

Equivalence Chains

Solution 1

(a)

Commutative(property)

$$\begin{aligned} p \wedge q \\ \Leftrightarrow q \wedge p \end{aligned} \quad [\text{commutative}]$$

(b)

De(Morgan)(law)

$$\begin{aligned} \neg(p \wedge q) \\ \Leftrightarrow \neg p \vee \neg q \quad [\text{De Morgan}] \\ \Leftrightarrow \neg p \vee \neg q \quad [\text{parentheses}] \end{aligned}$$

Solution 2

(a)

*Definition
of(implication)*

$$\begin{aligned} p \Rightarrow q \\ \Leftrightarrow \neg p \vee q \end{aligned} \quad [\text{definition}]$$

(b)

Multiple(equivalence)(steps)

$$\begin{aligned} (p \wedge q) \vee (p \wedge r) \\ \Leftrightarrow p \wedge (q \vee r) \quad [\text{distributive}] \\ \Leftrightarrow (q \vee r) \wedge p \quad [\text{commutative}] \\ \Leftrightarrow p \wedge (r \vee q) \quad [\text{commutative}] \end{aligned}$$

Solution 3

(a)

Equivalence(chain)(without)(justifications)

$$\begin{aligned} p \vee p \\ \Leftrightarrow p \end{aligned}$$

(b)

Mixed(justifications)

$$\begin{aligned} p \Leftrightarrow q \\ \Leftrightarrow (p \Rightarrow q) \wedge (q \Rightarrow p) \quad [\text{definition}] \\ \Leftrightarrow (\neg p \vee q) \wedge (\neg q \vee p) \end{aligned}$$