Sequence Operations

Example 1 : Sequence Length

The cardinality operator gives the length of a sequence:

$\#\langle\rangle$
$\#\langle a, b, c\rangle$
$\#s$

Example 2 : Pattern Matching for Head and Tail

In Z, decomposing a sequence uses pattern matching with concatenation.

Given a non-empty sequence s, if we have x and t such that:

$\langle x\rangle \frown t = s$
Then x is the first element (head) and t is the remaining sequence (tail).

Example 3 : User - Defined First and Rest Functions

You can define explicit functions for sequence decomposition:

$[X]$

$$
\begin{array}{|l}
fst : \mathrm{seq}_1\ X \longrightarrow X \\
rest : \mathrm{seq}_1\ X \longrightarrow \mathrm{seq}\ X \\
\hline
\forall\, x : X \bullet \forall\, s : \mathrm{seq}\ X \bullet fst(\langle x\rangle \frown s) = x \\
\forall\, x : X \bullet \forall\, s : \mathrm{seq}\ X \bullet rest(\langle x\rangle \frown s) = s
\end{array}
$$

Example 4 : User - Defined Last and Init Functions

$$
\begin{array}{|l}
lst : \mathrm{seq}_1\ X \longrightarrow X \\
init : \mathrm{seq}_1\ X \longrightarrow \mathrm{seq}\ X \\
\hline
\forall\, x : X \bullet \forall\, s : \mathrm{seq}\ X \bullet lst(s \frown \langle x\rangle) = x \\
\forall\, x : X \bullet \forall\, s : \mathrm{seq}\ X \bullet init(s \frown \langle x\rangle) = s
\end{array}
$$

Example 5 : Domain land Range

Sequences are functions from indices to elements:

$\mathrm{dom}\langle\rangle$
$\mathrm{dom}\langle a, b, c, d\rangle$
$\mathrm{ran}\langle a, b, c\rangle$

Example 6 : Concatenation

The concatenation operator joins sequences:

$s \frown t$
$\langle 1, 2\rangle \frown \langle 3, 4\rangle$
$\langle a\rangle \frown \langle b, c\rangle \frown \langle d\rangle$

Example 7 : Using Pattern Matching

Pattern matching extracts elements from sequences:

$\forall\, s : \mathrm{seq}_1\ \mathbb{N} \bullet \exists\, x : \mathbb{N} \bullet \exists\, t : \mathrm{seq}\ \mathbb{N} \bullet s = \langle x \rangle \frown t$
This asserts that every non-empty sequence can be decomposed.

Example 8 : Recursive Function Example

Define a sum function using pattern matching style:

$$
\begin{array}{|l}
sumSeq : \mathrm{seq}\ \mathbb{N} \longrightarrow \mathbb{N} \\
\hline
sumSeq(\langle\rangle) = 0 \\
\forall\, x : \mathbb{N} \bullet \forall\, s : \mathrm{seq}\ \mathbb{N} \bullet sumSeq(\langle x \rangle \frown s) = x + sumSeq(s)
\end{array}
$$