

Phase 5 : Proof Trees

Example 1 : Simple Implication

$$\frac{\frac{\frac{\neg p \wedge q^{\neg[1]}}{q} [\wedge \text{-elim-2}]}{p \wedge q \Rightarrow q} [\Rightarrow \text{-intro}^{[1]}]}$$

Example 2 : With Sibling Premises

$$\frac{\frac{\frac{\neg p \wedge (p \Rightarrow q)^{\neg[1]}}{p} [\wedge \text{-elim-1}] \quad \frac{\frac{\neg p \wedge (p \Rightarrow q)^{\neg[1]}}{p \Rightarrow q} [\wedge \text{-elim-2}]}{\frac{q}{p \wedge q} [\wedge \text{intro}]} [\Rightarrow \text{elim}]}{\frac{p \wedge (p \Rightarrow q) \Rightarrow (p \wedge q)}{p \wedge (p \Rightarrow q) \Rightarrow (p \wedge q)} [\Rightarrow \text{-intro}^{[1]}]}$$

Example 3 : Distribution with Cases

$$\frac{\frac{\frac{\neg \text{ [from above]} \quad \frac{p}{q} \text{ [from case]}}{p \wedge q} [\wedge \text{intro}] \quad \frac{\frac{\neg \text{ [from above]} \quad \frac{p}{r} \text{ [from case]}}{p \wedge r} [\wedge \text{intro}]}{(p \wedge q) \vee (p \wedge r)} [\vee \text{-intro-1}] \quad \frac{(p \wedge q) \vee (p \wedge r)}{(p \wedge q) \vee (p \wedge r)} [\vee \text{-intro-2}]}{\frac{(p \wedge q) \vee (p \wedge r)}{p \wedge (q \vee r) \Rightarrow (p \wedge q) \vee (p \wedge r)} [\vee \text{elim}]} [\Rightarrow \text{-intro}^{[1]}]$$

Example 4 : Modus Tollens

$$\frac{\frac{\frac{\neg(p \Rightarrow q) \wedge \neg q^{\neg[1]}}{p \Rightarrow q} [\wedge \text{-elim-1}] \quad \frac{\frac{\neg(p \Rightarrow q) \wedge \neg q^{\neg[1]}}{\neg q} [\wedge \text{-elim-2}] \quad \frac{\frac{\neg p^{\neg[2]}}{q} [\Rightarrow \text{elim}]}{false} [\text{contradiction}]}{\frac{\neg p}{(p \Rightarrow q) \wedge \neg q \Rightarrow \neg p} [\Rightarrow \text{-intro}^{[1]}]} [\text{negation-intro}^{[2]}]$$

Example 5 : Solution 18 Implication to Disjunction

$$\frac{\frac{\frac{\neg p \Rightarrow q^{\neg[1]}}{p \vee \neg p} [\text{excluded middle}] \quad \frac{\frac{\frac{\neg p^{\neg[2]}}{q} [\Rightarrow \text{elim}]}{\neg p \vee q} [\vee \text{-intro-2}] \quad \frac{\frac{\neg \neg p^{\neg[2]}}{\neg p \vee q} [\vee \text{-intro-1}]}{\neg p \vee q} [\vee \text{-elim}^{[2]}]}{(p \Rightarrow q) \Rightarrow (\neg p \vee q)} [\Rightarrow \text{-intro}^{[1]}]$$