The planets, comets and binary stars

An observer looking up at the night sky from the surface of the Earth sees an unchanging pattern of stars revolving slowly about the pole as the Earth spins on its axis. So great are the distances to the stars that the changing position of the Earth as it travels along its orbit around the Sun causes hardly any movement in the pattern, even in the course of six months. There are a few objects, however, which do appear to move a great deal with respect to this fixed background of stars. The objects are members of our Solar System, the planets, the asteroids and the comets. Eight major planets have been identified so far which, in order of increasing distance from the Sun, are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune. (Pluto was originally classified as a planet, but is now considered the largest member of another grouping of objects called the Kuiper belt.) These, together with other members of the Solar System, are all bound by the gravitational field of the Sun so that instead of moving off into space in different directions they are constrained to follow elliptical orbits about it. Their apparent motions in the sky are complicated because they are relatively close to us so that the position of the Earth in its own orbit needs to be taken into account. The next few sections contain methods for calculating the positions, angular sizes, distances, phases and brightnesses of the major planets. There are also sections describing how to calculate the orbit of a comet and the orbit of a binary star.

https://doi.org/10.1017/CBO9780511861161.007

53 The planetary orbits

Each planet in our Solar System describes an elliptical orbit about the Sun with the Sun at a focus of the ellipse. We discovered how to calculate the Sun–Earth orbit in Sections 44 to 47. This was a particularly simple case since the plane of the orbit defined the plane of the ecliptic; the ecliptic latitude was therefore always zero and the fundamental direction, the first point of Aries, was in the orbital plane. The other planets, however, do not move in the plane of the ecliptic but describe orbits inclined at small angles to it. Figure 63 shows the situation.

The Sun, S, is at the centre of the diagram and you are to imagine that you are looking at the path of a planet around the Sun from a great distance. The orbit of the planet is the small shaded ellipse N_1AP. The perihelion is marked A and the planet's present position is marked P. That part of the orbit which lies above the ecliptic is shown with solid lines, while that lying below it is shown with dashed lines. The large sphere is centred on the Sun and the plane of the planet's orbit is projected to cut the sphere along the circle $N_1'A'P'N_2'$. Here A' is the projection of A onto the sphere, P' the projection of P and so forth. Also shown in the diagram is the plane of the ecliptic $♈N_1'N_2'$, which contains the direction of ♈, the first point of Aries.

The planet moves along its orbit in the direction of the arrow. The point N_1 where it rises out of the plane of the ecliptic is called the **ascending node**. N_2, the point where it descends below the plane of the ecliptic, is called the **descending node**. Angles in the orbital plane are measured from the ascending node while longitudes are reckoned from the direction ♈ which is not in the orbital plane. Thus the perihelion is at an angle ω to the node (the 'argument' of the perihelion) and the present position of the planet is at an angle $\omega + \nu$. The corresponding longitudes are $\omega + \Omega$ and $\omega + \nu + \Omega$, where Ω is the longitude of the ascending node. Note that longitudes are the sum of two angles in different planes.

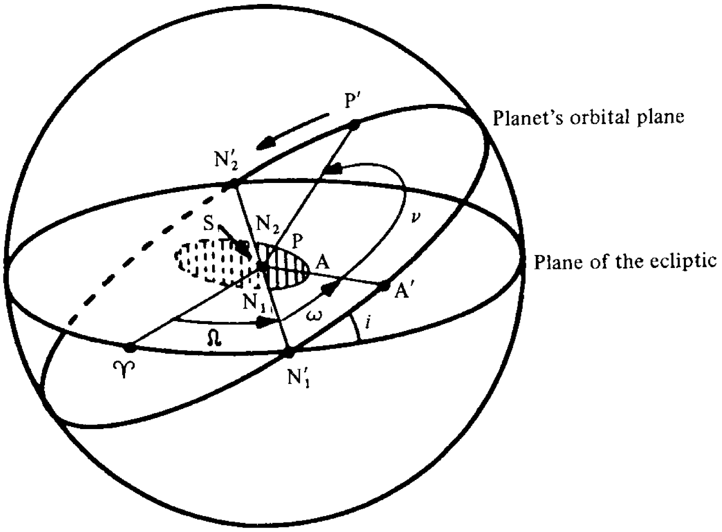


Figure 63. Defining the orbit of a planet.

54 Calculating the coordinates of a planet

Our calculation will proceed in three steps. The first is to calculate the position of the planet in its own orbital plane exactly as we did for the Sun–Earth orbit in Section 46. In the second step we will project the planet's calculated position onto the plane of the ecliptic and hence find its ecliptic longitude and latitude referred to the Sun (**heliocentric coordinates**). The third step will involve transforming from the Sun to the Earth to find the ecliptic coordinates referred to the Earth, from which we can find the right ascension and declination by the method given in Section 27.

As before, we choose our starting point, the epoch, as 2010.0. Having calculated the number of days, D, since the epoch, we find the mean anomaly, M, by the formula

$$M = \frac{360}{365.242\,191} \times \frac{D}{T_\mathrm{p}} + \varepsilon - \varpi \text{ degrees,}$$

where T_p is the orbital period of the planet in tropical years, ε is the mean longitude of the planet at the epoch, and ϖ is the longitude of the perihelion. These constants are listed for the planets in our Solar System in Table 8. This table is extracted from a list of **osculating elements** published on the web by the US Naval Observatory (see page 209).

Being osculating elements, they change with time and are valid only over a relatively short period. We can use the values in Table 8 for low-precision calculations, but should use the more-precise spreadsheet of Section 56 for extrapolations into the past or future of more than a few tens of years from 2010, or where higher accuracy is needed.

The mean anomaly refers to the motion of a fictitious planet, P_1, moving in a circle at constant speed with the same orbital period as the real planet (see Figure 64). We really want to know the value of the true anomaly, ν, which is the angle the real planet actually makes with the line joining the Sun to the perihelion. We can find ν from M using the equation of the centre:

$$\nu = M + \frac{360}{\pi} e \sin M \text{ degrees,}$$

where e is the eccentricity of the orbit (Table 8) and $\pi = 3.141\,592\,7$. Once again, this formula is an approximation that is good enough for most purposes; if you wish to make more precise calculations you can find the value of ν by solving Kepler's equation via the method of Section 47.

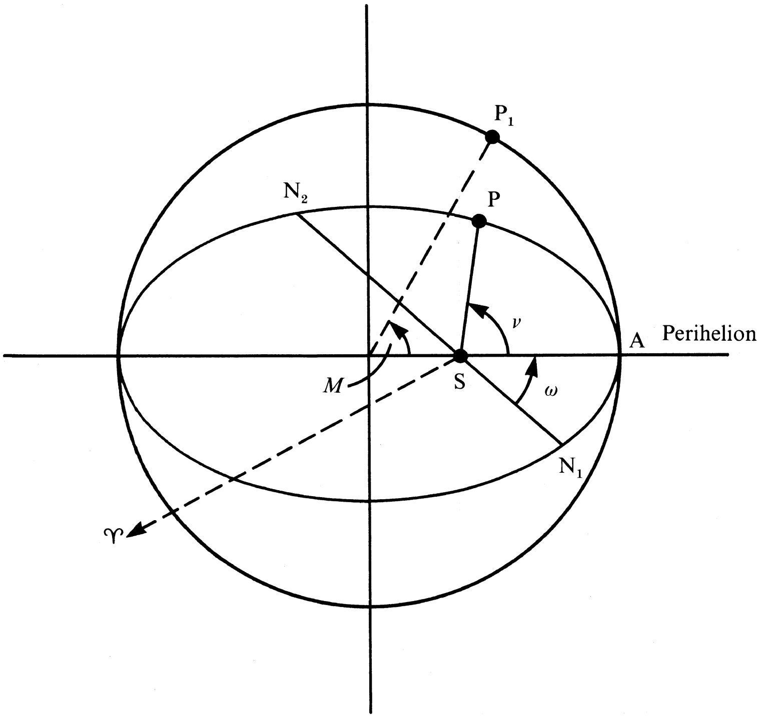


Figure 64. Mean and true anomalies.

	T_p (tropical years)	ε (degrees)	ϖ (degrees)	e	a (AU)	i (degrees)	Ω (degrees)	θ_0 (arcsec)	V_0
Mercury	0.240 85	75.567 1	77.612	0.205 627	0.387 098	7.005 1	48.449	6.74	−0.42
Venus	0.615 207	272.300 44	131.54	0.006 812	0.723 329	3.394 7	76.769	16.92	−4.40
Earth	0.999 996	99.556 772	103.205 5	0.016 671	0.999 985				
Mars	1.880 765	109.096 46	336.217	0.093 348	1.523 689	1.849 7	49.632	9.36	−1.52
Jupiter	11.857 911	337.917 132	14.663 3	0.048 907	5.202 78	1.303 5	100.595	196.74	−9.40
Saturn	29.310 579	172.398 316	89.567	0.053 853	9.511 34	2.487 3	113.752	165.60	−8.88
Uranus	84.039 492	271.063 148	172.884 833	0.046 321	19.218 14	0.773 059	73.926 961	65.80	−7.19
Neptune	165.845 39	326.895 127	23.07	0.010 483	30.198 5	1.767 3	131.879	62.20	−6.87

1 AU = 149.6×10^6 km.

T_p: period of orbit; ε: longitude at the epoch; ϖ: longitude of the perihelion; e: eccentricity of the orbit; a: semi-major axis of the orbit; i: orbital inclination; Ω: longitude of the ascending node; θ_0: angular diameter at 1 AU; V_0: visual magnitude at 1 AU.

Table 8. Elements of the planetary orbits at epoch 2010.0.

The next step is to calculate the heliocentric longitude, l, and this is simply given by

$$l = \nu + \varpi,$$

or

$$l = \left(\frac{360}{365.242\,191} \times \frac{D}{T_{\mathrm{p}}}\right) + \frac{360}{\pi} e \sin\left(\frac{360}{365.242\,191} \times \frac{D}{T_{\mathrm{p}}} + \varepsilon - \varpi\right) + \varepsilon \text{ degrees.}$$

We also need the length of the radius vector, r, calculated from

$$r = \frac{a\left(1-e^2\right)}{1+e\cos\nu},$$

where a is the semi-major axis of the orbit (Table 8).

The above calculations which you have made for the planet have to be repeated for the Earth as well. We shall denote the values derived for the planet by small letters and use capital letters for the Earth's values. Thus, we arrive at the figures for l and r for the planet and L and R for the Earth. In addition, we need the heliocentric latitude of the planet:

$$\psi = \sin^{-1}\left\{\sin\left(l - \Omega\right)\sin i\right\},$$

where i is the inclination of the orbit and Ω is the longitude of the ascending node (Table 8). The heliocentric latitude of the Earth is, of course, zero.

Now we need to project our calculations for the planet onto the plane of the ecliptic to find the projected heliocentric longitude, l', and the projected radius vector, r'. These are given by the formulas

$$\begin{aligned} l' &= \tan^{-1}\left\{\tan\left(l - \Omega\right)\cos i\right\} + \Omega, \\ r' &= r\cos\psi. \end{aligned}$$

The final step in the process is to refer the calculations to the Earth to find the geocentric ecliptic latitude, β, and longitude, λ, of the planet. Figure 65(a) describes the situation for an **outer planet**, whose orbit lies outside that of the Earth (i.e. Mars, Jupiter, Saturn, Uranus and Neptune), and Figure 65(b) is for an **inner planet** (the inner planets are Mercury and Venus). The plane of the paper represents the plane of the ecliptic. S is the Sun, E is the Earth and P_1 is the position of the planet projected onto the ecliptic. The first point of Aries is taken to be at a distance from the Solar System so large that the directions E♈ and S♈ are parallel. Then by application of a little simple geometry we have for the outer planets

$$\lambda = \tan^{-1}\left\{\frac{R\sin\left(l'-L\right)}{r' - R\cos\left(l'-L\right)}\right\} + l' \text{ degrees,}$$

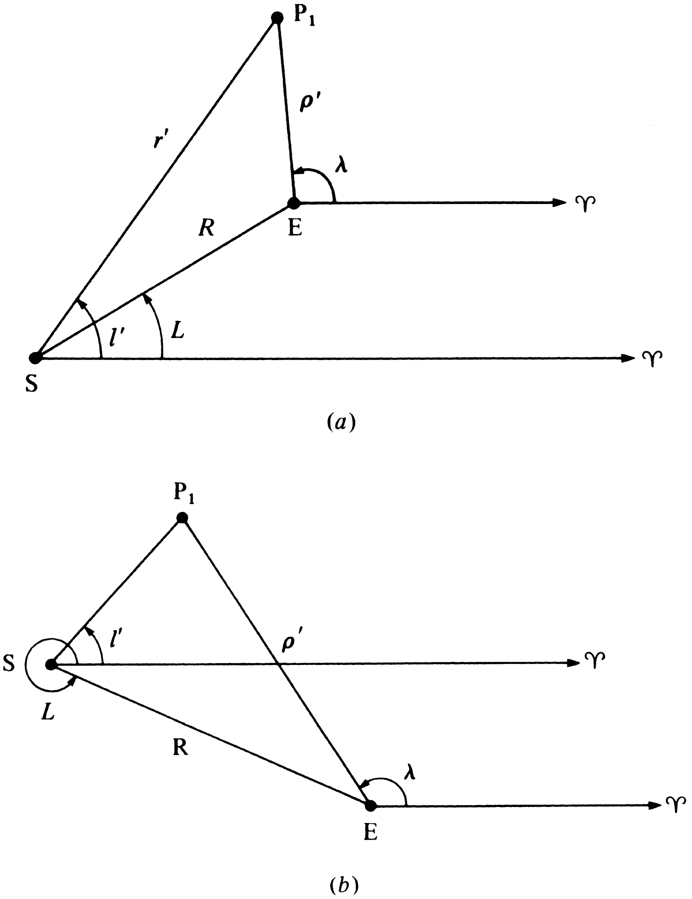
and for the inner planets

$$\lambda = 180 + L + \tan^{-1}\left\{\frac{r'\sin\left(L-l'\right)}{R - r'\cos\left(L-l'\right)}\right\} \text{ degrees,}$$

Figure 66 gives the diagram for calculating the latitude. Again, using simple geometry we find

$$\beta = \tan^{-1}\left\{\frac{r'\tan\psi\sin\left(\lambda - l'\right)}{R\sin\left(l'-L\right)}\right\} \text{ degrees,}$$

true for inner and outer planets alike.



(*a*)

(*b*)

Figure 65. Ecliptic geometry: (*a*) outer planet, (*b*) inner planet.

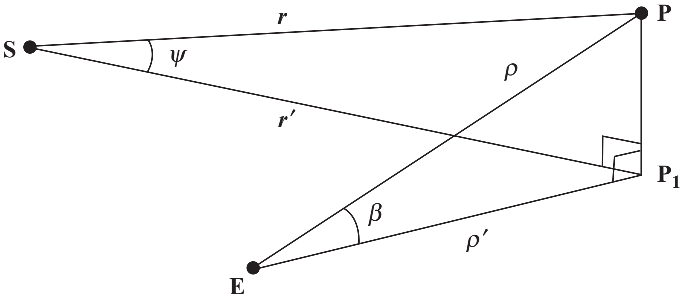


Figure 66. Projecting onto the ecliptic.

Let us consolidate these rather lengthy calculations with two examples, one for an outer planet, Jupiter, and the other for an inner planet, Mercury. For each of these two planets we shall calculate its right ascension and declination on 22 November 2003. For Jupiter (outer planet):

Method	*Example*
1. Find the number of days since 2010 January 0.0 (§3). The total is D.	22 November = $304+22$ = 326 $-2\,557$ D = $-2\,231$ days
For the planet, Jupiter:	
2. Calculate $N_{\mathrm{p}} = \frac{360}{365.242\,191} \times \frac{D}{T_{\mathrm{p}}}$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	N_{p} = 174.555 932 degrees
3. Find $M_{\mathrm{p}} = N_{\mathrm{p}} + \varepsilon - \varpi$.	M_{p} = 497.809 764 degrees
4. Calculate $\nu_{\mathrm{p}} = M_{\mathrm{p}} + \frac{360}{\pi} e \sin M_{\mathrm{p}}$ degrees. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	ν_{p} = 141.573 600 degrees
5. Find $l_{\mathrm{p}} = \nu_{\mathrm{p}} + \varpi$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	l_{p} = 156.236 900 degrees
6. Calculate $r = \frac{a\left(1-e^2\right)}{1+e\cos\nu_{\mathrm{p}}}$.	r = 5.397 121 AU
Now do the calculations for the Earth:	
7. Calculate $N_{\mathrm{E}} = \frac{360}{365.242\,191} \times \frac{D}{T_{\mathrm{E}}}$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	N_{E} = 321.011 952 degrees
8. Find $M_{\mathrm{E}} = N_{\mathrm{E}} + \varepsilon_{\mathrm{E}} - \varpi_{\mathrm{E}}$.	M_{E} = 317.363 223 degrees
9. Calculate $\nu_{\mathrm{E}} = M_{\mathrm{E}} + \frac{360}{\pi} e_{\mathrm{E}} \sin M_{\mathrm{E}}$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	ν_{E} = 316.069 248 degrees
10. Find $L = \nu_{\mathrm{E}} + \varpi_{\mathrm{E}}$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	L = 59.274 748 degrees
11. Calculate $R = \frac{a_{\mathrm{E}}\left(1-e_{\mathrm{E}}^2\right)}{1+e_{\mathrm{E}}\cos\nu_{\mathrm{E}}}$.	R = 0.987 847 AU

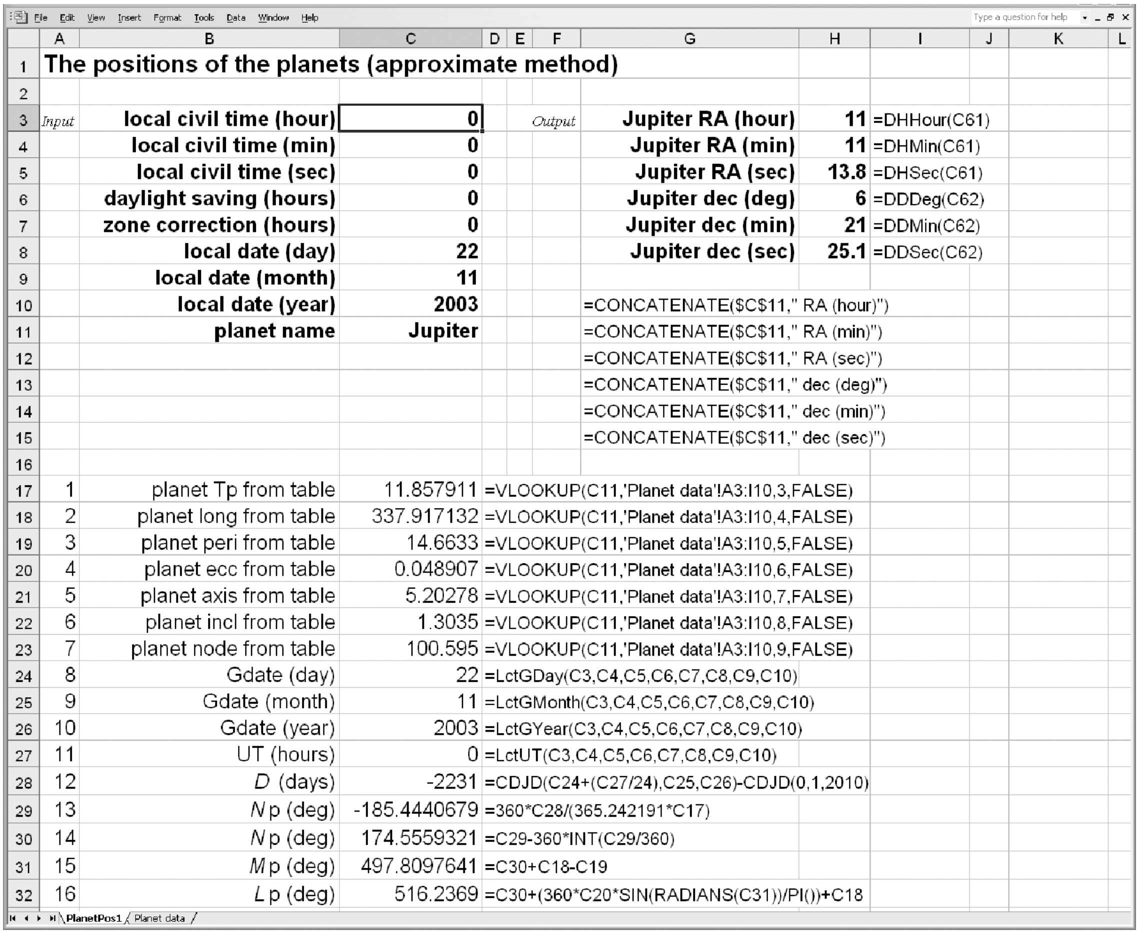
Method (continued)	*Example*
More calculations for the planet:	
12. Calculate $\psi = \sin^{-1}\left\{\sin\left(l_\mathrm{p} - \text{☊}\right)\sin i\right\}$.	ψ = 1.076 044 degrees
13. Find $y = \sin\left(l_\mathrm{p} - \text{☊}\right)\cos i$.	y = 0.825 313
14. Find $x = \cos\left(l_\mathrm{p} - \text{☊}\right)$.	x = 0.564 363
15. Find $\tan^{-1}\left(\frac{y}{x}\right)$ and remove the ambiguity by referring to Figure 29, adding or subtracting 180 degrees as indicated by the signs of x and y so that the result lies in the correct quadrant.	$\tan^{-1}\left(\frac{y}{x}\right)$ = 55.634 991 y and x both positive so the result is in the right quadrant
16. Add ☊ to get l' (check: l' should be nearly equal to l_p).	l' = 156.229 991 degrees
17. Find $r' = r\cos\psi$.	r' = 5.396 170 AU
Combine the calculations:	
18. Calculate $\lambda = \tan^{-1}\left\{\frac{R\sin(l'-L)}{r'-R\cos(l'-L)}\right\} + l'$. Bring the result into the range 0 to 360 by adding or subtracting 360. This is the planet's geocentric ecliptic longitude.	λ = 166.310 510 degrees
19. Find $\beta = \tan^{-1}\left\{\frac{r'\tan\psi\sin(\lambda-l')}{R\sin(l'-L)}\right\}$. This is the planet's geocentric ecliptic latitude.	β = 1.036 466 degrees
20. Finally, convert λ and β to right ascension and declination (§27).	α = **11h 11m 14s** δ = **6° 21′ 25″**

The *Astronomical Almanac* gives the apparent coordinates of Jupiter for this day as α = 11h 10m 30s and δ = 6° 25′ 56″. The error due to our approximation in counting only the first term of the equation of the centre could be reduced by solving Kepler's equation using the method in Section 47. We will see how to make more exact calculations in Section 56. For Mercury (inner planet):

Method	*Example*		
1. We proceed exactly as we did in the previous example, calculating l_p, v_p, r, ψ, l' and r' for Mercury, and L, v_E and R for the Earth.	l_p	=	288.012 253 degrees
	v_p	=	210.400 253 degrees
	r	=	0.450 657 AU
	ψ	=	−6.035 842 degrees
	l'	=	287.824 406 degrees
	r'	=	0.448 159 degrees
	L	=	59.274 748 degrees
	v_E	=	316.069 248 degrees
	R	=	0.987 847 AU
2. Now calculate $\lambda = 180 + L + \tan^{-1}\left\{\frac{r'\sin(L-l')}{R-r'\cos(L-l')}\right\}$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	λ	=	253.929 758 degrees
3. Find $\beta = \tan^{-1}\left\{\frac{r'\tan\psi\sin(\lambda-l')}{R\sin(l'-L)}\right\}$.	β	=	−2.044 057 degrees
4. Finally, convert λ and β to right ascension and declination (§27).	α	=	**16h 49m 12s**
	δ	=	**−24° 30′ 09″**

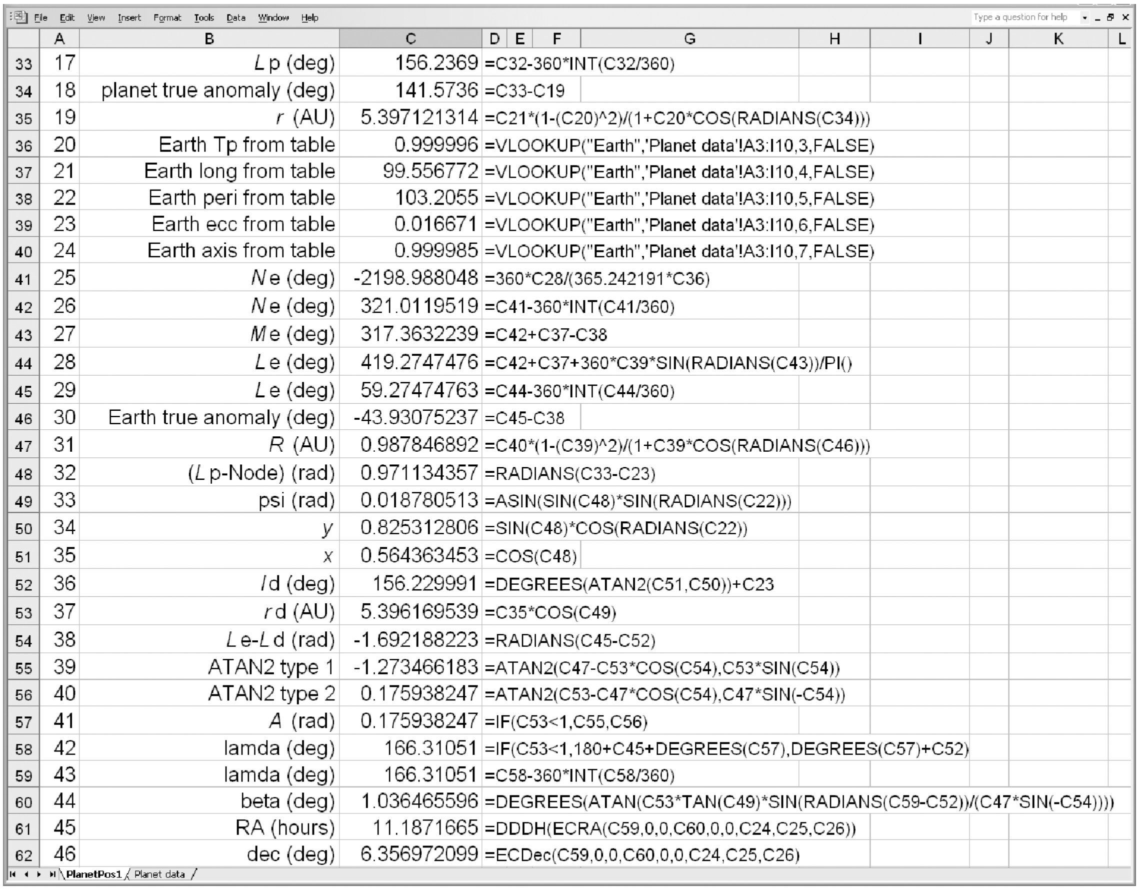
The *Astronomical Almanac* gives the apparent coordinates of planet Mercury as α = 16h 52m 02s and δ = −24° 38′ 41″. We should generally expect an error in α of a few minutes at most and in δ of a quarter of a degree, but the errors may be more for Mercury, for which $e = 0.2$. The inaccuracies arise because we have used only the first term in the equation of the centre, we have not allowed for the light travel time, and because of the slight perturbations to the orbits from other planets in the Solar System (see Section 56). We could reduce the error from the first cause by using the longer method of Section 47; see Figure 68 for a graph of the error incurred by the shorter method.

The spreadsheet for this calculation, called PlanetPos1, Figure 67 (three panels), uses a technique which we have not met previously in this book. The parameters of the orbits of the planets are presented in a table in a separate spreadsheet called Planet data (third panel). This table reproduces most of the data in Table 8, but note that the order of the planets is now alphabetical. The planet name is in column A and is used as the key to the corresponding row of data contained in columns C to I inclusive. If we number the columns in the table, then column 1 contains the planet name, 3 contains the orbital period, 4 the longitude at the epoch and so on. We can obtain any element of the data using the spreadsheet function VLOOKUP (e.g. row 17 of the first panel of Figure 67). This takes four arguments, which are the planet name (upper or lower case, or a mixture), the range of the table from top-left-hand corner to bottom-right-hand corner (e.g. 'Planet data'!A3:I10), the column number containing the required element (e.g. 3 for the orbital period, 4 for the longitude at the epoch), and a switch which is set to TRUE to find either an approximate or an exact match with the planet name in column 1, or FALSE if an exact match is required (as here). Using this formula makes it easy to change the orbital parameters without affecting the main spreadsheet calculation. Simply fill in the Planet data table with new numbers, and the spreadsheet will use those instead. The formulas contained in cells G3 to G8 are shown in cells G10 to G15 to save space.

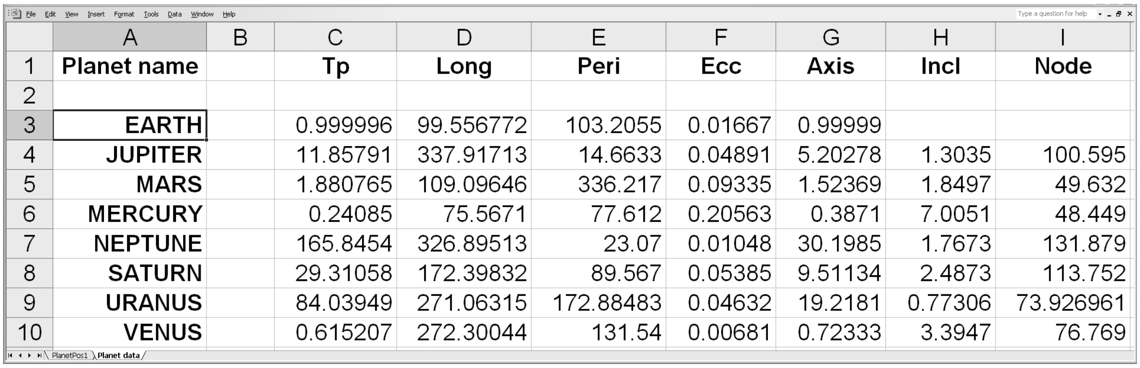


	A	B	C	D	E	F	G	H	I	J	K	L
1	The positions of the planets (approximate method)											
2												
3	*Input*	**local civil time (hour)**	**0**			*Output*	**Jupiter RA (hour)**	**11**	=DHHour(C61)			
4		**local civil time (min)**	**0**				**Jupiter RA (min)**	**11**	=DHMin(C61)			
5		**local civil time (sec)**	**0**				**Jupiter RA (sec)**	**13.8**	=DHSec(C61)			
6		**daylight saving (hours)**	**0**				**Jupiter dec (deg)**	**6**	=DDDeg(C62)			
7		**zone correction (hours)**	**0**				**Jupiter dec (min)**	**21**	=DDMin(C62)			
8		**local date (day)**	**22**				**Jupiter dec (sec)**	**25.1**	=DDSec(C62)			
9		**local date (month)**	**11**									
10		**local date (year)**	**2003**				=CONCATENATE(C11," RA (hour)")					
11		**planet name**	**Jupiter**				=CONCATENATE(C11," RA (min)")					
12							=CONCATENATE(C11," RA (sec)")					
13							=CONCATENATE(C11," dec (deg)")					
14							=CONCATENATE(C11," dec (min)")					
15							=CONCATENATE(C11," dec (sec)")					
16												
17	1	planet Tp from table	11.857911	=VLOOKUP(C11,'Planet data'!A3:I10,3,FALSE)								
18	2	planet long from table	337.917132	=VLOOKUP(C11,'Planet data'!A3:I10,4,FALSE)								
19	3	planet peri from table	14.6633	=VLOOKUP(C11,'Planet data'!A3:I10,5,FALSE)								
20	4	planet ecc from table	0.048907	=VLOOKUP(C11,'Planet data'!A3:I10,6,FALSE)								
21	5	planet axis from table	5.20278	=VLOOKUP(C11,'Planet data'!A3:I10,7,FALSE)								
22	6	planet incl from table	1.3035	=VLOOKUP(C11,'Planet data'!A3:I10,8,FALSE)								
23	7	planet node from table	100.595	=VLOOKUP(C11,'Planet data'!A3:I10,9,FALSE)								
24	8	Gdate (day)	22	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)								
25	9	Gdate (month)	11	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)								
26	10	Gdate (year)	2003	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)								
27	11	UT (hours)	0	=LctUT(C3,C4,C5,C6,C7,C8,C9,C10)								
28	12	*D* (days)	-2231	=CDJD(C24+(C27/24),C25,C26)-CDJD(0,1,2010)								
29	13	*N* p (deg)	-185.4440679	=360*C28/(365.242191*C17)								
30	14	*N* p (deg)	174.5559321	=C29-360*INT(C29/360)								
31	15	*M* p (deg)	497.8097641	=C30+C18-C19								
32	16	*L* p (deg)	516.2369	=C30+(360*C20*SIN(RADIANS(C31))/PI())+C18								

Figure 67. Finding the position of a planet by an approximate method; panels one and two show the main spreadsheet, and the third shows the data table (continued on the next page). The formulas contained in cells G3 to G8 are shown in cells G10 to G15 to save space.



	A	B	C	D
33	17	*L* p (deg)	156.2369	=C32-360*INT(C32/360)
34	18	planet true anomaly (deg)	141.5736	=C33-C19
35	19	*r* (AU)	5.397121314	=C21*(1-(C20)^2)/(1+C20*COS(RADIANS(C34)))
36	20	Earth Tp from table	0.999996	=VLOOKUP("Earth",'Planet data'!A3:I10,3,FALSE)
37	21	Earth long from table	99.556772	=VLOOKUP("Earth",'Planet data'!A3:I10,4,FALSE)
38	22	Earth peri from table	103.2055	=VLOOKUP("Earth",'Planet data'!A3:I10,5,FALSE)
39	23	Earth ecc from table	0.016671	=VLOOKUP("Earth",'Planet data'!A3:I10,6,FALSE)
40	24	Earth axis from table	0.999985	=VLOOKUP("Earth",'Planet data'!A3:I10,7,FALSE)
41	25	*N* e (deg)	-2198.988048	=360*C28/(365.242191*C36)
42	26	*N* e (deg)	321.0119519	=C41-360*INT(C41/360)
43	27	*M* e (deg)	317.3632239	=C42+C37-C38
44	28	*L* e (deg)	419.2747476	=C42+C37+360*C39*SIN(RADIANS(C43))/PI()
45	29	*L* e (deg)	59.27474763	=C44-360*INT(C44/360)
46	30	Earth true anomaly (deg)	-43.93075237	=C45-C38
47	31	*R* (AU)	0.987846892	=C40*(1-(C39)^2)/(1+C39*COS(RADIANS(C46)))
48	32	(*L* p-Node) (rad)	0.971134357	=RADIANS(C33-C23)
49	33	psi (rad)	0.018780513	=ASIN(SIN(C48)*SIN(RADIANS(C22)))
50	34	*y*	0.825312806	=SIN(C48)*COS(RADIANS(C22))
51	35	*x*	0.564363453	=COS(C48)
52	36	*l* d (deg)	156.229991	=DEGREES(ATAN2(C51,C50))+C23
53	37	*r* d (AU)	5.396169539	=C35*COS(C49)
54	38	*L* e-*L* d (rad)	-1.692188223	=RADIANS(C45-C52)
55	39	ATAN2 type 1	-1.273466183	=ATAN2(C47-C53*COS(C54),C53*SIN(C54))
56	40	ATAN2 type 2	0.175938247	=ATAN2(C53-C47*COS(C54),C47*SIN(-C54))
57	41	*A* (rad)	0.175938247	=IF(C53<1,C55,C56)
58	42	lamda (deg)	166.31051	=IF(C53<1,180+C45+DEGREES(C57),DEGREES(C57)+C52)
59	43	lamda (deg)	166.31051	=C58-360*INT(C58/360)
60	44	beta (deg)	1.036465596	=DEGREES(ATAN(C53*TAN(C49)*SIN(RADIANS(C59-C52))/(C47*SIN(-C54))))
61	45	RA (hours)	11.1871665	=DDDH(ECRA(C59,0,0,C60,0,0,C24,C25,C26))
62	46	dec (deg)	6.356972099	=ECDec(C59,0,0,C60,0,0,C24,C25,C26)



	A	B	C	D	E	F	G	H	I
1	**Planet name**		**Tp**	**Long**	**Peri**	**Ecc**	**Axis**	**Incl**	**Node**
2									
3	**EARTH**		0.999996	99.556772	103.2055	0.01667	0.99999		
4	**JUPITER**		11.85791	337.91713	14.6633	0.04891	5.20278	1.3035	100.595
5	**MARS**		1.880765	109.09646	336.217	0.09335	1.52369	1.8497	49.632
6	**MERCURY**		0.24085	75.5671	77.612	0.20563	0.3871	7.0051	48.449
7	**NEPTUNE**		165.8454	326.89513	23.07	0.01048	30.1985	1.7673	131.879
8	**SATURN**		29.31058	172.39832	89.567	0.05385	9.51134	2.4873	113.752
9	**URANUS**		84.03949	271.06315	172.88483	0.04632	19.2181	0.77306	73.926961
10	**VENUS**		0.615207	272.30044	131.54	0.00681	0.72333	3.3947	76.769

Figure 67. (Continued.)

55 Finding the approximate positions of the planets

The method of finding the equatorial coordinates of the planets given in the previous section is quite accurate but involves lengthy calculations. An amateur astronomer often only wants to know the approximate position of a planet so that he or she knows where to look for it in the sky, and does not want to have to spend 20 minutes beforehand submerged in a sea of figures obtaining the information. In that case it is usually sufficient to assume that the planets describe circular orbits about the Sun which all lie in the plane of the ecliptic. This leads to considerable simplifications in the calculations.

The heliocentric longitude, l, does not have to be corrected by the equation of the centre so that we may write

$$l = \frac{360}{365.242\,191} \times \frac{D}{T_\mathrm{p}} + \varepsilon \text{ degrees.}$$

We repeat this calculation for the Earth as before (giving L). Since the orbits are assumed to be circular with the Sun at the centre, the radius vector is constant. Hence

$$r = a.$$

The heliocentric (and therefore the geocentric) latitude of the planet is zero since we have assumed that the orbit lies in the ecliptic plane. Our final calculation is therefore

$$\lambda = \tan^{-1}\left\{\frac{\sin(l-L)}{a-\cos(l-L)}\right\} + l,$$

for the outer planets and

$$\lambda = 180 + L + \tan^{-1}\left\{\frac{a\sin(L-l)}{1-a\cos(L-l)}\right\},$$

for the inner planets, since we have assumed that $R = 1$ (the Earth's orbital radius is taken to be unity). This is the geocentric longitude of the planet from which the right ascension and declination can be found using the formulas of Section 27 (remember $\beta = 0$):

$$\alpha = \tan^{-1}\{\tan\lambda\cos\varepsilon\},$$

$$\delta = \sin^{-1}\{\sin\lambda\sin\varepsilon\},$$

where ε here is the obliquity of the ecliptic (about 23.5 degrees; see Section 27). In some cases it may even be possible to ignore the fact that the plane of the ecliptic is inclined at an angle to the plane of the equator and to write

$$\alpha = \lambda.$$

As an example, we will calculate again the coordinates of Jupiter on 22 November 2003 using this approximate method.

Method		*Example*		
1.	Find the number of days since 2010 January 0.0 (§3). The total is D.	22 November	=	$304+22$
			=	326
			–	2 557
		D	=	$-2\,231$ days
2.	Calculate $l=\left(\frac{360}{365.242\,191}\times\frac{D}{T_{\mathrm{p}}}\right)+\varepsilon$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	l	=	152.47 degrees
3.	Repeat step 2 for the Earth to find L.	L	=	60.57 degrees
4.	Calculate $\lambda=\tan^{-1}\left\{\frac{\sin(l-L)}{a-\cos(l-L)}\right\}+l$. Bring the result into the range 0–360 by adding or subtracting 360. This is the planet's geocentric ecliptic longitude.	λ	=	163.28 degrees
5.	Finally, convert λ and $\beta\,(=0)$ to right ascension and declination (§27).	α	=	**10h 58m**
		δ	=	**6° 34′**

56 Perturbations in a planet's orbit

Throughout the calculations to find the coordinates of a planet (Section 54), we assumed that its motion was controlled entirely by the gravitational field of the Sun so that the influences of other members of the Solar System were negligible. This is true to quite a high accuracy, but for more precision we need to take account of these **perturbations**, especially for the orbits of Jupiter and Saturn where the effects can be as large as 1 degree in longitude. The usual method of doing so is to apply a series of correction terms to the quantities calculated in Section 54. We have to make similar adjustments for the Moon in Section 65. Here, we shall consider only the most important terms in the orbits of Jupiter and Saturn where the corrections amount to more than about 0.04 degrees in longitude.

We must first calculate the time, T, in Julian centuries since the epoch 1900 January 0.5. This is given by

$$T=\frac{\mathrm{JD}-2\,415\,020.0}{36\,525},$$

where JD is the Julian date (Section 4).[†] Then we calculate the quantities:

$$A=\frac{T}{5}+0.1,$$

$$P=237.475\,55+3\,034.906\,1T\text{ degrees},$$

$$Q=265.916\,50+1\,222.113\,9T\text{ degrees},$$

$$V=5Q-2P,\text{ and}$$

$$B=Q-P.$$

[†] Note that this definition of T differs from that used in other parts of the book.

The principal terms for Jupiter and Saturn are then:

$$\begin{aligned}
\text{Jupiter}: \quad \Delta l &= (0.331\,4 - 0.010\,3A)\sin V - 0.064\,4A\cos V \text{ degrees.}\\
\text{Saturn}: \quad \Delta l &= (0.160\,9A - 0.010\,5)\cos V + (0.018\,2A - 0.814\,2)\sin V - 0.148\,8\sin B\\
&\quad - 0.040\,8\sin 2B + 0.085\,6\sin B\cos Q + 0.081\,3\cos B\sin Q \text{ degrees.}
\end{aligned}$$

The value of Δl must be added to the mean longitude l before proceeding with the calculation of Section 54.

Let us now recalculate the position of Jupiter on 22 November 2003, solving Kepler's equation properly (Section 47) and allowing for these principal terms of perturbation.

Method	*Example*
1. Calculate the Julian date (§4).	JD = 2 452 965.5 days
2. Find $T = \frac{\text{JD}-2415\,020.0}{36\,525}$.	T = 1.038 891 centuries
3. Find $A = \left(\frac{T}{5}\right) + 0.1$.	A = 0.307 778 centuries
4. Calculate $P = 237^{\circ}\!.475\,55 + 3\,034^{\circ}\!.906\,1\,T$.	P = 3 390.412 183 degrees
5. Find $Q = 265^{\circ}\!.916\,50 + 1\,222^{\circ}\!.113\,9\,T$.	Q = 1 535.559 632 degrees
6. Find $V = 5Q - 2P$.	V = 896.973 792 degrees
7. Calculate $\Delta l = (0.331\,4 - 0.010\,3A)\sin V - 0.064\,4A\cos V$.	Δl = 0.037 121 degrees
8. Now proceed as in the example of §54 to find M_p.	M_p = 497.809 764 degrees
9. Use the method of §47 to find v_p.	E_p = 2.436 915 radians
	v_p = 141.407 886 degrees
10. Find $l_\text{p} = v_\text{p} + \varpi$. Add or subtract multiples of 360 to bring the result into the range 0 to 360.	l_p = 156.071 186 degrees
11. Add Δl to get a better estimate of l_p.	l_p = 156.108 307 degrees
12. Calculate L, v_E and R for the Earth also using the method of §47.	E_E = 5.527 600 radians
	v_E = 316.049 185 degrees
	L = 59.254 685 degrees
	R = 0.987 847 AU
13. Now proceed with the calculations of §54 to find α and δ.	r = 5.396 627 AU
	ψ = 0.018 751 degrees
	l' = 156.101 386 degrees
	r' = 5.395 678 AU
	λ = 166.188 415 degrees
	β = 1.035 198 degrees
	α = **11h 10m 47s**
	δ = **6° 24′ 12″**

The error incurred in considering only the first term of the equation of the centre is plotted as a function of the mean anomaly, M, in Figure 68 for two values of the eccentricity, e.

The spreadsheet for finding the positions of the planets by a more precise method is called PlanetPos2 and is shown in Figure 69. We have used a full numerical model of the orbits of the planets in the two spreadsheet functions PlanetLong and PlanetLat which return, respectively, the named planet's ecliptic longitude and latitude in degrees. These functions appear in rows 20 and 21. They each take nine arguments, namely the local civil time expressed as hours, minutes, and seconds, the daylight saving correction and zone time offset in hours, the local calendar date expressed as day, month, year, and the full name of the planet as a string in upper or lower case. The example shown is for Jupiter on the 22 November 2003, and you can see that this method has provided the position correct within about 1 second in right ascension, and within a few arcseconds in declination. The formulas in cells G3 to G8 are shown in cells H10 to H15 to save space.

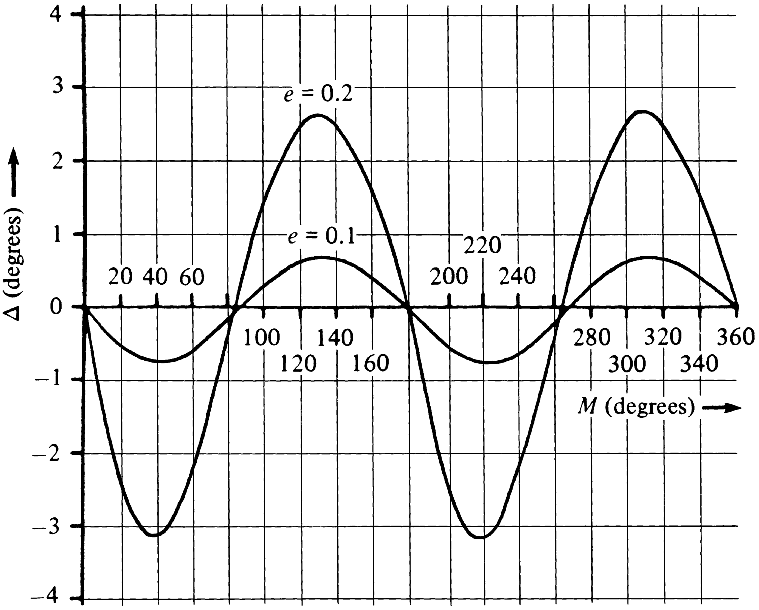
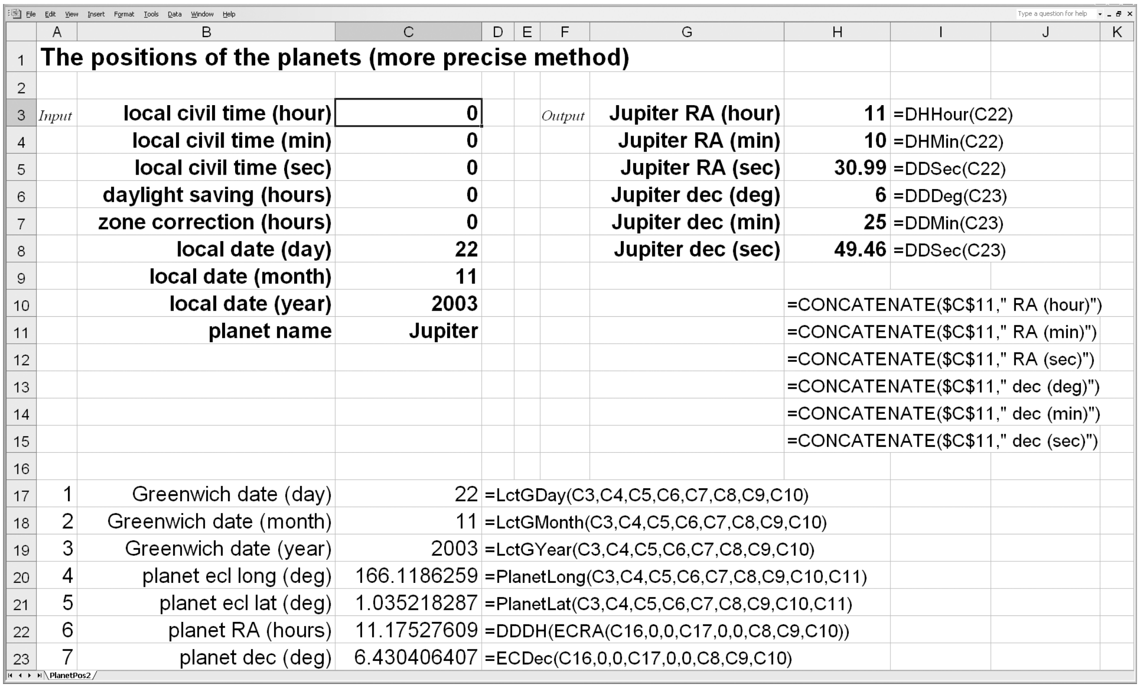


Figure 68. The error, Δ, incurred by taking $v = M + (360/\pi)\,e\sin M$ as an approximation to elliptical motion. The true anomaly should be $v - \Delta$.



	A	B	C	D	E	F	G	H	I	J	K
1	**The positions of the planets (more precise method)**										
2											
3	*Input*	**local civil time (hour)**	**0**			*Output*	**Jupiter RA (hour)**	**11**	=DHHour(C22)		
4		**local civil time (min)**	**0**				**Jupiter RA (min)**	**10**	=DHMin(C22)		
5		**local civil time (sec)**	**0**				**Jupiter RA (sec)**	**30.99**	=DDSec(C22)		
6		**daylight saving (hours)**	**0**				**Jupiter dec (deg)**	**6**	=DDDeg(C23)		
7		**zone correction (hours)**	**0**				**Jupiter dec (min)**	**25**	=DDMin(C23)		
8		**local date (day)**	**22**				**Jupiter dec (sec)**	**49.46**	=DDSec(C23)		
9		**local date (month)**	**11**								
10		**local date (year)**	**2003**					=CONCATENATE(C11," RA (hour)")			
11		**planet name**	**Jupiter**					=CONCATENATE(C11," RA (min)")			
12								=CONCATENATE(C11," RA (sec)")			
13								=CONCATENATE(C11," dec (deg)")			
14								=CONCATENATE(C11," dec (min)")			
15								=CONCATENATE(C11," dec (sec)")			
16											
17	1	Greenwich date (day)	22	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)							
18	2	Greenwich date (month)	11	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)							
19	3	Greenwich date (year)	2003	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)							
20	4	planet ecl long (deg)	166.1186259	=PlanetLong(C3,C4,C5,C6,C7,C8,C9,C10,C11)							
21	5	planet ecl lat (deg)	1.035218287	=PlanetLat(C3,C4,C5,C6,C7,C8,C9,C10,C11)							
22	6	planet RA (hours)	11.17527609	=DDDH(ECRA(C16,0,0,C17,0,0,C8,C9,C10))							
23	7	planet dec (deg)	6.430406407	=ECDec(C16,0,0,C17,0,0,C8,C9,C10)							

Figure 69. Finding the position of a planet by a more precise method.

57 The distance, light-travel time and angular size of a planet

During the course of our calculations in Section 54 to determine the position of a planet we found the distances r and R of the planet and the Earth, respectively, from the Sun. We can quite easily use these values together with the heliocentric longitudes l and L to calculate the planet's distance, ρ, from the Earth. The situation is drawn in Figure 70. The planet, P, does not lie in the plane of the ecliptic, so its heliocentric latitude, ψ, must be taken into account. The formula is

$$\rho^2 = R^2 + r^2 - 2Rr\cos(l - L)\cos\psi.$$

It is usual to express r and R in **astronomical units** (AU) where 1 AU is the semi-major axis of the Earth's orbit. ρ is then the distance of the planet from the Earth measured in AU.

Having calculated ρ, it is then an easy matter to find the light-travel time, τ, the time taken for the light to reach us from the planet. When we view a planet now, we see it in the position it occupied τ hours ago, given by

$$\tau = 0.1386\rho \text{ hours},$$

where ρ is expressed in AU.

We can also find the apparent angular diameter, θ, of the planet given by

$$\theta = \frac{\theta_0}{\rho},$$

where ρ is again expressed in AU and θ_0 is the angular diameter of the planet when it is at 1 AU from the Earth. Values of θ_0 are given in Table 8.

We shall calculate the distance, the light-travel time and the apparent angular diameter of Jupiter on 22 November 2003.

Method	*Example*
1. Find r, R, l_p, L and ψ as in §54.	r = 5.397 121 AU R = 0.987 847 AU l_p = 156.236 900 degrees L = 59.274 748 degrees ψ = 1.076 044 degrees
2. Calculate $\rho^2 = R^2 + r^2 - 2Rr\cos\left(l_p - L\right)\cos\psi$.	ρ^2 = 31.397 037 AU^2
3. Take the square root to find ρ.	ρ = **5.603 AU**
4. Multiply by 0.1386 to find τ; convert to minutes and seconds (§8).	τ = **46m 36s**
5. Find θ from $\theta = \frac{\theta_0}{\rho}$.	θ = **35.1 arcsec**

The *Astronomical Almanac* quotes $\rho = 5.60$ AU, τ = 46m 34s and $\theta = 35.0''$ for Jupiter on this day.

The spreadsheet for this calculation is called PlanetVis and includes other calculations to do with the visual aspect of a planet. It is given in Section 60, Figure 72. The corresponding spreadsheet functions are defined there too.

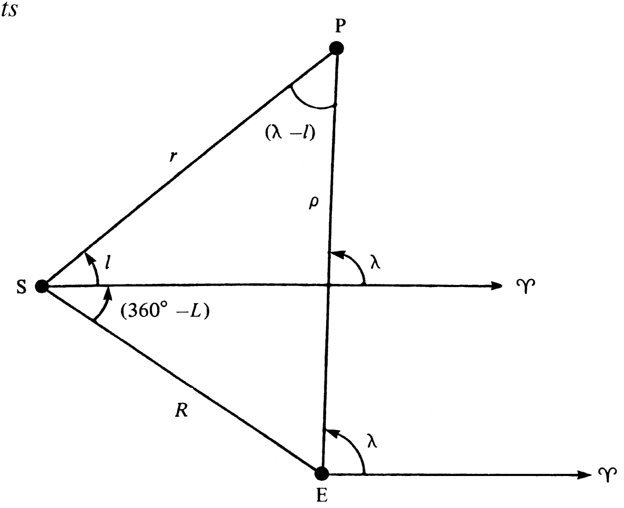


Figure 70. Finding the distance of a planet.

58 The phases of the planets

At any point in the orbit of a planet, the hemisphere which faces towards the Sun is brightly illuminated while the other half of the planet's surface is dark. The fraction of the surface that we can see from the Earth, however, is that part lying on the hemisphere facing the Earth which usually overlaps both the bright and the dark sides. We are presented therefore with a view of the planet's disc that is not uniformly illuminated but which contains a bright segment, the rest of the disc being dark and usually invisible. As the relative positions of the Earth, the planet and the Sun vary, so the area of the visible disc that is illuminated changes. The **phase** is defined to be the fraction of the visible disc that is illuminated.

In Figure 70, the angle $(\lambda - l)$ at P is the solar elongation of the Earth as measured at the planet. We represent this angle by d. Thus

$$d = \lambda - l.$$

The phase, F, is related to d by the formula

$$F = \frac{1}{2}(1 + \cos d).$$

F always lies in the range 0 to 1. When $F = 0$, the whole of the dark side of the planet is towards the Earth. This can only happen for the inner planets Mercury and Venus. When $F = 1$, the whole of the bright side faces the Earth.

We shall find the phases of Mercury and Jupiter on 22 November 2003 as our example.

Method	*Example*
1. Calculate $d = \lambda - l$ using the method outlined in §54 to find λ and l.	Mercury: $d_1 = -34.082°$ Jupiter: $d_2 = 10.736°$
2. Find $F = \frac{1}{2}(1+\cos d)$.	Mercury: $F_1 =$ **0.91** Jupiter: $F_2 =$ **0.99**

The *Astronomical Almanac* gives phase values of 0.90 and 0.99 for Mercury and Jupiter respectively.

The spreadsheet for this calculation is called PlanetVis and includes other calculations to do with the visual aspect of a planet. It is given in Section 60, Figure 72. The corresponding spreadsheet functions are defined there too.

59 The position-angle of the bright limb

Figure 71 shows the appearance of a planet whose phase is about $F = 0.7$. The dashed outline is of that part of the disc which is invisible, and the line NS is the projection of the Earth's axis onto the disc. The terminator, the line dividing night from day, is the curve AB. Position-angles are measured anticlockwise from the north. Thus points A and B are at position-angles θ_1 and θ_2. The point C, halfway between A and B on the circumference of the disc, is the midpoint of the bright side and it defines the position-angle, χ, of the bright limb. Hence

$$\chi = \frac{1}{2}\left(\theta_1 + \theta_2\right).$$

We can easily calculate χ provided we know the equatorial coordinates of the planet (α, δ) and of the Sun $(\alpha_\odot, \delta_\odot)$. Then

$$\chi = \tan^{-1}\left\{\frac{\cos\delta_\odot \sin\left(\alpha_\odot - \alpha\right)}{\cos\delta\sin\delta_\odot - \sin\delta\cos\delta_\odot\cos\left(\alpha_\odot - \alpha\right)}\right\}.$$

For example, what was the position-angle of the bright limb of Mercury on 22 November 2003? The Sun's coordinates were $\alpha_\odot = 15\text{h}\,48\text{m}\,13\text{s}$, $\delta_\odot = -19°\ 59'\ 32''$, and Mercury's coordinates were $\alpha = 16\text{h}\,52\text{m}\,02\text{s}$, $\delta = -24°\ 38'\ 41''$.

Method	*Example*
1. Find the right ascension and declination of the planet (§54).	α = 16h 52m 02s δ = $-24°\ 38'\ 41''$
2. Find the coordinates of the Sun (§46).	$\alpha_{\odot}$ = 15h 48m 13s $\delta_{\odot}$ = $-19°\ 59'\ 32''$
3. Convert both sets of coordinates to decimal form (§§7 and 21).	α = 16.867 222 hours δ = $-24.644\,722$ degrees $\alpha_{\odot}$ = 15.803 611 hours $\delta_{\odot}$ = $-19.992\,222$ degrees
4. Find $\Delta\alpha = \alpha_{\odot} - \alpha$. Convert to degrees by multiplying by 15 (§22).	$\Delta\alpha$ = $-15.954\,165$ degrees
5. Find $y = \cos\delta_{\odot}\sin\Delta\alpha$.	y = $-0.258\,304$
6. Find $x = \cos\delta\sin\delta_{\odot} - \sin\delta\cos\delta_{\odot}\cos\Delta\alpha$.	x = 0.066 018
7. Find $\chi = \tan^{-1}\left(\frac{y}{x}\right)$. Remove the ambiguity from taking inverse tan using the signs of x and y, referring to Figure 29, and adding or subtracting 180 if necessary to bring the result into the correct quadrant.	χ = $-75.663\,043$ degrees (already in correct quadrant; add 360 to bring into the range 0–360) χ = **284.3 degrees**

The spreadsheet for this calculation is called PlanetVis and includes other calculations to do with the visual aspect of a planet. It is given in Section 60, Figure 72. The corresponding spreadsheet functions are defined there too.

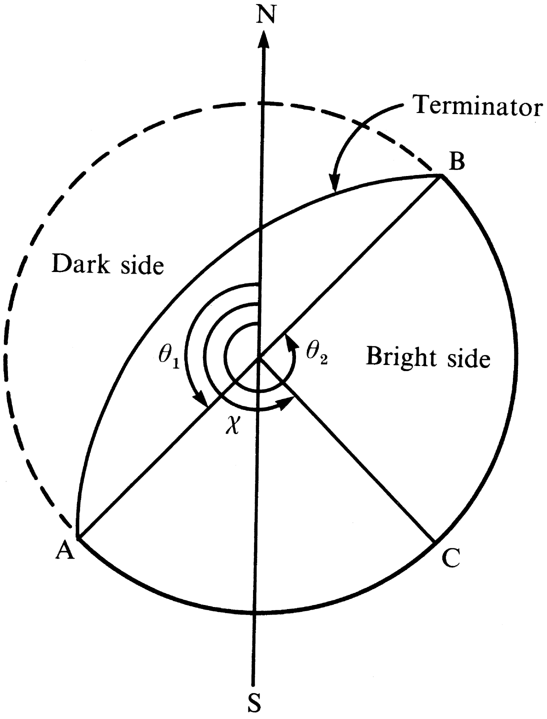


Figure 71. The position-angle of the bright limb.

60 The apparent brightness of a planet

Our calculations so far have given us the position, the solar elongation (Section 52), the distance from the Earth, the apparent angular diameter, the phase, and the position-angle of the bright limb of a planet. We need only add the apparent brightness to the list to obtain all the important parameters of the planet's **visual aspect**.

Brightness is usually measured in **magnitudes**, m, on a non-linear scale such that decreasing brightness goes with increasing magnitude. The brightest stars have a magnitude of about 1 while the faintest stars just visible with the unaided eye are of magnitude 6. The ratio in the light power flux between one magnitude and the next is about 2.5. The Sun, very bright at the Earth, has a visual magnitude of -26.74 while the Moon's magnitude at opposition is -12.73. The planets range from about $m = -4$ for Venus at its most brilliant to $m = +14$ for Pluto (strictly no longer classified as a planet) at its brightest.

The variation in a planet's brightness is caused by several factors. First the Sun's light flux on the planet varies inversely as the square of its distance, r, from the Sun. Then the amount of that light reradiated towards the Earth depends on the phase, F, and a 'brightness factor', V_0, the latter being a measure of the reflectivity of the planet combined with the area of the planet's disc. The larger the planet's area, the more light it intercepts from the Sun and hence the more it radiates towards the Earth. Finally, the light flux received from the planet varies inversely as the square of the planet's distance, ρ, from the Earth.

We can obtain an approximate value for the apparent magnitude of a planet from the formula

$$m = 5\log_{10}\left(\frac{r\rho}{\sqrt{F}}\right) + V_0,$$

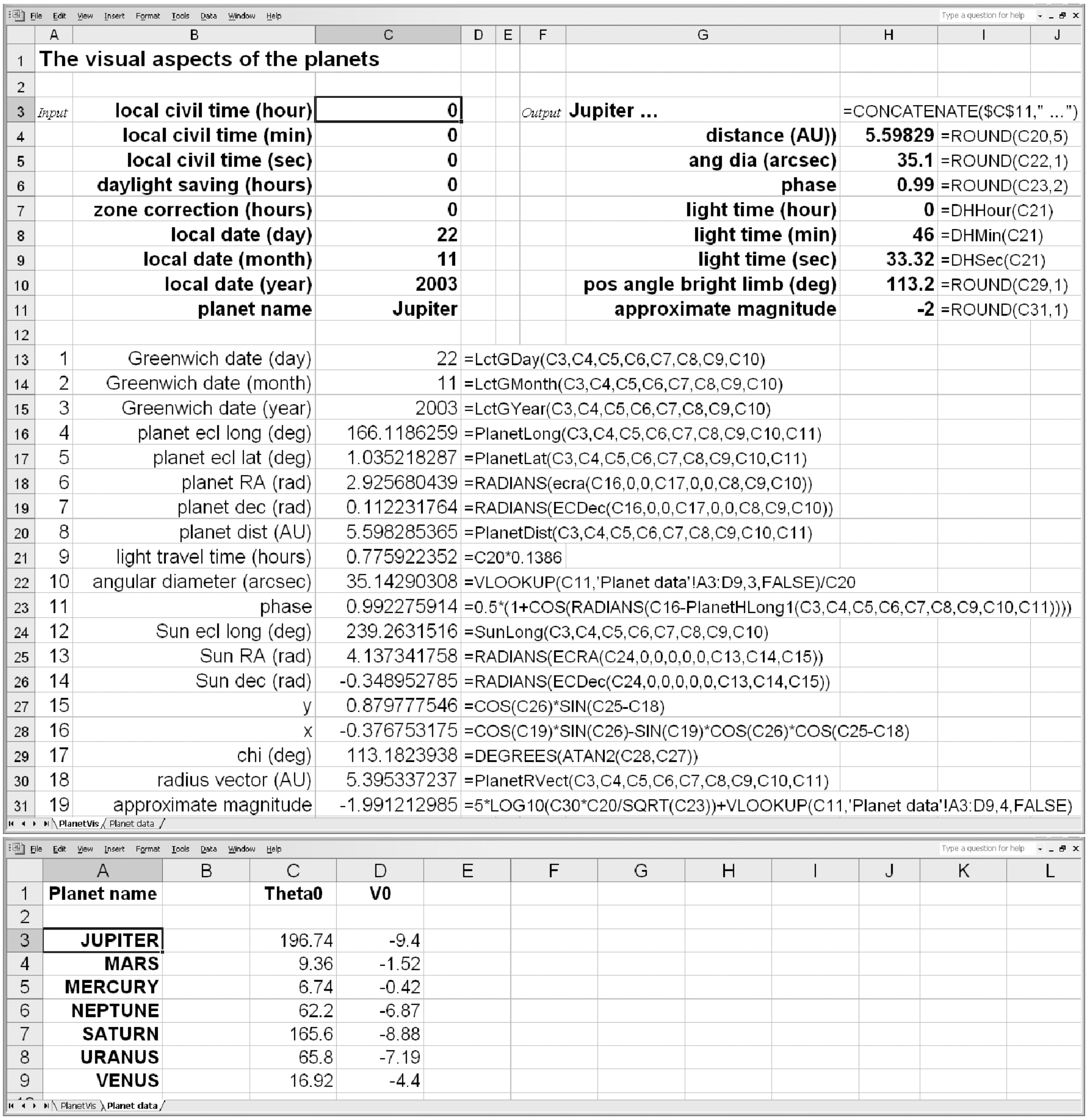
where r and ρ are measured in AU. The values of V_0 are listed in Table 8.

As an example, let us calculate the apparent magnitude of Jupiter on 22 November 2003.

Method		*Example*		
1.	Find the values of r, ρ and F using the methods given in §§54, 57 and 58.	r	=	5.397 AU
		ρ	=	5.603 AU
		F	=	0.99
2.	Calculate $m = 5\log_{10}\left(\frac{r\rho}{\sqrt{F}}\right) + V_0$. r and ρ must both be expressed in AU.	m	=	**−2**

The value of m given in the *Astronomical Almanac* for Jupiter on 22 November 2003 is $m = -1.9$. In general, our calculations should be correct to within a magnitude or so. No account has been taken of **atmospheric extinction** (see Section 43), which can increase the apparent magnitude of a star or planet near the horizon by 2 or 3. Nonetheless, our calculations will provide a fair guide of what to expect.

The distance, light time, angular diameter, phase, position-angle of the bright limb, and the approximate magnitude of a planet are all parts of its visual aspect (i.e. its appearance when viewed from Earth) and these calculations are swept up into one spreadsheet called PlanetVis, Figure 72. As in PlanetPos1 (Section 54), we make use of the spreadsheet function VLOOKUP to extract data from a table contained on a second spreadsheet (rows 22 and 31). We have also defined three additional spreadsheet functions to make life easier. The first of these to appear in the spreadsheet (row 20) is PlanetDist, taking nine arguments which are the local civil time expressed as hours, minutes, and seconds, the daylight saving and time zone offsets in hours, the local calendar date expressed as day, month, year, and the planet full name as a string (upper or lower case, or a combination). This function returns the Earth–planet distance in AU. The next new spreadsheet function to appear is called PlanetHLong1 (row 23) which returns the planet's heliocentric orbital longitude in degrees. It takes the same nine arguments as PlanetDist. The third new spreadsheet function is called PlanetRVect (row 30) and, as its name suggests, it returns the distance of the planet from the Sun (i.e. the length of its radius vector) in AU; it has the same nine arguments as PlanetDist.



	A	B	C	D	E	F	G	H	I	J
1	The visual aspects of the planets									
2										
3	*Input*	**local civil time (hour)**	**0**			*Output*	**Jupiter ...**	=CONCATENATE(C11," ...")		
4		**local civil time (min)**	**0**				**distance (AU))**	**5.59829**	=ROUND(C20,5)	
5		**local civil time (sec)**	**0**				**ang dia (arcsec)**	**35.1**	=ROUND(C22,1)	
6		**daylight saving (hours)**	**0**				**phase**	**0.99**	=ROUND(C23,2)	
7		**zone correction (hours)**	**0**				**light time (hour)**	**0**	=DHHour(C21)	
8		**local date (day)**	**22**				**light time (min)**	**46**	=DHMin(C21)	
9		**local date (month)**	**11**				**light time (sec)**	**33.32**	=DHSec(C21)	
10		**local date (year)**	**2003**				**pos angle bright limb (deg)**	**113.2**	=ROUND(C29,1)	
11		**planet name**	**Jupiter**				**approximate magnitude**	**-2**	=ROUND(C31,1)	
12										
13	1	Greenwich date (day)	22	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)						
14	2	Greenwich date (month)	11	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)						
15	3	Greenwich date (year)	2003	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)						
16	4	planet ecl long (deg)	166.1186259	=PlanetLong(C3,C4,C5,C6,C7,C8,C9,C10,C11)						
17	5	planet ecl lat (deg)	1.035218287	=PlanetLat(C3,C4,C5,C6,C7,C8,C9,C10,C11)						
18	6	planet RA (rad)	2.925680439	=RADIANS(ecra(C16,0,0,C17,0,0,C8,C9,C10))						
19	7	planet dec (rad)	0.112231764	=RADIANS(ECDec(C16,0,0,C17,0,0,C8,C9,C10))						
20	8	planet dist (AU)	5.598285365	=PlanetDist(C3,C4,C5,C6,C7,C8,C9,C10,C11)						
21	9	light travel time (hours)	0.775922352	=C20*0.1386						
22	10	angular diameter (arcsec)	35.14290308	=VLOOKUP(C11,'Planet data'!A3:D9,3,FALSE)/C20						
23	11	phase	0.992275914	=0.5*(1+COS(RADIANS(C16-PlanetHLong1(C3,C4,C5,C6,C7,C8,C9,C10,C11))))						
24	12	Sun ecl long (deg)	239.2631516	=SunLong(C3,C4,C5,C6,C7,C8,C9,C10)						
25	13	Sun RA (rad)	4.137341758	=RADIANS(ECRA(C24,0,0,0,0,0,C13,C14,C15))						
26	14	Sun dec (rad)	-0.348952785	=RADIANS(ECDec(C24,0,0,0,0,0,C13,C14,C15))						
27	15	y	0.879777546	=COS(C26)*SIN(C25-C18)						
28	16	x	-0.376753175	=COS(C19)*SIN(C26)-SIN(C19)*COS(C26)*COS(C25-C18)						
29	17	chi (deg)	113.1823938	=DEGREES(ATAN2(C28,C27))						
30	18	radius vector (AU)	5.395337237	=PlanetRVect(C3,C4,C5,C6,C7,C8,C9,C10,C11)						
31	19	approximate magnitude	-1.991212985	=5*LOG10(C30*C20/SQRT(C23))+VLOOKUP(C11,'Planet data'!A3:D9,4,FALSE)						

	A	B	C	D	E	F	G	H	I	J	K	L
1	**Planet name**		**Theta0**	**V0**								
2												
3	**JUPITER**		196.74	-9.4								
4	**MARS**		9.36	-1.52								
5	**MERCURY**		6.74	-0.42								
6	**NEPTUNE**		62.2	-6.87								
7	**SATURN**		165.6	-8.88								
8	**URANUS**		65.8	-7.19								
9	**VENUS**		16.92	-4.4								

Figure 72. Calculating some visual aspects of a planet. The upper panel shows the main spreadsheet, and the lower panel shows the data table.

61 Comets

In earlier sections we discovered how to calculate the orbit of any solid body moving around a central massive object, and we applied the method to the major planets of our Solar System. All we needed to know were the orbital elements of each planet. Likewise, we can calculate the position of a periodic comet given its orbital elements but the method needs to be modified slightly for two reasons:

(i) The longitude of the comet is not usually specified at a particular epoch. Rather, the epoch is given when the comet is at perihelion, the point of its closest approach to the Sun.
(ii) The eccentricity, e, of a comet is usually much more than 0.1 so that the equation of the centre does not apply. Instead, we have to solve Kepler's equation properly.

The orbital elements of some periodic comets are given in Table 9. Note that, as in the case of the planetary elements, we have specified ϖ, the longitude of the perihelion. Sometimes the **argument** of the perihelion is given instead. It has the symbol ω (very confusing) and is related to ϖ by $\varpi = \omega + \Omega$.

We begin, as before, by finding the mean anomaly, M, of the comet given by the formula

$$M = \frac{360}{365.242\,191} \times \frac{D}{T_\mathrm{p}} + \varepsilon - \varpi,$$

where D is the number of days since the epoch, T_p is the orbital period in years, ε is the longitude of the comet at the epoch and ϖ is the longitude of the perihelion. In this case, however, the epoch is the moment of perihelion so that $\varepsilon = \varpi$. Further, we do not need to specify the date in terms of days since the epoch. Rather, we can work in decimal years. Hence, M may be found from

$$M = \frac{360Y}{T_\mathrm{p}},$$

where Y is the number of tropical years since perihelion.

Next we have to solve Kepler's equation

$$E - e\sin E = M,$$

where e is the eccentricity and E is the eccentric anomaly. Both E and M are expressed in radians. A method of doing this was given in Section 47, *Routine R2*, in which the eccentricity was assumed to be less than 0.1 so that the first guess at the solution, $E = M$, was good enough for an accurate solution to be reached after only one or two iterations. Here, the eccentricity is much larger and although the routine would always converge eventually, many iterations might be needed. We can speed up the process if we begin with an approximate solution that is better than $E = M$. Kepler's graphs, Figure 73, are provided for this purpose. Given any value of e between 0 and 1 and the value of M (expressed in radians), you choose the corresponding value of Δ from the graphs. Then, in place of the first guess $E = E_0 = M$, use instead $E = E_0 = M + \Delta$ and proceed with the rest of *Routine R2* as before. You should find that only two or three iterations are needed whatever the values of e and M.

Alternatively, you may like to use the nomogram of Figure 74 (kindly provided by Mr S. J. Garvey) to find Δ. Place a ruler across the diagram joining the value of M (in radians) on the right-hand vertical scale with the value of e on the left-hand vertical scale. The point of intersection with the curve gives the magnitude of Δ/e. Multiply this by e to find Δ and give it the sign shown on the right-hand scale. For example, the line joining $M = 5.6$ with $e = 0.46$ cuts the curve at $|\Delta/e| = 0.9$. Thus $|\Delta| = 0.9 \times 0.46 = 0.41$ and its sign is negative, giving

$$\Delta = -0.41.$$

(Vertical bars either side of a quantity signify the absolute value.) Having found E, we can calculate the true anomaly, ν, from

$$\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$

(remember that you have found E in radians), and then carry on with the rest of the calculations of Section 54. If we find that r' is less than R, we must use the formula at the end which is appropriate for an **inner planet**, while if r' is greater than R, we must use the formula for an **outer planet**. In these calculations, bear in mind that the epoch for the comet and the epoch for the Earth are usually different.

Comet name	P	ϖ (degrees)	☊ (degrees)	T_{p} (years)	a (AU)	e	i (degrees)
Encke	1974.32	160.1	334.2	3.30	2.209	0.847	12.0
Temple 2	1972.87	310.2	119.3	5.26	3.024	0.549	12.5
Haneda–Campos	1978.77	12.016	131.700	5.37	3.066	0.641 52	5.805
Schwassmann–Wachmann 2	1974.70	123.3	126.0	6.51	3.489	0.386	3.7
Borrelly	1974.36	67.8	75.1	6.76	3.576	0.632	30.2
Whipple	1970.77	18.2	188.4	7.47	3.821	0.351	10.2
Oterma	1958.44	150.0	155.1	7.88	3.958	0.144	4.0
Schaumasse	1960.29	138.1	86.2	8.18	4.054	0.705	12.0
Comas Sola	1969.83	102.9	62.8	8.55	4.182	0.577	13.4
Schwassmann–Wachmann 1	1974.12	334.1	319.6	15.03	6.087	0.105	9.7
Neujmin 1	1966.94	334.0	347.2	17.93	6.858	0.775	15.0
Crommelin	1956.82	86.4	250.4	27.89	9.173	0.919	28.9
Olbers	1956.46	150.0	85.4	69.47	16.843	0.930	44.6
Pons–Brooks	1954.39	94.2	255.2	70.98	17.200	0.955	74.2
Halley	1986.112	170.011 0	58.154 0	76.008 1	17.943 5	0.967 3	162.238 4

P: epoch of the perihelion; ϖ: longitude of the perihelion; ☊: longitude of the ascending node; T_p: period of the orbit; a: semi-major axis of the orbit; e: eccentricity of the orbit; i: inclination of the orbit.

Table 9. The orbital elements of some periodic comets.

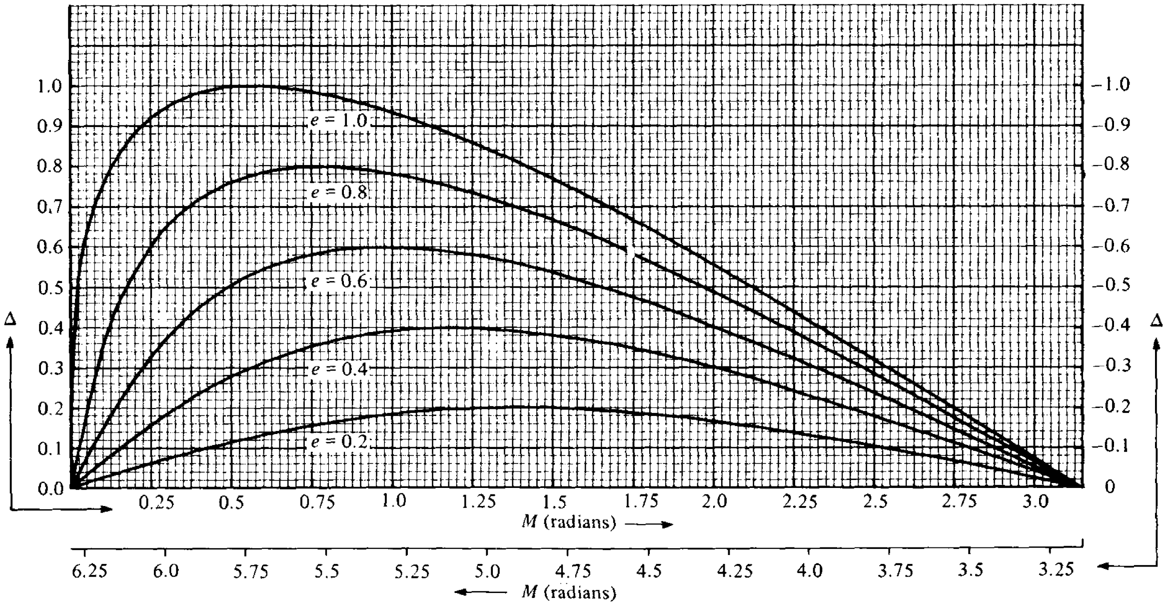


Figure 73. Kepler's graphs. Use the left-hand and upper M scales for values of M (in radians) between 0 and 3.14, and the right-hand and lower M scales for values of M between 3.14 and 6.28.

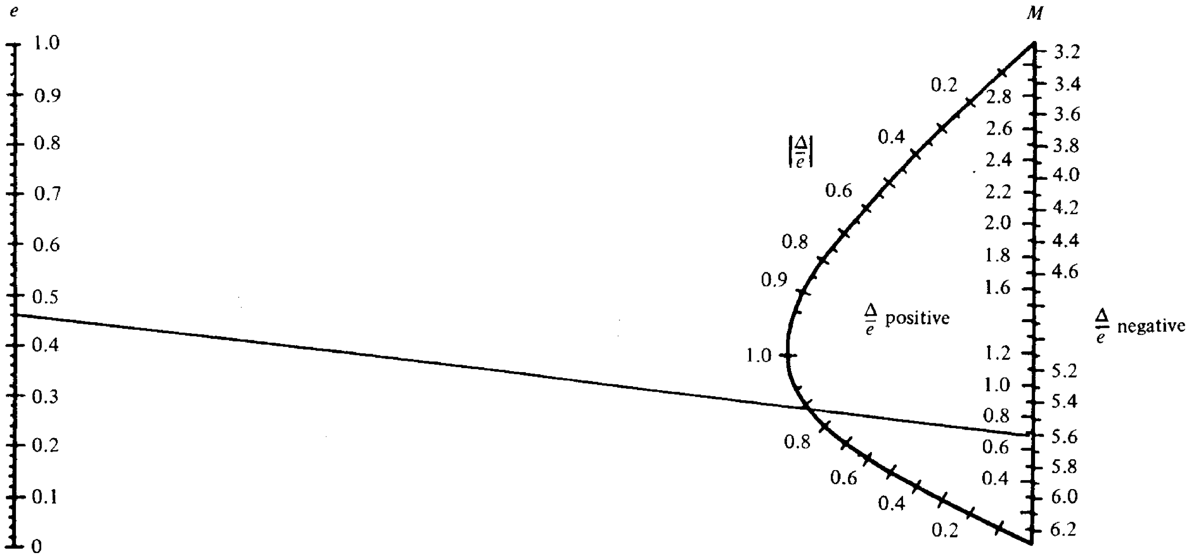


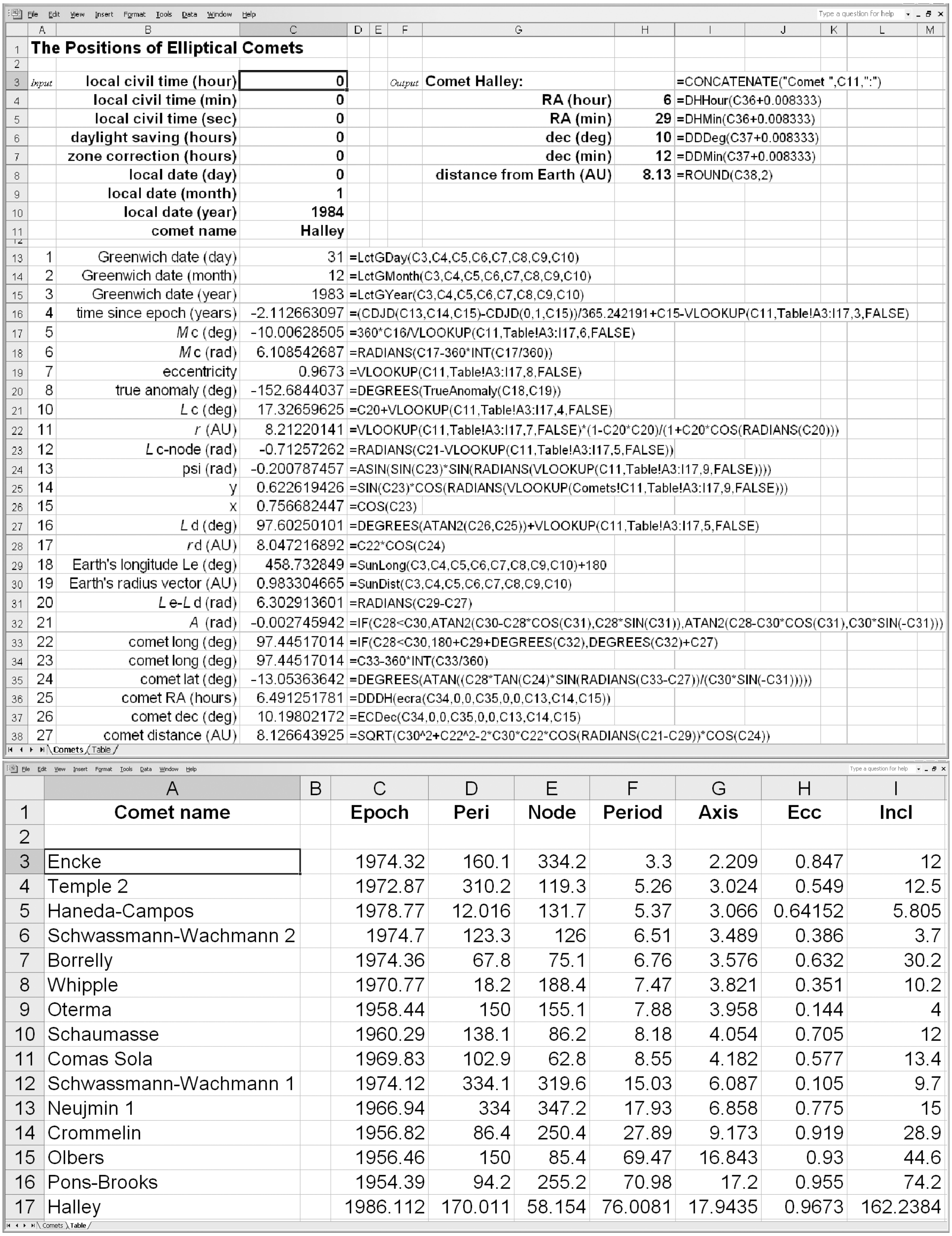
Figure 74. Nomogram to calculate Δ. Vertical bars around Δ/e signify its absolute value.

The method of finding the position of a comet is best clarified by an example. Let us calculate the position of comet Halley at the beginning of the year 1984.

Method		*Example*		
	The calculations for the comet			
1.	Find the number of years since the epoch.	Y	= =	1984.0 – 1986.112 -2.112
2.	Find $M_c = \frac{360Y}{T_p}$ degrees. Subtract multiples of 360 to bring the result back into the range 0 to 360, or if negative add 360.	M_c	=	349.996 856 degrees
3.	Convert M_c to radians by multiplying by $\frac{\pi}{180}$ ($\pi = 3.141\,592\,7$).	M_c	=	6.108 598 radians
4.	Now we solve Kepler's equation by the routine R2 (§47). First guess (from Kepler's graphs).	E_0	= =	$M + (-0.8)$ 5.31 radians
5.	Find a more accurate solution using R2.	E	=	5.307 696 radians
6.	Calculate $\tan\frac{v}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$ (all angles in radians).	$\tan\frac{v}{2}$	=	$-4.114\,741$
7.	Take inverse tan and double to get v.	v	=	$-152.680\,599$ degrees
8.	Find $l = v + \varpi$.	l	=	17.330 401 degrees
9.	Find $r = \frac{a(1-e^2)}{1+e\cos v}$.	r	=	8.210 480 AU
10.	Calculate $\psi = \sin^{-1}\{\sin(l - ☊)\sin i\}$.	ψ	=	$-11.503\,378$ degrees
11.	Find $y = \sin(l - ☊)\cos i$.	y	=	0.622 572
12.	Find $x = \cos(l - ☊)$.	x	=	0.756 726
13.	Find $\tan^{-1}\left(\frac{y}{x}\right)$ and remove the ambiguity by comparing the signs of x and y with Figure 29. If not in the correct quadrant, add or subtract 180 or 360.	$\tan^{-1}\left(\frac{y}{x}\right)$	=	39.444 727 degrees
14.	Add ☊ to find l'.	l'	=	97.598 727 degrees
15.	Find $r' = r\cos\psi$.	r'	=	8.045 555 AU
	Now do the calculations for the Earth			
16.	Find $N_E = \frac{360}{365.242\,191} \times \frac{D}{T_E}$, where D is the number of days since 1990.0 (see Table 3). Subtract multiples of 360 to bring the result into the range 0 to 360.	D N_E	= = =	-2192 $-2160.452\,587$ 359.547 413 degrees
17.	Calculate $M_E = N_E + \varepsilon - \varpi$ (see Table 8).	M_E	=	356.182 293 degrees
18.	Find $L = N_E + \frac{360}{\pi} e \sin\left(M_E\right) + \varepsilon$ ($\pi = 3.141\,592\,7$). If the result is more than 360, subtract 360. If the result is negative, add 360.	L	=	98.823 197 degrees
19.	Calculate $v_E = L - \varpi$.	v_E	=	$-3.945\,223$ degrees
20.	Find $R = \frac{a(1-e^2)}{1+e\cos v_E}$.	R	=	0.983 325 AU
21.	If r' is less than R, calculate λ by using equation (a); if r' is more than R, use equation (b): (a) $\lambda = 180 + L + \tan^{-1}\left\{\frac{r'\sin(L-l')}{R - r'\cos(L-l')}\right\}$, ($b$) $\lambda = \tan^{-1}\left\{\frac{R\sin(l'-L)}{r' - R\cos(l'-L)}\right\} + l'$. Add 360 to the result if negative.	r' λ	$>$ =	R; use equation (b) 97.428 255 degrees
22.	Find $\beta = \tan^{-1}\left\{\frac{r'\tan\psi\sin(\lambda - l')}{R\sin(l'-L)}\right\}$.	β	=	$-13.052\,959$ degrees
23.	Calculate the right ascension and declination using the method given in §27.	α δ	= =	**6h 29m** **10° 12′**
24.	Find the distance using the formula given in §57.	ρ	=	**8.13 AU**

The *Astronomical Almanac* gives these values as $\alpha = 6\text{h}\,29\text{m}$, $\delta = 10^\circ\ 13'$ and $\rho = 7.2$ AU. We must not expect great precision when dealing with comets as the orbital elements change all the time because of the perturbations to the comet's orbit by the gravitational fields of the planets.

The spreadsheet for this calculation, called Comets, is shown in Figure 75. Once again, we have made use of the spreadsheet function VLOOKUP (e.g. row 16) to look up data in a table contained on a separate page (here simply called Table). We have also provided the spreadsheet formula called TrueAnomaly (row 20) which solves Kepler's equation and returns the value of the true anomaly in radians. Its two arguments are the mean anomaly in radians, and the eccentricity.



	A	B	C	D	E	F	G	H	I
1	**The Positions of Elliptical Comets**								
2									
3	*Input*	**local civil time (hour)**	**0**			*Output*	**Comet Halley:**		=CONCATENATE("Comet ",C11,":")
4		**local civil time (min)**	**0**				**RA (hour)**	**6**	=DHHour(C36+0.008333)
5		**local civil time (sec)**	**0**				**RA (min)**	**29**	=DHMin(C36+0.008333)
6		**daylight saving (hours)**	**0**				**dec (deg)**	**10**	=DDDeg(C37+0.008333)
7		**zone correction (hours)**	**0**				**dec (min)**	**12**	=DDMin(C37+0.008333)
8		**local date (day)**	**0**				**distance from Earth (AU)**	**8.13**	=ROUND(C38,2)
9		**local date (month)**	**1**						
10		**local date (year)**	**1984**						
11		**comet name**	**Halley**						
12									
13	1	Greenwich date (day)	31	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)					
14	2	Greenwich date (month)	12	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)					
15	3	Greenwich date (year)	1983	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)					
16	4	time since epoch (years)	-2.112663097	=(CDJD(C13,C14,C15)-CDJD(0,1,C15))/365.242191+C15-VLOOKUP(C11,Table!A3:I17,3,FALSE)					
17	5	*M* c (deg)	-10.00628505	=360*C16/VLOOKUP(C11,Table!A3:I17,6,FALSE)					
18	6	*M* c (rad)	6.108542687	=RADIANS(C17-360*INT(C17/360))					
19	7	eccentricity	0.9673	=VLOOKUP(C11,Table!A3:I17,8,FALSE)					
20	8	true anomaly (deg)	-152.6844037	=DEGREES(TrueAnomaly(C18,C19))					
21	10	*L* c (deg)	17.32659625	=C20+VLOOKUP(C11,Table!A3:I17,4,FALSE)					
22	11	*r* (AU)	8.21220141	=VLOOKUP(C11,Table!A3:I17,7,FALSE)*(1-C20*C20)/(1+C20*COS(RADIANS(C20)))					
23	12	*L* c-node (rad)	-0.71257262	=RADIANS(C21-VLOOKUP(C11,Table!A3:I17,5,FALSE))					
24	13	psi (rad)	-0.200787457	=ASIN(SIN(C23)*SIN(RADIANS(VLOOKUP(C11,Table!A3:I17,9,FALSE))))					
25	14	y	0.622619426	=SIN(C23)*COS(RADIANS(VLOOKUP(Comets!C11,Table!A3:I17,9,FALSE)))					
26	15	x	0.756682447	=COS(C23)					
27	16	*L* d (deg)	97.60250101	=DEGREES(ATAN2(C26,C25))+VLOOKUP(C11,Table!A3:I17,5,FALSE)					
28	17	*r* d (AU)	8.047216892	=C22*COS(C24)					
29	18	Earth's longitude Le (deg)	458.732849	=SunLong(C3,C4,C5,C6,C7,C8,C9,C10)+180					
30	19	Earth's radius vector (AU)	0.983304665	=SunDist(C3,C4,C5,C6,C7,C8,C9,C10)					
31	20	*L* e-*L* d (rad)	6.302913601	=RADIANS(C29-C27)					
32	21	*A* (rad)	-0.002745942	=IF(C28<C30,ATAN2(C30-C28*COS(C31),C28*SIN(C31)),ATAN2(C28-C30*COS(C31),C30*SIN(-C31)))					
33	22	comet long (deg)	97.44517014	=IF(C28<C30,180+C29+DEGREES(C32),DEGREES(C32)+C27)					
34	23	comet long (deg)	97.44517014	=C33-360*INT(C33/360)					
35	24	comet lat (deg)	-13.05363642	=DEGREES(ATAN((C28*TAN(C24)*SIN(RADIANS(C33-C27))/(C30*SIN(-C31)))))					
36	25	comet RA (hours)	6.491251781	=DDDH(ecra(C34,0,0,C35,0,0,C13,C14,C15))					
37	26	comet dec (deg)	10.19802172	=ECDec(C34,0,0,C35,0,0,C13,C14,C15)					
38	27	comet distance (AU)	8.126643925	=SQRT(C30^2+C22^2-2*C30*C22*COS(RADIANS(C21-C29))*COS(C24))					

	A	B	C	D	E	F	G	H	I
1	**Comet name**		**Epoch**	**Peri**	**Node**	**Period**	**Axis**	**Ecc**	**Incl**
2									
3	Encke		1974.32	160.1	334.2	3.3	2.209	0.847	12
4	Temple 2		1972.87	310.2	119.3	5.26	3.024	0.549	12.5
5	Haneda-Campos		1978.77	12.016	131.7	5.37	3.066	0.64152	5.805
6	Schwassmann-Wachmann 2		1974.7	123.3	126	6.51	3.489	0.386	3.7
7	Borrelly		1974.36	67.8	75.1	6.76	3.576	0.632	30.2
8	Whipple		1970.77	18.2	188.4	7.47	3.821	0.351	10.2
9	Oterma		1958.44	150	155.1	7.88	3.958	0.144	4
10	Schaumasse		1960.29	138.1	86.2	8.18	4.054	0.705	12
11	Comas Sola		1969.83	102.9	62.8	8.55	4.182	0.577	13.4
12	Schwassmann-Wachmann 1		1974.12	334.1	319.6	15.03	6.087	0.105	9.7
13	Neujmin 1		1966.94	334	347.2	17.93	6.858	0.775	15
14	Crommelin		1956.82	86.4	250.4	27.89	9.173	0.919	28.9
15	Olbers		1956.46	150	85.4	69.47	16.843	0.93	44.6
16	Pons-Brooks		1954.39	94.2	255.2	70.98	17.2	0.955	74.2
17	Halley		1986.112	170.011	58.154	76.0081	17.9435	0.9673	162.2384

Figure 75. Finding the position of a comet. The upper panel shows the main spreadsheet, and the lower panel shows the data table.

62 Parabolic orbits

In preceding sections, we have calculated the orbits of members of the Solar System which are gravitationally bound to the Sun, like the planets and the periodic comets. These objects move in (more or less) elliptical orbits with the Sun at a focus of the ellipse, and in the absence of perturbations from other members of the Solar System or from external influences, would continue to move indefinitely along the same elliptical paths. However, some comets do not seem to be bound to the Sun. If unperturbed they would appear once and shoot off into space never to return again. Their orbits are often defined in terms of **parabolic motion** and we have to use a slightly different procedure for calculating their positions, given the parabolic orbital elements:

t_0 = the epoch of the perihelion (a calendar date);
q = perihelion distance (AU);
i = inclination of the orbit (degrees);
ω = argument of the perihelion (degrees; $\omega = \varpi - \Omega$); and
Ω = longitude of the ascending node (degrees).

The calculations proceed on much the same lines as for an elliptical orbit; once we have found the true anomaly, ν, and radius vector, r, we can use exactly the same method as in Section 61 to calculate the position of the comet. However, the calculations of ν and r are slightly different.

First, we find the value of the quantity W (in radians) from

$$W = \frac{0.036\,491\,162\,4}{q\sqrt{q}} \times d,$$

where d is the number of days since the comet passed through perihelion. Next we have to solve the equation

$$s^3 + 3s - W = 0.$$

This can be done by means of the iterative method shown in *Routine R3* below. Finally, calculate ν and r from

$$\nu = 2\tan^{-1} s,$$

$$r = q\left(1 + s^2\right).$$

(Note that s is in radians.) For example, the International Astronomical Union issued the following data on comet Kohler (designated 1977m; IAU circular number 3137):

t = 1977 November 10.5659
q = 0.990 662 AU
i = 48.719 6 degrees
ω = 163.479 9 degrees
Ω = 181.817 5 degrees (hence $\varpi = \omega + \Omega = 345.2974$ degrees).

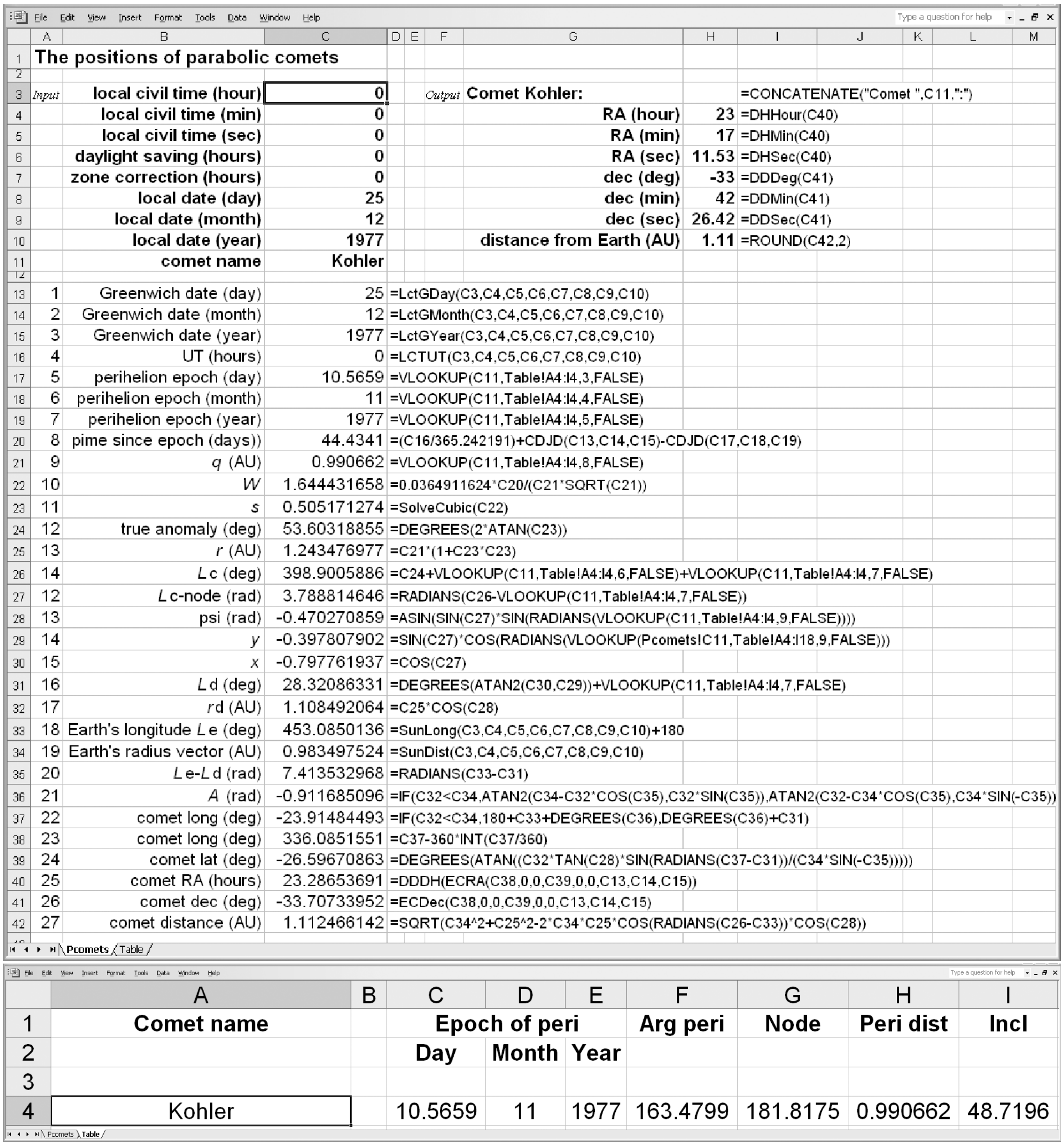
The values of i, ω and Ω quoted here were referred to the standard equinox of 1950.0. Strictly, we should refer all our calculations to the same equinox, but we shall ignore the small error introduced by not doing so. Let us calculate the position of the comet on Christmas Day 1977, assuming no perturbations to its orbit.

Method		*Example*		
1.	Calculate the number of days since the epoch of perihelion. This can be done (for example) by subtracting the Julian date (§4) of the epoch from the Julian date of the day in question.	10 Nov. 1977: JD_1	=	2 443 457.5
		25 Dec. 1977: JD_2	=	2 443 502.5
		$\mathrm{JD}_2 - \mathrm{JD}_1$	=	45 days
		Epoch	=	November 10.5659
		$\therefore d$	=	44.4341 days
2.	Find $W = \frac{0.03649116 24}{q\sqrt{q}} \times d$.	W	=	1.644 432
3.	Solve $s^3 + 3s - W = 0$ by using Routine R3.	First guess s	=	0.548 144
		s	=	0.505 171
4.	Find $v = 2\tan^{-1} s$ and $r = q\left(1+s^2\right)$.	v	=	53.603 189 degrees
		r	=	1.243 477 AU
5.	Carry on at instruction 8 (ignoring instruction 9) of §61 to find α and δ.	l	=	398.900 589 degrees
		ψ	=	−26.944 536 degrees
		l'	=	28.320 864 degrees
		r'	=	1.108 492 AU
	For the Earth:	D	=	−736 days
		N_{E}	=	−725.407 420 degrees
			=	354.592 580 degrees
		L	=	93.120 810 degrees
		R	=	0.983 503 AU
		r'	>	R; use equation (b)
		λ	=	336.099 109 degrees
		β	=	−26.585 505 degrees
	Here is the result:	α	=	**23h 17m**
		δ	=	**−33° 41′**

Comet Kohler was in the constellation of Sculptor on Christmas Day 1977.
Routine R3: To solve the equation $s^3 + 3s - W = 0$.

1. First guess put $s = s_0 = W/3$.
2. Calculate $\delta = s^3 + 3s - W$.
3. If $|\delta| < \varepsilon$, go to step 6.
   Otherwise proceed with step 4.
   ε is the required accuracy (e.g. 10^{-6} degree).
4. Calculate $s_1 = \frac{2s^3+W}{3\left(s^2+1\right)}$ (note that s^3 can be calculated from $s \times s^2$).
5. Set $s = s_1$ and go to step 2.
6. The current value of s is within $\pm\varepsilon$ of the correct value.

The spreadsheet for finding the position of a comet in a parabolic orbit is called Pcomets and is shown in Figure 76. Once again, we have used the spreadsheet function VLOOKUP to select an item of data from a table (e.g. row 17). In this case, the table contains data for only one comet, but you can easily extend it and add more data. Note that the comets need to be listed alphabetically in column A, and that the second argument (at present Table!A4:I4) needs to be altered so that the reference after the colon is to the bottom right-hand cell of the table. For example, if you add two more comets, the second argument would become Table!A4:I6. Do this for every instance of VLOOKUP in the main spreadsheet. We have also provided a new spreadsheet function called SolveCubic (row 23) that takes one argument, namely the value of W (in radians) as defined above and that returns the value of s (also in radians).

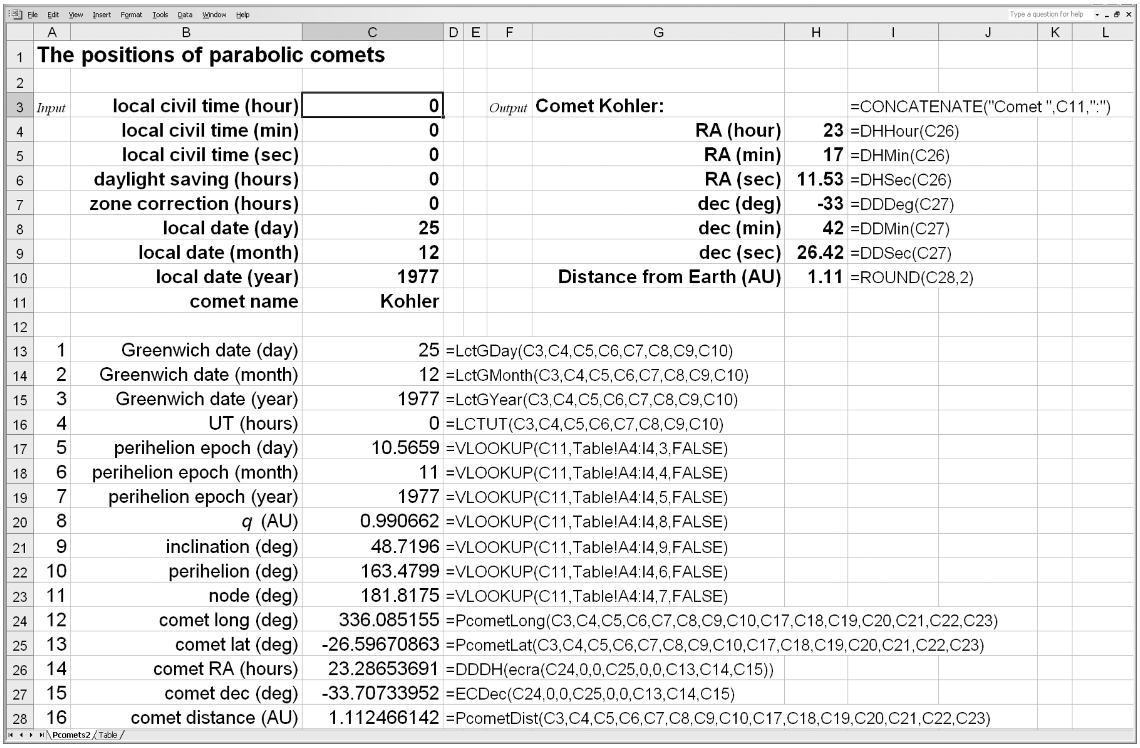


	A	B	C	D	E	F	G	H	I
1	The positions of parabolic comets								
2									
3	*Input*	local civil time (hour)	0			*Output*	Comet Kohler:		=CONCATENATE("Comet ",C11,":")
4		local civil time (min)	0				RA (hour)	23	=DHHour(C40)
5		local civil time (sec)	0				RA (min)	17	=DHMin(C40)
6		daylight saving (hours)	0				RA (sec)	11.53	=DHSec(C40)
7		zone correction (hours)	0				dec (deg)	-33	=DDDeg(C41)
8		local date (day)	25				dec (min)	42	=DDMin(C41)
9		local date (month)	12				dec (sec)	26.42	=DDSec(C41)
10		local date (year)	1977				distance from Earth (AU)	1.11	=ROUND(C42,2)
11		comet name	Kohler						
12									
13	1	Greenwich date (day)	25	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)					
14	2	Greenwich date (month)	12	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)					
15	3	Greenwich date (year)	1977	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)					
16	4	UT (hours)	0	=LCTUT(C3,C4,C5,C6,C7,C8,C9,C10)					
17	5	perihelion epoch (day)	10.5659	=VLOOKUP(C11,Table!A4:I4,3,FALSE)					
18	6	perihelion epoch (month)	11	=VLOOKUP(C11,Table!A4:I4,4,FALSE)					
19	7	perihelion epoch (year)	1977	=VLOOKUP(C11,Table!A4:I4,5,FALSE)					
20	8	pime since epoch (days))	44.4341	=(C16/365.242191)+CDJD(C13,C14,C15)-CDJD(C17,C18,C19)					
21	9	*q* (AU)	0.990662	=VLOOKUP(C11,Table!A4:I4,8,FALSE)					
22	10	*W*	1.644431658	=0.0364911624*C20/(C21*SQRT(C21))					
23	11	*s*	0.505171274	=SolveCubic(C22)					
24	12	true anomaly (deg)	53.60318855	=DEGREES(2*ATAN(C23))					
25	13	*r* (AU)	1.243476977	=C21*(1+C23*C23)					
26	14	*L* c (deg)	398.9005886	=C24+VLOOKUP(C11,Table!A4:I4,6,FALSE)+VLOOKUP(C11,Table!A4:I4,7,FALSE)					
27	12	*L* c-node (rad)	3.788814646	=RADIANS(C26-VLOOKUP(C11,Table!A4:I4,7,FALSE))					
28	13	psi (rad)	-0.470270859	=ASIN(SIN(C27)*SIN(RADIANS(VLOOKUP(C11,Table!A4:I4,9,FALSE))))					
29	14	*y*	-0.397807902	=SIN(C27)*COS(RADIANS(VLOOKUP(Pcomets!C11,Table!A4:I18,9,FALSE)))					
30	15	*x*	-0.797761937	=COS(C27)					
31	16	*L* d (deg)	28.32086331	=DEGREES(ATAN2(C30,C29))+VLOOKUP(C11,Table!A4:I4,7,FALSE)					
32	17	*r* d (AU)	1.108492064	=C25*COS(C28)					
33	18	Earth's longitude *L* e (deg)	453.0850136	=SunLong(C3,C4,C5,C6,C7,C8,C9,C10)+180					
34	19	Earth's radius vector (AU)	0.983497524	=SunDist(C3,C4,C5,C6,C7,C8,C9,C10)					
35	20	*L* e-*L* d (rad)	7.413532968	=RADIANS(C33-C31)					
36	21	*A* (rad)	-0.911685096	=IF(C32<C34,ATAN2(C34-C32*COS(C35),C32*SIN(C35)),ATAN2(C32-C34*COS(C35),C34*SIN(-C35))					
37	22	comet long (deg)	-23.91484493	=IF(C32<C34,180+C33+DEGREES(C36),DEGREES(C36)+C31)					
38	23	comet long (deg)	336.0851551	=C37-360*INT(C37/360)					
39	24	comet lat (deg)	-26.59670863	=DEGREES(ATAN((C32*TAN(C28)*SIN(RADIANS(C37-C31))/(C34*SIN(-C35)))))					
40	25	comet RA (hours)	23.28653691	=DDDH(ECRA(C38,0,0,C39,0,0,C13,C14,C15))					
41	26	comet dec (deg)	-33.70733952	=ECDec(C38,0,0,C39,0,0,C13,C14,C15)					
42	27	comet distance (AU)	1.112466142	=SQRT(C34^2+C25^2-2*C34*C25*COS(RADIANS(C26-C33))*COS(C28))					

	A	B	C	D	E	F	G	H	I
1	**Comet name**		**Epoch of peri**			**Arg peri**	**Node**	**Peri dist**	**Incl**
2			**Day**	**Month**	**Year**				
3									
4	Kohler		10.5659	11	1977	163.4799	181.8175	0.990662	48.7196

Figure 76. Finding the position of a comet in a parabolic orbit. The upper panel shows the main spreadsheet, and the lower panel shows the data table.

You can, if you prefer, simplify the main spreadsheet by using three other spreadsheet functions that we have provided for you. They are PcometLong, PcometLat and PcometDist, returning the geocentric ecliptic longitude (degrees), the geocentric ecliptic latitude (degrees) and the distance of the comet from Earth (AU) respectively. Each of them takes the same 15 arguments, namely the local civil time expressed as hours, minutes and seconds, daylight saving and time zone offsets in hours, the local civil date as day, month and year, the Greenwich epoch of perihelion expressed as day, month and year, the perihelion distance in AU, the inclination of the orbit in degrees, the argument of the perihelion in degrees, and finally the longitude of the ascending node in degrees. You will need to use the VLOOKUP function to obtain the arguments corresponding to the orbital elements (we suggest doing this first), and you will also need rows 40 and 41 to convert the ecliptic coordinates into right ascension and declination, but the spreadsheet should look much less complex as a result of using these functions. One example is shown in Figure 77.



	A	B	C	D	E	F	G	H	I	J	K	L
1	**The positions of parabolic comets**											
2												
3	*Input*	**local civil time (hour)**	**0**			*Output*	**Comet Kohler:**		=CONCATENATE("Comet ",C11,":")			
4		**local civil time (min)**	**0**				**RA (hour)**	**23**	=DHHour(C26)			
5		**local civil time (sec)**	**0**				**RA (min)**	**17**	=DHMin(C26)			
6		**daylight saving (hours)**	**0**				**RA (sec)**	**11.53**	=DHSec(C26)			
7		**zone correction (hours)**	**0**				**dec (deg)**	**-33**	=DDDeg(C27)			
8		**local date (day)**	**25**				**dec (min)**	**42**	=DDMin(C27)			
9		**local date (month)**	**12**				**dec (sec)**	**26.42**	=DDSec(C27)			
10		**local date (year)**	**1977**				**Distance from Earth (AU)**	**1.11**	=ROUND(C28,2)			
11		**comet name**	**Kohler**									
12												
13	1	Greenwich date (day)	25	=LctGDay(C3,C4,C5,C6,C7,C8,C9,C10)								
14	2	Greenwich date (month)	12	=LctGMonth(C3,C4,C5,C6,C7,C8,C9,C10)								
15	3	Greenwich date (year)	1977	=LctGYear(C3,C4,C5,C6,C7,C8,C9,C10)								
16	4	UT (hours)	0	=LCTUT(C3,C4,C5,C6,C7,C8,C9,C10)								
17	5	perihelion epoch (day)	10.5659	=VLOOKUP(C11,Table!A4:I4,3,FALSE)								
18	6	perihelion epoch (month)	11	=VLOOKUP(C11,Table!A4:I4,4,FALSE)								
19	7	perihelion epoch (year)	1977	=VLOOKUP(C11,Table!A4:I4,5,FALSE)								
20	8	q (AU)	0.990662	=VLOOKUP(C11,Table!A4:I4,8,FALSE)								
21	9	inclination (deg)	48.7196	=VLOOKUP(C11,Table!A4:I4,9,FALSE)								
22	10	perihelion (deg)	163.4799	=VLOOKUP(C11,Table!A4:I4,6,FALSE)								
23	11	node (deg)	181.8175	=VLOOKUP(C11,Table!A4:I4,7,FALSE)								
24	12	comet long (deg)	336.085155	=PcometLong(C3,C4,C5,C6,C7,C8,C9,C10,C17,C18,C19,C20,C21,C22,C23)								
25	13	comet lat (deg)	-26.59670863	=PcometLat(C3,C4,C5,C6,C7,C8,C9,C10,C17,C18,C19,C20,C21,C22,C23)								
26	14	comet RA (hours)	23.28653691	=DDDH(ecra(C24,0,0,C25,0,0,C13,C14,C15))								
27	15	comet dec (deg)	-33.70733952	=ECDec(C24,0,0,C25,0,0,C13,C14,C15)								
28	16	comet distance (AU)	1.112466142	=PcometDist(C3,C4,C5,C6,C7,C8,C9,C10,C17,C18,C19,C20,C21,C22,C23)								

Figure 77. Finding the position of a comet in a parabolic orbit using the Pcomet spreadsheet functions.

63 Binary-star orbits

Quite often an astronomer sees a pair of stars very close together in the telescope. This apparent closeness may be just because two quite unrelated stars happen to lie near to the same line of sight. Sometimes, however, the stars are actually close to one another in space and they may then form a **binary star** in which each is bound to the other by mutual gravitational attraction. The stars describe elliptical orbits about one another, just as Jupiter describes an elliptical orbit about the Sun. The brighter of the two stars is generally called the **primary** and the fainter is called the **companion**; we shall consider that the companion orbits about the primary which is fixed in space, although really both stars orbit about their common centre of mass.

Figure 78 shows the appearance of a binary star. A is the primary, B the companion, and the line NAS is the observer's meridian through A; AN therefore defines the direction north. The line joining A to B is at position-angle θ (measured anticlockwise as shown) and of length ρ. Provided that we know the elements of the binary orbit, we can calculate the values of θ and ρ and hence we can predict the appearance of the binary star at any time.

A binary-star orbit is drawn in Figure 79. The sphere is centred on the primary star, A, and its companion, B, describes an orbit about it shown by the small hatched ellipse in the centre. The great circle NL′DM′ shows where the plane through A perpendicular to the line of sight cuts the sphere. This plane is the plane of the sky as seen from the Earth. The line AN defines the direction north as in Figure 78. The great circle L′P′B′M′ shows where the plane of the true binary orbit cuts the sphere. The point L′ is the projection of the ascending node, L, onto the sphere, M′ the projection of the descending node, M, and P′ the projection of the point of closest approach, P, the **periastron**. The companion star is at B. Longitudes are reckoned from the ascending node, L, and the true anomaly, ν, is the angle between B and the periastron. We need the following elements to calculate the orbit:

T = the period of revolution;
t = the epoch of periastron;
e = the eccentricity of the orbit;
a = the semi-major axis of the orbit;
i = the inclination of the orbit to the plane of the sky;
Ω = the position-angle of the ascending node; and
ω = the longitude of the periastron.

All angles are measured in the direction of motion. The elements for some binary stars are listed in Table 10.

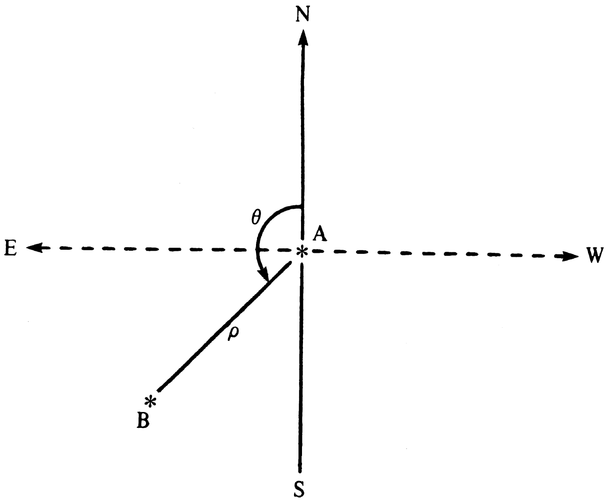


Figure 78. A binary star seen from the Earth.

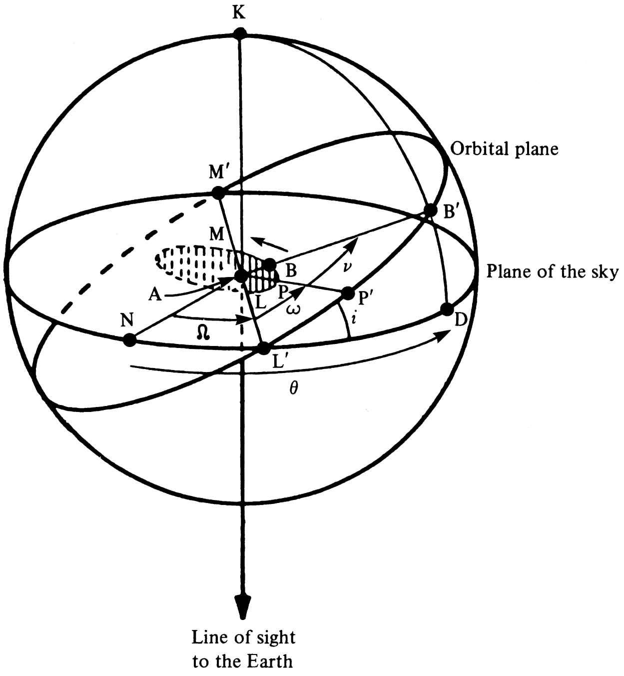


Figure 79. A binary-star orbit.

Name	T_p (years)	t	ω (degrees)	e	a (arcsec)	i (degrees)	☊ (degrees)
η Coronae Borealis	41.623	1934.008	219.907	0.2763	0.907	59.025	23.717
γ Virginis	171.37	1836.433	252.88	0.8808	3.746	146.05	31.78
η Cassiopeiae	480	1889.6	268.59	0.497	11.9939	34.76	278.42
ζ Orionis	1508.6	2070.6	47.3	0.07	2.728	72.0	155.5
α Canis Majoris (Sirius)	50.09	1894.13	147.27	0.5923	7.500	136.53	44.57
δ Geminorum	1200	1437	57.19	0.1100	6.9753	63.28	18.38
α Geminorum (Castor)	420.07	1965.3	261.43	0.33	6.295	115.94	40.47
α Canis Minoris (Procyon)	40.65	1927.6	269.8	0.40	4.548	35.7	284.3
α Centauri	79.920	1955.56	231.560	0.516	17.583	79.240	204.868
α Scorpionis (Antares)	900	1889.0	0.0	0.0	3.21	86.3	273.0

T_p: period; t: epoch of periastron; ω: longitude of periastron; e: eccentricity; a: semi-major axis of orbit; i: inclination of orbit; ☊: position angle of ascending node.

Table 10. The orbital elements of some binary stars.

The calculation of a binary-star orbit proceeds in much the same way as that of a planetary orbit. We first find the mean anomaly, M, from

$$M = \frac{360Y}{T} \text{ degrees,}$$

where Y is the number of years since the epoch of periastron. Next we must solve Kepler's equation

$$E - e\sin E = M \text{ radians,}$$

using the method given in Section 61. The true anomaly, ν, and radius vector, r, can then be found from

$$\nu = 2\tan^{-1}\left\{\sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}\right\},$$

and

$$r = a(1 - e\cos E),$$

(remembering that E has been found in radians). Finally, θ is given by

$$\theta = \tan^{-1}\left\{\frac{\sin(\nu+\omega)\cos i}{\cos(\nu+\omega)}\right\} + \Omega \text{ degrees,}$$

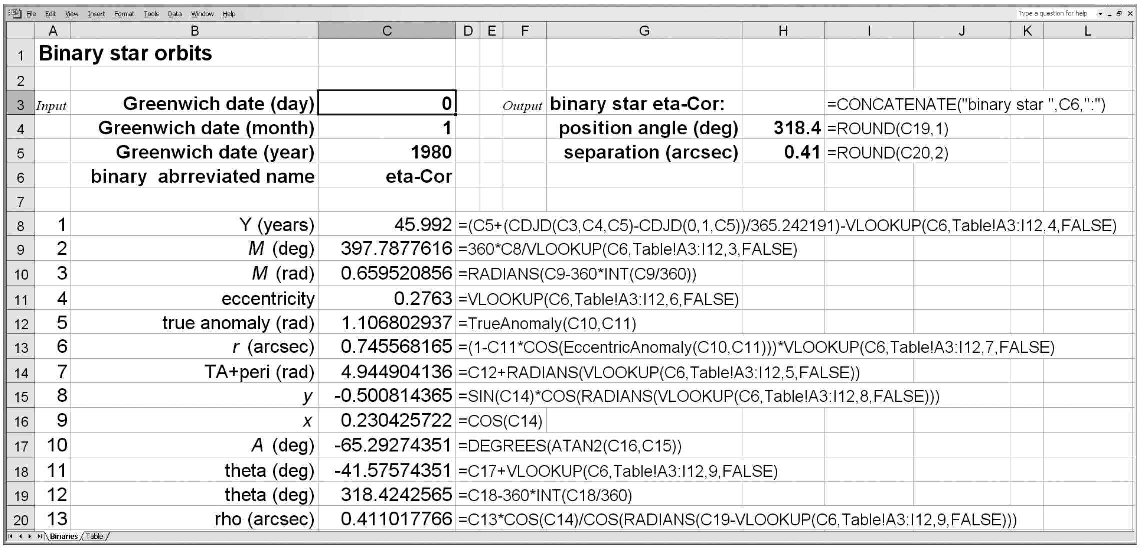
and ρ from

$$\rho = \frac{r\cos(\nu+\omega)}{\cos(\theta-\Omega)} \text{ degrees.}$$

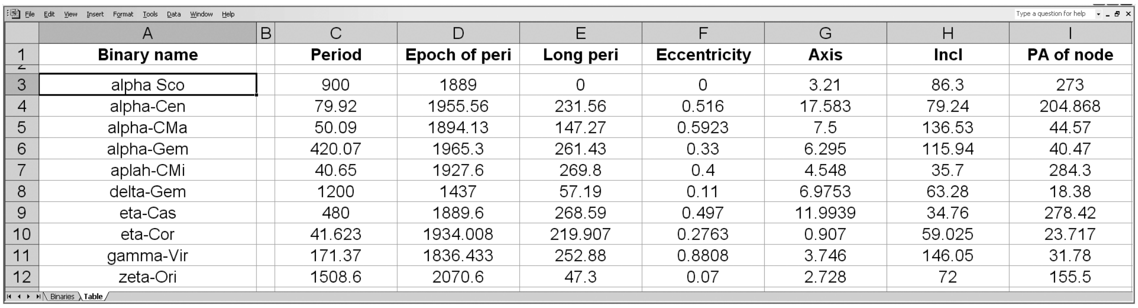
For example, let us calculate the visual aspect of the binary system η Coronae Borealis at the beginning of 1980.

Method		*Example*		
1.	Find the number of years since the epoch.	Y	=	1980.0 – 1934.008
			=	45.992 years
2.	Find $M = \frac{360Y}{T}$. Subtract multiples of 360	M	=	397.787 762 degrees
	to bring the result into the range 0 to 360.		=	37.787 762 degrees
3.	Convert to radians by multiplying by $\frac{\pi}{180}$ (π = 3.141 592 7).	M	=	0.659 521 radians
4.	Solve Kepler's equation $E - e\sin E = M$	First guess E_0	=	0.86 radians
	by the method outlined in §61.	Solution is E	=	0.870 858 radians
5.	Find $\nu = 2\tan^{-1}\left\{\sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}\right\}$, all angles in radians.	ν	=	1.106 803 radians
6.	Multiply by $\frac{180}{\pi}$ to convert to degrees (π = 3.141 592 7).	ν	=	63.415 137 degrees
7.	Find $r = a(1-e\cos E)$, remembering that E is expressed in radians.	r	=	0.745 568 arcsec
8.	Calculate $y = \sin(\nu+\omega)\cos i$.	y	=	−0.500 814
9.	Calculate $x = \cos(\nu+\omega)$.	x	=	0.230 426
10.	Find $\tan^{-1}\left(\frac{y}{x}\right)$ and remove the ambiguity by	$\tan^{-1}\left(\frac{y}{x}\right)$	=	−65.292 744
	reference to Figure 29. Add or subtract 180		+	360
	or 360 to bring the result into the correct quadrant,		=	294.707 256 degrees
	unless it is already in the correct quadrant.			
11.	Add Ω to find θ. Subtract 360 if more than 360. Add 360 if negative.	θ	=	**318.424 degrees**
12.	Find $\rho = \frac{r\cos(\nu+\omega)}{\cos(\theta-\Omega)}$.	ρ	=	**0.411 arcsec**

Figure 80 shows the spreadsheet for the binary-star calculation. As before, we have put the orbital elements of some binary stars into a data table on a separate page. This table reproduces the data of Table 10, but it represents only a small fraction of the known binary stars. You can add to the table as you wish. The VLOOKUP function (e.g. row 8) is used to select each item from the table as needed. If you extend the table, remember (i) to order the stars in alphabetical order, and (ii) to make sure that the cell reference after the colon in the second argument is to the bottom right-hand cell of the table. We have defined and used a new spreadsheet function EccentricAnomaly (row 13). This solves Kepler's equation and returns the value of the eccentric anomaly in radians for the two arguments, namely the mean anomaly in radians, and the eccentricity. We have already met the function TrueAnomaly (Section 61), which takes the same pair of arguments but returns the true anomaly.



	A	B	C	D	E	F	G	H	I
1	**Binary star orbits**								
2									
3	*Input*	**Greenwich date (day)**	**0**			*Output*	**binary star eta-Cor:**		=CONCATENATE("binary star ",C6,":")
4		**Greenwich date (month)**	**1**				**position angle (deg)**	**318.4**	=ROUND(C19,1)
5		**Greenwich date (year)**	**1980**				**separation (arcsec)**	**0.41**	=ROUND(C20,2)
6		**binary abrreviated name**	**eta-Cor**						
7									
8	1	Y (years)	45.992	=(C5+(CDJD(C3,C4,C5)-CDJD(0,1,C5))/365.242191)-VLOOKUP(C6,Table!A3:I12,4,FALSE)					
9	2	*M* (deg)	397.7877616	=360*C8/VLOOKUP(C6,Table!A3:I12,3,FALSE)					
10	3	*M* (rad)	0.659520856	=RADIANS(C9-360*INT(C9/360))					
11	4	eccentricity	0.2763	=VLOOKUP(C6,Table!A3:I12,6,FALSE)					
12	5	true anomaly (rad)	1.106802937	=TrueAnomaly(C10,C11)					
13	6	*r* (arcsec)	0.745568165	=(1-C11*COS(EccentricAnomaly(C10,C11)))*VLOOKUP(C6,Table!A3:I12,7,FALSE)					
14	7	TA+peri (rad)	4.944904136	=C12+RADIANS(VLOOKUP(C6,Table!A3:I12,5,FALSE))					
15	8	*y*	-0.500814365	=SIN(C14)*COS(RADIANS(VLOOKUP(C6,Table!A3:I12,8,FALSE)))					
16	9	*x*	0.230425722	=COS(C14)					
17	10	*A* (deg)	-65.29274351	=DEGREES(ATAN2(C16,C15))					
18	11	theta (deg)	-41.57574351	=C17+VLOOKUP(C6,Table!A3:I12,9,FALSE)					
19	12	theta (deg)	318.4242565	=C18-360*INT(C18/360)					
20	13	rho (arcsec)	0.411017766	=C13*COS(C14)/COS(RADIANS(C19-VLOOKUP(C6,Table!A3:I12,9,FALSE)))					



	A	B	C	D	E	F	G	H	I
1	**Binary name**		**Period**	**Epoch of peri**	**Long peri**	**Eccentricity**	**Axis**	**Incl**	**PA of node**
3	alpha Sco		900	1889	0	0	3.21	86.3	273
4	alpha-Cen		79.92	1955.56	231.56	0.516	17.583	79.24	204.868
5	alpha-CMa		50.09	1894.13	147.27	0.5923	7.5	136.53	44.57
6	alpha-Gem		420.07	1965.3	261.43	0.33	6.295	115.94	40.47
7	aplah-CMi		40.65	1927.6	269.8	0.4	4.548	35.7	284.3
8	delta-Gem		1200	1437	57.19	0.11	6.9753	63.28	18.38
9	eta-Cas		480	1889.6	268.59	0.497	11.9939	34.76	278.42
10	eta-Cor		41.623	1934.008	219.907	0.2763	0.907	59.025	23.717
11	gamma-Vir		171.37	1836.433	252.88	0.8808	3.746	146.05	31.78
12	zeta-Ori		1508.6	2070.6	47.3	0.07	2.728	72	155.5

Figure 80. Calculating a binary-star orbit. The upper panel shows the main spreadsheet, and the lower panel shows the data table.