Pattern Matching with Sequences

Example 1 : Empty Sequence Pattern

Pattern matching on the empty sequence:

$$f : \text{seq}\ \mathbb{N} \longrightarrow \mathbb{N}$$
$$f(\langle\rangle) = 0$$

Example 2 : Cons Pattern (Head land Tail)

Decomposing a sequence into head and tail:

$$g : \text{seq}\ \mathbb{N} \longrightarrow \mathbb{N}$$
$$g(\langle\rangle) = 0$$
$$\forall\, x : \mathbb{N} \bullet \forall\, s : \text{seq}\ \mathbb{N} \bullet g(\langle x \rangle \frown s) = x$$

Example 3 : Recursive Sum

Computing the sum of elements using pattern matching:

$$total : \text{seq}\ \mathbb{N} \longrightarrow \mathbb{N}$$
$$total(\langle\rangle) = 0$$
$$\forall\, x : \mathbb{N} \bullet \forall\, s : \text{seq}\ \mathbb{N} \bullet total(\langle x \rangle \frown s) = x + total(s)$$

Example 4 : Cumulative Total

A more descriptive example from the solutions:

$$cumulative\_total : \text{seq}\ \mathbb{N} \longrightarrow \mathbb{N}$$
$$cumulative\_total(\langle\rangle) = 0$$
$$\forall\, x : \mathbb{N} \bullet \forall\, s : \text{seq}\ \mathbb{N} \bullet cumulative\_total(\langle x \rangle \frown s) = x + cumulative\_total(s)$$

Example 5 : Filter Pattern

Filtering sequences based on a condition:

$$positives : \text{seq}\ \mathbb{Z} \longrightarrow \text{seq}\ \mathbb{Z}$$
$$positives(\langle\rangle) = \langle\rangle$$
$$\forall\, x : \mathbb{Z} \bullet \forall\, s : \text{seq}\ \mathbb{Z} \bullet positives(\langle x \rangle \frown s) = \text{if}\ x > 0$$
$$\text{then} \langle x \rangle \frown positives(s)$$
$$\text{else} positives(s)$$