

Conditional Expressions

Example 1 : Basic If - Then - Else

The if-then-else expression evaluates to one of two values based on a condition:

if $x > 0$ then 1 else -1

If x is positive, this evaluates to 1; otherwise, it evaluates to -1.

Example 2 : Absolute Value Function

$$\frac{abs : \mathbb{Z} \rightarrow \mathbb{N}}{\forall x : \mathbb{Z} \bullet abs(x) = \text{if } x \geq 0 \text{ then } x \text{ else } -x}$$

The absolute value function uses if-then-else to handle positive and negative cases.

Example 3 : Maximum Function

$$\frac{maximum : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}}{\forall a, b : \mathbb{N} \bullet maximum(a, b) = \text{if } a \geq b \text{ then } a \text{ else } b}$$

Returns the larger of two numbers.

Example 4 : Sign Function

$$\frac{sign : \mathbb{Z} \rightarrow \{-1, 0, 1\}}{\forall x : \mathbb{Z} \bullet sign(x) = \text{if } x > 0 \text{ then } 1 \text{ else if } x = 0 \text{ then } 0 \text{ else } -1}$$

Nested if-then-else for three-way branching.

Example 5 : Conditional elem Set Comprehension

$\{x : \mathbb{N} \mid x < 10 \bullet \text{if } x \bmod 2 = 0 \text{ then } x \text{ else } x + 1\}$

For each x less than 10, select x if it's even, otherwise select $x + 1$.

Example 6 : Pattern Matching with Conditionals

$$\frac{filterPositives : \text{seq } \mathbb{Z} \rightarrow \text{seq } \mathbb{Z}}{\begin{array}{l} filterPositives(\langle \rangle) = \langle \rangle \\ \forall x : \mathbb{Z} \bullet \forall s : \text{seq } \mathbb{Z} \bullet filterPositives(\langle x \rangle \frown s) = \text{if } x > 0 \\ \quad \text{then } \langle x \rangle \frown filterPositives(s) \\ \quad \text{else } filterPositives(s) \end{array}}$$

Filters a sequence to include only positive elements using conditional expressions.

Example 7 : Grade Assignment

$Grade ::= GradeA \mid GradeB \mid GradeC \mid GradeD \mid GradeF$

$scoreToGrade : \mathbb{N} \rightarrow Grade$	$\forall score : \mathbb{N} \bullet$ $scoreToGrade(score) = \text{if } score \geq 90 \text{ then } GradeA$ $\text{elseif } score \geq 80 \text{ then } GradeB$ $\text{elseif } score \geq 70 \text{ then } GradeC$ $\text{elseif } score \geq 60 \text{ then } GradeD$ $\text{else } GradeF$
---	---

Multi-way branching using nested if-then-else.

Example 8 : Piecewise Function

$piecewise : \mathbb{Z} \rightarrow \mathbb{Z}$	$\forall x : \mathbb{Z} \bullet$ $piecewise(x) = \text{if } x < -5 \text{ then } 0 \text{ elseif } x < 0 \text{ then } x + 5 \text{ elseif } x < 5 \text{ then } x \text{ elseif } x < 10 \text{ then } x - 5 \text{ else } 0$
---	---

A piecewise-defined function with multiple ranges.

Example 9 : Conditional elem Lambda

$\lambda x : \mathbb{N} \bullet \text{if } x > 100 \text{ then } 100 \text{ else } x$

Lambda function that caps values at 100.

Example 10 : Conditional Set Selection

$evenSet : \mathbb{P} \mathbb{N}$ $oddSet : \mathbb{P} \mathbb{N}$	$evenSet = \{ n : \mathbb{N} \mid n \bmod 2 = 0 \}$ $oddSet = \{ n : \mathbb{N} \mid n \bmod 2 = 1 \}$
---	---

Define sets that can be selected conditionally in expressions.

Example 11 : Practical Example - Tax Rate Selection

$taxRate : \mathbb{N} \rightarrow \mathbb{N}$	$\forall income : \mathbb{N} \bullet$ $taxRate(income) = \text{if } income < 20000 \text{ then } 10 \text{ elseif } income < 50000 \text{ then } 15 \text{ elseif } income < 100000 \text{ then } 20 \text{ else } 25$
---	---

Progressive tax rates with multi-level conditional.

Example 12 : Conditional with Complex Expression

$complexFunc : \mathbb{N} \rightarrow \mathbb{N}$	$\forall n : \mathbb{N} \bullet$ $complexFunc(n) = \text{if } n \bmod 3 = 0 \text{ then } n * n \text{ elseif } n \bmod 3 = 1 \text{ then } n + 10 \text{ else } n * 2$
---	--

Different computations based on remainder.

Example 13 : Guard Conditions elem Recursion

$$\left| \begin{array}{l} factorial : \mathbb{N} \rightarrow \mathbb{N} \\ \hline factorial(0) = 1 \\ \forall n : \mathbb{N} \bullet \\ \quad factorial(n) = \text{if } n > 0 \text{ then } n * factorial(n - 1) \text{ else } 1 \end{array} \right.$$

Conditional ensures factorial only recurses for positive n.

Example 14 : Ternary - Style Expression

Z notation's if-then-else is similar to C's ternary operator ($? :$):

$C : x > 0 ? 1 : -1$

$\mathbb{Z} : \text{if } x > 0 \text{ then } 1 \text{ else } -1$

Both evaluate to one of two values based on a condition.

Example 15 : Short - Circuit Evaluation Note

In Z notation, if-then-else expressions should be evaluated *lazily* : *only* the selected branch should be evaluated. This matters when branches have side effects or may be undefined.

If a key is not in a function's domain, accessing it would be undefined. A conditional ensures we only evaluate the function application when it's safe.

Example 16 : Best Practices

When using if-then-else:

1. Keep branches simple and readable
2. For many cases, consider using free types with pattern matching
3. Nest carefully—deep nesting hurts readability
4. Document complex conditions
5. Ensure both branches have the same type
6. Consider using helper predicates for complex conditions