DEMORGAN'S LAWS AND HEXAGONAL SYMMETRY

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Abstract. Here proofs for each of DeMorgan's two laws are given. A regular hexagon is constructed with compass and straightedge, and its symmetries are described.

1. DeMorgan's Laws

Theorem 1. *For any two sets A and B, $(A \cup B)^{\complement} = A^{\complement} \cap B^{\complement}$.*

Proof. Let $x \in (A \cup B)^{\complement}$. Then $x \notin A \cup B$. It must be the case that $x \notin A$ and $x \notin B$. Thus, $x \in A^{\complement}$ and $x \in B^{\complement}$, so $x \in A^{\complement} \cap B^{\complement}$. This means that $\forall x \in (A \cup B)^{\complement}$, $x \in A^{\complement} \cap B^{\complement}$. By the definition of set inclusion, $(A \cup B)^{\complement} \subset A^{\complement} \cap B^{\complement}$.

Let $y \in A^{\complement} \cap B^{\complement}$. Then $y \in A^{\complement}$ and $y \in B^{\complement}$. So $y \notin A$ and $y \notin B$. Therefore, $y \notin A \cup B$, so it must be that $y \in (A \cup B)^{\complement}$. Similar to above, it follows that $A^{\complement} \cap B^{\complement} \subset (A \cup B)^{\complement}$.

By definition of set equality, $(A \cup B)^{\complement} = A^{\complement} \cap B^{\complement}$. □

Theorem 2. *For any two sets A and B, $(A \cap B)^{\complement} = A^{\complement} \cup B^{\complement}$.*

Proof. Let $x \in (A \cap B)^{\complement}$. Then $x \notin A \cap B$. So $x \notin A$ or $x \notin B$. Therefore $x \in A^{\complement}$ or $x \in B^{\complement}$. Thus, $x \in A^{\complement} \cup B^{\complement}$. By the definition of set inclusion, $(A \cap B)^{\complement} \subset A^{\complement} \cup B^{\complement}$.

Let $y \in A^{\complement} \cup B^{\complement}$. It follows that $y \in A^{\complement}$ or $y \in B^{\complement}$. Then $y \notin A$ or $y \notin B$. So $y \notin A \cap B$, therefore $y \in (A \cap B)^{\complement}$. Similar to above, we find that $A^{\complement} \cup B^{\complement} \subset (A \cap B)^{\complement}$.

By definition of set equality, $(A \cap B)^{\complement} = A^{\complement} \cup B^{\complement}$. □

2. The Regular Hexagon