Jamison Lahman
PHYS 3701
Tutorial
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1) Determine the mean, RMS, number of counts, and error on the mean for the five prominent peaks in the Na-22, Co-60, and Cs-137 spectra. Quote the gamma ray energy and uncertainty for each of the peaks; include a citation of your reference source.

To find the mean of a peak which spans a total of n bins from j to k and which contains a total of N counts, we used the following equation,

$$< x >= \frac{1}{N} \sum_{i=j}^{k} (x_i \cdot count_i) \tag{1}$$

The RMS, or standard deviation σ_x is then

$$\sigma_x = \sqrt{< x^2 > - < x >^2} \tag{2}$$

where $< x^2 >$ is defined as

$$< x^2 >= \frac{1}{N} \sum_{i=j}^{k} (x_i^2 \cdot count_i) \tag{3}$$

The standard error of the mean is defined as

$$\sigma(\bar{x}) = \frac{\sigma_x}{\sqrt{N}} \tag{4}$$

Material	Mean	$\sigma(\bar{x})$	# of Counts	RMS	Gamma Energy (keV)	Uncertainty
Na	1511.977	0.056	2132	5.171	511.000	0.000
Na	3840.27	0.10	8125	4.736	1274.537	0.007
Cs	1971.994	0.060	3579	3.604	661.657	0.003
Co	3531.45	0.11	1414	4.051	1173.228	0.003
Co	4016.90	0.12	1132	4.023	1332.492	0.004

Table 1: Prominent peaks in Na-22, Co-60, and Cs-137 spectra. Gamma ray energies and uncertainties were taken from NuDat2.6.[1]

2) Determine the energy calibration using the five peaks in the Na-22, Co-60, and Cs-137 spectra. Determine the error on the calibration.

The uncertainty used for fitting, σ_i was found using the error on the mean and uncertainty on the gamma ray energy from literature, σ_γ

$$\sigma_i = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \sigma_\gamma^2} \tag{5}$$

Since the partial derivative of the least-squares fit function, $y(x) = ax + b$, with respect to x is a, making the formula implicit. To obtain a rough estimate of a, we applied a least-squares fit according to chapter 6 of Bevington.[2]

$$a = \frac{1}{\Delta} \left(\sum \frac{1}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} \right) \tag{6}$$

$$b = \frac{1}{\Delta} \left(\sum \frac{x_i^2}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} \right) \tag{7}$$

$$\Delta = \sum \frac{1}{\sigma_i^2} \sum \frac{x_i^2}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2} \right)^2 \tag{8}$$

The mean bin was fitted as the independent variable and the gamma ray energy as the dependent variable. With the initial guess using $\sigma_i^2 = (\sigma_x^2 + \sigma_\gamma^2)$, we got a value of 0.328 for a. Now, (5) can be solved for σ_i

$$\sigma_i = \sqrt{a^2\sigma_x^2 + \sigma_\gamma^2} \tag{9}$$

Material	Mean	σ_x	Gamma Energy (keV)[1]	σ_γ	σ_i
Na	1511.977	0.056	511.000	0.000	0.018
Na	3840.27	0.10	1274.537	0.007	0.034
Cs	1971.994	0.060	661.657	0.003	0.020
Co	3531.45	0.11	1173.228	0.003	0.035
Co	4016.90	0.12	1332.492	0.004	0.039

Table 2: Input data for the least-squares fitting.

Additionally, the errors of the fit are given by Nudat.[2]

$$\sigma_a^2 = \frac{1}{\Delta}\sum\frac{1}{\sigma_i^2} \tag{10}$$

$$\sigma_b^2 = \frac{1}{\Delta}\sum\frac{x_i^2}{\sigma_i^2} \tag{11}$$

$$\mathrm{cov}(a,b) = \frac{-1}{\Delta}\sum\frac{x_i}{\sigma_i^2} \tag{12}$$

Again using (7), (8), and (9), we were able to fit the data. The values from Table 2 produce the following fit parameters for the equation $y(x) = ax + b$

$$a = 0.327973 \pm 7.7 \times 10^{-5} \tag{13}$$

$$b = 15.02 \pm 0.20 \tag{14}$$

$$\mathrm{cov}(a,b) = -1.292 \times 10^{-5} \tag{15}$$
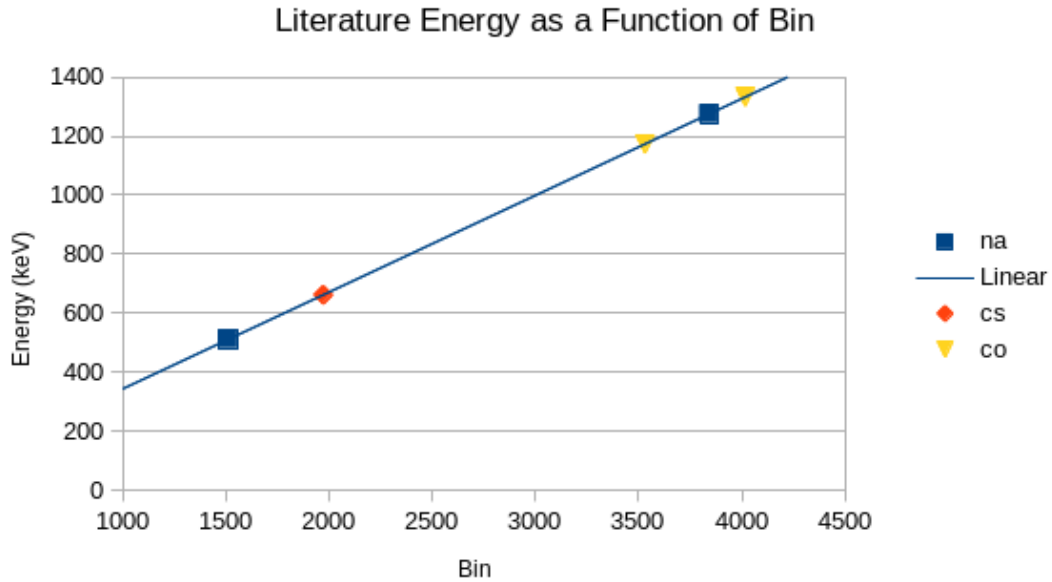
Which produces the following graph

Figure 1: The linear regression applied to the five points.

3) Find the energy residuals (residual = fit energy - literature energy) for your five calibration peaks. Comment on the size of the residuals compared to the calibration errors.

The standard error of the fit is defined as

$$\sigma_{fit} = \sqrt{x^2\sigma_a^2 + \sigma_b^2 + 2x \cdot \text{cov}(a,b)} \tag{16}$$

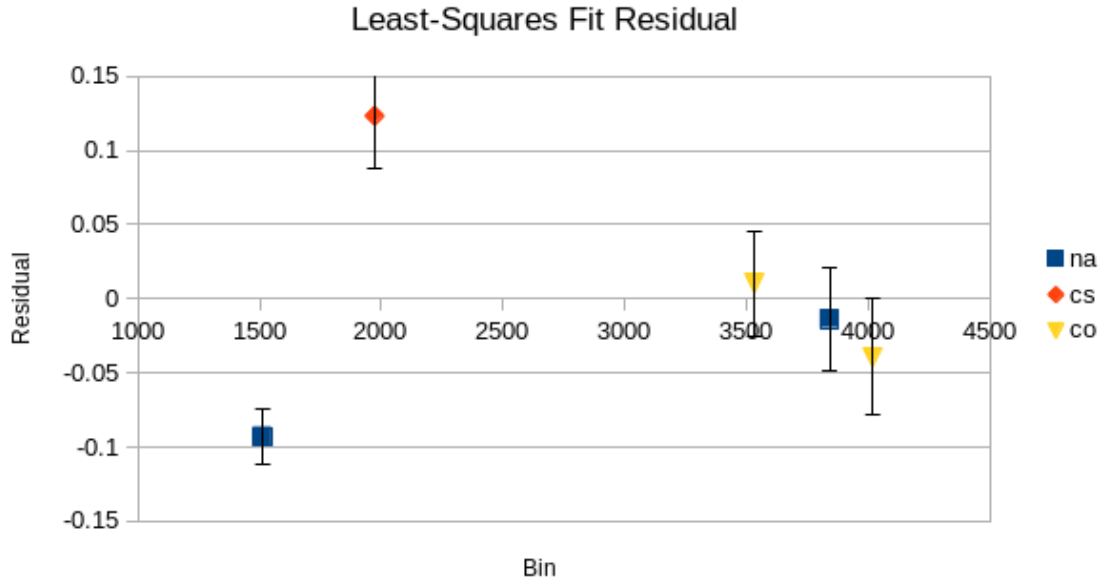| Material	Bin	σ_i	Fit Energy (keV)	σ_{fit}	Literature Energy (keV)[1]	Residual
Na	1511.977	0.018	510.91	0.11	511.000	-0.093
Na	3840.265	0.034	1274.523	0.087	1274.537	-0.014
Cs	1971.994	0.020	661.780	0.096	661.657	0.123
Co	3531.445	0.035	1173.24	0.11	1173.228	0.010
Co	4016.900	0.039	1332.45	0.12	1332.492	-0.039

Table 3: Analysis of the fit.

Figure 2: The residual of the Cesium-137 and the 511 keV peak in the Na-22 are both more than 5 standard errors away from the fit which is highly unlikely.

4) Compare the number of events in the two peaks in each of the Co-60 and Na-22 calibration spectra; comment on the agreement or disagreement of the ratio of the peaks to what you would expect based on literature information; include a citation of your literature source.

Material	Gamma Energy (keV)	Peak Count	σ_y	Intensity(%)	Intensity Uncertainty
Na	511.000	652	228.99	180.76	0.04
Na	1274.537	200	64.69	99.94	0.14
Co	1173.228	137	47.51	99.85	0.03
Co	1332.492	116	36.58	99.9826	0.0006

Table 4: Expected intensities of Co-60 and Na-22.[1] σ_y is found using (2) in the y-dimension.

3

The ratio of intensities and the error on the ratios are given respectively by

$$R \quad = \quad \frac{y_1}{y_2} = \frac{\text{intensity}_1}{\text{intensity}_2} \tag{17}$$

$$\Delta R \quad = \quad R \cdot \sqrt{\left(\frac{\sigma_{y_1}}{y_1}\right)^2 + \left(\frac{\sigma_{y_2}}{y_2}\right)^2} \tag{18}$$

Material	R	ΔR	Literature Ratio[1]
Na	3.26	1.56	1.81
Co	1.18	0.55	1.00

Table 5: Both measured intensities are within one standard deviation from the accepted values.

5) Determine the energy resolution of the detector. Does the energy resolution depend on gamma ray energy? If so, describe the energy dependence of the energy resolution.

The energy resolution can be found by multiplying the RMS by 2.35 to find the FWHM then multiply by the slope (13) of the calibration to find it in terms of energy units.[3]

Material	Gamma Energy (keV)	RMS	Energy Resolution (keV)
Na	511.000	5.167	3.98
Cs	661.657	3.604	2.78
Co	1173.228	4.023	3.12
Na	1274.537	4.736	3.65
Co	1332.492	4.051	3.10

Table 6: The energy resolutions do not appear to have an energy dependency.

6) Choose four prominent peaks in the europium spectrum, and determine the mean, RMS, number of counts, and error on the mean for each. Note that many of the peaks in the europium spectra will require background subtraction.

To deal with the background counts, a modified sideband subtraction methods was employed. Given a peak which spans a bin range of $[x_1, x_2]$, the average count over the intervals $[x_1 - \frac{x_2 - x_1}{2} - 1, x_1)$ and $(x_2, x_2 + \frac{x_2 - x_1}{2} + 1]$ are used as the y values to estimate a linear equation of the background over the peak interval, i.e.

$$y_{bkgd} = \left(\frac{(\bar{y}_-) - \bar{y}_+}{(\bar{x}_-) - \bar{x}_+}\right)(x_{bin} - (\bar{x}_-)) + (\bar{y}_-) \tag{19}$$

Peak Interval	$\bar{y}_-$	$\bar{y}_+$	Counts Subtracted	σ_{peak}	σ_{bkgd}
[312,335]	243.20	193.60	5271.36	1298.74	30.44
[686,711]	95.09	76.45	2241.10	199.28	13.34
[990,1017]	54.67	54.5	1528.04	462.03	7.34
[2320,2347]	25.08	20.67	643.08	73.72	4.74

Table 7: The average number of counts in the bands before and after the peak interval. The total number of counts subtracted are the predicted background counts over the peak interval given by (19). The standard deviations where found using (2) but in the y-dimension.
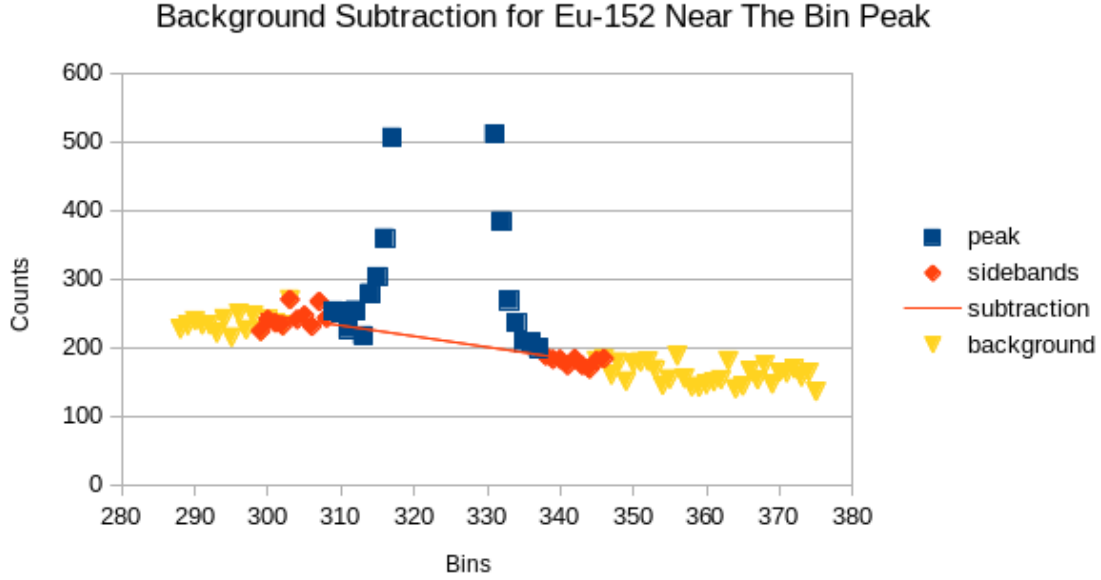
Figure 3: The Europium-152 background subtraction near the 330 bin peak. The values are from Table 7. The background subtraction function was calculated using (19).

Again, using (1-4), we found the mean, RMS, and error on the mean for the Europium peaks near bins 324, 700, 1000, 2333.

Mean Bin	$\sigma(\bar{x})$	# of Counts	RMS
323.991	0.019	26958.2	3.054
699.153	0.048	4106.68	3.068
1003.030	0.031	10046.92	3.127
2332.911	0.088	1883.79	3.811

Table 8: Prominent peaks in Na-22, Co-60, and Cs-137 spectra. Gamma ray energies and uncertainties were taken from NuDat2.6.[1]

7) Using your calibration, determine the energies and errors for your four europium peaks. Match your peaks to known gamma rays produced in the decay of Eu-152. Find the energy residuals residual = fit energy - literature energy) and errors.

Mean Bin	Calibration Energy	σ_y	σ_{fit}	Known Peak	Uncertainty	Residual
323.991	121.2795	0.0061	0.18	121.7817	0.0003	-0.1565
699.153	244.322	0.016	0.15	244.6974	0.0008	-0.029
1003.030	343.986	0.010	0.14	344.2785	0.0012	-0.292
2332.911	780.077	0.029	0.076	778.9045	0.0024	1.247

Table 9: Prominent peaks in the Europium-152 sample. The fitting was done with values from (13) and (14). σ_y and σ_{fit} are found using (9) and (16) respectively. The literature energies were taken from NuDat2.[1]

8) Using your four europium peaks, determine the detector efficiency as a function of energy. Include error analysis.

Detector efficiency is measured by comparing the count rates in each peak to the count rates expected from the known intensities of each gamma ray.[3]

$$\epsilon = \frac{N}{Intensity} \tag{20}$$

5

While the error is given by the equation

$$\Delta\epsilon = \sigma_\% \frac{N}{Intensity^2} \tag{21}$$

Mean Bin	Counts	Intensity[1]	Efficiency Uncertainty	Efficiency	Intensity Uncertainty
323.991	26958.2	0.2859	0.0016	94490.71	527.69
699.153	4106.68	0.0755	0.0004	54393.11 7	288.18
1003.030	10046.92	0.2659	0.0020	37784.58	284.20
2332.911	1883.79	0.1293	0.0008	14569.14	90.14

Table 10

Again using (6-8) and (10-12) we are able to fit the data with a linear regression.

$$a = -78.52 \pm 0.40 \tag{22}$$
$$b = 75339.4 \pm 290.1 \tag{23}$$
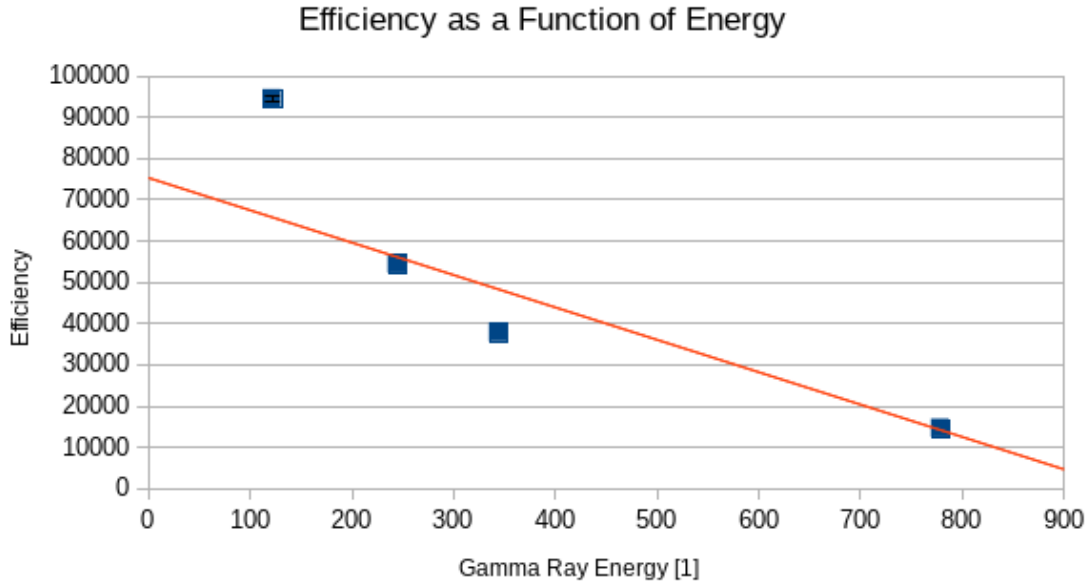$$\text{cov}(a,b) = -114.26 \tag{24}$$

Figure 4: The efficiencies as a function of the literature energy taken from NuDat2. A more proper fit would likely be a an exponentially decaying function. There is obviously a correlation between efficiency and energy of the ray.

References

[1] A. Sonzogni, *Nudat 2*, [Online; accessed 29-January-2017], http://www.nndc.bnl.gov/nudat2/.

[2] P. R. Bevington and D. K. Robinson, *Data reduction and error analysis for the physical sciences* (McGraw-Hill, 2003).

[3] Wikipedia, *Gamma spectroscopy — wikipedia, the free encyclopedia*, [Online; accessed 7-February-2017], (2016) https://en.wikipedia.org/w/index.php?title=Gamma_spectroscopy&oldid=757606924.