Phase 5 : Proof Trees

Example 1 : Simple Modus Ponens

- $p \Rightarrow q$ [premise]
- p [premise]
  - q [modus ponens]

Example 2 : Conjunction Elimination

- $p \land q$ [premise]
  - p [$\land$ elim left]
  - q [$\land$ elim right]

Example 3 : Nested Proof

- $(p \Rightarrow q) \land (q \Rightarrow r)$ [premise]
  - $p \Rightarrow q$ [$\land$ elim]
  - $q \Rightarrow r$ [$\land$ elim]
- p [premise]
  - q [modus ponens]
    - r [modus ponens]

Example 4 : Disjunction Cases

- $p \lor q$ [premise]
  - p [case 1]
    - $p \Rightarrow r$ [premise]
      - r [modus ponens]
  - q [case 2]
    - $q \Rightarrow r$ [premise]
      - r [modus ponens]

Example 5 : Proof with Quantifiers

- $\forall\, x : N \bullet x > 0$ [premise]
- $a \in N$ [premise]
  - $a > 0$ [universal instantiation]