Question 1

How many "yellow" wavelengths ($\lambda = 580$ nm) will fit into a distance equal to the thickness of a piece of paper (0.003 in)? How far would the same number of microwaves ($\nu = 1010$ Hz) extend?

The number of wavelengths with just be $N = \frac{d_{\text{paper}}}{\lambda}$, but need to convert $d_{paper} = 0.003\,\text{in}$ into nm:

$$d_{\text{paper}} = 0.003\,\text{in}\frac{2.54\,\text{cm}}{1\,\text{in}}\frac{10^7\,\text{nm}}{1\,\text{cm}} = 76\,200\,\text{nm} \rightarrow N_{\text{yellow}} = \frac{76\,200\,\text{nm}}{580\,\text{nm}} = 131.379$$

For the microwaves,

$$c = \lambda\nu \rightarrow \lambda = \frac{3\times10^8\,\text{m}\,\text{s}^{-1}}{10^{10}\,\text{Hz}} = 0.03\,\text{m} \rightarrow d = N\lambda = 131.379(0.03\,\text{m}) = 3.94\,\text{m}$$

Question 2

A vibrating hammer strikes the end of a long metal rod in such a way that a periodic compression wave with a wavelength of 4.3 m travels down the rod's length at a speed of 3.5 km/s. What was the frequency of vibration?

$$\nu = \frac{v}{\lambda} = \frac{3.5\times10^3\,\text{m}\,\text{s}^{-1}}{4.3\,\text{m}} = 813.953\,\text{Hz}$$

Question 3

The profile of a transverse harmonic wave traveling at 1.2 m/s on a string is given by

$$y = 0.02\sin(157x)$$

where x and y are given in meters. Determine the amplitude, wavelength, frequency, and period of the wave.

The amplitude A is the leading coefficient:

$$A = 0.02$$

The wavenumber k is

$$k = 157\,\text{m}^{-1} \rightarrow \lambda = \frac{2\pi}{k} = 0.04\,\text{m}$$

$$\nu = \frac{v}{\lambda} = \frac{1.2}{0.04} = 29.985\,\mathrm{Hz}$$

$$T = \frac{1}{\nu} = 0.033\,35\,\mathrm{s}$$

Question 4

Write the expression for the waveform of a harmonic wave of amplitude 10^4 V/m, period 2.2×10^{-15} s, and speed $3 \times 10^8\,\mathrm{m\,s^{-1}}$. The wave is propagating in the negative z direction and has a value of 10^3 V/m at $t = 0$ s and $x = 0$ m.

Our waveform will take the following functional form:

$$\Psi(\vec{\mathbf{r}}, t) = A\cos\Big(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t - \epsilon\Big)$$

So,

$$A = 10^4 V/m$$

$$\nu = \frac{1}{T} \to \omega = 2\pi\nu = \frac{2\pi}{T} = 2.86 \times 10^{15}\,\mathrm{rad\,s^{-1}}$$

$$v = \frac{\omega}{k} \to k = \frac{\omega}{v} = 9.52 \times 10^6\,\mathrm{m^{-1}}$$

but, $\vec{\mathbf{k}}$ is in direction of propagation, so $\vec{\mathbf{k}} = -9.52 \times 10^6\,\mathrm{m^{-1}}\hat{\mathbf{z}}$. So, we have the form

$$\Psi(\vec{\mathbf{r}}, t) = (10^4 V/m)\cos[(-9.52 \times 10^6\,\mathrm{m^{-1}}\hat{\mathbf{z}}) \cdot \vec{\mathbf{r}} - (2.86 \times 10^{15}\,\mathrm{rad\,s^{-1}})t - \epsilon]$$

$$\Psi(\vec{\mathbf{r}}, t) = (10^4 V/m)\cos[(-9.52 \times 10^6\,\mathrm{m^{-1}})z - (2.86 \times 10^{15}\,\mathrm{rad\,s^{-1}})t - \epsilon]$$