

## One - Point Rule Applications

### Example 1 : Simple One - Point Rule

When a quantifier binds a variable that is constrained by equality, we can eliminate the quantifier:

$$\begin{aligned}\exists y : \mathbb{N} \bullet y = 5 \wedge x > y \\ \Leftrightarrow 5 \in \mathbb{N} \wedge x > 5 \\ \Leftrightarrow x > 5\end{aligned}\quad [\text{one - point rule}]$$

### Example 2 : One - Point with Set Membership

$$\begin{aligned}\exists x, y : \mathbb{N} \bullet x + y = 4 \wedge x < y \\ \Leftrightarrow \exists x, y : \mathbb{N} \bullet y = 4 - x \wedge x < y \\ \Leftrightarrow \exists x : \mathbb{N} \bullet 4 - x \in \mathbb{N} \wedge x < 4 - x \\ \Leftrightarrow \text{true}\end{aligned}$$

The final equivalence holds because 0 in N, 4 - 0 in N, and 0 < 4.

### Example 3 : One - Point with Disjunction

$$\begin{aligned}\exists x : \mathbb{N} \bullet (x = 1 \wedge x > y) \vee (x = 2 \wedge x > z) \\ \Leftrightarrow (\exists x : \mathbb{N} \bullet x = 1 \wedge x > y) \vee (\exists x : \mathbb{N} \bullet x = 2 \wedge x > z) \\ \Leftrightarrow (1 \in \mathbb{N} \wedge 1 > y) \vee (\exists x : \mathbb{N} \bullet x = 2 \wedge x > z) \\ \Leftrightarrow (1 \in \mathbb{N} \wedge 1 > y) \vee (2 \in \mathbb{N} \wedge 2 > z) \\ \Leftrightarrow 1 > y \vee 2 > z\end{aligned}$$

### Example 4 : One - Point with Expression Part

$$\begin{aligned}\exists y : \mathbb{N} \bullet y = x + 1 \wedge y > 0 \\ \Leftrightarrow x + 1 \in \mathbb{N} \wedge x + 1 > 0 \\ \Leftrightarrow x + 1 > 0\end{aligned}\quad [\text{one - point rule}]$$