Equivalence Chains

Solution 1

(a) *Commutative property*

$$
\begin{aligned}
& p \wedge q \\
& \quad \Leftrightarrow q \wedge p && \text{commutative}
\end{aligned}
$$

(b) *De Morgan law*

$$
\begin{aligned}
& \neg\, p \wedge q \\
& \quad \Leftrightarrow \neg\, p \vee \neg\, q && \text{De Morgan} \\
& \quad \Leftrightarrow \neg\, p \vee \neg\, q && \text{parentheses}
\end{aligned}
$$

Solution 2

(a) *Definition of implication*

$$
\begin{aligned}
& p \Rightarrow q \\
& \quad \Leftrightarrow \neg\, p \vee q && \text{definition}
\end{aligned}
$$

(b) *Multiple equivalence steps*

$$
\begin{aligned}
& p \wedge q \vee p \wedge r \\
& \quad \Leftrightarrow p \wedge q \vee r && \text{distributive} \\
& \quad \Leftrightarrow q \vee r \wedge p && \text{commutative} \\
& \quad \Leftrightarrow p \wedge r \vee q && \text{commutative}
\end{aligned}
$$

Solution 3

(a) *Equivalence chain*

without justifications

$$
\begin{aligned}
& p \vee p \\
\Leftrightarrow\ & p
\end{aligned}
$$

(b) *Mixed justifications*

$$
\begin{aligned}
& p \Leftrightarrow q \\
\Leftrightarrow\ & p \Rightarrow q \wedge q \Rightarrow p && \text{definition} \\
\Leftrightarrow\ & \neg\, p \vee q \wedge \neg\, q \vee p
\end{aligned}
$$