

## Proof Tree Nesting Depth Test

**Test 1 : Simple ( depth 1 )**

$$\frac{\Gamma p \neg^{[1]} \quad \neg q}{p \Rightarrow q} \text{ [axiom] } \neg \text{ [}\Rightarrow\text{-intro}^{[1]}\text{]}$$

**Test 2 : Moderate ( depth 2 )**

$$\frac{\Gamma p \neg^{[1]} \quad \frac{\Gamma q \neg^{[2]} \quad \neg r}{q \Rightarrow r} \text{ [axiom] } \neg \text{ [}\Rightarrow\text{-intro}^{[2]}\text{]}}{p \Rightarrow (q \Rightarrow r)} \text{ [}\Rightarrow\text{-intro}^{[1]}\text{]}$$

**Test 3 : Deep ( depth 3 )**

$$\frac{\Gamma p \neg^{[1]} \quad \frac{\Gamma q \neg^{[2]} \quad \frac{\Gamma r \neg^{[3]} \quad \neg s}{r \Rightarrow s} \text{ [axiom] } \neg \text{ [}\Rightarrow\text{-intro}^{[3]}\text{]}}{q \Rightarrow (r \Rightarrow s)} \text{ [}\Rightarrow\text{-intro}^{[2]}\text{]}}{p \Rightarrow (q \Rightarrow (r \Rightarrow s))} \text{ [}\Rightarrow\text{-intro}^{[1]}\text{]}$$

**Test 4 : Very Deep ( depth 4 )**

$$\frac{\Gamma p \neg^{[1]} \quad \frac{\Gamma q \neg^{[2]} \quad \frac{\Gamma r \neg^{[3]} \quad \frac{\Gamma s \neg^{[4]} \quad \neg t}{s \Rightarrow t} \text{ [axiom] } \neg \text{ [}\Rightarrow\text{-intro}^{[4]}\text{]}}{r \Rightarrow (s \Rightarrow t)} \text{ [}\Rightarrow\text{-intro}^{[3]}\text{]}}{q \Rightarrow (r \Rightarrow (s \Rightarrow t))} \text{ [}\Rightarrow\text{-intro}^{[2]}\text{]}}{p \Rightarrow (q \Rightarrow (r \Rightarrow (s \Rightarrow t)))} \text{ [}\Rightarrow\text{-intro}^{[1]}\text{]}$$

**Test 5 : Horizontal siblings ( depth 2 )**

$$\frac{\Gamma p \wedge q \neg^{[1]} \quad \frac{\Gamma p \neg^{[1]} \quad \Gamma q \neg^{[1]}}{r} \text{ [}\wedge\text{ elim]}}{p \wedge q \Rightarrow r} \text{ [}\Rightarrow\text{-intro}^{[1]}\text{]}$$