Phys 141 Javier Duarte

Units

5/4/2023

One way to think about this is we are defining a new "ad hoc" set of custom units

$$\mathring{m},\ \mathring{\ell},\ \mathring{t},\ \mathring{v} = \mathring{\ell}/\mathring{t},\ \text{etc.}$$

We can choose 2 of these to be whatever we want, eg.

$$1\ \mathring{m} = 10^{41}\ M_\odot = 1.9891\times10^{41}\ \text{kg}$$

$$1\ \mathring{\ell} = 1.5\ \text{kpc} = 4.629\times10^{19}\ \text{m}$$

$$1\ \mathring{t} = ?$$

$$1\ \mathring{v} = ?$$

Then we can impose $G = 1$ in our units and relate that to SI units

$$G = 1\ \frac{(\mathring{\ell})^3}{(\mathring{m})(\mathring{t})^2} = 1\ \frac{(\mathring{\ell})(\mathring{v})^2}{(\mathring{m})}$$

$$= 6.6743\times10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$$

All other units, in this case $\mathring{t}$ or $\mathring{v}$, will be determined by this.

Explicitly,

$$1\mathring{v} = \left(\frac{G \cdot (1\mathring{m})}{1\mathring{\ell}}\right)^{1/2}$$

$$1\mathring{v} = \left(\frac{(6.6743\times10^{-11}\ m^3 kg^{-1} s^{-2})(1.9891\times10^{41}\ kg)}{4.629\times10^{19}\ m}\right)^{1/2}$$

$$= 5.3556\times10^{5}\ m/s = 5.477\times10^{-7}\ kpc/yr$$

$$1\mathring{t} = \frac{1\mathring{\ell}}{1\mathring{v}} = \frac{4.629\times10^{19}\ m}{5.3556\times10^{5}\ m/s}$$

$$= 8.6423\times10^{13}\ s = 2.7387\times10^{6}\ yr$$

So "1" in velocity units means $\sim 5.4\times10^{5}$ m/s