

## Phase 11 b : Function Application

Basic function application (single argument):

$f(x)$   
 $square(5)$   
 $parent(john)$

Multiple arguments:

$g(x, y, z)$   
 $sum(a, b, c, d)$   
 $max(x, y)$

Nested function application:

$f(g(x))$   
 $f(g(h(x)))$   
 $outer(middle(inner(value)))$

Empty argument list:

$f()$   
 $initialize()$

Function application in expressions:

$f(x) \wedge g(y)$   
 $f(x) \Rightarrow g(x)$   
 $f(x) > 0$   
 $height(m) < 10$

Function application in quantifiers:

$\forall x : \mathbb{N} \bullet f(x) > 0$   
 $\exists d : Dog \bullet gentle(d) \wedge neat(d)$   
 $\forall x, y : \mathbb{N} \bullet g(x, y) = g(y, x)$

Special Z notation functions (generic instantiation):

$\mid \quad items : seq \mathbb{N} possibilities : Itemcollection : bag \ Xsequence : iseq \mathbb{N}$

Function application with set membership:

$x \in f(S)$   
 $y \in X$   
 $z \in seq \mathbb{N}$

Complex predicates from Solution 5:

$\exists d : Dog \bullet gentle(d) \wedge neat(d)$   
 $\forall d : Dog \bullet neat(d) \Rightarrow attractive(d)$

Function application in definitions:

$$\frac{f : X \rightarrow Y}{\forall x : X \bullet f(x) \in Y}$$

$$\frac{\frac{PersonHeight}{height : Person \rightarrow \mathbb{N}}}{\forall p : Person \bullet height(p) > 0 \wedge height(p) < 300}$$

Function application with comparisons:

$$\begin{aligned} f(x, y) &> g(x, y) \\ max(x, y) &= max(y, x) \\ f(0) &= 1 \end{aligned}$$

Expression arguments:

$$\begin{aligned} f(x \wedge y) \\ g(x \vee y, p \Rightarrow q) \\ h(a > b, c < d) \end{aligned}$$

Function application with subscripts and superscripts:

$$\begin{aligned} f(x_i) \\ g(x^2) \\ h(a_1, a_2) \end{aligned}$$