MINIMIZING ANGULAR ERROR IN LABELED GRAPHS

GRAPHS

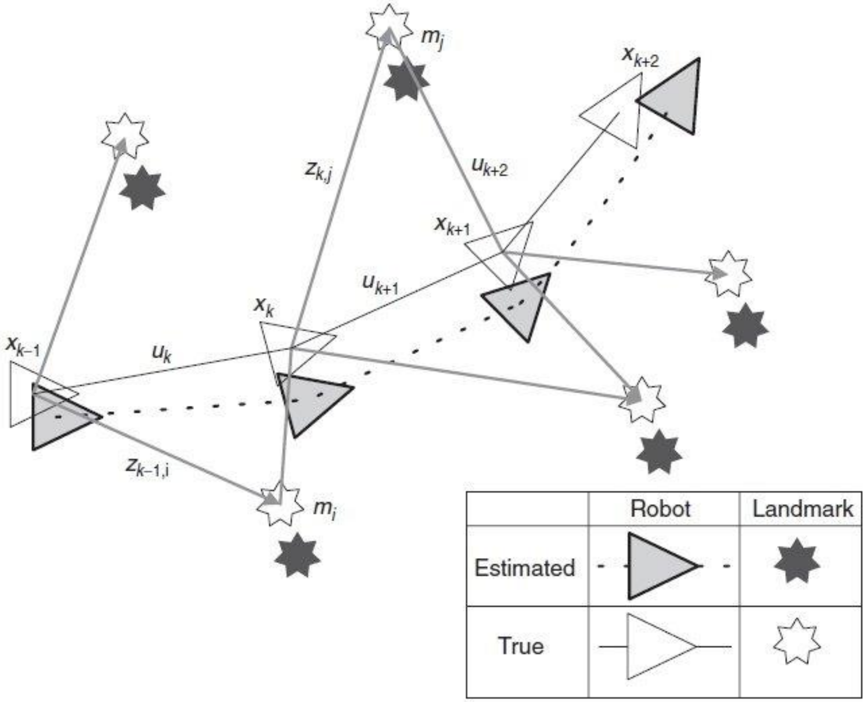
- Labeled Graph
- Directed Graph (Digraph), no edges to oneself
- Graph with angle orientation for each vertex

=> directed angular graph (DAG)

SLAM

"SLAM is a process by which a mobile robot can build a map of an environment and at the same time use this map to deduce its location.“

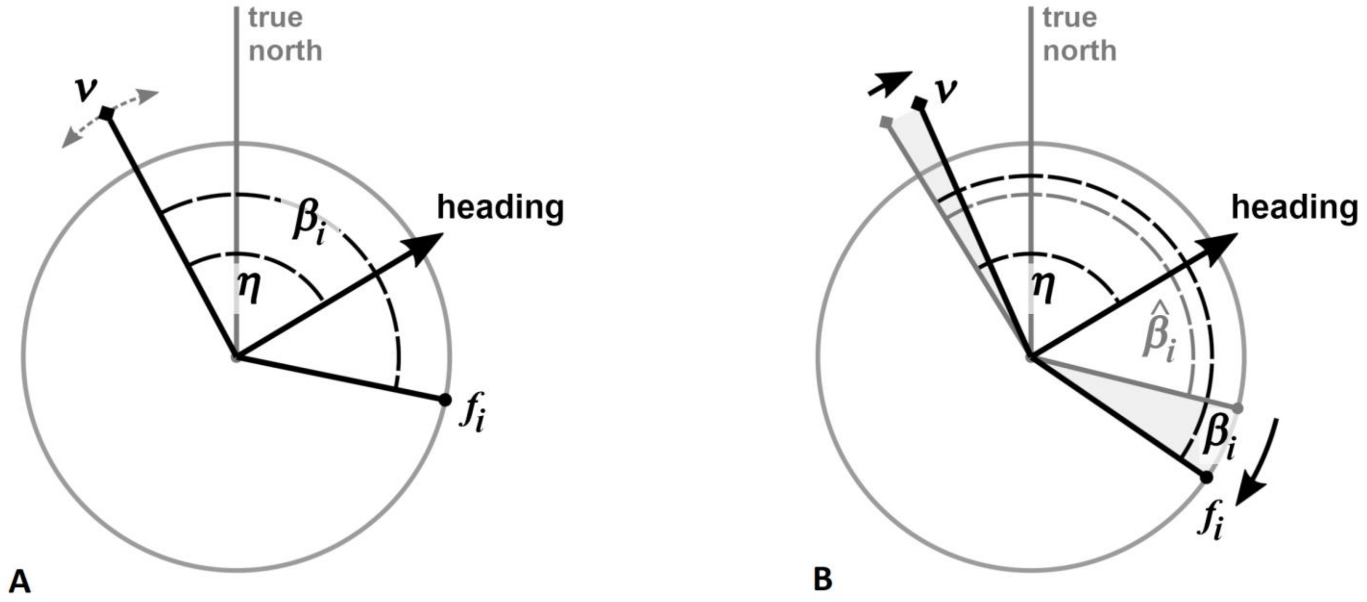
- **S**imultaneous **l**ocalization **a**nd **m**apping
- Cumulative error



DUAL POPULATION CODING

Navigation model by Tristan Baumann

- Graph based spatial representation
- Estimating the global reference direction at each point



DIRECTION DRIFT

PROBLEM

Given:

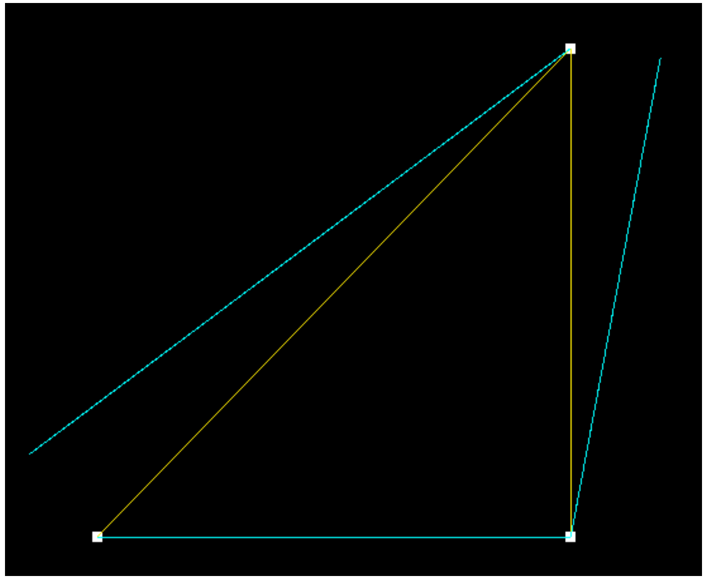
- Adjacency matrix $A = (a_{ij})$
  - directed Graph, as $a_{ij} \neq a_{ji}\ for\ some\ i, j$
- Internal reference directions v_i
- Angles ϕ_{ij} (in respect to reference direction)

Goal:

- Embedding of points $(x_1, \ldots, x_n), x_i \in \mathbb{R}^2$, such that $x_j - x_i \approx \lambda \begin{pmatrix} cos\phi_{ij} \\ sin\phi_{ij} \end{pmatrix} =: \lambda\, p_{ij}$
- Minimizing the Objective function: $f(x_1, \ldots, x_n) = \sum_{ij} a_{ij} \left(\frac{x_j - x_i}{||x_j - x_i||} - p_{ij} \right)^2$

PROBLEM

- Embedding of points $(x_1, \ldots, x_n), x_i \epsilon\ \mathbb{R}^2$, such that $x_j - x_i \approx \lambda \begin{pmatrix} cos\phi_{ij} \\ sin\phi_{ij} \end{pmatrix} =: \lambda\, p_{ij}$
- Minimization problem: $f(x_1, \ldots, x_n) = \sum_{ij} a_{ij} \left(\frac{x_j - x_i}{||x_j - x_i||} - p_{ij} \right)^2$



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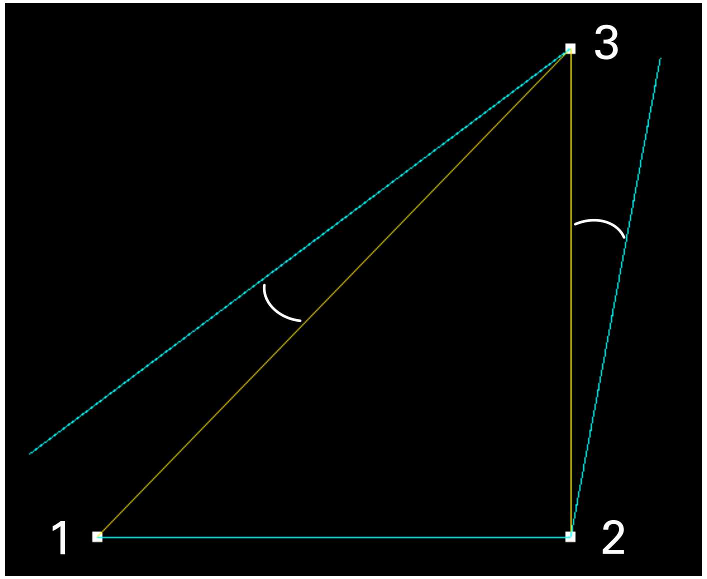

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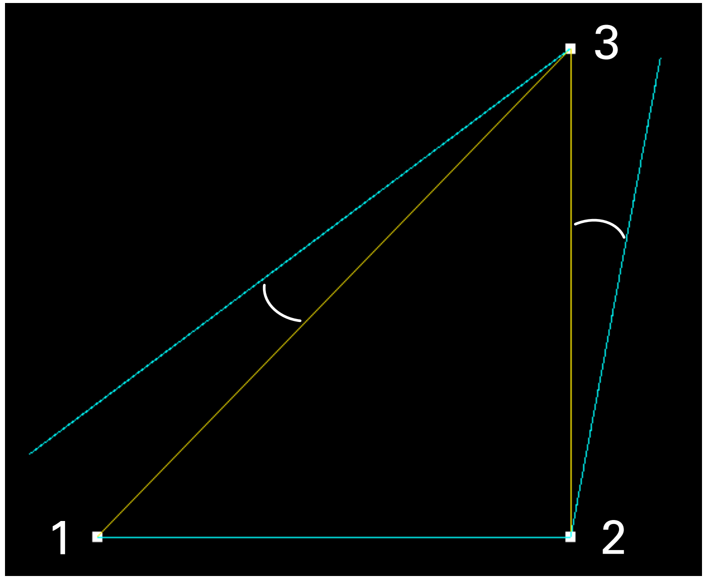

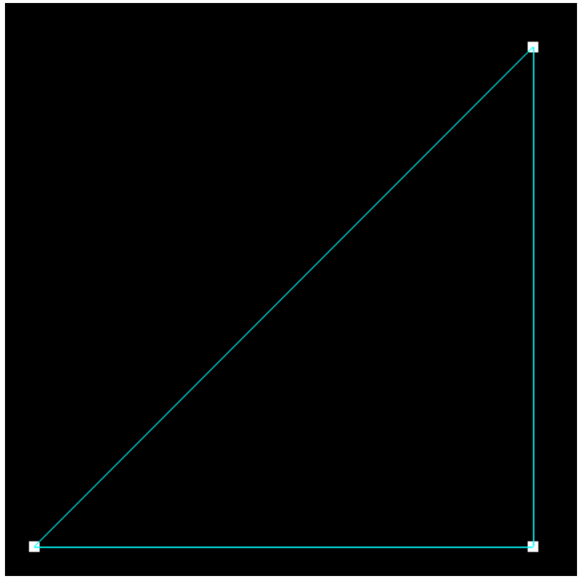

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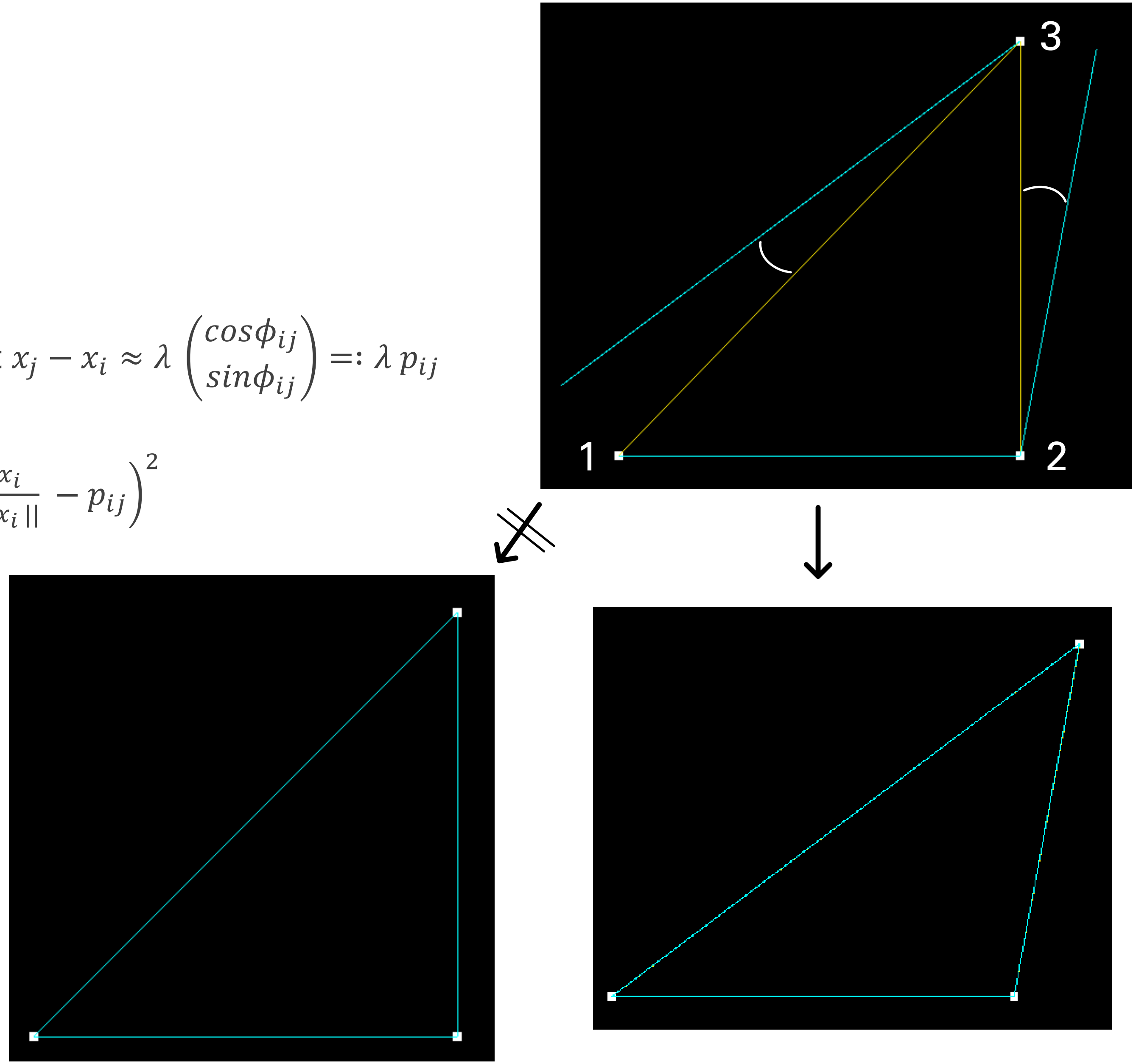

MDS – MULTIDIMENSIONAL SCALING

Classical MDS:

"Given a distance matrix with the distances between each pair of objects in a set [...] an MDS algorithm places each object into [...] [this] space such that the between-object distances are preserved as well as possible."

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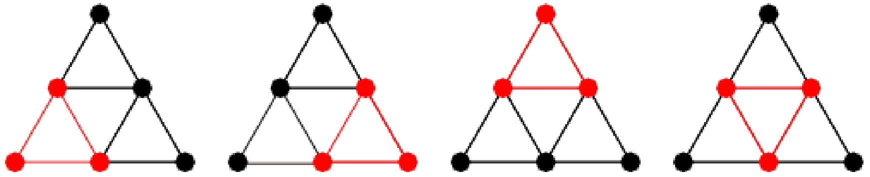
- Broad literature of different approaches
- MDS algorithm running time $O(n^2)$ not good for big graphs
  - Many approaches for (random) sampling of pivot points
- Looking for smallest cliques throughout the graph?



MDS – MULTIDIMENSIONAL SCALING

"A valid dissimilarity matrix must satisfy both of the following constraints:

(i) self-similarity $d_{ii} = 0$ and (ii) symmetry $d_{ij} = d_{ji}$“

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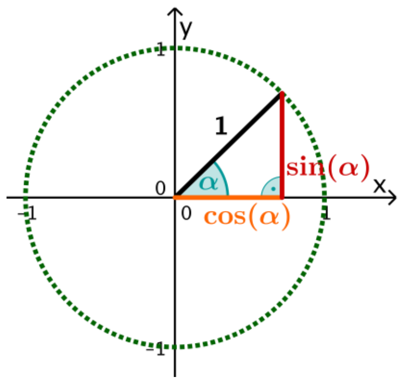
What are the implications for a classical MDS in our case?

Normalized $$f(x_1, \ldots, x_n) = \sum_{ij} a_{ij} \left(\frac{x_j - x_i}{||x_j - x_i||} - p_{ij} \right)^2$$

Non-normalized $$f(x_1, \ldots, x_n) = \sum_{ij} a_{ij} \left(x_j - x_i - p_{ij} \right)^2$$

MEASUREMENT

$$\left(\frac{x_j - x_i}{||x_j - x_i||} - p_{ij}\right)^2$$
$$= \left(x_{ij} - p_{ij}\right)^2$$
$$= x_{ij}^2 + p_{ij}^2 - 2\left(x_{ij} \circ p_{ij}\right)$$
$$= 1 + 1 - 2\left(x_{ij} \circ p_{ij}\right)$$
$$= 2 - 2 * \rho, mit\ \rho \in [-1,1]$$



$$f(x_1, \dots, x_n) = \sum_{ij} a_{ij} \left(\frac{x_j - x_i}{||x_j - x_i||} - p_{ij}\right)^2$$

with $\rho = 0$ we get the worst embedding (90°)

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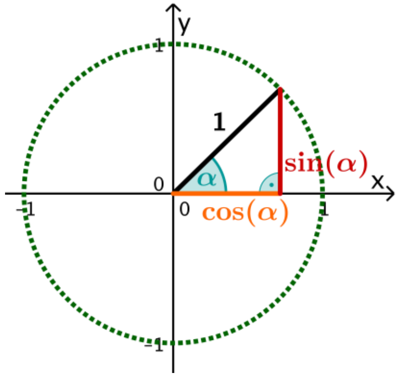


$$f(x_1, \ldots, x_n) = \sum_{ij} a_{ij} \left(\frac{x_j - x_i}{||x_j - x_i||} - p_{ij} \right)^2$$

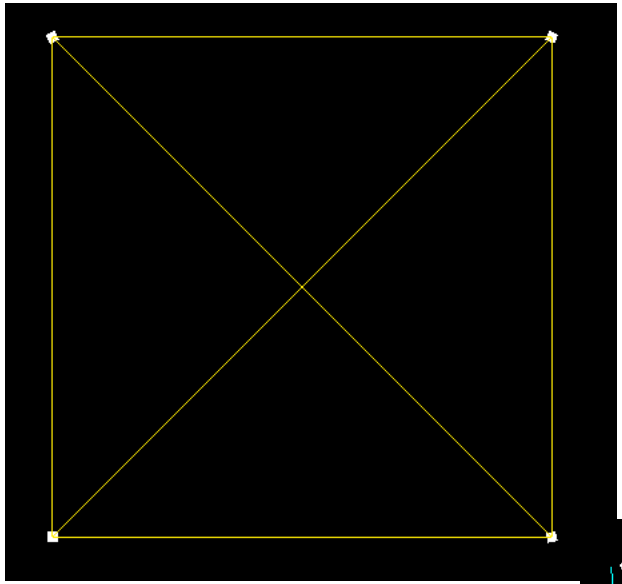
$$g(f) = \frac{f(x_1, \ldots, x_n)}{\sum_{ij} a_{ij}} \ \epsilon \ [0,4]$$

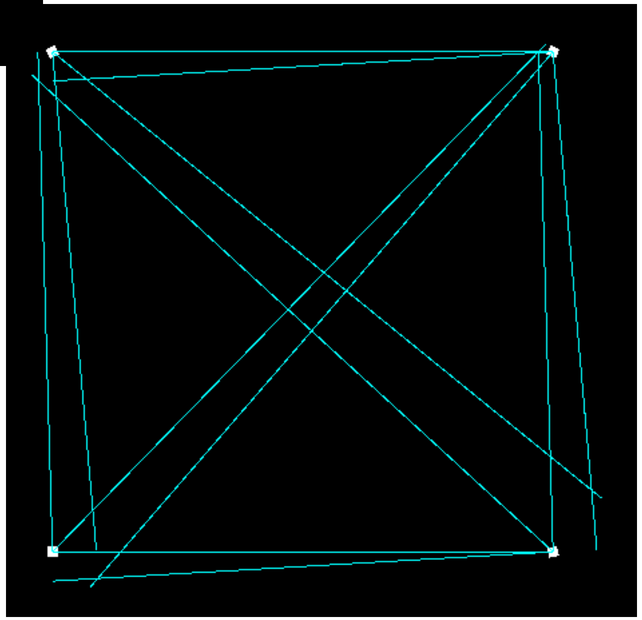
$$g(f, \xi) = |\ \xi * \left(\frac{f(x_1, \ldots, x_n)}{2 * \sum_{ij} a_{ij}} - 1 \right)| \ \epsilon \ [0, \xi]$$

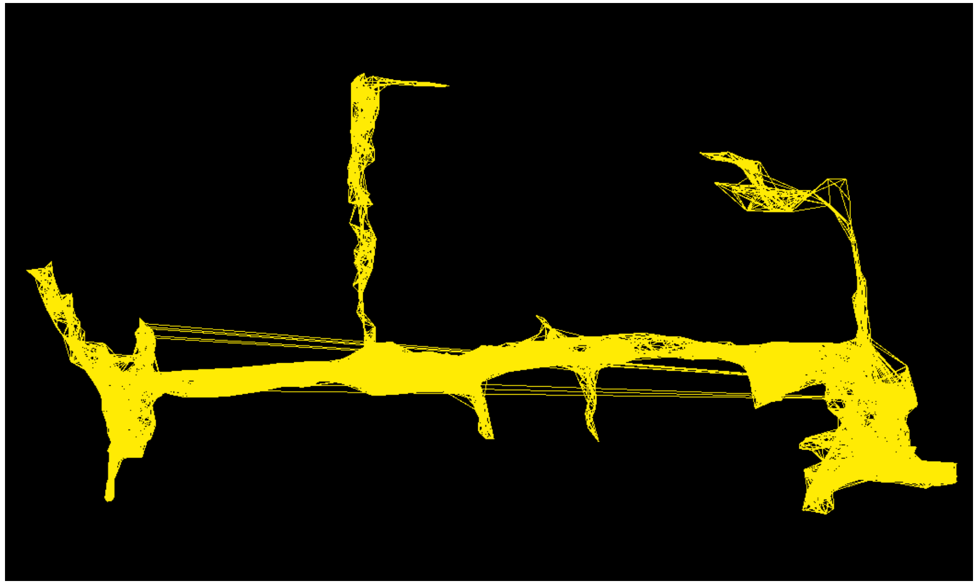
with $g(f, \xi) = \xi$
being perfect
measured value

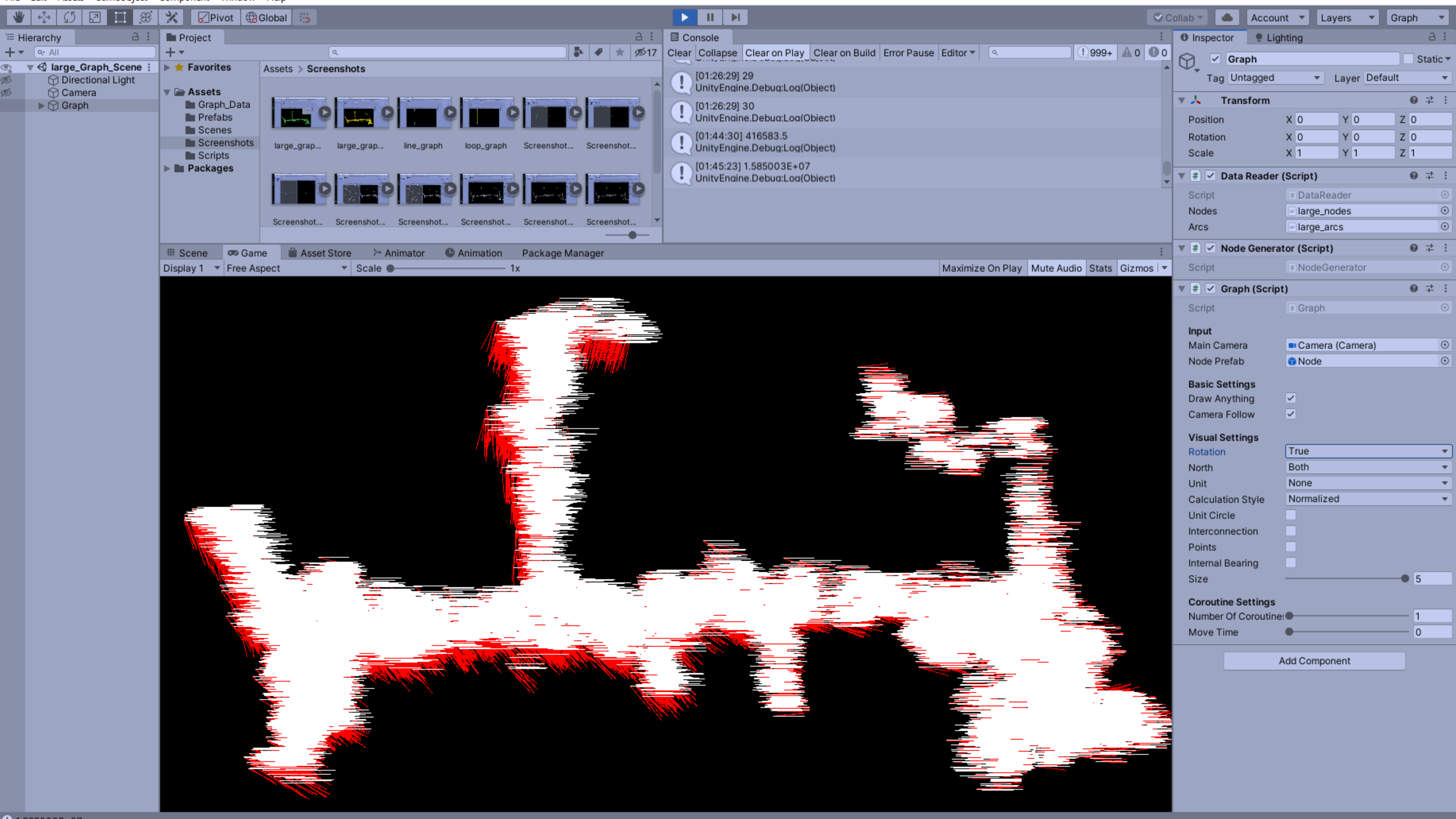
$$\left(\frac{x_j - x_i}{||x_j - x_i||} - p_{ij} \right)^2$$
$$= (x_{ij} - p_{ij})^2$$
$$= x_{ij}^2 + p_{ij}^2 - 2(x_{ij} \circ p_{ij})$$
$$= 1 + 1 - 2(x_{ij} \circ p_{ij})$$
$$= 2 - 2 * \rho, mit\ \rho\ \epsilon\ [-1,1]$$

with $\rho = 0$ we get the
worst embedding (all 90°)








Pivot
Global
Collab
Account
Layers
Graph
Hierarchy
All
large_Graph_Scene
Directional Light
Camera
Graph
Project
Favorites
Assets
Graph_Data
Prefabs
Scenes
Screenshots
Scripts
Packages
Assets > Screenshots
large_grap...
large_grap...
line_graph
loop_graph
Screenshot...
Screenshot...
Screenshot...
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Screenshot...
Screenshot...
Console
Clear
Collapse
Clear on Play
Clear on Build
Error Pause
Editor
999+
0
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UnityEngine.Debug:Log(Object)
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UnityEngine.Debug:Log(Object)
[01:44:30] 416583.5
UnityEngine.Debug:Log(Object)
[01:45:23] 1.585003E+07
UnityEngine.Debug:Log(Object)
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Game
Asset Store
Animator
Animation
Package Manager
Display 1
Free Aspect
Scale
1x
Maximize On Play
Mute Audio
Stats
Gizmos
Inspector
Lighting
Graph
Static
Tag Untagged
Layer Default
Transform
Position X 0 Y 0 Z 0
Rotation X 0 Y 0 Z 0
Scale X 1 Y 1 Z 1
Data Reader (Script)
Script DataReader
Nodes large_nodes
Arcs large_arcs
Node Generator (Script)
Script NodeGenerator
Graph (Script)
Script Graph
Input
Main Camera Camera (Camera)
Node Prefab Node
Basic Settings
Draw Anything
Camera Follow
Visual Settings
Rotation True
North Both
Unit None
Calculation Style Normalized
Unit Circle
Interconnection
Points
Internal Bearing
Size 5
Coroutine Settings
Number Of Coroutine: 1
Move Time 0
Add Component