Start: 29.4.2024
Return: 13.5.2024
https://classroom.github.com

Assignment 2
Introduction to Probabilistic Machine Learning

# Part 1: Factor Graphs and Message Passing in the SEIR Model (3 Points)

- Relevant source files: `factors.jl, seir.jl`
- Supporting library files from previous assignments: `discrete.jl, distributionbag.jl`
- Scripts to run the SEIR example: `run_seir.jl`
- Test files: `test*_factors.jl, test*_seir.jl`
- Suggested name for plot files: `plot_seir1.png` etc.

For the following exercise, please fill out the source files. Please also hand in the plots generated by your plotting library. The code for distributions and distribution bags is provided.

We consider an infection model – called the SEIR model – for single persons with the following 4 states:

| Index | State | Abbreviation | Description |
|---|---|---|---|
| 1 | S | Susceptible | The person is not infected yet |
| 2 | E | Exposed | The person is infected but not infectious yet |
| 3 | I | Infectious | The person is infectious (and can infect others) |
| 4 | R | Recovered | The person is recovered and not infectious anymore |

We want to study the dynamics of the SEIR model using factor graphs of chain type (see Unit 4, slide 10-13) and consider $N$ variables $x_i$, $i = 1, ..., N$ each describing the (discrete probability distribution) of the state (index) of a person in period $i = 1, ..., N$.

The prior distribution for the state of the person at period $i = 1$ is

$$f_1(x_1) = P(x_1) = \begin{cases} 1 & \text{if } x_1 = 1 \\ 0 & \text{if } x_1 = 2 \\ 0 & \text{if } x_1 = 3 \\ 0 & \text{if } x_1 = 4 \end{cases}$$

Further, the transition dynamics of the model from variable $x_i$ to $x_i + 1$ are given by, $i = 1, ..., N - 1$,

$$f_i(x_i, x_{i+1}) = P(x_{i+1}|x_i) = \mathbf{T}_{x_i,x_{i+1}} \qquad \text{where} \quad \mathbf{T} = \begin{pmatrix} 0.9 & 0.1 & 0 & 0 \\ 0 & 0.8 & 0.2 & 0 \\ 0 & 0 & 0.7 & 0.3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Implement the model and determine all marginals $p(x_i)$, $i = 1, ..., N$, and messages $m_{f_i \to x_i}(x_i)$, $i = 1, ..., N$, as well as $m_{f_{i+1} \to x_i}(x_i)$, $i = 1, ..., N - 1$. Use the steps 1.-4. of Unit 4, slide 13 and the structures provided in the example code.

1. Use the marginals $p(x_i)$ to plot the four state probabilities $P(x_i = 1)$, $P(x_i = 2)$, $P(x_i = 3)$, $P(x_i = 4)$ over time, i.e., all periods $i = 1, ..., N$, for $N = 100$ in one graph.
2. Solve a second version of the model (as done in a)) by considering the adjusted transition probabilities $P(x_{i+1} = 1|x_i = 1) = 0.95$, $P(x_{i+1} = 2|x_i = 1) = 0.05$ and plot the same graph.
3. Solve a third version of the model (as done in a)) by considering the adjusted transition probabilities $P(x_{i+1} = 1|x_i = 1) = 0.85$, $P(x_{i+1} = 2|x_i = 1) = 0.15$ and plot the same graph.

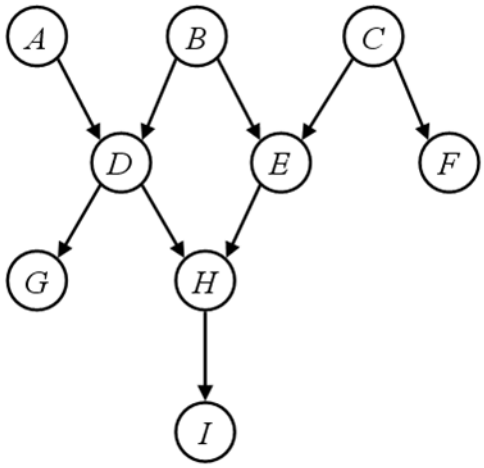


Figure 1: A directed graph with nodes A-I.

## Part 2: Determine the Conditional Independence of pairs of nodes in a complex Bayes Network (2 Points)

Consider the graph depicted in Figure 1. We look for pairs of variables that are conditionally independent!

a) Assume that node B and F are clamped/observed.

Are D and E conditionally independent?

Are A and E conditionally independent?

Are G and C conditionally independent?

b) Assume that node I and C are clamped/observed.

Are D and E conditionally independent?

Are A and E conditionally independent?

Are E and F conditionally independent?

c) Assume that node E and H are clamped/observed.

Are A and G conditionally independent?

Are C and I conditionally independent?

Are A and I conditionally independent?

d) Assume that node D is clamped/observed.

Are A and H conditionally independent?

Are C and I conditionally independent?

Are G and F conditionally independent?

e) Now we look for a graph that satisfies certain relations. Consider four random variables $(A, B, C, D)$. Find a graph that has the following relations.

1. A is a parent of D.
2. B is a child of C.
3. There is no edge between A and B.
4. D and C are conditionally independent when A is observed.
5. D and C are not conditionally independent when A is not observed.

Answer the questions for the different scenarios and explain your answer, e.g., by explaining whether or not blocked nodes (of type H2T, H2H, or T2T) exist at different places.