Simple Function Definitions

This file demonstrates basic function definitions in Z notation that work well with fuzz type checking.

Example 1 : Simple Total Functions

Total functions are defined over their entire domain.

$$square : \mathbb{N} \longrightarrow \mathbb{N}$$
$$\forall\, n : \mathbb{N} \bullet square(n) = n * n$$

$$successor : \mathbb{N} \longrightarrow \mathbb{N}$$
$$\forall\, n : \mathbb{N} \bullet successor(n) = n + 1$$

$$double : \mathbb{N} \longrightarrow \mathbb{N}$$
$$\forall\, n : \mathbb{N} \bullet double(n) = 2 * n$$

Example 2 : Partial Functions

Partial functions are not defined over their entire domain.

$$predecessor : \mathbb{N} \pfun \mathbb{N}$$
$$\forall\, n : \mathbb{N} \mid n > 0 \bullet predecessor(n) = n - 1$$

predecessor is partial on natural numbers since 0 has no predecessor in N. The bullet separator filters to positive numbers (where predecessor is defined), then specifies the function value.

Example 3 : Generic Functions

Generic functions work with any type parameter.

$$[X]$$
$$identity : X \longrightarrow X$$
$$\forall\, x : X \bullet identity(x) = x$$

$$[X, Y]$$
$$fst : X \times Y \longrightarrow X$$
$$\forall\, x : X \bullet \forall\, y : Y \bullet fst(x, y) = x$$

$$[X, Y]$$
$$snd : X \times Y \longrightarrow Y$$
$$\forall\, x : X \bullet \forall\, y : Y \bullet snd(x, y) = y$$

Example 4 : Functions with Given Types

$[Person, Department]$

$$assignment : Person \pfun Department$$

assignment is a partial function from Person to Department.

Example 5 : Functions on Numbers

$$triple : \mathbb{N} \fun \mathbb{N}$$
$$\forall n : \mathbb{N} \bullet triple(n) = 3 * n$$

$$addOne : \mathbb{Z} \fun \mathbb{Z}$$
$$\forall n : \mathbb{Z} \bullet addOne(n) = n + 1$$

Example 6 : Function with Given Types

$[Student, Grade]$

$$grades : Student \pfun Grade$$

grades maps students to their grades.

Example 7 : Function Composition Example

$$f : \mathbb{N} \fun \mathbb{N}$$
$$g : \mathbb{N} \fun \mathbb{N}$$
$$h : \mathbb{N} \fun \mathbb{N}$$
$$\forall n : \mathbb{N} \bullet f(n) = 2 * n$$
$$\forall n : \mathbb{N} \bullet g(n) = n + 1$$
$$h = f \circ g$$

f doubles its input, g adds one to its input. h is their forward *composition* : $h(n)$ = f(g(n)) = 2 * (n + 1).

Example 8 : Modulo Function

$$modulo3 : \mathbb{N} \fun \mathbb{N}$$
$$\forall n : \mathbb{N} \bullet modulo3(n) = n \bmod 3$$

For comprehensive function examples including recursive functions, higher-order functions, and advanced patterns, see examples/09_sequences/pattern_matching.txt.