One - Point Rule Applications

Example 1 : Simple One - Point Rule

When a quantifier binds a variable that is constrained by equality, we can eliminate the quantifier:

$$
\begin{array}{ll}
& \exists\, y : \mathbb{N} \bullet y = 5 \land x > y \\
\Leftrightarrow & 5 \in \mathbb{N} \land x > 5 \qquad \text{[one - point rule]} \\
\Leftrightarrow & x > 5
\end{array}
$$

Example 2 : One - Point with Set Membership

$$
\begin{array}{ll}
& \exists\, x, y : \mathbb{N} \bullet x + y = 4 \land x < y \\
\Leftrightarrow & \exists\, x, y : \mathbb{N} \bullet y = 4 - x \land x < y \\
\Leftrightarrow & \exists\, x : \mathbb{N} \bullet 4 - x \in \mathbb{N} \land x < 4 - x \\
\Leftrightarrow & \mathit{true}
\end{array}
$$

The final equivalence holds because 0 in N, 4 - 0 in N, and $0 < 4$.

Example 3 : One - Point with Disjunction

$$
\begin{array}{ll}
& \exists\, x : \mathbb{N} \bullet (x = 1 \land x > y) \lor (x = 2 \land x > z) \\
\Leftrightarrow & (\exists\, x : \mathbb{N} \bullet x = 1 \land x > y) \lor (\exists\, x : \mathbb{N} \bullet x = 2 \land x > z) \\
\Leftrightarrow & (1 \in \mathbb{N} \land 1 > y) \lor (\exists\, x : \mathbb{N} \bullet x = 2 \land x > z) \\
\Leftrightarrow & (1 \in \mathbb{N} \land 1 > y) \lor (2 \in \mathbb{N} \land 2 > z) \\
\Leftrightarrow & 1 > y \lor 2 > z
\end{array}
$$

Example 4 : One - Point with Expression Part

$$
\begin{array}{ll}
& \exists\, y : \mathbb{N} \bullet y = x + 1 \land y > 0 \\
\Leftrightarrow & x + 1 \in \mathbb{N} \land x + 1 > 0 \qquad \text{[one - point rule]} \\
\Leftrightarrow & x + 1 > 0
\end{array}
$$