Subscripts and Superscripts

Example 1 : Basic Subscripts

Subscripts are written with underscore:

$x_1 \in S$
$x_2 = x_1 + 1$
Common for indexed variables in sequences, arrays, or iterations.

Example 2 : Basic Superscripts (Exponentiation)

Superscripts denote exponentiation:

$x^2 = x * x$
$n^3 = n * n * n$
The caret symbol creates superscripts for powers.

Example 3 : Multiple Subscripted Variables

$$
\begin{array}{|l}
x_1 : \mathbb{N} \\
x_2 : \mathbb{N} \\
x_3 : \mathbb{N} \\
\hline
x_1 = 1 \\
x_2 = 2 \\
x_3 = 3
\end{array}
$$

Define multiple indexed variables.

Example 4 : Sequence Indexing

Subscripts are natural for sequence element notation:

$\forall i : 1 \mathinner{\ldotp\ldotp} n \bullet s\_i \in \mathbb{N}$
Here s_i represents the i-th element of a conceptual sequence.

Example 5 : Exponentiation in Expressions

$\{\, n : \mathbb{N} \mid n < 10 \bullet n^2 \,\}$
Set of squares.

Example 6 : Power Function for Nested Exponents

For nested exponents like power of a power, define a power function:

$$
\begin{array}{|l}
pow : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N} \\
\hline
\forall x : \mathbb{N} \bullet pow(x, 0) = 1 \\
\forall x, n : \mathbb{N} \bullet n > 0 \Rightarrow pow(x, n) = x * pow(x, n - 1)
\end{array}
$$

Then use it for nested exponentiation:

$pow(pow(x, 2), 3) = pow(x, 6)$
$pow(pow(n, 2), n + 1)$
This avoids LaTeX double superscript issues.

Example 7 : Polynomials

Subscripts for coefficients, superscripts for powers:

$a_0 + a_1 * x + a_2 * x^2 + a_3 * x^3$
Standard polynomial notation.

Example 8 : Combined Subscripts and Superscripts

Variable with subscript, raised to a power:

x_i^2
Meaning : *xsubi*, squared.

Example 9 : Cartesian Product Powers

Cartesian product repeated n times:

$S^2 = S \times S$
$S^3 = S \times S \times S$
Set of n-tuples from S.

Example 10 : Iteration Subscripts

Subscripts often denote iteration steps:

$x_0 = initial$
$x\_(i + 1) = f(x\_i)$
Defines a recurrence relation.

Example 11 : State Variable Subscripts

```
┌─ Counter ────────────────────────────────
│ count : ℕ
│ countNext : ℕ
├──────────────────────────────────────────
│ countNext = count + 1
└──────────────────────────────────────────
```

Next-state variables can use descriptive names like countNext instead of primed notation.

Example 12 : Indexed Family of Sets

$$
\begin{array}{|l}
S_1 : \mathbb{P}\,\mathbb{N} \\
S_2 : \mathbb{P}\,\mathbb{N} \\
S_3 : \mathbb{P}\,\mathbb{N} \\
\hline
S_1 = \{1, 2, 3\} \\
S_2 = \{4, 5, 6\} \\
S_3 = \{7, 8, 9\}
\end{array}
$$

Family of sets indexed by natural numbers.

Example 13 : Exponents in Constraints

$\forall\, n : \mathbb{N} \bullet n^2 \geq n$
Constraint using $exponentiation : everynaturalnumber's square$ is at least itself.

Example 14 : Complex Subscript Expressions

$x\_(i + 1) + x\_(i - 1) = 2 * x\_i$
Recurrence relation with subscript expressions.

Example 15 : Power Set Notation

Sometimes written with superscript:

$\mathbb{P}\,S \vee 2^S$
Both denote the power set of S.

Example 16 : Best Practices

Guidelines for subscripts and superscripts:

1. Use subscripts for indices, versions, or families

2. Use superscripts for exponents and powers

3. For nested exponents, define a pow function

4. Be consistent in indexing: start at 0 or 1, not mixed

5. Document what subscripts mean