Physics 373 – Introductory Laboratory – Nucleons

Rutherford Scattering

Purpose: The purpose of this experiment is to acquaint the student with the experimental procedures necessary to measure an experimental cross section that can be directly compared to a theoretical prediction. It is based on the measurement of the Rutherford scattering cross section in α+Au elastic scattering using the Edwards Accelerator Laboratory at Ohio University.

List of Goals:

A). Understand the experimental layout and related equipment.

B). Perform measurements of the α yield as a function of scattering angle with a silicon detector.

C). Compute a differential cross section for Rutherford scattering.

D). Compare a measured differential cross section to a theoretical calculation.

E). Keep a detailed logbook of all relevant parts of this measurement.

F). Prepare a detailed formal lab report on all aspects of this investigation.

I. Pre-Experiment Approach and Details

♢ Be sure that you have spent adequate time researching the general topic so that you can appreciate the basic ideas outlined above under the *"List of Goals"*.

♢ One of our graduate students will be assisting us in setting up the experiment and collecting the data. Your main work will be in analyzing the data once it is in hand as well as assisting in positioning the silicon detector for each run.

♢ Come up with a run plan for how many different angles are required to compare against the Rutherford formula. Talk to your classmates to optimize this plan. What should be the required level of statistics at each point? Note that in this lab we will all take data together and will not split up into teams.

♢ Do not be surprised if the experiment does not go smoothly or takes more than 2 hours to complete. Experimental nuclear physicists seem to spend a good deal of time waiting around for accelerator personnel to either deliver the beam or tune the beam to desired levels.

II. Pre-Experiment Checkout

a). After performing the essential pre-lab preparations, begin to understand the experimental setup. This includes understanding the purpose of each part of the system. The basic layout of the Rutherford scattering experiment is shown in Fig. 1.

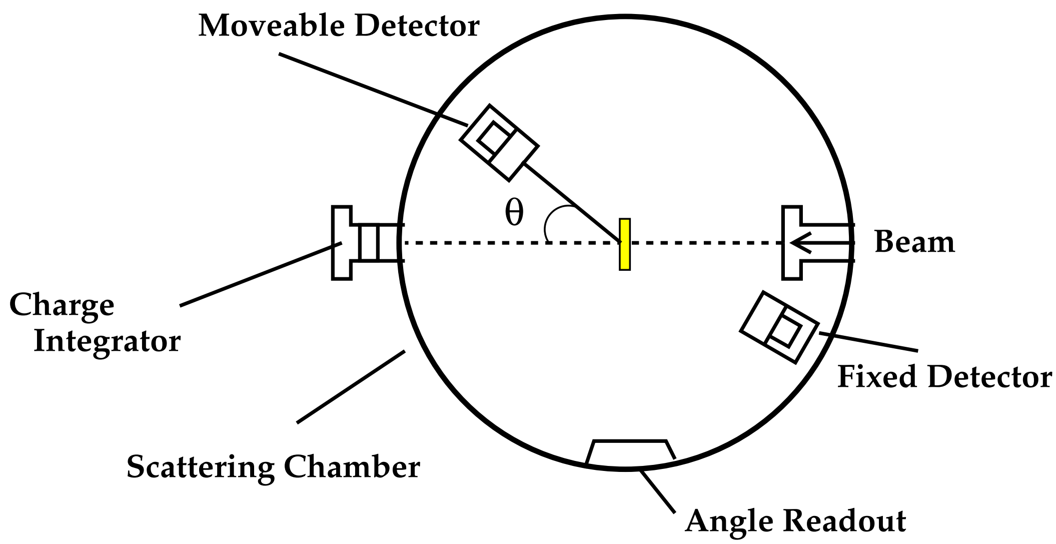


Figure 1: Major components of the Rutherford scattering experiment for this laboratory.

b). Go through the set-up step by step to understand how the electronics are laid out in both the experimental area and the control room. This basic layout and the purpose of each electronics module should be reasonably clear given the experience you have gained in the first part of this course. The manuals for each of the electronics used are available for review. A complete diagram is contained in Fig. 2. Be sure to provide clear and complete documentation in your logbook. Also begin to understand how to operate the data acquisition system and what the defined histograms mean. Make sure you understand where to record the computer dead time for each measurement. This is crucial in order to measure the Rutherford cross section.

c). Make sure you know how to set the angle of the movable silicon detector to the precise angle that you want and that the angle readback makes sense to you. Note that the movable detector is limited to angles in the range from 25° to 150° on one side of the beam and from 250° to 335° on the other.

d). Take measurements of the radial positions of the two silicon detectors. This is required to compute the detector solid angle.

e). Make sure you record the size of the apertures in from of the two silicon detectors. These are required for the solid angle computation.

f). Make sure that you understand how to read out the charge integrator for the alpha beam. This is required in order to determine the number of alphas incident on our gold foil.

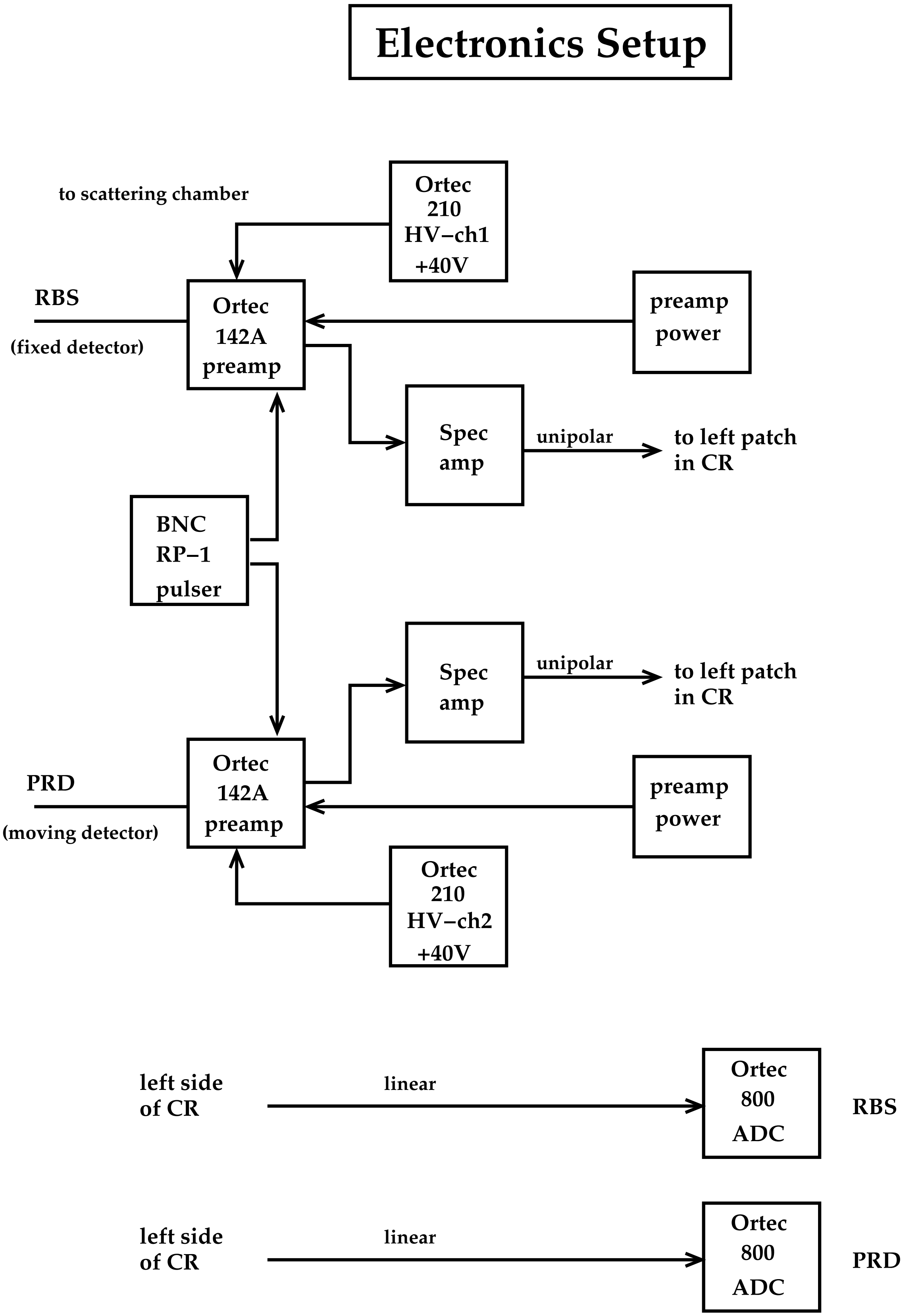


Figure 2: Electronics layout for the Rutherford scattering experiment.

g). Make sure you record the thickness of the gold foil (relevant units are μg/cm^2). This is necessary to compute the cross section.

h). Make sure you know how to set the angle of target (the gold foil). In this experiment we will want to minimize the material traversed by the scattered alpha particle in the gold foil. To accomplish this we will set the angle of the target to be $\theta_{scatt}/2$ for angles $\theta_\alpha \leq 90°$. Here θ_{scatt} is the defined angle of the silicon detector. For angles larger than 90°, we will set the target angle to be at 0° (i.e. normal to the incident beam flux).

III. Scattering Cross Section

The scattering of particles is described quantitatively by the differential cross section, $\frac{d\sigma}{d\Omega}$ [1]. We will consider the case when N_0 projectiles are incident on a target of uniform thickness described by n_1 target nuclei per unit area. The incident projectiles can in principle scatter to any angle with respect to the incident direction. The number of particles scattered into a solid angle $\Delta\Omega$ is given by

$$N_s = \frac{d\sigma}{d\Omega} N_0 \, n_1 \, \Delta\Omega. \tag{1}$$

In general, $\frac{d\sigma}{d\Omega}$ depends on the type of projectile, the type of target, the energy of the projectile, and the scattering angle. Note also that the above formula is technically only exactly true in the limit that $\Delta\Omega \to 0$, due to the fact that $\frac{d\sigma}{d\Omega}$ depends on the scattering angle. The number of scattered particles N_s is directly proportional to N_0, n_1, and $\Delta\Omega$ as one intuitively expects – in this picture, the differential cross section is the proportionality constant. It is often said that the differential cross section "contains the physics"...

The differential cross section has units of area divided by solid angle. While solid angle is technically dimensionless, we traditionally utilize the units of steradians (sr) to describe it, where $\Delta\Omega = 4\pi$ sr for an entire sphere. In nuclear and particle physics, it is also traditional to use the units of fermis (fm) to describe distances (1 fm=10^{-13} cm) and barns (b) to describe cross sections (1 b=10^{-24} cm^2).

IV. Rutherford Scattering Physics

The Rutherford scattering formula may be derived classically (or quantum-mechanically!) assuming the Coulomb potential between two particles (labeled 0 and 1):

$$U(r) = \frac{Z_0 Z_1 e^2}{4\pi\epsilon_0 r}, \tag{2}$$

where r is the distance between the particles, $Z_0 e$ is the charge of particle 0, $Z_1 e$ is the charge of particle 1, and ϵ_0 is usual electrostatics constant ($\epsilon_0 = 8.854187... \times 10^{-12}$ F/m). The resulting differential cross section is

$$\frac{d\sigma}{d\Omega_{\text{c.m.}}} = \frac{Z_0^2 Z_1^2 e^4}{16(4\pi\epsilon_0)^2 E_{\text{c.m.}}^2} \frac{1}{\sin^4(\theta_{\text{c.m.}}/2)},$$

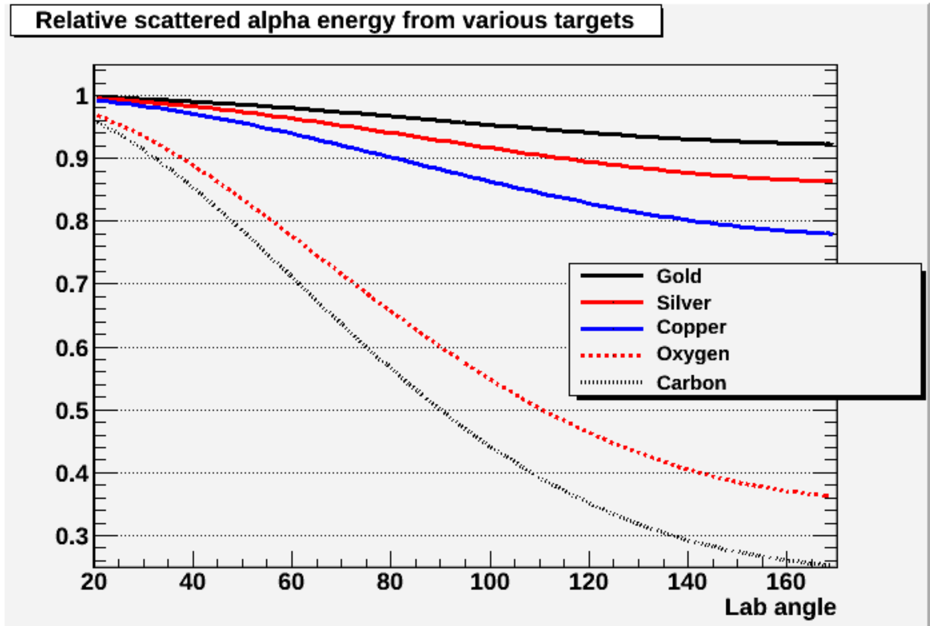


Figure 3: Relative scattered energies (K) for several nuclei present in the target foils as a function of laboratory angle.

where $E_{\text{c.m.}}$ and $\theta_{\text{c.m.}}$ are the center-of-mass kinetic energy and scattering angle, respectively. Note that the Rutherford differential cross section diverges as $\theta_{\text{c.m.}} \to 0$ and the total cross section is in fact infinite.

Since we will not be performing the experiment in the center-of-mass coordinate system, there is more work to do. The algebra of changing coordinate systems for the Rutherford formula is non-trivial and is seldom discussed in textbooks. The formulas below are taken from a paper by Sargood [2]. Let's now assume that label 0 refers to the projectile and label 1 refers to the target which is at rest. First we will define

$$K = \left(\frac{M_0 \cos\theta + \sqrt{M_1^2 - M_0^2 \sin^2\theta}}{M_0 + M_1} \right)^2, \tag{3}$$

where M_i are the masses and θ is the *laboratory* scattering angle, where $0°$ is by definition the direction of the incident projectile. We will also define the parameter

$$a = \sqrt{1 - \left(\frac{M_0 \sin\theta}{M_1} \right)^2}. \tag{4}$$

The differential cross section in the laboratory is given by:

$$\frac{d\sigma}{d\Omega} = \frac{Z_0^2 Z_1^2 e^4}{4a(4\pi\epsilon_0)^2 E_0^2} \frac{(a + \cos\theta)^2}{\sin^4\theta}, \tag{5}$$

where E_0 is the kinetic energy of the incident projectile. The energy of the scattered projectile is given by $E_0' = KE_0$. Since K is dependent on both the target mass and the scattering angle, particles scattered from different target nuclei will have different energies; Figure 3 shows the K values for several nuclei as a function of laboratory angle. Note that in the limit $M_0/M_1 \to 0$ ("light" projectile and "heavy" target) we have $K \to 1$ and $a \to 1$ and the lab system is also the center-of-mass system – one can also verify that the differential cross section formulas are equivalent in this limit using the trig identity

$$\frac{1+\cos\theta}{\sin^2\theta} = \frac{1}{2\sin^2(\theta/2)}. \tag{6}$$

It is convenient to expand Eqs. (4) and (5) in powers of M_0/M_1; the result is

$$\frac{d\sigma}{d\Omega} = \frac{Z_0^2 Z_1^2 e^4}{16(4\pi\epsilon_0)^2 E_0^2}\left[\frac{1}{\sin^4(\theta/2)} - 2\left(\frac{M_0}{M_1}\right)^2\right] + O\left[\left(\frac{M_0}{M_1}\right)^4\right]. \tag{7}$$

The fundamental constants can be inserted as follows

$$\begin{aligned}
\frac{e^4}{(4\pi\epsilon_0)^2} &= \left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)^2 (\hbar c)^2 & (8)\\
&= \left(\frac{1}{137.036}\right)^2 \times (197.327 \text{ MeV-fm})^2 \times \left(\frac{10 \text{ mb}}{\text{fm}^2}\right) & (9)\\
&= 20.735 \text{ mb-MeV}^2 & (10)
\end{aligned}$$

to yield "nuclear physics" units. The final formula is

$$\frac{d\sigma}{d\Omega} = 1.296\left(\frac{\text{mb-MeV}^2}{\text{sr}}\right)\left(\frac{Z_0 Z_1}{E_0}\right)^2\left[\frac{1}{\sin^4(\theta/2)} - 2\left(\frac{M_0}{M_1}\right)^2\right] \tag{11}$$

which can be used with E_0 in MeV to calculate the differential cross section in mb/sr.

V. Rutherford Scattering Experiment

The experiment will amount to acquiring data at a number of different scattering angles on either side of the incident beam direction. You will record the number of counts in each of our two silicon detectors. The naming convention is to call the fixed detector RBS and the movable detector PRD. In addition to summing up the counts in the ADC spectra, you will have to record the system live time (LT) for each measurement, as well as the real time duration (RT) of each run and the charge integrator value. The counts in the ADC spectra must be multiplied by the ratio RT/LT in order to determine the true number of particles which struck the detector during the real time duration of each run.

In defining E_0 for this experiment, use of the incident α beam energy is only an approximation. In general a better approximation is to use the average alpha energy in the target given by:

$$E_{\text{avg}} = \frac{E_i + E_f}{2}, \tag{12}$$

where E_i is the incident alpha energy and E_f is the alpha energy after passing through the gold foil. It would be relevant to compare the sensitivity of the final result with and without this correction.

Based on the previous discussion, the measured differential scattering cross section in units of cm^2/sr is given by:

$$\frac{d\sigma}{d\Omega} = \frac{N_s}{N_0 \cdot \Delta\Omega \cdot n_1} \tag{13}$$

where:

- N_s = the number of detected particles, corrected by the ratio RT/LT.
- N_0 = the number of beam particles which impinged on the scattering foil. This quantity is determined from the charge integrator. Remember that we are using a He^{2+} beam when converting the measured number of μC to the number of particles.
- $\Delta\Omega$ = the solid angle of the detector (sr). The solid angle is given by A_c/R^2, where A_c is the area of the detector (or the defining aperture) and R is the distance of separation between the aperture and the target.
- n_1 = the number of target nuclei per cm^2. This number will be given to you by your lab instructor.

Silicon Detectors – Background Information

Semiconductor charged particle detectors have been used extensively in experimental nuclear physics research for over 30 years, and have revolutionized nuclear particle detection. Silicon detectors can be used to measure a wide range of charged particles. This range includes protons and electrons as low as 20 keV up to fission fragments of energy over 100 MeV. The inherent resolution of ion-implanted and surface barrier detectors is surpassed only by magnetic spectrometers. The detector output pulses rise rapidly, hence they are well suited for fast (∼1 ns) timing with coincidence circuitry or time-to-amplitude converters.

The efficiency of silicon charged particle detectors for their active volume is essentially 100%, and their energy vs. pulse height curves are linear over a rather impressive range. They also have good long-term pulse height stability. This is particularly noticed when they are contrasted with scintillation counters, gas proportional counters, or ionization chambers. The high efficiency of these detectors allows for straightforward analysis of cross sections via the extraction of detected particle yields from the accumulated spectra (provided data acquisition/computer dead times are accounted for accurately).

Solid state detectors can be thought of as a solid state ionization detector. When a charged particle enters the depletion region of the detector, it loses energy primarily by making electron-hole pairs in the silicon. For each electron-hole pair that is made, the initial charged particle must lose 3.6 eV. In this experiment we will be measuring alpha particles that have an energy less than 3 MeV. If a 3 MeV alpha particle enters the detector, roughly 8×10^5 electron-hole pairs will be produced. The detector is reverse-biased and these electron-hole pairs are collected to produce the output pulse of the detector. Since a large number of charge carriers is produced, the statistical variation in the number collected is small and hence very good energy resolution is possible. The "partially depleted" silicon detector used in this experiment has a resolution of roughly 20 keV.

The three main parameters that define a silicon surface-barrier detector are its resolution, active area, and depletion depth. The shape of a typical detector is a circular disk. Thus the active area is simply the area of the face of this disk (provided it is placed normal to the incident flux of radiation). The depletion depth of a detector is synonymous with the sensitive depth of the detector. For any given experiment, this depth must be sufficient to completely stop all the incident charged particles that are to be measured. The detector's ability to do this is dependent upon both the energy and the particle type. Fig. 4 is a range-energy curve for five of the more common charged particles. From it, the maximum depth can be determined for the maximum energy of a particle type. From Fig. 4, note that a 3 MeV alpha particle is completely stopped with about 20 μm of silicon.

Acknowledgments

This document was written by Carl Brune and Daniel Carman (Ohio University). Many of the details in this write-up were extracted from the Rutherford scattering writeup contained in Ortec Application Note AN-34 *"Experiments in Nuclear Science"* (1976), as well as the Rutherford scattering writeup contained within *"Laboratory Investigations in Nuclear*

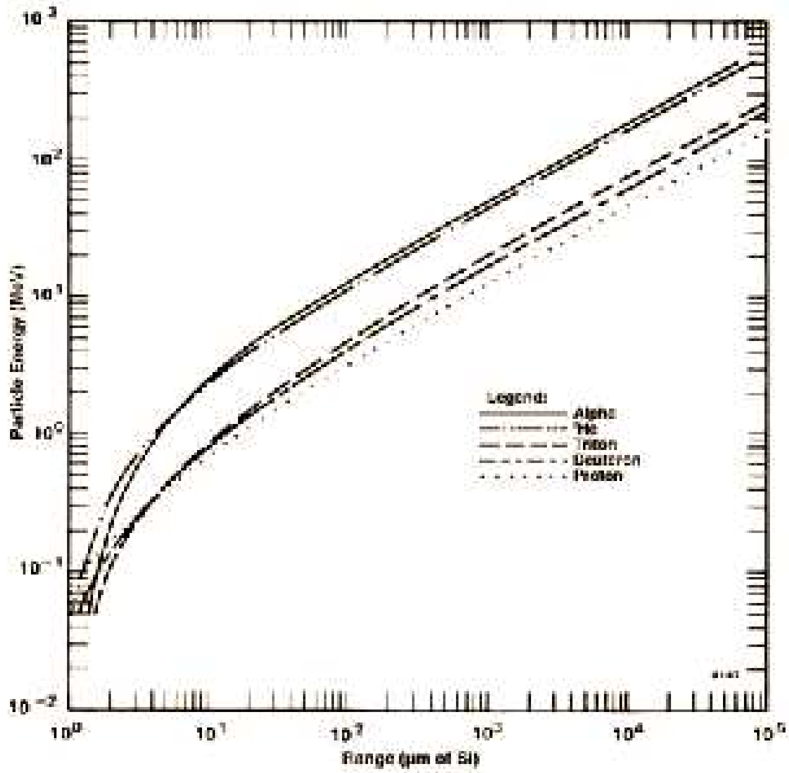


Figure 4: Energy-range curves for charged particles in silicon [3].

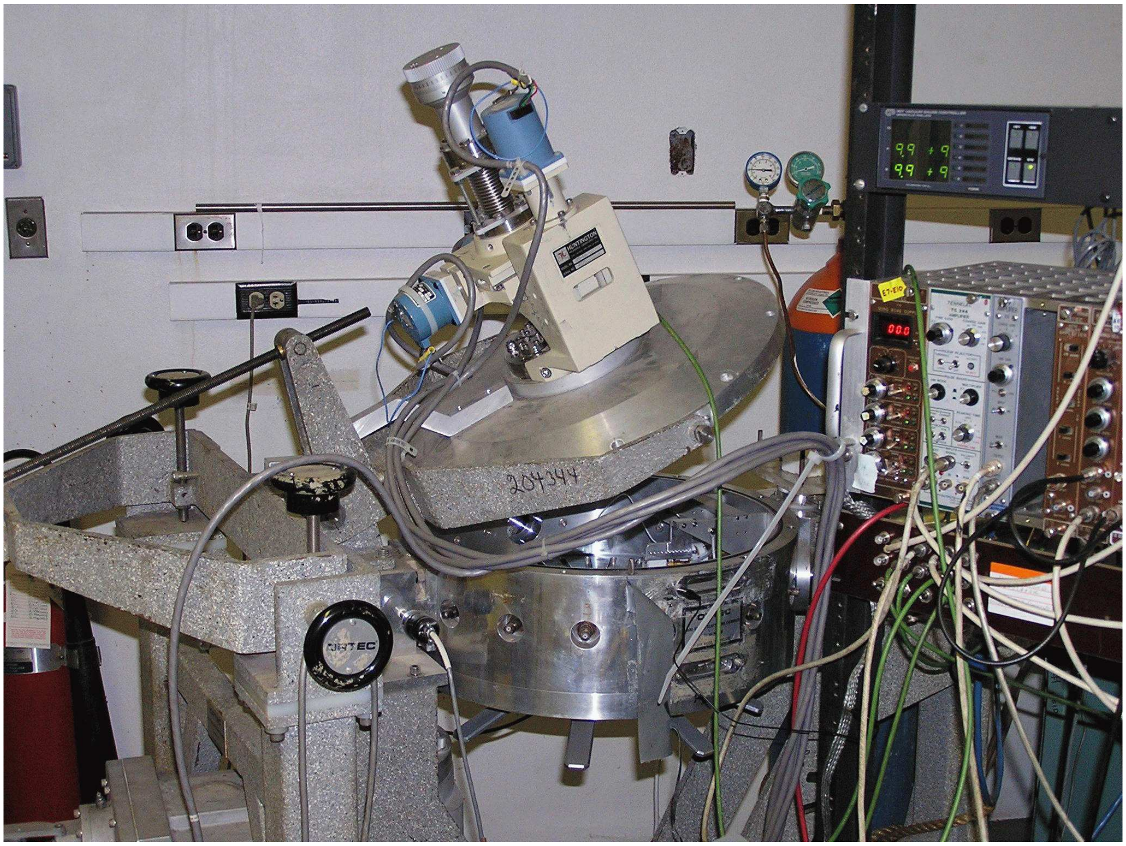
Science" by Jerome Duggan (1988).

References

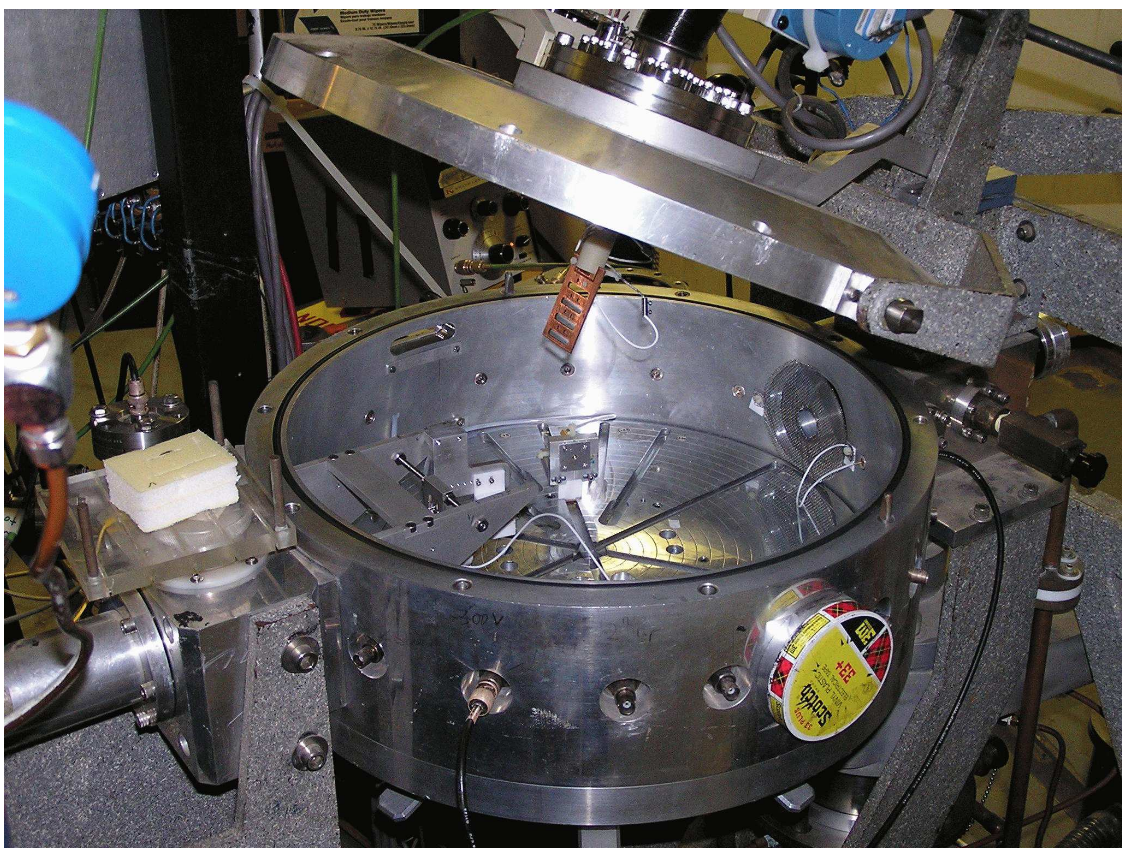
[1] W.S.C. Williams, *"Nuclear and Particle Physics"* (Clarendon Press, Oxford, 1991).

[2] D.G. Sargood, Physics Reports **93**, No. 2, 61-116 (1982).

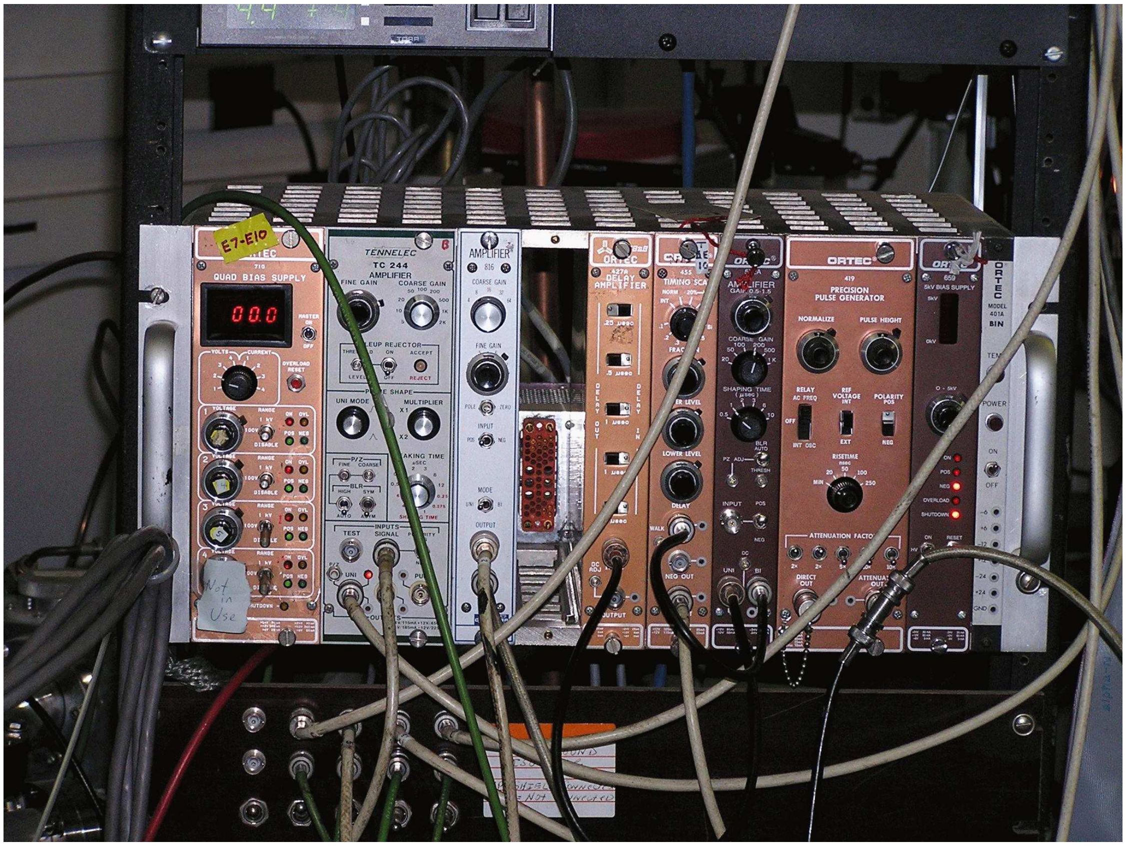
[3] C.F. Williamson *et al.*, *"Range of Stopping Power of Chemical Elements for Charged Particles of 0.5 to 500 MeV"*, CEA-R-3042, July 1966.



View of Scattering Chamber (External)



View of Scattering Chamber (Internal)



Experimental Area Electronics

Counting Room (CR) Electronics