

## Phase 11 b : Function Application

Basic function application (single argument):

$f(x)$   
 $\text{square}(5)$   
 $\text{parent}(\text{john})$

Multiple arguments:

$g(x, y, z)$   
 $\text{sum}(a, b, c, d)$   
 $\text{max}(x, y)$

Nested function application:

$f(g(x))$   
 $f(g(h(x)))$   
 $\text{outer}(\text{middle}(\text{inner}(\text{value})))$

Empty argument list:

$f()$   
 $\text{initialize}()$

Function application in expressions:

$f(x) \wedge g(y)$   
 $f(x) \Rightarrow g(x)$   
 $f(x) > 0$   
 $\text{height}(m) < 10$

Function application in quantifiers:

$\forall x: \mathbb{N} \bullet f(x) > 0$   
 $\exists d: \text{Dog} \bullet \text{gentle}(d) \wedge \text{neat}(d)$   
 $\forall x, y: \mathbb{N} \bullet g(x, y) = g(y, x)$

Special Z notation functions (generic instantiation):

|  $items : \text{seq } \mathbb{N}$   $possibilities : \mathbb{P}$   $Itemcollection : \text{bag } X$   $sequence : \text{iseq } \mathbb{N}$

Function application with set membership:

$x \in f(S)$   
 $y \in \mathbb{P} X$   
 $z \in \text{seq } \mathbb{N}$

Complex predicates from Solution 5:

$\exists d: \text{Dog} \bullet \text{gentle}(d) \wedge \text{neat}(d)$   
 $\forall d: \text{Dog} \bullet \text{neat}(d) \Rightarrow \text{attractive}(d)$

Function application in definitions:

$$\boxed{\begin{array}{l} f : X \rightarrow Y \\ \forall x: X \bullet f(x) \in Y \end{array}}$$

$$\boxed{\begin{array}{l} \text{PersonHeight} \\ \hline \text{height} : \text{Person} \rightarrow \mathbb{N} \\ \hline \forall p: \text{Person} \bullet \text{height}(p) > 0 \wedge \text{height}(p) < 300 \end{array}}$$

Function application with comparisons:

$$\begin{aligned} f(x, y) &> g(x, y) \\ \max(x, y) &= \max(y, x) \\ f(0) &= 1 \end{aligned}$$

Expression arguments:

$$\begin{aligned} f(x \wedge y) \\ g(x \vee y, p \Rightarrow q) \\ h(a > b, c < d) \end{aligned}$$

Function application with subscripts and superscripts:

$$\begin{aligned} f(x_i) \\ g(x^2) \\ h(a_1, a_2) \end{aligned}$$