# Aggregating Expert Ratings with IRT Models: The V-Dem Methodology

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V-Dem Institute & Durham University

10 February 2022

Based on work by V-Dem's measurement and data teams

- Dan Pemstein,
- Kyle L. Marquardt
- Eitan Tzelgov
- Yi-ting Wang
- Joshua Krusell
- Farhad Miri
- Johannes von Röemer

# The Data

# Elections (Austria)



1. Read Question. 2. Click & drag to select years. 3. Apply or Edit specific dates, if desired. 4. Apply or type response. 5. Rate Confidence. 6. Submit. 7. Repeat for remaining years. 8. Click "Next".

Undo

**(Regional government elected) :**
At the regional level, are government offices elected in practice?

"Government offices" here refers to a regional executive and a regional assembly, not a judiciary and not minor bureaucrats. An executive is a single individual (or a very small group) (e.g., a governor). An assembly is a larger body of officials, who may be divided into two chambers. "Elected" refers to offices that are directly elected by citizens or indirectly elected by a regional elected assembly. All other methods of obtaining office – including appointment by higher or lower levels of government – are considered to be non-elected. In classifying a position as elected one is making no judgments about the freeness/fairness of the election or the relative extent of suffrage. One is simply indicating that there is an election and that the winner of that election (however conducted) generally takes office.

| | 00 | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2005 | | | | | | 2005 | 2006 | 2007 | 2008 | 2009 |
| 2010 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | | |

Click and drag to select range of years.

Min: 0 Max: 5

- (0) Generally, offices at the regional level are not elected.
- (1) Generally, the regional executive is elected but not the assembly.
- (2) Generally, the regional assembly is elected but not the executive.
- (3) Generally, the regional executive is elected and there is no assembly.
- (4) Generally, the regional assembly is elected and there is no executive.
- (5) Generally, the regional executive and assembly are elected.

Specific Dates:

Add Del Edit

Legend

' ' : At least one date in this cell does not have an answer.
' ' : All dates in this cell have an answer, but at least one does not have a confidence rating.
' ' : All dates in this cell have both an answer and a confidence rating.

Confidence: 0%

I have no idea at all. [Any scores accompanied by a confidence level of zero will be treated as missing data.]

Submit

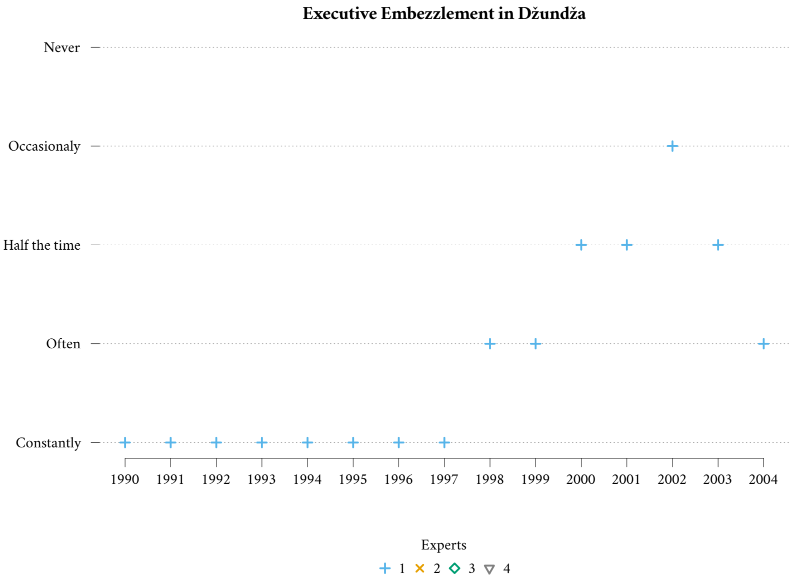
Jump To Question: :

Regional government elected

Previous Next

Exit

Executive Embezzlement in Džundža
Never
Occasionaly
Half the time
Often
Constantly
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
Experts
1
2
3
4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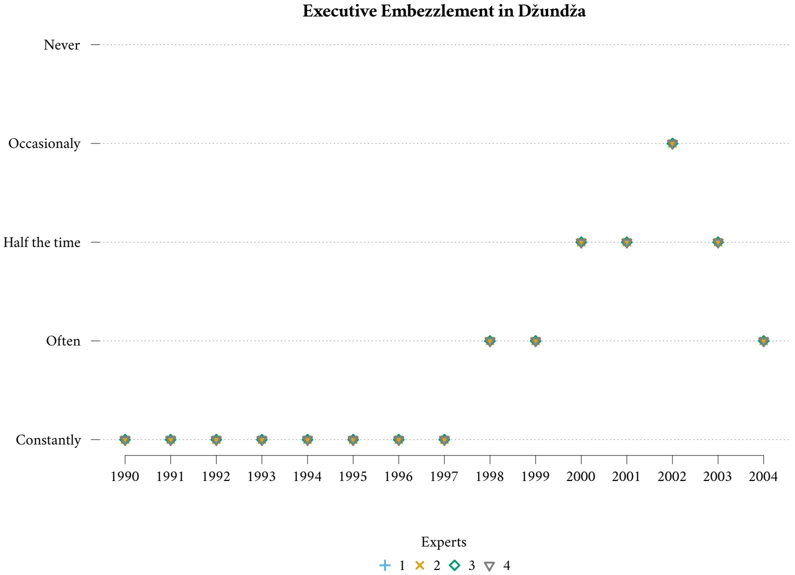

Executive Embezzlement in Džundža
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1990
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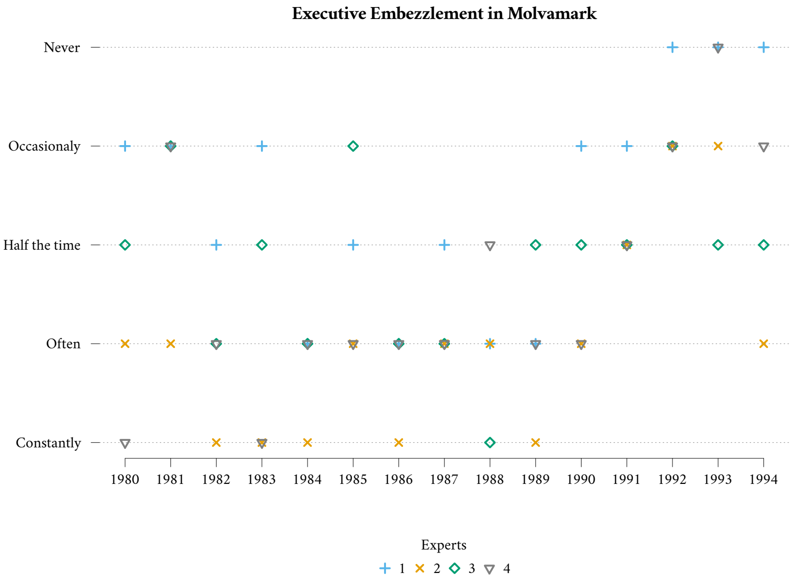
Executive Embezzlement in Molvamark
Never
Occasionaly
Half the time
Often
Constantly
1980
1981
1982
1983
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1994
Experts
1
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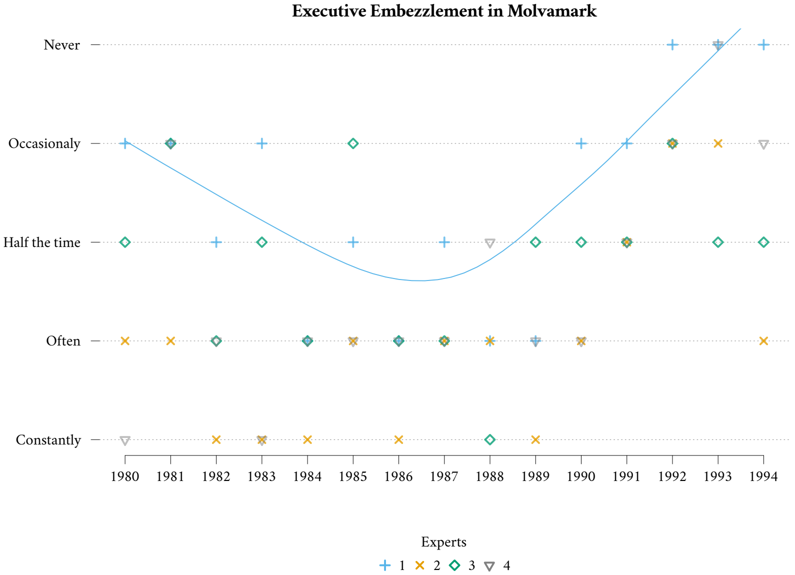
Executive Embezzlement in Molvamark
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Often
Constantly
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Experts
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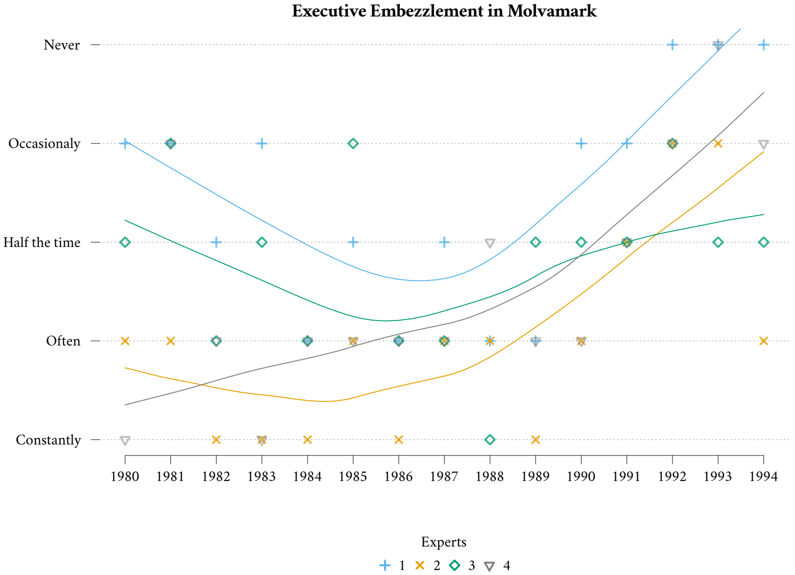

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Never
Occasionaly
Half the time
Often
Constantly
1980
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Experts
1
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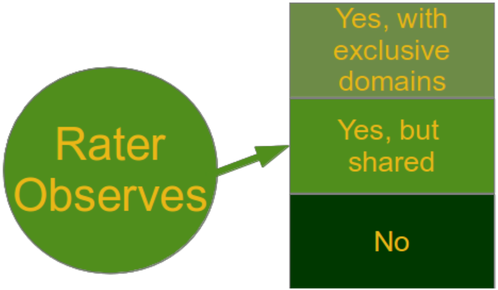
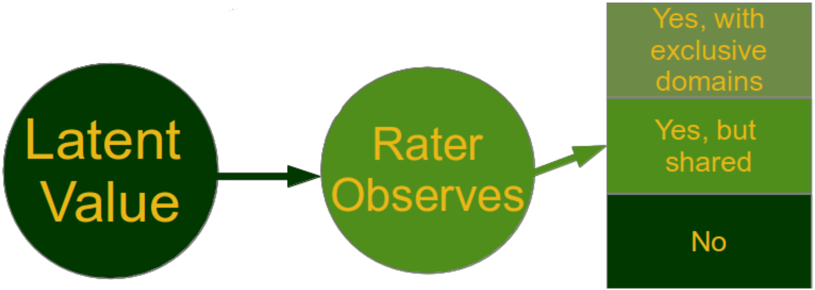
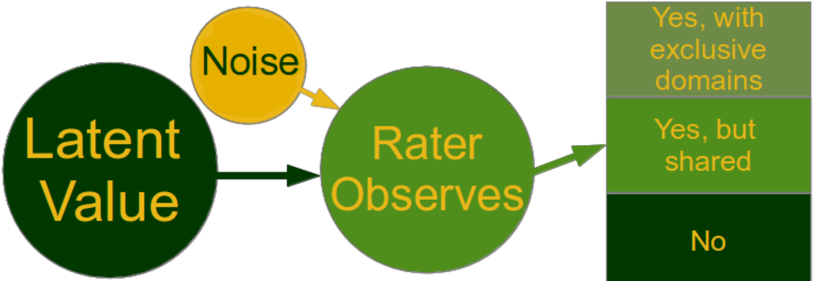
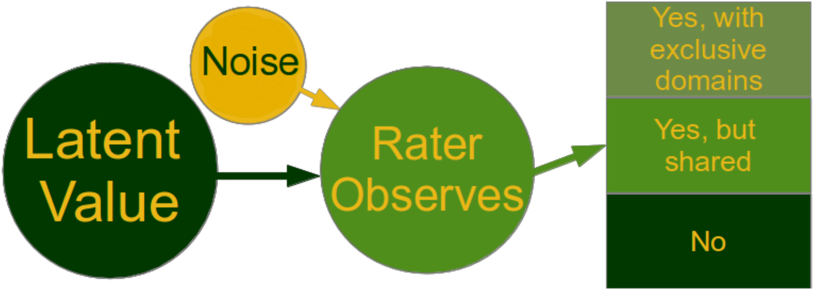
3
4

# The Problem

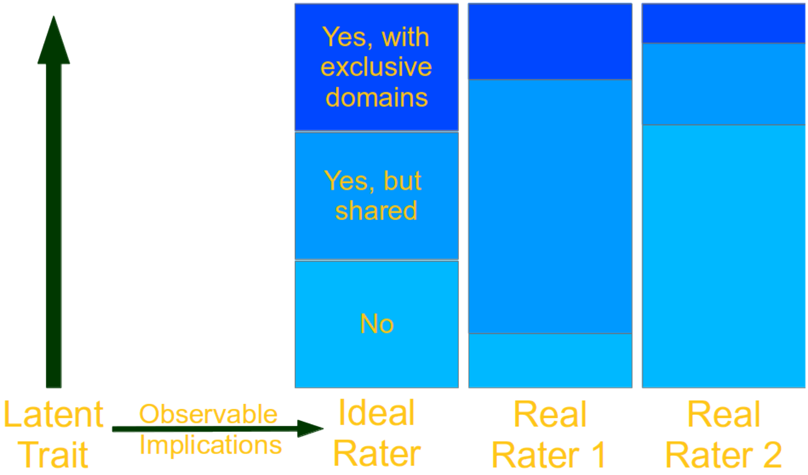

Rater Observes
Yes, with exclusive domains
Yes, but shared
No


Latent Value
Rater Observes
Yes, with exclusive domains
Yes, but shared
No


Noise
Latent Value
Rater Observes
Yes, with exclusive domains
Yes, but shared
No

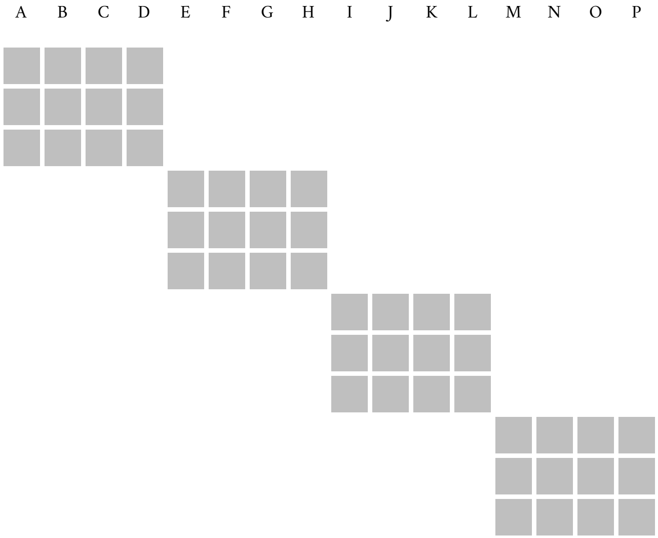


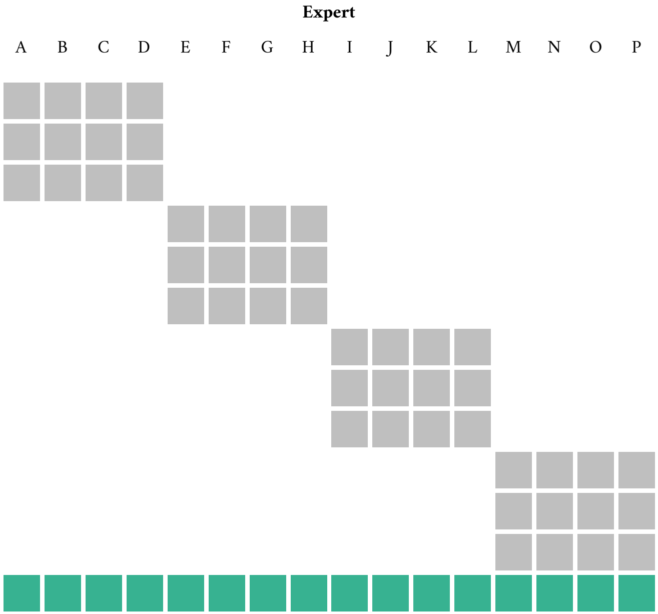

This is not all ...


Yes, with exclusive domains
Yes, but shared
No
Latent Trait
Observable Implications
Ideal Rater
Real Rater 1
Real Rater 2

# Survey Design

**Expert**




Expert
A
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D
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J
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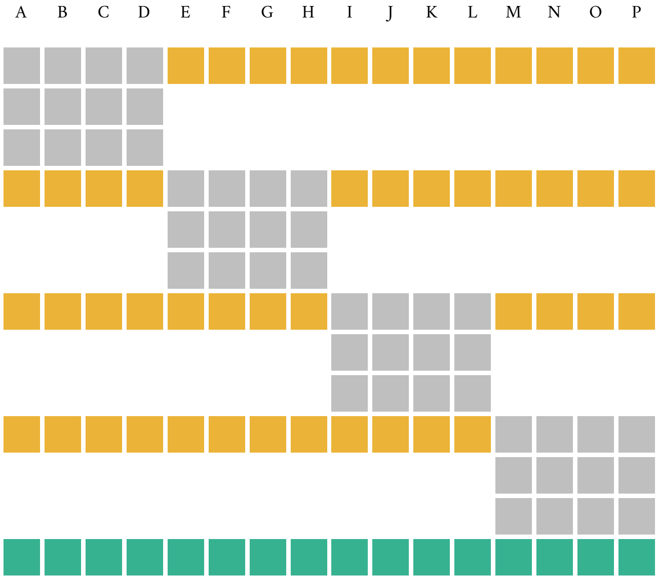
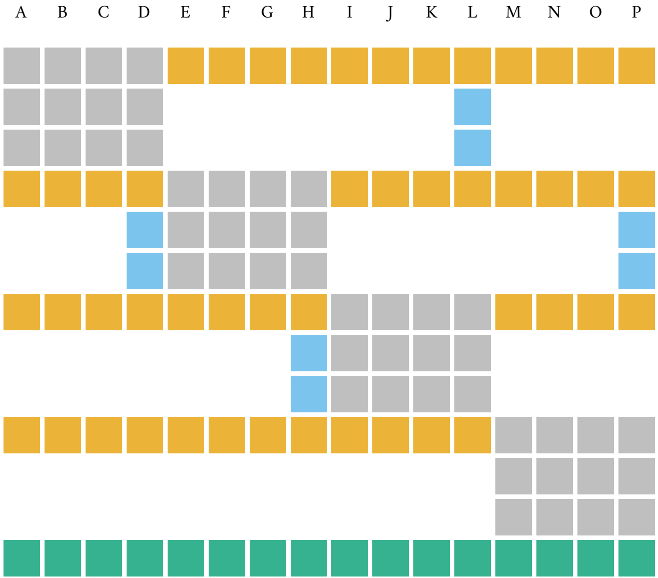
M
N
O
P

**Expert**

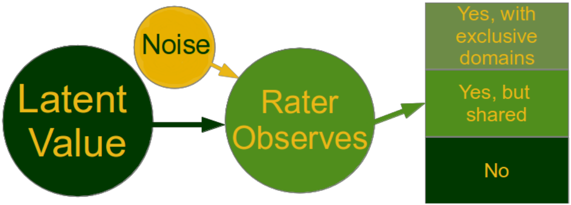


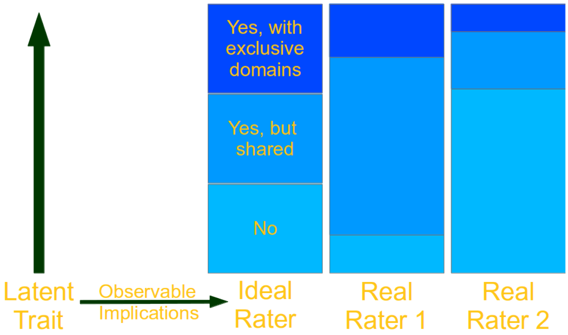
**Expert**



# The Model

Modeling The Data Generating Process



rating[object, expert] =
*f*(
truth[object],
reliability[expert],
thresholds[expert]
)

$$
\begin{aligned}
y_{ie} &\sim Categorical\left(\boldsymbol{p}_{ie}\right) \\
\boldsymbol{p}_{ie} &= \{p_{ie1}, \ldots, p_{ieK}\} \\
p_{iek} &= \phi\left(\tau_{e,k+1} - \beta_e \zeta_i\right) - \phi\left(\tau_{ek} - \beta_e \zeta_i\right) \\
\boldsymbol{\tau}_e &= \{\tau_{e1} = -\infty, \ldots, \tau_{eK-1}, \tau_{eK+1} = \infty\} \\
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\tau^{W}_{k} &\sim Uniform\left(-6, 6\right) \\
\beta_e &\sim Normal^{+}\left(1, 1\right) \\
\zeta_i &\sim Normal\left(z_i, 1\right)
\end{aligned}
$$

Google Scholar "item response theory"

Articles

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Since 2022
Since 2021
Since 2018
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Sort by relevance
Sort by date

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Review articles

include patents
include citations

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[BOOK] **Item response theory**
SE Embretson, SP Reise - 2013 - taylorfrancis.com
This book develops an intuitive understanding of IRT principles through the use of graphical displays and analogies to familiar psychological principles. It surveys contemporary IRT models, estimation methods, and computer programs. Polytomous IRT models are given central …
Save Cite Cited by 6890 Related articles All 8 versions

[BOOK] The basics of **item response theory**
FB Baker - 2001 - ERIC
… of **item response theory** would not be where it is today. … on **item response theory**, which resulted in the first edition of this book in 1985. This suggestion allowed me to fulfill a long-standing desire to develop an instructional software package dealing with **item response theory** for …
Save Cite Cited by 2490 Related articles All 2 versions

**Item response theory**.
F Drasgow, CL Hulin - 1990 - psycnet.apa.org
review a variety of IRT [**item response theory**] models that can be used with items scored dichotomously or polychotomously/methods of estimation and appropriate computer programs are also described/emphasis... is placed on applications of IRT to important measurement …
Save Cite Cited by 234 Related articles

[BOOK] **Item response theory**: Principles and applications
RK Hambleton, H Swaminathan - 2013 - books.google.com
… Despite these early research efforts, interest in **item response theory** lay dormant until the late 1960s and took a backseat to the emerging … ," gave rise to a resurgence of interest in **item response theory**. Impetus for the development of **item response theory** as we now know it was …
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- Bayesian hierarchical IRT model
  - IRT: each expert gets their own thresholds $\boldsymbol{\tau}$ and reliability $\beta$
  - Bayesian: full generative model with parameter distributions
  - Hierarchical: experts' thresholds are regularized towards their countries' thresholds which are in turn regularized towards the global thresholds
- Related to the Graded Response Model (Samejima)
- Alternative to Aldrich-McKelvey Scaling

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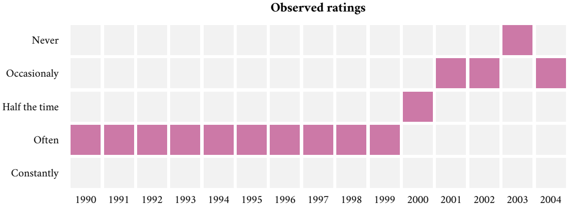


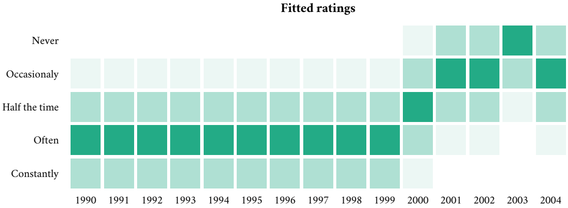

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\end{aligned}
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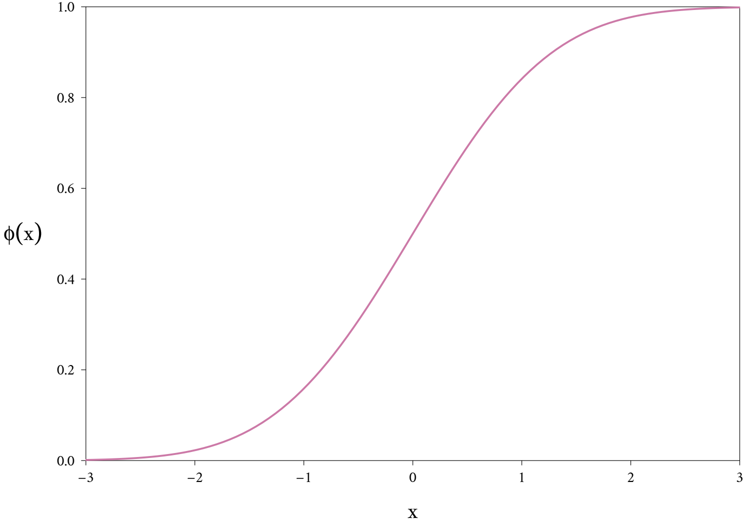
$$p_{iek} = \phi\left(\tau_{e,k+1} - \beta_e \zeta_i\right) - \phi\left(\tau_{ek} - \beta_e \zeta_i\right)$$



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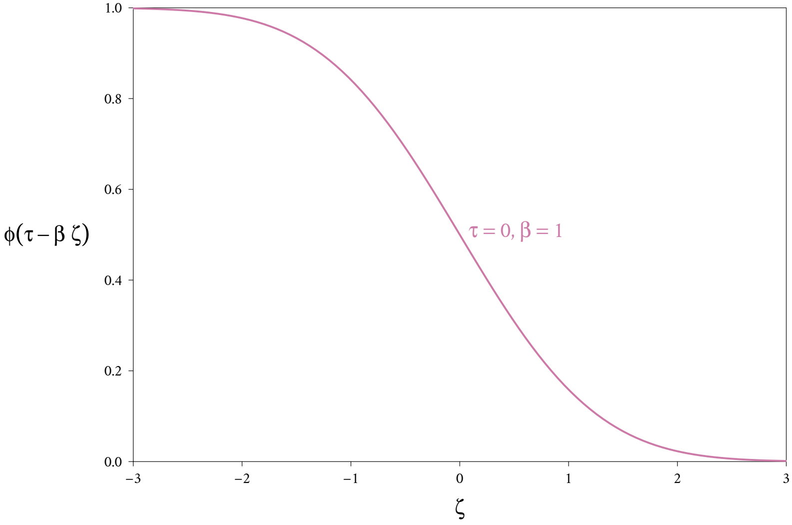

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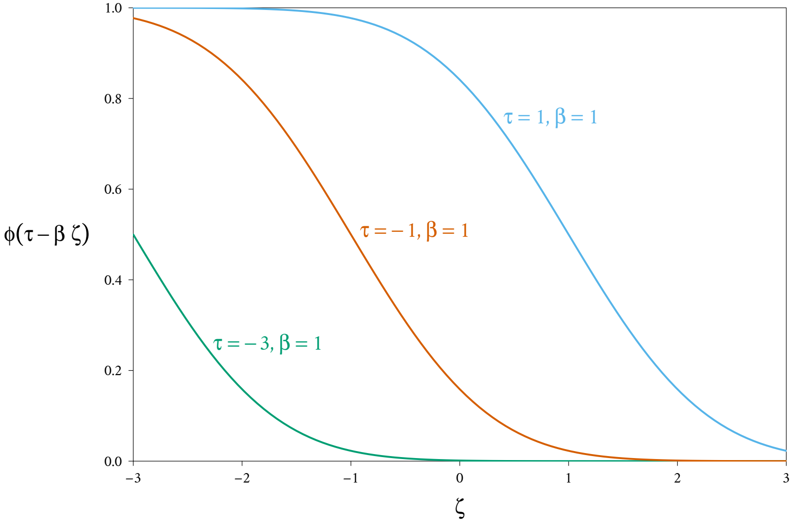

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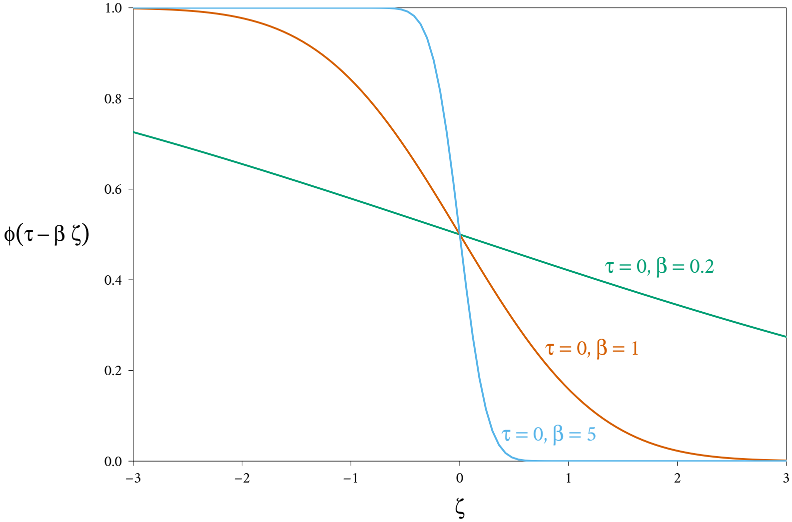
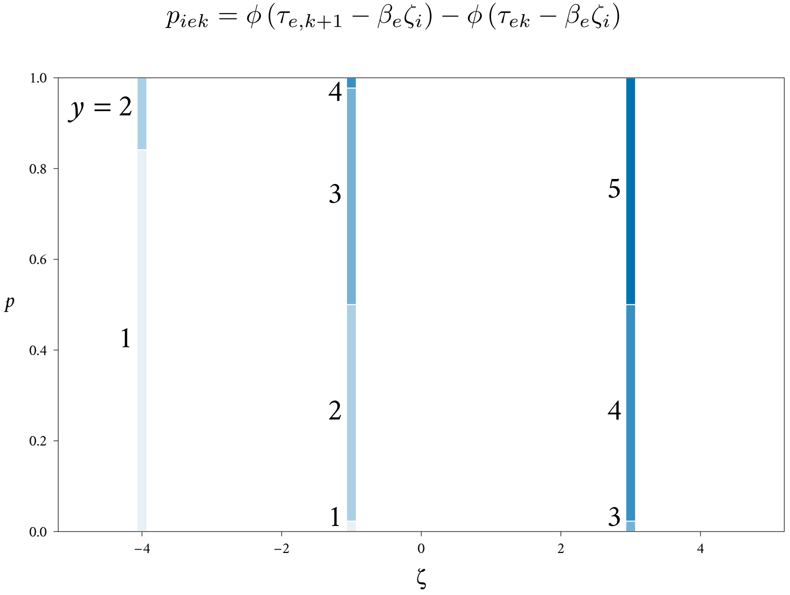
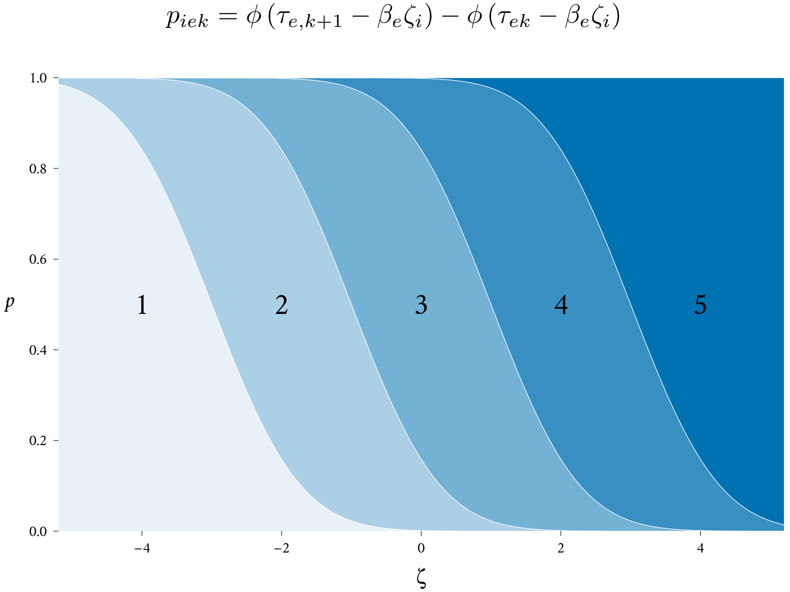


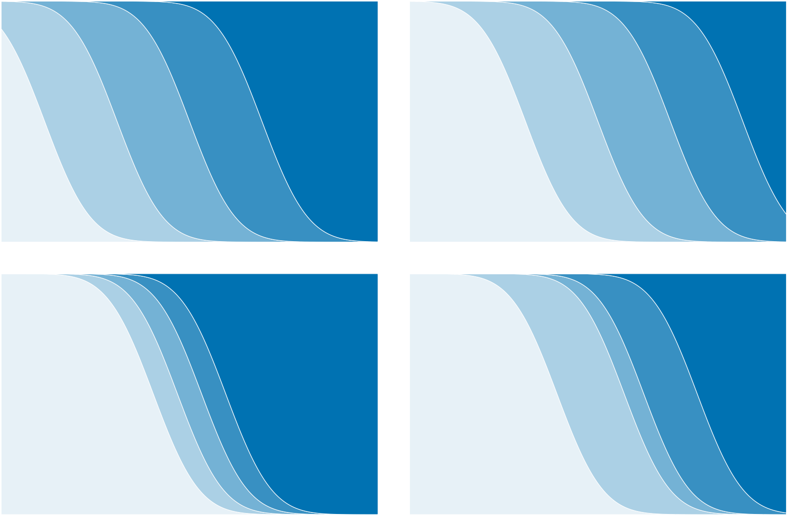


$p_{iek} = \phi\left(\tau_{e,k+1} - \beta_e \zeta_i\right) - \phi\left(\tau_{ek} - \beta_e \zeta_i\right)$
$y = 2$
1
4
3
2
1
5
4
3
1.0
0.8
0.6
0.4
0.2
0.0
$p$
−4
−2
0
2
4
$\zeta$


$p_{iek} = \phi\left(\tau_{e,k+1} - \beta_e \zeta_i\right) - \phi\left(\tau_{ek} - \beta_e \zeta_i\right)$
1.0
0.8
0.6
0.4
0.2
0.0
p
1
2
3
4
5
−4
−2
0
2
4
ζ

$$p_{iek} = \phi\left(\tau_{e,k+1} - \beta_e \zeta_i\right) - \phi\left(\tau_{ek} - \beta_e \zeta_i\right)$$

$$p_{iek} = \phi\left(\tau_{e,k+1} - \beta_e \zeta_i\right) - \phi\left(\tau_{ek} - \beta_e \zeta_i\right)$$



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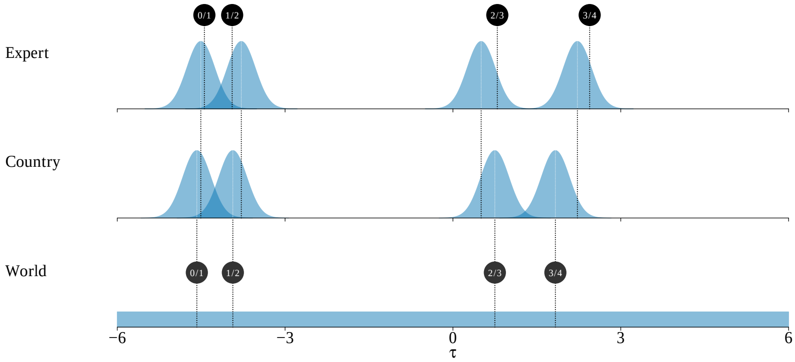

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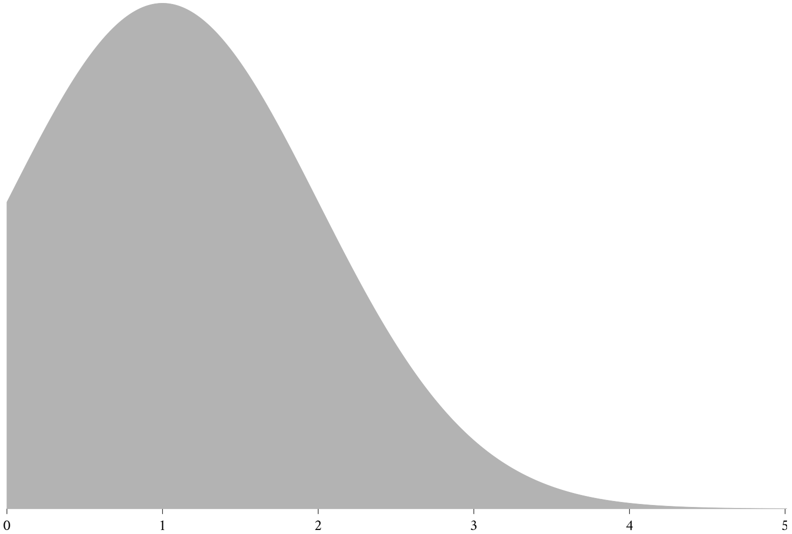
$$\beta_e \sim Normal^{+}(1,\, 1)$$



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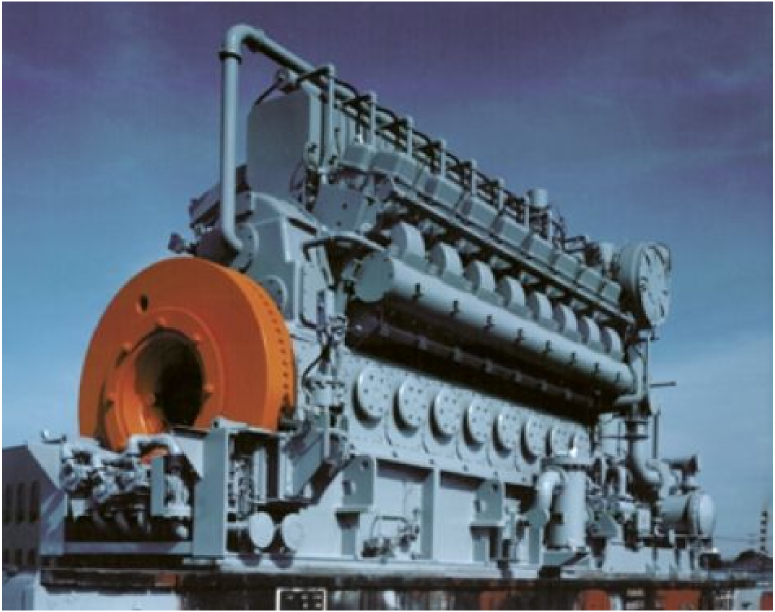
\beta_e &\sim Normal^+\left(1, 1\right) \\
\zeta_i &\sim Normal\left(z_i, 1\right)
\end{aligned}
$$

# The implementation

# Stan


- Probabilistic programming language
- Bayesian and penalized likelihood estimation
- No-U-turns sampler (NUTS), Hamiltonian Monte Carlo
- Many interfaces: `R`, `python`, `Stata` etc.

Stan



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\tau^W_k &\sim Uniform\left(-6, 6\right) \\
\beta_e &\sim Normal^+\left(1, 1\right) \\
\zeta_i &\sim Normal\left(z_i, 1\right)
\end{aligned}
$$

```
z_star ~ normal(0, 1);
for (j in 1:J)
  beta[j] ~ normal(1, 1)T[0,];
for (c in 1:C)
  tau_raw_country[c] ~ normal(tau_raw_world, 0.25);
for (j in 1:J)
  tau_raw_expert[j] ~ normal(tau_raw_country[expert_country[j]], 0.25);
for (r in 1:R) {
  p = Phi_approx(tau[j_id[r], y[r]+1] - z[cy_id[r]]*beta[j_id[r]])-
      Phi_approx(tau[j_id[r], y[r]] - z[cy_id[r]]*beta[j_id[r]]);
  target += log(p);
}
```

# Demonstration

- `https://github.com/jmedzihorsky/ELTE_29_01_2014/vdem_mm`
- `R` and `Stan` code
- Data: a small slice of old V-Dem data

Thank you!