Unwrapping Graphs with Local Metrics

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BSc Project for Jakob Meyer, September 2020

1 Problem

View-graphs with local metric are generated as representations of space in the PhD project of Tristan Baumann. The nodes of these graphs are small image features ("micro-snapshots") numbered $i = 1, ..., n$ with adjacencies $a_{ij} \in [0, 1]$. Each link is labeled with an angle ϕ_{ij} specifying the direction of node j as seen from node i, relative to a reference direction ν_i. During exploration of the environment, this agent continuously updates the reference direction and stores it with the node as a label. The updating process in error-prone.

2 Approach

The unwrapping problem can be stated as follows: Given an adjacency matrix $A = \{a_{ij}]\}$, a set of connection angles ϕ_{ij} for all pairs with $a_{ij} \neq 0$, and the local assumed reference directions ν_i, find an embedding $(\mathbf{x}_1, ...\mathbf{x}_n), \mathbf{x}_i \in \mathrm{IR}^2$ of the nodes satisfying

$$\mathbf{x}_j - \mathbf{x}_i \approx \lambda \begin{pmatrix} \cos\phi_{ij} \\ \sin\phi_{ij} \end{pmatrix} \tag{1}$$

where λ is the distance between the two nodes' embeddings.

In this constraint, ν_i does not show up. This is to say that it is implicitly set to zero. Once an embedding is found, we could say that ν_i has been rotated by the angle

$$\nu_i^* = \frac{\sum_j a_{ij}(\arctan(\mathbf{x}_j - \mathbf{x}_i) - \phi_{ij})}{\sum_j a_{ij}} \tag{2}$$

where the denominator is the out-degree of node i.

We now denote the unit vector in direction ϕ_{ij} as $\mathbf{p}_{ij}$, i.e.,

$$\mathbf{p}_{ij} := \begin{pmatrix} \cos\phi_{ij} \\ \sin\phi_{ij} \end{pmatrix}. \tag{3}$$

From the constraint equation (1), we can derive an objective function as

$$f(\mathbf{x}_1, ..., \mathbf{x}_n) = \sum_{ij} a_{ij} \left(\frac{\mathbf{x}_j - \mathbf{x}_i}{\|\mathbf{x}_j - \mathbf{x}_i\|} - \mathbf{p}_{ij} \right)^2 \tag{4}$$

which would also punish node distances differing from 1. Othe objective functions might be possible.

Since we now have implicitly assumed $\nu = 0$ for all nodes, solutions cannot be rotated. the only degree of freedom remaining is shift, which can be dealt with by setting $\mathbf{x}_1 = (0, 0)'$.

3 Next Steps

- Try out the objective function with simple graphs (chains, triangles, small loops, 4-cliques etc)
- Think about analytical approaches to minimizing the objective function
- Check MDS literature on related problems. (One is Hübner & Mallot, Autonomous Robots 2007, but there the angles are local, not with respect to a global reference direction. Still. the idea of an objective function working on triples rather than pairs may be useful.)