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ENGINEERING NOTE

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SUBJECT: Physics 2C 2/11/20

NAME:

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(1) Main Ch.20 Equations
(2) Degrees of Freedom ; Quantum Mechanics
(3) Adiabatic Processes (Redux)
(4) Entropy & Microstates

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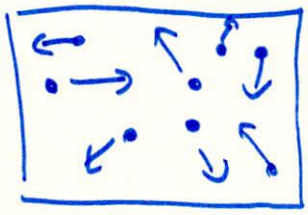
(1) Chapter 20 :

* Equipartition Thm: $\frac{1}{2} k_B T$ per d.o.f. per molecule
* $E_{th} = \frac{f}{2} kT$ per molecule
* $(KE)_{transl} = \frac{3}{2} k_B T$ per molecule

$\Rightarrow \frac{1}{2} m v_{rms}^2 = \frac{3}{2} k_B T$

$\Rightarrow \boxed{v_{rms} = \sqrt{\frac{3kT}{m}}}$

mono-atomic ideal gas



$H = E_{micro} = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2$

(Hamiltonian is homogeneous of degree 2 (quadratic) in a canonical momentum or position coordinate)
$\Rightarrow$ thermal contribution is $\frac{1}{2} k_B T$

$v_{rms} = \sqrt{\langle v^2 \rangle}$

$\boxed{p = \frac{1}{3} \frac{N}{V} m v_{rms}^2}$

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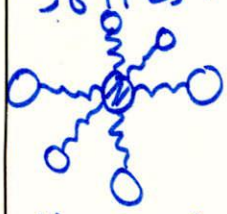
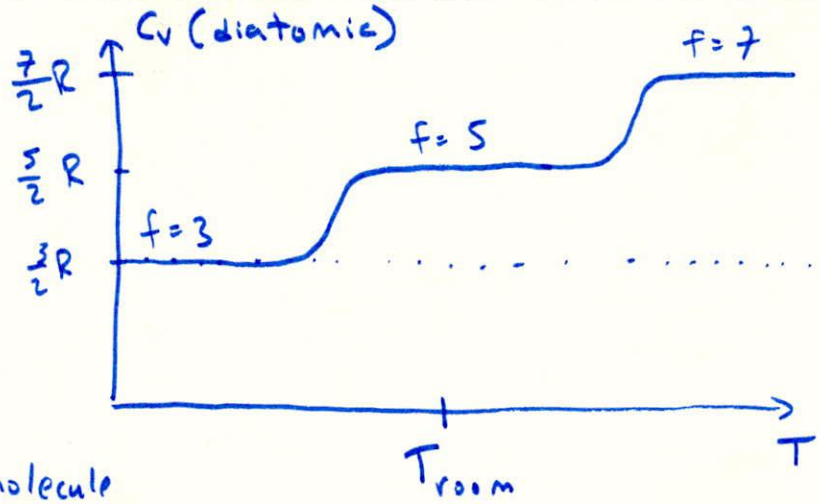
(2) D.o.f. (f)

Solids: $f = 6$



$H = \frac{1}{2}Kx^2 + \frac{p_x^2}{2m} + \ldots$

$\Rightarrow E_{th} = 3KT$ per molecule



<u>Diatomic gas</u>? Book says 8....

what do you think?

$H = \frac{\vec{p}_1^{\,2}}{2m_1} + \frac{\vec{p}_2^{\,2}}{2m_2} + \frac{1}{2}a(\Delta x)^2$

$3 + 3 + 1$

A) 8

B) 7

C) 5

D) 3

Alternatively

is not physical if we consider these to be <u>point</u> masses (in reality: I is so huge, it's inaccessible)

The reason $C_v \neq \frac{7}{2}R$ at all T is due to Quantum mechanics: energy needed to excite these vibrational/rotational modes is discrete and high!

<u>Summary</u>: $C_v = \frac{3}{2}R$ for monoatomic gas

$C_v = 3R$ for solids

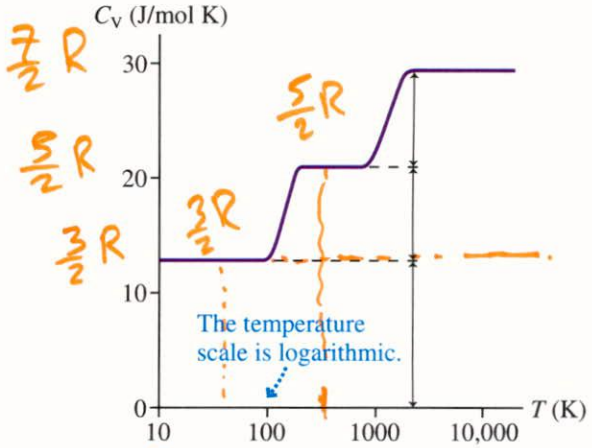
$$C_v = \begin{cases} \frac{3}{2}R & \text{low } T \\ \frac{5}{2}R & \text{med. } T \\ \frac{7}{2}R & \text{high } T \end{cases}$$

because QM

$E_{th} = C_V n T$ $\quad C_V = \frac{f}{2} R$

Is the following graph of heat capacity vs. temperature that of a monoatomic gas or a diatomic gas?

A) Monoatomic
B) Diatomic
C) Not enough info



$T_{room} \sim 300$ K

Suppose I have 10 J of energy that I want to give to a sample of air via heat in a constant-pressure process. In order to get the largest temperature change, should I give this heat to the gas at 50 K or 300 K?

$C_p = C_V + R$

$Q = n C_p \Delta T$

A) 50 K
B) 300 K
C) No difference

$\Delta T = \left(\frac{1}{n C_p}\right) Q$

For a monoatomic gas, what fraction of its total internal energy is translational kinetic energy?

$KE = E_{th} = \frac{3}{2} k_B T$

A) 33.3%
B) 50.0%
C) 60.0%
D) 66.7%
E) 100%

$KE = \frac{3}{2} k_B T$

For a diatomic gas, what fraction of its total internal energy is translational kinetic energy at room temperature?

$E_{th} = \frac{5}{2} k_B T$

$\Rightarrow 3/5 = 60\%$

A) 33.3%
B) 50.0%
C) 60.0%
D) 66.7%
E) 100%

$Q_v = \frac{5}{2} nR (\Delta T)_v = \frac{7}{2} \cdot Rn(\Delta T)_p = Q_p$

$C_v = \frac{5}{2} R$

$C_p = \frac{7}{2} R$

$\Delta E_{th} = \frac{5}{2} nR\Delta T$

$(\Delta T)_v = \frac{7}{2} (\Delta T)_p$

$\Rightarrow$

An ideal diatomic gas, with molecular rotation but without any vibration, loses a certain amount of energy as heat Q. Is the resulting decrease in the internal energy of the gas greater if the loss occurs in a constant-volume process or in a constant-pressure process?

A) Constant-volume process
B) Constant-pressure process
C) No difference

Take a diatomic gas at room temperature ($\gamma = 1.40$) and quickly compress it to half its volume.

(a) What's $\left(\frac{p_f}{p_i}\right)$?

$p_f V_f^{\gamma} = p_i V_i^{\gamma}$, $V_f = \frac{1}{2} V_i$

$p_f = (2)^{\gamma} p_i = 2^{1.4} p_i \approx 2.64 p_i$

(b) What's $\left(\frac{T_f}{T_i}\right)$?

$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$

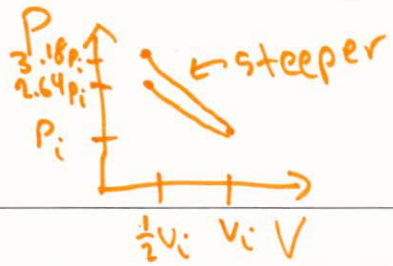
$T_f = (2)^{\gamma-1} T_i = 2^{0.4} T_i \approx 1.32 T_i$

(c) Would your answers increase or decrease if we used a monoatomic gas? Draw a pV diagram!

monoatomic $\gamma = 5/3 \approx 1.67$

$p_f \approx 3.18 p_i$

$T_f \approx 1.59 T_i$



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③ Adiabatic Processes

$$E_{th} = n C_v T = \frac{f}{2} n R T = \frac{f}{2} N k T$$

equipartition

$$C_v = \frac{f}{2} R$$

$$C_p = C_v + R = \frac{f+2}{2} R$$

$$\Rightarrow \boxed{\gamma = \frac{C_p}{C_v} = \frac{f+2}{f}}$$

$$\boxed{T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}}$$

$$\boxed{P_i V_i^{\gamma} = P_f V_f^{\gamma}}$$

f	γ
3	$5/3 \approx 1.67$
5	$7/5 \approx 1.40$
7	$9/7 \approx 1.29$

$1 < \gamma \leq 5/3$

④ Entropy & Microstates

2nd law : $\Delta S \geq 0$

$\Leftrightarrow$ (Hotter System) $\xRightarrow{\text{heat}}$ (Colder System)

$\Leftrightarrow$ arrow of time is defined by increase in entropy

increasing entropy $\Leftrightarrow$ increasing randomness $\Leftrightarrow$ increasing prob.

entropy is just another way of quantifying most likely states

$$S = k_B \log \Omega$$

Ω = number of accessible microstates consistent w/ macrostate

Note: fundamental assumption: all accessible microstates are equally likely

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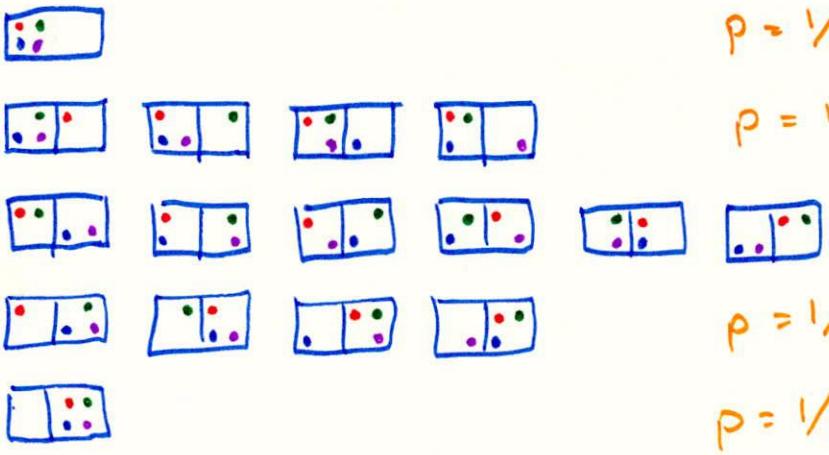
Example: Consider # of ways to distribute 4 particles among 2 boxes: $2^4 = 16$



$p = 1/16$

$p = 1/4$

$p = 3/8$

$p = 1/4$

$p = 1/16$

Most probable dist. is the one w/ greatest number of microstates and thus greatest entropy!

Yes, how many (effective) d.of. is this?

$\gamma = \frac{f+2}{f} = 1.5 \Rightarrow f = 4$

$n = n_1 + n_2$

$E_{tot} = \frac{3n_1RT}{2} + \frac{5n_2RT}{2} \Rightarrow \left(\frac{3n_1+5n_2}{2}\right)RT \stackrel{?}{=} \frac{4n}{2}RT$

if $n_1 = n_2 = \frac{n}{2}$

Is it possible to have $\gamma = 1.50$ for a gas? Could you produce it with a mixture of monoatomic and diatomic gases at room temperature?

A) Yes, 50% monoatomic / 50% diatomic
B) Yes, 38% monoatomic / 62% diatomic
C) Yes, 62% monoatomic / 38% diatomic
D) No, not possible

N total balls, N_1 in box 1, N_2 in box 2

36. ‖ Two containers hold several balls. Once a second, one of the balls is chosen at random and switched to the other container. After a long time has passed, you record the number of balls in each container every second. In 10,000 s, you find 80 times when all the balls were in one container (either one) and the other container was empty.

a. How many balls are there?

b. What is the most likely number of balls to be found in one of the containers?

measured prob

a) $\text{prob}\left(\begin{matrix}\text{all balls}\\ \text{in one}\\ \text{box}\end{matrix}\right) = \frac{2}{2^N} = \frac{80}{10{,}000}$

$\Rightarrow N = \log_2(250) = 7.97$ but it must be integer!

$\boxed{N = 8}$

b) $N_1 = N_2 = \frac{N}{2} = \boxed{4}$