

## Equivalence Chains

### Solution 1

(a)

*Commutative(property)*

$$\begin{aligned} p \wedge q \\ \Leftrightarrow q \wedge p \quad & [\text{commutative}] \end{aligned}$$

(b)

*De(Morgan)(law)*

$$\begin{aligned} \neg (p \wedge q) \\ \Leftrightarrow \neg p \vee \neg q \quad & [\text{De Morgan}] \\ \Leftrightarrow \neg p \vee \neg q \quad & [\text{parentheses}] \end{aligned}$$

### Solution 2

(a)

*Definition  
of(implication)*

$$\begin{aligned} p \Rightarrow q \\ \Leftrightarrow \neg p \vee q \quad & [\text{definition}] \end{aligned}$$

(b)

*Multiple(equivalence)(steps)*

$$\begin{aligned} (p \wedge q) \vee (p \wedge r) \\ \Leftrightarrow p \wedge (q \vee r) \quad & [\text{distributive}] \\ \Leftrightarrow (q \vee r) \wedge p \quad & [\text{commutative}] \\ \Leftrightarrow p \wedge (r \vee q) \quad & [\text{commutative}] \end{aligned}$$

### Solution 3

(a)

*Equivalence(chain)(without)(justifications)*

$$\begin{aligned} p \vee p \\ \Leftrightarrow p \end{aligned}$$

(b)

*Mixed(justifications)*

$$\begin{aligned} p \Leftrightarrow q \\ \Leftrightarrow (p \Rightarrow q) \wedge (q \Rightarrow p) \quad & [\text{definition}] \\ \Leftrightarrow (\neg p \vee q) \wedge (\neg q \vee p) \end{aligned}$$