

# A Latent Class Framework for Roll Call Analysis

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## Abstract

This paper introduces to roll call analysis a latent class framework that abandons the standard assumption that the model holds in the whole population. The framework splits the population into a class that belongs to the model and a minimized residual class, and draws inferences from the size and composition of the latter. The framework generalizes two widely used measures, the Rice cohesion index and the party unity score, and generates new methods that identify votes inconsistent with group cohesion defined in terms of ideal points, or with non-partisan voting. In ideal point analysis, the framework also aids validation and substantive interpretation with a differential item functioning diagnostic. The methods are demonstrated on congressional roll calls on the Civil Rights Act of 1964.

## The Mixture Index of Fit

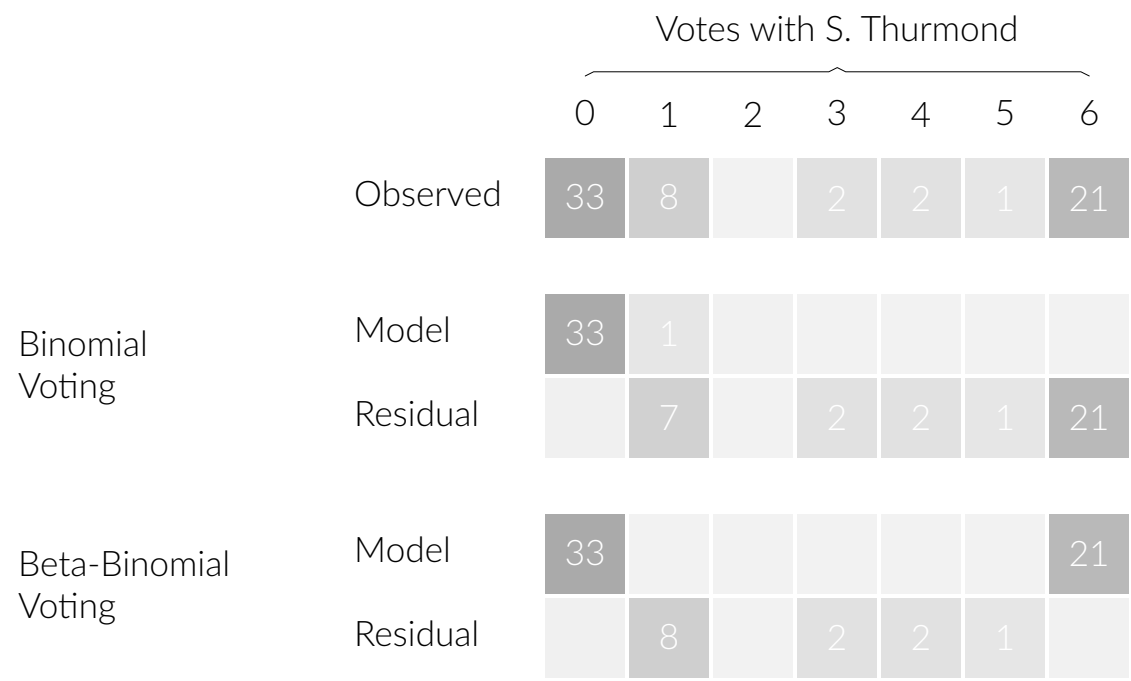
The  $\pi^*$  mixture index [1] measures the misfit of a model by the smallest fraction of the population that it cannot describe,

$$\pi^*(X, \mathcal{M}) = \inf\{\pi: X = (1 - \pi) M + \pi R, \pi \in [0, 1], M \in \mathcal{M}, R \text{ unspecified}\}, \tag{1}$$

where  $\pi^*$  is the size of the undescribed fraction, X is the observed distribution, M an element from the model  $\mathcal{M}$ , and R an unspecified residual distribution. The index rests on always true assumptions, is easy and intuitive to interpret, and provides a new kind of substantively informative residual analysis. The mixture index framework extends the toolbox of legislative scholars in two ways. First, it allows to define new measures that are easy to interpret and based on always true assumptions. These measures rest on comparing the data with a substantively interesting baseline model, and capturing their distance as the smallest fraction of the observations that cannot be described by the baseline model. This paper presents such measures of group cohesion and partisan voting. Second, it provides a differential item functioning diagnostic that can aid the validation and substantive interpretation of ideal point estimates.

## Ideal Point Group Cohesion

From the ideal point perspective, a group can be cohesive even if its legislators do not always vote the same, provided they have similar ideal points. This notion of cohesion can be measured with the mixture index in two different ways. Under the first, the legislators are sorted into those who share an ideal point and those who do not. The second represents a more relaxed notion of cohesion, dividing the legislators into those with ideal points from the same restrictive distribution and those from an unrestricted one. The shared ideal point or the parameters of the restrictive distribution are estimated in the process. Figure 1 shows an example of both, using binomial and beta-binomial voting, respectively [2, 3].



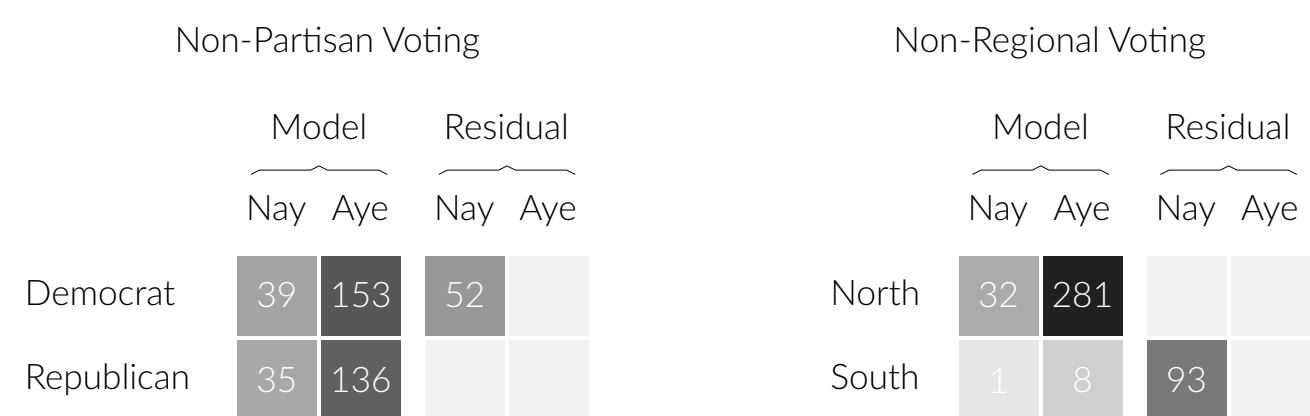
**Figure 1:** The group-fixed and Beta distributed ideal points models applied to the data on voting among the Democratic senators with S. Thurmond in six selected roll calls on the Civil Rights Act of 1964.

## Group Voting

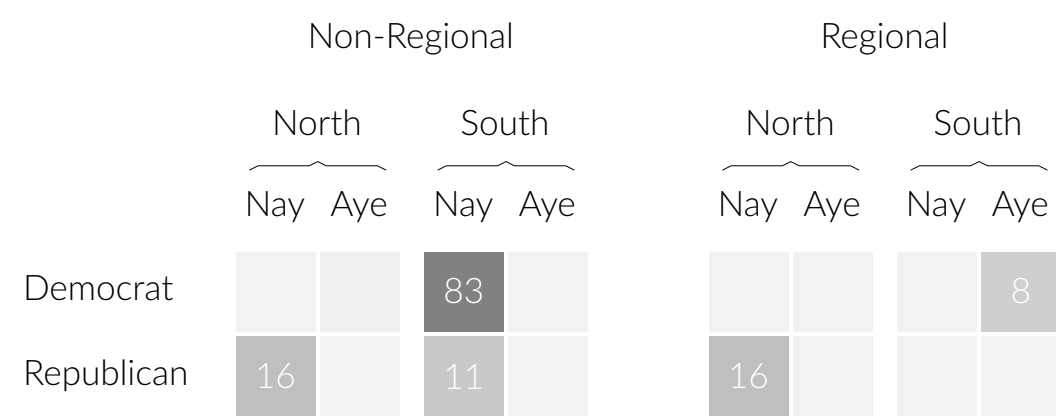
The mixture index provides an intuitive measure of group voting that rests on splitting the votes into those independent of the group and those associated with it, maximizing the size of the former class. This measure applies to any number of options, groups, as well as multiple potentially related group memberships. For any number of options and parties non-partisan voting corresponds to the log-linear model of independence

$$\ln m_{op} = \lambda^0 + \lambda_o^O + \lambda_p^P, \tag{2}$$

where  $m_{op}$  are the non-partisan votes for option o in party p,  $\lambda^0$  the main term, and  $\lambda^O$  and  $\lambda^P$  the option and party terms, respectively. The smallest possible fraction of non-partisan votes is computed using this model as  $\mathcal{M}$  in Equation 1. Figure 2 illustrates this approach with two decompositions of the final House passage of the CRA. The first splits the votes into those independent of the party and those associated with it and the second does the same for the region. Figure 4 contrasts the  $\pi^*$  measure of partisan voting with the difference between party unity scores on 120 CRA-related Senate roll calls, highlighting five patterns of party unity and partisan voting.



**Figure 2:** Two minimum mixture decompositions of the final House passage of the CRA.

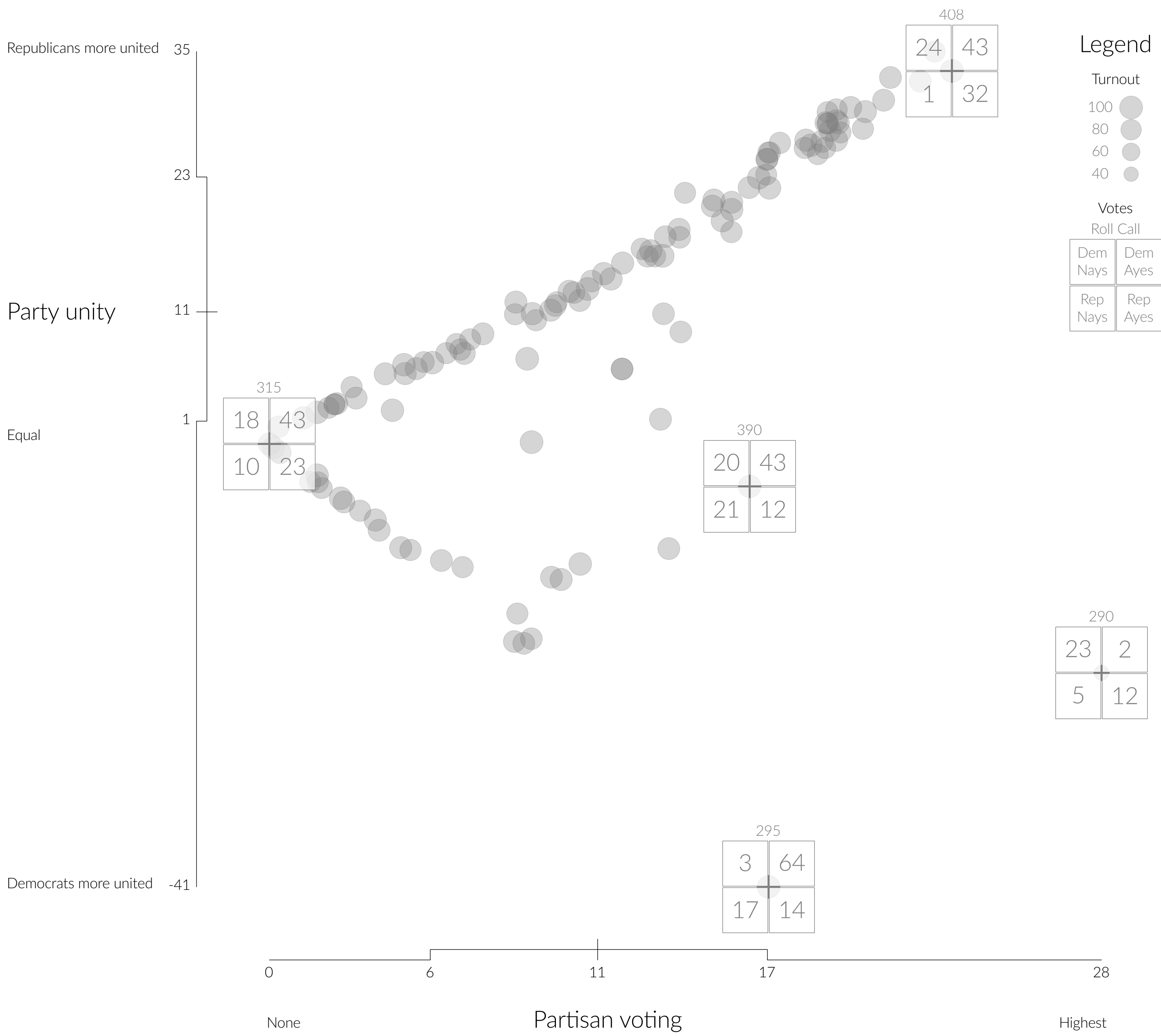


**Figure 3:** Residuals under two models of non-partisan voting in the final House passage of the CRA.

The approach extends easily to multiple group memberships. For the case of party and region

$$\ln m_{opr} = \lambda^0 + \lambda_o^O + \lambda_p^P + \lambda_r^R + \lambda_{pr}^{PR}, \tag{3}$$

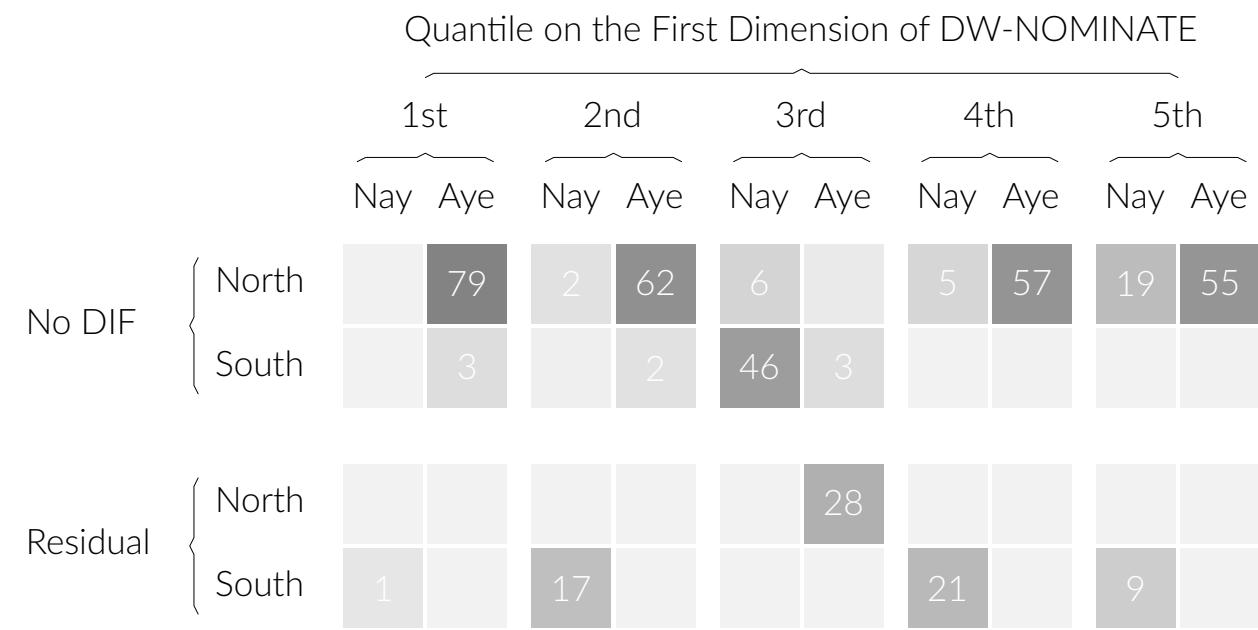
where  $\lambda^P$  and  $\lambda^R$  are the party and region terms, respectively, and  $\lambda^{PR}$  the party-region interaction accounting for different party shares in the regions. Under this model voting is neither partisan nor regional. Partisan voting can be introduced with the option-party interaction  $\lambda^{OP}$  and regional voting with the option-region interaction  $\lambda^{OR}$ . Figure 3 shows the application of two such baseline models—non-partisan non-regional voting, and non-partisan regional voting—to the final House passage of the CRA.



**Figure 4:** Partisan voting and difference in party unity for 120 Senate roll calls related to the Civil Rights Act of 1964. The axes show the minima and maxima, and 25 per cent, 50 per cent, and 75 per cent quantiles.

## Differential Item Functioning Diagnostics for Ideal Point Scaling

In a differentially functioning roll call legislators with similar ideal points will vote differently conditional on some attribute of theirs such as party or gender. Thus, DIF detection can aid ideal point scaling. Roll call data is typically small and covers the whole population, limiting the usefulness of NHST-based diagnostics. The minimum mixture DIF diagnostic [4] is free of these drawbacks and identifies DIF-affected responses. It rests on two nested log-linear models. Under the first, no DIF, model the responses are associated with the ideal points, but not with the attribute. The second model allows a uniform association of the attribute and the response over the latent dimension. Figure 5 illustrates the first model.



**Figure 5:** DIF diagnostics for the first DW-NOMINATE dimension [5] in the final House passage of the CRA. Decomposition into a DIF-free and residual components.

## Discussion

- Interval estimates available via jackknife [6] and bias-corrected bootstrap [7].
- The mixture index has been generalized to missing data [8].
- Smoothing sparse roll call data to facilitate the application of the mixture index benefits from sensitivity analysis.
- Latent class IRT models [9] facilitate the application of the mixture index in roll call scaling, but this is currently computationally too demanding for general use.
- Computation with the **pistar** package [10], with EM nested in line-splitting or Brent root search.
- Ongoing work on frequentist and Bayesian computation of the mixture index with Hamiltonian Monte Carlo.

## References

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