Support Violations for Difference-in-Differences

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Support Issues for 2-period DiD

1. Adding distributional information on the outcomes to PT plots can reveal situations where quantile-based CiC (Athey and Imbens 2006) will be preferred to standard DiD.
2. Plots of imputed PO distributions to assess the robustness of ATT findings.
3. The ATT can be bounded by a support-adjusted estimate.
4. Because time-invariant and group-invariant confounding are not equivalent assumptions on the quantile scale, simultaneous presentation of CiC and reverse CiC, in addition to support-adjusted estimates for both, allow for a more robust check on DiD when one of these assumptions is not preferred to the other.
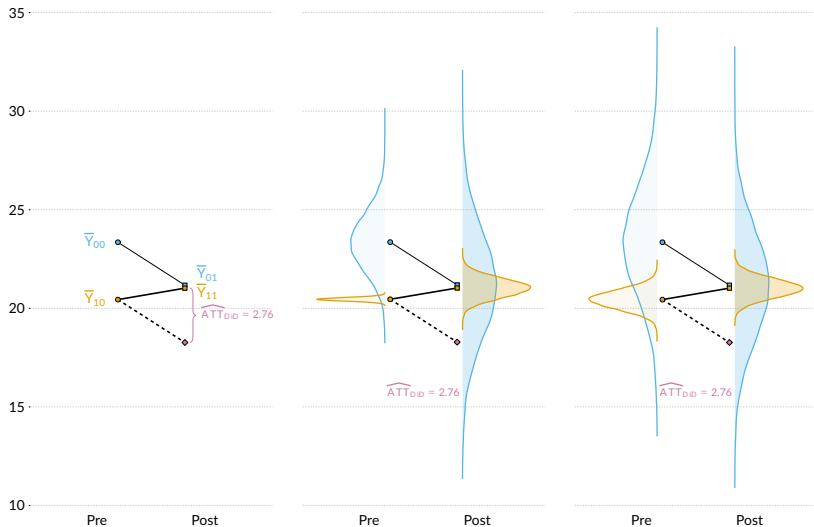
Mean Based Parallel Trends Plot Shows Large Effect
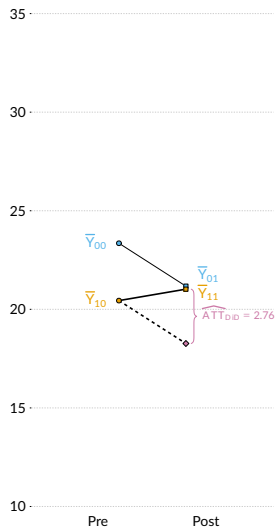
Some Consistent Distributions also Imply Large Effect
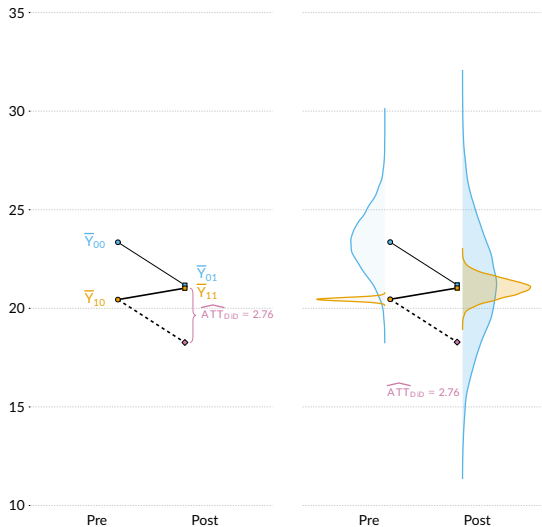
Some Consistent Distributions Imply Small Effect

Minimum Wage Notation and Decomposed Estimators

As our example, we use the Card and Kreuger (1994) analysis of full-time equivalent (FTE) employment in fast food stores in NJ and PA before and after the 1992 NJ minimum wage increase.

Y_{q11}: FTE employment in restaurant q in NJ after the min wage increase.

Y_{r10}: FTE employment in restaurant r in NJ before the min wage increase.

Y_{s01}: FTE employment in restaurant s in PA after the min wage increase.

Y_{t00}: FTE employment in restaurant t in PA before the min wage increase.

Under the linear DiD estimator:

$$\widehat{ATT}_{DiD} = \frac{1}{n_{11}} \sum_{q=1}^{n_{11}} Y_{q11} - \frac{1}{n_{10}} \sum_{r=1}^{n_{10}} \left\{ Y_{r10} + \underbrace{\left[\frac{1}{n_{01}} \sum_{s=1}^{n_{01}} Y_{s01} - \frac{1}{n_{00}} \sum_{t=1}^{n_{00}} Y_{t00} \right]}_{\text{Imputed Potential Outcome}} \right\}$$

Minimum Wage Notation and Decomposed Estimators

Assuming time-invariant confounding on the quantile scale (Sofer et al. 2016):

$$F_{Y_{01}}\left(F_{Y_{11}(0)}^{-1}(v)\right) = F_{Y_{00}}\left(F_{Y_{10}}^{-1}(v)\right), v \in [0, 1]$$

where a (group) = 1 for NJ, = 0 for PA, and t (time) = 1 for after, = 0 for before the minimum wage increase, Athey and Imbens (2006) propose:

$$\widehat{ATT}_{CiC} = \frac{1}{n_{11}} \sum_{q=1}^{n_{11}} Y_{q11} - \frac{1}{n_{10}} \sum_{r=1}^{n_{10}} \left\{ \underbrace{\widehat{F}_{Y_{01}}^{-1}\left(\widehat{F}_{Y_{00}}(Y_{r10})\right)}_{\text{Imputed Potential Outcome}} \right\}$$

where

n_{at}: number of restaurants in group a at time t

$\widehat{F}_{Y_{at}}^{-1}$: inverse empirical CDF for employment in group a at time t.

$\widehat{F}_{Y_{at}}$: empirical CDF for employment in group a at time t.
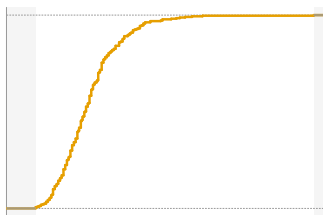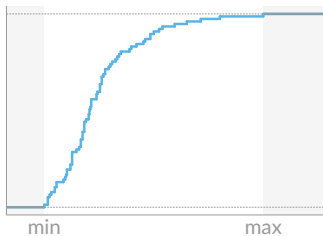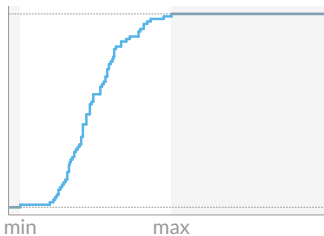
Loop Plot: Changes-in-Changes
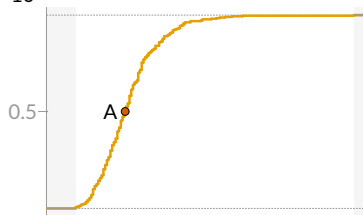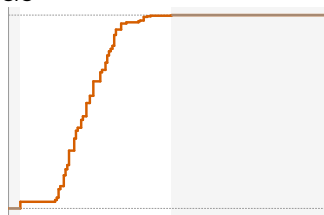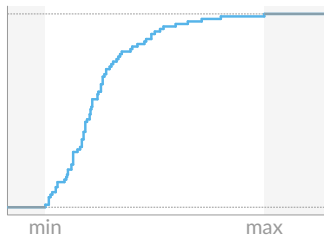

Y_{10}

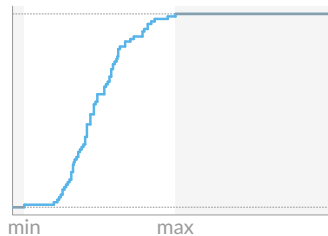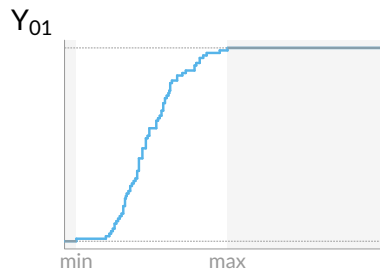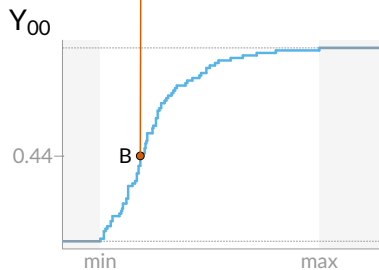$\widehat{Y_{11}(0)}_{CiC}$

Y_{00}

Y_{01}

min   max

Loop Plot: Changes-in-Changes

Y_{10}

0.5 — A

$\widehat{Y_{11}(0)}_{CiC}$

Y_{00}

min   max

Y_{01}

min   max
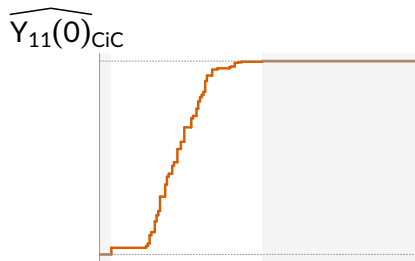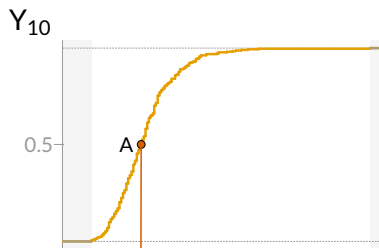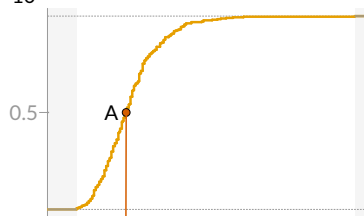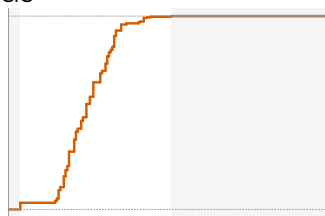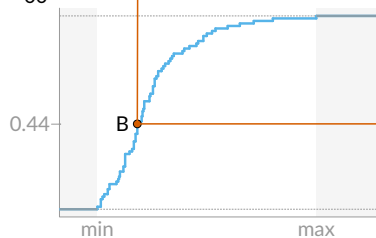
Loop Plot: Changes-in-Changes

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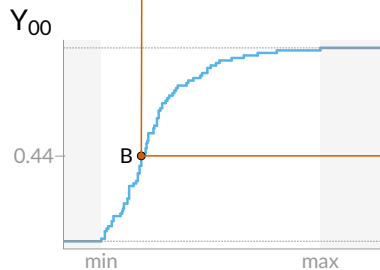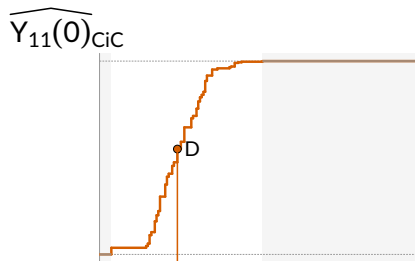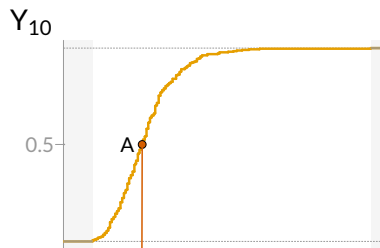
Diagonal Lines Show Direction of Support Violation

Imputed Potential Outcomes Plot

Legend:
- CiC, with support
- CiC, too large
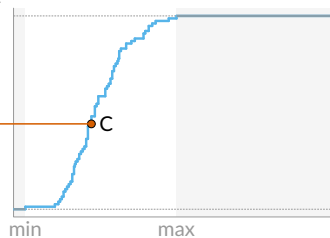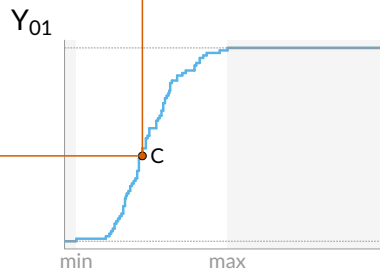- CiC, too small
- CiC, adjusted
- DiD

$\widehat{Y_{11}(0)}$ (y-axis)

Y_{10} (x-axis)

mean(Y_{11})
mean($\widehat{Y_{11}(0)_{CiC}}$)
mean($\widehat{Y_{11}(0)_{DiD}}$)
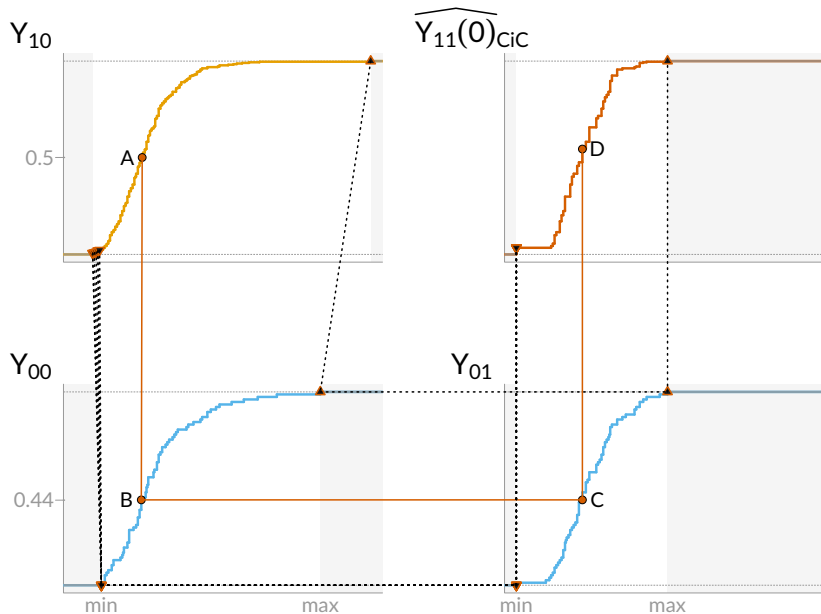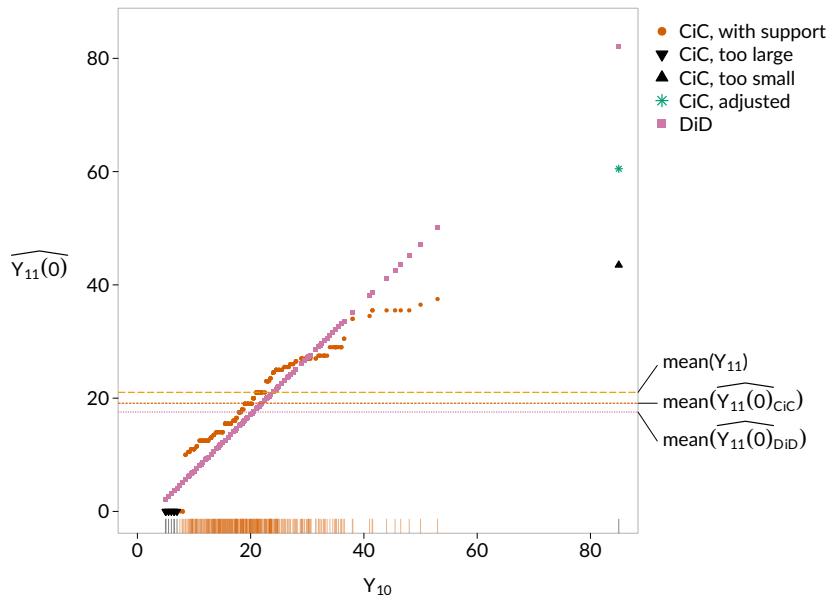
Plot demonstrates CiC and DiD methods of extrapolation for observations without support.

Bounding

- Lack of support implies that extrapolation is necessary for point identification.
- For a lower bound on ATT, we only need to provide an upper bound for the triangle on the right. We use the maximum post-treatment Y value.
- Estimates:

$\widehat{ATT}_{DiD}$	2.75
$\widehat{ATT}_{CiC}$	1.92
$\widehat{ATT}_{CiC}$ (lower bound using max $Y_{\cdot 1}$)	1.87
$\widehat{ATT}_{CiC-r}$	1.86

- The balance of the evidence is that the DiD estimate is probably too high.

Time vs Group Invariance

$\overline{Y}_{00}$

23.33

$\overline{Y}_{01}$
21.03
$\overline{Y}_{11}$
20.44
$\overline{Y}_{10}$

18.27
$\widehat{\overline{Y}_{11}(0)}_{DiD}$

All
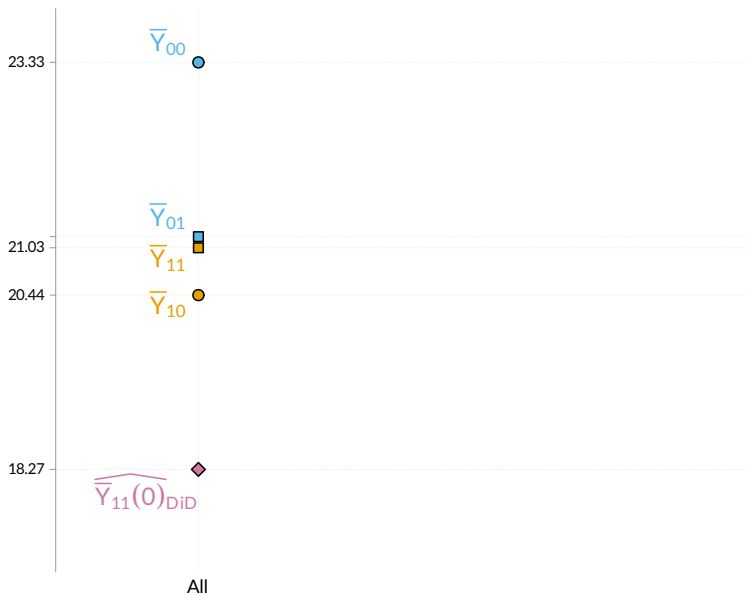
Assume Time Invariant Confounding

$\overline{Y}_{00}$

$\overline{Y}_{01}$

$\overline{Y}_{11}$

$\overline{Y}_{10}$

Control

Treated

Parallel Trends

$\widehat{ATT}_{DiD}$

$\widehat{\overline{Y}_{11}(0)}_{DiD}$

23.33

21.03

20.44

18.27

All   Pre   Post
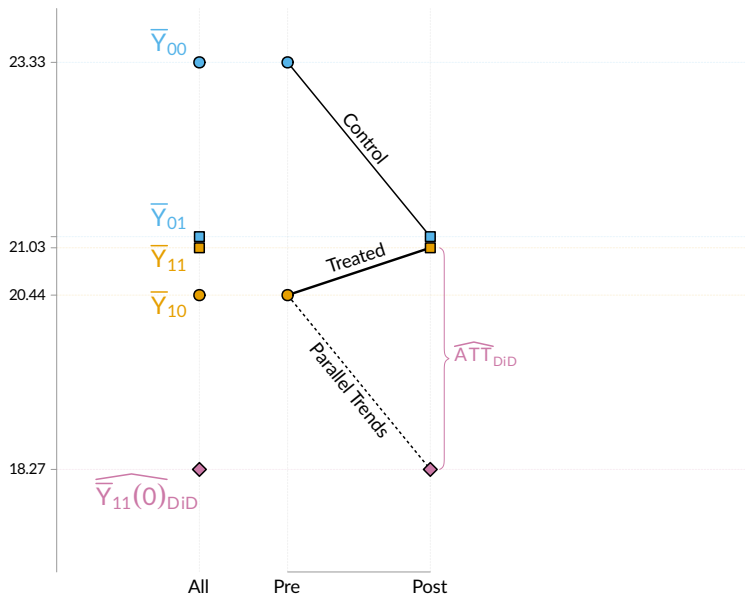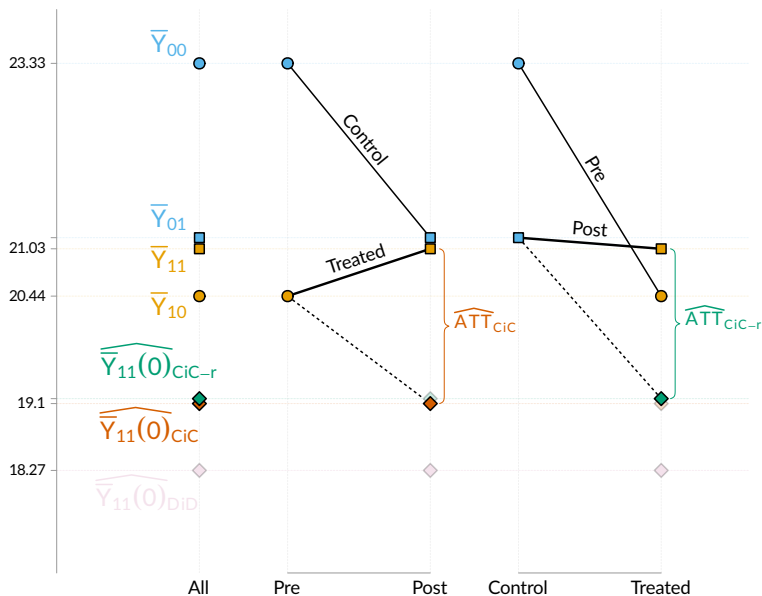
Assumption of Group Invariant Confounding Provides Same DiD Estimate
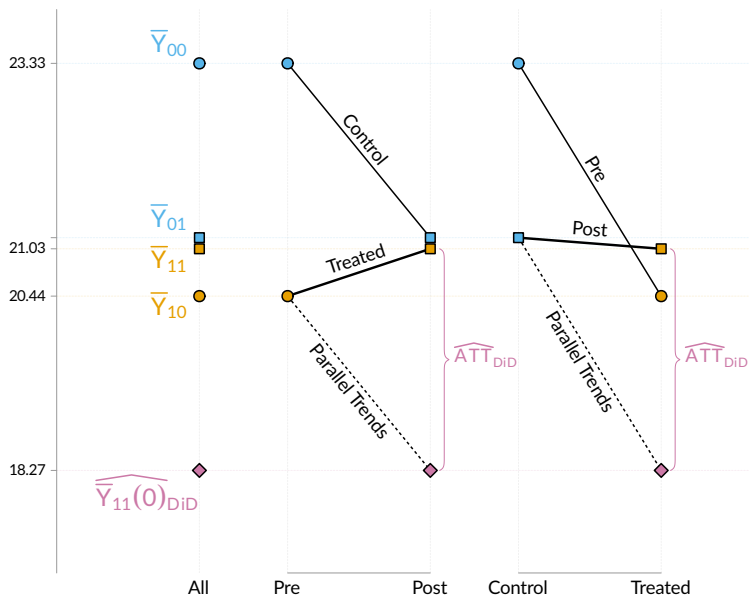
Time and Group Invariance Provide Different CiC Type Estimates

Summary

- Even with only two-time periods, distributional information can reveal issues with DiD, including support problems.
- Loops plots and plots of imputed PO distributions to assess support issues.
- The ATT can be bounded by a support-adjusted CiC estimates. In the minimum wage example CiC estimate provides a lower bound.
- Time-invariant and group-invariant confounding are not equivalent on the quantile scale, simultaneous presentation of CiC and reverse CiC, in addition to support-adjusted estimates for both. In the minimum wage example, CiC-r has no support issues.
- CiC and CiC-r estimates (with and without support adjustments) for the minimum wage example indicate that DiD estimate is likely too high.

Thank you!