Subscripts and Superscripts

Example 1 : Basic Subscripts

Subscripts are written with underscore:

$x_1 \in S$
$x_2 = x_1 + 1$
Common for indexed variables in sequences, arrays, or iterations.

Example 2 : Basic Superscripts (Exponentiation)

Superscripts denote exponentiation:

$x^2 = x * x$
$n^3 = n * n * n$
The caret symbol creates superscripts for powers.

Example 3 : Multiple Subscripted Variables

$$\begin{array}{|l} x_1 : \mathbb{N} \\ x_2 : \mathbb{N} \\ x_3 : \mathbb{N} \\ \hline x_1 = 1 \\ x_2 = 2 \\ x_3 = 3 \end{array}$$

Define multiple indexed variables.

Example 4 : Sequence Indexing

Subscripts are natural for sequence element notation:

$\forall\, i : 1 \,..\, n \bullet s\_i \in \mathbb{N}$
Here s_i represents the i-th element of a conceptual sequence.

Example 5 : Exponentiation in Expressions

$\{\, n : \mathbb{N} \mid n < 10 \bullet n^2 \,\}$
Set of squares.

Example 6 : Power Function for Nested Exponents

For nested exponents like power of a power, define a power function:

$$\begin{array}{|l} pow : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N} \\ \hline \forall\, x : \mathbb{N} \bullet pow(x, 0) = 1 \\ \forall\, x, n : \mathbb{N} \bullet n > 0 \Rightarrow pow(x, n) = x * pow(x, n - 1) \end{array}$$

Then use it for nested exponentiation:

$pow(pow(x, 2), 3) = pow(x, 6)$
$pow(pow(n, 2), n + 1)$
This avoids LaTeX double superscript issues.

Example 7 : Polynomials

Subscripts for coefficients, superscripts for powers:

$a_0 + a_1 * x + a_2 * x^2 + a_3 * x^3$
Standard polynomial notation.

Example 8 : Combined Subscripts and Superscripts

Variable with subscript, raised to a power:

x_i^2
Meaning : *xsubi*, squared.

Example 9 : Cartesian Product Powers

Cartesian product repeated n times:

$S^2 = S \times S$
$S^3 = S \times S \times S$
Set of n-tuples from S.

Example 10 : Iteration Subscripts

Subscripts often denote iteration steps:

$x_0 = initial$
$x\_(i + 1) = f(x\_i)$
Defines a recurrence relation.

Example 11 : State Variable Subscripts

Counter

$count : \mathbb{N}$
$countNext : \mathbb{N}$

$countNext = count + 1$

Next-state variables can use descriptive names like countNext instead of primed notation.

Example 12 : Indexed Family of Sets

$$
\begin{array}{|l}
S_1 : \mathbb{P}\,\mathbb{N} \\
S_2 : \mathbb{P}\,\mathbb{N} \\
S_3 : \mathbb{P}\,\mathbb{N} \\
\hline
S_1 = \{1, 2, 3\} \\
S_2 = \{4, 5, 6\} \\
S_3 = \{7, 8, 9\}
\end{array}
$$

Family of sets indexed by natural numbers.

Example 13 : Exponents in Constraints

$\forall\, n : \mathbb{N} \bullet n^2 \geq n$
Constraint using *exponentiation* : $every natural number's square$ is at least itself.

Example 14 : Complex Subscript Expressions

$x\_(i + 1) + x\_(i - 1) = 2 * x\_i$
Recurrence relation with subscript expressions.

Example 15 : Power Set Notation

Sometimes written with superscript:

$\mathbb{P}\, S \vee 2^S$
Both denote the power set of S.

Example 16 : Best Practices

Guidelines for subscripts and superscripts:

1. Use subscripts for indices, versions, or families
2. Use superscripts for exponents and powers
3. For nested exponents, define a pow function
4. Be consistent in indexing: start at 0 or 1, not mixed
5. Document what subscripts mean