# Phase 39 : Strict Subset Operator

The strict subset operator (psubset, $\subset$) represents proper subset relationships, where A is a subset of B AND A is not equal to B.

Distinction from subset ($\subseteq$):

- A subset B means: every element of A is $\in$ B (may be equal)

- A psubset B means: every element of A is $\in$ B AND $A \neq B$ (strict inequality)

## Example 1 : Basic Strict Subset

$$\begin{array}{|l}
A : \mathbb{P}\,\mathbb{N} \\
B : \mathbb{P}\,\mathbb{N} \\
\hline
A = \{1, 2, 3\} \land B = \{1, 2, 3, 4, 5\} \land A \subset B
\end{array}$$

Here A psubset B holds because all elements of A are in B, and A contains fewer elements than B.

## Example 2 : Comparing subset land psubset

$$\begin{array}{|l}
X : \mathbb{P}\,\mathbb{N} \\
Y : \mathbb{P}\,\mathbb{N} \\
W : \mathbb{P}\,\mathbb{N} \\
\hline
X = \{1, 2\} \land Y = \{1, 2, 3\} \land W = \{1, 2\} \land X \subset Y \land X \subseteq W
\end{array}$$

X psubset Y is true (X is strictly contained in Y)

X subset W is true (X is a subset of W, including the case where $X = W$)

X psubset W would be FALSE (they are equal sets)

## Example 3 : Empty Set

$$\begin{array}{|l}
emptySet : \mathbb{P}\,\mathbb{N} \\
anySet : \mathbb{P}\,\mathbb{N} \\
\hline
emptySet = \{\} \land anySet = \{1, 2, 3\} \land emptySet \subset anySet
\end{array}$$

The empty set is a strict subset of any non-empty set.

## Example 4 : Transitive Property

If A psubset B $\land$ B psubset C, then A psubset C.

$$\begin{array}{|l}
A1 : \mathbb{P}\,\mathbb{N} \\
B1 : \mathbb{P}\,\mathbb{N} \\
C1 : \mathbb{P}\,\mathbb{N} \\
\hline
A1 = \{1\} \land B1 = \{1, 2\} \land C1 = \{1, 2, 3\} \land A1 \subset B1 \land B1 \subset C1 \land A1 \subset C1
\end{array}$$

## Example 5 : Set Hierarchy

$[PersonSS]$

$$
\begin{array}{|l}
students : \mathbb{P}\ PersonSS \\
grads : \mathbb{P}\ PersonSS \\
phds : \mathbb{P}\ PersonSS \\
\hline
phds \subset grads \land grads \subset students
\end{array}
$$

PhD students are a strict subset of graduate students, which are a strict subset of all students.