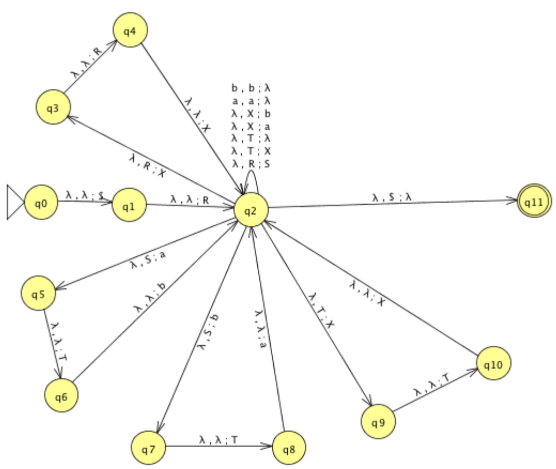
Due: Tuesday, October 14

Collaborators: LIST YOUR COLLABORATORS OR SAY NONE

E-2.12

Convert the CFG G given in Exercise 2.3 to an equivalent PDA, using the procedure given in Theorem 2.20. (You may use multiple character input - as in Figure 2.24)

Answer:



E-2.30

Use the pumping lemma to show that the following languages are not context free.

a. $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$
Answer: let $L = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$. Assume for sake of contradiction that L is a context-free language. Since L is a context-free language, there exists a pumping length p such that any string in language L of length p can be divided into 5 pieces, $s = uvxyz$, satisfying the following conditions
1) for each $i \geq 0$, $uv^i xy^i z \in L$
2) $|vy| > 0$
3) $|vxy| \leq p$
let $s = 0^p 1^p 0^p 1^p$. We know that $s \in L$ however there exists no way to break string s into $uvxyz$. Since $|vxy| \leq p$ we know that $vxy \leq 0^p$, $vxy \leq 1^p$, or $vxy = 1^i 0^j$, where i and j are less than p. Knowing this we know that there exists no way to split s into $uvxyz$ due to the inability to pump up, $uv^2 xy^2 z$ no matter what way s is divided (no mater what way the string is divided pumping up will always lead to an imbalance in the number of zeros and ones). We have a contradiction, thus L is not a context free language.

d. $\{t_1\#t_2\#\cdots\#t_k \mid k \geq 2, \text{each } t_i \in \{a,b\}^*, \text{and } t_i = t_j \text{ for some } i \neq j\}$
Answer: let $L = \{t_1\#t_2\#\cdots\#t_k \mid k \geq 2, \text{each } t_i \in \{a,b\}^*, \text{and } t_i = t_j \text{ for some } i \neq j\}$. Assume for sake of contradiction that L is a context-free language. Since L is a context-free language, there exists a pumping length p such that any string in language L of length p can be divided into 5 pieces, $s = uvxyz$, satisfying the following conditions
1) for each $i \geq 0$, $uv^ixy^iz \in L$
2) $|vy| > 0$
3) $|vxy| \leq p$
let $s = a^pb^p\#a^pb^p$ where $t_1 = a^pb^p$ before the $\#$ and $t_2 = a^pb^p$ after the $\#$. Knowing rules 2 and 3 of the pumping lemma, we know that vxy can consist of either exclusively a or b characters from either t_1 or t_2, a combination of a and b characters a^fb^g from either t_1 or t_2, a singular $\#$, or finally a combination of b characters from t_1, the $\#$ and a characters from t_2: $b^t\#a^u$ where $t, u \geq 0$. The pumping lemma states that we can always pump down to the string $s = vxz \in L$. However, no matter how the string is partitioned, $vxz \notin L$. We have our contradiction. L is not a regular language.

E-2.31

Let B be the language of all palindromes over $\{0, 1\}$ containing equal numbers of 0s and 1s. Show that B is not context free.

Answer: Assume for sake of contradiction that B is a context-free language. Since B is a context-free language, there exists a pumping length p such that any string in language B of length p can be divided into 5 pieces, $s = uvxyz$, satisfying the following conditions
1) for each $i \geq 0$, $uv^ixy^iz \in L$
2) $|vy| > 0$
3) $|vxy| \leq p$
let $s = 0^p1^{2p}0^p \in B$. We know from rules 2 and 3 of the pumping lemma that vxy can be only cannot include 0 characters from both before and after the 1 characters. vxy can be either entirely 0 characters before or after the 1 characters 0^i1^j (where $i, j > 0$), or 1^i0^j where $i, j > 0$). However, in all of these cases the string cannot be pumped down to yield a string $vxz \in B$. As such we have a contradiction, B is not a context free grammar.