Homework 4 Report

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***Abstract*— This homework is to practice and observe the simulation of a helical coil as a 3D Discrete Elastic Rod. This will be compared to the measured steady-state displacement of the coil under given loads to measure the stiffness coefficient of the coil and comparing the simulated values with the theoretical values.**

I. INTRODUCTION

This homework covers the simulation of a helix coil using the concepts of reference twist, curvature, and reference frames to create the 3D Discrete Elastic Rod Simulation (DER).

The values of helix coil are as followed:

Measurement	Value
Helix diameter	0.04 m
Pitch	0.002 m
Number of Turns	5
Arc length	0.6284 m
Axial length	0.01 m
Young's Modulus	10 MPa
Poisson ratio	0.5

Table 1: Given Values of the Helix Rod

This is to compare the simulation tests of helix coil/spring with that of the theoretical calculations of the textbook to see how accurate the results are. The initial Force Characteristic is in Equation 1.

$$F_{char} = \frac{EI}{L^2} \quad [1]$$

Equations 2 and 3 are the equations the simulation will be using to calculate the stiffness coefficient and the theoretical stiffness coefficient respectfully.

$$F = k\,\delta^*_z \quad [2]$$

$$k_{text} = \frac{Gd^4}{8ND^3} \quad [4]$$

II. RESULTS

Figure 1 shows the displacement of the end node until it reaches the stead- state of the load for part 1 of measurements and Figure 2 shows the snapshots of the coil during the simulation. The steady-state is determined by measuring the relative change of the last 50 measurements taken by the simulation. It takes the difference of the maximum and minimum values of the window and compares it with the mean of the value window taken. If the change is within 10% of the mean value, it considers the mean value the estimated steady state of the system. For the starting perimeters, the stead-state displacement was -1.415 mm.

The code uses larger timesteps to cause the system to converge faster. The function would need to be adjusted to use a larger window of the most recent values for smaller timesteps to be more accurate as the difference value between the nodes is much smaller and converges slower that it might not get an accurate maximum and minimum value of the oscillations to calculate an accurate steady state value. The code perimeter, the timestep and window, was made for calculating the steady state faster for all parts of the homework.

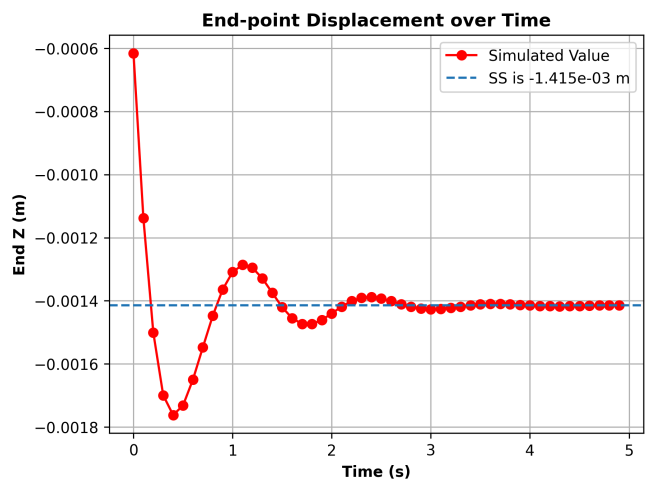
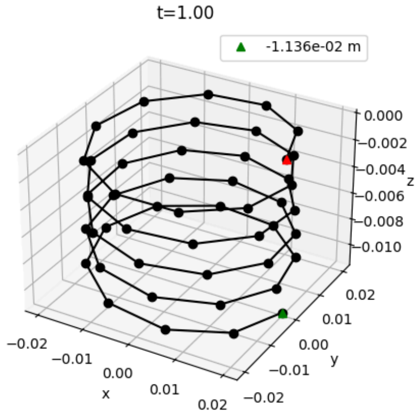
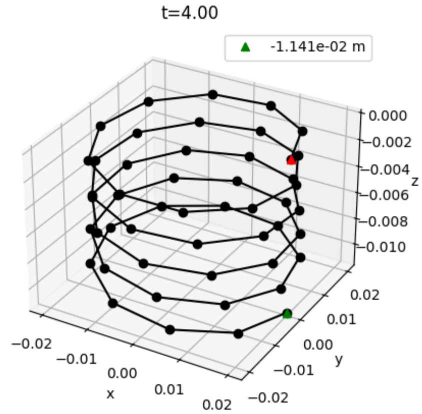
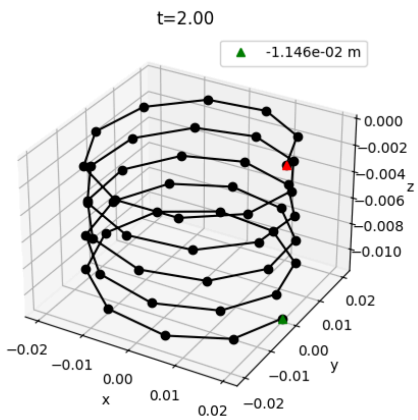
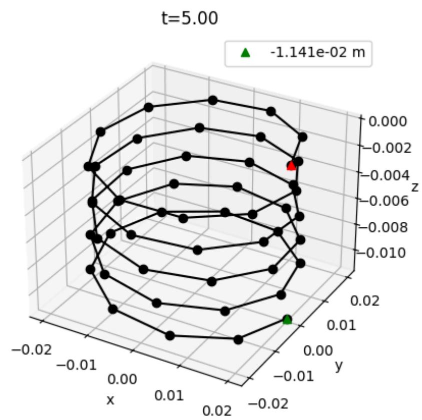


Figure 1: Z displacement of End Node of the Coil vs Time.









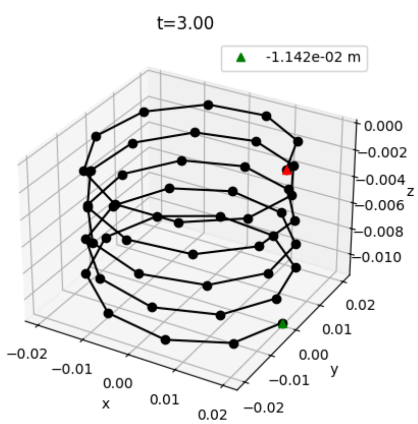


Figure 2: Snapshots of the system over time (a) t=1 s, (b) t=2 s, © t=3 , (d) t=4 s, and (e) t=5 s

Part 2 uses a force sweep function to determine the stiffness constant of the coil at the given diameter. The code uses 7 forces between 0.1 times the characteristic force load and 10 times the characteristic force load. The force sweep function goes through a for loop of the part 1 until it finds and stores the steady-state value before moving on to the next force.

The steady-state values can be used in comparison to the force values to find the line of best fit starting from the origin at zero. The slope of the line is the estimated stiffness coefficient. In the code, all the forces steady-state values are used to calculate the slope except the 10 times the characteristic force, because it seems to barely fall outside the small-displacement region. The plot can be seen in Figure 3. The stiffness coefficient, k, was 0.01377 N/m.

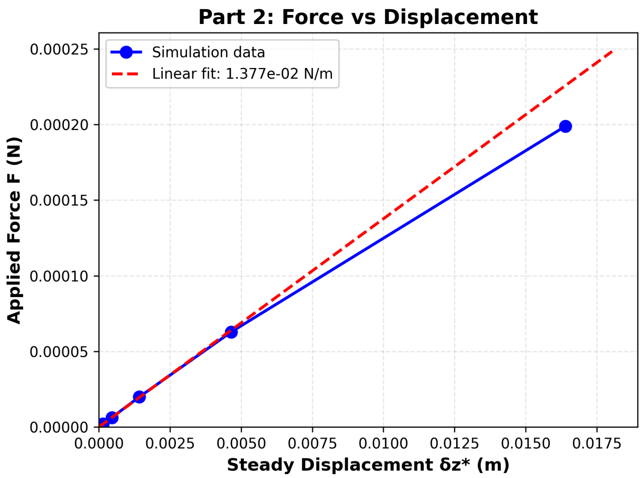


Figure 3: Force Load vs Steady-State Displacement with the calculated stiffness coefficient

Part 3 is a diameter sweep of the helix using 10 values from 0.01 m to 0.05 m. These diameters used a modified version of the for loop in Part 2 of the force sweep to calculate the stiffness coefficient at the given diameter. Then, these stiffness coefficient values will be compared to the theoretical stiffness coefficient values from the textbook equation. The plot comparing the values in Figure 4 and Table 2 has the calculated stiffness coefficients from DER.

It seems that compared to the theoretical values, it seems that the DER values for the stiffness are smaller than the calculated. However, it seems that the trend seems to closely follow that of the theoretical value in a linear manner with a growing divergence with a smaller slope. With the larger values of the diameter a larger difference.

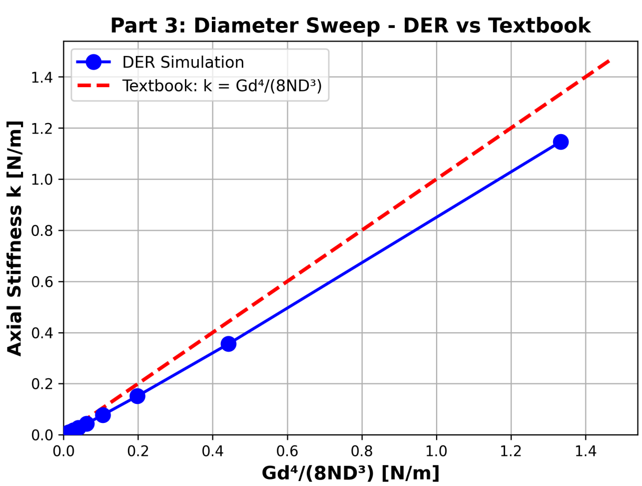


Figure 4: Stiffness Coefficient of DER vs the Textbook ratio.

Diameter (m)	**DER K (N/m)**	**Textbook K (N/m)**
0.0100	1.146603e+00	1.333333e+00
0.0144	3.555685e-01	4.424215e-01
0.0189	1.511429e-01	1.978425e-01
0.0233	7.709307e-02	1.049563e-01
0.0278	4.415822e-02	6.220800e-02
0.0322	2.767080e-02	3.985403e-02
0.0367	1.842784e-02	2.704733e-02
0.0411	1.291972e-02	1.918939e-02
0.0456	9.333662e-03	1.410310e-02
0.0500	7.058316e-03	1.066667e-02

Table 2: Table of Stiffness Coefficients at the Given Helix Diameter