

## Simple Function Definitions

This file demonstrates basic function definitions in Z notation that work well with fuzz type checking.

### Example 1 : Simple Total Functions

Total functions are defined over their entire domain.

$$\frac{\text{square} : \mathbb{N} \rightarrow \mathbb{N}}{\forall n : \mathbb{N} \bullet \text{square}(n) = n * n}$$

$$\frac{\text{successor} : \mathbb{N} \rightarrow \mathbb{N}}{\forall n : \mathbb{N} \bullet \text{successor}(n) = n + 1}$$

$$\frac{\text{double} : \mathbb{N} \rightarrow \mathbb{N}}{\forall n : \mathbb{N} \bullet \text{double}(n) = 2 * n}$$

### Example 2 : Partial Functions

Partial functions are not defined over their entire domain.

$$\frac{\text{predecessor} : \mathbb{N} \rightarrow \mathbb{N}}{\forall n : \mathbb{N} \mid n > 0 \bullet \text{predecessor}(n) = n - 1}$$

predecessor is partial on natural numbers since 0 has no predecessor in N. The bullet separator filters to positive numbers (where predecessor is defined), then specifies the function value.

### Example 3 : Generic Functions

Generic functions work with any type parameter.

$$\frac{\begin{array}{l} [X] \\ \text{identity} : X \rightarrow X \end{array}}{\forall x : X \bullet \text{identity}(x) = x}$$

$$\frac{\begin{array}{l} [X, Y] \\ \text{fst} : X \times Y \rightarrow X \end{array}}{\forall x : X \bullet \forall y : Y \bullet \text{fst}(x, y) = x}$$

$$\frac{\begin{array}{l} [X, Y] \\ \text{snd} : X \times Y \rightarrow Y \end{array}}{\forall x : X \bullet \forall y : Y \bullet \text{snd}(x, y) = y}$$

## Example 4 : Functions with Given Types

$[Person, Department]$

$| \quad assignment : Person \rightarrow Department$

assignment is a partial function from Person to Department.

## Example 5 : Functions on Numbers

$$\frac{triple : \mathbb{N} \rightarrow \mathbb{N}}{\forall n : \mathbb{N} \bullet triple(n) = 3 * n}$$
$$\frac{addOne : \mathbb{Z} \rightarrow \mathbb{Z}}{\forall n : \mathbb{Z} \bullet addOne(n) = n + 1}$$

## Example 6 : Function with Given Types

$[Student, Grade]$

$| \quad grades : Student \rightarrow Grade$

grades maps students to their grades.

## Example 7 : Function Composition Example

$$\frac{\begin{array}{l} f : \mathbb{N} \rightarrow \mathbb{N} \\ g : \mathbb{N} \rightarrow \mathbb{N} \\ h : \mathbb{N} \rightarrow \mathbb{N} \end{array}}{\begin{array}{l} \forall n : \mathbb{N} \bullet f(n) = 2 * n \\ \forall n : \mathbb{N} \bullet g(n) = n + 1 \\ h = f \circ g \end{array}}$$

f doubles its input, g adds one to its input. h is their forward *composition* :  $h(n) = f(g(n)) = 2 * (n + 1)$ .

## Example 8 : Modulo Function

$$\frac{modulo3 : \mathbb{N} \rightarrow \mathbb{N}}{\forall n : \mathbb{N} \bullet modulo3(n) = n \bmod 3}$$

For comprehensive function examples including recursive functions, higher-order functions, and advanced patterns, see [examples/09\\_sequences/pattern\\_matching.txt](#).