

Phase 5 : Proof Trees

Example 1 : Simple Implication

$$\frac{\frac{p \wedge q \neg[1]}{q} \quad p \wedge q \Rightarrow q}{p \wedge q \neg[1]} \text{ [}\neg\text{-intro}^{[1]}\text{]}$$

Example 2 : With Sibling Premises

$$\frac{\frac{\frac{r}{p \wedge (p \Rightarrow q) \neg^{[1]}} [\wedge\text{-elim-1}] \quad \frac{r}{p \wedge (p \Rightarrow q) \neg^{[1]}} [\wedge\text{-elim-2}]}{p} [\Rightarrow\text{ elim}]}{q} [\wedge\text{ intro}]$$

Example 3 : Distribution with Cases

$$\frac{\frac{\frac{\neg p \text{ [from above]} \quad \neg q \text{ [from case]}}{\frac{p \wedge q}{p \wedge q \vee p \wedge r} \text{ [\vee-intro-1]}} \quad \frac{\neg p \text{ [from above]} \quad \neg r \text{ [from case]}}{\frac{p \wedge r}{p \wedge q \vee p \wedge r} \text{ [\vee-intro-2]}}}{\frac{p \wedge q \vee p \wedge r}{p \wedge (q \vee r) \Rightarrow p \wedge q \vee p \wedge r} \text{ [\Rightarrow-intro}^{[1]} \text{]}} \text{ [\vee elim]}$$

Example 4 : Modus Tollens

$$\frac{\frac{\frac{\vdash (p \Rightarrow q) \wedge \neg q^{\neg[1]}}{p \Rightarrow q} [\wedge\text{-elim-1}] \quad \frac{\vdash (p \Rightarrow q) \wedge \neg q^{\neg[1]}}{\neg q} [\wedge\text{-elim-2}]}{\neg p} [\Rightarrow\text{-intro}^{[1]}]}{(p \Rightarrow q) \wedge \neg q \Rightarrow \neg p} [\Rightarrow\text{-elim}]$$

[contradiction]

[negation-intro^[2]]

Example 5 : Solution 18 Implication to Disjunction

$$\frac{\frac{\frac{\vdash p \Rightarrow q \neg [1]}{p \vee \neg p} \text{ [excluded middle]}}{\vdash p \Rightarrow q \Rightarrow \neg p \vee q} \quad \frac{\frac{\vdash p \neg [2]}{q} \text{ [⇒ elim]}}{\vdash \neg p \vee q} \text{ [∨-intro-2]} \quad \frac{\frac{\vdash \neg p \vee q}{\vdash \neg p \vee q} \text{ [∨-intro-1]}}{\vdash \neg p \vee q} \text{ [∨-elim}^{[2]}\text{]}}{\vdash p \Rightarrow q \Rightarrow \neg p \vee q} \text{ [⇒-intro}^{[1]}\text{]}$$