Phase 5 : Proof Trees

Example 1 : Simple Implication

$$\dfrac{\dfrac{\ulcorner p \wedge q \urcorner^{[1]}}{q}\ [\wedge\text{-elim-2}]}{p \wedge q \Rightarrow q}\ [\Rightarrow\text{-intro}^{[1]}]$$

Example 2 : With Sibling Premises

$$\dfrac{\dfrac{\dfrac{\ulcorner p \wedge (p \Rightarrow q) \urcorner^{[1]}}{p}\ [\wedge\text{-elim-1}] \quad \dfrac{\ulcorner p \wedge (p \Rightarrow q) \urcorner^{[1]}}{p \Rightarrow q}\ [\wedge\text{-elim-2}]}{q}\ [\Rightarrow \text{ elim}]}{p \wedge q}\ [\wedge \text{ intro}]}{p \wedge (p \Rightarrow q) \Rightarrow p \wedge q}\ [\Rightarrow\text{-intro}^{[1]}]$$

Example 3 : Distribution with Cases

$$\dfrac{\dfrac{\dfrac{p \quad q}{p \wedge q}\ [\wedge \text{ intro}]}{p \wedge q \vee p \wedge r}\ [\vee\text{-intro-1}] \quad \dfrac{\dfrac{p \quad r}{p \wedge r}\ [\wedge \text{ intro}]}{p \wedge q \vee p \wedge r}\ [\vee\text{-intro-2}]}{p \wedge q \vee p \wedge r}\ [\vee \text{ elim}]}{p \wedge (q \vee r) \Rightarrow p \wedge q \vee p \wedge r}\ [\Rightarrow\text{-intro}^{[1]}]$$

Example 4 : Modus Tollens

$$\dfrac{\dfrac{\ulcorner (p \Rightarrow q) \wedge \neg\, q \urcorner^{[1]}}{p \Rightarrow q}\ [\wedge\text{-elim-1}] \quad \dfrac{\ulcorner (p \Rightarrow q) \wedge \neg\, q \urcorner^{[1]}}{\neg\, q}\ [\wedge\text{-elim-2}] \quad \dfrac{\dfrac{\ulcorner p \urcorner^{[2]}}{q}\ [\Rightarrow \text{ elim}]}{\mathit{false}}\ [\text{contradiction}]}{\neg\, p}\ [\text{negation-intro}^{[2]}]}{(p \Rightarrow q) \wedge \neg\, q \Rightarrow \neg\, p}\ [\Rightarrow\text{-intro}^{[1]}]$$

Example 5 : Solution 18 Implication to Disjunction

$$\dfrac{\dfrac{\ulcorner p \Rightarrow q \urcorner^{[1]}}{p \vee \neg\, p}\ [\text{excluded middle}] \quad \dfrac{\dfrac{\dfrac{\ulcorner p \urcorner^{[2]}}{q}\ [\Rightarrow \text{ elim}]}{\neg\, p \vee q}\ [\vee\text{-intro-2}] \quad \dfrac{\ulcorner \neg\, p \urcorner^{[2]}}{\neg\, p \vee q}\ [\vee\text{-intro-1}]}{\neg\, p \vee q}\ [\vee\text{-elim}^{[2]}]}{(p \Rightarrow q) \Rightarrow \neg\, p \vee q}\ [\Rightarrow\text{-intro}^{[1]}]$$