Question 1

Two thin lenses having focal lengths of $+15.0cm$ and -15.0 cm respectively are positioned 60.0 cm apart. A page of print is held 25.0 cm in front of the positive lens. Describe, in detail, the image of the print (i.e. insofar as it's paraxial).

$s_1 = 25.0$ cm, $f_1 = 15.0$ cm, $f_2 = -15.0$ cm, and $d = 60.0$ cm.

$$\frac{1}{f_1} = \frac{1}{s_1} + \frac{1}{p_1} \to \frac{1}{p_1} = \frac{1}{15.0} - \frac{1}{25.0}$$
$$p_1 = 37.5 \text{ cm}$$
$$\frac{1}{f_2} = \frac{1}{d - p_1} + \frac{1}{p_2} \to \frac{1}{p_2} = -\frac{1}{15.0} - \frac{1}{60 - 37.5}$$
$$p_2 = -9.0 \text{ cm}$$

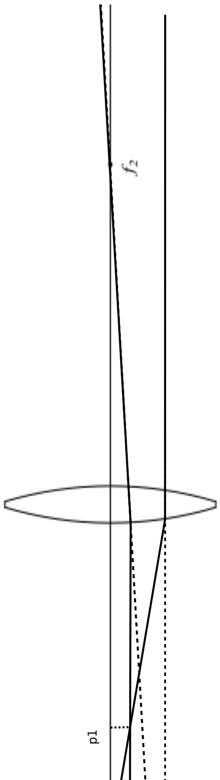
What about magnification?

$$m = m_1 m_2 = \left(-\frac{p_1}{s_1}\right)\left(-\frac{p_2}{s_2}\right)$$
$$m = \left(-\frac{37.5}{25.0}\right)\left(\frac{9}{60 - 37.5}\right) = -0.5984$$

So, the image will be virtual, 9.0 cm in front of the negative lens (between the lenses), 0.5984 times smaller, and inverted.

Question 2

Draw a ray diagram for the combination of two positive lenses wherein their separation equals the sum of their respective focal lengths. Do this carefully, with a ruler. You may do your sketch on the arrangement shown on the last page of this assignment



Question 5

Show that the triple scalar product $(\vec{\mathbf{A}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{C}}$ can be written as

$$(\vec{\mathbf{A}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{C}} = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

Write the vector's in term's of their components:

$$\vec{\mathbf{A}} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

and similarly for $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$.

So,

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \times \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = \hat{\mathbf{i}}(A_2B_3 - A_3B_2) + \hat{\mathbf{j}}(A_3B_1 - A_1B_3) + \hat{\mathbf{k}}(A_1B_2 - A_2B_1)$$

by cofactor expansion of the determinant. Then,

$$(\vec{\mathbf{A}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{C}} = C_1(A_2B_3 - A_3B_2) + C_2(A_3A_1 - A_1A_3) + C_3(A_1A_2 - A_2A_1)$$

We can reverse the cofactor expansion, giving

$$(\vec{\mathbf{A}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{C}} = \begin{vmatrix} C_1 & C_2 & C_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

Switching a row of the determinant switches the sign of the result:

$$-(\vec{\mathbf{A}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{C}} = \begin{vmatrix} A_1 & A_2 & A_3 \\ C_1 & C_2 & C_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

Switching again, the sign once again becomes positive:

$$(\vec{\mathbf{A}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{C}} = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

Question 6

Suppose we have a positive meniscus lens of radii 6 and 10 cm and a thickness of 3 cm, made with a material of index of refraction $n = 1.5$. Determine its focal length and the locations of its principle points.

Positive lens, so

$$R_1 = 6 \text{ cm}, \ R_2 = 10 \text{ cm}, \ d = 3 \text{ cm, and } n = 1.5$$

First, we need f

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right) = (1.5-1)\left(\frac{1}{6} - \frac{1}{10} + \frac{(1.5-1)(3)}{(1.5)(6)(10)}\right)$$

$$f = 25 \text{ cm}$$

$$h_1 = -\frac{f(n-1)d}{nR_2} = -\frac{(25)(1.5-1)(3)}{(1.5)(10)} = -2.5 \text{ cm}$$

$$h_2 = -\frac{f(n-1)d}{nR_1} = -\frac{(25)(1.5-1)(3)}{(1.5)(6)} = -4.1667 \text{ cm}$$

Question 7

A spherical glass bottle 20 cm in diameter with walls that are negligibly thin is filled with water. The bottle is sitting on the back seat of a car on a nice, sunny day. What is the focal length of the "lens?"

$$R_1 = 10 \text{ cm}, \ R_2 = -10 \text{ cm}, \ d = 20 \text{ cm, and } n = 1.333$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right) = (1.333-1)\left(\frac{1}{10} + \frac{1}{10} + \frac{(1.333-1)(20)}{(1.333)(10)(-10)}\right)$$

$$f = 20 \text{ cm}$$

Question 8

It is found that sunlight is focused to a spot 29.6 cm from the back face of a thick lens, which has principle points $h_1 = 0.2$ cm and $h_2 = -0.4$ cm. Determine the location of the image of a candle that is placed 49.8 cm in front of this lens.

Assuming the rays of light from the sun are parallel, this means the $s \to \infty$ and $p = f$, but f and p are measured from h_2, making $p = 29.6 + 0.4 = 30.0 \text{ cm} = f$. So,

$$\frac{1}{s} + \frac{1}{p} = \frac{1}{f} \to \frac{1}{p} = \frac{1}{30} - \frac{1}{s}$$

But, s is measured from the h_1, so $s = 49.8 + 0.2 = 50.0$ cm.

$$\frac{1}{p} = \frac{1}{30} - \frac{1}{50.0} \to p = 75.0 \text{ cm}$$

So, the image will be located $75.0 - 0.4 = 74.6$ cm from the back of the lens.

Question 9

A crown glass double-convex lens that is 4.0 cm thick has an index of refraction of 3/2. Given that its radii are 4.0 cm and 15 cm, locate its principle points and compute its focal length. If a television screen is placed 1.0 m from the front of the lens, where will the real image of the picture appear?

Since the lens is double-convex, R_2 is negative.

$$R_1 = 4.0 \text{ cm},\ R_2 = -15 \text{ cm},\ d = 4.0 \text{ cm, and } n = 3/2$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right) = (1.5-1)\left(\frac{1}{4} + \frac{1}{15} + \frac{(1.5-1)(4)}{(1.5)(4)(-15)}\right)$$

$$f = 6.79$$

$$h_1 = -\frac{f(n-1)d}{nR_2} = -\frac{(6.79)(1.5-1)(4)}{(1.5)(-15)} = 0.604 \text{ cm}$$

$$h_2 = -\frac{f(n-1)d}{nR_1} = -\frac{(6.79)(1.5-1)(4)}{(1.5)(4)} = -2.264 \text{ cm}$$

$$\frac{1}{s} + \frac{1}{p} = \frac{1}{f} \to \frac{1}{p} = \frac{1}{6.79} - \frac{1}{1.0 + 0.604}$$

$$p = -2.1 \text{ cm}$$

But p is measured from h_2, so the image is $-2.264 - 2.1 = -4.36$ cm from the back of the lens, which is 0.36 cm in front of the lens.

Question 10

Given the following matrices,

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

work out the product $\boldsymbol{AB}$.

$$\boldsymbol{AB} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(2)+(2)(0)+(-1)(1) & (1)(1)+(2)(-1)+(-1)(1) & (1)(0)+(2)(2)+(-1)(3) \\ (0)(2)+(3)(0)+(1)(1) & (0)(1)+(3)(-1)+(1)(1) & (0)(0)+(3)(2)+(1)(3) \\ (2)(2)+(0)(0)+(1)(1) & (2)(1)+(0)(-1)+(1)(1) & (2)(0)+(0)(2)+(1)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 1 \\ 1 & -2 & 9 \\ 5 & 3 & 3 \end{bmatrix}$$