Homework 2 Report

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***Abstract*— This homework covers the 2D simulation of a load on a hollow, aluminum beam constrained on the x and y-axes and the other the y-axis. The beam is represented by a mass-spring system with the nodes having mass.**

I. INTRODUCTION

This homework simulates Euler-Bernoulli Beam theory using a load on an aluminum beam. This beam will be analyzed using vertical displacement over time and displacement over multiple loads, and compared with the theoretical prediction from Euler beam theory.

The values of beam are as followed:

Measurement	Value
Beam Length [L]	1 (m)
Outer Radius [R]	0.013 (m)
Inner Radius [r]	0.011 (m)
Young's Modulus [E]	70 (GPa)
Density [ρ]	2700 ($\frac{kg}{m^3}$)
Load Force [P]	2000 (N)
Load Position [d]	0.75 (m)

Table 1: Given Values of the Hollow Aluminum Beam

The code uses the modified implicit functions that we have used in the previous lectures, focusing on the iterations of the one for the falling beam simulations from Lecture 6. The modified use of the implicit function uses load force and the position the load is applied to simulate the force applied to the beam. The implicit function then returns the new positions of the arrays for calculation.

The main function will set up the values and measurements for the simulations. The first loop takes a time array from 0 seconds to 1 second separated by steps of 0.01 s or 10 ms. The loop will measure the positions of the nodes over time. It will then store the value of the maximum vertical (y) displacement in an array the size of the number of steps of the given positions recorded. After the loop is finished, it will make a maximum vertical displacement versus time chart.

The code then compares the value of maximum vertical displacement of the simulation with Euler beam theory. After it will then make an array of loads, from 20 N to 20,000 N, to go into a loop function. It will have two storage arrays for the displacement values of the simulation and the theoretical equation. The loop will use the implicit function only changing the load for the simulation and the Euler beam equation for the theoretical. After the loop, the arrays will be compared in a plot with each other over the changing loads.

II. MATH

The following equations calculate the values needed for the mass at each node as well as the moment of Inertia for the cross-section of the beam:

$$m = \frac{\pi(R2 - r2)l\rho}{(N-1)} \quad [1]$$

$$I = \frac{4}{\pi}(R^4 - r^4) \quad [2]$$

The homework will use the implicit methods from the lecture as well as force balancing to account for the load:

$$F_{net} = F_{inertia} - F_{elastic} - P_{Load} \quad [3]$$

$$J = J_{inertia} - J_{elastic} \quad [4]$$

The following is the equation for Euler beam theory which will calculate the theoretical values to compare with the simulation:

$$y_{max} = \frac{Pc(l^2 - c^2)^{1.5}}{9\sqrt{3}EIl} \quad [5]$$

Where: $c = min(d,\ l - d)$ [6]

III. QUESTIONS

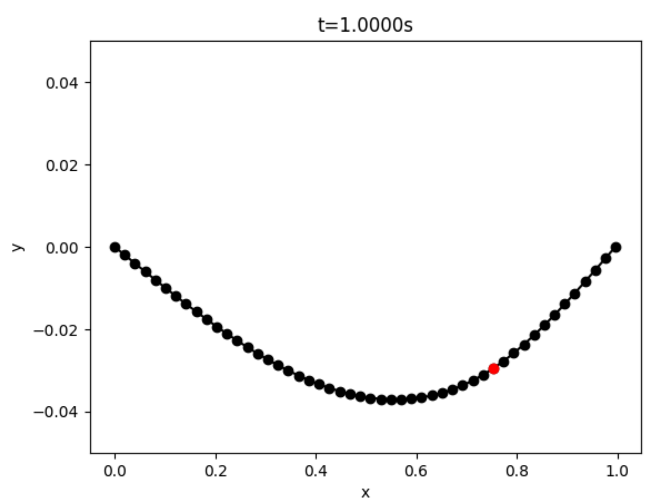

Figure 1: Shape of the Beam 1 second after a Load of 2,000 N at d =0.75m

A. Question 1

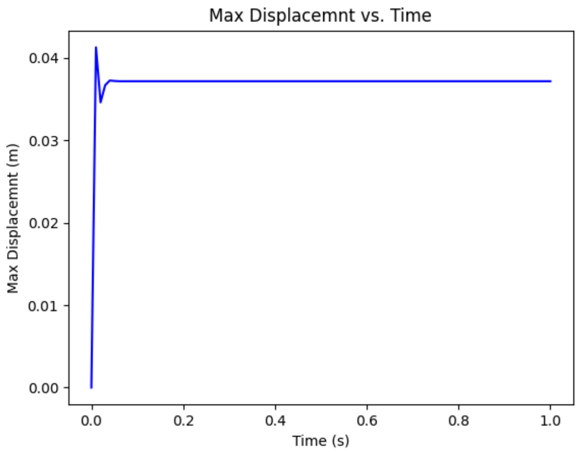

Figure 2: Maximum Vertical Displacement over time with a Load of 2,000 N at d=0.75m

When plotting the maximum vertical displacement over time, it seems that the vertical displacement does stabilize rather quickly. The time to converge seems to be less than 0.1 seconds, not oscillating very much after the load is applied.

Since the system is being simulated with a mass-spring system, it can be observed that the system seems to display the qualities of critical to over-damped oscillation as it stabilizes. This makes sense for the simulation medium and for the displacement of an aluminum beam which is not a very elastic material.

The final value of the simulated displacement is 0.03717482 m measured 1 second after the load is applied, the point well after stabilizing. Using Euler beam theory Equation [5] with the given measurements of the beam, the theoretical value comes to 0.0380442 m. This is a difference of 0.00086938 m or 0.086938 cm which is a nearly accurate difference. It is not exact, but they are close enough.

B. Question 2

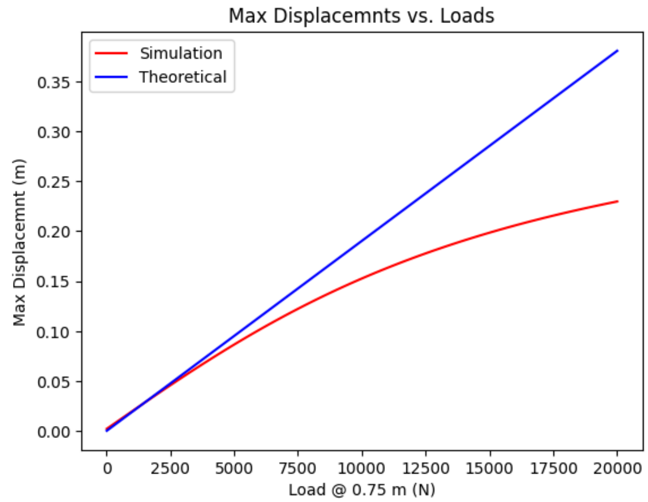

Figure 3: Maximum Vertical Displacement of Simulation compared to Euler Beam Theory Equation with different Loads at d = 0.75m

Simulating maximum displacement of both the simulated beam and the theoretical beam, it seems to diverge significantly past a load of 2500 N. It does seem like the Theoretical equation is better for smaller load measurements as it shares a similar behavior with the simulation. However, at larger loads they will diverge as they will display different properties.

It seems the Euler Beam Theory Equation [5] displays a linear trend as more load is applied which makes sense as the Load is a scalar value. In the simulation, the beam seems to come to stable value, being more accurate at larger loads as the beam bends only so far with a given force.