# A Mixture Framework for Scaling with Anchoring Items

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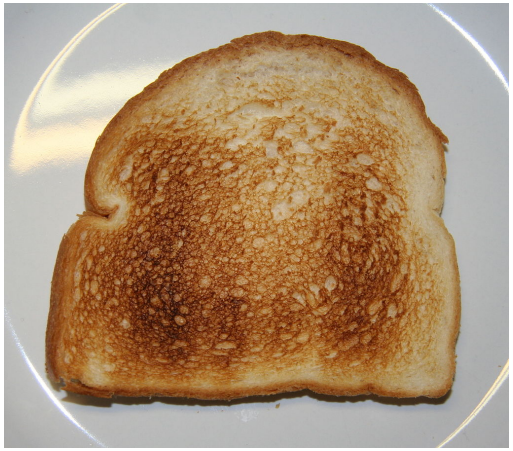
# The Problem

How crunchy do you like your toast?

1. Very crunchy
2. Crunchy
3. Neither crunchy nor soft
4. Soft
5. Very soft

# The Solution



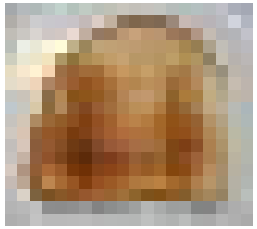
Source: Wikimedia Commons

How crunchy is this toast?

1. Very crunchy
2. Crunchy
3. Neither crunchy nor soft
4. Soft
5. Very soft

Source: Wikimedia Commons

# But did it work?



How crunchy is this toast?

1. Very crunchy
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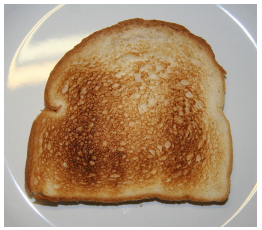
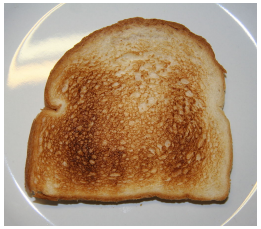


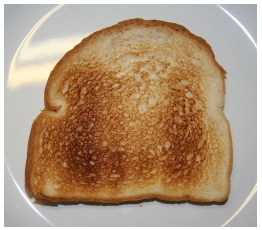
How [redacted] is this toast?

1. Very [redacted]
2. [redacted]
3. Neither [redacted] nor [redacted]
4. [redacted]
5. Very [redacted]



How ~~crunchy~~ **hot** is this toast?

1. Very ~~crunchy~~ **hot**
2. ~~Crunchy~~ **Hot**
3. Neither ~~crunchy~~ **hot** nor ~~soft~~ **cold**
4. ~~Soft~~ **Cold**
5. Very ~~soft~~ **cold**



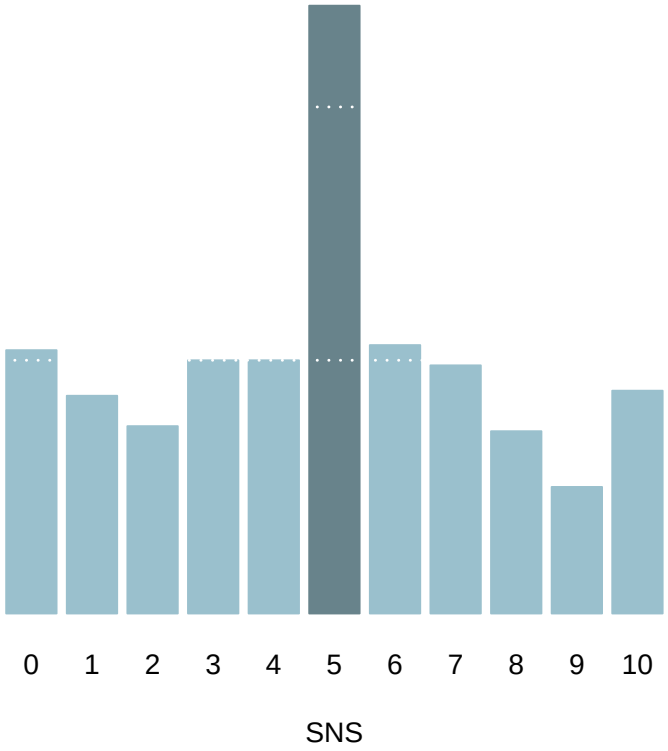
How crunchy is this toast?

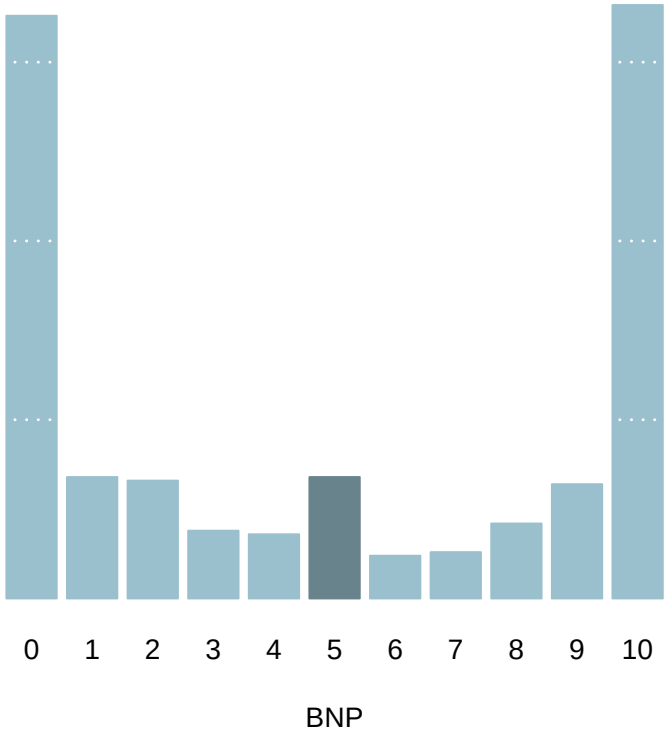
1. Very crunchy
2. Crunchy
3. Neither crunchy nor soft
4. Soft
5. Very soft

Source: telegraph.co.uk

# How bad is it?

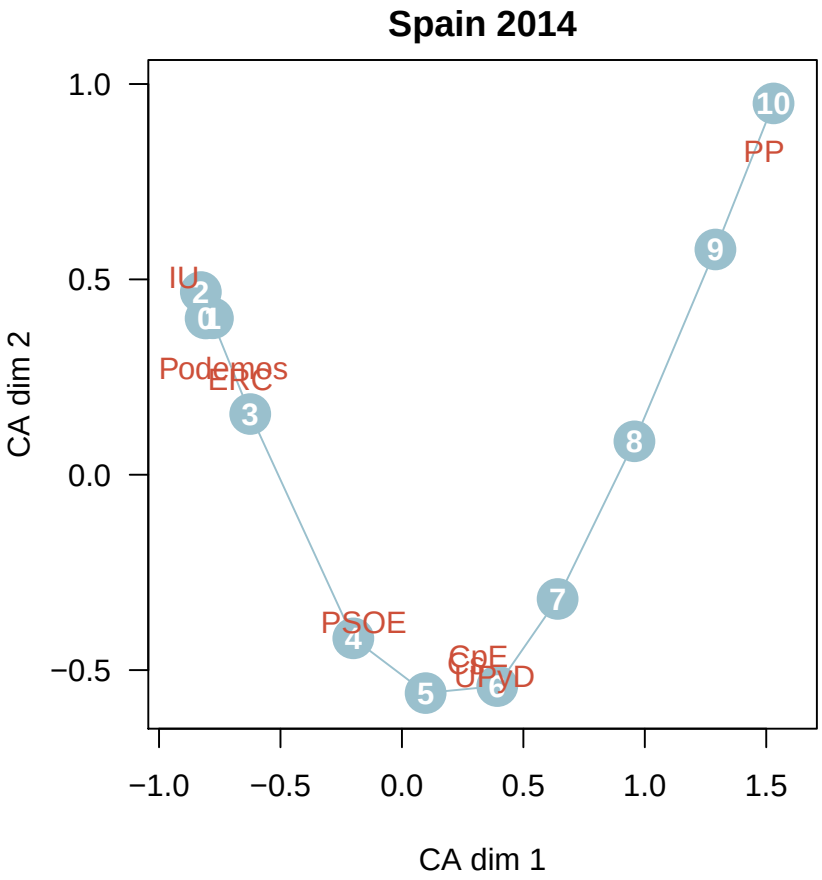
# EES voter study L/R batteries



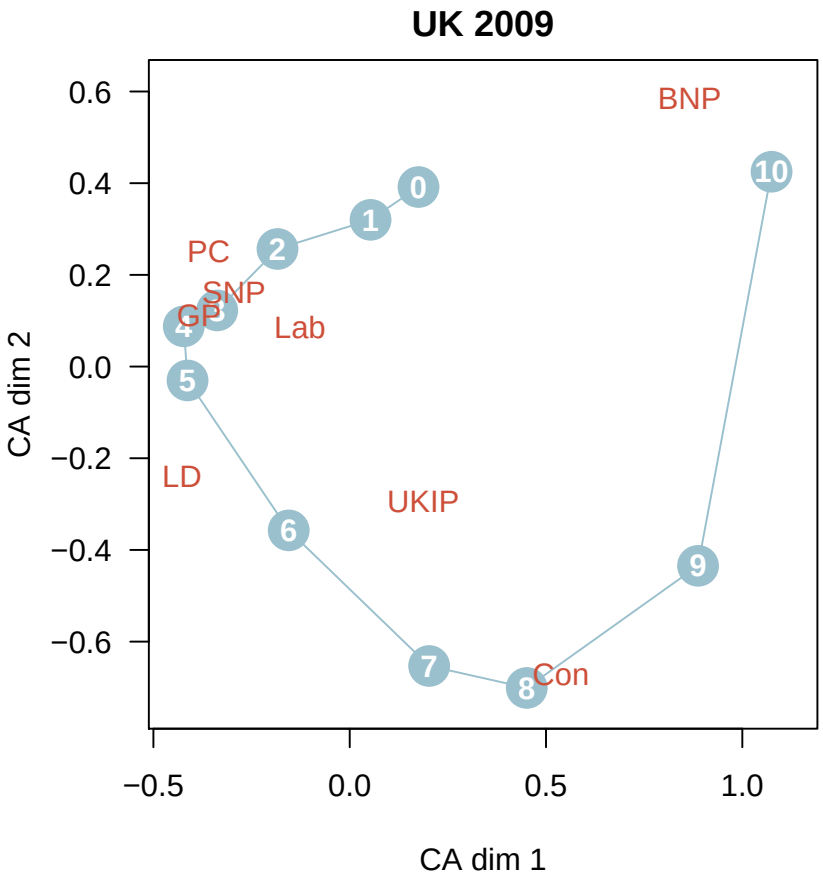



# Diagnostics with correspondence analysis

✔




✘



# Parametric Scaling

# Aldrich-McKelvey Scaling

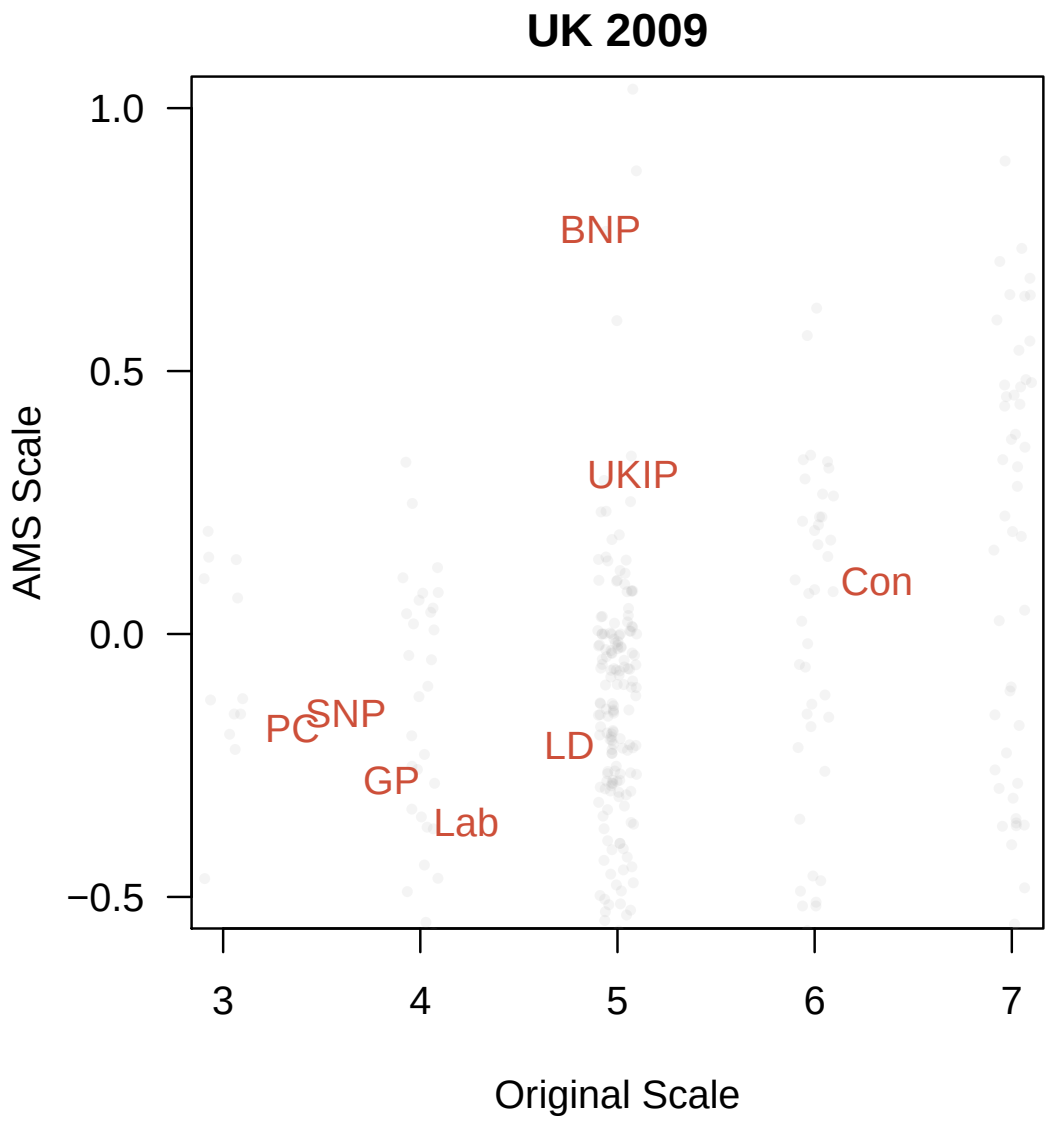
## Anchoring items model

$$
\begin{aligned}
y_{ro} &\sim \text{Normal}(\hat{y}_{ro}, \sigma) \\
\hat{y}_{ro} &= \alpha_r + \beta_r \theta_o
\end{aligned}
$$

## Scaling self-placements

$$
\zeta_r = \frac{s_r - \alpha_r}{\beta_r}
$$

# AMS: EES VS L/R Batteries




44% respondents have flipped scales

# Aldrich-McKelvey Scaling

# Aldrich-McKelvey Scaling

# Bayesian AMS (Hare at al. 2015)

$$
\begin{aligned}
y_{ro} &\sim \text{Normal}(\alpha_r + \beta_r \theta_o,\ \sigma_r \sigma_o) \\
\sigma_o^{-1/2} &\sim \text{Gamma}(0.1,\ 0.1) \\
\sigma_r^{-1/2} &\sim \text{Gamma}(\gamma_1,\ \gamma_2) \\
\boldsymbol{\gamma} &\sim \text{Gamma}(0.1,\ 0.1) \\
\boldsymbol{\theta} &\sim \text{Normal}(0,\ 1) \\
\boldsymbol{\alpha} &\sim \text{Uniform}(-100,\ 100) \\
\boldsymbol{\beta} &\sim \text{Uniform}(-100,\ 100)
\end{aligned}
$$

# Fitting complex latent variable models

Peter Fischli and David Weiss's The First Blush of Morning, 1984.
Source: https://www.wmagazine.com/story/peter-fischli-david-weiss-merry-pranksters

# A Mixture Framework

$$O = \pi M + (1 - \pi) C$$

$O$ observed responses

$M$ *informative* responses

$C$ *uninformative* responses

$\pi$ mixing weight, $\pi \in [0, 1]$

# Measurement model

$$
\begin{aligned}
y_{ro} &\sim \text{Categorical}\left(\boldsymbol{p}_{ro}\right) \\
p_{rok} &= \text{OrdLogit}\left(\left(\tau_{rk} - \gamma_r \theta_o\right) \beta_o\right) \\
\boldsymbol{\tau}_{\mathbf{r}} &\sim \text{Logistic}(0,\ 1),\ \tau_{rk} < \tau_{r,k+1} \\
P(\gamma_r = -1) &\sim \text{Beta}(0.5,\ 0.5),\ \gamma_r \in \{-1,\ +1\} \\
\ln \beta_o &\sim \text{Normal}(0,\ 1) \\
\theta_o &\sim \text{Normal}(0,\ \sigma) \\
\zeta_r &\sim \text{Normal}(0,\ \sqrt{R}\sigma) \\
\sigma &\sim \text{HalfNormal}^{+}(0,\ 1)
\end{aligned}
$$

. . . itself a mixture if $\{\gamma_r\}$ included.

# Mixing patterns

## None

**Item**

| A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 5 | 4 | 8 | 6 | 6 | 5 | 5 | 2 |
| 5 | 5 | 7 | 5 | 5 | 10 | 10 | 8 |
| 1 | 5 | 2 | 6 | 8 | 1 | 5 | 5 |
| 5 | 4 | 5 | 7 | 5 | 2 | 4 | 5 |
| 9 | 5 | 5 | 9 | 7 | 5 | 5 | 5 |
| 7 | 5 | 5 | 4 | 7 | 0 | 8 | 1 |
| 0 | 6 | 2 | 2 | 2 | 8 | 5 | 8 |
| 2 | 3 | 5 | 0 | 3 | 0 | 10 | 6 |
| 2 | 7 | 5 | 9 | 5 | 5 | 10 | 6 |
| 5 | 4 | 2 | 10 | 3 | 9 | 8 | 9 |
| 0 | 5 | 4 | 1 | 10 | 5 | 0 | 0 |
| 6 | 7 | 3 | 7 | 0 | 5 | 3 | 0 |
| 10 | 0 | 5 | 9 | 6 | 4 | 8 | 5 |

## Respondent

**Item**

| A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 7 | 2 | 6 | 8 | 10 | 2 | 5 | 5 |
| 0 | 9 | 3 | 2 | 4 | 5 | 5 | 7 |
| 10 | 6 | 5 | 4 | 8 | 4 | 10 | 4 |
| 2 | 3 | 10 | 0 | 5 | 2 | 4 | 0 |
| 3 | 5 | 2 | 8 | 2 | 7 | 9 | 5 |
| 4 | 9 | 0 | 5 | 8 | 5 | 7 | 5 |
| 8 | 5 | 5 | 7 | 4 | 4 | 4 | 10 |
| 1 | 6 | 7 | 1 | 4 | 8 | 5 | 7 |
| 10 | 5 | 7 | 7 | 8 | 8 | 9 | 9 |
| 5 | 5 | 5 | 9 | 3 | 2 | 8 | 0 |
| 0 | 7 | 8 | 5 | 5 | 5 | 7 | 7 |
| 2 | 5 | 9 | 6 | 2 | 5 | 10 | 0 |
| 9 | 5 | 0 | 5 | 9 | 0 | 6 | 4 |

## Answer

**Item**

| A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 9 | 1 | 9 | 5 | 6 | 5 | 3 | 8 |
| 7 | 2 | 10 | 0 | 5 | 10 | 6 | 6 |
| 1 | 1 | 5 | 6 | 9 | 2 | 3 | 1 |
| 0 | 0 | 8 | 7 | 1 | 5 | 9 | 8 |
| 5 | 10 | 8 | 5 | 8 | 6 | 6 | 5 |
| 5 | 5 | 6 | 8 | 1 | 5 | 5 | 7 |
| 5 | 9 | 7 | 5 | 5 | 4 | 5 | 5 |
| 8 | 7 | 5 | 4 | 7 | 0 | 8 | 1 |
| 10 | 6 | 0 | 6 | 2 | 2 | 2 | 8 |
| 5 | 5 | 10 | 6 | 2 | 3 | 5 | 3 |
| 5 | 2 | 10 | 3 | 9 | 8 | 9 | 2 |
| 0 | 5 | 0 | 0 | 5 | 1 | 10 | 5 |
| 0 | 5 | 6 | 4 | 8 | 5 | 7 | 3 |

## Both

**Item**

| A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 9 | 1 | 9 | 5 | 6 | 5 | 3 | 8 |
| 7 | 2 | 10 | 0 | 5 | 10 | 6 | 6 |
| 1 | 1 | 5 | 6 | 9 | 2 | 3 | 1 |
| 0 | 0 | 8 | 7 | 1 | 5 | 9 | 8 |
| 5 | 10 | 8 | 5 | 8 | 6 | 6 | 5 |
| 5 | 5 | 6 | 8 | 1 | 5 | 5 | 7 |
| 5 | 9 | 7 | 5 | 5 | 4 | 5 | 5 |
| 8 | 7 | 5 | 4 | 7 | 0 | 8 | 1 |
| 10 | 6 | 0 | 6 | 2 | 2 | 2 | 8 |
| 5 | 5 | 10 | 6 | 2 | 3 | 5 | 3 |
| 5 | 2 | 10 | 3 | 9 | 8 | 9 | 2 |
| 0 | 5 | 0 | 0 | 5 | 1 | 10 | 5 |
| 0 | 5 | 6 | 4 | 8 | 5 | 7 | 3 |

# Mixing patterns

By **respondent**

$$g_r \sim \text{Categorical}\left(\boldsymbol{\pi}\right)$$

By **answer**

$$g_{ro} \sim \text{Categorical}\left(\boldsymbol{\pi} \text{ or } \boldsymbol{\pi}_o\right)$$

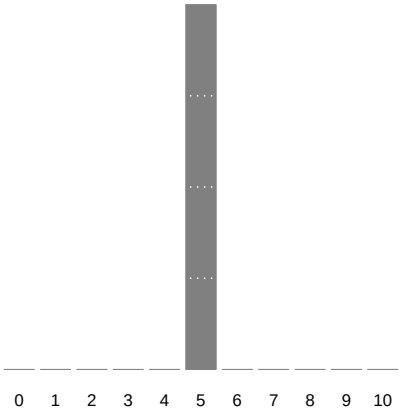
with

$$\boldsymbol{\pi} \text{ or } \boldsymbol{\pi}_o \sim \text{Dirichlet}\left(0.5, \ldots, 0.5\right)$$

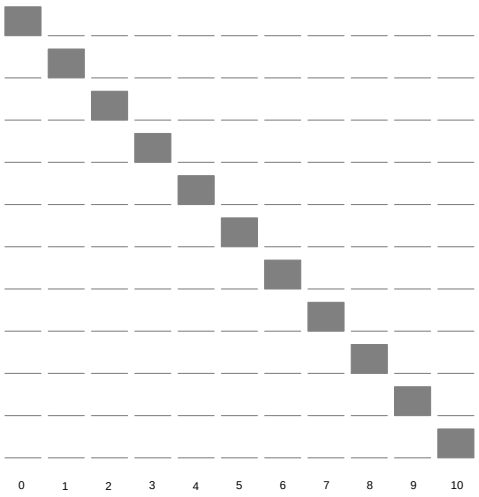
# Contamination models

## i. Pseudoguessing

## ii. Midpoint




## iii. Straightlining

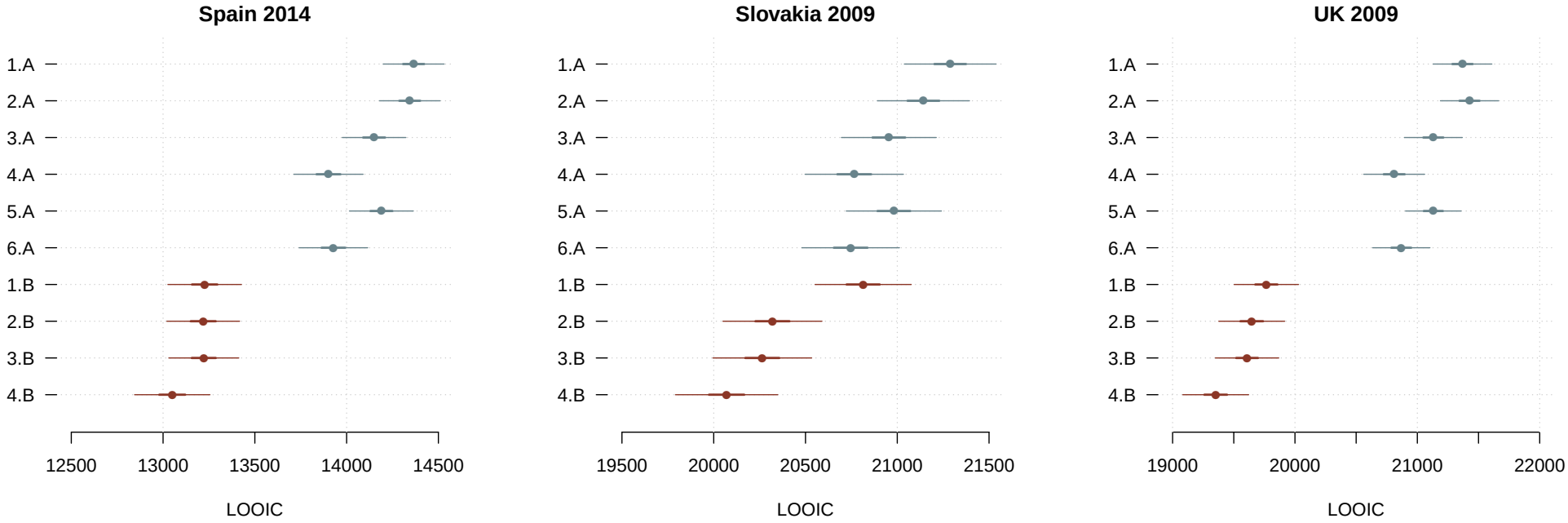



## iv. Between-respondent multidimensionality

# EES: Models

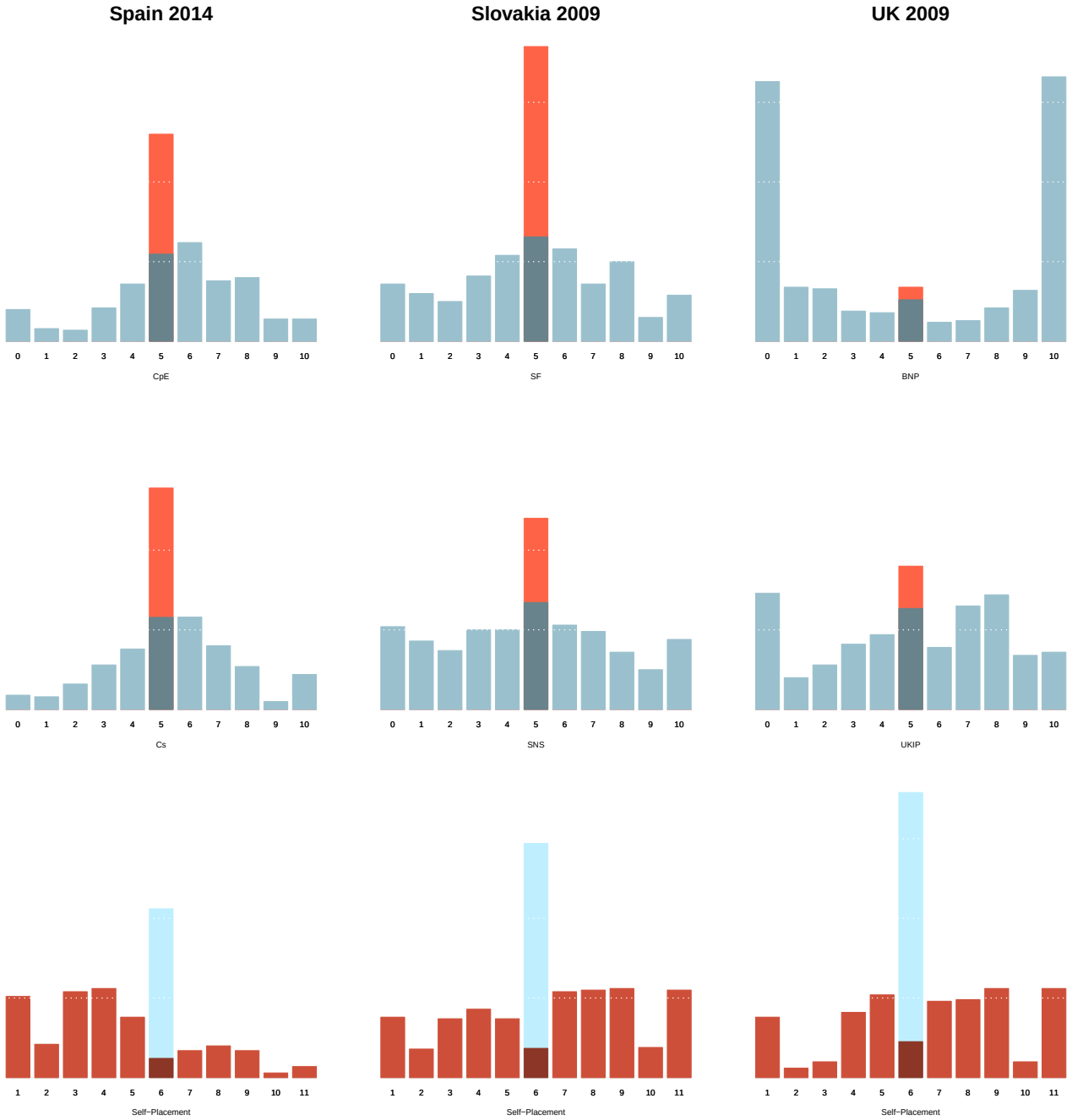
| | Scale flipping | Midpoint inflation | Pseudo-guessing | Item slope |
|---|---|---|---|---|
| 1.A | | | | |
| 2.A | Y | | | |
| 3.A | Y | aggr. | | |
| 4.A | Y | item | | |
| 5.A | Y | aggr. | item | |
| 6.A | Y | item | item | |
| 1.B | | | | Y |
| 2.B | Y | | | Y |
| 3.B | Y | aggr. | | Y |
| 4.B | Y | item | | Y |

# EES: Model fit

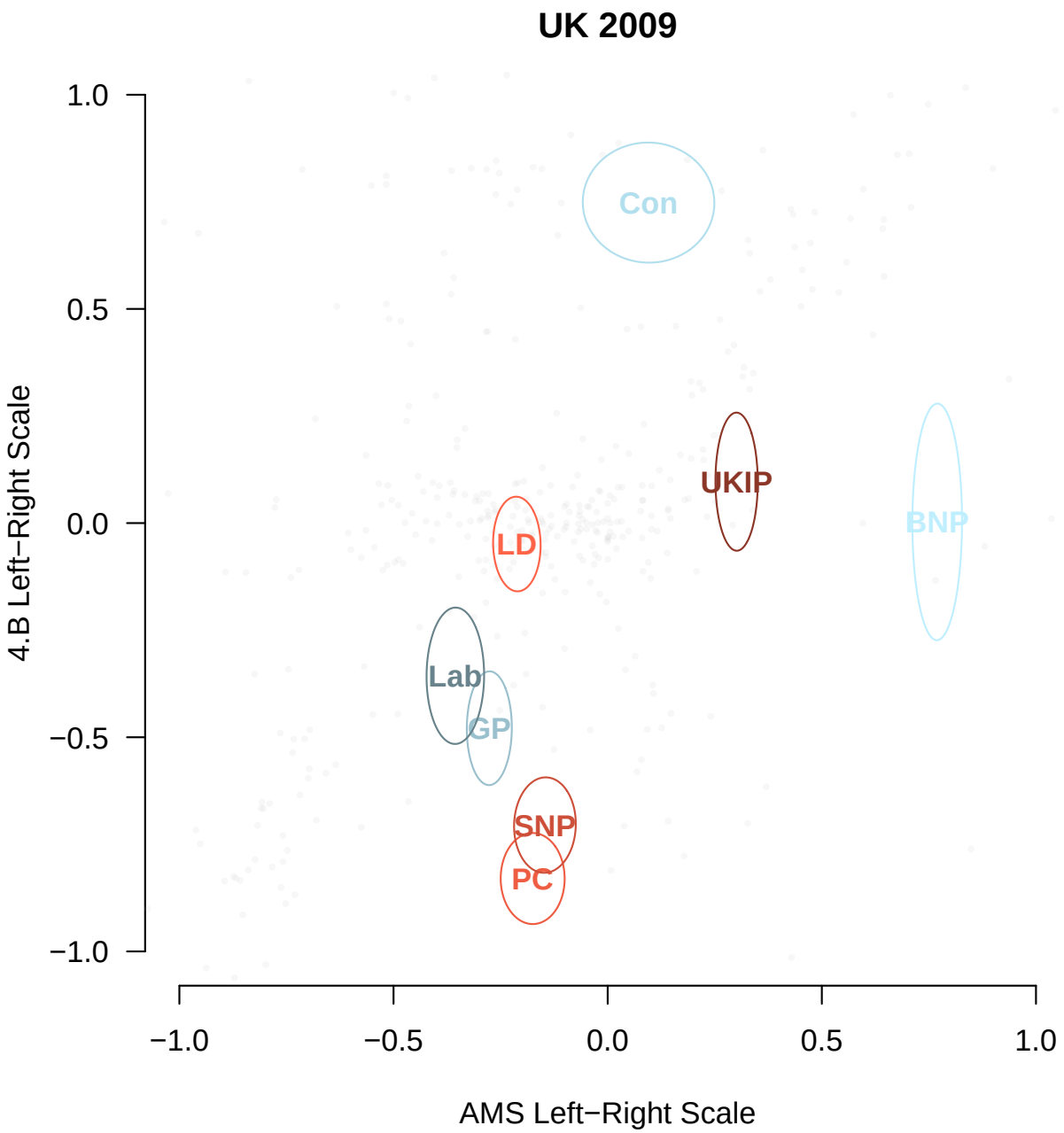



4.B: scale flipping, midpoint inflation, item discrimination

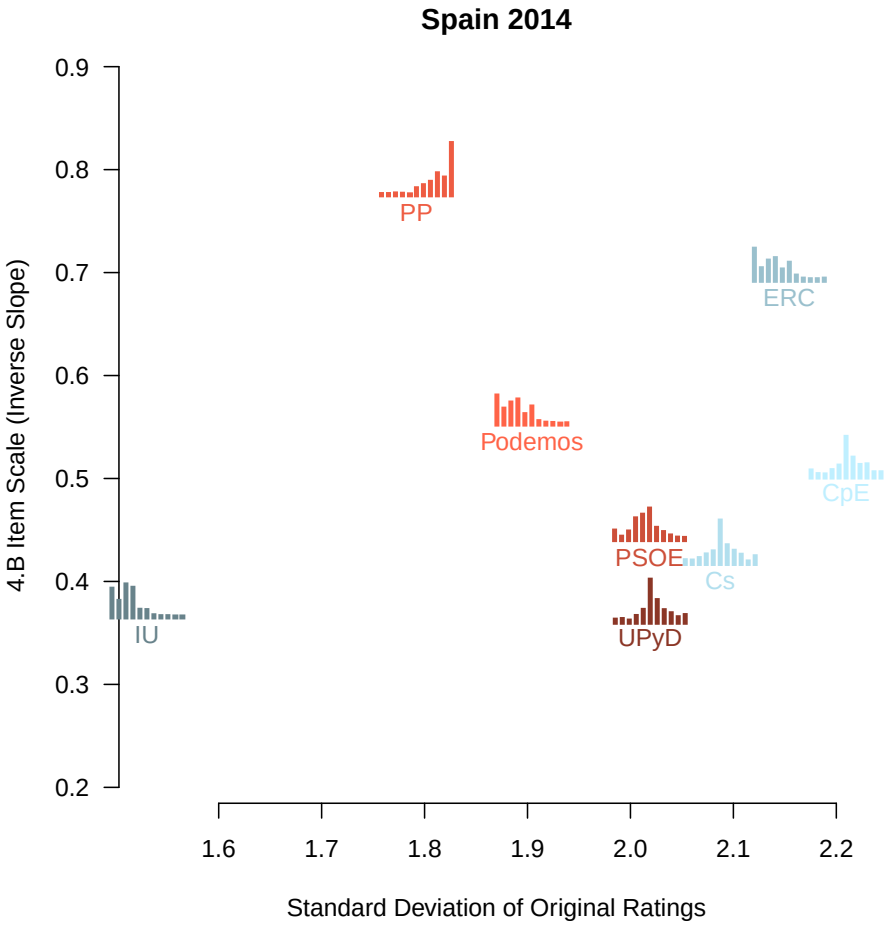
# EES: Midpoint inflation

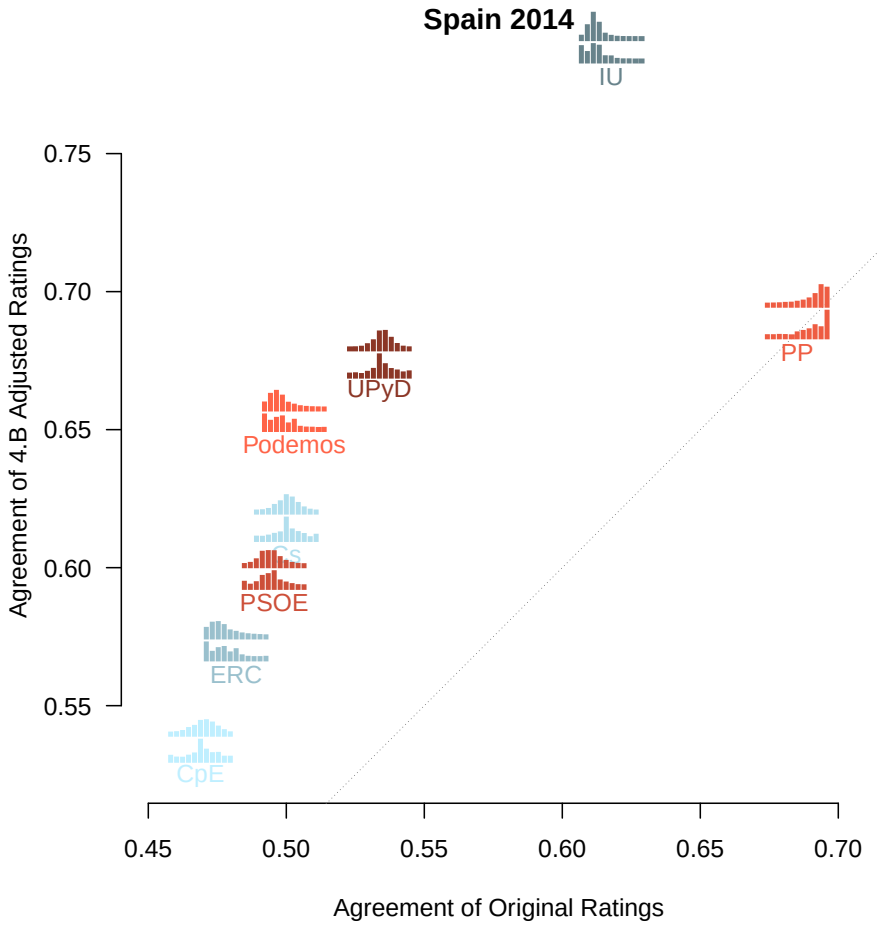


# 4.B vs. AMS: Item locations



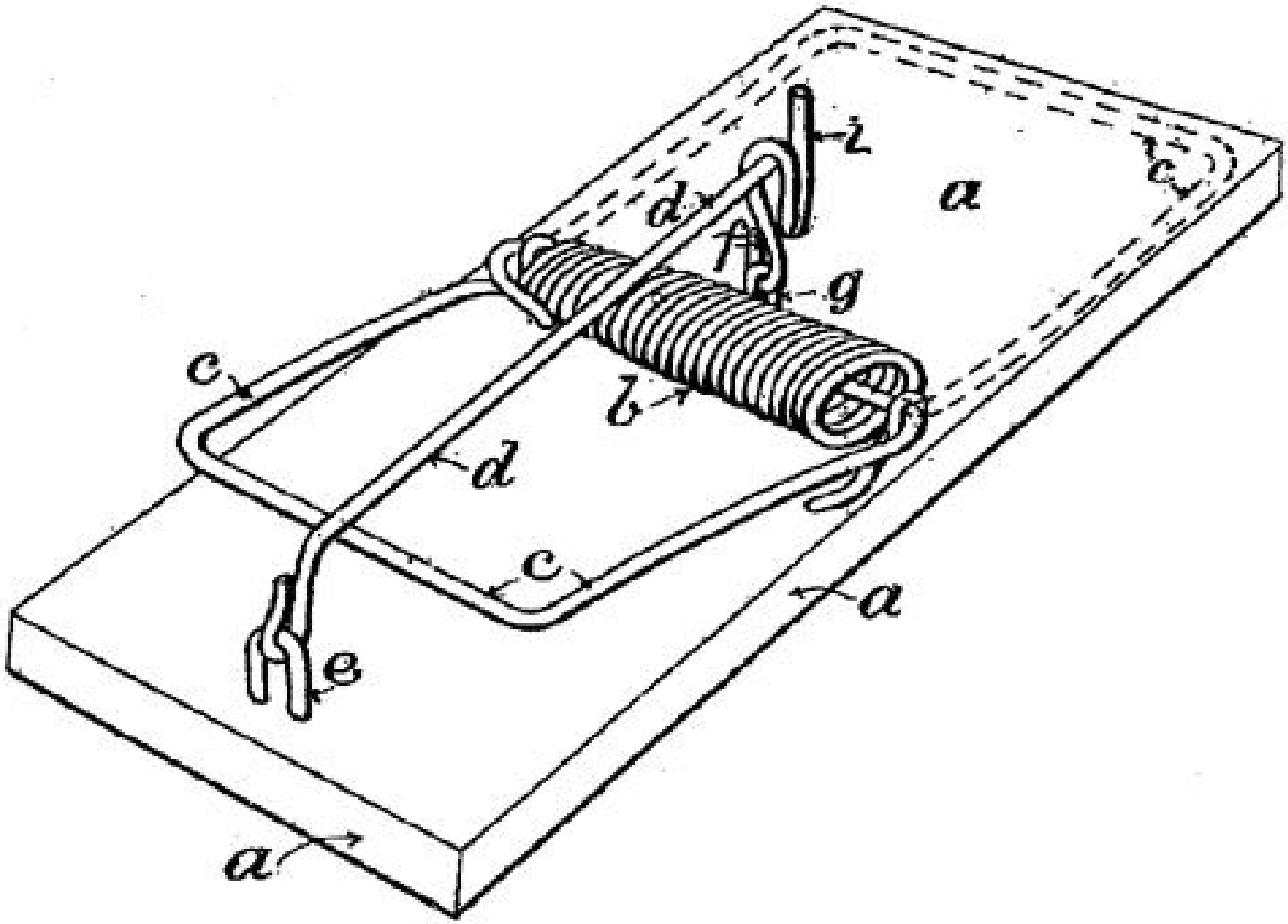
# Item discrimination, rater agreement, party ambiguity



# Conclusion

# You're selling a mousetrap, so where are the regressions?



# Conclusion

No free lunch

- bias-variance trade-off
- estimating *well* costs

What *didn't* work so well deserves a paper

- GP cut-points
- respondent & item slopes

Next

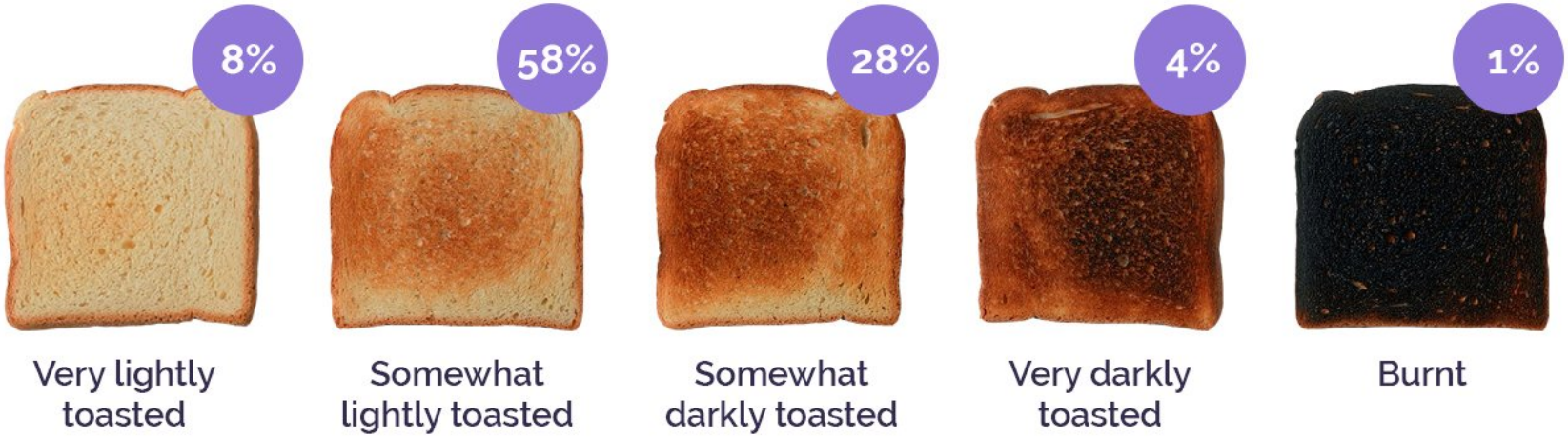
- missing answers
- more structured memberships

# Thank you!

@medzihorsky

## Most Americans prefer their bread to be somewhat lightly toasted

What is your MOST preferred level of toasting for bread? (%)




YouGov

August 27 - 28, 2019