

Sequence Operations

Example 1 : Sequence Length

The cardinality operator gives the length of a sequence:

```
#⟨⟩  
#⟨a, b, c⟩  
#s
```

Example 2 : Pattern Matching for Head and Tail

In Z, decomposing a sequence uses pattern matching with concatenation.

Given a non-empty sequence s, if we have x and t such that:

$$\langle x \rangle \cap t = s$$

Then x is the first element (head) and t is the remaining sequence (tail).

Example 3 : User - Defined First and Rest Functions

You can define explicit functions for sequence decomposition:

[X]

$$\boxed{\begin{array}{l} fst : \text{seq}_1 X \rightarrow X \\ rest : \text{seq}_1 X \rightarrow \text{seq } X \\ \hline \forall x : X \bullet \forall s : \text{seq } X \bullet fst(\langle x \rangle \cap s) = x \\ \forall x : X \bullet \forall s : \text{seq } X \bullet rest(\langle x \rangle \cap s) = s \end{array}}$$

Example 4 : User - Defined Last and Init Functions

$$\boxed{\begin{array}{l} lst : \text{seq}_1 X \rightarrow X \\ init : \text{seq}_1 X \rightarrow \text{seq } X \\ \hline \forall x : X \bullet \forall s : \text{seq } X \bullet lst(s \cap \langle x \rangle) = x \\ \forall x : X \bullet \forall s : \text{seq } X \bullet init(s \cap \langle x \rangle) = s \end{array}}$$

Example 5 : Domain and Range

Sequences are functions from indices to elements:

```
dom⟨⟩  
dom⟨a, b, c, d⟩  
ran⟨a, b, c⟩
```

Example 6 : Concatenation

The concatenation operator joins sequences:

```
s ∩ t  
⟨1, 2⟩ ∩ ⟨3, 4⟩  
⟨a⟩ ∩ ⟨b, c⟩ ∩ ⟨d⟩
```

Example 7 : Using Pattern Matching

Pattern matching extracts elements from sequences:

$$\forall s : \text{seq}_1 \mathbb{N} \bullet \exists x : \mathbb{N} \bullet \exists t : \text{seq } \mathbb{N} \bullet s = \langle x \rangle \cap t$$

This asserts that every non-empty sequence can be decomposed.

Example 8 : Recursive Function Example

Define a sum function using pattern matching style:

$$\frac{\begin{array}{c} \text{sumSeq} : \text{seq } \mathbb{N} \rightarrow \mathbb{N} \\ \text{sumSeq}(\langle \rangle) = 0 \\ \forall x : \mathbb{N} \bullet \forall s : \text{seq } \mathbb{N} \bullet \text{sumSeq}(\langle x \rangle \cap s) = x + \text{sumSeq}(s) \end{array}}{\forall x : \mathbb{N} \bullet \forall s : \text{seq } \mathbb{N} \bullet \text{sumSeq}(s) = \dots}$$