A Mixture Framework for Scaling with Anchoring Items

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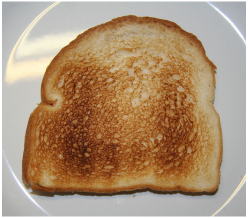
The Problem

How crunchy do you like your toast?

1. Very crunchy
2. Crunchy
3. Neither crunchy nor soft
4. Soft
5. Very soft

The Solution



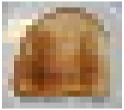
Source: Wikimedia Commons

How crunchy is this toast?

1. Very crunchy
2. Crunchy
3. Neither crunchy nor soft
4. Soft
5. Very soft

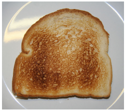
Source: Wikimedia Commons

But did it work?



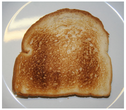
How crunchy is this toast?

1. Very crunchy
2. Crunchy
3. Neither crunchy nor soft
4. Soft
5. Very soft



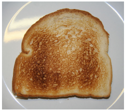
How [redacted] is this toast?

1. Very [redacted]
2. [redacted]
3. Neither [redacted] nor [redacted]
4. [redacted]
5. Very [redacted]



How ~~crunchy~~ **hot** is this toast?

1. Very ~~crunchy~~ **hot**
2. ~~Crunchy~~ **Hot**
3. Neither ~~crunchy~~ **hot** nor ~~soft~~ **cold**
4. ~~Soft~~ **Cold**
5. Very ~~soft~~ **cold**



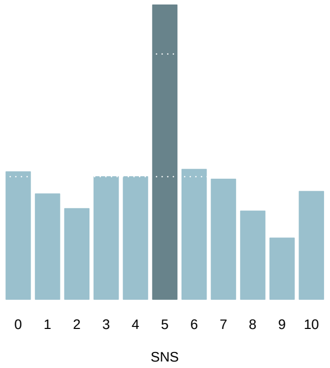
How crunchy is this toast?

1. Very crunchy
2. Crunchy
3. Neither crunchy nor soft
4. Soft
5. Very soft

Source: telegraph.co.uk

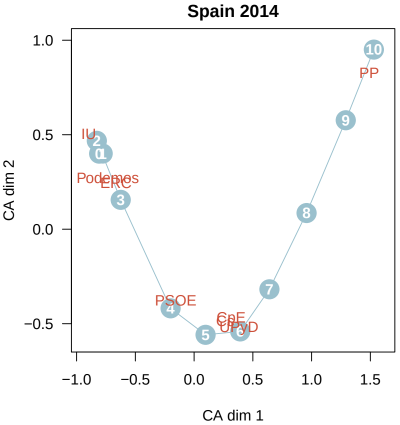
How bad is it?

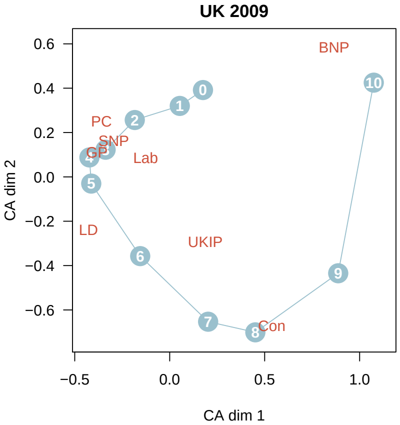
EES voter study L/R batteries



Diagnostics with correspondence analysis

✔ ✘





Parametric Scaling

Aldrich-McKelvey Scaling

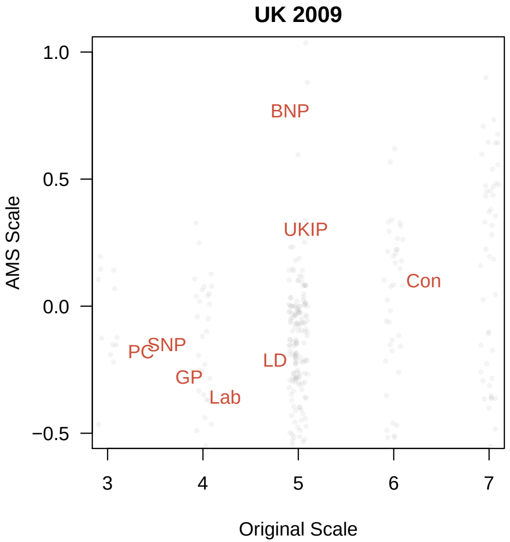
Anchoring items model

$$\begin{aligned} y_{ro} &\sim \text{Normal}(\hat{y}_{ro},\ \sigma) \\ \hat{y}_{ro} &= \alpha_r + \beta_r \theta_o \end{aligned}$$

Scaling self-placements

$$\zeta_r = \frac{s_r - \alpha_r}{\beta_r}$$

CA: EES VS L/R Batteries



44% respondents have flipped scales

Bayesian AMS (Hare at al. 2015)

$$
\begin{aligned}
y_{ro} &\sim \mathrm{Normal}(\alpha_r + \beta_r\theta_o,\ \sigma_r\sigma_o) \\
\sigma_o^{-1/2} &\sim \mathrm{Gamma}(0.1,\ 0.1) \\
\sigma_r^{-1/2} &\sim \mathrm{Gamma}(\gamma_1,\ \gamma_2) \\
\boldsymbol{\gamma} &\sim \mathrm{Gamma}(0.1,\ 0.1) \\
\boldsymbol{\theta} &\sim \mathrm{Normal}(0,\ 1) \\
\boldsymbol{\alpha} &\sim \mathrm{Uniform}(-100,\ 100) \\
\boldsymbol{\beta} &\sim \mathrm{Uniform}(-100,\ 100)
\end{aligned}
$$

Fitting complex latent variable models

Peter Fischli and David Weiss's The First Blush of Morning, 1984.
Source: https://www.wmagazine.com/story/peter-fischli-david-weiss-merry-pranksters

A Mixture Framework

$$O = \pi M + (1 - \pi)C$$

- O observed responses
- M *informative* responses
- C *uninformative* responses
- π mixing weight, $\pi \in [0, 1]$

Measurement model

$$
\begin{aligned}
y_{ro} &\sim \text{Categorical}\left(\boldsymbol{p}_{ro}\right) \\
p_{rok} &= \text{OrdLogit}\left(\left(\tau_{rk} - \gamma_r \theta_o\right) \beta_o\right) \\
\boldsymbol{\tau}_{\mathbf{r}} &\sim \text{Logistic}(0,\ 1),\ \tau_{rk} < \tau_{r,k+1} \\
P(\gamma_r = -1) &\sim \text{Beta}(0.5,\ 0.5),\ \gamma_r \in \{-1,\ +1\} \\
\ln \beta_o &\sim \text{Normal}(0,\ 1) \\
\theta_o &\sim \text{Normal}(0,\ \sigma) \\
\zeta_r &\sim \text{Normal}(0,\ \sqrt{R}\sigma) \\
\sigma &\sim \text{HalfNormal}^{+}(0,\ 1)
\end{aligned}
$$

... itself a mixture if $\{\gamma_r\}$ included.

Mixing patterns

None

Item

A	B	C	D	E	F	G	H
5	4	8	6	6	5	5	2
5	5	7	5	5	10	10	8
1	5	2	6	8	1	5	5
5	4	5	7	5	2	4	5
9	5	5	9	7	5	5	5
7	5	5	4	7	0	8	1
0	6	2	2	2	8	5	8
2	3	5	0	3	0	10	6
2	7	5	9	5	5	10	6
5	4	2	10	3	9	8	9
0	5	4	1	10	5	0	0
6	7	3	7	0	5	3	0
10	0	5	9	6	4	8	5

Respondent

Item

A	B	C	D	E	F	G	H
7	2	6	8	10	2	5	5
0	9	3	2	4	5	5	7
10	6	5	4	8	4	10	4
2	3	10	0	5	2	4	0
3	5	2	8	2	7	9	5
4	9	0	5	8	5	7	5
8	5	5	7	4	4	4	10
1	6	7	1	4	8	5	7
10	5	7	7	8	8	9	9
5	5	5	9	3	2	8	0
0	7	8	5	5	5	7	7
2	5	9	6	2	5	10	0
9	5	0	5	9	0	6	4

Answer

Item

A	B	C	D	E	F	G	H
9	1	9	5	6	5	3	8
7	2	10	0	5	10	6	6
1	1	5	6	9	2	3	1
0	0	8	7	1	5	9	8
5	10	8	5	8	6	6	5
5	5	6	8	1	5	5	7
5	9	7	5	5	4	5	5
8	7	5	4	7	0	8	1
10	6	0	6	2	2	2	8
5	5	10	6	2	3	5	3
5	2	10	3	9	8	9	2
0	5	0	0	5	1	10	5
0	5	6	4	8	5	7	3

Both

Item

A	B	C	D	E	F	G	H
9	1	9	5	6	5	3	8
7	2	10	0	5	10	6	6
1	1	5	6	9	2	3	1
0	0	8	7	1	5	9	8
5	10	8	5	8	6	6	5
5	5	6	8	1	5	5	7
5	9	7	5	5	4	5	5
8	7	5	4	7	0	8	1
10	6	0	6	2	2	2	8
5	5	10	6	2	3	5	3
5	2	10	3	9	8	9	2
0	5	0	0	5	1	10	5
0	5	6	4	8	5	7	3

Mixing patterns

By **respondent**

$$g_r \sim \text{Categorical}\left(\boldsymbol{\pi}\right)$$

By **answer**

$$g_{ro} \sim \text{Categorical}\left(\boldsymbol{\pi} \text{ or } \boldsymbol{\pi}_o\right)$$

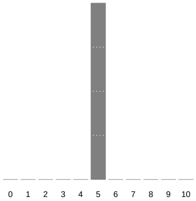
with

$$\boldsymbol{\pi} \text{ or } \boldsymbol{\pi}_o \sim \text{Dirichlet}\left(0.5, \ldots, 0.5\right)$$

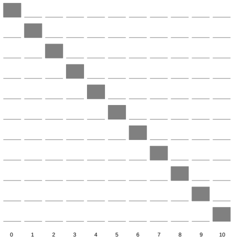
Contamination models

i. Pseudoguessing

ii. Midpoint



iii. Straightlining

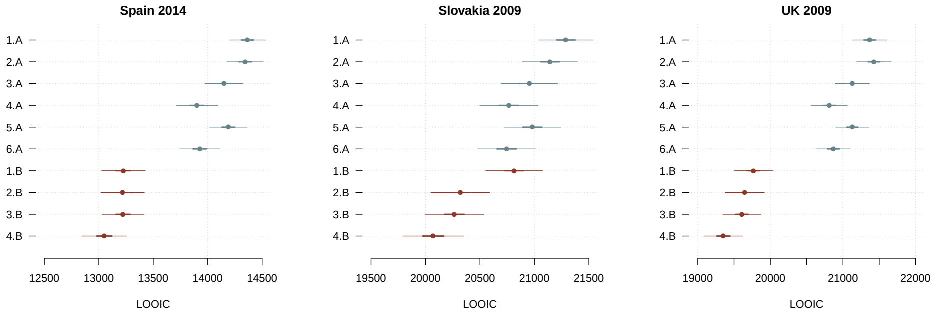


iv. Between-respondent multidimensionality

EES: Models

	Scale flipping	Midpoint inflation	Pseudo-guessing	Item slope
1.A				
2.A	Y			
3.A	Y	aggr.		
4.A	Y	item		
5.A	Y	aggr.	item	
6.A	Y	item	item	
1.B				Y
2.B	Y			Y
3.B	Y	aggr.		Y
4.B	Y	item		Y

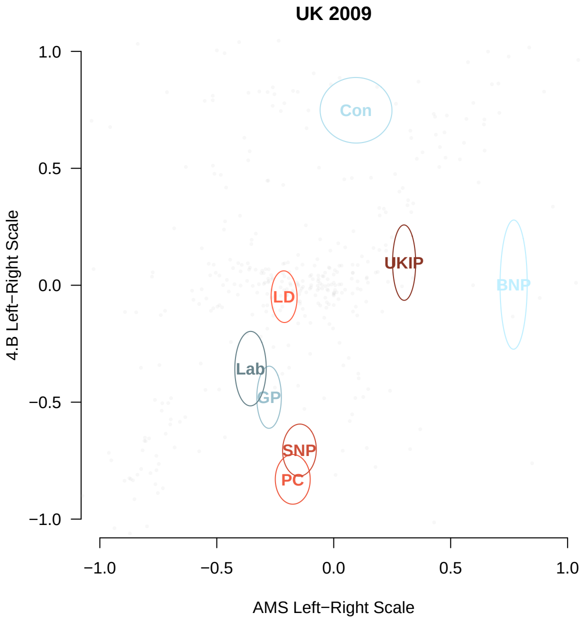
EES: Model fit



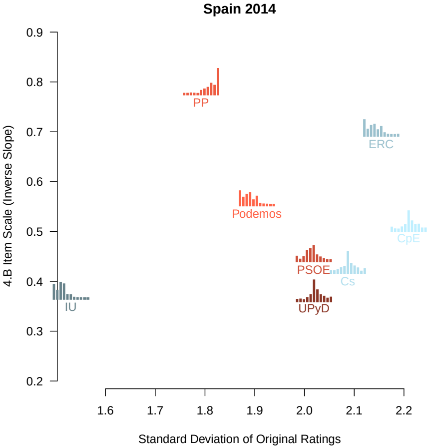
4.B: scale flipping, midpoint inflation, item discrimination

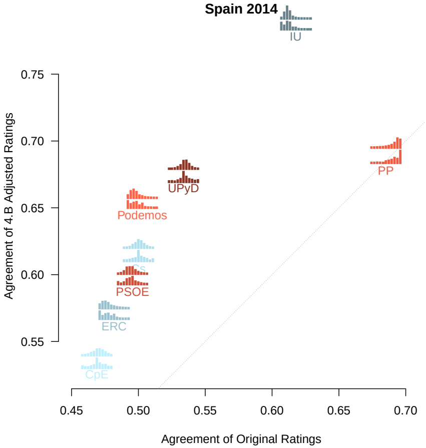
EES: Midpoint inflation

4.B vs. AMS: Item locations



Item discrimination, rater agreement, party ambiguity





Conclusion

No free lunch

- bias-variance trade-off
- estimating *well* costs

What didn't work so well deserves a paper

- GP cut-points
- respondent & item slopes

Next

- missing answers
- more structured memberships

Thank you!

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