INTR 5057 Research Design & Methods in IR
Day 09

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C E U

18 November 2016

Outline

Homework #2

... where were we?

Samples and Surveys

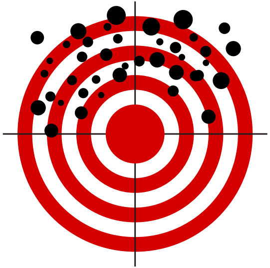
Probability

Homework #2

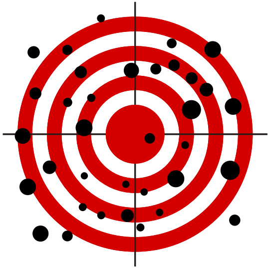
Short individualized exercises on probability. Individualized means each student gets the same exercises with slightly different numbers. If you do not see your homework on Moodle, email me. Deadline: 9 am Budapest time of Day 11 of the course.

The Three Steps in Measurement

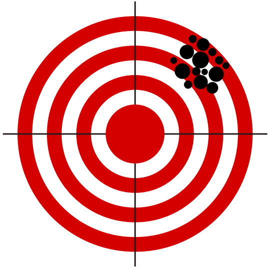
1. Conceptualize.
2. Operationalize.
3. Collect data.



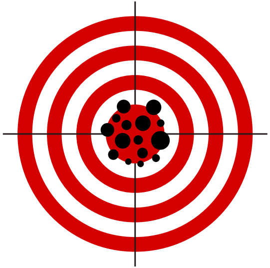
Unreliable & Invalid



Unreliable, But Valid



Reliable, Not Valid



Both Reliable & Valid

Four Levels of Measurement

- **N**
- **O**
- **I**
- **R**

Four Levels of Measurement

- **N**ominal
- **O**rdinal
- **I**nterval
- **R**atio

The Three Attributes

- Ranked?
- Meaningful distance between categories?
- True zero?

N.O.I.R.

	Ranked	*Meaningful distances*	*True 0*
Nominal			
Ordinal	$\surd$		
Interval	$\surd$	$\surd$	
Ratio	$\surd$	$\surd$	$\surd$

Samples and Surveys

- **How** do sample surveys work?
- **Why** do sample surveys work?

Sample Surveys

- What is a **population**?
- What is a **sample**?

Sample Surveys

- In a **census** we ask – i.e. observe – a whole **population**.
- In a **sample survey** we ask a **sample**.

Parameters & Estimates

We are interested in what % of a population has a car. We ask them a question that can be answered only 'yes' or 'no.' Then we count all of those who do have a car and divide it by the overall number: $\frac{\textit{car owners}}{\textit{all asked}}$ and we get a number.

Parameters & Estimates

- If we ask the **whole population** the number is a **parameter**.
- If we ask a **sample** this number is an **estimate** of the population parameter which we do not know.

Parameters & Estimates

$$\text{estimate} = \text{parameter} + \text{bias} + \text{chance error}$$

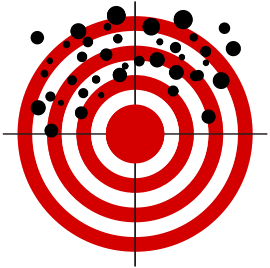
Parameters & Estimates

$$\text{estimate} = \text{parameter} + \text{systematic error} + \text{chance error}$$

Errors, Validity, and Reliability

Errors, Validity, and Reliability

- Systematic error - validity problem.
- Chance error - reliability problem.



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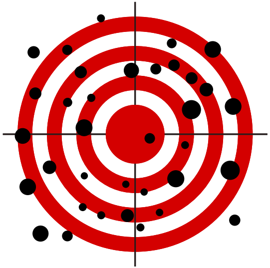

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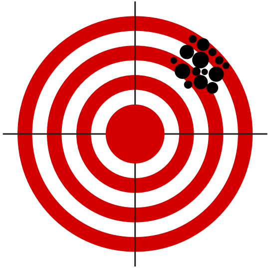

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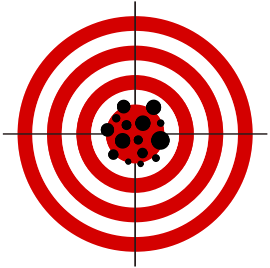

Both Reliable & Valid

Sample Surveys

- How can we get a sample from a population?

Convenience Sampling

- When we ask people who we have approach to.

Convenience Sampling

- When we ask people who we have approach to.
- What problems might occur there?

Convenience Sampling

- When we ask people who we have approach to.
- What problems might occur there?
- People we know might not be representative of the population we are interested in.

Quota Sampling

- We want a representative sample, so we define the attributes we think that matter. We know how the population looks like and set up quotas. The data is collected simply to fill in the quotas.

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- What problems might occur there?

Quota Sampling

- We want a representative sample, so we define the attributes we think that matter. We know how the population looks like and set up quotas. The data is collected simply to fill in the quotas.
- What problems might occur there?
- People who will get interviewed might be different from those who do not.

Random Sampling

- What does it mean that we take a **random sample** of the population?

Random Sampling

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- All members of the population have the same chance to end up in the sample.

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- If the country has 10 million inhabitants and we make a random sample of them, what chance does each of them have to end up in the random sample?

Random Sampling

- What does it mean that we take a **random sample** of the population?
- All members of the population have the same chance to end up in the sample.
- If the country has 10 million inhabitants and we make a random sample of them, what chance does each of them have to end up in the random sample?
- $$\frac{1}{10,000,000}$$

Simple Random Sampling

- Drawing at random without replacement.

Stratified Random Sampling

- First we draw a town from a country at random.
- The we draw a ward from that town at random.
- Then we draw a building from that ward at random.
- And finally we draw a person from that building at random.

Sample Size

- Does size of the sample matter?

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- For random samples larger size decreases the uncertainty attached to the parameter estimate.

Sample Size

- Does size of the sample matter?
- For random samples larger size decreases the uncertainty attached to the parameter estimate.
- In non-random samples it might not help. If the sampling procedure is not good, then no matter how large the sample is, it will give us a biased estimate.

Size of Random Samples

- Larger samples mean less uncertainty. But this relationship is not linear. That is why e.g. opinion polls have 1000-2000 respondents. Having more would decrease the uncertainty but it is not worth the money.

Non-Response

- When a respondent we want to interview does not give one or all answers.
- A respondent is sometimes called more generally a **unit**. If she refuses to answer all questions, it is **unit non-response**.
- A question is sometimes called more generally an **item**. If a unit does not answer a question, it is **item non-response**.

Full Response

Respondent	Q1	Q2	Q3	Q4
1	Y	5	1	a
2	N	3	1	b
3	Y	2	0	a
4	Y	1	1	c
5	N	3	0	b
6	N	2	1	c
7	Y	4	0	a

Unit Non-Response

Respondent	Q1	Q2	Q3	Q4
1	Y	5	1	a
2	N	3	1	b
3	Y	2	0	a
4				
5	N	3	0	b
6	N	2	1	c
7	Y	4	0	a

Item Non-Response

Respondent	Q1	Q2	Q3	Q4
1	Y	5	1	a
2		3	1	b
3		2	0	a
4	Y	1	1	c
5		3	0	b
6	N	2	1	c
7		4	0	a

Non-Response

- Why is non-response a problem?

Non-Response

- Why is non-response a problem?
- Units that refuse to answer might be different from those who do answer.

Non-Response

- Why is non-response a problem?
- Units that refuse to answer might be different from those who do answer.
- Ignoring non-response might bias the estimates.

Probability

Write down – **as a number** – what is the probability that:

- It will rain somwhere on the CEU campus at 9 am on 25 November 2016.
- George Washington was the first President of the USA.

Probability

Write down – **as a number** – what is the probability that:

- It will rain somwhere on the CEU campus at 9 am on 25 November 2016.
- George Washington was the first President of the USA.
- Who wrote 0 for at least one of them?

Probability

Write down – **as a number** – what is the probability that:

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- Who wrote 0 for at least one of them?
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Probability

Write down – **as a number** – what is the probability that:

- It will rain somwhere on the CEU campus at 9 am on 25 November 2016.
- George Washington was the first President of the USA.
- Who wrote 0 for at least one of them?
- Who wrote 1 for at least one of them?
- What other numbers did you write?

Probability

- In classical statistics unique events either do or do not happen.

Two Views of Probability in Statistics

- **Classical**: probability is a **frequency**.
- **Bayesian**: probability is a **degree of belief**.
- For the most of the course we will be dealing with classical – i.e. **frequentist** – statistics.

Probability in Classical Statistics.

Probability is the frequency to which something converges over a long run of repetitions.

Probability in Classical Statistics.

The **frequency theory** of probability works the best for processes repeated

- **independently** – i.e. one repetition does not affect the other –
- under **identical** conditions.

Chance

- One way how to express probability.

Chance

- Chances can range from 0% to 100%
- In other words

  $0\% \leq \text{chance of something} \leq 100\%$
- So as an interval it is ...

Chance

- Chances can range from 0% to 100%
- In other words

  $0\% \leq \text{chance of something} \leq 100\%$
- So as an interval it is ...

  $[0\%; 100\%]$, a closed interval.

Chance

- If a chance of something happening is 40% what is the chance of it not happening?

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  $100\% - 40\% = 60\%$

Chance

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- What is the rule working here?

Chance

- If a chance of something happening is 40% what is the chance of it not happening?
  $100\% - 40\% = 60\%$
- What is the rule working here?
- The chance of something happening plus the chance of it not happening equals 100%.

Random Drawing

- Suppose we have a box with 4 cubes.
- What does it mean to **draw at random** a single cube from the box?

Random Drawing

- Suppose we have a box with 4 cubes.
- What does it mean to **draw at random** a single cube from the box?
- That all cubes in the box have an equal chance of being drawn.

Ex.

- There are 5 cubes in a box. 2 are red and 3 are white. You reach into the box without looking and take 1 cube out. What is the chance that it will be a white cube?

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- There are 5 cubes in a box. 2 are red and 3 are white. You reach into the box without looking and take 1 cube out. What is the chance that it will be a white cube?
- 60% Why?

Ex.

- There are 5 cubes in a box. 2 are red and 3 are white. You reach into the box without looking and take 1 cube out. What is the chance that it will be a white cube?
- 60% Why?
- $$\frac{\text{No. of white cubes}}{\text{No. of all cubes}}$$

Repeated Drawing

Repeated drawing can be either

- **without** replacement, or
- **with** replacement.

Drawing Without Replacement

- The drawn item is not replaced with another item.
- E.g. after a draw of a single item from a box with 4 items only 3 items are left.

Drawing Without Replacement

- There are 4 cubes in a box. 2 are yellow and 2 are green.
- If we draw 1 cube from the box without replacement, how many cubes are left in the box?

Drawing Without Replacement

- There are 4 cubes in a box. 2 are yellow and 2 are green.
- If we draw 1 cube from the box without replacement, how many cubes are left in the box?
- 3

Drawing With Replacement

- The drawn item is replaced with the same item.
- E.g. after a draw of a single item from a box with 4 items there are 4 items with the same characteristics left in the box.

Drawing With Replacement

- There are other kinds of replacement – e.g. by an item of a different type – but most of the time when we say just "drawn with replacement" we mean replaced by an item with the same characteristics.

Drawing Without Replacement

- There are 4 cubes in a box. 2 are yellow and 2 are green.
- If we draw 1 cube from the box with replacement, how many cubes are left in the box?

Drawing Without Replacement

- There are 4 cubes in a box. 2 are yellow and 2 are green.
- If we draw 1 cube from the box with replacement, how many cubes are left in the box?
- 4
- How many are green and how many are yellow?

Drawing Without Replacement

- There are 4 cubes in a box. 2 are yellow and 2 are green.
- If we draw 1 cube from the box with replacement, how many cubes are left in the box?
- 4
- How many are green and how many are yellow?
- 2 are green and 2 are yellow.

Probability Notation

- Probability of event X happening can be expressed as

$$Probability(\text{event X})$$

or simply

$$P(\text{X})$$

Conditional Probability

- Probability of event X happening given event Y happens.
- Can be expressed as

$$P(X|Y)$$

We read this as "*probability of X given Y*"

Independence

- What does it mean that two events are **independent**?

Independence

- What does it mean that two events are **independent**?
- Whether one happens or not has no relation to whether the other does happen or not.

Independence

- We can understand independence as **lack of association**.

Independence & Probability

- Suppose we know that event X is independent of event Y.
- Can you express this in terms of probabilities?

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$$P(X|\mathrm{Y}) = P(X|\text{not Y})$$

Independence & Probability

- Suppose we know that event X is independent of event Y.
- Can you express this in terms of probabilities?
- Probability of X is the same if Y did happen and if it did not happen.
- How about a more formal expression?

$$P(X|\mathrm{Y}) = P(X|\text{not Y})$$

- Works the other way around as well.

Independence & Drawing

- When drawing **with** replacement the draws are **independent**
- When drawing **without** replacement the draws are **dependent**

Ex.

- There are 4 cubes in a box. 1 is yellow and 3 are green.
- If we draw 1 cube from the box **with** replacement, what is the chance that it will be yellow?

Ex.

- There are 4 cubes in a box. 1 is yellow and 3 are green.
- If we draw 1 cube from the box **with** replacement, what is the chance that it will be yellow?
- 25%
- Suppose we have drawn a yellow cube and we draw another cube. What is the chance that it will be green?

Ex.

- There are 4 cubes in a box. 1 is yellow and 3 are green.
- If we draw 1 cube from the box **with** replacement, what is the chance that it will be yellow?
- 25%
- Suppose we have drawn a yellow cube and we draw another cube. What is the chance that it will be green?
- 75% Why?

Ex.

- There are 4 cubes in a box. 1 is yellow and 3 are green.
- If we draw 1 cube from the box **with** replacement, what is the chance that it will be yellow?
- 25%
- Suppose we have drawn a yellow cube and we draw another cube. What is the chance that it will be green?
- 75% Why?
- We replaced the drawn yellow cube with another yellow cube.

Ex.

- There are 4 cubes in a box. 1 is yellow and 3 are green.
- If we draw 1 cube from the box **without** replacement, what is the chance that it will be yellow?

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- 25%
- Suppose we have drawn a yellow cube and we draw another cube. What is the chance that it will be green?
- 100% Why?

Ex.

- There are 4 cubes in a box. 1 is yellow and 3 are green.
- If we draw 1 cube from the box **without** replacement, what is the chance that it will be yellow?
- 25%
- Suppose we have drawn a yellow cube and we draw another cube. What is the chance that it will be green?
- 100% Why?
- Only 3 green cubes were left in the box.

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- At least how many silver coins are there?

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- At least how many silver coins are there?
- 0. Why?

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- At least how many silver coins are there?
- 0. Why?
- Both coins can be golden.

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- At most how many silver coins are there?

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- At most how many silver coins are there?
- 2, Why?

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- At most how many silver coins are there?
- 2, Why?
- Both coins can be silver.

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- We draw 1 item at random. What is the chance it will be a coin?

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- We draw 1 item at random. What is the chance it will be a coin?
- 50% i.e. $\frac{1}{2}$.

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
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- 50% i.e. $\frac{1}{2}$.

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- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- We draw 1 item at random. What is the chance it will be a golden coin?

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- We draw 1 item at random. What is the chance it will be a golden coin?
- That depends on whether material and shape are independent.

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- How would the contents of the box look if material and shape were independent?

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- How would the contents of the box look if material and shape were independent?
- 1 golden coin, 1 silver coin, 1 golden cube, 1 silver cube.

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes. 2 are silver and 2 are gold.
- How would the contents of the box look if material and shape were independent?
- 1 golden coin, 1 silver coin, 1 golden cube, 1 silver cube.
- Can you write this as a table?

Ex.

- There are 8 items in a box. 4 are coins and 4 are cubes. 4 are silver and 4 are gold. Material and shape are independent.
- We draw 1 item at random. What is the chance it will be a golden coin?

Ex.

- There are 8 items in a box. 4 are coins and 4 are cubes. 4 are silver and 4 are gold. Material and shape are independent.
- We draw 1 item at random. What is the chance it will be a golden coin?
- 25% or $\frac{1}{4}$. Why?

Ex.

- There are 8 items in a box. 4 are coins and 4 are cubes. 4 are silver and 4 are gold. Material and shape are independent.
- We draw 1 item at random. What is the chance it will be a golden coin?
- 25% or $\frac{1}{4}$. Why?

  $P(\text{golden coin}) = P(\text{golden}) \times P(\text{coin})$

  $P(\text{golden coin}) = \frac{4}{8} \times \frac{4}{8} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Multiplication Rule

- If 2 events are independent their joint probability is equal to the product of their probabilities.
- I.e. if

$$P(\text{A}|\text{B}) = P(\text{A}|\text{not B})$$

then

$$P(\text{A \& B}) = P(\text{A}) \times P(\text{B})$$

Ex.

- There are 7 items in a box. 3 are coins and 4 are cubes. 5 are silver and 2 are gold. Material and shape are independent.
- We draw 1 item at random. What is the chance it will be a golden coin?

Ex.

- There are 7 items in a box. 3 are coins and 4 are cubes. 5 are silver and 2 are gold. Material and shape are independent.
- We draw 1 item at random. What is the chance it will be a golden coin?

  $P(\text{golden coin}) = P(\text{golden}) \times P(\text{coin})$

  $P(\text{golden coin}) = \frac{2}{7} \times \frac{3}{7} = \frac{6}{49}$

Ex.

- There are 12 items in a box. 3 are coins and 9 are cubes. 10 are silver and 2 are gold. Material and shape are independent.
- We draw 1 item at random. What is the chance it will be a golden coin?

Ex.

- There are 78 items in a box. 33 are coins and 45 are cubes. 37 are silver and 41 are gold. Material and shape are independent.
- We draw 1 item at random. Write down what is the chance it will be a golden coin.

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes.
- We draw 1 item at random with replacement. Then we draw another item at random with replacement.
- What is the chance that both will be coins?

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes.
- We draw 1 item at random with replacement. Then we draw another item at random with replacement.
- What is the chance that at least one of them will be a coin?

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes.
- We draw 1 item at random with replacement. Then we draw another item at random with replacement.
- What is the chance that at least one of them will be a coin?
- The opposite of never getting a coin.

Ex.

- There are 4 items in a box. 2 are coins and 2 are cubes.
- We draw 1 item at random with replacement. Then we draw another item at random with replacement.
- What is the chance that at least one of them will be a coin?
- The opposite of never getting a coin.
- $1 - (\frac{2}{4} \times \frac{2}{4}) = 1 - \frac{1}{4} = \frac{3}{4}$

Probability of ‘At least 1 in X repetitions’

- Probability of getting a certain outcome at least once in several repetitions.
- It is the opposite of never getting the outcome.
- 1 minus the probability of never getting the outcome.

Ex.

- There are 6 items in a box. 3 are coins and 3 are cubes.
- We draw 1 item at random with replacement. Then we draw another item at random with replacement.
- What is the chance that none of them will be a coin?

Ex.

- There are 6 items in a box. 3 are coins and 3 are cubes.
- We draw 1 item at random with replacement. Then we draw another item at random with replacement.
- What is the chance that none of them will be a coin?
- $P(Cube) \times P(Cube) = \frac{3}{6} \times \frac{3}{6} = \frac{9}{36} = \frac{1}{4}$

Ex.

- There are 6 items in a box. 3 are coins and 3 are cubes.
- We draw 1 item at random with replacement. Then we draw another item at random with replacement.
- What is the chance that at least of them will be a coin?

Ex.

- There are 6 items in a box. 3 are coins and 3 are cubes.
- We draw 1 item at random with replacement. Then we draw another item at random with replacement.
- What is the chance that at least of them will be a coin?
- $1 - \frac{3}{6} \times \frac{3}{6} = 1 - \frac{9}{36} = 1 - \frac{1}{4} = \frac{3}{4}$

Ex.

- There are 4 items in a box. 1 is a coin, 1 is a cube and 2 are sticks.
- We draw 1 item at random.
- What is the chance that it is a coin or a cube?

Ex.

- There are 4 items in a box. 1 is a coin, 1 is a cube and 2 are sticks.
- We draw 1 item at random.
- What is the chance that it is a coin or a cube?
- $P(Cube) + P(Coin) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Addition Rule

- Two events are **mutually exclusive** if the occurence of one prevents the occurence of the other one.
- If two events are **mutually exclusive** we can calculate the probability that at least one of them happens by adding up their probabilities.

Exam Strategy Example

- 20 possible questions, 4 drawn at random on the exam.
- Strategy: pick 10 at random.

Exam Strategy Example

- 20 possible questions, 4 drawn at random on the exam.
- Strategy: pick 10 at random.
- #1: What is the chance of getting all right?
- #2: What is the expected grade?

Peer Review Example

- What is peer review in scientific journals?

Peer Review Example

- What is peer review in scientific journals?
- A journal receives 100 submissions. 80 of them are bad, the rest is good.
- The chance of a bad one getting published is 10%.
- The chance of a good one getting published is 90%.
- How many good and bad articles will get published?

Two Views of Probability in Statistics

Two Views of Probability in Statistics

- Classical: probability is a **frequency**.
- Bayesian: probability is a **degree of belief**.

Probability in Classical Statistics

Probability is the frequency to which something converges over a long run of repetitions.

Probability in Classical Statistics.

The **frequency theory** of probability works for processes repeated

- **independently** – i.e. one repetition does not affect the other –
- under **identical** conditions.

Chance

- Chances can range from 0% to 100%

  $$0\% \leq \text{chance of something} \leq 100\%$$

Repeated Drawing

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Repeated drawing can be either

- **without** replacement, or
- **with** replacement.

Probability Notation

- Probability of event X happening can be expressed as

$$Probability(\text{event X})$$

or simply

$$P(\text{X})$$

Conditional Probability

Conditional Probability

- Probability of event X happening given event Y happens.
- Can be expressed as

$$P(X|Y)$$

We read this as "*probability of X given Y*"

Independence

- What does it mean that two events are **independent**?

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- Whether one happens or not has no relation to whether the other does happen or not.

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- We can understand independence as **absence of association**.

Independence & Probability

- Suppose we know that event X is independent of event Y.
- Can you express this in terms of probabilities?

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- Suppose we know that event X is independent of event Y.
- Can you express this in terms of probabilities?
- Probability of X is the same if Y happens and if it does not happen.

Independence & Probability

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$$P(X|\text{Y}) = P(X|\text{not Y})$$

Independence & Drawing

- How is drawing with/without replacement related to independence.

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- How is drawing with/without replacement related to independence.
- When drawing **with** replacement the draws are **independent**.
- When drawing **without** replacement the draws are **dependent**.

Multiplication Rule

- If 2 events are independent their joint probability is equal to the product of their probabilities.
- I.e. if

$$P(\text{A}|\text{B}) = P(\text{A}|\text{not B})$$

then

$$P(\text{A \& B}) = P(\text{A}) \times P(\text{B})$$

Example

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- We draw 1 item at random with replacement. Then we draw another item at random with replacement.
- What is the chance that at least one of them will be a coin?

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- The opposite of never getting a coin.
- $1 - (\frac{2}{4} \times \frac{2}{4}) = 1 - \frac{1}{4} = \frac{3}{4}$

Probability of ‘At least 1 in X repetitions’

- Probability of getting a certain outcome at least once in several repetitions.
- It is the opposite of never getting the outcome.
- 1 minus the probability of never getting the outcome.

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- $P(no = 2) + P(no = 4) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

Addition Rule

- Two events are **mutually exclusive** if the occurrence of one prevents the occurrence of the other one.
- If two events are **mutually exclusive** we can calculate the probability that at least one of them happens by adding up their probabilities.