

Equivalence Chains

Solution 1

(a) *Commutative(property)*

$$\begin{aligned} p \wedge q \\ \Leftrightarrow q \wedge p \quad [\text{commutative}] \end{aligned}$$

(b) *De(Morgan)(law)*

$$\begin{aligned} \neg (p \wedge q) \\ \Leftrightarrow \neg p \vee \neg q \quad [\text{De Morgan}] \\ \Leftrightarrow \neg p \vee \neg q \quad [\text{parentheses}] \end{aligned}$$

Solution 2

(a) *Definition
of(implication)*

$$\begin{aligned} p \Rightarrow q \\ \Leftrightarrow \neg p \vee q \quad [\text{definition}] \end{aligned}$$

(b) *Multiple(equivalence)(steps)*

$$\begin{aligned} p \wedge q \vee p \wedge r \\ \Leftrightarrow p \wedge (q \vee r) \quad [\text{distributive}] \\ \Leftrightarrow (q \vee r) \wedge p \quad [\text{commutative}] \\ \Leftrightarrow p \wedge (r \vee q) \quad [\text{commutative}] \end{aligned}$$

Solution 3

(a) *Equivalence(chain)(without)(justifications)*

$$\begin{aligned} p \vee p \\ \Leftrightarrow p \end{aligned}$$

(b) *Mixed(justifications)*

$$\begin{aligned} p \Leftrightarrow q \\ \Leftrightarrow (p \Rightarrow q) \wedge (q \Rightarrow p) \quad [\text{definition}] \\ \Leftrightarrow (\neg p \vee q) \wedge (\neg q \vee p) \end{aligned}$$