

## Unique Existence

### Example 1 : Unique Quantifier ( exists1 )

The unique existence quantifier asserts there is exactly one value satisfying the predicate:

$$\begin{aligned}\exists_1 x : \mathbb{N} \bullet x = 5 \\ \exists_1 n : \mathbb{N} \bullet \forall m : \mathbb{N} \bullet n \leq m\end{aligned}$$

### Example 2 : Multiple Conditions

Unique existence with compound predicates:

$$\begin{aligned}\exists_1 y : \mathbb{N} \bullet y \in \{0, 1\} \wedge y \neq 1 \\ \exists_1 z : \mathbb{Z} \bullet z^2 = 4 \wedge z > 0\end{aligned}$$

### Example 3 : One - Point Rule

When a unique value is specified by equality, we can apply the one-point rule:

$$\begin{aligned}\exists y : \mathbb{N} \bullet y \in \{0, 1\} \wedge y \neq 1 \wedge x \neq y \\ \Leftrightarrow \exists y : \mathbb{N} \bullet y = 0 \wedge x \neq y & \quad [\text{arithmetic}] \\ \Leftrightarrow 0 \in \mathbb{N} \wedge x \neq 0 & \quad [\text{one - point rule}] \\ \Leftrightarrow x \neq 0\end{aligned}$$

### Example 4 : Unique elem Domain

Testing uniqueness within a specific domain:

$$\begin{aligned}\exists_1 p : \mathbb{P}\{0, 1\} \bullet \#p = 1 \\ \exists_1 x : \{1, 2, 3, 4\} \bullet x > 3\end{aligned}$$