

Phase 5 : Proof Trees

Example 1 : Simple Implication

$$\frac{\frac{\frac{\lceil p \wedge q \rceil^{[1]}}{q} [\wedge\text{-elim-2}]}{p \wedge q \Rightarrow q} [\Rightarrow\text{-intro}^{[1]}]}$$

Example 2 : With Sibling Premises

$$\frac{\frac{\frac{\lceil p \wedge (p \Rightarrow q) \rceil^{[1]}}{p} [\wedge\text{-elim-1}] \quad \frac{\frac{\lceil p \wedge (p \Rightarrow q) \rceil^{[1]}}{p \Rightarrow q} [\wedge\text{-elim-2}]}{q} [\Rightarrow\text{elim}]}{\frac{q}{p \wedge q} [\wedge\text{intro}]} [\Rightarrow\text{-intro}^{[1]}]$$

Example 3 : Distribution with Cases

$$\frac{\frac{\frac{\overline{p} \text{ [from above]} \quad \overline{q} \text{ [from case]}}{p \wedge q} [\wedge\text{intro}]}{p \wedge q \vee p \wedge r} [\vee\text{-intro-1}] \quad \frac{\frac{\frac{\overline{p} \text{ [from above]} \quad \overline{r} \text{ [from case]}}{p \wedge r} [\wedge\text{intro}]}{p \wedge q \vee p \wedge r} [\vee\text{-intro-2}]}{p \wedge q \vee p \wedge r} [\vee\text{elim}]}{p \wedge (q \vee r) \Rightarrow p \wedge q \vee p \wedge r} [\Rightarrow\text{-intro}^{[1]}]$$

Example 4 : Modus Tollens

$$\frac{\frac{\frac{\lceil (p \Rightarrow q) \wedge \neg q \rceil^{[1]}}{p \Rightarrow q} [\wedge\text{-elim-1}] \quad \frac{\frac{\lceil (p \Rightarrow q) \wedge \neg q \rceil^{[1]}}{\neg q} [\wedge\text{-elim-2}]}{\frac{\frac{\lceil p \rceil^{[2]}}{q} [\Rightarrow\text{elim}]}{false} [\text{contradiction}]} [\neg\text{-intro}^{[1]}]}{p \Rightarrow q \wedge \neg q \Rightarrow \neg p} [\Rightarrow\text{-intro}^{[1]}]$$

Example 5 : Solution 18 Implication to Disjunction

$$\frac{\frac{\frac{\lceil p \Rightarrow q \rceil^{[1]}}{p \vee \neg p} [\text{excluded middle}] \quad \frac{\frac{\frac{\lceil p \rceil^{[2]}}{q} [\Rightarrow\text{elim}]}{\neg p \vee q} [\vee\text{-intro-2}] \quad \frac{\frac{\lceil \neg p \rceil^{[2]}}{\neg p \vee q} [\vee\text{-intro-1}]}{\neg p \vee q} [\vee\text{-elim}^{[2]}]}{p \Rightarrow q \Rightarrow \neg p \vee q} [\Rightarrow\text{-intro}^{[1]}]$$