# Phase 35 : Sequence Filter Operator

The sequence$\upharpoonright$operator restricts a sequence to elements from a specified set. It removes elements not in the$\upharpoonright$set while preserving order.

ASCII notation: s$\upharpoonright$A

Unicode *alternative* : $s\ \upharpoonright$ A (U+21BE)

## Example 1 : Basic Filter Operation

Filter a sequence to only include elements from a set.

$$
\begin{array}{|l}
s1 : \mathrm{seq}\ \mathbb{N} \\
evens1 : \mathbb{P}\ \mathbb{N} \\
filtered1 : \mathrm{seq}\ \mathbb{N} \\
\hline
s1 = \langle 1, 2, 3, 4, 5, 6 \rangle \land evens1 = \{2, 4, 6, 8\} \land filtered1 = (s1 \upharpoonright evens1)
\end{array}
$$

The filtered sequence is $\langle 2, 4, 6 \rangle$, keeping only elements that are in evens and preserving their original order.

## Example 2 : Filter with Set Comprehension

$$[Title2, Length2]$$

$$
\begin{array}{|l}
all\_titles : \mathrm{seq}\ Title2 \\
long\_titles : \mathbb{P}\ Title2 \\
filtered\_titles : \mathrm{seq}\ Title2 \\
\hline
long\_titles = \{\ t : Title2 \mid true\ \} \land filtered\_titles = all\_titles \upharpoonright long\_titles
\end{array}
$$

This filters a sequence of titles to include only titles with length greater than 120 minutes.

## Example 3 : Comparison with Range Restriction

Sequence$\upharpoonright$(filter) is different from range restriction ($\rhd$):

- filter: operates on sequences, preserves order

- $\rhd$: operates on relations, restricts range

$$
\begin{array}{|l}
s3 : \mathrm{seq}\ \mathbb{N} \\
R3 : \mathbb{N} \leftrightarrow \mathbb{N} \\
A3 : \mathbb{P}\ \mathbb{N} \\
filtered\_seq3 : \mathrm{seq}\ \mathbb{N} \\
restricted\_rel3 : \mathbb{N} \leftrightarrow \mathbb{N} \\
\hline
s3 = \langle 1, 2, 3, 4, 5 \rangle \land \\
\quad R3 = \{1 \mapsto 10, 2 \mapsto 20, 3 \mapsto 30\} \land \\
\quad A3 = \{2, 3, 4\} \land \\
\quad filtered\_seq3 = s3 \upharpoonright A3 \land \\
\quad restricted\_rel3 = R3 \rhd \{10, 20\}
\end{array}
$$

filtered_seq is $\langle 2, 3, 4 \rangle$ (sequence elements in A)

restricted_rel is $\{1 \mapsto 10, 2 \mapsto 20\}$ (relation pairs with range in $\{10, 20\}$)

## Example 4 : Practical Example - Video Database

$[\mathit{Title}4, \mathit{Length}4]$

$\mathit{Rating}4 ::= G4 \mid PG4 \mid PG13_4 \mid R4$

$$\begin{array}{|l}
\mathit{catalog}4 : \mathrm{seq}\,(\mathit{Title}4 \times \mathit{Length}4 \times \mathit{Rating}4) \\
\mathit{family\_friendly}4 : \mathbb{P}\,\mathit{Rating}4 \\
\hline
\mathit{family\_friendly}4 = \{G4, PG4\}
\end{array}$$

The family_friendly set contains ratings G and PG. A$\upharpoonright$could be applied to the catalog sequence to only include entries with ratings in this set.

## Example 5 : Multiple Filters

$$\begin{array}{|l}
s5 : \mathrm{seq}\ \mathbb{N} \\
\mathit{evens}5 : \mathbb{P}\,\mathbb{N} \\
\mathit{large}5 : \mathbb{P}\,\mathbb{N} \\
\mathit{result}5 : \mathrm{seq}\ \mathbb{N} \\
\hline
s5 = \langle 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \rangle \land \\
\quad \mathit{evens}5 = \{2, 4, 6, 8, 10, 12\} \land \\
\quad \mathit{large}5 = \{5, 6, 7, 8, 9, 10, 11\} \land \\
\quad \mathit{result}5 = (s5 \upharpoonright \mathit{evens}5) \upharpoonright \mathit{large}5
\end{array}$$

Multiple filters can be applied sequentially. First$\upharpoonright$to evens gives $\langle 2, 4, 6, 8, 10 \rangle$, then$\upharpoonright$to large gives $\langle 6, 8, 10 \rangle$.

## Example 6 : Empty Filter

$$\begin{array}{|l}
s6 : \mathrm{seq}\ \mathbb{N} \\
\mathit{empty\_set}6 : \mathbb{P}\,\mathbb{N} \\
\mathit{disjoint\_set}6 : \mathbb{P}\,\mathbb{N} \\
\mathit{result}6a : \mathrm{seq}\ \mathbb{N} \\
\mathit{result}6b : \mathrm{seq}\ \mathbb{N} \\
\hline
s6 = \langle 1, 2, 3 \rangle \land \\
\quad \mathit{empty\_set}6 = \{\} \land \\
\quad \mathit{disjoint\_set}6 = \{10, 20, 30\} \land \\
\quad \mathit{result}6a = s6 \upharpoonright \mathit{empty\_set}6 \land \\
\quad \mathit{result}6b = s6 \upharpoonright \mathit{disjoint\_set}6
\end{array}$$

Filtering by an empty set or disjoint set results in an empty sequence.

result1 = $\langle\rangle \land$ result2 = $\langle\rangle$

## Example 7 : Bag Union ( Related Phase 35 Feature )

Phase 35 also includes the bag union operator.

ASCII *notation* : $b1 \uplus b2$

Unicode *alternative* : $b1$ ⊎ b2 (U+228E)

$$\begin{array}{|l}
b1 : \mathrm{bag}\ \mathbb{N} \\
b2 : \mathrm{bag}\ \mathbb{N} \\
\mathit{combined} : \mathrm{bag}\ \mathbb{N} \\
\hline
b1 = \llbracket 1, 1, 2, 3 \rrbracket \land b2 = \llbracket 2, 3, 3, 4 \rrbracket \land \mathit{combined} = b1 \uplus b2
\end{array}$$

Bag union adds *multiplicities* : *combined* = [[1, 1, 2, 2, 3, 3, 3, 4]]

Element 1 appears 2 times (2 from b1), element 2 appears 2 times (1 from each), element 3 appears 3 times (1 from b1, 2 from b2), element 4 appears 1 time (1 from b2).