

Mu Operator (μ)

Example 1 : Basic Mu Expression

The mu operator returns "the unique value that" satisfies a predicate:

$$\begin{aligned} &(\mu x : \mathbb{N} \mid x = 5) \\ &(\mu n : \mathbb{N} \mid \forall m : \mathbb{N} \bullet n \leq m) \end{aligned}$$

Example 2 : Mu with Precondition

The \exists_1 quantifier provides a precondition to ensure mu is well-defined:

Since the proposition

$\exists_1 n : \mathbb{N} \bullet \forall m : \mathbb{N} \bullet n \leq m$
is equivalent to true, we can be certain that

$(\mu n : \mathbb{N} \mid \forall m : \mathbb{N} \bullet n \leq m)$
will return a result (which is 0).

Example 3 : Mu elem Expressions

Using mu to select unique values:

$$\begin{aligned} &(\mu a : \mathbb{N} \mid a = a) = 0 \\ &(\mu z : \mathbb{Z} \mid z = 10) = 10 \end{aligned}$$

Example 4 : Undefined Mu Expressions

When no unique value \exists , mu is undefined:

$$(\mu b : \mathbb{N} \mid b = b)$$

This is undefined because the property holds for all natural numbers, not just one.

$$(\mu c : \mathbb{N} \mid c > c)$$

This is also undefined because no natural number satisfies this property.