

Pattern Matching with Sequences

Example 1 : Empty Sequence Pattern

Pattern matching on the empty sequence:

$$\frac{f : \text{seq } \mathbb{N} \rightarrow \mathbb{N}}{f(\langle \rangle) = 0}$$

Example 2 : Cons Pattern (Head land Tail)

Decomposing a sequence into head and tail:

$$\frac{g : \text{seq } \mathbb{N} \rightarrow \mathbb{N}}{\begin{array}{l} g(\langle \rangle) = 0 \\ \forall x : \mathbb{N} \bullet \forall s : \text{seq } \mathbb{N} \bullet g(\langle x \rangle \cdot s) = x \end{array}}$$

Example 3 : Recursive Sum

Computing the sum of elements using pattern matching:

$$\frac{\text{total} : \text{seq } \mathbb{N} \rightarrow \mathbb{N}}{\begin{array}{l} \text{total}(\langle \rangle) = 0 \\ \forall x : \mathbb{N} \bullet \forall s : \text{seq } \mathbb{N} \bullet \text{total}(\langle x \rangle \cdot s) = x + \text{total}(s) \end{array}}$$

Example 4 : Cumulative Total

A more descriptive example from the solutions:

$$\frac{\text{cumulative_total} : \text{seq } \mathbb{N} \rightarrow \mathbb{N}}{\begin{array}{l} \text{cumulative_total}(\langle \rangle) = 0 \\ \forall x : \mathbb{N} \bullet \forall s : \text{seq } \mathbb{N} \bullet \text{cumulative_total}(\langle x \rangle \cdot s) = x + \text{cumulative_total}(s) \end{array}}$$

Example 5 : Filter Pattern

Filtering sequences based on a condition:

$$\frac{\text{positives} : \text{seq } \mathbb{Z} \rightarrow \text{seq } \mathbb{Z}}{\begin{array}{l} \text{positives}(\langle \rangle) = \langle \rangle \\ \forall x : \mathbb{Z} \bullet \forall s : \text{seq } \mathbb{Z} \bullet \text{positives}(\langle x \rangle \cdot s) = \begin{array}{l} \text{if } x > 0 \\ \text{then} \langle x \rangle \cdot \text{positives}(s) \\ \text{else} \text{positives}(s) \end{array} \end{array}}$$