University of Tübingen
Faculty of Science
Departement of Biology
Cognitive Neuroscience

Bachelor's thesis in Cognitive Science

Minimizing Angular Error in Labeled Graphs

Jakob Meyer

April 2, 2021

Supervisor

Prof. Dr. Hanspeter A. Mallot
Cognitive Neuroscience, Faculty of Science
University of Tübingen

Meyer, Jakob:
Minimizing Angular Error in Labeled Graphs
Bachelor's thesis in Cognitive Science
Eberhard Karls University Tübingen
Processing period: 17.09.2020-April 2, 2021

Summary

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Abstract

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Thanksgiving

I want to thank my parents for having me as a child and not leave me hanging despite many moments of unyielding stubbornness, which made me grow up in a safe and supporting environment.

I want to thank my siblings for getting on my nerves all the time as kids, which made me become a human able to withstand hardships in any phases of my young life as well as looking at life from the bright side. As a young adult i might appreciate them ever so slightly more as we are growing older each day for that.

I want to thank all the people i have met through my life, mostly the people who took up with me for quite some time or at an intense level of love over maybe not so long a time that would have been wished for.

TODO: Kappa Thanksgiving so far

Contents

1 Introduction

As far as the field of robotics goes back in time, a prime goal has been not only to model and build robots and machines that can accomplish useful tasks that might be of too much work for a mere human to do, but to build and reconstruct the human person itself. From a biological or chemical perspective, there is no clear-cut distinction of external stimuli being received, the forwarding of this information as electrical impulses and its effect on an abundant number of neurons in the brain. Every 'step' in this information processing is in one form or the other done on a purely electrical basis. While perception and processing go wholly intertwined in the human brain, to model what is going on, we need to break down the vastly complex actions our body and brain are taking into small chunks ready to be analyzed and made sense of. This is true as well for the understanding of spatial representations of our surrounding world in our brain. How can we orientate our own person in surroundings familiar to us, if desired without help of our visual sense? Do we have integrated a map of named familiar surroundings embedded in another, bigger map of our whole ever perceived world? If so, how is the information of this map stored?

One probably has wandered around in the dark in his or her own house, myself for certain on multiple occasions. In most of these times I had no trouble finding the first step up or down the stairs or avoiding walking into the next wall head first, maybe while holding some beverage for an late hour working session. While pleasant, most of the time it's not worth mentioning or boasting of. Or, at least, so I tend to think. Admittedly, boasting should not be necessary, but mentioning and thinking about what underlies this capability of orientation on the other hand is quite interesting.

While one does not see its surroundings in the pitch black (let's assume with closed eyes, as otherwise the next light source out the window might shed enough light to grasp some visual information) this does not mean that same person is without sensory input about it. A scratch in a wooden floorboard, the ticking of some well-placed clock or even possibly the scent of previously brewed coffee on the kitchenette might give cues about the whereabouts of ones self. And cues as sensory input reach us all the time.

In this thesis, what is of interest here, is how we as humans do use cues of our surroundings to update our (spatial) representation of precisely these surroundings...

TODO: there is a long way to go still TODO: put all sources in bib! TODO: from Tristan over Slam and Mallot to this thesis...

2 Theory

In this chapter we aspire to find a reliable way of calculating good embeddings of angle-labeled graphs.

The visualization and calculation of embeddings and the definition of itself presupposes a mathematical background, that will be established in section 2.1. The term "good embedding" is open to definition and will therefore be investigated in the subsequent section 2.2. With this fundamentals, the process of calculating embeddings can be stated and differentiated in sections 2.3 and 2.4. At the end of this chapter (section 2.5) we test our theoretical procedure of finding "good embeddings" for error-prone view-graphs. As this thesis takes up on the master's thesis of Baumann (2019), its final view-graph will be, besides procedurally generated graphs, subject to this test.

The overarching theoretical substance developed in this chapter will finally be applied for the analysis of a small survey in chapter 3. TODO: ausführung davon

2.1 Mathematical Background

2.1.1 General Graph Theory

In this section, let us establish the relevant subjects of *graph theory* we need for this thesis.

A graph is a mathematical construct of the form of an ordered pair

$$G = (V, E) \tag{2.1}$$

which consists of a set of vertices V and a set of edges E connecting vertices in a pairwise fashion. As we want to represent our graphs as directed, we define E as a subset of the cartesian product over the set of vertices V.

$$E \subseteq \{(v_1, v_2) | (v_1, v_2) \in V^2 \text{ and } v_1 \neq v_2\} \tag{2.2}$$

Let us denote here the number of vertices and edges in the graph as

$$\mathbf{n} := |V| \tag{2.3}$$
$$\mathbf{m} := |E| \tag{2.4}$$

An *embedding* of a graph does represent the mathematical structure of G in some space Σ. One example might be, that the embedding associates vertices $v \in V$ of G with points in the 2-dimensional Euclidean space $\mathbb{R}^2$ and edges $e \in E$ of G with straight lines or vectors connecting these points. As the view-graphs we want to analyse are basically top-down seen maps of the environment, these restrictions to Σ suffice.

Mark here, that any one graph can have an infinite number of different embeddings. The art of finding an aesthetic embedding for a given graph, from which one can easily deduce some of the graphs properties, is named the field and study of *graph drawing*.

For more detailed information not covered here see Bender and Williamson (2010); Diestel (2017); Pich (2009).

2.1.2 Angular Labels

Taking up the work of Baumann (2019), we have to introduce the notion of angular directions to the graph and its vertices. These angles are assumed to be error-prone for the sake of real world examples.

As is the case in much literature of *simultaneous localization and mapping (SLAM)*, a robotic device, which is able to move around on its own term, is to map unfamiliar surroundings. (TODO: cite here some sources!) This robot, while moving around and dodging obstacles, is progressively trying to map his explored surroundings by means of measuring distances as well as angles between landmarks. (TODO: cite again) While the measurement is fairly accurate for distances, integrated lasers might suffice for that, the angles of rotations the robot actually performs are slightly off of its internally perceived measure thereof. These small measurement errors in angles are accumulating over the course of its explorations and can greatly disturb the actual map. The intrinsic map the robot builds, or in our case its underlying graph representation by means of angles, is therefore error-prone in regard to its own rotations.

Let us define a global reference direction $\nu = 0$, which can be seen as the objective *"north"* direction of a compass. In Baumann (2019) the accumulating angular deviation from the global reference direction is simulated and updated while the agent explores the environment. It is stored as label ν_i with a vertex v_i upon latter's finding in the mapping process.

Relative to ν_i we then are given the angular directions ϕ_{ij} from vertex v_i to each of its neighboring vertices v_j.

Figure 2.1: Compass direction drift over a large explored area. The ν estimate may deviate substantially (over 90 degree) from its starting value, but remains locally consistent.
Figure and caption reproduced with permission from Baumann (2019).

The overarching effort of this thesis is to reverse the accumulating angular errors ν_i from ν for each vertex of the graph of Baumann (2019) seen in figure 2.1. As the main requirement for our embeddings in all following sections we therefore establish

$$\forall i \in \{1, ..., \mathbf{n}\} : \nu_i = \nu \tag{2.5}$$

2.2 Finding embeddings

2.2.1 Problem

Given a graph $G = (V, E)$ with adjacency matrix $A = \{a_{ij}\}$ and a set of connection angles ϕ_{ij} for all pairs $(v_i, v_j) \in E$ (or $a_{ij} \neq 0$), find an embedding $\mathbf{X} = (\mathbf{x}_1 \mathbf{x}_2 \ldots \mathbf{x_n})^T \in \mathbb{R}^{\mathbf{n}\times 2}, \mathbf{x}_i \in \mathbb{R}^2$ for the vertices satisfying

$$\mathbf{x}_j - \mathbf{x}_i \approx \left\|\mathbf{x}_j - \mathbf{x}_i\right\| \cdot \begin{pmatrix} \cos\,\phi_{ij} \\ \sin\,\phi_{ij} \end{pmatrix} \tag{2.6}$$

$$\iff \quad \vec{\mathbf{x}}_{ij} \approx \lambda_{ij} \cdot \hat{\mathbf{p}}_{ij} \tag{2.7}$$

$$\iff \quad \hat{\mathbf{x}}_{ij} \approx \hat{\mathbf{p}}_{ij} \tag{2.8}$$

Notably, equation 2.6 only exists for such $i, j \in \{1, ..., n\}$ for which $a_{ij} \neq 0$ and, therefore, ϕ_{ij} is given. Whenever $\hat{\mathbf{p}}_{ij}$ is used in the following, this exact notion would to have be implicit, which makes of poor mathematical syntax. Therefore, we establish

$$(v_i, v_j) \notin E \implies \phi_{ij} = 0 \tag{2.9}$$

2.2.2 Objective Functions

TODO: Mallot hat hier x_2 genommen, statt x_1, um direkt anzudeuten, dass x_1 auf 0 liegt...
TODO: Das Quadrat wird im folgenden als die squared magnitude beschrieben, vielleicht sollte ich das nochmal mit Tristan und Mallot absprechen, ob das so gemeint war, außerdem vielleicht dafür ein Symbol einführen?
From the constraint equation 2.6 we can derive for a given graph G objective functions of the form

$$\Omega(G) : \mathbb{R}^{\mathbf{n}\times 2} \longrightarrow \mathbb{R}, \mathbf{X} \longmapsto \omega \tag{2.10}$$

such as

$$\Omega_{init}(G, \mathbf{X}) = \sum_{ij} a_{ij} \left(\vec{\mathbf{x}}_{ij} - \hat{\mathbf{p}}_{ij}\right)^2 \tag{2.11}$$

$$\Omega_{norm}(G, \mathbf{X}) = \sum_{ij} a_{ij} \left(\hat{\mathbf{x}}_{ij} - \hat{\mathbf{p}}_{ij}\right)^2 \tag{2.12}$$

that assign a single numerical value, the *objective value* ω, to one specific graph embedding $\mathbf{X}$ of a given graph G. Which objective value is implying a good embedding (see subsection 2.2.3) is dependent on the Ω it is calculated upon.

To get a straightforward assessment of the objective value in the normalized case of equation 2.12, we can fit this function to a range of values $W = [0, 1]$, basically resulting in an additional objective function. As $\hat{\mathbf{x}}_{ij}$ and $\hat{\mathbf{p}}_{ij}$ are both unit vectors with length one, equation 2.12 can be simplified through equation 2.13, which states as follows:

Be s,t unit vectors $\in \mathbb{R}^2$ and the scalar product given as $\langle s, t\rangle \in [-1, 1]$, then

$$\begin{aligned} \|s - t\|^2 &= (s_x - t_x)^2 + (s_y - t_y)^2 \\ &= s_x^2 - 2s_x t_x + t_x^2 + s_y^2 - 2s_y t_y + t_y^2 \\ &= \|s\|^2 + \|t\|^2 - 2 \cdot (s_x t_x + s_y t_y) \\ &= 2 \cdot (1 - \langle s, t\rangle) \end{aligned} \tag{2.13}$$

It follows that

$$\Omega_{norm}(G,\mathbf{X}) = \sum_{ij} a_{ij} \left\|\hat{\mathbf{x}}_{ij} - \hat{\mathbf{p}}_{ij}\right\|^2 \tag{2.14}$$

$$\iff \quad \Omega_{norm}(G,\mathbf{X}) = \sum_{ij} 2a_{ij} \left(1 - \langle\hat{\mathbf{x}}_{ij}, \hat{\mathbf{p}}_{ij}\rangle\right) \in [0, 4\mathbf{m}] \tag{2.15}$$

$$\iff \quad \frac{\Omega_{norm}(G,\mathbf{X})}{2} = \mathbf{m} - \sum_{ij} a_{ij} \langle\hat{\mathbf{x}}_{ij}, \hat{\mathbf{p}}_{ij}\rangle \in [0, 2\mathbf{m}] \tag{2.16}$$

$$\iff \quad 1 - \frac{1}{2\mathbf{m}} \cdot \Omega_{norm}(G,\mathbf{X}) = \frac{\sum_{ij} a_{ij} \langle\hat{\mathbf{x}}_{ij}, \hat{\mathbf{p}}_{ij}\rangle}{\mathbf{m}} \in [-1, 1] \tag{2.17}$$

$$\iff \quad \left|1 - \frac{1}{2\mathbf{m}} \cdot \Omega_{norm}(G,\mathbf{X})\right| = \left|\frac{\sum_{ij} a_{ij} \langle\hat{\mathbf{x}}_{ij}, \hat{\mathbf{p}}_{ij}\rangle}{\mathbf{m}}\right| \in [0, 1] \tag{2.18}$$

TODO: Can the norm of the last equation be taken so the range goes from [0,1] with 1 being good and 0 bad embeddings? 0 would be 90° across all scalar products, whereas 1 would mean either overlapping (0°) or mirrored (180°) bad embeddings. The latter might be swapped along the north direction to get the "more positive" partner embedding...?
which we will denote as

$$\Omega_{fitted}(G,\mathbf{X}) = \left|\frac{\sum_{ij} a_{ij} \langle\hat{\mathbf{x}}_{ij}, \hat{\mathbf{p}}_{ij}\rangle}{\mathbf{m}}\right|, W = [0, 1] \tag{2.19}$$

2.2.3 "Good" embeddings

As briefly stated in subsection 2.2.2, the meaning of a given objective value ω depends on the objective function Ω its calculated from. For example, an objective value of $\omega_{fitted} = 0$ would describe a *worst possible embedding* of perpendicularity between each pair of $(\hat{\mathbf{x}}_{ij}, \hat{\mathbf{p}}_{ij})$ of a given graph G, whereas $\omega_{fitted} = 1$ would announce its perfect embedding. On the other hand, the perfect embedding for Ω_{norm} would be described by the value $\omega_{norm} = 0$. And for Ω_{init} there is no value for the *worst possible embedding*.

As can be seen, the notion of ω is not unified and has to be interpreted for each objective function differently. In addition, the construction of different objective functions can be founded on any measurable criterion of a graph embedding (not for the graph itself, as we look for the best embedding of one fixed graph). In our objective functions thus far we used as concept the *spatial* distances between "should be" and "in fact" positions of the neighbouring vertices (see figure 2.2). Additional concepts could be *angular*, which evaluates the angular error rather than the distance, or *areal*, which spans areas by means of the vector(cross?)-product and thus combines the spatial and angular concepts.

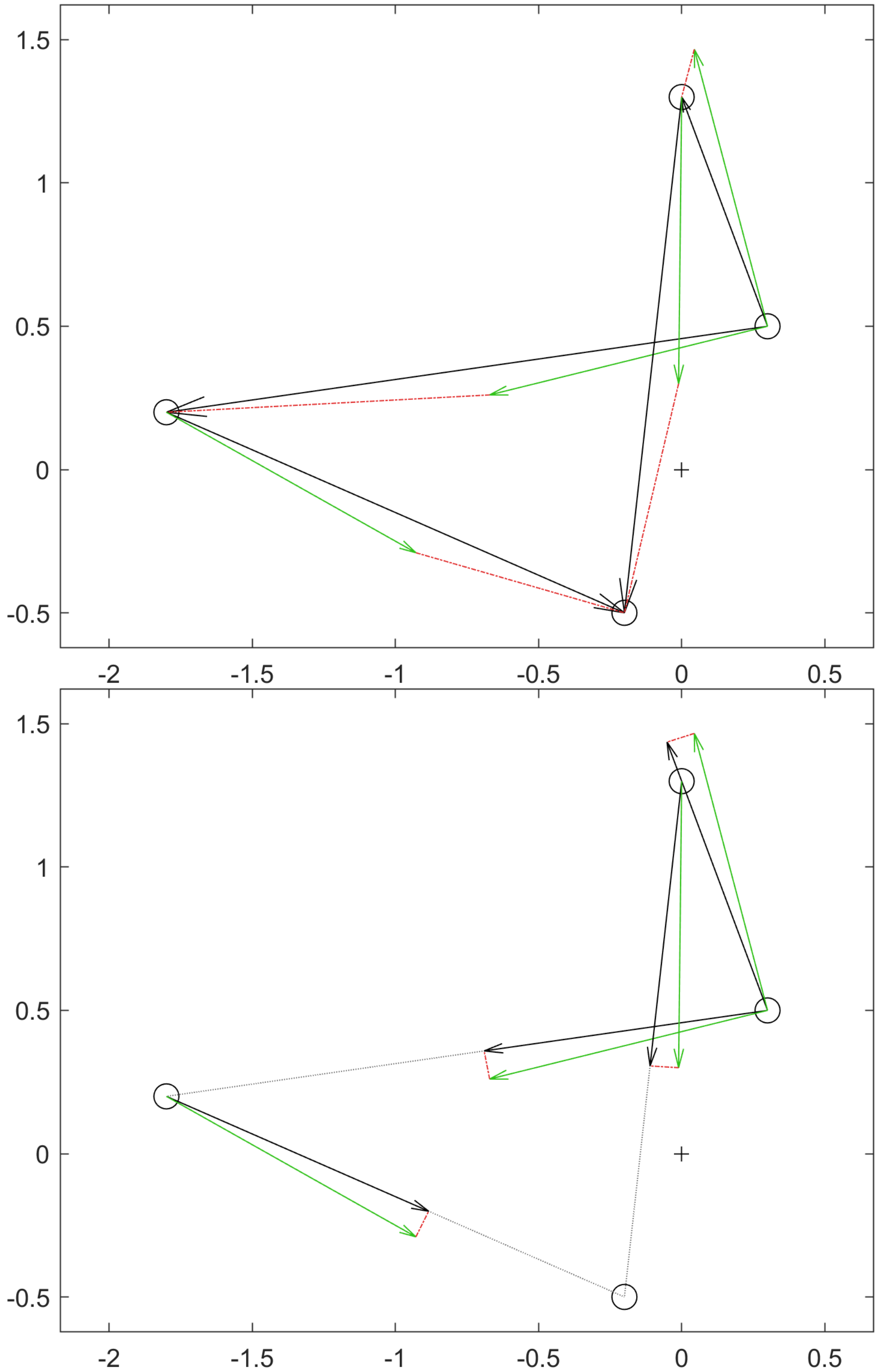

Figure 2.2: A random embedding of a directed graph with 4 vertices, its edges (black arrows) and the respective $\hat{\mathbf{p}}_{ij}$ unit vectors (green arrows) are drawn. These unit vectors are derived from the angular labels ϕ_{ij} of each vertex. **(top)** The differences between $\hat{\mathbf{p}}_{ij}$ and the non-normalized vector $\vec{\mathbf{x}}_{ij}$ are drawn as dotted red lines. This can be seen as the visualization of Ω_{init} (equation 2.11). **(bottom)** Here the edges are normalized and the differences drawn as dotted red lines are therefore of the respective unit vectors $\hat{\mathbf{x}}_{ij} - \hat{\mathbf{p}}_{ij}$ instead. Ω_{norm} (equation 2.12) is visualized here.

2.2.4 Analytical solution for the non-normalized case

For the non-normalized version of the objective function in equation 2.11, the ideal solution should satisfy

$$\hat{\mathbf{p}}_{ij} = \vec{\mathbf{x}}_{ij} \quad \text{only when } a_{ij} = 1 \tag{2.20}$$

For the construction of the respective matrix form we will need the indexing functions

$$i(k) = \left\lceil \frac{k}{\mathbf{n}-1} \right\rceil \tag{2.21}$$

$$j(k) = \begin{cases} \mathrm{mod}(k, \mathbf{n}-1) & \text{for} \quad \mathrm{mod}(k, \mathbf{n}-1) < i(k) \\ 1 + \mathrm{mod}(k, \mathbf{n}-1) & \text{for} \quad \mathrm{mod}(k, \mathbf{n}-1) \geq i(k) \end{cases} \tag{2.22}$$

With these we formulate equation 2.20 as

$$P = M\mathbf{X} \tag{2.23}$$

where $\forall\ k \in \{1, 2, ..., \mathbf{n}(\mathbf{n}-1)\}$:

$$a_{i(k),j(k)} \neq 0 \implies P_k = \hat{\mathbf{p}}^T_{i(k),j(k)} \quad \text{and} \quad M_{kl} = \begin{cases} -1 & \text{for } i(k) = l \\ 1 & \text{for } j(k) = l \\ 0 & \text{otherwise} \end{cases} \tag{2.24}$$

The dimensions of the three matrices are $P \in \mathbb{R}^{\mathbf{m}\times 2}$, $M \in \mathbb{R}^{\mathbf{m}\times\mathbf{n}}$, $\mathbf{X} \in \mathbb{R}^{\mathbf{n}\times 2}$.

With the construction of the matrices, we are able to compute directly the optimal embedding minimizing distances between $\hat{\mathbf{p}}_{ij}$ and $\vec{\mathbf{x}}_{ij}$ by means of the *method of least sqaures* (see ()). The *least sqaure solution* is given as

$$\mathbf{X}^* = (M^T M)^{-1} P \tag{2.25}$$

For this method the $\mathbf{n} \times \mathbf{n}$ matrix $M^T M$ has to be invertible. If it is singular instead, we remove $\mathbf{x}_1$ from X as well as clear the matrix M of its first column, which will result in an implicit fixing of $\mathbf{x}_1$ to point $(0, 0)$.

2.3 Embedding calculation and assessment

2.4 Embedding with Normalization

2.5 Applying the Theory

2.6 Comparing different graphs

TODO:
idea for a new approach for the objective function:

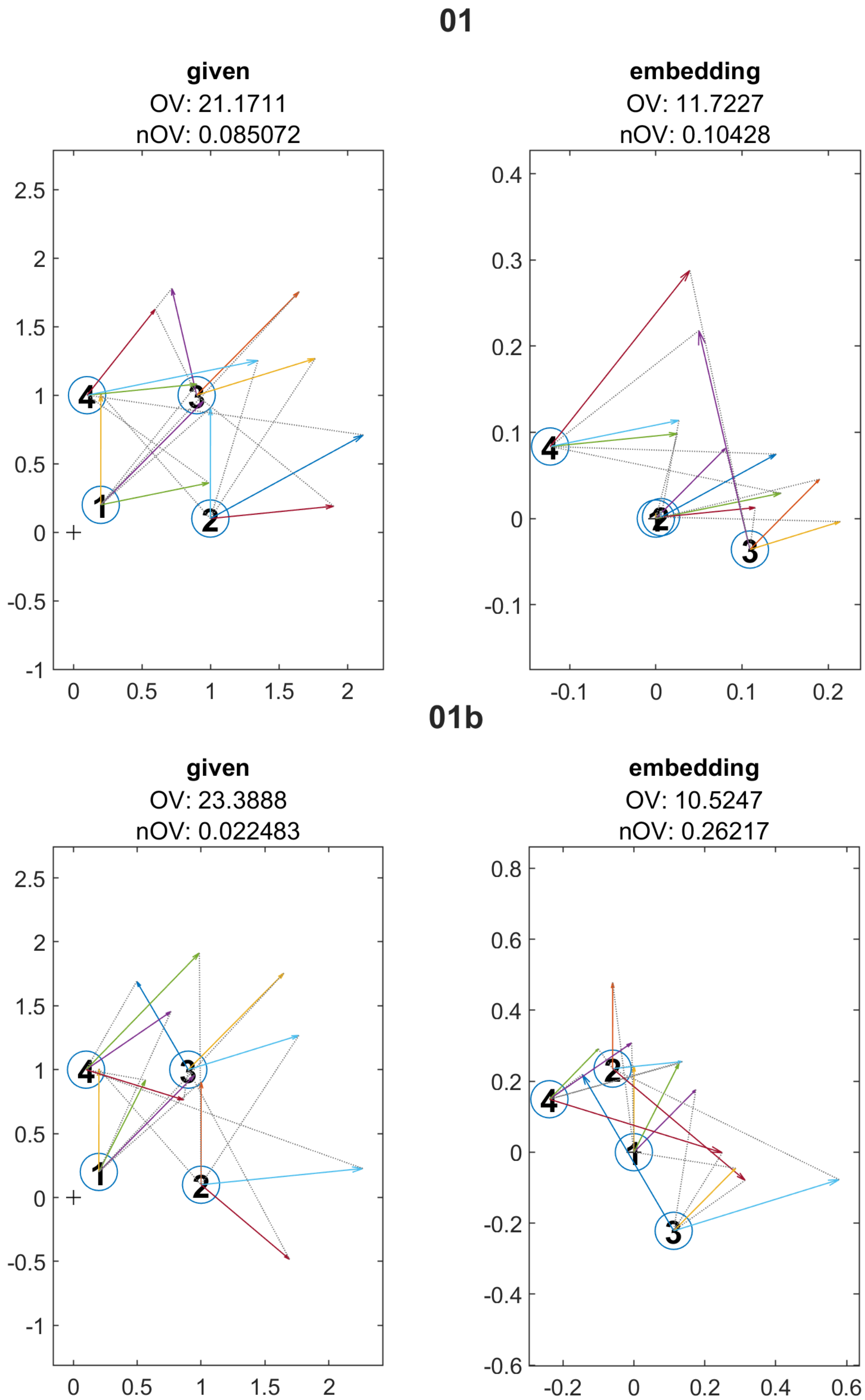

Figure 2.3: Comparisons of example graphs **01** and **01b** as given (left column) and the optimized embeddings calculated minimizing the objective function (right column). **OV** and **nOV** refer to the objective values in non-normalized and normalized fashion respectively. Between **01** and **01b** only the angular values were tweaked in a random fashion.

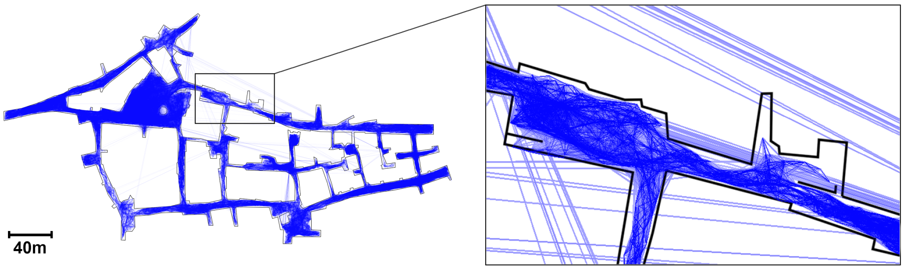

Figure 2.4: Map of the testing environment "Virtual Tuebingen" with view graph. The view graph can be embedded into a map by placing each feature at the agent's position from where it was first detected, and drawing the graph's edges between them (blue lines). The shown graph completely maps the virtual environment and consists of 222,433 nodes and 3,492,096 edges. Some of the edges connect very distant features (long blue lines crossing the empty white space). These are wrong connections resulting from aliasing.
Figure and caption reproduced with permission from Baumann (2019).

the length between each pair of vertices in relation to the length of the difference between the real and estimated endpoint given by the angular perception $\frac{length of all ? errors}{length of all ? intervertex distances}$

3 Practice

To apply the theoretical manners of constructing north-directed view-graphs from the previous chapter 2 in practice, a small survey was conducted. It was designed to collect data of subjective estimates of the direction from one location to another in a familiar surrounding. From the so called angular direction estimates (*ADEs*) view-graphs were constructed. The survey will be described in 3.1 and its collected data visually presented and analyzed in **??**. The raw data can be found in appendix A.

TODO: Hypothesis

3.1 Survey design

3.1.1 Participants

The survey was performed by 6 participants (3 female and 3 male) being of age 13,19,22,27,51 and 66. Familiarity with the surrounding the survey took place in and locations thereof was confirmed by each participant. No monetary compensation was received, but oral consent has been given prior to participating.

3.1.2 Surroundings

The actual distances between each pair of landmarks were measured with the distance-measurement-tool provided by Google Earth (Google, 2020) and one angle between the landmarks Loeschenmuehle and Unterkaierberg was calculated with help of the suitable measurement tool (SunEarthTools, 2009) as 177.8°.

The respective distance matrix representing the pairwise landmark distances, we will denote it by D, is given in the appendix in table A.1. The survey surroundings therefore were confined within a radius of 2.6km around Loeschenmuehle, Bavaria. The maximum and minimum distances between landmarks for which angular estimations were done are 3.5km and 736m respectively.

With D it was possible to construct a metric embedding of the surroundings of Loeschenmuehle by means of multi-dimensional-scaling (Runkler, 2015; Torgerson, 1952, 1958). The calculated angle then was used to rotate the generated graph to fit the real world equivalent in orientation. In addition, the starting point X_0, e.g. Loeschenmuehle, is set as the origin of the graph representation. This final embedding of used landmarks as well as their position on a map of the respective surroundings is given in figure 3.1. In the following sections this constructed embedding will be referred to as *correct embedding.*

3.1.3 Measurement of an angular direction estimation

All *ADEs* were measured by the participant itself by means of a compass, namely the "Präzisions-Kompaß für Sport und Freizeit von Eschenbach Optik, Nürnberg, Germany". This compass was simply used as a protractor, where the angles measured are in respect to the north direction of the compass.

To put it simple, the participants did execute angular measurements by means of a protractor (see figure 3.2):
The participants were positioned at one spot at the current landmark and were allowed to rotate around their own axis. Once in position, they were given the name of the landmark for which the *ADE* was to be measured. The participants then did rotate themselves to align their bodily *sagittal axis* (e.g. their egocentric view) along the estimated direction of the called landmark, while holding the compass in front of them. After adjusting the compass base to fit the compass needle, the *ADE* could be read off the rotary dial.

3.1.4 Procedure

The survey was constructed as a measure-travel-procedure (MTP) in the sense, that the participants conduct the desired measurements at one location and then travel to the next location to repeat these steps.

The exact procedure is explained in the following with 4 phases, from which phases 2 to 4 are repeated the necessary number of times. In this survey only 3 repetitions were done, as thereby collected data was sufficient for constructing view-graphs.

All angular measurements in the following are done according to 3.1.3.

Phase 1: Preparation

Starting at point X_0, the participants were given the true north direction $\nu = 0$ as well as a list of n locations in the surrounding area by name. These locations along with the starting point are called *landmarks* and denoted by X.

$$X = \{X_i\} \text{ for } i \in \{0, ..., n\} \tag{3.1}$$

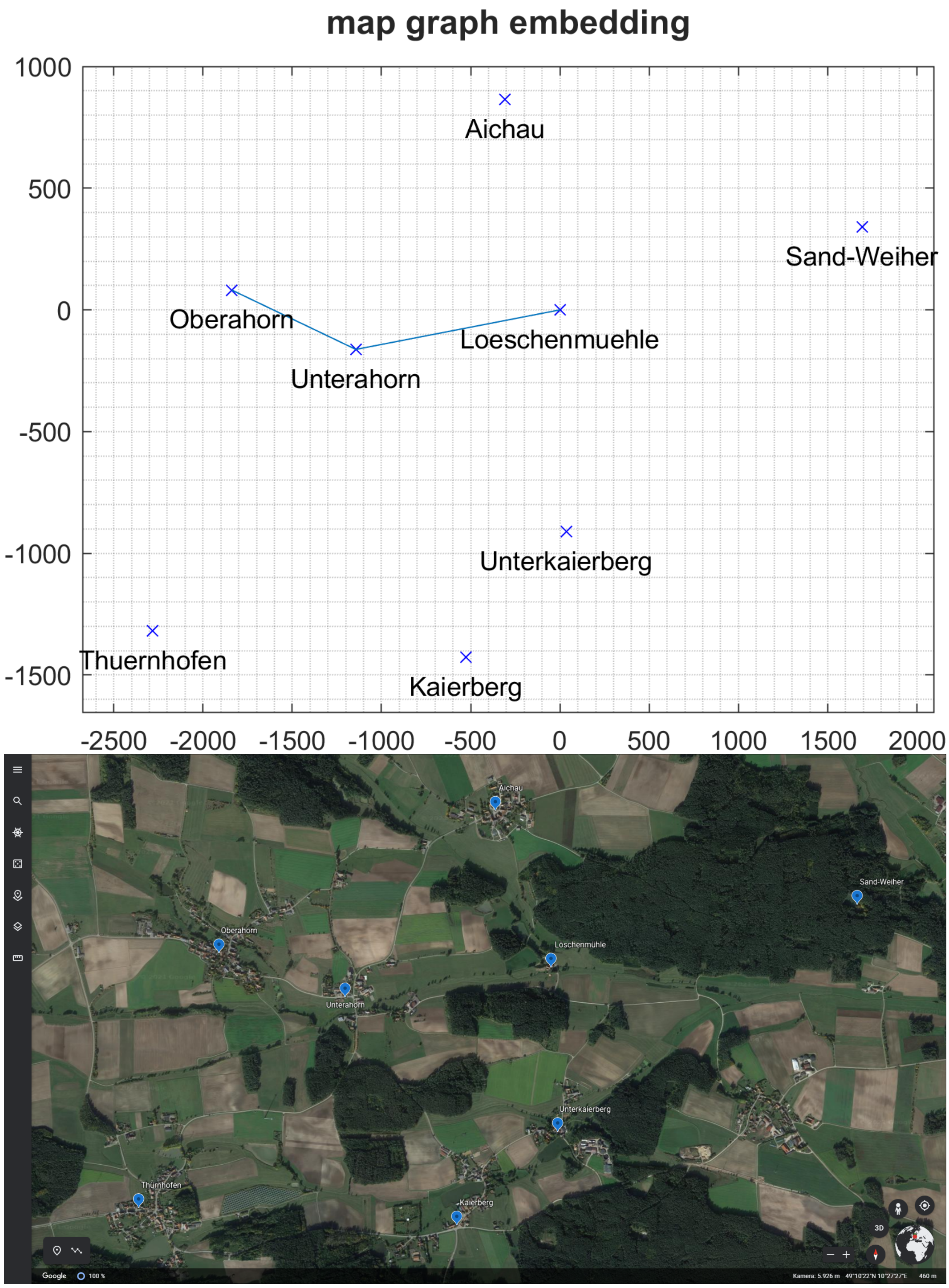

Figure 3.1: top: Graph embedding generated with distance matrix and angle between points 'Loeschenmuehle' and 'Unterkaierberg'. The route the participants did take is drawn as a blue line. Starting point was 'Loeschenmuehle' and endpoint 'Oberahorn'. **bottom:** Satellite map taken with Google Earth (Google, 2020), within which the respective landmarks are marked as blue flags.

Figure 3.2: Classic measurement of an angle by means of a protractor: **top:** Alignment of sagittal axis and compass along the angular direction estimation. **bottom:** Fitting the compass base to align the compass needle. The estimated angle can then be read off the dial.

Phase 2: North Estimation

Upon arriving at the current landmark X_i, the participants were to give an estimate ϕ_i of the true (compass) north direction ν. ϕ_i was first signaled through an outstretched arm, bodily alignment or fixation of an object in the visible surroundings and therewith compared to the true north.

Phase 3: Measurement

In the measurement phase the participants were to give an estimate of the angular direction for each landmark other than the one they were measuring at.

$$\alpha_i = (\alpha_{ij})^T \text{ with } i \in \{1, ..., n\},\ j \in \{0, ..., n\} \text{ and } i \neq j \tag{3.2}$$

The *ADEs* for the landmarks were done in the same order over all participants.

Phase 4: Travel

The participants did travel between the 3 landmarks, at which measurement was conducted, either by car or bike. The routes taken between the landmarks were consistent over all participants regardless of the means of transportation. While neither blindfolded nor naive to their task, the participants were able to update their assessment of their surroundings and relations between landmarks during the travel.

At the locations of measurement (Loeschenmuehle, Unterahorn, Oberahorn) other landmarks (e.g. streets, tree lines, houses) were visible and internal odometric calculation (or its estimation) is to be assumed. One participant visibly imagined following the street toward the target landmarks, while others might have done so as well, although not in a visible manner.

The target landmarks were *not* visible at the 3 measurement locations.

3.2 Data visualization

In the following, in contrast to the correct embedding (section 3.1.2 or figure 3.1), we refer to an embedding we calculated from the collected angular data for any one participant (see appendix A.1) by means of the least square solution 2.25 as a *subjective embedding.*

In figure 3.3 we present these subjective embeddings. Figure 3.4 then puts each of them next to the correct embedding and marks differences between them. Here we scale the correct embedding to equalize the distances between the points representing the landmarks "Loeschenmuehle" and "Unterahorn" for both embeddings. We rotated neither the correct nor the subjective embeddings and the vertices representing the landmark "Loeschenmuehle" in both embeddings are still at the origin point. This is done to be able to make an intuitive visual comparison. Below in section 3.3 this comparison is quantified.

The objective values (OV and nOV) are calculated with equations 2.11 and 2.19 respectively with the input given as the *ADEs* from each participant (see appendix A.1).

3.3 Data comparison

3.4 Conclusion

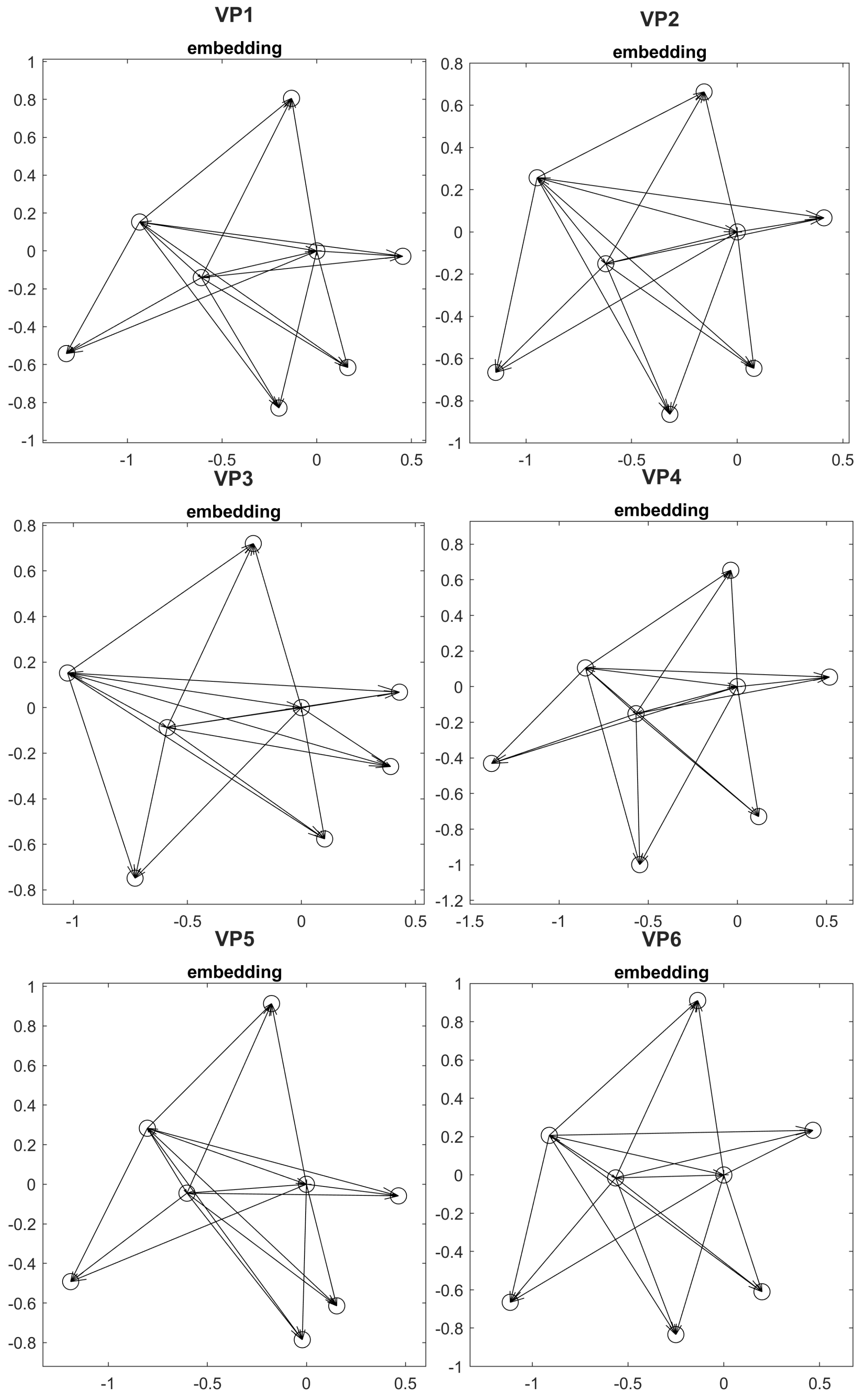

Figure 3.3: Depicted are the subjective embeddings calculated by means of the least-square-solution (equation 2.25) and therefore minimizing the objective function 2.11. Each embedding is calculated from the collected *ADEs* of the respective participant. Note that the scaling of the axes do slightly differ.

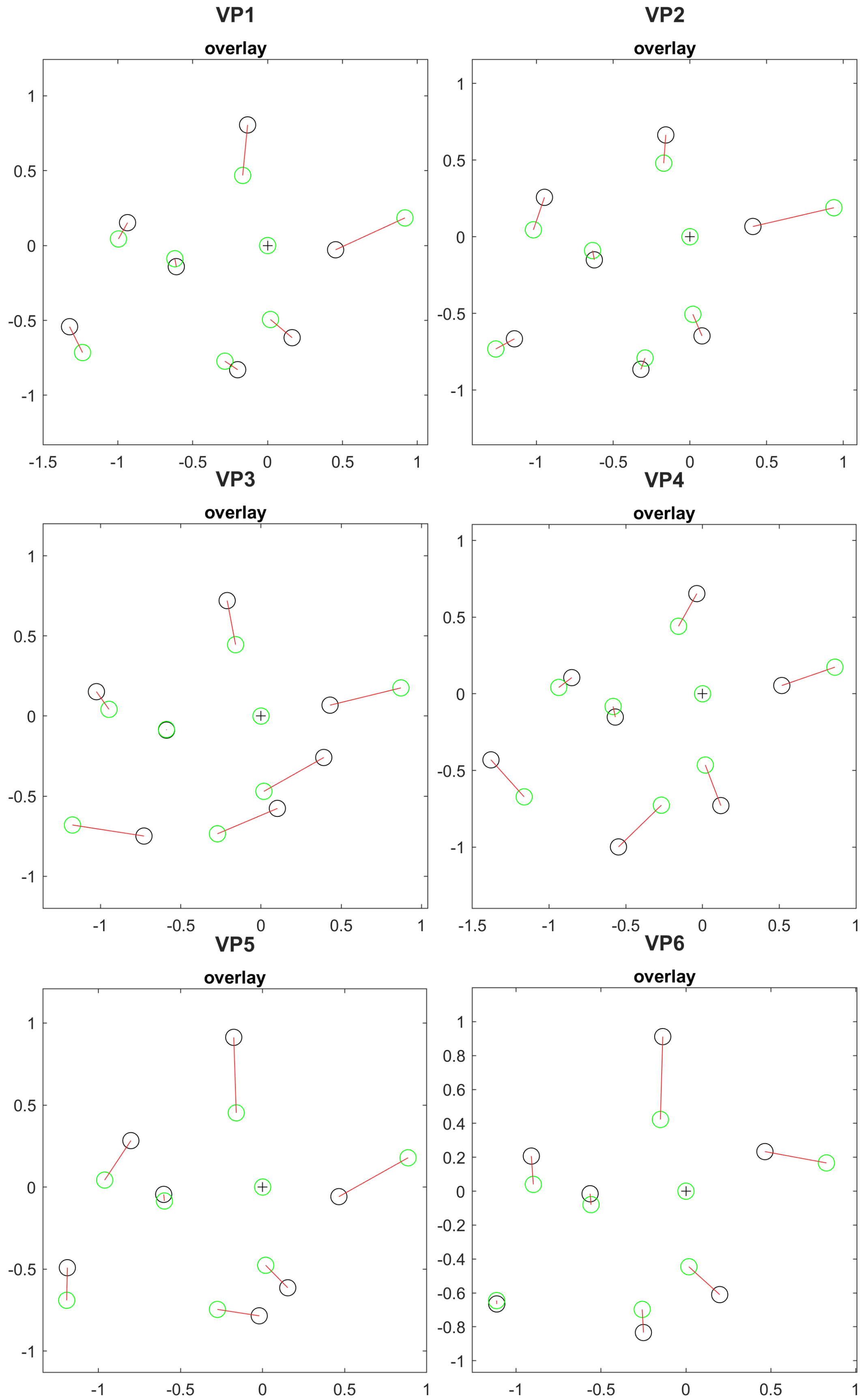

Figure 3.4: For each participant we compare their subjective embedding (**black** circles, see figure 3.3) with the correct embedding (**green** circles, see figure 3.1). This correct embedding is scaled to equalize over both embeddings the distances between the vertices representing the landmarks "Loeschenmuehle" and "Unterahorn" (again see figure 3.1). No rotations have been applied and the embeddings are fixed at the origin. The **red** lines visualize the error between both

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A | Supplementary data

In the following all the data for the survey conducted around Loeschenmuehle is listed.

A.1 Distance matrix and survey measurements

The distance matrix D is here depicted as the upper triangle only. All numbers are given in meters ($\mathbf{m}$).

The measurements for the angular perception are given in degree ($^\circ$) from the north direction in clockwise fashion (standard cardinal points notation).

	Mühle	Sand-Weiher	Unterkaierberg	Kaierberg	Thürnhofen	Oberahorn	Unterahorn	Aichau
Mühle		1.725,92	912,4	1.519,91	2.634,79	1.840,3	1.154,89	917,42
Sand-Weiher			2.077,01	2.836,04	4.305,45	3.539,84	2.879,6	2.066,88
Unterkaierberg				761,46	2.351,29	2.120,43	1.396,49	1.808,77
Kaierberg					1.757,09	1.997,92	1.405,71	2.300,56
Thürnhofen						1.468,92	1.622,28	2.943,1
Oberahorn							736,54	1.719,53
Unterahorn								1.324,01
Aichau								

Theo	Norden	Mühle	Sand-Weiher	Unterkaierb.	Kaierberg	Thürnhofen	Oberahorn	Unterahorn	Aichau
Mühle	0		84	164	198	254	270	264	346
Oberahorn	8	110	112	126	146	224		114	48
Unterahorn	330	72	80	108	136	230	286		40

Jakob	Norden	Mühle	Sand-Weiher	Unterkaierb.	Kaierberg	Thürnhofen	Oberahorn	Unterahorn	Aichau
Mühle	0		80	170	192	244	270	270	330
Oberahorn	18	118	117	136	172	194		124	76
Unterahorn	350	70	68	110	137	226	278		39

Ronja	Norden	Mühle	Sand-Weiher	Unterkaierb.	Kaierberg	Thürnhofen	Oberahorn	Unterahorn	Aichau
Mühle	0		81	117	175	233	267	288	327
Oberahorn	357	110	107	116	115	142		119	59
Unterahorn	334	78	74	87	112	203	278		42

Helmut	Norden	Mühle	Sand-Weiher	Unterkaierb.	Kaierberg	Thürnhofen	Oberahorn	Unterahorn	Aichau
Mühle	0		82	162	197	248	256	260	341
Oberahorn	90	102	96	148	182	250		129	70
Unterahorn	0	82	80	110	174	238	279		44

Leia	Norden	Mühle	Sand-Weiher	Unterkaierb.	Kaierberg	Thürnhofen	Oberahorn	Unterahorn	Aichau
Mühle	0		88	162	170	264	278	268	340
Oberahorn	42	130	126	142	150	222		132	30
Unterahorn	342	76	82	110	138	210	276		46

Birgit	Norden	Mühle	Sand-Weiher	Unterkaierb.	Kaierberg	Thürnhofen	Oberahorn	Unterahorn	Aichau
Mühle	0		77	154	196	244	274	282	350
Oberahorn	4	116	98	132	150	210		114	44
Unterahorn	356	78	65	117	150	208	277		33

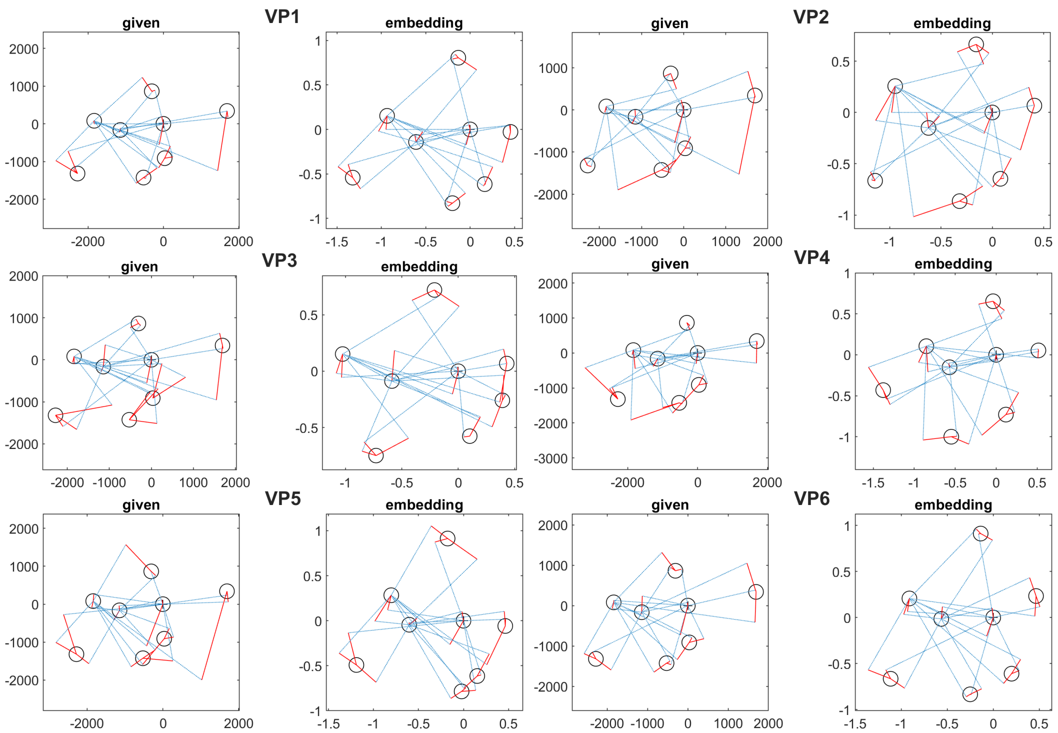
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Figure A.1

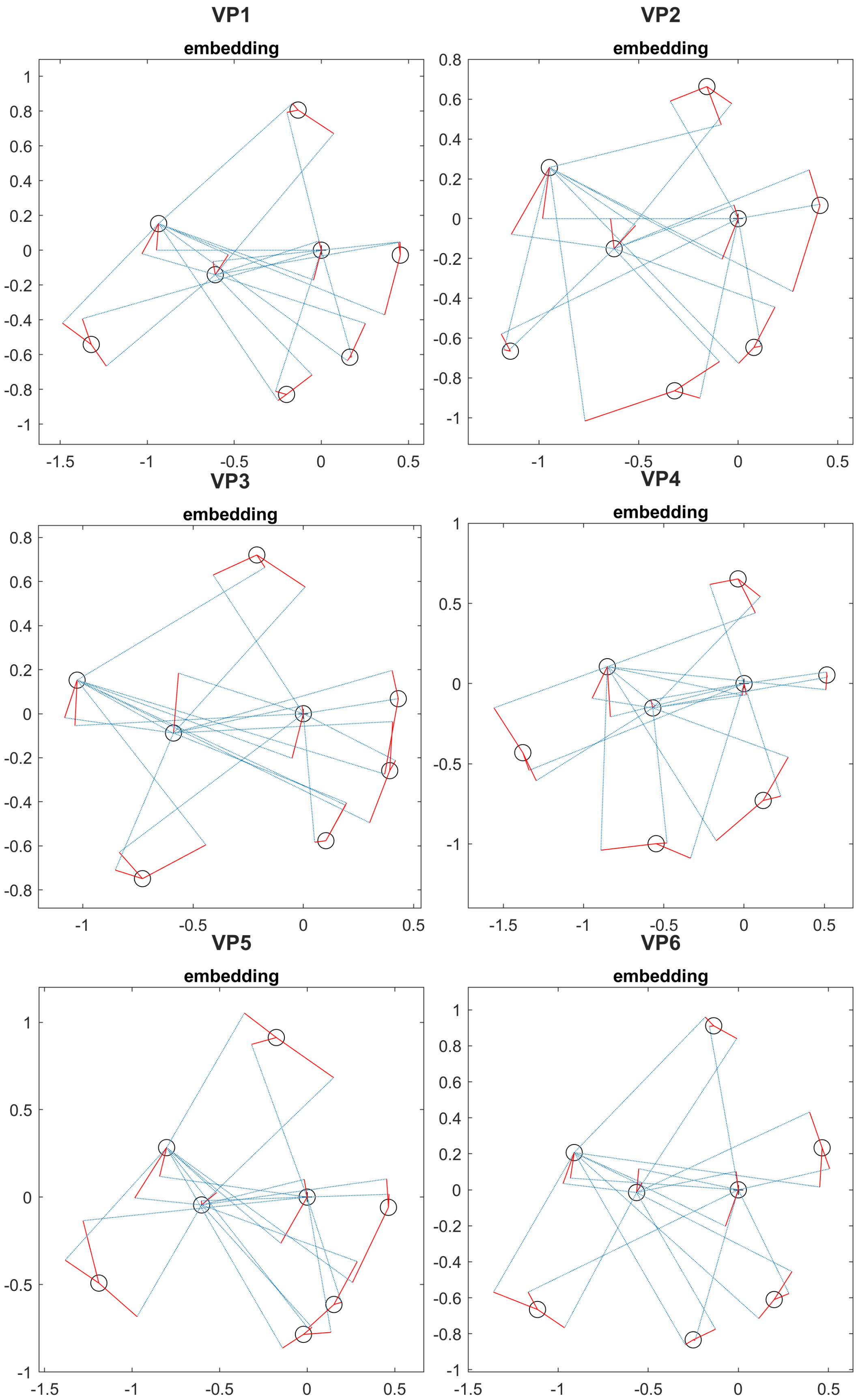
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Figure A.2

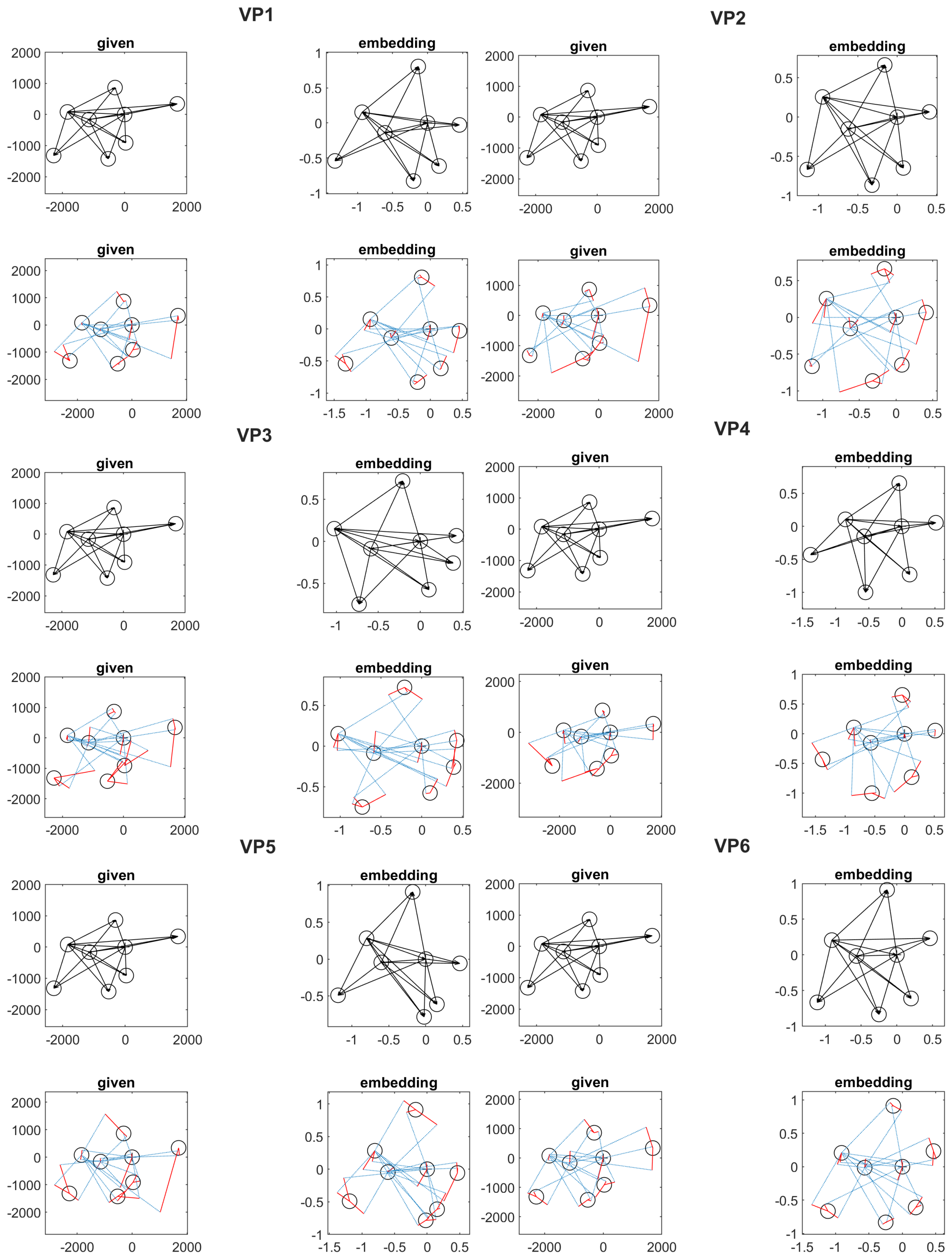

Figure A.3

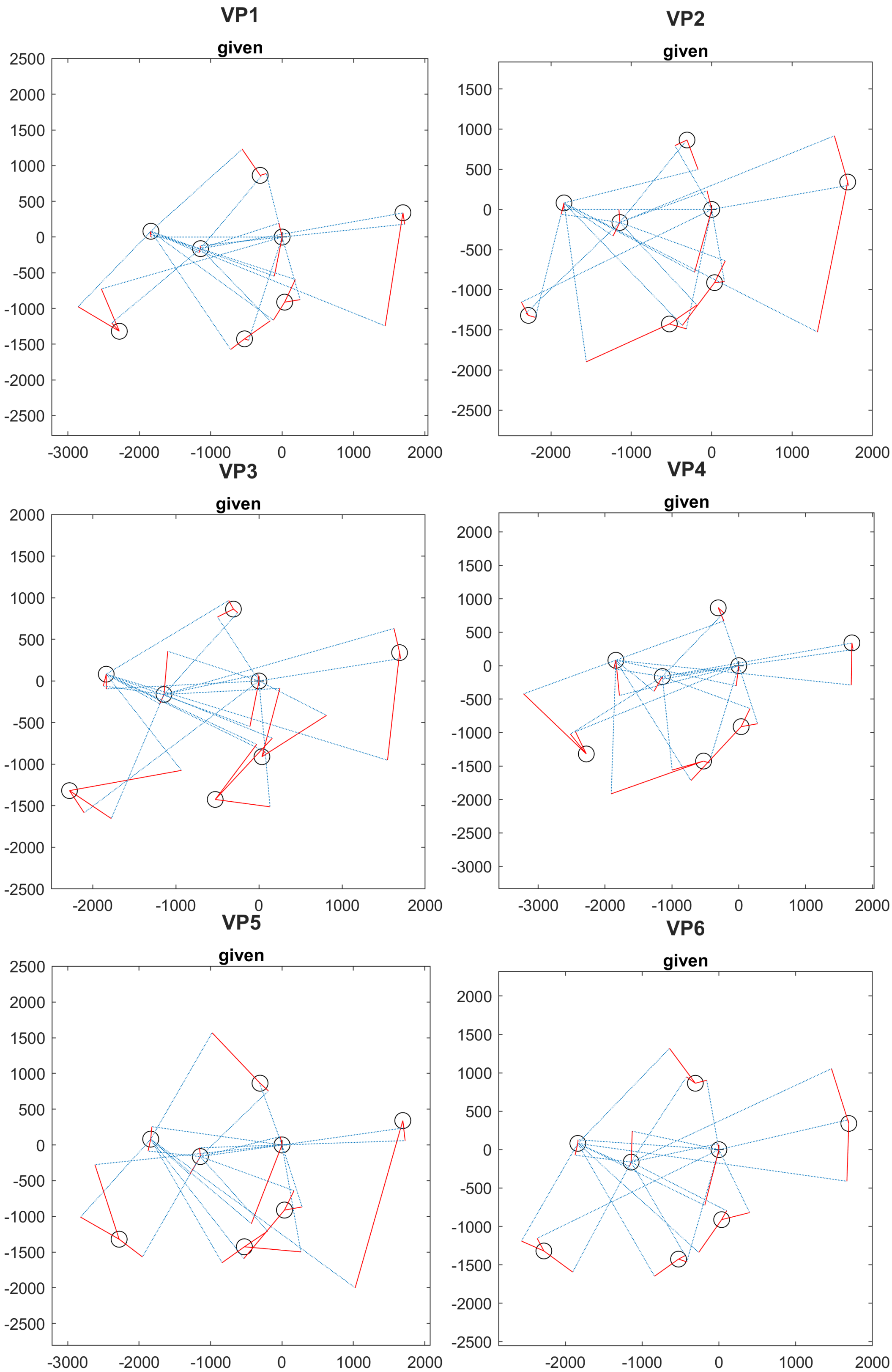

Figure A.4

Selbstständigkeitserklärung

Hiermit versichere ich, dass ich die vorliegende Bachelorarbeit selbstständig und nur mit den angegebenen Hilfsmitteln angefertigt habe und dass alle Stellen, die dem Wortlaut oder dem Sinne nach anderen Werken entnommen sind, durch Angaben von Quellen als Entlehnung kenntlich gemacht worden sind.
Diese Bachelorarbeit wurde in gleicher oder ähnlicher Form in keinem anderen Studiengang als Prüfungsleistung vorgelegt.

Ort, Datum Unterschrift