Problem 3

E < V0 (Set the resulting equation equal to zero and that is the transcendental equation

In[1]:=
```
Clear[CC]
CC = CC /.
    Solve[{A + B == CC + DD, -i k A + i k B == -i k2 CC + i k2 DD, A e^(i k a) + B e^(-i k a) == 0}, {A, B, CC}];
Simplify[((CC e^(-i k2 a) + DD e^(i k2 a)) /. DD → 1) // ExpToTrig]
```

Out[3]=
$$\left\{\frac{2\,i\,(k2\,\text{Cos}[a\,k2]\,\text{Sin}[a\,k] + k\,\text{Cos}[a\,k]\,\text{Sin}[a\,k2])}{k\,\text{Cos}[a\,k] + i\,k2\,\text{Sin}[a\,k]}\right\}$$

E > V0 (Set the resulting equation equal to zero and that is the transcendental equation

In[4]:=
```
Clear[CC]
CC =
  CC /. Solve[{A + B == CC + DD, - k A + k B == -i k2 CC + i k2 DD, A e^(k a) + B e^(-k a) == 0}, {A, B, CC}];
Simplify[((CC e^(-i k2 a) + DD e^(i k2 a)) /. DD → 1) // ExpToTrig]
```

Out[6]=
$$\{(i\,(-k2\,\text{Cos}[a\,k2] + k\,\text{Sin}[a\,k2] + \text{Cosh}[2\,a\,k]\,(k2\,\text{Cos}[a\,k2] + k\,\text{Sin}[a\,k2]) + (k2\,\text{Cos}[a\,k2] + k\,\text{Sin}[a\,k2])\,\text{Sinh}[2\,a\,k]))\,/\,((\text{Cosh}[a\,k] + \text{Sinh}[a\,k])\,(k\,\text{Cosh}[a\,k] + i\,k2\,\text{Sinh}[a\,k]))\}$$

Problem 4

In[7]:=
$$\frac{\frac{1}{2^6\,(3!)^2}}{\frac{1}{2^6\,(3!)^2} + \frac{1}{2^8\,(4!)^2}}\;\frac{\frac{1}{2^8\,(4!)^2}}{\frac{1}{2^6\,(3!)^2} + \frac{1}{2^8\,(4!)^2}}$$

Out[7]=
$$\frac{64}{65}$$

Out[8]=
$$\frac{1}{65}$$

Problem 6

In[9]:=
```
$Assumptions = c > 0;
A = (m ω / (π ħ 2^(2*3) (3!)^2))^(1/4);
c = m ω / ħ;
y = Sqrt[c] x;
ψ3 = A e^(-c x^2/2) (-12 (y - 2/3 y^3));
ψ2 = B e^(-c x^2/2) (-2 (1 - 2 y^2));
Integrate[ψ3 ψ2, {x, -Infinity, Infinity}]
Integrate[ψ3 ψ3, {x, -Infinity, Infinity}]
```

Out[15]= 0

Out[16]= 1

Problem 7

```
ψ0 = 1/Sqrt[2] (((m ω)/(π ħ))^(1/4) e^(-(m ω x^2)/(2 ħ)) + 2 ((m ω)/ħ)^(1/2) x ((m ω)/(π ħ 4))^(1/4) e^(-(m ω x^2)/(2 ħ)));
Integrate[ψ0^2 x, {x, -Infinity, Infinity}] (* <X(0)> *)
Integrate[ψ0 (-i ħ D[ψ0, x]), {x, -Infinity, Infinity}] (* <P(0)> *)
U = ((m ω)/(2 π i ħ Sin[ω T]))^(1/2) e^((i m ω)/ħ ((x^2+xp^2) Cos[ω T]-2 x xp)/(2 Sin[ω T]));
xt = Integrate[(ψ0 U)^2, {x, -Infinity, Infinity}]
pt = Integrate[(ψ0 U) (-i ħ D[ψ0 U, x]), {x, -Infinity, Infinity}]
```

Out[56]=
$$\frac{1}{\sqrt{2}\sqrt{\frac{m\omega}{\hbar}}}$$

Out[57]= 0

Out[59]=
$$-\frac{i e^{-\frac{m xp^2 \omega}{\hbar}} \left(\frac{m\omega}{\hbar}\right)^{3/2} \text{Csc}[T\omega]\left(-2 i m xp^2 \omega + 2 i \hbar + \left(2 m xp^2 \omega + \hbar\right)\text{Cot}[T\omega] + 2\sqrt{2}\, xp \sqrt{\frac{m\omega}{\hbar}}\, \hbar\, \text{Csc}[T\omega]\right)}{4\pi\hbar\sqrt{\frac{m\omega(1-i\,\text{Cot}[T\omega])}{\hbar}}\,(i + \text{Cot}[T\omega])}$$

Out[60]= 0

In[63]:=
```
D[xt, T] - pt // TrigToExp // Simplify
D[pt, T] + m ω^2 xt // TrigToExp // Simplify
```

Out[63]=
$$-\frac{1}{4\left(-1+e^{2 i T\omega}\right) m \pi} i e^{-3 i T\omega - \frac{m xp^2 \omega}{\hbar}} \left(\frac{m\omega}{\hbar}\right)^{3/2} \sqrt{-\frac{e^{2 i T\omega} m\omega}{2\hbar - 2 e^{2 i T\omega}\hbar}} \left(4\left(-2+3e^{2 i T\omega}\right) m xp^2 \omega + \left(2 - 3 e^{2 i T\omega} + 3 e^{4 i T\omega} - 4\sqrt{2}\, e^{i T\omega}\, xp \sqrt{\frac{m\omega}{\hbar}} + 8\sqrt{2}\, e^{3 i T\omega}\, xp \sqrt{\frac{m\omega}{\hbar}}\right)\hbar\right)$$

Out[64]=
$$\frac{e^{-3 i T\omega - \frac{m xp^2 \omega}{\hbar}}\, \omega \left(\frac{m\omega}{\hbar}\right)^{3/2} \sqrt{-\frac{e^{2 i T\omega} m\omega}{2\hbar - 2 e^{2 i T\omega}\hbar}} \left(4 m xp^2 \omega + \left(-1 + 3 e^{2 i T\omega} + 4\sqrt{2}\, e^{i T\omega}\, xp \sqrt{\frac{m\omega}{\hbar}}\right)\hbar\right)}{4\pi}$$

Problem 8

In[109]:=
```
ψ0 = A e^(-c x^2/2)
XHat0 = ψ0 - Integrate[x ψ0^2, {x, -Infinity, Infinity}]
PHat0 = -i ħ D[ψ0, x] - Integrate[ψ0 (-i ħ D[ψ0, x]), {x, -Infinity, Infinity}]
XHat0
-----
PHat0
```

Out[109]=
$$\frac{e^{-\frac{m x^2 \omega}{2 \hbar}} \left(\frac{2^{-2 n} m \omega}{\hbar (n!)^2}\right)^{1/4}}{\pi^{1/4}}$$

Out[110]=
$$\frac{e^{-\frac{m x^2 \omega}{2 \hbar}} \left(\frac{2^{-2 n} m \omega}{\hbar (n!)^2}\right)^{1/4}}{\pi^{1/4}}$$

Out[111]=
$$\frac{i\, e^{-\frac{m x^2 \omega}{2 \hbar}}\, m x \omega \left(\frac{2^{-2 n} m \omega}{\hbar (n!)^2}\right)^{1/4}}{\pi^{1/4}}$$

Out[112]=
$$-\frac{i}{m x \omega}$$

Problem 10

In[113]:=
```
197.327
-------
2 × 114*^4
```

Out[113]= 0.0000865469