Equivalence Chains

Solution 1

(a)

Commutative property

$$
\begin{aligned}
& p \wedge q \\
\Leftrightarrow\ & q \wedge p && \text{[commutative]}
\end{aligned}
$$

(b)

De Morgan law

$$
\begin{aligned}
& \neg\,(p \wedge q) \\
\Leftrightarrow\ & \neg\, p \vee \neg\, q && \text{[De Morgan]} \\
\Leftrightarrow\ & \neg\, p \vee \neg\, q && \text{[parentheses]}
\end{aligned}
$$

Solution 2

(a)

Definition of implication

$$
\begin{aligned}
& p \Rightarrow q \\
\Leftrightarrow\ & \neg\, p \vee q && \text{[definition]}
\end{aligned}
$$

(b)

Multiple equivalence steps

$$
\begin{aligned}
& p \wedge q \vee p \wedge r \\
\Leftrightarrow\ & p \wedge (q \vee r) && \text{[distributive]} \\
\Leftrightarrow\ & (q \vee r) \wedge p && \text{[commutative]} \\
\Leftrightarrow\ & p \wedge (r \vee q) && \text{[commutative]}
\end{aligned}
$$

Solution 3

(a)

Equivalence chain without justifications

$$\begin{aligned} & p \vee p \\ \Leftrightarrow\ & p \end{aligned}$$

(b)

Mixed justifications

$$\begin{aligned} & p \Leftrightarrow q \\ \Leftrightarrow\ & (p \Rightarrow q) \wedge (q \Rightarrow p) && \text{[definition]} \\ \Leftrightarrow\ & (\neg\, p \vee q) \wedge (\neg\, q \vee p) \end{aligned}$$