Bullet Separator Examples

Example 1 : Set Comprehension with Bullet

The bullet separator distinguishes the constraint (filter) from the term (selector):

$\{ x : \mathbb{N} \mid x > 0 \bullet x * x \}$
This reads: "the set of x squared, for all x in N where x is positive". The constraint $x > 0$ filters the domain, and x * x is the value selected.

Example 2 : Set Comprehension Without Bullet

Without the bullet, the set contains the values that satisfy the predicate:

$\{ x : \mathbb{N} \mid x > 0 \land x < 10 \}$
This is the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. No transformation is applied.

Example 3 : Comparing With land Without Bullet

With bullet (maps/transforms values):

$\{ n : \mathbb{N} \mid n < 5 \bullet n * n \}$
Result: $\{0, 1, 4, 9, 16\}$ (squares of 0, 1, 2, 3, 4)

Without bullet (filters values):

$\{ n : \mathbb{N} \mid n < 5 \}$
Result: $\{0, 1, 2, 3, 4\}$ (the values themselves)

Example 4 : Complex Set Comprehension

$\{ x : \mathbb{Z} \mid x \mathrel{\mathsf{mod}} 2 = 0 \land x \geq -10 \land x \leq 10 \bullet x(div)(2) \}$
This gives the set $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ by taking even integers from -10 to 10 and dividing by 2.

Example 5 : Mu Operator with Bullet

The mu operator finds the unique value satisfying a condition:

$(\mu\, x : \mathbb{N} \mid x > 5 \land x < 7 \bullet x)$
This evaluates to 6, the unique natural number strictly between 5 and 7. The bullet separates the uniqueness constraint from the selected expression.

Example 6 : Mu with Complex Expression

$(\mu\, n : \mathbb{N} \mid n * n = 16 \bullet n + 1)$
This finds the unique n where n squared equals 16 (which is $n = 4$), then evaluates n + 1, giving 5.

Example 7 : Nested Expressions

$\{ x : \mathbb{N} \mid x < 3 \bullet \{ y : \mathbb{N} \mid y < x \bullet (x, y) \} \}$
This creates a set of sets of tuples. For each $x < 3$, we create the set of pairs (x, y) where $y < x$.

Example 8 : Quantifiers with Bullet (Limited Support)

Standard Z notation supports bullet in quantifiers to separate constraints from body:

PROPOSED: $\forall x : \mathbb{N} \bullet x > 0.\ x \geq 0$

This would read: "for all positive x, x is non-negative". The constraint $x > 0$ restricts the quantification domain.

NOTE: Full bullet support in $\forall/\exists$ quantifiers is not yet implemented. Currently use implication instead:

$\forall x : \mathbb{N} \bullet x > 0 \Rightarrow x \geq 0$

Example 9 : Partial Functions with Bullet

In pattern matching and function definitions:

$$\begin{array}{|l} \mathit{predecessor} : \mathbb{N} \rightarrowtail\!\!\!\!\!+ \ \mathbb{N} \\ \hline \forall n : \mathbb{N} \mid n > 0 \bullet \mathit{predecessor}(n) = n - 1 \end{array}$$

The bullet indicates predecessor is only defined when the constraint $n > 0$ holds.

Example 10 : Practical Example - Data Filtering

$[\mathit{Employee}]$

$$\begin{array}{|l} \mathit{salaries} : \mathit{Employee} \longrightarrow \mathbb{N} \\ \mathit{high\_earners} : \mathbb{P}\ \mathit{Employee} \\ \mathit{high\_salaries} : \mathbb{P}\ \mathbb{N} \\ \hline \mathit{high\_earners} = \{ e : \mathit{Employee} \mid \mathit{salaries}(e) > 100000 \} \\ \mathit{high\_salaries} = \{ e : \mathit{Employee} \mid \mathit{salaries}(e) > 100000 \bullet \mathit{salaries}(e) \} \end{array}$$

high_earners is the set of employees earning over 100K (the employees themselves). high_salaries is the set of salary values over 100K (the transformed values).

Example 11 : Mathematical Sets

The set of perfect squares less than 100:

$\{ n : \mathbb{N} \mid n < 10 \bullet n * n \}$
Result: $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$

The set of even numbers:

$\{ n : \mathbb{N} \mid n \bmod 2 = 0 \}$
Result: 0, 2, 4, 6, 8, ... (infinite set, no bullet needed)

Example 12 : Design Pattern

The bullet separator follows a consistent pattern across Z notation:

Before *bullet* : *constraint* (declaration and filter)

After *bullet* : *term* (what to compute or select)

This separation makes specifications clearer by explicitly distinguishing the domain restriction from the value computation.