Coordinate systems

To fix the position of any astronomical object, we must have a frame of reference, or coordinate system, which assigns a different pair of numbers to every point in the sky. The two numbers, or ***coordinates****, usually refer to 'how far round' and 'how far up', just as do the longitude and latitude of an object on the Earth's surface. There are several such coordinate systems which you may meet, and we shall be concerned with four of these, namely the horizon system, the equatorial system, the ecliptic system and the galactic system. Each system takes its name from the fundamental plane which it uses as a reference; for instance, the ecliptic coordinate system makes all its measurements with respect to the plane of the ecliptic, the plane of the Earth's orbit about the Sun. In the next few sections, we shall find how to convert any position given in one system into the equivalent coordinates of another system. We shall also find how to describe positions on the surfaces of the Sun and Moon, how to deal with the problems of calculating the time of rising and setting, and with the effects of the Earth's precession, nutation, aberration, atmospheric refraction, and parallax on the apparent position of a celestial body.*

 https://doi.org/10.1017/CBO9780511861161.005

17 Horizon coordinates

The horizon coordinates, azimuth and altitude, of an object in the sky are referred to the plane of the observer's horizon (see Figure 17). Imagine an observer standing at point O; then his or her horizon is the circle NESW, where the letters refer to the north, east, south and west points of the horizon respectively. The direction north, by the way, relates to the direction of the north pole on the Earth's rotation axis and not to the magnetic north pole. You must imagine the stars as fixed on the surface of the hemisphere with the observer at the centre as in Figure 17; the whole sphere of which this hemisphere is part is called the **celestial sphere**. The point Z directly over the observer's head is called the **zenith**; the direction OZ is the direction defined by a plumb line held by the observer. Now consider a star X and imagine a great circle (i.e. a circle drawn on the surface of the sphere whose centre is the same as that of the sphere) going through Z and X; it meets the horizon at point B. The **altitude**, a, of the star is then the angle subtended at O by the points X and B. The **azimuth**, A, is the angle subtended by the points N and B. Hence, the altitude is 'how far up' in degrees (negative if below the horizon) and the azimuth is 'how far round' from the north direction, also measured in degrees. A increases from 0° to 360° as you go around in the sense NESW, N being 0°, S being 180°, etc.

The altitudes and azimuths of all heavenly bodies except geostationary satellites are continually changing with time as the Earth rotates. This coordinate system then, marvellous for setting the direction of your telescope, is not much good for fixing the positions of the stars. Another frame of reference, independent of the Earth's motion, is needed to do that. It is described in the next section.

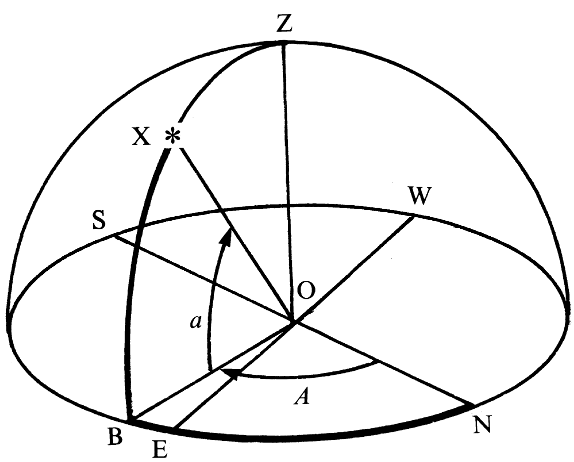
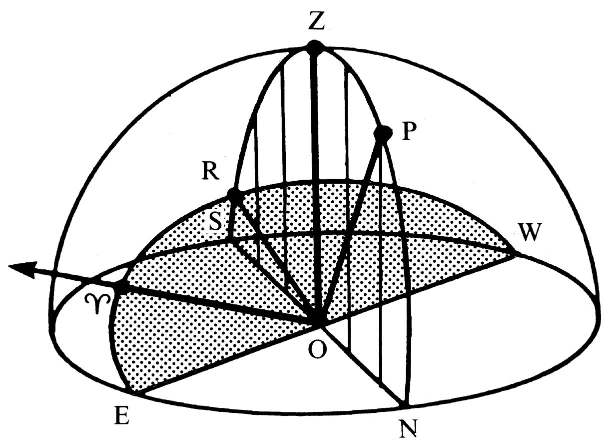


Figure 17. Horizon coordinates.

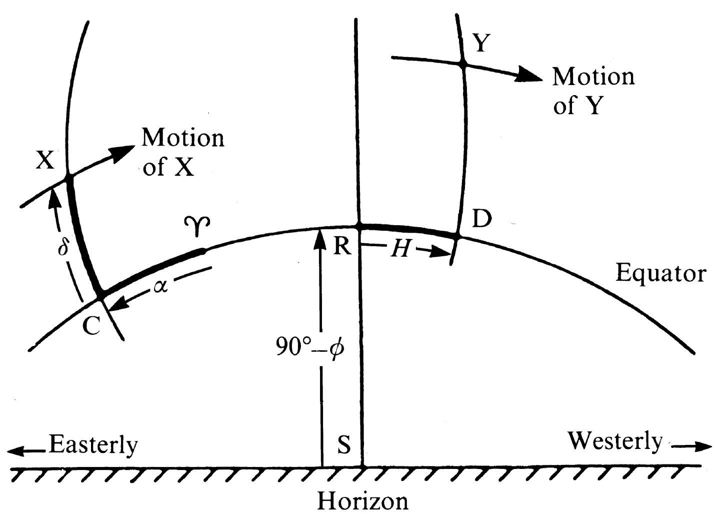
18 Equatorial coordinates

As their name suggests, these coordinates are referred to the plane of the Earth's equator (see Figure 18(*a*)). The observer (assumed to be in the northern hemisphere) is at O and the plane containing the circle NESW is again the horizon with Z the zenith point. You are to imagine now that the figure represents the view obtained at a vast distance from the Earth. The Earth, with the observer standing on it, has shrunk to a tiny dot at the centre of the diagram, but the plane of the equator has been extended to cut the celestial sphere along the circle E♈RW. This is the equatorial plane and is inclined at the angle $90° - \phi$ to the horizon, where ϕ is the observer's geographical latitude. For observations at latitude 52° N this angle is 38°. At right angles to the equatorial plane along the line OP lies the axis of rotation of the Earth; it intersects the celestial sphere at P, the **north celestial pole**, or **north pole** for short. Since this is the line about which the Earth spins, all the stars appear to describe circles in the sky about P.

Figure 18(*b*) shows the situation as seen by the observer O looking up into the sky. The south point, S, of



(*a*)



(*b*)

Figure 18. Equatorial coordinates: (*a*) on the celestial sphere, and (*b*) as seen from the ground.

the horizon is marked and so is the imaginary trace of the equator, C♈RD. The arc extending down through R and S is the great circle which goes through NPZRS in Figure 18(*a*). The arc extending down through XC is another great circle, not marked in Figure 18(*a*), which goes through PXC. Consider the star at X. The arc CX, or the angle subtended at O by the points C and X, is called the **declination**, δ, of X, defining 'how far up' from, or north of, the equator. The other coordinate, 'how far round', is defined with respect to a fixed direction in the sky, marked by the symbol ♈. This direction, called the **vernal equinox** or the **first point of Aries**, lies along the line of the intersection of the plane of the Earth's equator with that of the Earth's orbit around the Sun. But we needn't worry about such definitions at the moment. All we need to know is that the direction ♈ remains fixed with respect to the stars (except for the effects of **precession** and **nutation** – see Sections 34 and 35), and that we measure the other coordinate with respect to it. This coordinate is called the **right ascension**, α, and is the angle subtended at O by the points ♈ and C.

Throughout the course of the day the star X moves steadily westwards along a circle centred on P, completing one revolution in 24 hours of sidereal time (see Section 11 for a description of sidereal time). Since this circle is a circle parallel to that of the equator the declination does not change. Furthermore, since the direction ♈ is fixed in the heavens, it appears to move along the equator at exactly the same rate as X moves along the circle. Hence the right ascension does not change either. Thus α and δ are ideal coordinates for describing the positions of the stars and other 'fixed' heavenly bodies.

Related to the right ascension is another 'how far round' coordinate called the **hour angle**, H (see Figure 18(*b*)). For the star Y it is defined as the angle subtended at O by the points R and D and is a measure of how far the star has travelled along the equator from the southern point R, that is a measure of the time since it crossed the meridian. H increases uniformly as the day proceeds; when H is zero, the star crosses the great circle NPZRS (Figure 18(*a*)). This circle is called the **meridian** and the star is said to **transit** or **culminate**. Its altitude (Section 17) is then maximum and its azimuth† is 180° (provided that its declination is less than the geographical latitude).

The declination is measured in degrees, positive north of the equator and negative south of it. The hour angle and the right ascension may also be measured in degrees, 0° to 360°. α is measured in the sense that it increases as you move *east* from ♈; the point ♈ itself is at 0°. (Note that this is in the *opposite* sense to that in which H is measured.) More commonly, however, these two coordinates H and α are measured in hours, minutes and seconds of time from 0 to 24 hours. One complete revolution, 360°, corresponds to 24 hours of sidereal time; thus 1 hour is equivalent to 15°. The two statements 'the right ascension of X is 90°' and 'the right ascension of X is 6 h' are entirely equivalent. To convert from one to the other simply multiply or divide by 15.

A useful result of measuring the right ascension in time is that the star transits when the local sidereal time is equal to the right ascension.

†Some authors measure azimuth from the south point rather than the north point, in which case $A = 0°$ at transit.

19 Ecliptic coordinates

The plane containing the Earth's orbit around the Sun is called the **ecliptic** and the other planets in our Solar System also move in orbits close to this plane. When making calculations on objects in the Solar System it is therefore often convenient to define positions with respect to the ecliptic, that is, to use the ecliptic coordinate system. This system, like the equatorial system described in Section 18, also uses the vernal equinox, ♈, as its reference direction. Figure 19, which is similar to Figure 18(*b*), shows how it goes.

The imaginary traces of the planes of the equator and the ecliptic are drawn on the sky, and their point of intersection is the vernal equinox, ♈. The two planes are inclined to each other at an angle of about 23.5 degrees, called the **obliquity** of the ecliptic and given the symbol ε. (See Section 27 for a formula for calculating ε.) This angle is the tilt of the Earth's NS axis from the perpendicular to the plane of the ecliptic. Also marked in Figure 19 is a planet, V. Part of the trace of the imaginary great circle from the pole of the ecliptic (i.e. the point where the line drawn through the Sun perpendicular to the ecliptic meets the celestial sphere) down through V is marked and this cuts the ecliptic at F. Then the **ecliptic longitude**, λ, of V is defined to be the angle subtended by the points ♈ and F, and the **ecliptic latitude**, β, the angle subtended by the points F and V.

As with equatorial coordinates, β is positive if the planet is above (i.e. north of) the ecliptic and negative if it is below it. The sense of λ is such that λ increases as you move eastwards along the ecliptic. Both λ and β are usually measured in degrees.

During the course of the year the Sun moves eastwards along the trace of the ecliptic. By definition, its ecliptic latitude is always zero. On about 21 March, it is at the position ♈ and its right ascension and declination are both zero. Its ecliptic longitude is also zero. Thereafter, its ecliptic longitude steadily increases until three months later it is 90°, midsummer in the northern hemisphere. After the course of 1 year, the Sun has returned to its starting position having traversed 360° of ecliptic longitude.

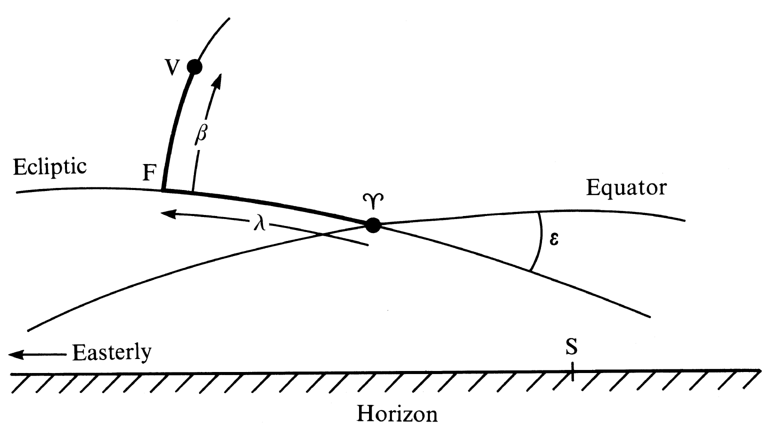


Figure 19. Ecliptic coordinates as seen from the ground (northern hemisphere).

20 Galactic coordinates

Astronomers occasionally need to describe the relationships between stars or other celestial objects within our own Galaxy and to do so it is convenient to use the galactic coordinate system. This time, the fundamental plane is the plane of the Galaxy and the fundamental direction is the line joining our Sun to the centre of the Galaxy. Figure 20 describes the situation. The point marked S represents the Sun, G is the centre of the Galaxy, and X a star which does not lie in the galactic plane. In equatorial coordinates, the position of G is $\alpha = 17\text{h}\,42.4\text{m}$ and $\delta = -28^\circ\,55'$. The lines SG and SX′ both lie in the plane of the Galaxy; the point X′ is the projection of the star's position onto the plane. The **galactic longitude** is defined to be the angle l measured in the plane, and the **galactic latitude** is defined to be the angle b measured perpendicular to it. The longitude increases from 0° to 360° in the same direction as increasing right ascension, and the latitude ranges from 0° to 90° north of the plane and from 0° to −90° south of it. These coordinates may be used, for example, to express the position of a star in the Milky Way.

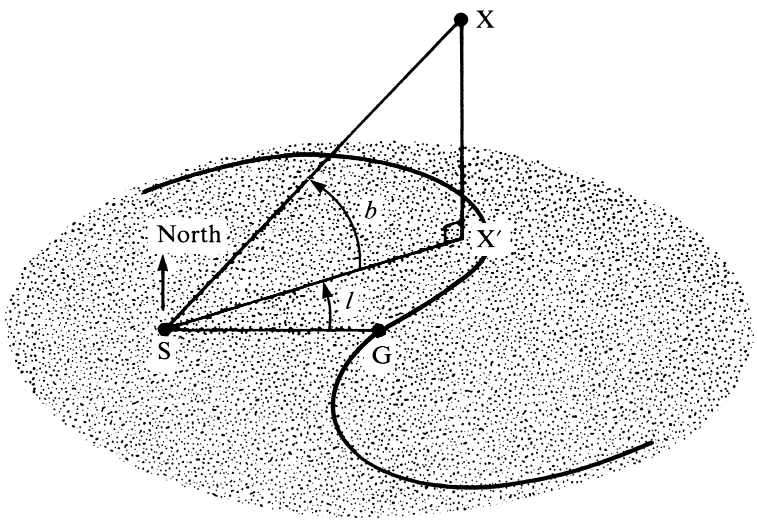


Figure 20. Galactic coordinates.

21 Converting between decimal degrees and degrees, minutes and seconds

Angles are often expressed as degrees, minutes and seconds; the minutes and seconds are called minutes and seconds of arc to distinguish them from time. Calculations are best done, however, with decimal degrees and the methods of conversion between these two forms are exactly the same as the methods for conversion between hours, minutes and seconds and decimal hours (Sections 7 and 8).

As an example, the angle $182° 31' 27''$ is equal to 182.524 167 degrees, seen as follows:

Method	*Example*		
1. Take the number of seconds and divide by 60.	27/60	=	0.450 000
2. Add this to the number of minutes and divide by 60.	31.45/60	=	0.524 167
3. Add the number of degrees.	+182.0	=	**182.524 167**

Going the other way is equally straightforward. Continuing our example, express the angle 182.524 167 in degrees, minutes and seconds form.

Method	*Example*		
1. Take the fractional part and multiply by 60. The integer part of the result is the number of minutes.	$0.524\,167 \times 60$	=	**31**.450 020
2. Take the fractional part of the result and multiply by 60. This gives the number of seconds.	$0.450\,020 \times 60$	=	**27**.001 200
Add the number of degrees.			$\mathbf{182° 31' 27''}$

The spreadsheets shown in Figures 21 and 22, DMSDD, short for Degrees, Minutes and Seconds to Decimal Degrees conversion, and DDDMS, short for Decimal Degrees to Degrees, Minutes and Seconds conversion, will carry out these calculations, as will also the corresponding spreadsheet functions DMSDD(D,M,S), and DDDeg(D), DDMin(D), and DDSec(D). The first of these, DMSDD(D,M,S), takes three arguments D, M, S corresponding to the angle expressed in degrees, minutes and seconds, and returns the equivalent angle expressed in decimal degrees. Thus you could delete rows 7 to 10 of the spreadsheet shown in Figure 21 (save a copy) and insert the formula =DMSDD(C3,C4,C5) into cell H3.

The three functions DDDeg(D), DDMin(D), and DDSec(D) return the degrees part, the minutes part and the seconds part respectively of the angle given by the argument, D, expressed in decimal degrees. You can therefore delete rows 7 to 14 of the spreadsheet shown in Figure 22 (having saved a copy) and insert the following formulas into cells H3, H4 and H5 respectively:

```
=DDDeg(C3)
=DDMin(C3)
=DDSec(C3).
```

Why not try this for yourself?

	A	B	C	D	E	F	G	H	I
1	**Converting degrees, minutes and seconds to decimal degrees**								
2									
3	*Input*	**degrees**	**182**			*Output*	**decimal degrees**	**182.5241667**	=C10
4		**minutes**	**31**						
5		**seconds**	**27**						
6									
7	1	*A*	0.45	=ABS(C5)/60					
8	2	*B*	0.524166667	=(ABS(C4)+C7)/60					
9	3	*C*	182.5241667	=ABS(C3)+C8					
10	4	*D*	182.5241667	=IF((C3<0)+(C4<0)+(C5<0),-C9,C9)					

Figure 21. Converting angles expressed in degrees, minutes and seconds into decimal degrees.

	A	B	C	D	E	F	G	H	I
1	**Converting decimal degrees to degrees, minutes and seconds**								
2									
3	*Input*	**decimal degrees**	**182.5241667**			*Output*	**degrees**	**182**	=C14
4							**minutes**	**31**	=C12
5							**seconds**	**27**	=C10
6									
7	1	unsigned decimal	182.5241667	=ABS(C3)					
8	2	total seconds	657087.0001	=C7*3600					
9	3	seconds (2 dp)	27	=ROUND(MOD(C8,60),2)					
10	4	corrected seconds	27	=IF(C9=60,0,C9)					
11	5	corrected remainder	657087.0001	=IF(C9=60,C8+60,C8)					
12	6	minutes	31	=MOD(TRUNC(C11/60),60)					
13	7	unsigned degrees	182	=TRUNC(C11/3600)					
14	8	signed degrees	182	=IF(C3<0,-1*C13,C13)					

Figure 22. Converting angles expressed in decimal degrees into degrees, minutes and seconds.

22 Converting between angles expressed in degrees and angles expressed in hours

It is common astronomical practice to express the hour angle or right ascension of a star in hours, minutes and seconds of time rather than in degrees. We can transform one to the other by noting that 360° of Earth's rotation takes place in 1 day, or 24 hours. Thus 360° is equivalent to 24 hours or 15° to 1 hour. Table 4 illustrates this equivalence more completely. To convert between angles expressed in *decimal* hours and angles expressed in *decimal* degrees, simply multiply or divide by 15. For example, the right ascension 9h 36m 10.2s is equivalent to 144° 02′ 33″.

Unit of time	Equivalent angle
1 day	360 degrees
1 hour	15 degrees
1 minute	15 arcmin
1 second	15 arcsec
Unit of angle	**Equivalent time**
1 radian	3.819 719 hours
1 degree	4 minutes
1 arcmin	4 seconds
1 arcsec	0.066 667 seconds

Table 4. Expressing angles in degrees or units of time.

We have not included spreadsheets for such simple calculations, but we have provided the spreadsheet functions DDDH(D), short for Decimal Degrees to Decimal Hours conversion, and DHDD(H), short for Decimal Hours to Decimal Degrees conversion. Each takes a single argument giving the angle expressed in decimal degrees or decimal hours respectively. The following spreadsheet formulas will carry out the example given above:

=DDDeg(DHDD(HMSDH(9,36,10.2))), which returns 144°;
=DDMin(DHDD(HMSDH(9,36,10.2))), which returns 02′; and
=DDSec(DHDD(HMSDH(9,36,10.2))), which returns 33″.

Here we have nested several spreadsheet functions together. To work out how these work, always start from the middle and work outwards. Thus, in the first of the formulas above, HMSDH(9,36,10.2) returns the time 9h 36m 10.2s expressed in decimal hours. This value becomes the argument for the function DHDD that converts the decimal hours to decimal degrees. Its result is then the argument for the outer function DDDeg that returns the degrees part of the angle expressed as degrees, minutes and seconds. To convert the other way round you could write:

=DHHour(DDDH(DMSDD(144,2,33))), which returns 9h;
=DHMin(DDDH(DMSDD(144,2,33))), which returns 36m; and
=DHSec(DDDH(DMSDD(144,2,33))), which returns 10.2s.

23 Converting between one coordinate system and another

It is very often necessary to convert the coordinates of a heavenly body expressed in one coordinate system into the equivalent coordinates of another system. This is the case when, for example, you have found the position of a planet in ecliptic coordinates and you then wish to convert to horizon coordinates to see where to look in the sky. The formulas for conversion between the equatorial system and any of the other three systems, horizon, ecliptic, or galactic, are relatively straightforward. The conversion, therefore, is often best done via the equatorial system, as illustrated in Figure 23. The arrows indicate the conversions treated explicitly in this book in the section specified by the number. For example, to convert from galactic coordinates to horizon coordinates, first convert to equatorial coordinates (Section 30) and then to horizon coordinates (Section 25).

An alternative to using explicit formulas for each conversion, as in Sections 25 to 30, is to adopt the matrix method of generalised coordinate transformation. This is described in Section 31, and may be used to convert from any system to any other system directly, once you have worked out the appropriate matrix.

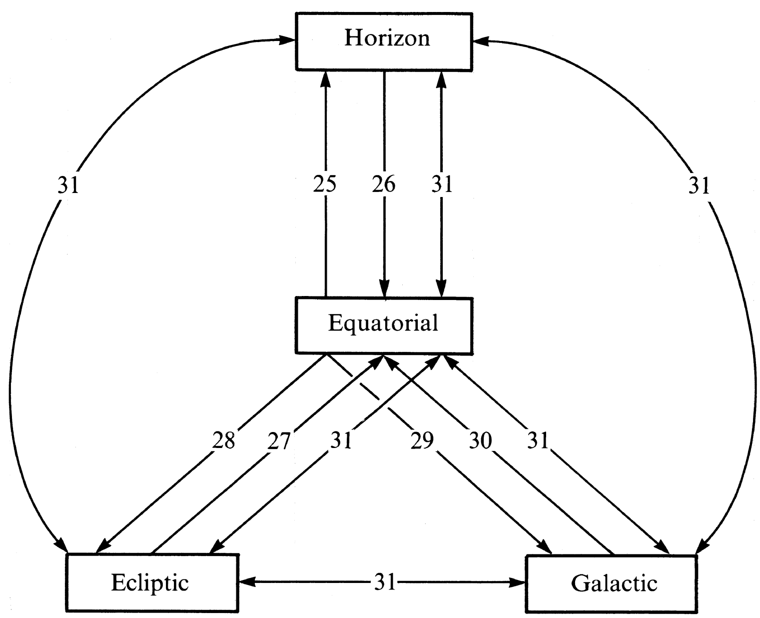


Figure 23. Converting between coordinate systems.

24 Converting between right ascension and hour angle

The hour angle, H, and the right ascension, α, are related by the simple formula

$$H = \text{LST} - \alpha,$$

where LST is the local sidereal time. All quantities must be expressed in the same units, i.e. as degrees or as hours. Let us take as an example the problem of finding the local hour angle of a star whose right ascension is α = 18h 32m 21s, at a point whose longitude is 64° W, in time zone −4 h when daylight saving was not in operation, on local calendar date 22 April 1980 at local civil time 14h 36m 51.67s.

	Method	*Example*		
1.	Find the UT and Greenwich calendar date corresponding to the local time and date (§9).	UT	=	18.614 353
		GDay	=	22
		GMonth	=	4
		GYear	=	1980
2.	Find the corresponding Greenwich sidereal time (§12)...	GST	=	8.679 071
3.	...hence find the local sidereal time (§14).	LST	=	4.412 404
4.	Express the right ascension α in decimal hours (§7).	α	=	18.539 167 hours
5.	Subtract this from the LST. If the answer is	$H1$	=	−14.126 763
	negative, add 24. This is the hour angle.	H	=	9.873 237 hours
6.	Convert to hours, minutes and seconds form (§8).	H	=	**9h 52m 23.66s**

Figure 24 shows the spreadsheet for this calculation. We have made use of several spreadsheet functions, such as LCTUT, LCTGDay etc., to carry out much of the work. These functions are defined in their corresponding sections. The spreadsheet is called RAHA (Right Ascension to Hour Angle conversion). We have also provided a spreadsheet function of the same name with 12 arguments:

RAHA(RH,RM,RS,LH,LM,LS,DS,ZC,D,M,Y,GL).

RH, RM, RS are the right ascension in hours, minutes and seconds respectively, LH, LM, LS are the local civil time in hours, minutes, and seconds respectively, DS and ZC are the daylight saving and zone correction in hours, D, M, Y is the local calendar date as day, month, year, and GL is the geographical longitude in degrees, west negative. Having saved a copy, you can delete rows 16 to 24 of the spreadsheet and insert the following formulas in cells H3, H4, and H5 respectively:

```
=DHHour(RAHA(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DHMin(RAHA(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DHSec(RAHA(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
```

to achieve the same result with a lot less typing!

File Edit View Insert Format Tools Data Window Help — Type a question for help

	A	B	C	D	E	F	G	H	I	J
1	**Conversion of right ascension to hour angle**									
2										
3	*Input*	**RA hours**	**18**			*Output*	**H hours**	**9**	=DHHour(C24)	
4		**RA mins**	**32**				**H mins**	**52**	=DHMin(C24)	
5		**RA secs**	**21**				**H secs**	**23.66**	=DHSec(C24)	
6		**LCT hours**	**14**							
7		**LCT mins**	**36**							
8		**LCT secs**	**51.67**							
9		**daylight saving**	**0**							
10		**zone correction**	**-4**							
11		**local day**	**22**							
12		**local month**	**4**							
13		**local year**	**1980**							
14		**geog long**	**-64**							
15										
16	1	**UT**	**18.61435278**	=LCTUT(C6,C7,C8,C9,C10,C11,C12,C13)						
17	5	**Gday**	**22**	=LCTGDay(C6,C7,C8,C9,C10,C11,C12,C13)						
18	6	**Gmonth**	**4**	=LCTGMonth(C6,C7,C8,C9,C10,C11,C12,C13)						
19	7	**Gyear**	**1980**	=LCTGYear(C6,C7,C8,C9,C10,C11,C12,C13)						
20	8	**GST**	**8.679070963**	=UTGST(C16,0,0,C17,C18,C19)						
21	12	**LST**	**4.412404296**	=GSTLST(C20,0,0,C14)						
22	13	**RA**	**18.53916667**	=HMSDH(C3,C4,C5)						
23	14	**H1**	**-14.12676237**	=C21-C22						
24	15	**H**	**9.873237629**	=IF(C23<0,24+C23,C23)						

RAHA

Figure 24. Converting right ascension to local hour angle.

Converting an hour angle back to its equivalent right ascension is a very similar process. Continuing the example, what was the right ascension of the star whose local hour angle was 9h 52m 23.66s on local calendar date 22 April 1980 when observed in time zone −4 h from longitude 64° W at local time 14h 36m 51.67s, when daylight saving was not in operation?

Method		*Example*		
1.	Find the UT and Greenwich calendar date corresponding to the local time and date (§9).	UT	=	18.614 353
		GDay	=	22
		GMonth	=	4
		GYear	=	1980
2.	Find the corresponding Greenwich sidereal time (§12)...	GST	=	8.679 071
3.	...hence find the local sidereal time (§14).	LST	=	4.412 404
4.	Express the hour angle H in decimal hours (§7).	H	=	9.873 239 hours
5.	Subtract this from the LST. If the answer is	α_1	=	−5.460 835
	negative, add 24. This is the right ascension.	α	=	18.539 165 hours
6.	Convert to hours, minutes and seconds form (§8).	α	=	**18h 32m 21s**

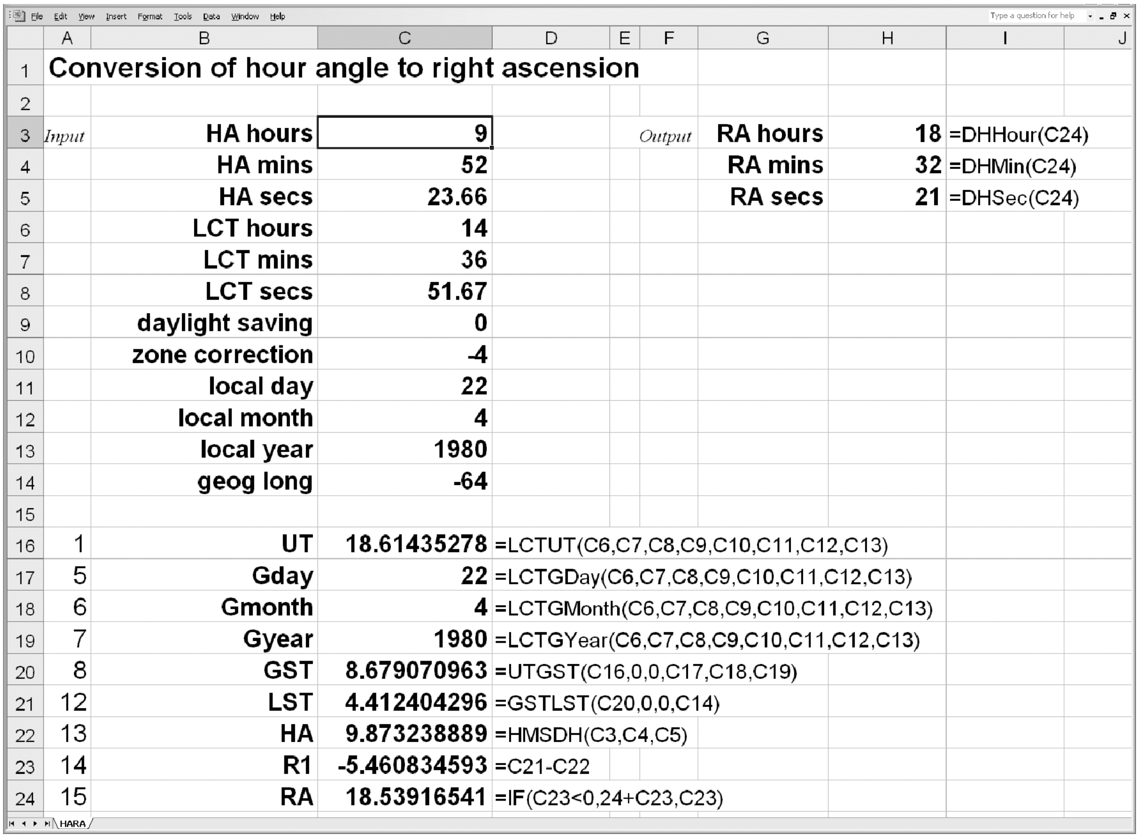
The spreadsheet for making this calculation is shown in Figure 25 and is called HARA (for Hour Angle to Right Ascension conversion). We have also provided a spreadsheet function with the same name which takes 12 arguments as follows:

HARA(HH,HM,HS,LH,LM,LS,DS,ZC,D,M,Y,GL).

The first three, HH, HM, HS take the hours, minutes and seconds of the hour angle, the next three, LH, LM, LS take the hours, minutes and seconds of the local civil time, then DS and ZC take the daylight saving and zone correction values in hours, D, M, Y take the day, month and year of the local calendar date, and finally GL takes the observer's geographical longitude in degrees, west negative. You can use this function to replace rows 16 to 24 of Figure 25 (save a copy first). Delete those rows and insert the following formulas into cells H3, H4, and H5 respectively:

```
=DHHour(HARA(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DHMin(HARA(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DHSec(HARA(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14)).
```

As usual, the arguments are references to cells in the spreadsheet which contain the relevant values. Thus the first argument, which is the hours part of the hour angle expressed as hours, minutes, seconds, points to cell C3 which actually contains that number.



	A	B	C	D	F	G	H	I
1	Conversion of hour angle to right ascension							
2								
3	*Input*	HA hours	9		*Output*	RA hours	18	=DHHour(C24)
4		HA mins	52			RA mins	32	=DHMin(C24)
5		HA secs	23.66			RA secs	21	=DHSec(C24)
6		LCT hours	14					
7		LCT mins	36					
8		LCT secs	51.67					
9		daylight saving	0					
10		zone correction	-4					
11		local day	22					
12		local month	4					
13		local year	1980					
14		geog long	-64					
15								
16	1	UT	18.61435278	=LCTUT(C6,C7,C8,C9,C10,C11,C12,C13)				
17	5	Gday	22	=LCTGDay(C6,C7,C8,C9,C10,C11,C12,C13)				
18	6	Gmonth	4	=LCTGMonth(C6,C7,C8,C9,C10,C11,C12,C13)				
19	7	Gyear	1980	=LCTGYear(C6,C7,C8,C9,C10,C11,C12,C13)				
20	8	GST	8.679070963	=UTGST(C16,0,0,C17,C18,C19)				
21	12	LST	4.412404296	=GSTLST(C20,0,0,C14)				
22	13	HA	9.873238889	=HMSDH(C3,C4,C5)				
23	14	R1	-5.460834593	=C21-C22				
24	15	RA	18.53916541	=IF(C23<0,24+C23,C23)				

Figure 25. Converting local hour angle to right ascension.

25 Equatorial to horizon coordinate conversion

The formulas describing the relationships between hour angle, H, declination, δ, azimuth, A, and altitude, a, are:

$$\sin a = \sin\delta \sin\phi + \cos\delta \cos\phi \cos H,$$

$$\cos A = \frac{\sin\delta - \sin\phi \sin a}{\cos\phi \cos a},$$

where ϕ is the observer's geographical latitude. (The hour angle may be found from the right ascension by the method of Section 24.) These may be dealt with in the following way using the example 'what are the altitude and azimuth of a star whose hour angle is 5h 51m 44s and declination is $+23^\circ\ 13'\ 10''$?'. The observer's latitude is 52° N.

Method		*Example*		
1.	Convert hour angle to decimal hours (§7).	H	=	5.862 222 hours
2.	Multiply by 15 to convert H to degrees (§22).	H	=	87.933 333 degrees
3.	Convert δ into decimal degrees (§21).	δ	=	23.219 444 degrees
4.	Find $\sin a = \sin\delta \sin\phi + \cos\delta \cos\phi \cos H$.	$\sin a$	=	0.331 080
5.	Take inverse sin to find a.	a	=	19.334 345 degrees
6.	Find $\cos A = \frac{\sin\delta - \sin\phi \sin a}{\cos\phi \cos a}$.	$\cos A$	=	0.229 558
7.	Take inverse cos to find A'.	A'	=	76.728 973
8.	Find $\sin H$.	$\sin H$	=	0.999 350
	If negative, the true azimuth is $A = A'$.			(positive)
	If positive, the true azimuth is $A = 360 - A'$.	A	=	283.271 027 degrees
9.	Convert a and A to degrees, minutes	a	=	**19° 20′ 3.64″**
	and seconds (§21).	A	=	**283° 16′ 15.70″**

Step 8 is necessary because calculators can only return inverse trigonometrical functions correctly (inverse sin, inverse cos and inverse tan, which we will denote $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ respectively) over half the range of 0° to 360°. For example, try cos 147°. The answer is −0.8387 which reverts to 147° when you take inverse cos. But now try cos 213°. The answer is again −0.8387 which, when you take inverse cos, gives 147°. Hence, whenever the inverse is taken an ambiguity arises that has to be cleared up by another means.

As an alternative to steps 6 to 8 you can calculate

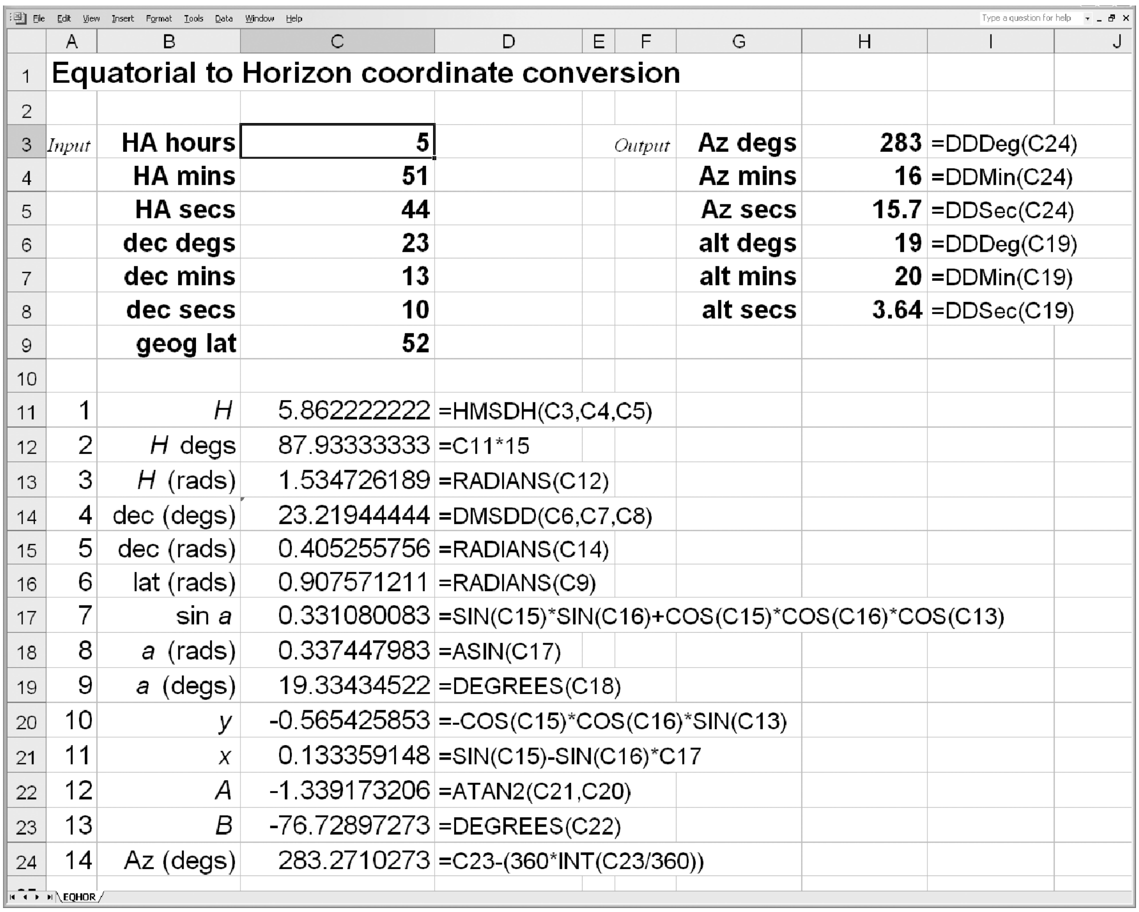
$$\tan A' = \frac{y}{x} = \frac{-\cos\delta \cos\phi \sin H}{\sin\delta - \sin\phi \sin a}.$$

The azimuth is then found in the correct quadrant by using the rules (i) if x is negative, add 180° to A', and (ii) if x is positive but y is negative, add 360° to A'.

Note that negative angles can be transformed back into the range 0° to 360° by simply adding 360. An example is −87.23 degrees which is the same as $360 - 87.23 = 272.77$ degrees.

The spreadsheet for converting from Equatorial coordinates (right ascension and declination) to Horizon coordinates is called EQHOR and is shown in Figure 26. Note that, in spreadsheet trigonometric formulas, the angles must be expressed in **radians** rather than degrees. We make extensive use of the functions RADIANS(D) and DEGREES(R) which convert, respectively, D given in degrees into radians, and R given in radians into degrees. Examples can be found in rows 13 and 19. Another trick is to use the built-in function ATAN2(x,y) to find the inverse tangent of y/x (row 22). In this case, the values of x and y are given explicitly in the two arguments, and the function returns the angle in its correct quadrant (in radians), thus overcoming the ambiguity on taking inverse tan. Some calculators also have a special key to do this. On ours, it is labelled $(y,x) \rightarrow (\theta,R)$. This is for converting the Cartesian coordinates of a point in two-dimensional space, (x,y), into its equivalent polar coordinates (R,θ). Don't worry about the details. Suffice it to say that if you use the key on the values of x and y you get the angle, θ, in its correct quadrant.

The spreadsheet functions are EqAz(H,M,S,DD,DM,DS,GP) and EqAlt(H,M,S,DD,DM,DS,GP). Both take the same seven arguments H, M, S, the hour angle expressed as hours, minutes and seconds, DD, DM, DS, the declination expressed as degrees, minutes, seconds, and GP, the observer's geographical latitude in decimal degrees (S negative). EqAz returns the azimuth, and EqAlt returns the altitude, both in decimal degrees. You can try these functions for yourself by first saving a copy of the spreadsheet EQHOR (in case



	A	B	C	D	E	F	G	H	I
1	Equatorial to Horizon coordinate conversion								
2									
3	*Input*	**HA hours**	**5**			*Output*	**Az degs**	**283**	=DDDeg(C24)
4		**HA mins**	**51**				**Az mins**	**16**	=DDMin(C24)
5		**HA secs**	**44**				**Az secs**	**15.7**	=DDSec(C24)
6		**dec degs**	**23**				**alt degs**	**19**	=DDDeg(C19)
7		**dec mins**	**13**				**alt mins**	**20**	=DDMin(C19)
8		**dec secs**	**10**				**alt secs**	**3.64**	=DDSec(C19)
9		**geog lat**	**52**						
10									
11	1	*H*	5.862222222	=HMSDH(C3,C4,C5)					
12	2	*H* degs	87.93333333	=C11*15					
13	3	*H* (rads)	1.534726189	=RADIANS(C12)					
14	4	dec (degs)	23.21944444	=DMSDD(C6,C7,C8)					
15	5	dec (rads)	0.405255756	=RADIANS(C14)					
16	6	lat (rads)	0.907571211	=RADIANS(C9)					
17	7	sin *a*	0.331080083	=SIN(C15)*SIN(C16)+COS(C15)*COS(C16)*COS(C13)					
18	8	*a* (rads)	0.337447983	=ASIN(C17)					
19	9	*a* (degs)	19.33434522	=DEGREES(C18)					
20	10	*y*	-0.565425853	=-COS(C15)*COS(C16)*SIN(C13)					
21	11	*x*	0.133359148	=SIN(C15)-SIN(C16)*C17					
22	12	*A*	-1.339173206	=ATAN2(C21,C20)					
23	13	*B*	-76.72897273	=DEGREES(C22)					
24	14	Az (degs)	283.2710273	=C23-(360*INT(C23/360))					

Figure 26. Converting equatorial to horizon coordinates.

you want it again), second deleting rows 11 to 24 inclusive, and third inserting the following formulas into cells H3 to H8:

```
=DDDeg(EqAz(C3,C4,C5,C6,C7,C8,C9))
=DDMin(EqAz(C3,C4,C5,C6,C7,C8,C9))
=DDSec(EqAz(C3,C4,C5,C6,C7,C8,C9))
=DDDeg(EqAlt(C3,C4,C5,C6,C7,C8,C9))
=DDMin(EqAlt(C3,C4,C5,C6,C7,C8,C9))
=DDSec(EqAlt(C3,C4,C5,C6,C7,C8,C9)).
```

The calculations of rows 11–24 are carried out by the BASIC programs behind the spreadsheet functions, and should return precisely the same values as the spreadsheet in Figure 26.

26 Horizon to equatorial coordinate conversion

This problem is the reverse of that of the preceding section, namely given a star's altitude, a, and azimuth, A, what are its declination, δ, and hour angle, H? The appropriate formulas are:

$$\sin\delta = \sin a \sin\phi + \cos a \cos\phi \cos A,$$

$$\cos H = \frac{\sin a - \sin\phi \sin\delta}{\cos\phi \cos\delta},$$

where ϕ is the observer's geographical latitude. Notice that these formulas are exactly the same as those given in Section 25 except that δ and H have been substituted for a and A and vice-versa. This fact is useful when writing a program for a programmable calculator since exactly the same program can be used to convert δ, H to a, A or a, A to δ, H.

Let us take the following example: a star is observed by an observer at latitude 52° N to have an altitude of 19° 20′ 03.64″ and an azimuth of 283° 16′ 15.7″. What are its hour angle and declination? If the observer is on the Greenwich meridian and the GST is 0h 24m 05s, what is the right ascension?

Method		*Example*		
1.	Convert azimuth to decimal degrees (§21).	A	=	283.271 028 degrees
2.	Convert altitude to decimal degrees (§21).	a	=	19.334 344 degrees
3.	Find $\sin\delta = \sin a \sin\phi + \cos a \cos\phi \cos A$.	$\sin\delta$	=	0.394 254
4.	Take inverse sin to find δ.	δ	=	23.219 444 degrees
5.	Find $\cos H = \frac{\sin a - \sin\phi \sin\delta}{\cos\phi \cos\delta}$.	$\cos H$	=	0.036 062
6.	Take inverse cos to find H'.	H'	=	87.933 334 degrees
7.	Find $\sin A$.	$\sin A$	=	−0.973 295
	If negative, the true hour angle is $H = H'$.			(negative)
	If positive, the true hour angle is $H = 360 - H'$.	H	=	87.933 334 degrees
8.	Convert H into hours by dividing by 15 (§22).	H	=	5.862 222 hours
9.	Convert H and δ into minutes and	H	=	**5h 51m 44s**
	seconds form (§§8 and 21).	δ	=	**23° 13′ 10″**

Again, step 7 is necessary to remove the ambiguity introduced by taking the inverse of cos. Alternatively, calculate the hour angle from

$$\tan H' = \frac{y}{x} = \frac{-\cos a \cos\phi \sin A}{(\sin a - \sin\phi \sin\delta)}.$$

H is then found in the correct quadrant by adding 180° to H' if x is negative, or 360° to H' if x is positive and y is negative.

The spreadsheet for converting Horizon to Equatorial coordinates is called HOREQ and is shown in Figure 27. Please see the notes in the previous section about converting between degrees and radians, and about using the ATAN2 function to remove the ambiguity on taking the inverse of a trigonometric quantity.

The spreadsheet functions

HorHa(AZD,AZM,AZS,ALD,ALM,ALS,GP) and
HorDec(AZD,AZM,AZS,ALD,ALM,ALS,GP)

will convert the azimuth expressed as degrees, minutes and seconds (AZD, AZM, AZS) and altitude also

	A	B	C	D	E	F	G	H	I	J
1	**Horizon to Equatorial coordinate conversion**									
2										
3	*Input*	**Az degs**	**283**			*Output*	**HA hours**	**5**	=DHHour(C24)	
4		**Az mins**	**16**				**HA mins**	**51**	=DHMin(C24)	
5		**Az secs**	**15.7**				**HA secs**	**44**	=DHSec(C24)	
6		**alt degs**	**19**				**dec degs**	**23**	=DDDeg(C18)	
7		**alt mins**	**20**				**dec mins**	**13**	=DDMin(C18)	
8		**alt secs**	**3.64**				**dec secs**	**10**	=DDSec(C18)	
9		**geog lat**	**52**							
10										
11	1	Az (degs)	283.2710278	=DMSDD(C3,C4,C5)						
12	2	alt (degs)	19.33434444	=DMSDD(C6,C7,C8)						
13	3	alt (rads)	0.337447969	=RADIANS(C12)						
14	4	lat (rads)	0.907571211	=RADIANS(C9)						
15	5	Az (rads)	4.94401211	=RADIANS(C11)						
16	6	sin dec	0.394253809	=SIN(C13)*SIN(C14)+COS(C13)*COS(C14)*COS(C15)						
17	7	dec (rads)	0.405255751	=ASIN(C16)						
18	8	dec (degs)	23.21944417	=DEGREES(C17)						
19	9	*y*	0.565425854	=-COS(C13)*COS(C14)*SIN(C15)						
20	10	*x*	0.020403829	=SIN(C13)-SIN(C14)*C16						
21	11	*A*	1.534726206	=ATAN2(C20,C19)						
22	12	*B*	87.93333428	=DEGREES(C21)						
23	13	HA (degs)	87.93333428	=C22-(360*INT(C22/360))						
24	14	HA (hours)	5.862222286	=DDDH(C23)						

HOREQ

Figure 27. Converting horizon to equatorial coordinates.

expressed as degrees, minutes and seconds (ALD, ALM, ALS) into the corresponding hour angle in decimal hours and declination in decimal degrees respectively. The last argument, GP, is the observer's geographical latitude in decimal degrees (south negative). You can try these functions for yourself by first saving a copy of the spreadsheet HOREQ (in case you want it again), second deleting rows 11 to 24 inclusive, and third inserting the following formulas into cells H3–H8:

```
=DDDeg(HorHa(C3,C4,C5,C6,C7,C8,C9))
=DDMin(HorHa(C3,C4,C5,C6,C7,C8,C9))
=DDSec(HorHa(C3,C4,C5,C6,C7,C8,C9))
=DDDeg(HorDec(C3,C4,C5,C6,C7,C8,C9))
=DDMin(HorDec(C3,C4,C5,C6,C7,C8,C9))
=DDSec(HorDec(C3,C4,C5,C6,C7,C8,C9)).
```

The second part of the problem, converting the hour angle into the right ascension, was covered in Section 24. By applying the method shown there, you should obtain the result that the right ascension is 18h 32m 21s. Looking in our star atlas (see Bibliography on page 208), we find a sixth-magnitude star in the constellation of Hercules listed near this position.

27 Ecliptic to equatorial coordinate conversion

The ecliptic longitude, λ, and the ecliptic latitude, β, may be converted into right ascension, α, and declination, δ, using the formulas:

$$\alpha = \tan^{-1}\left\{\frac{\sin\lambda\cos\varepsilon - \tan\beta\sin\varepsilon}{\cos\lambda}\right\},$$

$$\delta = \sin^{-1}\{\sin\beta\cos\varepsilon + \cos\beta\sin\varepsilon\sin\lambda\},$$

where ε is the **obliquity of the ecliptic**, the angle between the planes of the equator and the ecliptic. This angle changes slowly with time and for high accuracy the appropriate value should be used. If, for example, α and δ are referred to the standard epoch of 2000.0 (see Section 34), then ε should have its 2000.0 value. In the examples given here and in the following section, we make the calculation for a given Greenwich calendar date. Since the value of ε changes so slowly, it is often sufficient to find its value in the middle of a given year and use that in all calculations for that year. The method of calculating the mean obliquity of the ecliptic for any date is given by the equation

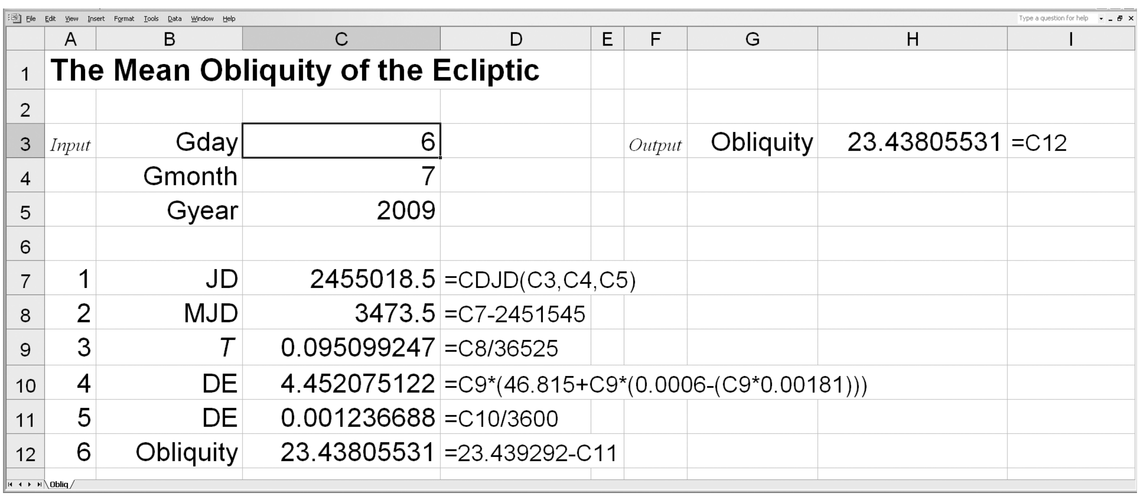
$$\varepsilon = 23^\circ\,26'\,21.45'' - 46.815''T - 0.0006''T^2 + 0.00181''T^3,$$

where T is the number of Julian centuries since epoch 2000 January 1.5 (Julian date 2 451 545.0). For example, what was the mean obliquity of the ecliptic on Greenwich calendar date 6 July 2009?

Method		*Example*		
1.	Calculate the Julian date (§4).	JD	=	2 455 018.5
2.	Subtract 2 451 545.0 (= JD for 2000 January 1.5).	MJD	=	3 473.5 days
3.	Divide by 36 525.0. The result is T.	T	=	0.095 099 247 centuries
4.	Calculate $\mathrm{DE} = 46.815T + 0.0006T^2 - 0.00181T^3$.	DE	=	4.452 075 122 arcsec
5.	Divide by 3600 to convert to degrees.	DE	=	0.001 236 688 degrees
6.	Subtract DE from 23.439 292 to find ε.	ε	=	23.438 055 31 degrees
7.	If necessary, convert to degrees, minutes and seconds (§21).	ε	=	**23° 26′ 17″**

The spreadsheet Obliq, Figure 28, will make this calculation. We have also provided the spreadsheet function Obliq(D,M,Y), which returns the mean obliquity of the ecliptic calculated for the Greenwich calendar date, D days, M months and Y years. Whenever you need the obliquity in a spreadsheet, you could therefore insert the formula =Obliq(C3,C4,C5) where, for example, the day, month and year were in the cells C3, C4, C5 respectively. For very precise calculations, you also want to make allowance for **nutation** (see Section 35) by adding the result of the function NutatObl(D,M,Y). The spreadsheet function Obliq already adds in this correction for you so you do not need to do so separately.

Having obtained the value of the obliquity of the ecliptic, we are now in a position to convert ecliptic coordinates into equatorial coordinates. Our example this time is: what were the right ascension and the declination of a planet whose ecliptic coordinates were longitude 139° 41′ 10″ and latitude 4° 52′ 31″ on 6 July 2009?



	A	B	C	D	E	F	G	H	I
1	The Mean Obliquity of the Ecliptic								
2									
3	*Input*	Gday	6			*Output*	Obliquity	23.43805531	=C12
4		Gmonth	7						
5		Gyear	2009						
6									
7	1	JD	2455018.5	=CDJD(C3,C4,C5)					
8	2	MJD	3473.5	=C7-2451545					
9	3	*T*	0.095099247	=C8/36525					
10	4	DE	4.452075122	=C9*(46.815+C9*(0.0006-(C9*0.00181)))					
11	5	DE	0.001236688	=C10/3600					
12	6	Obliquity	23.43805531	=23.439292-C11					

Figure 28. Calculating the mean obliquity of the ecliptic.

Method		*Example*		
1.	Convert λ and β into decimal degrees (§21).	λ	=	139.686 111 degrees
		β	=	4.875 278 degrees
2.	Find $\sin\delta = \sin\beta\cos\varepsilon + \cos\beta\sin\varepsilon\sin\lambda$ (with ε = 23.438 055 degrees[a]).	$\sin\delta$	=	0.334 383
3.	Take inverse sin to find δ in decimal degrees.	δ	=	19.535 003 degrees
4.	Find $y = \sin\lambda\cos\varepsilon - \tan\beta\sin\varepsilon$.	y	=	0.559 666
5.	Find $x = \cos\lambda$.	x	=	−0.762 512
6.	Find $\alpha' = \tan^{-1}\left(\frac{y}{x}\right)$.	α'	=	−36.277 799 degrees
7.	We have to remove the ambiguity which arises from taking the inverse tan. The rule is that α should lie in the quadrant indicated by the signs of x and y in Figure 29. Add or subtract 180 or 360 to α' to bring it into the correct quadrant, unless it is already there, in which case $\alpha = \alpha'$.			x negative y positive
		$\therefore \alpha$	=	$\alpha' + 180.0$
			=	143.722 173 degrees
8.	Convert α to hours by dividing by 15 (§22).	α	=	9.581 478 hours
9.	Convert α and δ to minutes and seconds form (§§21 and 8).	α	=	**9h 34m 53.32s**
		δ	=	**19° 32′ 6.01″**

[a]Includes the nutation correction.

The spreadsheet for carrying out this calculation is shown in Figure 30 and is called ECEQ, short for Ecliptic coordinates to Equatorial coordinates conversion. The corresponding spreadsheet functions are EcRA and EcDec, returning the right ascension and declination in decimal degrees respectively. Each function takes the same nine arguments which are the ecliptic longitude as degrees, minutes, seconds, the ecliptic latitude as degrees, minutes, seconds, and the Greenwich calendar date as day, month, year. Thus you could replace the calculation part of the spreadsheet shown in Figure 30 using this function as follows. Save a copy of the spreadsheet, then delete rows 13 to 27 inclusive, and finally insert the following formulas into cells H3 to H8 respectively:

```
=DHHour(DDDH(EcRA(C3,C4,C5,C6,C7,C8,C9,C10,C11)))
=DHMin(DDDH(EcRA(C3,C4,C5,C6,C7,C8,C9,C10,C11)))
=DHSec(DDDH(EcRA(C3,C4,C5,C6,C7,C8,C9,C10,C11)))
=DDDeg(EcDec(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDMin(EcDec(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDSec(EcDec(C3,C4,C5,C6,C7,C8,C9,C10,C11)).
```

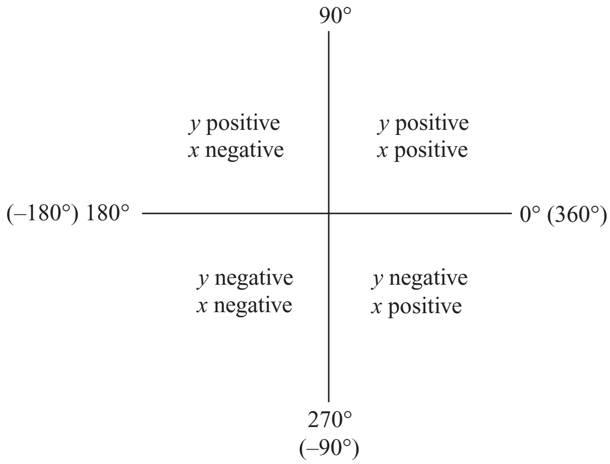


Figure 29. Removing the ambiguity on taking $\tan^{-1}(y/x)$.

	A	B	C	D	E	F	G	H	I	J
1	**Ecliptic to Equatorial Coordinate Conversion**									
2										
3	*Input*	**ecl long deg**	**139**			*Output*	**RA hour**	**9**	=DHHour(C27)	
4		**ecl long min**	**41**				**RA min**	**34**	=DHMin(C27)	
5		**ecl long sec**	**10**				**RA sec**	**53.4**	=DHSec(C27)	
6		**ecl lat deg**	**4**				**dec deg**	**19**	=DDDeg(C21)	
7		**ecl lat min**	**52**				**dec min**	**32**	=DDMin(C21)	
8		**ecl lat sec**	**31**				**dec sec**	**8.52**	=DDSec(C21)	
9		**G day**	**6**							
10		**G month**	**7**							
11		**G year**	**2009**							
12										
13	1	eclon (deg)	139.6861111	=DMSDD(C3,C4,C5)						
14	2	eclat (deg)	4.875277778	=DMSDD(C6,C7,C8)						
15	3	eclon (rad)	2.437982558	=RADIANS(C13)						
16	4	eclat (rad)	0.085089649	=RADIANS(C14)						
17	5	obliq (deg)	23.43923176	=Obliq(C9,C10,C11)						
18	6	obliq (rad)	0.409091768	=RADIANS(C17)						
19	7	sin dec	0.334394125	=SIN(C16)*COS(C18)+COS(C16)*SIN(C18)*SIN(C15)						
20	8	dec (rad)	0.340962273	=ASIN(C19)						
21	9	dec (deg)	19.53569924	=DEGREES(C20)						
22	10	*y*	0.559659329	=SIN(C15)*COS(C18)-TAN(C16)*SIN(C18)						
23	11	*x*	-0.762511521	=COS(C15)						
24	12	RA (rad)	2.508430994	=ATAN2(C23,C22)						
25	13	RA (deg)	143.7225092	=DEGREES(C24)						
26	14	RA (deg)	143.7225092	=C25-360*INT(C25/360)						
27	15	RA (hours)	9.58150061	=DDDH(C26)						

ECEQ

Figure 30. Converting from ecliptic to equatorial coordinates.

28 Equatorial to ecliptic coordinate conversion

The reverse problem of the previous section is to find the celestial longitude and latitude, λ and β, given the right ascension and declination, α and δ. The formulas are:

$$\lambda = \tan^{-1}\left\{\frac{\sin\alpha\cos\varepsilon + \tan\delta\sin\varepsilon}{\cos\alpha}\right\},$$

$$\beta = \sin^{-1}\{\sin\delta\cos\varepsilon - \cos\delta\sin\varepsilon\sin\alpha\},$$

where ε is the obliquity of the ecliptic (see Section 27). These formulas are very nearly identical to those of the previous section with λ, β in place of α, δ and vice-versa; the symmetry is not quite complete, however, as the sign appearing in each formula is reversed.

Consider the example: what are the ecliptic coordinates of a planet whose right ascension and declination are given as α = 9h 34m 53.32s and $\delta = 19^\circ\ 32'\ 6.01''$ when the Greenwich calendar date is 6 July 2009?

	Method	*Example*		
1.	Convert α and δ into decimal degrees (§§21, 22 and 7).	δ α	= =	19.535 003 degrees 143.722 167 degrees
2.	Find $\sin\beta = \sin\delta\cos\varepsilon - \cos\delta\sin\varepsilon\sin\alpha$ (with, in 2009, ε = 23.438 055 degrees[a]).	$\sin\beta$	=	0.084 987
3.	Take inverse sin to find β in decimal degrees.	β	=	4.875 276 degrees
4.	Find $y = \sin\alpha\cos\varepsilon + \tan\delta\sin\varepsilon$.	y	=	0.684 007
5.	Find $x = \cos\alpha$.	x	=	−0.806 157
6.	Calculate $\lambda' = \tan^{-1}\left(\frac{y}{x}\right)$.	λ'	=	−40.313 894 degrees
7.	We have to remove the ambiguity which arises from taking the inverse tan. The rule is that λ should lie in the quadrant indicated by the signs of x and y in Figure 29. Add or subtract 180 or 360 to bring it into the correct quadrant, unless it is already there, in which case, $\lambda = \lambda'$.	 $\therefore\ \lambda$	 = =	x negative y positive $\lambda' + 180$ 139.686 106 degrees
8.	Convert λ and β to minutes and seconds form (§§21 and 8).	λ β	= =	**139° 41′ 9.98″** **4° 52′ 30.99″**

[a]Includes the nutation correction.

Figure 31 shows the spreadsheet for this conversion, called EQEC (Equatorial to Ecliptic conversion). The corresponding spreadsheet functions are EqElong and EqElat returning, respectively, the ecliptic longitude and the ecliptic latitude, both in decimal degrees. They each take the same nine arguments, namely the right ascension in hour, minutes, seconds, the declination in degrees, minutes, seconds, and the Greenwich calendar date as day, month, year. You could therefore delete rows 13 to 26 inclusive of the spreadsheet shown in Figure 31 (having saved a copy), and insert into cells H3 to H8 respectively the following formulas:

```
=DDDeg(EqElong(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDMin(EqElong(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDSec(EqElong(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDDeg(EqElat(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDMin(EqElat(C3,C4,C5,C6,C7,C8,C9,C10,C11))
=DDSec(EqElat(C3,C4,C5,C6,C7,C8,C9,C10,C11)).
```

	A	B	C	D	E	F	G	H	I
1	**Equatorial to Ecliptic Coordinate Conversion**								
2									
3	*Input*	**RA (hour)**	**9**			*Output*	**ecl long (deg)**	**139**	=DDDeg(C26)
4		**RA (min)**	**34**				**ecl long (min)**	**41**	=DDMin(C26)
5		**RA (sec)**	**53.4**				**ecl long(sec)**	**9.97**	=DDSec(C26)
6		**dec (deg)**	**19**				**ecl lat (deg)**	**4**	=DDDeg(C21)
7		**dec (min)**	**32**				**ecl lat (min)**	**52**	=DDMin(C21)
8		**dec (sec)**	**8.52**				**ecl lat (sec)**	**30.99**	=DDSec(C21)
9		**Gday**	**6**						
10		**Gmonth**	**7**						
11		**Gyear**	**2009**						
12									
13	1	RA (deg)	143.7225	=DHDD(HMSDH(C3,C4,C5))					
14	2	dec (deg)	19.5357	=DMSDD(C6,C7,C8)					
15	3	RA (rad)	2.508430834	=RADIANS(C13)					
16	4	dec (rad)	0.340962287	=RADIANS(C14)					
17	5	obliq (deg)	23.43923176	=Obliq(C9,C10,C11)					
18	6	obliq (rad)	0.409091768	=RADIANS(C17)					
19	7	sin ecl lat	0.084986972	=SIN(C16)*COS(C18)-COS(C16)*SIN(C18)*SIN(C15)					
20	8	ecl lat (rad)	0.085089613	=ASIN(C19)					
21	9	ecl lat (deg)	4.875275723	=DEGREES(C20)					
22	10	*y*	0.684010234	=SIN(C15)*COS(C18)+TAN(C16)*SIN(C18)					
23	11	*x*	-0.806160703	=COS(C15)					
24	12	ecl long (rad)	2.437982411	=ATAN2(C23,C22)					
25	13	ecl long (deg)	139.6861027	=DEGREES(C24)					
26	14	ecl long (deg)	139.6861027	=C25-360*INT(C25/360)					

EQEC

Figure 31. Converting from equatorial to ecliptic coordinates.

29 Equatorial to galactic coordinate conversion

Occasionally we need to know the position of a star in relation to the rest of the stars in our Galaxy and to do this we can use the galactic coordinate system. The conversion formulas are:

$$b = \sin^{-1}\{\cos\delta\cos(27.4^\circ)\cos(\alpha - 192.25^\circ) + \sin\delta\sin(27.4^\circ)\},$$

$$l = \tan^{-1}\left\{\frac{\sin\delta - \sin b\sin(27.4^\circ)}{\cos\delta\sin(\alpha - 192.25^\circ)\cos(27.4^\circ)}\right\} + 33^\circ.$$

The numbers come from the following facts about our Galaxy: north galactic pole coordinates are $\alpha = 192^\circ\ 15'$, $\delta = +27^\circ\ 24'$; ascending node of the galactic plane on equator $l = 33^\circ$.

The example is: what are the galactic coordinates of a star whose right ascension and declination are $\alpha = 10\text{h}\,21\text{m}\,00\text{s}$ and $\delta = 10^\circ\ 03'\ 11''$?

Method		*Example*		
1.	Convert α, δ into decimal form (§§21 and 7).	δ	=	10.053 056 degrees
		α	=	10.350 000 hours
2.	Convert α into degrees by multiplying by 15 (§22).	α	=	155.250 000 degrees
3.	Find $\sin b = \cos\delta\cos(27.4) \times \cos(\alpha - 192.25) + \sin\delta\sin(27.4)$.	$\sin b$	=	0.778 487
4.	Take inverse sin to find b in degrees.	b	=	51.122 268 degrees
5.	Find $y = \sin\delta - \sin b \sin(27.4)$ and note its sign.	y	=	$-0.183\,700$ (negative)
6.	Find $x = \cos\delta\sin(\alpha - 192.25) \times \cos(27.4)$ and note its sign.	x	=	$-0.526\,097$ (negative)
7.	Divide y by x.	y/x	=	0.349 174
8.	Take inverse tan. Now we have to remove the ambiguity which arises from taking the inverse tan. To do so, look at Figure 29 and add or subtract 180 or 360 to bring the result into the correct quadrant, unless it is already in the correct quadrant.	$\tan^{-1}\left(\frac{y}{x}\right)$	=	19.247 881 degrees
				From Fig. 29:
			+	180.0
9.	Add 33 to get l.		+	33.0
		l	=	232.247 881 degrees
10.	Convert l and b into minutes and seconds form (§21).	l	=	**232° 14′ 52″**
		b	=	**51° 07′ 20″**

Figure 32 shows the spreadsheet, named EQGAL (Equatorial to Galactic conversion), which will carry out this conversion. We have also provided spreadsheet functions EqGlong and EqGlat which return the galactic longitude and latitude respectively, both in decimal degrees. Each function takes the same six arguments, namely the right ascension in hours, minutes, seconds, and the declination in degrees, minutes, seconds. The spreadsheet in Figure 32 could be modified to use these functions instead of the calculation part. To do this, save a copy of the spreadsheet, delete rows 10 to 20, then insert the following formulas into cells H3 to H8 inclusive:

```
=DDDeg(EqGlong(C3,C4,C5,C6,C7,C8))
=DDMin(EqGlong(C3,C4,C5,C6,C7,C8))
=DDSec(EqGlong(C3,C4,C5,C6,C7,C8))
=DDDeg(EqGlat(C3,C4,C5,C6,C7,C8))
=DDMin(EqGlat(C3,C4,C5,C6,C7,C8))
=DDSec(EqGlat(C3,C4,C5,C6,C7,C8)).
```

	A	B	C	D	E	F	G	H	I	J
1	**Equatorial to Galactic Coordinate Conversion**									
2										
3	*Input*	**RA (hour)**	**10**			*Output*	**Gal long (deg)**	**232**	=DDDeg(C20)	
4		**RA (min)**	**21**				**Gal long (min)**	**14**	=DDMin(C20)	
5		**RA (sec)**	**0**				**Gal long (sec)**	**52.38**	=DDSec(C20)	
6		**dec (deg)**	**10**				**Gal lat (deg)**	**51**	=DDDeg(C16)	
7		**dec (min)**	**3**				**Gal lat (min)**	**7**	=DDMin(C16)	
8		**dec (sec)**	**11**				**Gal lat (sec)**	**20.16**	=DDSec(C16)	
9										
10	1	RA (deg)	155.25	=DHDD(HMSDH(C3,C4,C5))						
11	2	dec (deg)	10.05305556	=DMSDD(C6,C7,C8)						
12	3	RA (rad)	2.709623664	=RADIANS(C10)						
13	4	dec (rad)	0.175458919	=RADIANS(C11)						
14	5	sin *b*	0.778487145	=COS(C13)*COS(RADIANS(27.4))*COS(C12-RADIANS(192.25))+SIN(C13)*SIN(RADIANS(27.4))						
15	6	*b* (radians)	0.892251894	=ASIN(C14)						
16	7	*b* (deg)	51.1222678	=DEGREES(C15)						
17	8	*y*	-0.183699587	=SIN(C13)-C14*SIN(RADIANS(27.4))						
18	9	*x*	-0.52609727	=COS(C13)*SIN(C12-RADIANS(192.25))*COS(RADIANS(27.4))						
19	10	long (deg)	-127.7521165	=DEGREES(ATAN2(C18,C17))+33						
20	11	long (deg)	232.2478835	=C19-360*INT(C19/360)						

EQGAL

Figure 32. Converting from equatorial to galactic coordinates.

30 Galactic to equatorial coordinate conversion

Given the galactic coordinates, l and b, of a star, what are the corresponding equatorial coordinates, α and δ? To answer this question we need the conversion formulas:

$$\delta = \sin^{-1}\{\cos b \cos(27.4^\circ)\sin(l-33^\circ)+\sin b \sin(27.4^\circ)\},$$

$$\alpha = \tan^{-1}\left\{\frac{\cos b \cos(l-33^\circ)}{\sin b \cos(27.4^\circ)-\cos b \sin(27.4^\circ)\sin(l-33^\circ)}\right\}+192.25^\circ,$$

where both α and δ are expressed in degrees. As an example we shall find the right ascension and declination of the star whose galactic coordinates are $l = 232^\circ\ 14'\ 52''$ and $b = 51^\circ\ 07'\ 20''$.

	Method	*Example*		
1.	Convert l and b into decimal form (§21).	b	=	51.122 222 degrees
		l	=	232.247 778 degrees
2.	Find $\sin\delta = \cos b \cos(27.4) \times \sin(l-33) + \sin b \sin(27.4)$.	$\sin\delta$	=	0.174 560
3.	Take inverse sin to find δ in degrees.	δ	=	10.053 087 degrees
4.	Find $y = \cos b \cos(l-33)$ and note its sign.	y	=	$-0.592\,576$ (negative)
5.	Find $x = \sin b \cos(27.4) - \cos b \sin(27.4) \sin(l-33)$ and note its sign.	x	=	0.786 373 (positive)
6.	Divide y by x.	y/x	=	$-0.753\,556$
7.	Take inverse tan. We have to remove the ambiguity which arises from taking the inverse tan. To do so, look at Figure 29 and add or subtract 180 or 360 to bring the result into the correct quadrant, unless it is already there.	$\tan^{-1}\left(\frac{y}{x}\right)$	=	$-37.000\,075$ (already in correct quadrant)
8.	Add 192.25 to find α in degrees.		+	192.25
		α	=	155.249 925 degrees
9.	Divide by 15 to find α in hours (§22).	α	=	10.349 995 hours
10.	Convert α and δ to minutes and seconds form (§§21 and 8).	α	=	**10h 21m 00s**
		δ	=	**10° 03′ 11″**

The spreadsheet, GALEQ (Galactic to Equatorial conversion), for this calculation is shown in Figure 33. We have also provided the spreadsheet functions GalRA and GalDec which return the right ascension in decimal degrees and the declination in decimal degrees respectively. These functions each take the same six arguments, being the galactic longitude in degrees, minutes and seconds, and the galactic latitude in degrees, minutes and seconds. If you wish, you can delete rows 10 to 21 of the spreadsheet shown in Figure 33 and insert the following formulas in cells H3 to H8 respectively.

```
=DHHour(DDDH(GalRA(C3,C4,C5,C6,C7,C8)))
=DHMin(DDDH(GalRA(C3,C4,C5,C6,C7,C8)))
=DHSec(DDDH(GalRA(C3,C4,C5,C6,C7,C8)))
=DDDeg(GalDec(C3,C4,C5,C6,C7,C8))
=DDMin(GalDec(C3,C4,C5,C6,C7,C8))
=DDSec(GalDec(C3,C4,C5,C6,C7,C8))
```

Don't forget to save a copy of the original spreadsheet first.

	A	B	C	D	E	F	G	H	I	J
1	**Galactic to Equatorial Coordinate Conversion**									
2										
3	*Input*	**Gal long (deg)**	**232**			*Output*	**RA (hour)**	**10**	=DHHour(C21)	
4		**Gal long (min)**	**14**				**RA (min)**	**21**	=DHMin(C21)	
5		**Gal long (sec)**	**52.38**				**RA (sec)**	**0**	=DHSec(C21)	
6		**Gal lat (deg)**	**51**				**dec (deg)**	**10**	=DDDeg(C16)	
7		**Gal lat (min)**	**7**				**dec (min)**	**3**	=DDMin(C16)	
8		**Gal lat (sec)**	**20.16**				**dec (sec)**	**11**	=DDSec(C16)	
9										
10	1	Glong (deg)	232.2478833	=DMSDD(C3,C4,C5)						
11	2	Glat (deg)	51.12226667	=DMSDD(C6,C7,C8)						
12	3	Glong (rad)	4.053490245	=RADIANS(C10)						
13	4	Glat (rad)	0.892251874	=RADIANS(C11)						
14	5	sin dec	0.17456002	=COS(C13)*COS(RADIANS(27.4))*SIN(C12-RADIANS(33))+SIN(C13)*SIN(RADIANS(27.4))						
15	6	dec (radians)	0.17545891	=ASIN(C14)						
16	7	dec (deg)	10.05305504	=DEGREES(C15)						
17	8	*y*	-0.592575092	=COS(C13)*COS(C12-RADIANS(33))						
18	9	*x*	0.786373677	=SIN(C13)*COS(RADIANS(27.4))-COS(C13)*SIN(RADIANS(27.4))*SIN(C12-RADIANS(33))						
19	10	RA (deg)	155.249999	=DEGREES(ATAN2(C18,C17))+192.25						
20	11	RA (deg)	155.249999	=C19-360*INT(C19/360)						
21	12	RA (hours)	10.34999993	=DDDH(C20)						

GALEQ

Figure 33. Converting from galactic to equatorial coordinates.

31 Generalised coordinate transformations

The methods described in Sections 24 to 30 for converting between one coordinate system and another are quite satisfactory for normal use where you wish to make a single calculation. However, if you have a computer, or a programmable calculator that can handle matrices, you may be interested in a more general method of converting between one system and another. You can then write a single program that converts between any two systems, for example directly from galactic to horizon coordinates, or from horizon to ecliptic coordinates. The method makes use of **matrices**, ordered sets of numbers set out in rows and columns. The method of manipulation of these numbers by the computer is always the same. Conversion between different coordinate systems is achieved merely by changing the numbers in the matrices.

The matrices which we shall have to deal with are of size 3×3, that is they consist of nine numbers set out in three rows of three like this:

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix},$$

where each of the letters represents a number. We can specify the whole matrix by an upper-case letter in a bold sans-serif typeface. Let $\mathsf{\mathbf{A}}$ represent the above matrix. For example, if b, c, d, f, g and h are all zero, and a, e, i are all 1, then

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The coordinates of the point that we wish to convert are specified by means of a **column vector**. This is really just a simple case of a matrix which has three rows with a single number in each row. If x, y and z are the **elements** of the vector, we can write it as

$$\boldsymbol{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix},$$

where the bold-face, italic lower-case letter $\boldsymbol{v}$ represents the vector. Our program simply has to put the correct numbers into $\mathbf{A}$ and $\boldsymbol{v}$, and then multiply them together to form a new column vector $\boldsymbol{w}$. We can then extract the new coordinates from $\boldsymbol{w}$. In symbols we can write this as

$$\boldsymbol{w} = \mathbf{A} \cdot \boldsymbol{v}.$$

The multiplication of $\boldsymbol{v}$ by $\mathbf{A}$ follows a strict set of rules. Let the elements of $\boldsymbol{w}$ be m, n and p. Then we have

$$\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

where $m = ax + by + cz$, $n = dx + ey + fz$ and $p = gx + hy + iz$. You can remember the rules as follows. To find the value of the element of the resultant column vector (e.g. n), point with a finger of your left hand to the first element of the corresponding row in the matrix (in this case d) and point with a finger of your right hand to the first element of the column vector (x). Multiply the elements together and add up the products as you move your left hand across the row, and your right hand down the column. All this sounds a bit complicated, but you'll soon get the hang of it, and remember that the computer or your calculator will be doing the manipulating in any case.

Matrices for conversion between (H, δ) and (A, a), (α, δ) and (H, δ), (α, δ) and (λ, β) and between (l, b) and (α, δ) are all given in Table 5. If you wish to make transformations between other combinations of coordinates, say (A, a) and (λ, β), either you can do so by repeated operations using the given matrices, or you can form a new matrix first, and then use it. For example, let us convert from ecliptic coordinates (λ, β), represented by $\boldsymbol{v}$, to horizon coordinates (A, a), represented by $\boldsymbol{w}$. This can be written as

$$\boldsymbol{w} = \mathbf{A} \cdot \mathbf{B} \cdot \mathbf{C}' \cdot \boldsymbol{v}.$$

You would carry out the operations as follows. First multiply $\mathbf{C}'$ and $\boldsymbol{v}$ to form a new column vector $\boldsymbol{s}$. Then multiply $\mathbf{B}$ and $\boldsymbol{s}$ to form a second column vector $\boldsymbol{r}$. Finally, multiply $\mathbf{A}$ and $\boldsymbol{r}$ to find $\boldsymbol{w}$. Note that the order in which you carry out the operations is important. $\mathbf{A} \cdot \mathbf{B}$ is not necessarily the same as $\mathbf{B} \cdot \mathbf{A}$.

The alternative method of proceeding is first to make a new matrix $\mathbf{E}$ that is the product of $\mathbf{A} \cdot \mathbf{B} \cdot \mathbf{C}'$, and then use it in a single stage of calculation:

$$\boldsymbol{w} = \mathbf{E} \cdot \boldsymbol{v}.$$

The rules for multiplying two matrices are similar to those for multiplying a matrix and a column vector. Let the elements of **A** be a, b, c, d, e, f, g, h, i as before, and of **B**, j, k, l, m, n, o, p, q and r. Then we have

$$\mathbf{F} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \begin{pmatrix} j & k & l \\ m & n & o \\ p & q & r \end{pmatrix}.$$

Think of **F** as being made up of three column vectors. Then the first column vector of **F** is the product of **A** and the first column of **B**, and so on like this:

$$\mathbf{F} = \begin{pmatrix} (aj+bm+cp) & (ak+bn+cq) & (al+bo+cr) \\ (dj+em+fp) & (dk+en+fq) & (dl+eo+fr) \\ (gj+hm+ip) & (gk+hn+iq) & (gl+ho+ir) \end{pmatrix}.$$

To find **E** you would need to make two matrix multiplications of this sort.

The other things which we need to know in order to perform all this magic are, first, how to convert a given pair of coordinates (μ, ν) into the column vector $\boldsymbol{v}$. Let (μ, ν) represent any pair, e.g. (A, a) or (l, b), etc. Then, having done the matrix operations to find $\boldsymbol{w}$, how do we convert back to the new coordinates (θ, ψ)? The procedure is quite simple and is as follows. If (μ, ν) represent the coordinates to be transformed, then the column vector $\boldsymbol{v}$ is given by

$$\boldsymbol{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos(\mu)\cos(\nu) \\ \sin(\mu)\cos(\nu) \\ \sin(\nu) \end{pmatrix}.$$

Having found $\boldsymbol{w}$, the new coordinates (θ, ψ) are extracted from its elements m, n and p using the formulas

$$\boldsymbol{w} = \begin{pmatrix} m \\ n \\ p \end{pmatrix}; \quad \theta = \tan^{-1}\left(\frac{n}{m}\right) \text{ and } \psi = \sin^{-1}(p).$$

Let us clarify the methods by means of an example: what are the azimuth (A) and altitude (a) of a planet whose ecliptic coordinates are longitude $97^\circ\ 38'\ 17.228''$ and latitude $-17^\circ\ 51'\ 28.688''$ for an observer at geographical latitude $52^\circ\ 10'\ 31.0''$ at local sidereal time 5h 9m 21.103s? We shall assume that the obliquity of the ecliptic is $23^\circ\ 26'\ 46.45''$. We do this first by successive matrix multiplications, and then repeat the calculation using one matrix formed from several others.

$$\mathbf{A}: \quad \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{H,\delta} = \begin{pmatrix} -\sin(\phi) & 0 & \cos(\phi) \\ 0 & -1 & 0 \\ \cos(\phi) & 0 & \sin(\phi) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{A,a}$$

$$\mathbf{A}: \quad \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{A,a} = \begin{pmatrix} -\sin(\phi) & 0 & \cos(\phi) \\ 0 & -1 & 0 \\ \cos(\phi) & 0 & \sin(\phi) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{H,\delta}$$

$$\mathbf{B}: \quad \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{H,\delta} = \begin{pmatrix} \cos(\mathrm{ST}) & \sin(\mathrm{ST}) & 0 \\ \sin(\mathrm{ST}) & -\cos(\mathrm{ST}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha,\delta}$$

$$\mathbf{B}: \quad \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{\alpha,\delta} = \begin{pmatrix} \cos(\mathrm{ST}) & \sin(\mathrm{ST}) & 0 \\ \sin(\mathrm{ST}) & -\cos(\mathrm{ST}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{H,\delta}$$

$$\mathbf{C}: \quad \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{\lambda,\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon) & \sin(\varepsilon) \\ 0 & -\sin(\varepsilon) & \cos(\varepsilon) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha,\delta}$$

$$\mathbf{C}': \quad \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{\alpha,\delta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon) & -\sin(\varepsilon) \\ 0 & \sin(\varepsilon) & \cos(\varepsilon) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\lambda,\beta}$$

where ϕ is the geographical latitude, ST is the local sidereal time (expressed in radians or degrees) and ε is the obliquity of the ecliptic. The matrices for converting between galactic and equatorial coordinates are constant and best expressed by decimal numbers as follows:

$$\mathbf{D}: \quad \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{l,b} = \begin{pmatrix} -0.0669887 & -0.8727558 & -0.4835389 \\ 0.4927285 & -0.4503470 & 0.7445846 \\ -0.8676008 & -0.1883746 & 0.4601998 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha,\delta}$$

$$\mathbf{D}': \quad \begin{pmatrix} m \\ n \\ p \end{pmatrix}_{\alpha,\delta} = \begin{pmatrix} -0.0669887 & 0.8727558 & -0.4835389 \\ -0.4927285 & -0.4503470 & -0.7445846 \\ -0.8676008 & 0.1883746 & 0.4601998 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{l,b}$$

The column vector x, y, z is formed from the coordinates (μ, ν) as follows:

$$\boldsymbol{\nu}: \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mu,\nu} = \begin{pmatrix} \cos(\mu)\cos(\nu) \\ \sin(\mu)\cos(\nu) \\ \sin(\nu) \end{pmatrix}.$$

The new coordinates (θ, ψ) are extracted from the column vector m, n, p by means of the equations:

$$\theta = \tan^{-1}\left(\frac{n}{m}\right); \quad \psi = \sin^{-1}(p).$$

Table 5. Matrix conversions.

First method	*Example*
1. Convert λ and β into decimal form (§§7 and 21).	$\lambda = 97.638\,119$ degrees $\beta = -17.857\,969$ degrees
2. Form the column vector $\boldsymbol{v}$ using the equations given in Table 5.	$\boldsymbol{v} = \begin{pmatrix} -0.126\,512 \\ 0.943\,374 \\ -0.306\,658 \end{pmatrix}$
3. Convert ε to degrees and construct the matrix $\mathbf{C}'$ (see Table 5).	$\varepsilon = 23.446\,236$ degrees $\mathbf{C}' = \begin{pmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 0.917\,434 & -0.397\,888 \\ 0.0 & 0.397\,888 & 0.917\,434 \end{pmatrix}$
4. Multiply $\mathbf{C}'$ and $\boldsymbol{v}$ to form column vector $\boldsymbol{s}$.	$\boldsymbol{s} = \begin{pmatrix} -0.126\,512 \\ 0.987\,499 \\ 0.094\,019 \end{pmatrix}$
5. Convert the local sidereal time into hours, and then into degrees by multiplying by 15. Construct the matrix $\mathbf{B}$ (see Table 5).	ST $= 5.155\,862$ hours ST $= 77.337\,930$ degrees $\mathbf{B} = \begin{pmatrix} 0.219\,200 & 0.975\,680 & 0.0 \\ 0.975\,680 & -0.219\,200 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{pmatrix}$
6. Multiply $\mathbf{B}$ and $\boldsymbol{s}$ to form column vector $\boldsymbol{r}$.	$\boldsymbol{r} = \begin{pmatrix} 0.935\,752 \\ -0.339\,895 \\ 0.094\,019 \end{pmatrix}$
7. Convert the geographical latitude to degrees and construct the matrix $\mathbf{A}$ (see Table 5).	$\phi = 52.175\,278$ degrees $\mathbf{A} = \begin{pmatrix} -0.789\,890 & 0.0 & 0.613\,248 \\ 0.0 & -1.0 & 0.0 \\ 0.613\,248 & 0.0 & 0.789\,890 \end{pmatrix}$
8. Multiply $\mathbf{A}$ and $\boldsymbol{r}$ to find column vector $\boldsymbol{w}$.	$\boldsymbol{w} = \begin{pmatrix} -0.681\,485 \\ 0.339\,895 \\ 0.648\,113 \end{pmatrix}$
9. Find the azimuth, A, and altitude, a, from the elements of $\boldsymbol{w}$ (see Table 5). Remove the ambiguity on taking inverse tan: look at Figure 29 and add or subtract 180 or 360 to bring the result into the correct quadrant.	$\tan^{-1}\left(\frac{n}{m}\right) = -26.508\,056$ degrees $+ \; 180.0$ (from Fig. 29) $\therefore A = 153.491\,944$ degrees $a = \sin^{-1}(p)$ $= 40.399\,444$ degrees
10. Convert A and a into degrees, minutes and seconds form (§21).	$A = \mathbf{153°\,29'\,31''}$ $a = \mathbf{40°\,23'\,58''}$

 https://doi.org/10.1017/CBO9780511861161.005

	Second method	*Example*
1.	Convert λ and β into decimal form (§§7 and 21).	$\lambda = 97.638\,119$ degrees $\beta = -17.857\,969$ degrees
2.	Form the column vector $\boldsymbol{v}$ using the equations given in Table 5.	$\boldsymbol{v} = \begin{pmatrix} -0.126\,512 \\ 0.943\,374 \\ -0.306\,658 \end{pmatrix}$
3.	Convert ε to degrees and construct the matrix $\mathbf{C}'$ (see Table 5).	$\varepsilon = 23.446\,236$ degrees $\mathbf{C}' = \begin{pmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 0.917\,434 & -0.397\,888 \\ 0.0 & 0.397\,888 & 0.917\,434 \end{pmatrix}$
4.	Convert the local sidereal time into hours, and then into degrees by multiplying by 15. Construct the matrix $\mathbf{B}$ (see Table 5).	$\mathrm{ST} = 5.155\,862$ hours $\mathrm{ST} = 77.337\,930$ degrees $\mathbf{B} = \begin{pmatrix} 0.219\,200 & 0.975\,680 & 0.0 \\ 0.975\,680 & -0.219\,200 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{pmatrix}$
5.	Multiply $\mathbf{B}$ and $\mathbf{C}'$ to form a new matrix $\mathbf{F}$.	$\mathbf{F} = \mathbf{B} \cdot \mathbf{C}' = \begin{pmatrix} 0.219\,200 & 0.895\,122 & -0.388\,212 \\ 0.975\,680 & -0.201\,102 & 0.087\,217 \\ 0.0 & 0.397\,888 & 0.917\,434 \end{pmatrix}$
6.	Convert the geographical latitude to degrees and construct the matrix $\mathbf{A}$ (see Table 5).	$\phi = 52.175\,278$ degrees $\mathbf{A} = \begin{pmatrix} -0.789\,890 & 0.0 & 0.613\,248 \\ 0.0 & -1.0 & 0.0 \\ 0.613\,248 & 0.0 & 0.789\,890 \end{pmatrix}$
7.	Multiply $\mathbf{A}$ and $\mathbf{F}$ to form a new matrix $\mathbf{E}$.	$\mathbf{E} = \mathbf{A} \cdot \mathbf{F} = \begin{pmatrix} -0.173\,144 & -0.463\,044 & 0.869\,259 \\ -0.975\,680 & 0.201\,102 & -0.087\,217 \\ 0.134\,424 & 0.863\,220 & 0.486\,602 \end{pmatrix}$
8.	Multiply $\mathbf{E}$ and $\boldsymbol{v}$ to find column vector $\boldsymbol{w}$.	$\boldsymbol{w} = \begin{pmatrix} -0.681\,485 \\ 0.339\,895 \\ -0.648\,113 \end{pmatrix}$
9.	Find the azimuth, A, and altitude, a, from the elements of $\boldsymbol{w}$ (see Table 5). Remove the ambiguity on taking inverse tan: look at Figure 29 and add or subtract 180 or 360 to bring the result into the correct quadrant.	$\tan^{-1}\left(\frac{n}{m}\right) = -26.508\,056$ degrees $+$ 180.0 (from Fig. 29) $\therefore A = 153.491\,944$ degrees $a = \sin^{-1}(p)$ $= 40.399\,444$ degrees
10.	Convert A and a into degrees, minutes and seconds form (§21).	$A =$ **153° 29′ 31″** $a =$ **40° 23′ 58″**

We have not included a spreadsheet or spreadsheet functions to carry out generalised coordinate transformations.

32 The angle between two celestial objects

Sometimes it is of interest to know what is the angle between two objects in the sky, and this can be calculated very easily provided their equatorial coordinates (α, δ) or ecliptic coordinates (λ, β) are known. The formula is:

$$\cos d = \sin\delta_1 \sin\delta_2 + \cos\delta_1 \cos\delta_2 \cos\left(\alpha_1 - \alpha_2\right)$$

or

$$\cos d = \sin\beta_1 \sin\beta_2 + \cos\beta_1 \cos\beta_2 \cos\left(\lambda_1 - \lambda_2\right),$$

where d is the angle between the objects whose coordinates are α_1, δ_1 (or λ_1, β_1) and α_2, δ_2 (or λ_2, β_2). These formulas are exact and mathematically correct for any values of α, δ or λ, β. However, when d becomes either very small, or close to 180°, your calculator may not have enough precision to return the correct answer, in which case a better expression is

$$d = \sqrt{(\cos\delta \times \Delta\alpha)^2 + \Delta\delta^2}$$

or

$$d = \sqrt{(\cos\beta \times \Delta\lambda)^2 + \Delta\beta^2},$$

where $\Delta\alpha$, $\Delta\delta$ (or $\Delta\lambda$, $\Delta\beta$) are the differences in the two coordinates (i.e. $\Delta\alpha = \alpha_1 - \alpha_2$, etc.). These expressions may be used for values of d within about 10 arcmin of 0° or 180°. Both $\Delta\alpha$ ($\Delta\lambda$) and $\Delta\delta$ ($\Delta\beta$) must be expressed in the same units (e.g. arcseconds) and d will then be returned in those units.

For example, what is the angular distance between the stars β Orionis (α = 5h 13m 31.7s; $\delta = -8°\ 13'\ 30''$) and α Canis Majoris (α = 6h 44m 13.4s; $\delta = -16°\ 41'\ 11''$)?

Method		*Example*		
1.	Convert both sets of coordinates to decimal form (§§7 and 21).	α_1	=	5.225 472 hours
		δ_1	=	−8.225 000 degrees
		α_2	=	6.737 056 hours
		δ_2	=	−16.686 389 degrees
2.	Find $\alpha_1 - \alpha_2$, and convert to degrees by multiplying by 15 (§22).	$\alpha_1 - \alpha_2$	=	−1.511 583 hours
			=	−22.673 750 degrees
3.	Calculate $\cos d = \sin\delta_1 \sin\delta_2 + \cos\delta_1 \cos\delta_2 \cos\left(\alpha_1 - \alpha_2\right)$.	$\cos d$	=	0.915 846
4.	Take inverse cos to find d. Convert to minutes and seconds form if required (§21).	d	=	23.673 850 degrees
			=	**23° 40′ 26″**

The spreadsheet, labelled Angle, is shown in Figure 34. It can use coordinates expressed either in equatorial or ecliptic form, specified via a switch in cell C15. Set this to H (as here) if the coordinates are α, δ (i.e. α is in Hours, minutes and seconds) or D if the coordinates are λ, β (i.e. λ is in Degrees, minutes and seconds). The corresponding spreadsheet function is also called Angle, and it takes the same 13 arguments as entered in the spreadsheet in cells C3 to C15, i.e. the right ascension/longitude of the first object expressed in hours/degrees, minutes, seconds, the declination/latitude of the first object expressed in degrees, minutes and seconds, the same again for the second object, and finally the character H or D specifying the coordinate format.

	A	B	C	D	E	F	G	H	I	J
1	**The angle between two objects**									
2										
3	*Input*	**RA/long 1 (hour/deg)**	**5**			*Output*	**angle (deg)**	**23**	=DDDeg(C29)	
4		**RA/long 1 (min)**	**13**				**angle(min)**	**40**	=DDMin(C29)	
5		**RA/long 1 (sec)**	**31.7**				**angle(sec)**	**25.86**	=DDSec(C29)	
6		**dec/lat 1 (deg)**	**-8**							
7		**dec/lat 1 (min)**	**13**							
8		**dec/lat 1 (sec)**	**30**							
9		**RA/long 2 (hour/deg)**	**6**							
10		**RA/long 2 (min)**	**44**							
11		**RA/long 2 (sec)**	**13.4**							
12		**dec/lat 2 (deg)**	**-16**							
13		**dec/lat 2 (min)**	**41**							
14		**dec/lat 2 (sec)**	**11**							
15		**Hour/degree [H or D]**	**H**							
16										
17	1	RA/long 1 (decimal)	5.225472222	=IF(C15="H",HMSDH(C3,C4,C5),DMSDD(C3,C4,C5))						
18	2	RA/long 1 (deg)	78.38208333	=IF(C15="H",DHDD(C17),C17)						
19	3	RA/long 1 (rad)	1.368025429	=RADIANS(C18)						
20	4	dec/lat 1 (deg)	-8.225	=DMSDD(C6,C7,C8)						
21	5	dec/lat 1 (rad)	-0.143553331	=RADIANS(C20)						
22	6	RA/long 2 (decimal)	6.737055556	=IF(C15="H",HMSDH(C9,C10,C11),DMSDD(C9,C10,C11))						
23	7	RA/long 2 (deg)	101.0558333	=IF(C15="H",DHDD(C22),C22)						
24	8	RA/long 2 (rad)	1.76375702	=RADIANS(C23)						
25	9	dec/lat 2 (deg)	-16.68638889	=DMSDD(C12,C13,C14)						
26	10	dec/lat 2 (rad)	-0.291232426	=RADIANS(C25)						
27	11	cos(*d*)	0.915845952	=SIN(C21)*SIN(C26)+COS(C21)*COS(C26)*COS(C19-C24)						
28	12	*d* (rad)	0.413186619	=ACOS(C27)						
29	13	*d* (deg)	23.67384942	=DEGREES(C28)						

Angle

Figure 34. Finding the angle between two celestial objects.

Thus you could delete rows 17 to 29 of the spreadsheet shown in Figure 34 (having saved a copy), and insert into cells H3, H4 and H5 the following formulas:

```
=DDDeg(Angle(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15))
=DDMin(Angle(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15))
=DDSec(Angle(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14,C15)).
```

33 Rising and setting

During the course of a sidereal day, the stars and other 'fixed' celestial objects appear to move in circles about the rotation axis of the Earth, making one complete revolution in 24 hours. At the moment, there is a star called Polaris very close to the north pole of the Earth's axis so that stars in the northern sky appear to revolve about Polaris. There is nothing special about this star, however, and no corresponding object exists for the south pole. In any case, the poles are gradually changing their positions in the sky because of precession (see the next section) so that Polaris will no longer be the pole star in a few thousand years.

The apparent radius of a star's rotation depends, of course, on the angular separation, or **polar distance**, between it and the pole; those stars with a small enough polar distance never dip below the horizon during the course of their rotation. Such stars are called **circumpolar**. As the polar distance increases, however, a point comes when the star just touches the horizon at some time during the day. Stars with polar distances greater than this spend part of their time below the horizon, out of sight to the observer. When the star crosses the horizon on the way down it is said to **set** and as it reappears it is said to **rise**.

There are several effects, including atmospheric refraction (Section 37) and parallax (for bodies relatively close to the Earth: Sections 38 and 39), that shift an object's apparent position and this may alter the apparent times of rising or setting by several minutes. The situation at rising or setting is shown in Figure 35. The celestial body appears to cross the horizon at B, although its 'true' position, as calculated from its uncorrected coordinates, is at A. Provided we know the vertical shift†, v, we can include its effects on the circumstances of rising and setting.

The local sidereal times of rising and setting, and the azimuths at which they occur, can be calculated using the formulas

$$\cos H = -\frac{(\sin v + \sin\phi \sin\delta)}{\cos\phi\cos\delta},$$

$$\mathrm{LST_r} = \alpha - H,$$

$$\mathrm{LST_s} = \alpha + H,$$

$$\cos A_\mathrm{r} = \frac{\sin\delta + \sin v \sin\phi}{\cos v \cos\phi},$$

$$A_\mathrm{s} = 360^\circ - A_\mathrm{r},$$

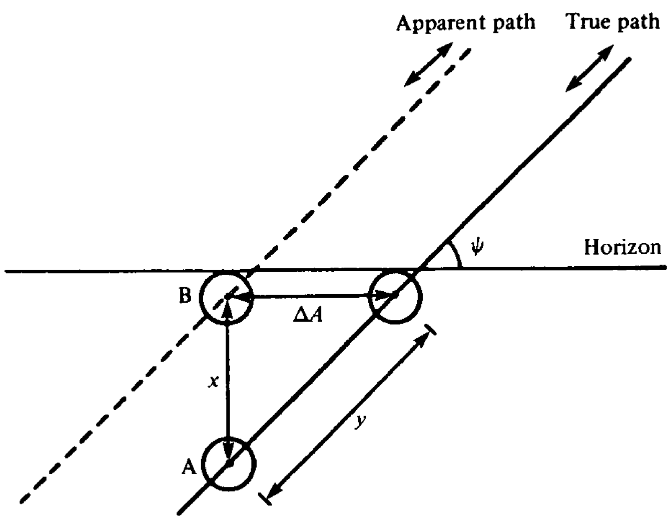


Figure 35. The true and apparent positions of a celestial object at rising or setting.

† v is positive if the star stays longer above the horizon.

where the subscripts r and s correspond to rising and setting respectively, A is the azimuth, LST is the local sidereal time in hours, α is the right ascension, δ is the declination, ϕ is the observer's geographical latitude and H is the hour angle. The value of $\cos H$ can be used as an indicator of whether the star never rises, or is circumpolar. If $\cos H$ is greater than 1, the star is permanently below the horizon and never rises. If $\cos H$ is more negative than -1, the star is permanently above the horizon and never sets (i.e. is circumpolar).

The LST can be converted to UT and hence to the local civil time by the methods given in Sections 15, 13 and 10. Hence, all the circumstances of a star's rising and setting can be calculated. However, there is a difficulty that you may need to overcome if you live far away from the Greenwich meridian. In order to convert the Greenwich sidereal time into the UT, you need to know the calendar date at Greenwich on which the rising or setting occurs. But in order to find this from your local calendar date you need to know the UT at which the rising or setting occurs. This difficulty is easily overcome however. If you take the date at Greenwich to be the same as your local calendar date, the times of rising and setting will usually not be more than a few minutes out. You can then use those times to recalculate the calendar date(s) at Greenwich and iterate until there are no further changes.

As an example, let us calculate the UTs of rising and setting over a sea horizon of a star whose equatorial coordinates are α = 23h 39m 20s and δ = 21° 42′ 00″ on 24 August 2010, and find the corresponding azimuths. The geographical latitude is 30° N, and the longitude is 64° E, and the value of v due to atmospheric refraction is 34 arcmin.

	Method	*Example*		
1.	Convert α and δ into decimal form (§§7 and 21).	α δ	= =	23.655 558 hours 21.700 000 degrees
2.	Find $\cos H = -\frac{(\sin v + \sin\phi\sin\delta)}{\cos\phi\cos\delta}$.	$\cos H$	=	$-0.242\,047$
3.	If $\cos H$ is between -1 and $+1$, take the inverse cos to find[a] H.	H	=	6.933 827 hours
4.	Find $\mathrm{LST_r} = \alpha - H$. Restore to the range 0 to 24 by adding or subtracting 24.	$\mathrm{LST_r}$	=	16.721 728 hours
5.	Find $\mathrm{LST_s} = \alpha + H$. Restore to the range 0 to 24 by adding or subtracting 24.	$\mathrm{LST_s}$	=	6.589 383 hours
6.	Find $A_\mathrm{r} = \cos^{-1}\left\{\frac{\sin\delta + \sin v \sin\phi}{\cos v \cos\phi}\right\}$. Restore to the range 0 to 360 by adding or subtracting 360.	A_r	=	**64.362 348 degrees**
7.	Find $A_\mathrm{s} = 360 - A_\mathrm{r}$.	A_s	=	**296.637 652 degrees**
8.	Convert the LST values to GST values, then to universal times (§§15 and 13).	$\mathrm{UT_r}$ $\mathrm{UT_s}$	= =	14.271 670 hours 4.166 990 hours
9.	Finally, express the times as hours, minutes and seconds (§8).	$\mathrm{UT_r}$ $\mathrm{UT_s}$	= =	**14h 16m** **4h 10m**

[a] If the star's declination is such that it never rises above the horizon, or if it is circumpolar, then you will find that you will be trying to take inverse cos of a number greater than 1 or less than -1. This is impossible and your calculator should respond with 'error'.

Note that the UTs you calculate are appropriate for the date you have applied. As here, the setting time on a given date may be earlier than the rising time.

Figure 36 shows the spreadsheet for making this lengthy calculation. We have used several techniques that are worthy of note. First, in rows 29 and 30, we have used the trick of adding 30 s (= 0.008 333 hours) to the UTs so that, when displayed as hours and minutes, the time will be rounded correctly to the nearest

	A	B	C	D	F	G	H	I
1	Rising and Setting							
2								
3	*Input*	**RA (hour)**	**23**		*Output*	**Rise/set status**	**OK**	=IF(C20>1,"** never rises",IF(C20<-1,"** circumpolar","OK"))
4		**RA (min)**	**39**			**UT rise:**	**14:16**	=IF(H3="OK",CONCATENATE(DHHour(C29),":",DHMin(C29)),"")
5		**RA (sec)**	**20**			**UT set:**	**4:10**	=IF(H3="OK",CONCATENATE(DHHour(C30),":",DHMin(C30)),"")
6		**dec (deg)**	**21**			**Azimuth rise:**	**64.36**	=IF(H3="OK",ROUND(C25,2),"")
7		**dec (min)**	**42**			**Azimuth set:**	**295.64**	=IF(H3="OK",ROUND(C26,2),"")
8		**dec (sec)**	**0**					
9		**Greenwich date (day)**	**24**				=IF(H3="OK","UT rise:","")	
10		**Greenwich date (month)**	**8**				=IF(H3="OK","UT set:","")	
11		**Greenwich date (year)**	**2010**				=IF(H3="OK","Azimuth rise:","")	
12		**geog longitude (deg)**	**64**				=IF(H3="OK","Azimuth set:","")	
13		**geog lat (deg)**	**30**					
14		**vertical shift (deg)**	**0.5667**					
15								
16	1	RA (hours)	23.6555556	=HMSDH(C3,C4,C5)				
17	2	dec (rad)	0.37873645	=RADIANS(DMSDD(C6,C7,C8))				
18	3	vertical displ (radians)	0.00989078	=RADIANS(C14)				
19	4	geo lat (radians)	0.52359878	=RADIANS(C13)				
20	5	cos (*H*)	-0.2420474	=-(SIN(C18)+SIN(C19)*SIN(C17))/(COS(C19)*COS(C17))				
21	6	*H* (hours)	6.93382728	=DDDH(DEGREES(ACOS(C20)))				
22	7	LST rise (hours)	16.7217283	=(C16-C21)-24*INT((C16-C21)/24)				
23	8	LST set (hours)	6.58938284	=(C16+C21)-24*INT((C16+C21)/24)				
24	9	*A* (deg)	64.362348	=DEGREES(ACOS((SIN(C17)+SIN(C18)*SIN(C19))/(COS(C18)*COS(C19))))				
25	10	Az rise (deg)	64.362348	=C24-360*INT(C24/360)				
26	11	Az set (deg)	295.637652	=(360-C24)-360*INT((360-C24)/360)				
27	12	UT rise (hours)	14.2716699	=GSTUT(LSTGST(C22,0,0,C12),0,0,C9,C10,C11)				
28	13	UT set (hours)	4.16699015	=GSTUT(LSTGST(C23,0,0,C12),0,0,C9,C10,C11)				
29	14	UT rise adjusted (hours)	14.2800029	=C27+0.008333				
30	15	UT set adjusted (hours)	4.17532315	=C28+0.008333				

RiseSet

Figure 36. Finding the circumstances of rising and setting.

minute. If the seconds part is less than 30, then adding 30 s will not take the result over the minute boundary and the minutes part will be unaffected. If the seconds part is 30 or more, then adding 30 s will cause the minutes part to increment by 1, as required when rounding to the nearest minute.

Second, we have used $\cos H$ as an indicator of whether the star is circumpolar, or never rises. In cell H3, we use an IF formula to return a status word. This is OK if the star rises and sets, ** never rises if it is permanently below the horizon, and ** circumpolar if it never sets. We then test the status word in cells G4 to G7, and H4 to H7, not displaying anything in these cells unless the status word is OK. This avoids rather ugly error messages appearing in the output cells when there is no rising and setting. Note that the formulas in cells G4 to G7 are displayed in cells H9 to H12. Finally, we have used the spreadsheet formula =CONCATENATE(a,b,c,. . .) in cells H4 and H5 to display the time formatted as hh:mm. The formula interprets each of its arguments a, b, c,. . . as text, strings them all together without any spaces in between, and displays the result. Thus =CONCATENATE(DHHour(C29),'':'',DHMin(C29)) displays the hour part of the time contained in cell C29, then a colon, then the minute part.

There are five spreadsheet functions provided for this calculation. Each takes eight arguments corresponding to the right ascension in hours, minutes, and seconds, the declination in degrees, minutes and seconds, the vertical shift in decimal degrees, and the geographical latitude in decimal degrees. The functions are RSLSTR, RSLSTS, RSAZR, RSAZS and eRS, returning respectively the local sidereal times of rising and setting in hours, the azimuths of rising and setting in degrees, and a status word of OK, ** never rises, or ** circumpolar as appropriate. Parts of the spreadsheet of Figure 36 could therefore be replaced with these functions as follows (save a copy first). Delete rows 16 to 21 inclusive, and row 24. Insert the following spreadsheet formulas into the cells which were C22, C23, C25 and C26:

```
=RSLSTR(C3,C4,C5,C6,C7,C8,C14,C13)
=RSLSTS(C3,C4,C5,C6,C7,C8,C14,C13)
=RSAZR(C3,C4,C5,C6,C7,C8,C14,C13)
=RSAZS(C3,C4,C5,C6,C7,C8,C14,C13).
```

Finally, you need to insert =eRS(C3,C4,C5,C6,C7,C8,C14,C13) into cell H3. Try this for yourself.

34 Precession

In Section 18 we found that equatorial coordinates were ideal for fixing the positions of the stars because they were independent of the Earth's motion and therefore constant. This is true to quite a high accuracy, but we find that the coordinates do in fact change slowly with time. This is because of a gyrating motion of the Earth's axis. Rather as the rotation axis of a quickly-spinning top revolves slowly about a vertical line, so the rotation axis of the Earth rotates slowly about a fixed direction in space. The motion is called **luni-solar precession**, or **precession of the equinoxes**, and it is caused by the gravitational effects of the Moon and Sun on the Earth. We need not be worried by the details. It is sufficient to say that the effect is small over periods of a few years, the north pole of the Earth making one complete circuit in 25 800 years, but for high precision we must be able to allow for it. In this section you will find two methods of doing so. The first, which is suitable for most purposes (i.e. precession over periods of a few tens of years), is an approximate method that is quite easy to apply. The second is a rigorous method, correct for long or short periods, which uses matrices (see Section 31 for a description of the use of matrices).

Low-precision method

The coordinates α and δ of the stars and galaxies are given in catalogues correct at some particular time or **epoch**. The ones you are quite likely to see at present will be correct at the epoch 1950.0 (strictly 1950 January 0.923) or 2000.0 (2000 January 1.5). For example, you can convert coordinates from their 1950.0 values to the values they will have at some other date using the formulas:

$$\alpha_1 = \alpha_0 + \left(3\overset{\mathrm{s}}{.}07327 + 1\overset{\mathrm{s}}{.}33617 \sin\alpha_0 \tan\delta_0\right) \times N,$$

$$\delta_1 = \delta_0 + \left(20\overset{\prime\prime}{.}0426 \cos\alpha_0\right) \times N,$$

where N is the number of years since 1950.0, α_0 and δ_0 are the coordinates at 1950.0, and α_1 and δ_1 are the new coordinates. These formulas may not work well for regions around the north and south poles where the magnitude of $\tan\delta$ tends towards infinity. You must use the rigorous method (see below) in such cases.

To convert from coordinates given at an epoch other than 1950.0, use the following formulas with the values of m, n and n' given in Table 6:

$$\alpha_1 = \alpha_0 + \left(m + n \sin\alpha_0 \tan\delta_0\right) \times N,$$

$$\delta_1 = \delta_0 + \left(n' \cos\alpha_0\right) \times N.$$

Epoch	m (seconds)	n (seconds)	n' (arcsec)
1900.0	3.072 34	1.336 45	20.046 8
1950.0	3.073 27	1.336 17	20.042 6
2000.0	3.074 20	1.335 89	20.038 3
2050.0	3.075 13	1.335 60	20.034 0

Table 6. Precessional constants.

For our example we shall work out the 1979.5 coordinates of a star whose 1950.0 coordinates were α_0 = 9h 10m 43s and δ_0 = 14° 23′ 25″.

	Method	*Example*		
1.	Convert α_0, δ_0 into decimal form (§§21 and 7).	α_0^{h}	=	9.178 611 hours
		δ_0	=	14.390 278 degrees
2.	Convert α_0 to degrees by multiplying by 15 (§22).	α_0^{d}	=	137.679 167 degrees
3.	Find $S_1 = \left(3.07327 + 1.33617 \sin\alpha_0^{\mathrm{d}} \tan\delta_0\right) \times N$ (where $N = 1979.5 - 1950.0 = 29.5$).	S_1	=	97.470 656 seconds
4.	Divide by 3600 to convert to hours.	S_1^{h}	=	0.027 075 hours
5.	Add S_1^{h} to α_0^{h} to get α_1^{h}.	α_1^{h}	=	9.205 686 hours
6.	Convert to hours, minutes and seconds (§8).	α_1	=	**9h 12m 20s**
7.	Find $S_2 = \left(20.0426 \cos\alpha_0^{\mathrm{d}}\right) \times N$.	S_2	=	−437.167 123 arcsec
8.	Divide by 3600 to convert to degrees.	S_2^{d}	=	−0.121 435 degrees
9.	Add S_2^{d} to δ_0 to get δ_1.	δ_1	=	14.268 842 degrees
10.	Convert to degrees, minutes and seconds (§21).	δ_1	=	**14° 16′ 08″**

Figure 37 shows the spreadsheet for carrying out the low-precision method, called Precess1. It differs slightly from the method given in the table in that it calculates the precession constants explicitly in rows 19 and 20 rather than looking them up in the table. The small differences in the answers are also in part caused by rounding errors. The spreadsheet functions provided for this section use the rigorous method (see below).

Rigorous method

The rigorous reduction of coordinates from one epoch to another makes use of matrices. These were described in Section 31, and if you are not familiar with them, you should read and understand that section first.

The method proceeds in two parts. First, we convert the given coordinates α_1, δ_1, which are appropriate to date (or epoch) number 1, into the corresponding coordinates of 2000.0, i.e. 2000 January 1.5, epoch 0. Second, we convert from epoch 0 to the required date, epoch 2.

	A	B	C	D	E	F	G	H	I	J
1	**Low-precision Precession**									
2										
3	*Input*	**RA (hour)**	**9**			*Output*	**corrected RA (hour)**	**9**	=DHHour(C23)	
4		**RA (min)**	**10**				**corrected RA (min)**	**12**	=DHMin(C23)	
5		**RA (sec)**	**43**				**corrected RA (sec)**	**20.18**	=DHSec(C23)	
6		**dec (deg)**	**14**				**corrected Dec (deg)**	**14**	=DDDeg(C25)	
7		**dec (min)**	**23**				**corrected Dec (min)**	**16**	=DDMin(C25)	
8		**dec (sec)**	**25**				**corrected Dec (sec)**	**9.12**	=DDSec(C25)	
9		**epoch1 (day)**	**0.923**							
10		**epoch1 (month)**	**1**							
11		**epoch1 (year)**	**1950**							
12		**epoch2 (day)**	**1**							
13		**epoch2 (month)**	**6**							
14		**epoch2 (year)**	**1979**							
15										
16	1	RA 1 (rad)	2.40295477	=RADIANS(DHDD(HMSDH(C3,C4,C5)))						
17	2	dec 1 (rad)	0.251157727	=RADIANS(DMSDD(C6,C7,C8))						
18	3	T (centuries)	0.499997892	=(CDJD(C9,C10,C11)-2415020)/36525						
19	4	*m* (sec)	3.073269996	=3.07234+(0.00186*C18)						
20	5	*n'* (arcsec)	20.04255002	=20.0468-(0.0085*C18)						
21	6	*N* (years)	29.41294182	=(CDJD(C12,C13,C14)-CDJD(C9,C10,C11))/365.25						
22	7	S1 (hours)	0.02699528	=((C19+(C20*SIN(C16)*TAN(C17)/15))*C21)/3600						
23	8	RA 2 (hours)	9.205606391	=HMSDH(C3,C4,C5)+C22						
24	9	S2 (deg)	-0.121076639	=(C20*COS(C16)*C21)/3600						
25	10	dec 2 (deg)	14.26920114	=DMSDD(C6,C7,C8)+C24						

Precess1

Figure 37. Low-precision precession.

We begin by calculating the precessional variables ζ_A, z_A and θ_A, for the date at which the coordinates are specified (epoch 1). These are given in degrees by the following formulas:

$$\begin{aligned}
\zeta_A &= 0.6406161T + 0.0000839T^2 + 0.0000050T^3,\\
z_A &= 0.6406161T + 0.0003041T^2 + 0.0000051T^3,\\
\theta_A &= 0.5567530T - 0.0001185T^2 - 0.0000116T^3,
\end{aligned}$$

where T is the number of Julian centuries of 36 525 days at epoch 1 since the epoch J2000.0. The value of T may be calculated from

$$T = \frac{(\mathrm{JD1} - 2\,451\,545)}{36\,525},$$

where JD1 is the Julian date of epoch 1. We then construct the matrix $\mathbf{P}'$ which allows us to convert the coordinates from epoch 1 to epoch 0. The matrix $\mathbf{P}'$ is given by

$$\mathbf{P}' = \begin{pmatrix} CX\cdot CT\cdot CZ - SX\cdot SZ & CX\cdot CT\cdot SZ + SX\cdot CZ & CX\cdot ST \\ -SX\cdot CT\cdot CZ - CX\cdot SZ & -SX\cdot CT\cdot SZ + CX\cdot CZ & -SX\cdot ST \\ -ST\cdot CZ & -ST\cdot SZ & CT \end{pmatrix}$$

where $CX = \cos\zeta_A$, $SX = \sin\zeta_A$, $CZ = \cos z_A$, $SZ = \sin z_A$, $CT = \cos\theta_A$, and $ST = \sin\theta_A$. We next calculate the column vector, $\boldsymbol{v}$, corresponding to the coordinates at epoch 1, α_1 and δ_1, from

$$\boldsymbol{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\alpha_1,\delta_1} = \begin{pmatrix} \cos\alpha_1 \cos\delta_1 \\ \sin\alpha_1 \cos\delta_1 \\ \sin\delta_1 \end{pmatrix}.$$

Now we multiply P' and $\boldsymbol{v}$ to form the column vector $\boldsymbol{s}$ corresponding to the epoch-0 coordinates,

$$\boldsymbol{s} = \mathsf{P}' \cdot \boldsymbol{v}.$$

The second part of the process is to convert from epoch 0 to the epoch at which the coordinates are actually required, epoch 2. We need to repeat the above procedure, calculating values for ζ_{A}, z_{A} and θ_{A} using T appropriate to epoch 2 (i.e. use JD2 instead of JD1). Then we construct the matrix P, which is just the **transpose** of P', from

$$\mathsf{P} = \begin{pmatrix} CX\cdot CT\cdot CZ - SX\cdot SZ & -SX\cdot CT\cdot CZ - CX\cdot SZ & -ST\cdot CZ \\ CX\cdot CT\cdot SZ + SX\cdot CZ & -SX\cdot CT\cdot SZ + CX\cdot CZ & -ST\cdot SZ \\ CX\cdot ST & -SX\cdot ST & CT \end{pmatrix}.$$

The transpose of a 3×3 matrix is found by interchanging the rows and columns, ‘flipping’ them about the diagonal. For example, the transpose of

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \text{ is } \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}.$$

Now, the column vector, $\boldsymbol{w}$, corresponding to the coordinates at epoch 2, can be found by multiplying P and $\boldsymbol{s}$:

$$\boldsymbol{w} = \mathsf{P} \cdot \boldsymbol{s}.$$

Finally, we extract the new coordinates, α_2, δ_2, from the elements of $\boldsymbol{w}$ using

$$\alpha_2 = \tan^{-1}\left(\frac{n}{m}\right) \text{ and } \delta_2 = \sin^{-1}(p).$$

Let us clarify this procedure by repeating the example used for the low-precision method.

Method		*Example*		
1.	Find the Julian date corresponding to epoch 1 (1,1,1950; §4).	JD1	=	2 433 282.5
2.	Find $T = (\text{JD1} - 2\,451\,545)/36\,525$.	T	=	−0.500 000
3.	Calculate ζ_A, z_A and θ_A for JD1 using the formulas given above.	ζ_A	=	−0.320 288 degrees
		z_A	=	−0.320 233 degrees
		θ_A	=	−0.278 405 degrees
4.	Construct the matrix P'.			
		P'	=	$\begin{pmatrix} 0.999\,926 & -0.011\,179 & -0.004\,859 \\ 0.011\,179 & 0.999\,938 & -0.000\,027 \\ 0.004\,859 & -0.000\,027 & 0.999\,988 \end{pmatrix}$
5.	Convert α_1 and δ_1 into decimal degrees (§§21 and 22).	α_1	=	137.679 167 degrees
		δ_1	=	14.390 278 degrees
6.	Construct the column vector $\boldsymbol{v}$.	x	=	−0.716 188
		y	=	0.652 157
		z	=	0.248 526
7.	Multiply P' and $\boldsymbol{v}$ to find $\boldsymbol{s}$ (§31).	$\boldsymbol{s}$	=	$\begin{pmatrix} -0.724\,633 \\ 0.644\,104 \\ 0.245\,025 \end{pmatrix}$
8.	Find the Julian date corresponding to epoch 2 (1,6,1979; §4).	JD2	=	2 444 025.5
9.	Calculate $T = (\text{JD2} - 2\,451\,545)/36\,525$.	T	=	−0.205 873
10.	Now find new values of ζ_A, z_A and θ_A, appropriate to JD2.	ζ_A	=	−0.131 882 degrees
		z_A	=	−0.131 873 degrees
		θ_A	=	−0.114 625 degrees
11.	Construct the matrix P.			
		P	=	$\begin{pmatrix} 0.999\,987 & 0.004\,603 & 0.002\,001 \\ -0.004\,603 & 0.999\,989 & -0.000\,005 \\ -0.002\,001 & -0.000\,005 & 0.999\,998 \end{pmatrix}$
12.	Multiply P and $\boldsymbol{s}$ to find $\boldsymbol{w}$, corresponding to the coordinates at epoch 2 (§31).	m	=	−0.721 169
		n	=	0.647 432
		p	=	0.246 471
13.	Find the new coordinates using $\alpha_2 = \tan^{-1}\left(\frac{n}{m}\right)$ and $\delta_2 = \sin^{-1}(p)$. Remove the ambiguity on taking $\tan^{-1}$ by adding or subtracting 180 or 360 according to Figure 29.	α_2	=	−41.916 009 degrees
			+	180.0
		α_2	=	138.083 991 degrees
		δ_2	=	14.268 792 degrees
14.	Convert α_2 and δ_2 to minutes and seconds form (§§21 and 22).	α_2	=	**9h 12m 20.16s**
		δ_2	=	**14° 16′ 7.65″**

You can see that the low-precision method gave quite a good result in this case.

We have not provided a spreadsheet for this calculation. Manipulation of matrices in spreadsheets is possible, but is a complication that is instead carried out more easily in the background by the spreadsheet functions Precess2RA and Precess2Dec, which return the right ascension in decimal hours and the declination in decimal degrees respectively, each corrected for precession using the rigorous method. Both functions take the same 12 arguments, being the right ascension in hours, minutes and seconds, the declination in degrees, minutes and seconds, the calendar date at which those coordinates were specified (epoch 1) as day, month, year, and finally the calendar date to which the coordinates must be precessed (epoch 2) as day, month, and year.

We can now simplify the spreadsheet Precess1 and, at the same time, make it more accurate by replacing the calculation part by these functions. Save a copy of the spreadsheet shown in Figure 37 and delete rows 16–22 and 24. In the cells which were C23 and C25 (now C16 and C17 after the deletion) insert the following spreadsheet formulas respectively:

```
=Precess2RA(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14)
=Precess2Dec(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14).
```

You should also change the title of the spreadsheet (cell A1) to Rigorous precession and rename the spreadsheet Precess2.

35 Nutation

The combined gravitational fields of the Sun and the Moon acting on the non-spherical Earth cause the direction of the Earth's rotation axis to gyrate slowly with a period of about 25 800 years. We saw how to allow for this effect, called **precession**, in Section 34. Superimposed on the regular motion there are also small additional periodic terms caused by the varying distances and relative directions of the Moon and Sun, which continuously alter the strength and direction of the gravitational field. This slight wobbling motion is called **nutation**, and can be taken into account by calculating its effects on ecliptic longitude, $\Delta\psi$, and on the mean obliquity of the ecliptic, $\Delta\varepsilon$.

The method described here uses the pre-1984 theory of nutation. We consider only the most important terms to achieve an accuracy of about half an arcsecond. We first calculate the number of Julian centuries, T, on the date in question since 1900 January 0.5 using

$$T = \frac{\mathrm{JD} - 2\,415\,020.0}{36\,525},$$

where JD is the Julian date. Note that this definition of T is different from that in other parts of the book where the fundamental epoch 2000 January 1.5 is used. Next, we find the values of the Sun's mean longitude, L, and the longitude of the Moon's ascending node, Ω, from the formulas

$$L = 279.6967 + 360.0 \times (A - \mathrm{INT}(A)) \text{ degrees},$$

where

$$A = 100.002\,136 \times T,$$

and

$$\Omega = 259.1833 - 360.0 \times (B - \mathrm{INT}(B)) \text{ degrees},$$

where

$$B = 5.372\,617 \times T.$$

(See Section 4 about the meaning of INT.) The effects of nutation on the ecliptic longitude and obliquity of the ecliptic are then given by

$$\begin{aligned}\Delta\psi &= -17.2\sin(\Omega) - 1.3\sin(2L)\ \text{arcsec},\\ \Delta\varepsilon &= 9.2\cos(\Omega) + 0.5\cos(2L)\ \text{arcsec}.\end{aligned}$$

As an example we calculate the nutation on 1 September 1988.

Method		*Example*		
1.	Find the Julian day number for this calendar date.	JD	=	2 447 405.5
2.	Calculate T = (JD−2 415 020.0)/36 525.0.	T	=	0.886 667
3.	Find $A = 100.002\,136T$.	A	=	88.668 560
4.	Calculate the Sun's mean longitude $L = 279.6967 + 360.0\,(A - \text{INT}\,(A))$. Reduce to the range 0 to 360 if necessary by adding or subtracting multiples of 360.	L	=	160.378 686 degrees
5.	Find $B = 5.372\,617T$.	B	=	4.763 720
6.	Calculate the Moon's node $\Omega = 259.1833 - 360.0\,(B - \text{INT}\,(B))$. Reduce to the range 0 to 360 if necessary by adding or subtracting multiples of 360.	Ω	=	344.243 954 degrees
7.	Calculate nutation in longitude $\Delta\psi = -17.2\sin(\Omega) - 1.3\sin(2L)$.	$\Delta\psi$	=	**5.5″**
8.	Calculate nutation in obliquity $\Delta\varepsilon = 9.2\cos(\Omega) + 0.5\cos(2L)$.	$\Delta\varepsilon$	=	**9.2″**

The *Astronomical Almanac* gives the values $\Delta\psi$ = 5.1″ and $\Delta\varepsilon$ = 9.2″ for this date, so we are well within the accuracy claimed for this method of half an arcsecond. $\Delta\psi$ must be added to the ecliptic longitude, and $\Delta\varepsilon$ to the mean obliquity, to allow for nutation.

The spreadsheet for making this calculation is shown in Figure 38. It gives the small amounts to be added to the ecliptic longitude and the mean obliquity of the ecliptic in degrees in cells H3 and H4. The corresponding quantities in arcseconds are in cells C18 and C19. We have also provided the spreadsheet functions NutatLong(D,M,Y) and NutatObl(D,M,Y) giving, respectively, the nutation amount to be added in ecliptic longitude and in mean obliquity, both in degrees. Both functions take the calendar date as arguments expressed as day, month and year (strictly, the calendar date at Greenwich, although your local calendar date will usually do). Note that when you use the function Obliq (Section 27), the nutation term is already included in the result so you will hardly ever need to use NutatObl explicitly.

You can use these two functions to simplify the spreadsheet shown in Figure 38. Save a copy in case you want it later (or you make a mistake and need to start again), then delete rows 7 to 19 (the calculation part), and insert the following formulas into cells H3 and H4:

```
=NutatLong(C3,C4,C5)
=NutatObl(C3,C4,C5).
```

	A	B	C	D	E	F	G	H	I
1	**Nutation in ecliptic longitude and obliquity**								
2									
3	*Input*	**Gday**	**1**			*Output*	**nut in long (deg)**	**0.001525808**	=C18/3600
4		**Gmonth**	**9**				**nut in obl (deg)**	**0.0025671**	=C19/3600
5		**Gyear**	**1988**						
6									
7	1	JD (days)	2447405.5	=CDJD(C3,C4,C5)					
8	2	*T* (centuries)	0.886666667	=(C7-2415020)/36525					
9	3	*A* (deg)	88.66856041	=100.0021358*C8					
10	4	*L* 1 (deg)	279.6969382	=279.6967+(0.000303*C8*C8)					
11	5	*L* (deg)	520.3786856	=C10+360*(C9-INT(C9))					
12	6	*L* (deg)	160.3786856	=C11-360*INT(C11/360)					
13	7	*L* (rad)	2.799136113	=RADIANS(C12)					
14	8	*B* (deg)	4.763720407	=5.372617*C8					
15	9	*N* (deg)	-15.7560464	=259.1833-360*(C14-INT(C14))					
16	10	*N* (deg)	344.2439536	=C15-360*(INT(C15/360))					
17	11	*N* (rad)	6.00819042	=RADIANS(C16)					
18	12	nut in long (arcsec)	5.492910079	=-17.2*SIN(C17)-1.3*SIN(2*C13)					
19	13	nut in obl (arcsec)	9.24156161	=9.2*COS(C17)+0.5*COS(2*C13)					

Figure 38. Calculating nutation in ecliptic longitude and obliquity.

36 Aberration

There are several small effects which must be taken into account to improve the accuracy of our calculations. One of them is called **aberration**, and is caused by the speed of the Earth in its orbit around the Sun. Since light does not travel at infinite speed, the motion of the Earth causes the apparent direction of a celestial body to be shifted slightly from its true direction, just as rain falling vertically downwards appears to come at an angle to a cyclist moving through it. The correction is small, amounting to a maximum shift of 20.5 arcsec (0.000 569 degrees). We can find its effect on the ecliptic longitude, $\Delta\lambda$, and on the ecliptic latitude, $\Delta\beta$, using the formulas

$$\Delta\lambda = -20.5\cos\left(\lambda_\odot - \lambda\right)/\cos\beta \text{ arcsec}$$

and

$$\Delta\beta = -20.5\sin\left(\lambda_\odot - \lambda\right)\sin\beta \text{ arcsec},$$

where λ and β are the true ecliptic longitude and latitude, and $\lambda_\odot$ is the ecliptic longitude of the Sun (see Section 46). The apparent longitude and latitude are then given by

$$\lambda' = \lambda + \Delta\lambda \text{ and } \beta' = \beta + \Delta\beta.$$

Corrections to the true right ascension, $\Delta\alpha$, and to the declination, $\Delta\delta$, of a body can be calculated using the formulas

$$\Delta\alpha = -20.5 \frac{\cos\alpha\cos\lambda_{\odot}\cos\varepsilon + \sin\alpha\sin\lambda_{\odot}}{\cos\delta} \text{ arcsec},$$

$$\Delta\delta = -20.5 \left[\cos\lambda_{\odot}\cos\varepsilon\,(\tan\varepsilon\cos\delta - \sin\alpha\sin\delta) + \cos\alpha\sin\delta\sin\lambda_{\odot}\right] \text{ arcsec},$$

where α and δ are the true coordinates and ε is the obliquity of the ecliptic (see Section 27). These corrections must be added to the true coordinates to find the apparent coordinates.

Let's take as an example the effect of aberration on the position of Mars on 8 September 1988. Its true ecliptic coordinates were $\lambda = 352° \ 37' \ 10.1''$ and $\beta = -1° \ 32' \ 56.4''$. The longitude of the Sun on that day (at 0h TT) was $165° \ 33' \ 44.1''$.

Method		*Example*		
1.	Convert λ, β and $\lambda_{\odot}$ to decimal degrees (§21).	λ	=	352.619 472 degrees
		β	=	−1.549 000 degrees
		$\lambda_{\odot}$	=	165.562 250 degrees
2.	Calculate $\Delta\lambda = -20.5\cos\left(\lambda_{\odot} - \lambda\right)/\cos\beta$.	$\Delta\lambda$	=	20.352 128 arcsec
3.	Calculate $\Delta\beta = -20.5\sin\left(\lambda_{\odot} - \lambda\right)\sin\beta$.	$\Delta\beta$	=	0.068 084 arcsec
4.	Convert $\Delta\lambda$ and $\Delta\beta$ to degrees by dividing by 3600.	$\Delta\lambda$	=	0.005 653 degrees
		$\Delta\beta$	=	0.000 019 degrees
5.	Add the corrections to λ and β to find the apparent coordinates.	λ'	=	352.625 126 degrees
		β'	=	−1.548 981 degrees
6.	Convert back to minutes and seconds (§21).	λ'	=	**352° 37′ 30.5″**
		β'	=	**−1° 32′ 56.3″**

Notice that $\Delta\beta$ is negligible (much less than 1 arcsecond) for ecliptic latitudes close to zero, as in this case.

Our spreadsheet for this calculation is shown in Figure 39 and is called Aberration. Of special note is that we have anticipated the method of Section 46 for calculating the ecliptic longitude of the Sun and have used the spreadsheet function SunLong in row 18. Here, we have also supplied the spreadsheet functions AbLong and AbLat which return, respectively, the ecliptic longitude in degrees corrected for the effect of aberration (Aberration in Longitude) and the ecliptic latitude in degrees corrected for the effect of aberration (Aberration in Latitude). They both take 12 arguments, these being the universal time in hours, minutes and seconds, the Greenwich calendar date as day, month, year, the true ecliptic longitude in degrees, minutes and seconds, and the true ecliptic latitude in degrees, minutes and seconds. If you wish, you can therefore simplify the spreadsheet of Figure 39 (save a copy first) by deleting rows 16 to 22 inclusive, and inserting the following spreadsheet formulas into cells H3 to H8 inclusive:

```
=DDDeg(AbLong(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DDMin(AbLong(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DDSec(AbLong(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DDDeg(AbLat(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DDMin(AbLat(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14))
=DDSec(AbLat(C3,C4,C5,C6,C7,C8,C9,C10,C11,C12,C13,C14)).
```

	A	B	C	D	E	F	G	H	I	J
1	**Correcting for Aberration**									
2										
3	*Input*	**UT (hour)**	**0**			*Output*	**apparent ecl long (deg)**	**352**	=DDDeg(C21)	
4		**UT (min)**	**0**				**apparent ecl long (min)**	**37**	=DDMin(C21)	
5		**UT (sec)**	**0**				**apparent ecl long (sec)**	**30.45**	=DDSec(C21)	
6		**G date (day)**	**8**				**apparent ecl lat (deg)**	**-1**	=DDDeg(C22)	
7		**G date (month)**	**9**				**apparent ecl lat (min)**	**32**	=DDMin(C22)	
8		**G date (year)**	**1988**				**apparent ecl lat (sec)**	**56.33**	=DDSec(C22)	
9		**true ecl long (deg)**	**352**							
10		**true ecl long (min)**	**37**							
11		**true ecl long (sec)**	**10.1**							
12		**true ecl lat (deg)**	**-1**							
13		**true ecl lat (min)**	**32**							
14		**true ecl lat (sec)**	**56.4**							
15										
16	1	true long (deg)	352.6194722	=DMSDD(C9,C10,C11)						
17	2	true lat (deg)	-1.549	=DMSDD(C12,C13,C14)						
18	3	Sun true long (deg)	165.5633044	=Sunlong(C3,C4,C5,0,0,C6,C7,C8)						
19	4	dlong (arcsec)	20.35217443	=-20.5*COS(RADIANS(C18-C16))/COS(RADIANS(C17))						
20	5	dlat (arcsec)	0.068073433	=-20.5*SIN(RADIANS(C18-C16))*SIN(RADIANS(C17))						
21	6	apparent long (deg)	352.6251256	=C16+(C19/3600)						
22	7	apparent lat (deg)	-1.548981091	=C17+(C20/3600)						

Aberration

Figure 39. Correcting ecliptic coordinates for the effects of aberration.

37 Refraction

In all our calculations so far, we have assumed that the light from distant objects reaches us by the most direct route, a straight line. This is not actually the case (except for observations made at the zenith) as the Earth's atmosphere bends the light a little, making the rays reach the ground at a slightly different angle from that which they would have had if the atmosphere had not been there (see Figure 40). This is called **atmospheric refraction** and its effect is to make the star appear to be closer to the zenith than it really is. The amount of refraction depends on the **zenith angle** or **zenith distance** ($90^\circ - $ altitude) and on the atmospheric conditions, particularly the temperature and pressure. If we observe a star with zenith angle ζ from the surface of the Earth, its true zenith angle, z, is given by $z = \zeta + R$, where R is the refraction angle. An approximate expression for R that is suitable for altitudes above 15° is

$$R = 0.004\,52 P \tan z / (273 + T) \text{ degrees,}$$

where T is the temperature in degrees centigrade and P is the barometric pressure in millibars, both measured at the observation point. This formula is usually accurate to about 6 arcsec for altitudes greater than 15°. At lower altitudes, better results can be obtained using the approximate formula

$$R = \frac{P\left(0.1594 + 0.0196a + 0.000\,02a^2\right)}{(273 + T)\left(1 + 0.505a + 0.0845a^2\right)} \text{ degrees,}$$

where a is the altitude in degrees.†

†Strictly, a is the apparent altitude as measured through the atmosphere, rather than the true altitude as measured with no atmosphere.

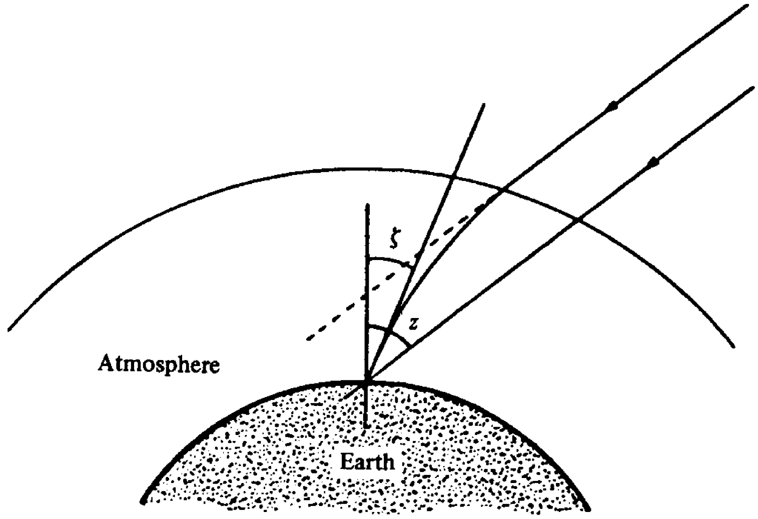


Figure 40. Atmospheric refraction.

The effect of refraction on true equatorial, ecliptic and galactic coordinates is best computed by first converting to horizon coordinates, increasing the altitude by adding R, and then converting back to the original coordinate system to find the apparent position. We will now illustrate this by calculating the refraction for a star whose true hour angle is 5h 51m 44s and true declination +23° 13′ 10″ as observed at a geographical latitude of 52° N. The temperature is 13° C and the pressure is 1008 mbar.

Method		*Example*		
1.	Calculate the true altitude and azimuth of the star (§25).	a	=	19.334 345 degrees
		A	=	283.271 027 degrees
2.	Find the refraction angle R from the formula appropriate	z	=	70.665 655 degrees
	to the altitude: $a > 15°$, $R = 0.00452 P \tan z / (273 + T)$.	R	=	0.045 403 degrees
3.	Add R to the altitude to find the apparent altitude a'.	a'	=	19.379 748 degrees
4.	Convert A and a' back into equatorial coordinates (§26).	H'	=	**5h 51m 36s**
		δ'	=	**23° 15′ 14″**

The magnitude of R right at the horizon is usually assumed to be 34 arcmin. (Its actual value may be different depending on atmospheric conditions.) Since its effect is to increase the apparent altitude, the times of rising and setting will be earlier and later, respectively, than they would have been without the atmosphere. The effective length of the day, therefore, is increased by atmospheric refraction. We can calculate its effects on the azimuths and times of rising and setting by the method given in Section 33. Alternatively, we can calculate the effect on the hour angle, H, at rising or setting by

$$\Delta H = \frac{34}{15 \cos\phi \cos\delta \sin H} \text{ minutes of time,}$$

where ΔH is the amount by which the true hour angle is reduced.

The spreadsheet for calculating the effects of refraction is labelled Refract and is shown in Figure 41. It goes some way beyond the method given in the method table above in that it handles conversion from apparent to true coordinates as well as from true to apparent, it converts right ascension and declination, and it makes use of the spreadsheet function Refract to do the refraction calculation. This function, used in row 26, takes four arguments, the altitude to be corrected for the effect of refraction, a switch which is set to TRUE or APPARENT depending on the altitude type, the atmospheric pressure in millibars, and the atmospheric temperature in degrees centigrade. In the example shown in the spreadsheet, the true right ascension and declination are converted to the apparent right ascension and declination, this direction being specified by the text in cell C9. You can check that the conversion in the reverse direction is working properly by writing down the corrected coordinates on a piece of paper, then inserting them into cells C3 to C8 in place of the true coordinates, and changing the text in cell C9 to APPARENT (actually, anything which does not begin with T or t will do as the function only looks at the first character). You should see that the true coordinates appear in the output, cells H3 to H8, correct to the second decimal place in the seconds part.

	A	B	C	D	E	F	G	H	I	J
1	**Atmospheric refraction**									
2										
3	*Input*	**true RA (hour)**	**23**			*Output*	**corrected RA (hour)**	**23**	=DHHour(C28)	
4		**true RA (min)**	**14**				**corrected RA (min)**	**13**	=DHMin(C28)	
5		**true RA (sec)**	**0**				**corrected RA (sec)**	**44.74**	=DHSec(C28)	
6		**true dec (deg)**	**40**				**corrected dec (deg)**	**40**	=DDDeg(C29)	
7		**true dec (min)**	**10**				**corrected dec (min)**	**19**	=DDMin(C29)	
8		**true dec(sec)**	**0**				**corrected dec (sec)**	**45.76**	=DDSec(C29)	
9		**.. these coordinates are of type**	**TRUE**							
10		**geog longitude (deg)**	**0.17**							
11		**geog latitude (deg**	**51.203611**							
12		**daylight saving (hours)**	**0**							
13		**time zone (hours)**	**0**							
14		**local calendar date (day)**	**23**							
15		**local calendar date (month)**	**3**							
16		**local calendar date (year)**	**1987**							
17		**local civil time (hour)**	**1**							
18		**local civil time (min)**	**1**							
19		**local civil time (sec)**	**24**							
20		**atmospheric pressure (mBar)**	**1012**							
21		**atmospheric temperature (C)**	**21.7**							
22										
23		HA (hour)	13.8009048	=RAHA(C3,C4,C5,C17,C18,C19,C12,C13,C14,C15,C16,C10)						
24		azimuth (deg)	20.3710605	=EqAz(C23,0,0,C6,C7,C8,C11)						
25		altitude (deg)	4.36733747	=EQAlt(C23,0,0,C6,C7,C8,C11)						
26		corrected altitude (deg)	4.53712816	=Refract(C25,C9,C20,C21)						
27		corrected HA (hour)	13.8051426	=HorHa(C24,0,0,C26,0,0,C11)						
28		corrected RA (hour)	23.2290955	=HARA(C27,0,0,C17,C18,C19,C12,C13,C14,C15,C16,C10)						
29		corrected dec (deg)	40.3293776	=HORDec(C24,0,0,C26,0,0,C11)						

Figure 41. Correcting equatorial coordinates for the effects of refraction.

38 Geocentric parallax and the figure of the Earth

In later sections of this book we calculate the coordinates of the Sun, the Moon and other members of our Solar System. These coordinates are the ones which would be observed from the centre of the Earth, called **geocentric coordinates**, and if the celestial body is at a very great distance from the Earth, they are also the coordinates which would be measured by anyone on the Earth's surface. However, objects relatively close at hand like the Sun, and especially the Moon, appear to be at slightly different positions depending upon the exact viewpoint of the observer. This is illustrated in Figure 42 where two observers, O_1 and O_2, are viewing the Moon, M, from the surface of the Earth, E. Each measures the angle between the Moon and a very distant star in the direction S. Since this star is so far away the lines of sight to the star, O_1S and O_2S, are parallel so that both observers see it in the same place in the sky relative to other stars. However, they do not measure the same angles, a_1 and a_2, and hence do not agree about the Moon's apparent position. If a_0 represented, say, the right ascension of the Moon as calculated from the Earth's centre, then each observer would have to add a different correction to a_0 to get a_1 or a_2, the apparent right ascension at each place. This apparent shift of position is known as **geocentric parallax** and we often need to be able to correct for it as, for example, when we wish to calculate the circumstances of an eclipse.

The problem is complicated slightly by the fact that the Earth is not quite spherical, but is, instead, more like a **spheroid of revolution**, being flattened along the line joining the north and south poles. A cross-section through the Earth along any line of longitude would be approximately elliptical, while a cross-section along any line of latitude would be circular. We have to take account of the **figure of the Earth**, its deviation from a perfect sphere, if we are to make precise corrections for parallax. The situation is shown much exaggerated in Figure 43 where the Earth, E, is drawn with its north and south poles, N and S. An observer at O locates his zenith by means of a plumb line to be along the dashed line OZ; the angle this makes with the equator defines his **geographical** or **astronomical latitude**, ϕ. Since the Earth is not

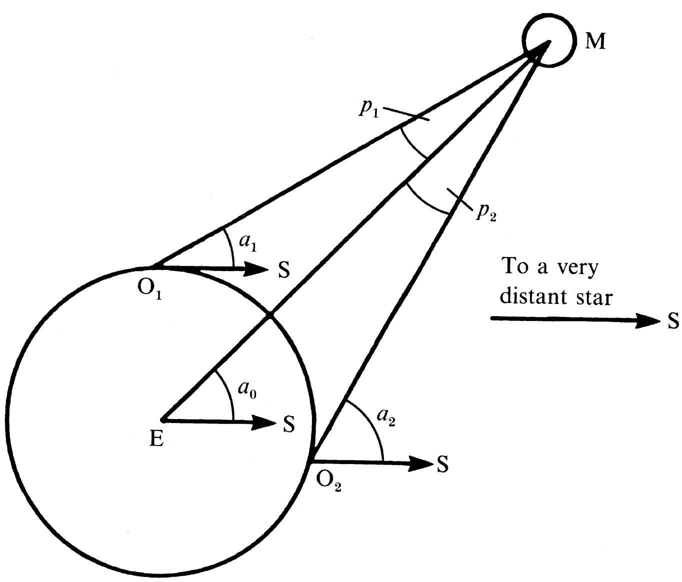


Figure 42. Geocentric parallax.

quite spherical, his geocentric vertical, EZ′, is slightly different and so too is his **geocentric latitude**, ϕ'. In calculations of the effect of parallax, we need to know the quantities $\rho\sin\phi'$ and $\rho\cos\phi'$, where ρ is the distance of the observer from the centre of the Earth in units of the Earth's equatorial radius. For a place whose height above sea-level is h metres, we have

$$\rho\sin\phi' = 0.996\,647\sin u + \frac{h}{6\,378\,140}\sin\phi,$$

$$\rho\cos\phi' = \cos u + \frac{h}{6\,378\,140}\cos\phi,$$

where

$$u = \tan^{-1}\{0.996\,647\tan\phi\}.$$

ϕ must be reckoned as positive in the northern hemisphere and negative in the southern hemisphere. For example, let us calculate the values of $\rho\sin\phi'$ and $\rho\cos\phi'$ for an observer whose height above sea-level is 60 metres at longitude 100° W and latitude 50° N.

Method	*Example*		
1. Calculate $u = \tan^{-1}\{0.996\,647\tan\phi\}$.	ϕ	=	+50.0 degrees
	u	=	49.905 217 degrees
2. Calculate $h' = \frac{h}{6378140}$.	h	=	60.0 metres
	h'	=	0.000 009
3. Calculate $\rho\sin\phi' = 0.996\,647\sin u + h'\sin\phi$.	$\rho\sin\phi'$	=	**0.762 422**
4. Calculate $\rho\cos\phi' = \cos u + h'\cos\phi$.	$\rho\cos\phi'$	=	**0.644 060**

In Figure 43, it is the angle p which is formally called the geocentric parallax. This is the angle between the observer and the Earth's centre as seen by the celestial body in question. If the observer views the body right at his horizon (i.e. the zenith angle = 90°), then p is called the **horizontal parallax**. Further, if the observer is also on the equator, this angle becomes the **equatorial horizontal parallax**, and is given the symbol P. (We will meet parallax again in Section 69.) The spreadsheet for parallax calculations is given in the next section (Figure 44).

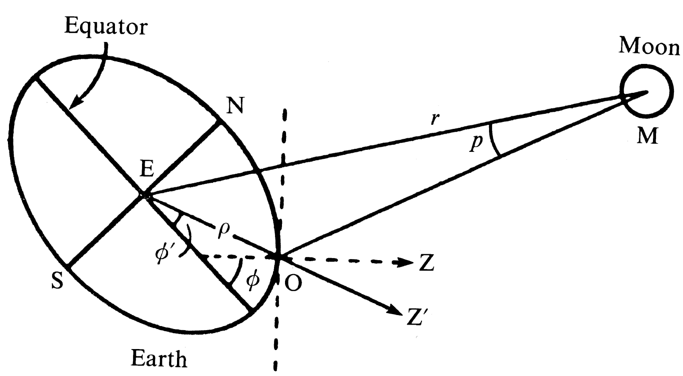


Figure 43. Allowing for the figure of the Earth.

39 Calculating corrections for parallax

If a body has geocentric hour angle, H, and geocentric right ascension, α, then its apparent hour angle, H', and right ascension, α' (taking account of parallax), are given by

$$H' = H + \Delta,$$

$$\alpha' = \alpha - \Delta,$$

with

$$\Delta = \tan^{-1}\left\{\frac{\rho\cos\phi'\sin H}{r\cos\delta - \rho\cos\phi'\cos H}\right\},$$

where $\rho\cos\phi'$ is the quantity calculated in Section 38 and r is the distance of the body from the centre of the Earth measured in units of (equatorial) Earth radii, 6378.14 km. If r' is this distance in kilometres, then

$$r = \frac{r'}{6378.14}.$$

r can also be found from the equatorial horizontal parallax of the body, P. Thus

$$r = \frac{1}{\sin P}.$$

The formula for finding the apparent declination, δ', from the geocentric declination, δ, is

$$\delta' = \tan^{-1}\left\{\cos H' \frac{r\sin\delta - \rho\sin\phi'}{r\cos\delta\cos H - \rho\cos\phi'}\right\}.$$

Again, $\rho\sin\phi'$ and $\rho\cos\phi'$ can be found by the method described in Section 38.

As an example, let us calculate the apparent right ascension and declination of the Moon on 26 February 1979 at 16h 45m UT when observed from a location 60 metres above sea-level on longitude 100° W and latitude 50° N. The geocentric coordinates were α = 22h 35m 19s and $\delta = -7^\circ\,41'\,13''$, and the Moon's equatorial horizontal parallax was $1^\circ\,01'\,09''$.

Method		*Example*		
1.	Convert UT to GST and hence to LST by the methods of §§12 and 14.	UT	=	16h 45m
		GST	=	3.145 778 hours
		LST	=	20.479 111 hours
2.	Convert α and δ to decimal form (§§7 and 21).	α	=	22.588 611 hours
		δ	=	−7.686 944 degrees
3.	Find the hour angle, H (§24), and convert to degrees (§22).	H	=	−2.109 500 hours
			=	−31.642 500 degrees
4.	Find $\rho\cos\phi'$ and $\rho\sin\phi'$ (§38).	$\rho\cos\phi'$	=	0.644 060
		$\rho\sin\phi'$	=	0.762 422
5.	Find $r = (1/\sin P)$ (remember to convert P to decimal degrees first (§21)).	P	=	1.019 167 degrees
		r	=	56.221 228 Earth-radii
6.	Calculate $\Delta = \tan^{-1}\left\{\frac{\rho\cos\phi'\sin H}{r\cos\delta-\rho\cos\phi'\cos H}\right\}$.	Δ	=	−0.350 915 degrees
7.	Find $H' = H + \Delta$.	H'	=	−31.994 414 degrees
8.	Convert Δ to hours by dividing by 15 (§22).	Δ	=	−0.023 394 hours
9.	Subtract Δ from α to find α'.	α'	=	22.612 005 hours
10.	Calculate $\delta' = \tan^{-1}\left\{\cos H' \frac{r\sin\delta-\rho\sin\phi'}{r\cos\delta\cos H-\rho\cos\phi'}\right\}$.	δ'	=	−8.538 165 degrees
11.	Convert α' and δ' to minutes and second form (§§8 and 21).	α'	=	**22h 36m 43s**
		δ'	=	**−8° 32′ 17″**

Such lengthy calculations are strictly only necessary for the Moon which has a very large parallax. The Sun, planets and comets usually have much smaller values, enabling us to simplify the formulas slightly without serious loss of accuracy. Let r again denote the distance of the body from the centre of the Earth, but this time measured in astronomical units (AU). Then

$$\pi = \frac{8.794}{r}\ \text{arcsec}$$

and

$$\alpha' = \alpha - \frac{\pi\sin H \times \rho\cos\phi'}{\cos\delta},$$

$$\delta' = \delta - \pi\left(\rho\sin\phi'\cos\delta - \rho\cos\phi'\cos H\sin\delta\right).$$

π is the symbol often used for parallax. Take care to distinguish its use for parallax from its use to represent the circular constant 3.141 592 654.

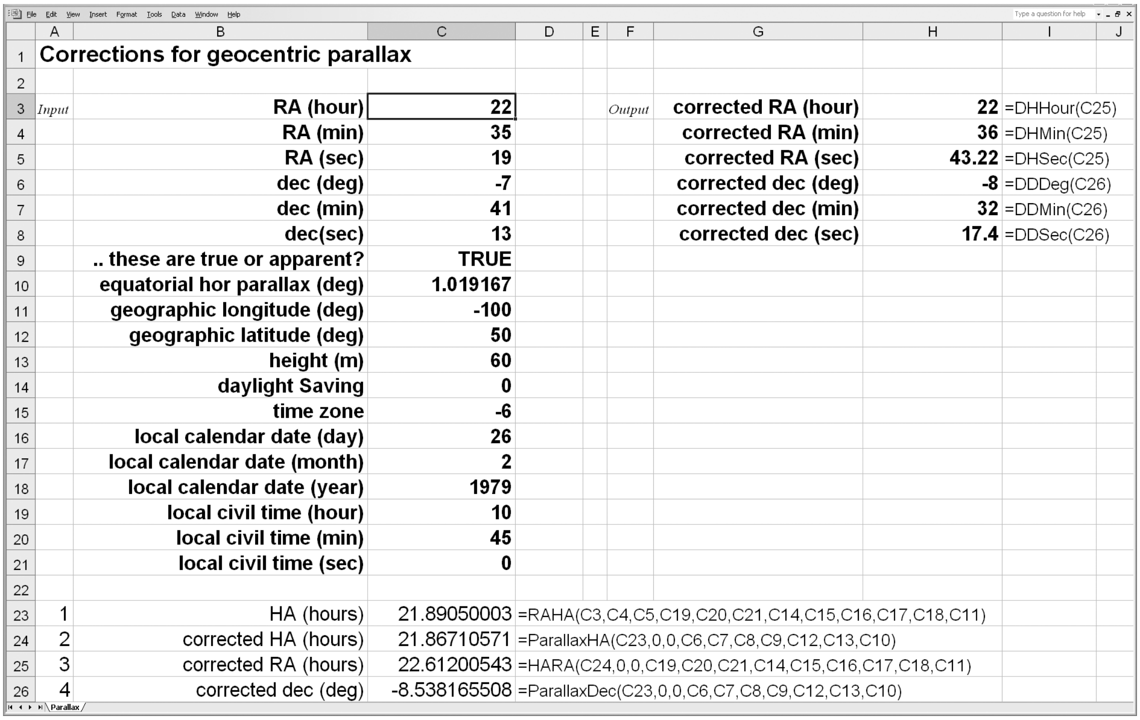
Let us now calculate the apparent position of the Sun when observed by the same observer at the same time as in the previous example. The geocentric right ascension of the Sun was 22h 36m 44s and its declination was −8° 44′ 24″. Its distance was 0.9901 AU.

Method		*Example*		
1.	Calculate $\pi = \frac{8.794}{r}$.	π	=	8.881 931 arcsec
2.	Convert to degrees by dividing by 3600.	π	=	0.002 467 degrees
3.	Convert to hours by dividing by 15 (§22).	π	=	0.000 164 hours
4.	Convert UT to GST and hence to	GST	=	3.145 778 hours
	LST by the methods of §§12 and 14.	LST	=	20.479 111 hours
5.	Convert α and δ to decimal form	α	=	22.612 222 hours
	(§§7 and 21).	δ	=	−8.740 000 degrees
6.	Find $\rho\cos\phi'$ and $\rho\sin\phi'$ (§38).	$\rho\cos\phi'$	=	0.644 060
		$\rho\sin\phi'$	=	0.762 422
7.	Find the hour angle, H, and	H	=	−2.133 111 hours
	convert to degrees (§§24 and 22).		=	−31.996 666 degrees
8.	Calculate $\Delta_1 = \frac{\pi \sin H \rho\cos\phi'}{\cos\delta}$ (π expressed in hours).	Δ_1	=	−0.000 051 hours
9.	Subtract from α to get α'.	α'	=	22.612 279 hours
10.	Find $\Delta_2 = \pi(\rho\sin\phi'\cos\delta - \rho\cos\phi'\cos H\sin\delta)$ (π expressed in degrees).	Δ_2	=	0.002 064 degrees
11.	Subtract from δ to get δ'.	δ'	=	−8.742 064 degrees
12.	Convert α' and δ' to minutes and	α'	=	**22h 36m 44s**
	seconds form (§§8 and 21).	δ'	=	**−8° 44′ 31″**

Note that the correction for parallax has had hardly any effect in this case. Except for the Moon, geocentric parallax can often be ignored. Note also that the Sun and Moon have almost the same apparent positions in this example; we have chosen the moment of a total solar eclipse (see Section 74).

The calculations of Sections 38 and 39 have been swept up into just one spreadsheet, called Parallax, shown in Figure 44. Rather than carrying out all the calculations of the last three method tables explicitly, we have put them into two spreadsheet functions called ParallaxHA and ParallaxDec, returning respectively the corrected hour angle in decimal hours and the corrected declination in decimal degrees. Each takes the same ten arguments, these being the hour angle as hours, minutes, seconds, the declination as degrees, minutes, seconds, a text word set to TRUE or APPARENT specifying whether the hour angle and declination are true coordinates or are apparent coordinates, the geographical latitude in degrees, the height above sea-level in metres, and the horizontal parallax in degrees. These functions are used in rows 24 and 26 of the spreadsheet. The example given in Figure 44 is the same as that of the first method table of this section, except that the local civil time has been specified for time zone −6 h; the result is the same.

Although the usual calculation is to find the apparent coordinates given the true coordinates (setting the switch in cell C9 to TRUE), the spreadsheet also allows the calculation to be done the other way around. Try this for yourself. Write down the apparent coordinates and then enter them into the spreadsheet in cells C3 to C8, setting also cell C9 to APPARENT (or anything which does not begin with T or t). You should find the true (geocentric) coordinates returned in cells H3 to H8 correct to within the second decimal place in the seconds part.



	A	B	C	D	E	F	G	H	I
1	Corrections for geocentric parallax								
2									
3	*Input*	RA (hour)	22			*Output*	corrected RA (hour)	22	=DHHour(C25)
4		RA (min)	35				corrected RA (min)	36	=DHMin(C25)
5		RA (sec)	19				corrected RA (sec)	43.22	=DHSec(C25)
6		dec (deg)	-7				corrected dec (deg)	-8	=DDDeg(C26)
7		dec (min)	41				corrected dec (min)	32	=DDMin(C26)
8		dec(sec)	13				corrected dec (sec)	17.4	=DDSec(C26)
9		.. these are true or apparent?	TRUE						
10		equatorial hor parallax (deg)	1.019167						
11		geographic longitude (deg)	-100						
12		geographic latitude (deg)	50						
13		height (m)	60						
14		daylight Saving	0						
15		time zone	-6						
16		local calendar date (day)	26						
17		local calendar date (month)	2						
18		local calendar date (year)	1979						
19		local civil time (hour)	10						
20		local civil time (min)	45						
21		local civil time (sec)	0						
22									
23	1	HA (hours)	21.89050003	=RAHA(C3,C4,C5,C19,C20,C21,C14,C15,C16,C17,C18,C11)					
24	2	corrected HA (hours)	21.86710571	=ParallaxHA(C23,0,0,C6,C7,C8,C9,C12,C13,C10)					
25	3	corrected RA (hours)	22.61200543	=HARA(C24,0,0,C19,C20,C21,C14,C15,C16,C17,C18,C11)					
26	4	corrected dec (deg)	-8.538165508	=ParallaxDec(C23,0,0,C6,C7,C8,C9,C12,C13,C10)					

Figure 44. Correcting equatorial coordinates for the effects of parallax.

40 Heliographic coordinates

Heliographic coordinates enable us to define the position of any point (such as a sunspot) on the surface of the Sun. As with any other set of astronomical coordinates, latitudes are referred to a fundamental plane and longitudes to a fixed point in that plane. In this case the fundamental plane is taken to be the solar equator, inclined at an angle $I = 7° \ 15'$ to the ecliptic, and the fixed point is the present position of the point occupied by the ascending node of the solar equator on the ecliptic at noon on 1 January 1854 (JD 2 398 220.0). There are no permanent features on the Sun's disc by which we can locate this point, so we have to work out its position assuming a rotation period of 25.38 days.

The situation is illustrated in Figure 45. The sphere represents the surface of the Sun and the great circle ONJ the solar equator. P_NC is the rotation axis of the Sun and any point on the equator rotates in the direction from O to N. The plane of the ecliptic intersects the Sun's surface along the great circle ♈EN; the point N is therefore the ascending node of the solar equator on the plane of the ecliptic. Imaginary lines drawn from the centre of the Sun, C, towards the Earth and towards the vernal equinox cut the Sun's surface at E and ♈ respectively. O is the point which, at midday on 1 January 1854, was at N. A sunspot at X has heliographic latitude B (positive north of the equator, negative south of it) and heliographic longitude L, reckoned in the same sense as the solar rotation and measured along the equator from O.

When we observe the Sun (which, to avoid permanent injury to the eye, must only be by projection onto a screen, or using proper solar filters) we see a flat disc, the centre of which is the point E. This is shown in Figure 46, together with the point P_N, the north pole of the Sun's rotation axis, and X, the position of the sunspot. The dashed line N′S′ represents the projection of the Earth's axis of rotation, NS, onto the disc. We define the position of X by the coordinates ρ_1 and θ. The trick is to turn ρ_1 and θ into B and L.

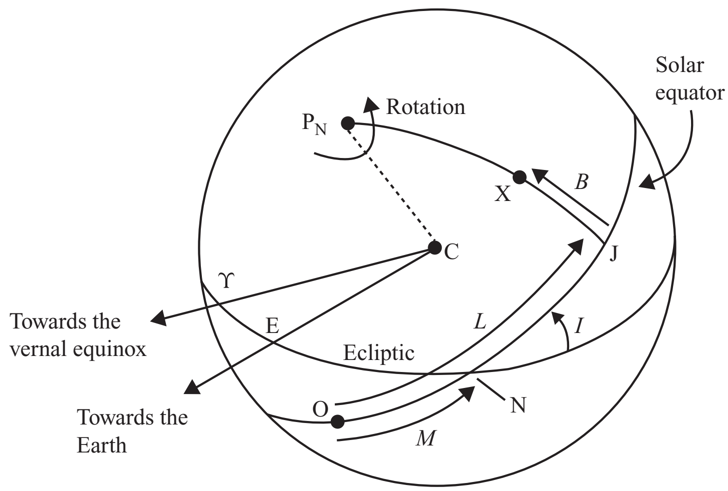


Figure 45. Defining heliographic coordinates.

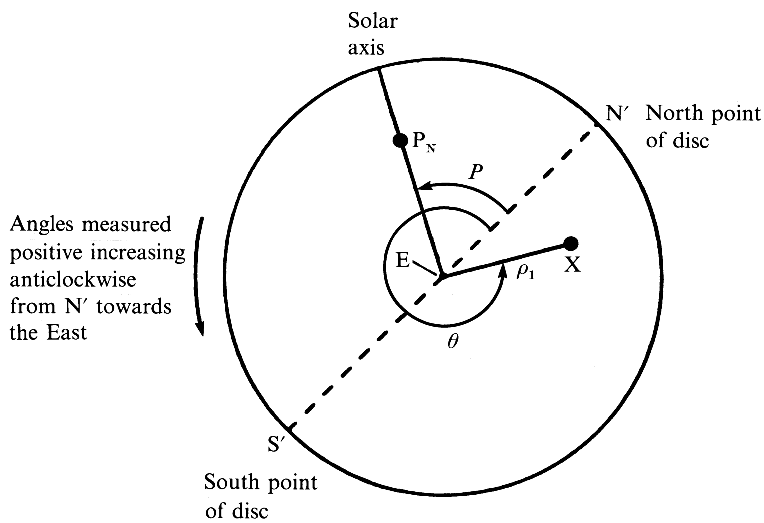


Figure 46. The Sun's disc.

To do this we need first to calculate the heliographic coordinates, B_0 and L_0, of the centre of the disc, E. The equations are:

$$L_0 = \tan^{-1}\left\{\frac{\sin\left(\Omega - \lambda_{\odot}\right)\cos I}{-\cos\left(\Omega - \lambda_{\odot}\right)}\right\} + M,$$

$$B_0 = \sin^{-1}\left\{\sin\left(\lambda_{\odot} - \Omega\right)\sin I\right\},$$

where $\lambda_{\odot}$ is the geocentric ecliptic longitude of the Sun, I is the inclination of the solar equator to the plane of the ecliptic (= $7^\circ\ 15'$), Ω is the longitude of the ascending node (the angle ♈N in Figure 45), and M is the angle between O and N (Figure 45; here and elsewhere the angles on the celestial sphere are subtended at the centre C, and measured along the corresponding great-circle arcs.). Ω is given by

$$\Omega = 74^\circ 22' + 84' T,$$

where T is the number of Julian centuries since the epoch 1900 January 0.5†. M is given by

$$M = 360 - M',$$

with

$$M' = \frac{360}{25.38}\,(\mathrm{JD} - 2\,398\,220.0)\,,$$

where JD is the Julian date. M' must be reduced to the range 0 to 360 by subtracting integral multiples of 360.

For example, let us calculate the heliographic coordinates of the centre of the solar disc on 1 May 1988. The geocentric longitude of the Sun, $\lambda_{\odot}$, can be found by the method described in Section 46; its value at 0h UT on this day was $40^\circ\ 50'\ 37''$.

†Note that this definition of T is different from that used elsewhere in the book.

Method		*Example*		
1.	Find the Julian date (§4).	JD	=	2 447 282.5
2.	Subtract 2 415 020 and divide by 36 525 to find T in centuries since 1900 January 0.5.	T	=	0.883 299 centuries
3.	Calculate $\Delta = 84T$. Divide by 60 to convert to degrees.	Δ	=	74.197 125 arcmin
			=	1.236 619 degrees
4.	Convert 74° 22′ to decimal degrees (§21) and add to Δ to find Ω.	Ω	=	75.603 285 degrees
5.	Convert $\lambda_\odot$ to decimal degrees (§21). (You can calculate $\lambda_\odot$ by the method of §46.)	$\lambda_\odot$	=	40.843 611 degrees
6.	Find $y = \sin\left(\Omega - \lambda_\odot\right)\cos I$ (I = 7.25 degrees).	$\Omega - \lambda_\odot$	=	34.759 674 degrees
		y	=	0.565 577
7.	Find $x = -\cos\left(\Omega - \lambda_\odot\right)$.	x	=	−0.821 551
8.	Find $A' = \tan^{-1}\left(\frac{y}{x}\right)$. We have to remove the ambiguity introduced by taking inverse tan. To do so, look up Figure 29 and add or subtract 180 or 360 until A' is in the correct quadrant. If it is already in the correct quadrant, $A = A'$.	A'	=	−34.544 552
				y positive
				x negative
			+	180.0
		A	=	145.455 448
9.	Calculate $M' = \frac{360}{25.38}(\text{JD} - 2\,398\,220)$. Subtract multiples of 360 to bring it back into the range 0 to 360.	M'	=	695 921.985 816
			−	360 × 1933
			=	41.985 816 degrees
10.	Find $M = 360 - M'$.	M	=	318.014 184 degrees
11.	Add M to A to find L_0. Subtract 360 if more than 360.	L_0	=	**103.47 degrees**
12.	Calculate $B_0 = \sin^{-1}\left\{\sin\left(\lambda_\odot - \Omega\right)\sin I\right\}$.	B_0	=	**−4.13 degrees**

The *Astronomical Almanac* lists these values as L_0 = 103.47 degrees and B_0 = −4.12 degrees.

In addition to B_0 and L_0 we also need the position-angle of the Sun's rotation axis, the angle P in Figure 46. This is given by

$$P = \theta_1 + \theta_2,$$

with

$$\theta_1 = \tan^{-1}\left\{-\cos\lambda_\odot \tan\varepsilon\right\}$$

and

$$\theta_2 = \tan^{-1}\left\{-\cos\left(\Omega - \lambda_\odot\right)\tan I\right\},$$

where ε is the obliquity of the ecliptic (see Section 27).

For example, what was the value of P on 1 May 1988?

Method		*Example*		
	Referring to the previous example for values of $\lambda_\odot$, Ω and I:	$\lambda_\odot$	=	40.843 611 degrees
		Ω	=	75.603 285 degrees
		I	=	7.25 degrees
1.	Calculate $\theta_1 = \tan^{-1}\left\{-\cos\lambda_\odot \tan\varepsilon\right\}$ (with ε = 23.442 degrees).	θ_1	=	−18.160 747 degrees
2.	Calculate $\theta_2 = \tan^{-1}\left\{-\cos\left(\Omega - \lambda_\odot\right)\tan I\right\}$.	θ_2	=	−5.966 575 degrees
3.	Find $P = \theta_1 + \theta_2$.	P	=	**−24.127 321 degrees**

The *Astronomical Almanac* gives $P = -24.11$ degrees.

We are now in a position to calculate the heliographic coordinates of the sunspot X, given its position-angle, θ (see Figure 46), and ρ_1, the angle subtended at the Earth by X and E. The formulas are as follows:

$$B = \sin^{-1}\left\{\sin B_0 \cos\rho + \cos B_0 \sin\rho \cos(P-\theta)\right\},$$

$$L = \sin^{-1}\left\{\frac{\sin\rho \sin(P-\theta)}{\cos B}\right\} + L_0,$$

with

$$\rho = \sin^{-1}\left\{\frac{\rho_1}{S}\right\} - \rho_1,$$

where S is the angular radius of the Sun.

Continuing our example, what were the heliographic coordinates of a sunspot measured at position-angle $\theta = 220^\circ$ and displacement $\rho_1 = 10.5$ arcminutes on 1 May 1988? The angular radius of the Sun was $15' 52''$.

Method		*Example*		
1.	Calculate L_0, B_0 and P (see previous examples).	L_0	=	103.47 degrees
		B_0	=	−4.13 degrees
		P	=	−24.11 degrees
2.	Find $\sin^{-1}\left(\frac{\rho_1}{S}\right)$ (remember to	S	=	15.867 arcmin
	convert S to decimal arcminutes first).	$\sin^{-1}\left(\frac{\rho_1}{S}\right)$	=	41.435 degrees
3.	Convert ρ_1 to decimal degrees (§21)	ρ_1	=	0.175 degrees
	and subtract to find ρ.	ρ	=	41.260 degrees
4.	Calculate $B = \sin^{-1}\{\sin B_0 \cos\rho +$	$(P-\theta)$	=	−244.127 degrees
	$\cos B_0 \sin\rho \cos(P-\theta)\}$.	B	=	**−19.945 degrees**
5.	Calculate $A = \sin^{-1}\left\{\frac{\sin\rho\sin(P-\theta)}{\cos B}\right\}$.	A	=	39.141 degrees
6.	Add L_0 to find L; subtract 360 if L is greater than 360.	L	=	**142.611 degrees**

All of the calculations of this section are made in one spreadsheet called Heliographic (see Figure 47). It makes a forward reference to finding the position of the Sun, and uses the spreadsheet function SunLong referred to in Figure 47 (row 12). The small differences between the answers calculated by the spreadsheet and those calculated in the method table above are caused by rounding errors, and by the difference between the solar longitudes calculated by SunLong (higher precision) and the method of Section 46 (lower precision). We have also provided spreadsheet functions HeliogLong and HeliogLat to carry out the calculation. These return, respectively, the Heliographic Longitude and the Heliographic Latitude, both in degrees. Each function takes the same five arguments. They are the heliographic position angle in degrees, the heliographic displacement from the centre of the Sun's disc in arcminutes, and the Greenwich calendar date as day, month and year. You can simplify the spreadsheet by using these functions instead of the calculation part. Save a copy, and then delete rows 9 to 29 inclusive. Insert the following spreadsheet formulas into cells H3 and H4:

```
=ROUND(HeliogLong(C3,C4,C5,C6,C7),2)
=ROUND(HeliogLat(C3,C4,C5,C6,C7),2).
```

	A	B	C	D	E	F	G	H	I	J
1	**Heliographic coordinates**									
2										
3	*Input*	**helio position angle (deg)**	**220**			*Output*	**helio long (deg)**	**142.59**	=ROUND(C29,2)	
4		**helio displacement (arcmin)**	**10.5**				**helio lat (deg)**	**-19.94**	=ROUND(C27,2)	
5		**Greenwich date day**	**1**							
6		**Greenwich date month**	**5**							
7		**Greenwich date year**	**1988**							
8										
9	1	Julian date (days)	2447282.5	=CDJD(C5,C6,C7)						
10	2	*T* (centuries)	0.88329911	=(C9-2415020)/36525						
11	3	Long asc node (deg)	75.60328542	=DMSDD(74,22,0)+(84*C10/60)						
12	4	Sun long (deg)	40.84263343	=SunLong(0,0,0,0,0,C5,C6,C7)						
13	5	*y*	0.565591133	=SIN(RADIANS(C11-C12))*COS(RADIANS(DMSdd(7,15,0)))						
14	6	*x*	-0.821540954	=-COS(RADIANS(C11-C12))						
15	7	*A* (deg)	145.4544725	=DEGREES(ATAN2(C14,C13))						
16	8	*M* (deg)	-695561.9858	=360-(360*(C9-2398220)/25.38)						
17	9	*M* (deg)	318.0141844	=C16-360*INT(C16/360)						
18	10	*L* 0 (deg)	463.4686569	=C17+C15						
19	11	*L* 0 (deg)	103.4686569	=C18-360*INT(C18/360)						
20	12	*B* 0 (rad)	-0.07201451	=ASIN(SIN(RADIANS(C12-C11))*SIN(RADIANS(DMSdd(7,15,0))))						
21	13	theta1 (rad)	-0.316988267	=ATAN(-COS(RADIANS(C12))*TAN(RADIANS(obliq(C5,C6,C7))))						
22	14	theta2 (rad)	-0.104135152	=ATAN(-COS(RADIANS(C11-C12))*TAN(RADIANS(DMSdd(7,15,0))))						
23	15	*P* (deg)	-24.12859453	=DEGREES(C21+C22)						
24	16	rho1 (deg)	0.175	=C4/60						
25	17	rho (rad)	0.71975524	=ASIN(2*C24/SunDia(0,0,0,0,0,C5,C6,C7))-RADIANS(C24)						
26	18	*B* (rad)	-0.347984957	=ASIN(SIN(C20)*COS(C25)+COS(C20)*SIN(C25)*COS(RADIANS(C23-C3)))						
27	19	*B* (deg)	-19.93806936	=DEGREES(C26)						
28	20	*L* (deg)	502.5889605	=DEGREES(ASIN(SIN(C25)*SIN(RADIANS(C23-C3))/COS(C26)))+C18						
29	21	*L* (deg)	142.5889605	=C28-360*INT(C28/360)						

Figure 47. The calculation of heliographic coordinates.

41 Carrington rotation numbers

Solar rotations are numbered by the Carrington rotation number, CRN, the first of which began on 9 November 1853. One rotation is the period during which the value of L_0 (Section 40) decreases by 360°, and its mean length is 27.2753 days. We can calculate CRN quite accurately by noting from the *Astronomical Ephemeris* that rotation number 1690 began on 1979 December 27.84. Thus

$$\mathrm{CRN} = 1690 + \left[\frac{\mathrm{JD} - 2\,444\,235.34}{27.2753}\right],$$

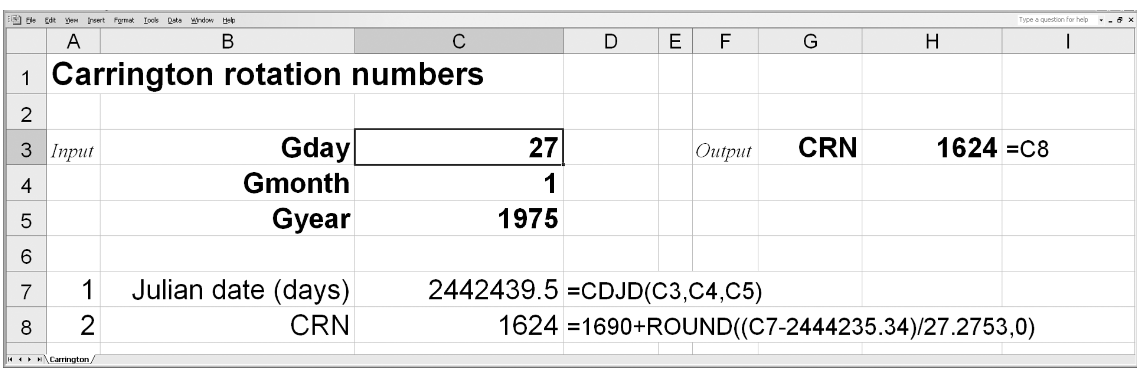
where JD is the Julian date. Round the result to the nearest integer. You may be in error by ± 1 just at the point where the rotation number changes.

For example, what was the CRN on 27 January 1975?

Method		*Example*		
1.	Calculate the Julian date (§4).	JD	=	2 442 439.50
2.	Find CRN $= 1690 + \left[\frac{\mathrm{JD}-2\,444\,235.34}{27.2753}\right]$, rounding the result to the nearest integer.	CRN	=	**1624**

The spreadsheet for this straightforward calculation is shown in Figure 48. We have also provided the spreadsheet function CRN which returns the Carrington Rotation Number for the given Greenwich calendar date specified in the three arguments as day, month, and year. Thus you could, if you wished, get the same result by deleting rows 7 and 8 of the spreadsheet and inserting the following formula in cell H3 (save a copy of the spreadsheet first!):

=CRN(C3,C4,C5).



	A	B	C	D	E	F	G	H	I
1	**Carrington rotation numbers**								
2									
3	*Input*	**Gday**	**27**			*Output*	**CRN**	**1624**	=C8
4		**Gmonth**	**1**						
5		**Gyear**	**1975**						
6									
7	1	Julian date (days)	2442439.5	=CDJD(C3,C4,C5)					
8	2	CRN	1624	=1690+ROUND((C7-2444235.34)/27.2753,0)					

Figure 48. Carrington rotation number calculation.

42 Selenographic coordinates

The position of any point on the surface of the Moon can be described by means of a pair of **selenographic coordinates**. As with all other astronomical systems, latitudes are referred to a fundamental plane, and longitudes to a fixed point in that plane. In this case, the plane is taken to be the Moon's equator, inclined at an angle of $I = 1° \ 32' \ 32.7''$ to the plane of the ecliptic. However, there is a problem with defining a fixed point of zero longitude because the Moon wobbles about so much. In particular, the rotation axis of the Moon oscillates about its mean position, an effect known as **physical libration**, and the mean position itself does not have a fixed sidereal direction, but is subject to a regular precession with a period of 18.6 years. Nevertheless, we can identify a point on the lunar surface which has an average position exactly in the centre of the apparent disc as it would be seen from the centre of the Earth. This is the reference point for measuring longitudes. The actual centre of the apparent lunar disc at any time may be as much as 8° in longitude and 6° in latitude away from the average position.

The selenocentric celestial sphere (i.e. the celestial sphere centred on the Moon) is shown in Figure 49. The traces of the planes of the ecliptic and the lunar equator are drawn, and they intersect with one another at angle I. The point P_0 represents the pole of the equator, and M is the mean centre of the Moon's apparent disc as observed from the centre of the Earth. The great circle through P_0 and M is the prime meridian, longitude 0°. Point R is a crater on the surface of the Moon, and the great circle through P_0 and R cuts the equator at X. Then the selenographic longitude of R is the angle l measured from M along the equator to X, and the selenographic latitude is the angle b subtended at the centre of the Moon by X and R. Longitudes increase to the west (towards Mare Crisium) and latitudes to the north, as seen on the apparent disc.

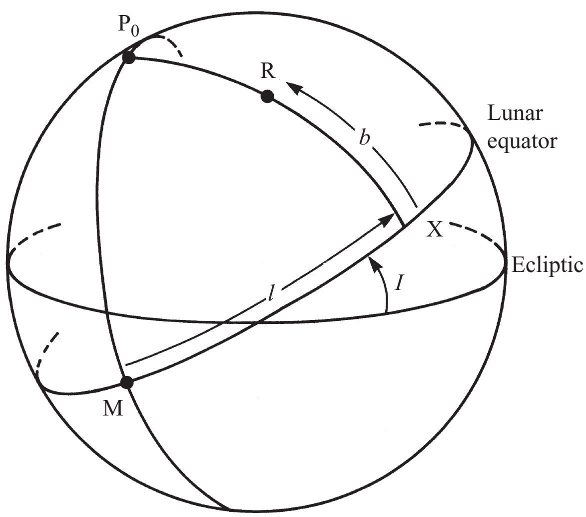


Figure 49. Selenographic coordinates.

The point which, at any moment, is in the centre of the apparent lunar disc is called the **sub-Earth point**. Its selenographic coordinates, l_e, and b_e, may be calculated from the equations

$$b_e = \sin^{-1}\left[-\cos I \sin\beta + \sin I \cos\beta \sin(\Omega - \lambda_m)\right],$$

$$l_e = \tan^{-1}\left\{\frac{-\sin\beta\sin I - \cos\beta\cos I\sin(\Omega - \lambda_m)}{\cos\beta\cos(\Omega - \lambda_m)}\right\} - F,$$

where λ_m and β are the apparent geocentric coordinates of the Moon, Ω is the longitude of the ascending node of the Moon's mean orbit on the ecliptic, and $F = L - \Omega$, where L is the mean longitude of the Moon. Ω and F can be found from the approximate relations

$$\Omega = 125.044\,522 - 1934.136\,261 T \text{ degrees},$$

$$F = 93.271\,910 + 483\,202.017\,5 T \text{ degrees},$$

where T is the number of Julian centuries of 36 525 days since the epoch 2000 January 1.5, i.e.

$$T = (\mathrm{JD} - 2451\,545.0)/36\,525.$$

The position-angle, C, of the pole of the lunar equator (P_0) is measured in the same way as position-angles on the surface of the Sun, i.e. anticlockwise from the north point of the apparent lunar disc. We can calculate it from the following set of equations:

$$C = C_1 + C_2,$$

$$C_1 = \tan^{-1}\left\{\frac{\cos(\Omega - \lambda_m)\sin I}{\cos\beta\cos I + \sin\beta\sin I\sin(\Omega - \lambda_m)}\right\},$$

$$C_2 = \tan^{-1}\left\{\frac{\sin\varepsilon\cos\lambda_m}{\sin\varepsilon\sin\beta\sin\lambda_m - \cos\varepsilon\cos\beta}\right\},$$

where ε is the obliquity of the ecliptic (Section 27).

We shall find as an example of these calculations the values of l_e, b_e and C on 1 May 1988. The Moon's geocentric longitude was 209.12°, and latitude −3.08°. The obliquity was 23.4433°.

Method		*Example*		
1.	Convert the date to the Julian day number (§4).	JD	=	2 447 282.5
2.	Calculate $T = (\mathrm{JD} - 2\,451\,545.0)/36\,525$.	T	=	$-0.116\,701$
3.	Find the value of Ω using the equation given in this section. Reduce to the range 0 to 360 if necessary by adding or subtracting multiples of 360.	Ω	=	350.759 945 degrees
4.	Find the value of F using the equation given in this section. Reduce to the range 0 to 360.	F	=	223.166 514 degrees
5.	Find $\sin b_\mathrm{e} = (-\cos I \sin\beta + \sin I \cos\beta \sin(\Omega - \lambda_\mathrm{m}))$.	$\sin b_\mathrm{e}$	=	0.070 391
6.	Take the inverse sine to find b_e.	b_e	=	**4.04 degrees**
7.	Find, and note the sign of, $y = (-\sin\beta \sin I - \cos\beta \cos I \sin(\Omega - \lambda_\mathrm{m}))$.	y	=	$-0.618\,034$ (negative)
8.	Find, and note the sign of, $x = \cos\beta \cos(\Omega - \lambda_\mathrm{m})$.	x	=	$-0.782\,994$ (negative)
9.	Take $A = \tan^{-1}\left(\frac{y}{x}\right)$. Consult Figure 29 and add or subtract 180 or 360 to bring the result into the quadrant specified by the signs of x and y.	A	=	38.284 805 degrees
			+	180.0
		A	=	218.284 805 degrees
10.	Subtract F from A and reduce the result to the range -180 to $+180$ by adding or subtracting 360. This is l_e.	l_e	=	**−4.88 degrees**
11.	Calculate the values of C_1 and C_2 using the equations given in this section. There is no need to worry about ambiguities of 180° when taking inverse tan since, in this case, the results will always be in the correct quadrants.	C_1	=	$-1.212\,401$ degrees
		C_2	=	20.993 347 degrees
12.	Add C_1 and C_2 to find the position angle C.	C	=	**19.78 degrees**

The *Astronomical Almanac* gives values of $l_\mathrm{e} = -4.918$, $b_\mathrm{e} = 4.051$ and $C = 19.761$, having taken account also of physical libration.

The spreadsheet for finding the coordinates of the sub-Earth point and the position-angle of the equator is shown in Figure 50, labelled Selenographic1. It steals spreadsheet functions from Section 65 to find the Moon's longitude and latitude (rows 12 and 13), but otherwise follows the method given in the method table. We have not provided a corresponding spreadsheet function.

The selenographic coordinates of the Sun (strictly, the **sub-solar** point) can also be found quite easily. We can use the same equations as for calculating l_e and b_e, but with λ_m and β replaced by the heliocentric ecliptic coordinates of the Moon, λ'_m, and β'. Their values may be found by the approximations

$$\lambda'_\mathrm{m} = \lambda_\odot + 180 + \left\{ \frac{26.4 \cos\beta \sin\left(\lambda_\odot - \lambda_\mathrm{m}\right)}{\pi R} \right\} \text{ degrees},$$

$$\beta' = \frac{0.146\,66\beta}{\pi R} \text{ degrees},$$

where π is the equatorial horizontal parallax of the Moon (see Section 69) expressed in arcminutes, R is the Sun–Earth distance in astronomical units (Section 48), and $\lambda_\odot$ is the true geocentric longitude of the Sun (Section 46). Note that the Sun's selenographic longitude is often expressed as the (selenographic) **colongitude**, which is just 90° − longitude (reduced to the range 0 to 360 by adding or subtracting 360). If we know the colongitude of the Sun, then the selenographic longitude of the morning **terminator** (the division between night and morning on the Moon) is approximately 360° − colongitude of the Sun, and the selenographic longitude of the evening terminator is approximately 180° − colongitude of the Sun.

	A	B	C	D	F	G	H	I
1	**Selenographic coordinates 1**							
2								
3	*Input*	**Greenwich date (day)**	**1**		*Output*	**sub-Earth longitude**	**-4.88**	=ROUND(C22,2)
4		**Greenwich date (month)**	**5**			**sub-Earth latitude**	**4.04**	=ROUND(C17,2)
5		**Greenwich date (year)**	**1988**			**position angle of pole**	**19.78**	=ROUND(C26,2)
6								
7	1	Julian date (days)	2447282.5	=CDJD(C3,C4,C5)				
8	2	*T* (centuries)	-0.11670089	=(C7-2451545)/36525				
9	3	long asc node (deg)	350.7599447	=125.044522-1934.136261*C8				
10	4	*F*	-56296.83349	=93.27191+483202.0175*C8				
11	5	*F*	223.1665139	=C10-360*INT(C10/360)				
12	6	Geocentric Moon long (deg)	209.1175282	=MoonLong(0,0,0,0,0,C3,C4,C5)				
13	7	Geocentric Moon lat (rad)	-0.053838535	=RADIANS(MoonLat(0,0,0,0,0,C3,C4,C5))				
14	8	inclination (rad)	0.026920249	=RADIANS(DMSDD(1,32,32.7))				
15	9	node-long (rad)	2.472126528	=RADIANS(C9-C12)				
16	10	sin(be)	0.070472641	=-COS(C14)*SIN(C13)+SIN(C14)*COS(C13)*SIN(C15)				
17	11	sub-Earth lat (deg)	4.041134615	=DEGREES(ASIN(C16))				
18	12	*A* (rad)	-2.473441682	=ATAN2(COS(C13)*COS(C15),-SIN(C13)*SIN(C14)-COS(C13)*COS(C14)*SIN(C15))				
19	13	*A* (deg)	-141.7177693	=DEGREES(C18)				
20	14	sub-Earth long (deg)	-364.8842831	=C19-C11				
21	15	sub-Earth long (deg)	355.1157169	=C20-360*INT(C20/360)				
22	16	sub-Earth long (deg)	-4.884283136	=IF(C21>180,C21-360,C21)				
23	17	*C* 1 (rad)	-0.021161235	=ATAN(COS(C15)*SIN(C14)/(COS(C13)*COS(C14)+SIN(C13)*SIN(C14)*SIN(C15)))				
24	18	obliquity (rad)	0.409163585	=RADIANS(OBLIQ(C3,C4,C5))				
25	19	*C* 2 (rad)	0.366418907	=ATAN(SIN(C24)*COS(RADIANS(C12))/(SIN(C24)*SIN(C13)*SIN(RADIANS(C12))-COS(C24)*COS(C13)))				
26	20	*C* (deg)	19.78180743	=DEGREES(C23+C25)				

Selenographic1

Figure 50. Calculating the selenographic coordinates of the sub-Earth point and the position angle of the lunar equator.

Continuing our previous example: what were the selenographic coordinates of the Sun on 1 May 1988 at 0h UT? The equatorial horizontal parallax of the Moon was 55.952 arcminutes, the Sun–Earth distance was 1.0076 AU, and the true geocentric longitude of the Sun was 40.8437°.

Method		*Example*		
1.	Find the values of λ'_m and β' using the expressions given above.	λ'_m	=	220.749 degrees
		β'	=	−0.008 degrees
2.	Repeat the calculations in steps 1–10 in the previous example, using λ'_m and β' instead of λ_m and β.	b_s	=	**1.19 degrees**
		y	=	−0.766
		x	=	−0.643
		A	=	229.978 degrees
		l_s	=	**6.811 degrees**
3.	Subtract l_s from 90 and reduce to the range 0 to 360 by adding or subtracting 360. This is the colongitude.	colong.	=	**83.19 degrees**

The *Astronomical Almanac* gives the coordinates as colongitude = 83.16° and latitude = 1.2°.

The spreadsheet for calculating the selenographic coordinates of the sub-solar point is shown in Figure 51, called Selenographic2. It follows the method given in the method table above except that it makes use of five spreadsheet functions defined in later sections to obtain the Sun's longitude (row 12; Section 47), the Moon's equatorial horizontal parallax (row 13; Section 65), the distance between the Sun and the Earth (row 14; Section 48), and the Moon's latitude and longitude (rows 15 and 16; Section 65). We have not provided a corresponding spreadsheet function for this calculation.

	A	B	C	D	E	F	G	H	I	J
1	**Selenographic coordinates 2**									
2										
3	*Input*	**Greenwich date (day)**	1			*Output*	**sub-solar longitude**	**6.81**	=ROUND(C27,2)	
4		**Greenwich date (month)**	5				**sub-solar colongitude**	**83.19**	=ROUND(C28,2)	
5		**Greenwich date (year)**	**1988**				**sub-solar latitude**	**1.19**	=ROUND(C22,2)	
6										
7	1	Julian date (days)	2447282.5	=CDJD(C3,C4,C5)						
8	2	*T* (centuries)	-0.11670089	=(C7-2451545)/36525						
9	3	long asc node (deg)	350.7599447	=125.044522-1934.136261*C8						
10	4	*F*	-56296.83349	=93.27191+483202.0175*C8						
11	5	*F*	223.1665139	=C10-360*INT(C10/360)						
12	6	Sun geocentric long (deg)	40.84263343	=SunLong(0,0,0,0,0,C3,C4,C5)						
13	7	Moon equ hor parallax (arc min)	55.95238522	=MoonHP(0,0,0,0,0,C3,C4,C5)*60						
14	8	Sun-Earth dist (AU)	1.00760326	=SunDist(0,0,0,0,0,C3,C4,C5)						
15	9	geocentric Moon lat (rad)	-0.053838535	=RADIANS(MoonLat(0,0,0,0,0,C3,C4,C5))						
16	10	geocentric Moon long (deg)	209.1175282	=MoonLong(0,0,0,0,0,C3,C4,C5)						
17	11	adjusted Moon long (deg)	220.7476113	=C12+180+(26.4*COS(C15)*SIN(RADIANS(C12-C16))/(C13*C14))						
18	12	adjusted Moon lat (rad)	-0.000140054	=0.14666*C15/(C13*C14)						
19	13	inclination (rad)	0.026920249	=RADIANS(DMSdd(1,32,32.7))						
20	14	node-long (rad)	2.269143285	=RADIANS(C9-C17)						
21	15	sin(bs)	0.020755895	=-COS(C19)*SIN(C18)+SIN(C19)*COS(C18)*SIN(C20)						
22	16	sub-solar lat (deg)	1.189310602	=DEGREES(ASIN(C21))						
23	17	*A* (rad)	-2.269324173	=ATAN2(COS(C18)*COS(C20),-SIN(C18)*SIN(C19)-COS(C18)*COS(C19)*SIN(C20))						
24	18	*A* (deg)	-130.0226975	=DEGREES(C23)						
25	19	sub-solar long (deg)	-353.1892113	=C24-C11						
26	20	sub-solar long (deg)	6.810788676	=C25-360*INT(C25/360)						
27	21	sub-solar long (deg)	6.810788676	=IF(C26>180,C26-360,C26)						
28	22	sub-solar colong (deg)	83.18921132	=90-C27						

Figure 51. Calculating the selenographic coordinates of the Sun.

43 Atmospheric extinction

The light that reaches us on the surface of the Earth from heavenly bodies first has to pass through the atmosphere where some of it is scattered by dust, electrons, oxygen and nitrogen molecules, and other sundry particles. The amount of this **Rayleigh scattering** depends on the physical conditions in the atmosphere (it will be enhanced, for example, by extra dust from a volcanic eruption) and on the wavelength of the light. In general, the shorter wavelengths (blue) are scattered much more than the longer wavelengths (red); for this reason, the sky looks blue (we see the scattered light) and the apparent colour of a star observed from the Earth's surface is reddened. If we take the visual wavelengths as a whole, we can make a rough estimate of the amount of absorption to expect when the atmosphere is clear, from

$$\Delta m = \frac{0.2}{\cos z} \text{ magnitudes,}$$

where Δm is the quantity to be added to the **magnitude**, and z is the zenith angle ($z = 90° -$ altitude). For example, a planet whose altitude is 15° may appear dimmer by about 0.8 magnitudes in good conditions when the atmosphere is clear; in general this will be an underestimate since there are additional causes of absorption. The formula breaks down for zenith angles greater than about 85°.