

## Pattern Matching with Sequences

### Example 1 : Empty Sequence Pattern

Pattern matching on the empty sequence:

$$\left| \begin{array}{l} f : \text{seq } \mathbb{N} \rightarrow \mathbb{N} \\ \hline f(\langle \rangle) = 0 \end{array} \right|$$

### Example 2 : Cons Pattern ( Head land Tail )

Decomposing a sequence into head and tail:

$$\left| \begin{array}{l} g : \text{seq } \mathbb{N} \rightarrow \mathbb{N} \\ \hline g(\langle \rangle) = 0 \\ \forall x : \mathbb{N} \bullet \forall s : \text{seq } \mathbb{N} \bullet g(\langle x \rangle \frown s) = x \end{array} \right|$$

### Example 3 : Recursive Sum

Computing the sum of elements using pattern matching:

$$\left| \begin{array}{l} total : \text{seq } \mathbb{N} \rightarrow \mathbb{N} \\ \hline total(\langle \rangle) = 0 \\ \forall x : \mathbb{N} \bullet \forall s : \text{seq } \mathbb{N} \bullet total(\langle x \rangle \frown s) = x + total(s) \end{array} \right|$$

### Example 4 : Cumulative Total

A more descriptive example from the solutions:

$$\left| \begin{array}{l} cumulative\_total : \text{seq } \mathbb{N} \rightarrow \mathbb{N} \\ \hline cumulative\_total(\langle \rangle) = 0 \\ \forall x : \mathbb{N} \bullet \forall s : \text{seq } \mathbb{N} \bullet cumulative\_total(\langle x \rangle \frown s) = x + cumulative\_total(s) \end{array} \right|$$

### Example 5 : Filter Pattern

Filtering sequences based on a condition:

$$\left| \begin{array}{l} positives : \text{seq } \mathbb{Z} \rightarrow \text{seq } \mathbb{Z} \\ \hline positives(\langle \rangle) = \langle \rangle \\ \forall x : \mathbb{Z} \bullet \forall s : \text{seq } \mathbb{Z} \bullet positives(\langle x \rangle \frown s) = \text{if } x > 0 \\ \quad \text{then } \langle x \rangle \frown positives(s) \\ \quad \text{else } positives(s) \end{array} \right|$$