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0.1 Exclusive Buyer Mechanism

We can now develop a formal description of the exclusive buyer mechanism by considering an analog of the single dimensional case. In the single dimensional case, a good is allocated according to a bidder's *virtual value*, which is a function of the of the bidder's type representing the surplus that can be extracted from the bidder (see Myerson 1981). In the multidimensional case of a single good with multiple quality levels we can define a multidimensional analog of a single dimensional virtual value. For each bidder i and each quality grade j of the good we define a virtual value $\beta^i_j(x)$, which is also a function of each other bidder's types. For each bidder, we define $\beta^i = \max_j \beta^i_j$ as the maximum of the quality grade-specific virtual values. (Note, although the virtual values may depend on the reserve price, we omit the notational dependence on r for clarity.) The key idea behind an exclusive buyer mechanism is that the good is allocated to the bidder i with the largest β^i.

This formulation of the exclusive buyer mechanism has its origins in the work of Brusco, Lopomo, and Marx (2011), who first explored a similar mechanism in the specific case of two quality levels. Their mechanism can be understood as an auction where the buyers compete in a second price or ascending-bid auction (with reserve prices) for the right to be the only buyer and choose which quality grade to purchase. If a bidder wins the auction they can select between the lower quality grade of the item or the higher quality grade (and pay an additional price). Their mechanism was further elaborated in a follow up work (Belloni, Lopomo, and Wang 2010) from which a number of a number of conjectures in this chapter are drawn.

Formally, in our more general context, we can define the set of bidders who are allocated the good as follows. Allowing for ties, let M denote the set of bidders with the largest β^i:

$$M(x) = \{i \mid \beta^i > \beta^{i'} \ \forall i' \neq i \text{ and } \beta^i \geq 0\} \tag{1}$$

Then the allocation q is defined as:

$$q_j^i(x) = \begin{cases} \frac{1}{|M(x)|} & i \in M(x) \text{ and } \beta_j^i = \max_{j'} \beta_{j'}^i \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Notice the similarity with the canonical single dimensional formulation of Myerson (1981). Again, it is worth emphasizing that at this juncture we have neither constrained the shape of function β_j^i nor have we restricted its domain (it may also depend on other bidders' types). However, in this chapter, we consider the specific case of *linear* virtual values defined by

$$\beta_j^i = x_j^i - r_j \tag{3}$$

where r_j is the reserve price associated with quality level j. Surprisingly, as will become clear in Section **??**, this simple functional form is a promising point of departure to explore the optimality of the exclusive buyer mechanism.

We can now express the revenue as a function of the allocation. Supposing there are no ties, note that the *iterim* or *expected* allocation Q_j^i can be written:

$$\begin{aligned} Q_j^i(x^i) &= \int_{X^{-i}} q_j^i(x^i, x^{-i}) dF^{-i}(x^{-i}) && (4) \\ &= \underbrace{\mathbb{1}\{\beta_j^i \geq \beta_{j'}^i \ \forall j' \neq j \text{ and } \beta_j^i \geq 0\}}_{j \text{ is } i\text{'s preferred quality grade}} \cdot \underbrace{\int_{X^{-i}} \mathbb{1}\{\beta^i > \beta^{i'} \ \forall i \neq i'\} dF^{-i}(x^{-i})}_{\text{probability } i \text{ wins}} && (5) \\ &= \mathbb{1}\{\beta_j^i \geq \beta_{j'}^i \ \forall j' \neq j \text{ and } \beta_j^i \geq 0\} \cdot F(\max\{\overline{x}_1, r_1 + \beta_j^i\}, \ldots, x_j^i, \ldots, \max\{\overline{x}_J, r_J + \beta_j^i\})^{N-1} && (6) \end{aligned}$$

Where we make use of the fact that, when bidders' valuations are independent and identically distributed, $F^{N-1}(x) = F(x)^{N-1}$. Since[1]

$$p^i(x) = \max_j \int_0^{x^i} q_j^i(t, x^{-i}) dt \tag{7}$$

We can write the iterim price $P^i(x^i)$ as

$$P^i(x^i) = P^i(0) + \max_j \int_0^{x^i} Q_j^i(t) dt \tag{8}$$

[1] Doesn't this depend on the 1D result that $p^i(0) = 0$ (ie, that IR binds at $\underline{x}$)?

Furthermore, we can rewrite the objective **??** as follows:

$$\textbf{??} = \max_{p,q} \int_X \left(\sum_i p^i(x) - \sum_i \sum_j r_j q_j^i(x) \right) dF(x) \tag{9}$$

$$= \max_{p,q} \int_{X^i} \int_{X^{-i}} \sum_i \big(p^i(x^i, x^{-i}) - r \cdot q^i(x^i, x^{-i}) \big) dF^{-i}(x^{-i}) dF^i(x^i) \tag{10}$$

$$= \max_{p,q} \int_{X^i} \sum_i \big(P^i(0) + \max_j Q_j^i(x^i) - r \cdot Q^i(x^i) \big) dF^i(x^i) \tag{11}$$

Note that, unlike in the single dimensional case, although $P^i(0)$ is a constant it is endogenously determined by the mechanism and causes complications for the optimization problem[2]. Ultimately, the expression above captures the seller's optimization problem when bidder valuations are independent and identically distributed and their virtual values β_j^i are linear.

1 References

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[2]For a related exposition of these complications see (Jullien 2000).