

Phase 20 Demo : bigcup (Distributed Union)

Example 1 : Basic bigcup

bigcup S is the union of all sets in the set of sets S:

$[X]$

abbrev

$S == \mathbb{P} \mathbb{P} X$

$$\frac{\begin{array}{c} allElements : S \rightarrow \mathbb{P} X \\ \hline \forall s : S \bullet allElements(s) = \bigcup s \end{array}}{\forall s : S \bullet allElements(s) = \bigcup s}$$

Example 2 : bigcup with ran

Combining bigcup with ran to get all elements from a relation's range:

$[UserId, SongId]$

$$\frac{\begin{array}{c} loved : UserId \rightarrow \mathbb{F} SongId allLovedSongs : \mathbb{F} SongId \\ \hline allLovedSongs = \bigcup \text{ran } loved \end{array}}{\forall s : S \bullet allLovedSongs(s) = \bigcup \text{ran } loved}$$

Example 3 : Set difference with bigcup (Solution 50 a)

Users who are not subscribers of any playlist:

$[PlaylistId, UserId]$

$$\frac{\begin{array}{c} users : \mathbb{F} UserId playlistSubscribers : PlaylistId \rightarrow \mathbb{F}_1 UserId nonSubscribers : \mathbb{F} UserId \\ \hline nonSubscribers = users \setminus \bigcup \text{ran } playlistSubscribers \end{array}}{\forall s : S \bullet nonSubscribers(s) = users(s) \setminus \bigcup \text{ran } playlistSubscribers(s)}$$

Example 4 : Nested bigcup

bigcup can be nested for multi-level set structures:

$[N]$

$$\frac{\begin{array}{c} nested : \mathbb{P} \mathbb{P} \mathbb{P} N \\ \hline \bigcup \bigcup nested \subseteq qN \end{array}}{\bigcup \bigcup nested \subseteq qN}$$