

Phase 39 : Strict Subset Operator

The strict subset operator (psubset, \subset) represents proper subset relationships, where A is a subset of B AND A is not equal to B.

Distinction from subset (\subseteq):

- A subset B means: every element of A is \in B (may be equal)
- A psubset B means: every element of A is \in B AND $A \neq B$ (strict inequality)

Example 1 : Basic Strict Subset

$$\begin{array}{|l} A : \mathbb{P} \mathbb{N} \\ B : \mathbb{P} \mathbb{N} \\ \hline A = \{1, 2, 3\} \wedge B = \{1, 2, 3, 4, 5\} \wedge A \subset B \end{array}$$

Here A psubset B holds because all elements of A are in B, and A contains fewer elements than B.

Example 2 : Comparing subset and psubset

$$\begin{array}{|l} X : \mathbb{P} \mathbb{N} \\ Y : \mathbb{P} \mathbb{N} \\ W : \mathbb{P} \mathbb{N} \\ \hline X = \{1, 2\} \wedge Y = \{1, 2, 3\} \wedge W = \{1, 2\} \wedge X \subset Y \wedge X \subseteq W \end{array}$$

X psubset Y is true (X is strictly contained in Y)

X subset W is true (X is a subset of W, including the case where $X = W$)

X psubset W would be FALSE (they are equal sets)

Example 3 : Empty Set

$$\begin{array}{|l} emptySet : \mathbb{P} \mathbb{N} \\ anySet : \mathbb{P} \mathbb{N} \\ \hline emptySet = \{\} \wedge anySet = \{1, 2, 3\} \wedge emptySet \subset anySet \end{array}$$

The empty set is a strict subset of any non-empty set.

Example 4 : Transitive Property

If A psubset B \wedge B psubset C, then A psubset C.

$$\begin{array}{|l} A1 : \mathbb{P} \mathbb{N} \\ B1 : \mathbb{P} \mathbb{N} \\ C1 : \mathbb{P} \mathbb{N} \\ \hline A1 = \{1\} \wedge B1 = \{1, 2\} \wedge C1 = \{1, 2, 3\} \wedge A1 \subset B1 \wedge B1 \subset C1 \wedge A1 \subset C1 \end{array}$$

Example 5 : Set Hierarchy

$[PersonSS]$

$students : \mathbb{P} PersonSS$
$grads : \mathbb{P} PersonSS$
$phds : \mathbb{P} PersonSS$
$phds \subset grads \wedge grads \subset students$

PhD students are a strict subset of graduate students, which are a strict subset of all students.