

## Proof Tree Nesting Depth Test

### Test 1 : Simple ( depth 1 )

$$\frac{\Gamma p \neg^{[1]} \quad \neg q}{p \Rightarrow q} \text{ [axiom] } \Rightarrow \neg\text{-intro}^{[1]}$$

### Test 2 : Moderate ( depth 2 )

$$\frac{\Gamma p \neg^{[1]} \quad \frac{\Gamma q \neg^{[2]} \quad \neg r}{q \Rightarrow r} \text{ [axiom] } \Rightarrow \neg\text{-intro}^{[2]}}{p \Rightarrow (q \Rightarrow r)} \text{ [axiom] } \Rightarrow \neg\text{-intro}^{[1]}$$

### Test 3 : Deep ( depth 3 )

$$\frac{\Gamma p \neg^{[1]} \quad \frac{\Gamma q \neg^{[2]} \quad \frac{\Gamma r \neg^{[3]} \quad \neg s}{r \Rightarrow s} \text{ [axiom] } \Rightarrow \neg\text{-intro}^{[3]}}{q \Rightarrow (r \Rightarrow s)} \text{ [axiom] } \Rightarrow \neg\text{-intro}^{[2]}}{p \Rightarrow (q \Rightarrow (r \Rightarrow s))} \text{ [axiom] } \Rightarrow \neg\text{-intro}^{[1]}$$

### Test 4 : Very Deep ( depth 4 )

$$\frac{\Gamma p \neg^{[1]} \quad \frac{\Gamma q \neg^{[2]} \quad \frac{\Gamma r \neg^{[3]} \quad \frac{\Gamma s \neg^{[4]} \quad \neg t}{s \Rightarrow t} \text{ [axiom] } \Rightarrow \neg\text{-intro}^{[4]}}{r \Rightarrow (s \Rightarrow t)} \text{ [axiom] } \Rightarrow \neg\text{-intro}^{[3]}}{q \Rightarrow (r \Rightarrow (s \Rightarrow t))} \text{ [axiom] } \Rightarrow \neg\text{-intro}^{[2]}}{p \Rightarrow (q \Rightarrow (r \Rightarrow (s \Rightarrow t)))} \text{ [axiom] } \Rightarrow \neg\text{-intro}^{[1]}$$

### Test 5 : Horizontal siblings ( depth 2 )

$$\frac{\Gamma p \wedge q \neg^{[1]} \quad \frac{\Gamma p \neg^{[1]} \quad \Gamma q \neg^{[1]}}{r} \text{ [\wedge elim]}}{(p \wedge q) \Rightarrow r} \text{ [axiom] } \Rightarrow \neg\text{-intro}^{[1]}$$