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Assignment 2

Sommersemester 2024

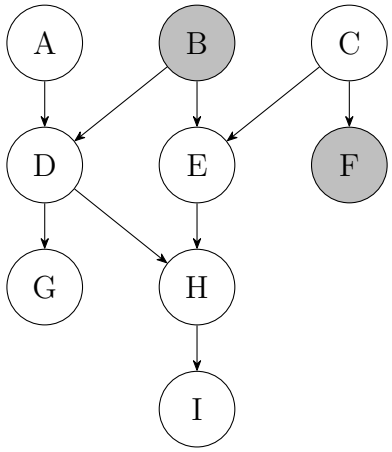
Part 2:

To show the conditional indepence of two variables, every undirected path in the network between them needs to have at least one blocked node.

A node is said to be blocked, if ...

- it's a H2T or T2T node and the node is observed.
- it's a H2H node and neither the node nor any of its descendants are observed.

(a) **Assume that node B and F are observed.**

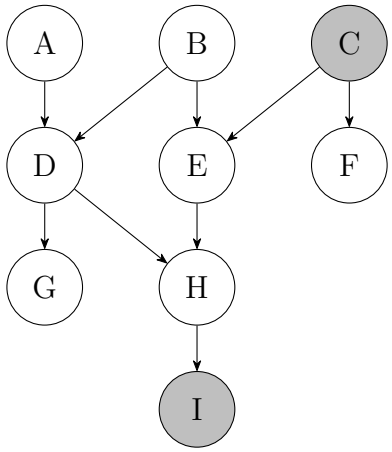


(i) D, E are conditionally independent because B is a blocked T2T node on path DBE and H is a blocked H2H node on path DHE.

(ii) A, E are conditionally independent because the paths ADHE and ADBE have the same blocked nodes as (i).

(iii) G, C are conditionally independent because the paths GDHEC and GDBEC have the same blocked nodes as (i).

(b) **Assume that node I and C are observed.**

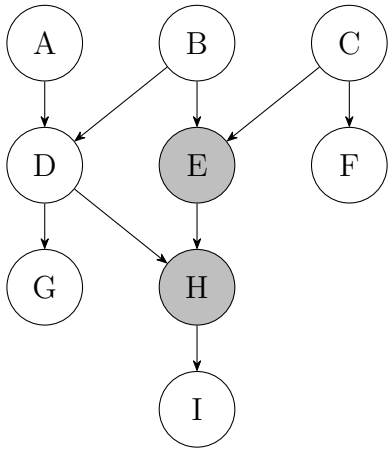


(i) D, E are dependent because B is a non-blocked T2T node on the path DBE and therefore the whole path is missing a blocked node.

(ii) A, E are dependent because B is a non-blocked T2T and D a non-blocked H2H node on the path ADBE and therefore the whole path is missing a blocked node.

(iii) E, F are conditionally independent because C is a blocked T2T node on the only path ECF.

(c) **Assume that node E and H are observed.**

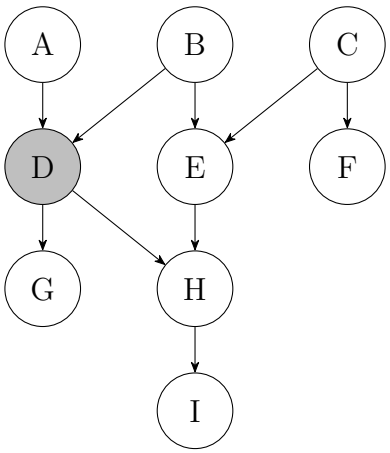


(i) A, G are dependent because D is a non-blocked H2T node on the path ADG and therefore the whole path is missing a blocked node.

(ii) C, I are conditionally independent because H is a blocked H2T node on the path CEHI and on the path CEBDHI.

(iii) A, I are conditionally independent because H is a blocked H2T node on the path ADHI and on the path ADBEHI.

(d) **Assume that node D is observed.**



(i) A, H are dependent because D is a non-blocked H2H, B a non-blocked T2T and E a non-blocked H2T node on the path ADBEH.

(ii) C, I are dependent because E and H are non-blocked H2T nodes on the path CEHI.

(iii) G, F are conditionally independent because D is a blocked H2T node on the path GDBECF and a blocked T2T node on the path GDHECF.

(e) **A graph meeting the 5 conditions**
A is a H2T node, which gurantees the conditional independence of C and D when A is observed and makes them dependent when A is not observed.