HOMEWORK 7

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93. We can use the isomorphism theorem by showing that there is a homomorphism $f : R[x]/ \to R$, demonstrating the kernel of f to be (x), and showing that the image of f is R itself. So first, choose f such that $r(x) = r_0 + \ldots + r_n x^n \mapsto r_0$; in other words, f is the evaluation function that sends polynomials $r(x)$ to $r(0)$.

This map is well defined since equality of polynomials is defined by equality of coefficients. The map is a homomorphism since

$$f(r(x) + s(x)) = r_0 + s_0 = f(r(x)) + f(s(x))$$

and

$$f(r(x)s(x)) = r_0 s_0 = f(r_0)f(s_0),$$

and furthermore, the zero polynomial obviously maps to zero in R. Thus f is a homomorphism.

The kernel of f is given by

$$\begin{aligned} \ker f &= \{r(x) \in R[x] : f(r(x)) = 0\} \\ &= \{r_1 x + \ldots + r_n x^n\} \\ &= (x) \end{aligned}$$

since polynomials without constant terms are divisible by x without remainder. Finally, the image of f is clearly the entirety of R since one can choose any constant polynomial $r(x) = r_0$ with $r_0 \in R$. Thus by the first isomorphism theorem, there exists an isomorphism between $R[x]/(x)$ and R.