Homework #1

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Due: Tuesday, September 9

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E-0.1

Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

a. $\{1, 3, 5, 7, \ldots\}$
Answer: The set of all numbers that can be expressed as the sum of one and two times a nonnegative integer n.

b. $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$
Answer: The set of all numbers that can be expressed as the product of two and an integer n.

c. $\{n | n = 2m$ for some m in $\mathcal{N}\}$
Answer: The set of all numbers n, where n can be expressed as the product of two and a number m, where m is an element of set N.

d. $\{n | n = 2m$ for some m in $\mathcal{N}$, and $n = 3k$ for some k in $\mathcal{N}\}$
Answer: The set of all numbers n, where n can be expressed as BOTH two times a number m, where m is an element of set N, AND three times a number k, where k is an element of set N.

E-0.2

Write formal descriptions of the following sets:

a. The set containing the number 1, 10, and 100
Answer: $\{1, 10, 100\}$

b. The set containing all integers that are greater than 5
Answer: $\{6, 7, 8, \ldots\}$

c. The set containing all natural numbers that are less than 5
Answer: $\{1, 2, 3, 4\}$

d. The set containing the string `aba`
Answer: $\{aba\}$

E-1.3

The formal description of a DFA M is $(\{q_1, q_2, q_3, q_4, q_5\}, \{\mathtt{u}, \mathtt{d}\}, \delta, q_3, \{q_3\})$, where δ is given by the following table. Give the state diagram of this machine.

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

Answer: Save this as the file `1_3.jff`

E-1.4

Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.

a. $\{w | w$ has at least three `a`'s and at least two `b`'s$\}$

c. $\{w | w$ has an even number of `a`'s and one or two `b`'s$\}$

e. $\{w | w$ starts with an `a` and has at most one `b`$\}$

Answer: You only have to show the final diagram for each problem. (If you are having difficulty combining, then save and submit the intermediate diagrams as well.) Save these as the files `1_4_a.jff, 1_4_c.jff, 1_4_e.jff`

E-1.5

Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.

c. $\{w | w$ contains neither the substrings `ab` nor `ba`$\}$

g. $\{w | w$ is any string that doesn't contain exactly two `a`'s$\}$

Answer: You only have to show the final diagram for each problem. Save these as the files `1_5_c.jff, 1_5_g.jff`

E-1.6

Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{0, 1\}$.

a. $\{w | w$ begins with a 1 and ends with a 0$\}$

b. $\{w | w$ contains at least three 1s$\}$

c. $\{w | w$ contains the substring 0101 (i.e., $w = x0101y$ for some x and y)$\}$

d. $\{w | w$ has length at least 3 and its third symbol is a 0$\}$

Answer: Save these as the files `1_6_a.jff, 1_6_b.jff, 1_6_c.jff, 1_6_d.jff`