Question 1

A beam of light in air strikes the surface of a smooth piece of plastic having an index of refraction 1.55 at an angle of incidence of 20°. The incident light has an electric field with component parallel to the plane of incidence of 10 V/m and component perpendicular to the plane of incidence of 20 V/m. Determine the corresponding reflected electric field amplitudes.

As derived in class, for the component parallel to the plane of incidence:

$$r_{\parallel} = \frac{E_{r,\parallel}}{E_{i,\parallel}} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

and

$$r_{\perp} = \frac{E_{r,\perp}}{E_{i,\perp}} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t},$$

where $n_i = 1.00$, $n_t = 1.55$, $\theta_i = 20°$, and

$$\theta_t = \sin^{-1}\left(\frac{n_i \sin\theta_i}{n_t}\right) = 12.75°.$$

Thus,

$$r_{\parallel} = 0.198,$$

and

$$r_{\perp} = -0.233.$$

Considering the amplitudes,

$$E_{r,\parallel} = r_{\parallel} E_{i,\parallel} = 0.198(10\,\text{V/m}) = 1.98\,\text{V/m},$$

and

$$E_{r,\perp} = r_{\perp} E_{i,\perp} = 0.233(20\,\text{V/m}) = 4.67\,\text{V/m}$$

Question 2

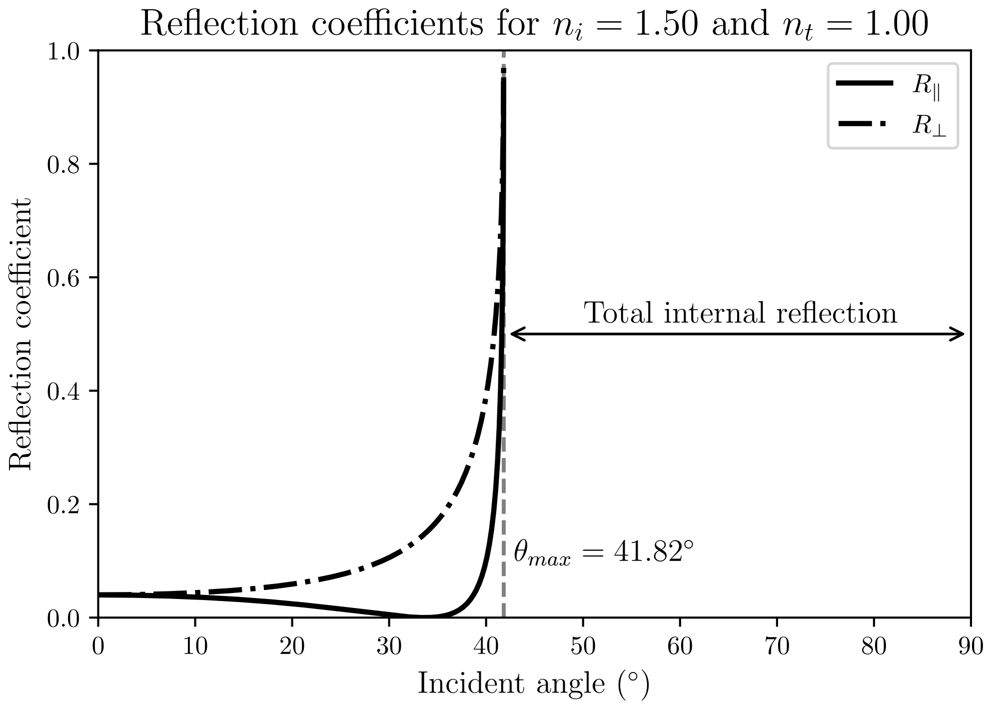

Figure 1: $R_{\parallel}$ and $R_{\perp}$ for $n_i = 1.5$ and $n_t = 1.0$.

Question 3

Show that

$$T_{\parallel} = \frac{\sin 2\theta_i \sin 2\theta_t}{\sin^2(\theta_i + \theta_t)\cos^2(\theta_i - \theta_t)}$$

and

$$T_{\perp} = \frac{\sin 2\theta_i \sin 2\theta_t}{\sin^2(\theta_i + \theta_t)}$$

$$\begin{aligned}
T_{\parallel} &= \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t_{\parallel}^2 = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} \frac{4n_i^2 \cos^2\theta_i}{n_t^2 \cos^2\theta_i + n_i^2 \cos^2\theta_t + 2n_i n_t \cos\theta_i \cos\theta_t} \\
&= \frac{4n_i n_t \cos\theta_i \cos\theta_t}{n_t^2 \cos^2\theta_i + n_i^2 \cos^2\theta_t + 2n_i n_t \cos\theta_i \cos\theta_t} \\
&= \frac{4\left(\frac{n_i^2 \sin\theta_i}{\sin\theta_t}\right)\cos\theta_i \cos\theta_t}{\frac{n_i^2 \sin^2\theta_i}{\sin^2\theta_t}\cos^2\theta_i + n_i^2 \cos^2\theta_t + 2\left(\frac{n_i^2 \sin\theta_i}{\sin\theta_t}\right)\cos\theta_i \cos\theta_t}\left(\frac{\sin^2\theta_t}{\sin^2\theta_t}\right) \\
&= \frac{\sin 2\theta_i \sin 2\theta_t}{\frac{1}{4}\sin^2 2\theta_i + \frac{1}{4}\sin^2 2\theta_t + \frac{1}{2}\sin 2\theta_i \sin 2\theta_t} = \frac{\sin 2\theta_i \sin 2\theta_t}{\frac{1}{4}\left(\sin 2\theta_i + \sin 2\theta_t\right)^2} \\
&= \frac{\sin 2\theta_i \sin 2\theta_t}{\sin^2(\theta_i + \theta_t)\cos^2(\theta_i - \theta_t)}
\end{aligned}$$

and

$$\begin{aligned}
T_\perp &= \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t_\perp^2 = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} \frac{4n_i^2 \cos^2\theta_i}{n_i^2 \cos^2\theta_i + n_t^2 \cos^2\theta_t + 2n_i n_t \cos\theta_i \cos\theta_t} \\
&= \frac{4n_i n_t \cos\theta_i \cos\theta_t}{n_i^2 \cos^2\theta_i + n_t^2 \cos^2\theta_t + 2n_i n_t \cos\theta_i \cos\theta_t} \\
&= \frac{4\left(\frac{n_i^2 \sin\theta_i}{\sin\theta_t}\right)\cos\theta_i \cos\theta_t}{n_i^2 \cos^2\theta_i + \frac{n_i^2 \sin^2\theta_i}{\sin^2\theta_t}\cos^2\theta_t + 2\left(\frac{n_i^2 \sin\theta_i}{\sin\theta_t}\right)\cos\theta_i \cos\theta_t} \left(\frac{\sin^2\theta_t}{\sin^2\theta_t}\right) \\
&= \frac{\sin 2\theta_i \sin 2\theta_t}{\cos^2\theta_i \sin^2\theta_t + \cos^2\theta_t \sin^2\theta_i + \frac{1}{2}\sin 2\theta_i \sin 2\theta_t} \\
&= \frac{\sin 2\theta_i \sin 2\theta_t}{\sin^2(\theta_i + \theta_t)}
\end{aligned}$$

Question 4

Show that

$$R_\parallel + T_\parallel = 1$$

and

$$R_\perp + T_\perp = 1$$

For $R_\parallel + T_\parallel = 1$,

$$\begin{aligned}
R_\parallel + T_\parallel &= r_\parallel^2 + \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t_\parallel^2 = \left(\frac{n_i \cos\theta_t - n_t \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}\right)^2 + \frac{n_t \cos\theta_t}{n_i \cos\theta_i} \frac{4n_i^2 \cos^2\theta_i}{(n_i \cos\theta_t + n_t \cos\theta_i)^2} \\
&= \frac{n_i^2 \cos^2\theta_t + n_t^2 \cos^2\theta_i - 2n_i n_t \cos\theta_i \cos\theta_t + 4n_i n_t \cos\theta_i \cos\theta_t}{(n_i \cos\theta_t + n_t \cos\theta_i)^2} \\
&= \frac{n_i^2 \cos^2\theta_t + n_t^2 \cos^2\theta_i + 2n_i n_t \cos\theta_i \cos\theta_t}{(n_i \cos\theta_t + n_t \cos\theta_i)^2} \\
&= \frac{(n_i \cos\theta_t + n_t \cos\theta_i)^2}{(n_i \cos\theta_t + n_t \cos\theta_i)^2} = 1
\end{aligned}$$

Now for $R_\perp + T_\perp = 1$,

$$\begin{aligned}
R_\perp + T_\perp &= r_\perp^2 + \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t_\perp^2 = \left(\frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}\right)^2 + \frac{n_t \cos\theta_t}{n_i \cos\theta_i} \frac{4n_i^2 \cos^2\theta_i}{(n_i \cos\theta_i + n_t \cos\theta_t)^2} \\
&= \frac{n_i^2 \cos^2\theta_i + n_t^2 \cos^2\theta_t - 2n_i n_t \cos\theta_i \cos\theta_t + 4n_i n_t \cos\theta_i \cos\theta_t}{(n_i \cos\theta_i + n_t \cos\theta_t)^2} \\
&= \frac{n_i^2 \cos^2\theta_i + n_t^2 \cos^2\theta_t + 2n_i n_t \cos\theta_i \cos\theta_t}{(n_i \cos\theta_i + n_t \cos\theta_t)^2} \\
&= \frac{(n_i \cos\theta_i + n_t \cos\theta_t)^2}{(n_i \cos\theta_i + n_t \cos\theta_t)^2} = 1
\end{aligned}$$

Question 5

Show that

$$\vec{\boldsymbol{\nabla}}(\ln|\vec{\boldsymbol{r}}|) = \frac{\vec{\boldsymbol{r}}}{r^2}$$

In spherical coordinates,

$$\vec{\boldsymbol{\nabla}}(f(r,\theta,\varphi)) = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}}.$$

Thus,

$$\vec{\boldsymbol{\nabla}}(\ln|\vec{\boldsymbol{r}}|) = \vec{\boldsymbol{\nabla}}(\ln r) = \hat{\boldsymbol{r}}\frac{\partial}{\partial r}\ln r = \frac{\hat{\boldsymbol{r}}}{r} = \frac{\vec{\boldsymbol{r}}}{r^2}.$$