

Proof Tree Nesting Depth Test

Test 1 : Simple (depth 1)

$$\frac{\frac{\lceil p \neg^{[1]} \quad \bar{q} \rceil}{p \Rightarrow q} \quad \text{[axiom]}}{\lceil \Rightarrow \text{-intro}^{[1]} \rceil}$$

Test 2 : Moderate (depth 2)

$$\frac{\frac{\lceil p \neg^{[1]} \quad \frac{\frac{\lceil q \neg^{[2]} \quad \bar{r} \rceil}{q \Rightarrow r} \text{[axiom]}}{q \Rightarrow r} \text{[}\Rightarrow \text{-intro}^{[2]}\text{]}}{p \Rightarrow (q \Rightarrow r)} \text{[}\Rightarrow \text{-intro}^{[1]}\text{]}}$$

Test 3 : Deep (depth 3)

$$\frac{\frac{\lceil p \neg^{[1]} \quad \frac{\frac{\frac{\lceil q \neg^{[2]} \quad \frac{\frac{\lceil r \neg^{[3]} \quad \bar{s} \rceil}{r \Rightarrow s} \text{[axiom]}}{r \Rightarrow s} \text{[}\Rightarrow \text{-intro}^{[3]}\text{]}}{q \Rightarrow (r \Rightarrow s)} \text{[}\Rightarrow \text{-intro}^{[2]}\text{]}}{q \Rightarrow (r \Rightarrow s)} \text{[}\Rightarrow \text{-intro}^{[1]}\text{]}}{p \Rightarrow (q \Rightarrow (r \Rightarrow s))} \text{[}\Rightarrow \text{-intro}^{[1]}\text{]}}$$

Test 4 : Very Deep (depth 4)

$$\frac{\frac{\lceil p \neg^{[1]} \quad \frac{\frac{\frac{\frac{\frac{\lceil q \neg^{[2]} \quad \frac{\frac{\frac{\frac{\lceil r \neg^{[3]} \quad \frac{\frac{\lceil s \neg^{[4]} \quad \bar{t} \rceil}{s \Rightarrow t} \text{[axiom]}}{s \Rightarrow t} \text{[}\Rightarrow \text{-intro}^{[4]}\text{]}}{r \Rightarrow (s \Rightarrow t)} \text{[}\Rightarrow \text{-intro}^{[3]}\text{]}}{q \Rightarrow (r \Rightarrow (s \Rightarrow t))} \text{[}\Rightarrow \text{-intro}^{[2]}\text{]}}{q \Rightarrow (r \Rightarrow (s \Rightarrow t))} \text{[}\Rightarrow \text{-intro}^{[1]}\text{]}}{p \Rightarrow (q \Rightarrow (r \Rightarrow (s \Rightarrow t)))} \text{[}\Rightarrow \text{-intro}^{[1]}\text{]}}{p \Rightarrow (q \Rightarrow (r \Rightarrow (s \Rightarrow t)))} \text{[}\Rightarrow \text{-intro}^{[1]}\text{]}}$$

Test 5 : Horizontal siblings (depth 2)

$$\frac{\frac{\lceil p \wedge q \neg^{[1]} \quad \frac{\frac{\lceil p \neg^{[1]} \quad \lceil q \neg^{[1]} \rceil}{p \wedge q} \text{[}\wedge \text{elim]}}{(p \wedge q) \Rightarrow r} \text{[}\Rightarrow \text{-intro}^{[1]}\text{]}}{(p \wedge q) \Rightarrow r} \text{[}\Rightarrow \text{-intro}^{[1]}\text{]}}$$