

## Mu Operator ( $\mu$ )

### Example 1 : Basic Mu Expression

The mu operator returns "the unique value that" satisfies a predicate:

$$(\mu x : \mathbb{N} \mid x = 5)$$
$$(\mu n : \mathbb{N} \mid \forall m : \mathbb{N} \bullet n \leq m)$$

### Example 2 : Mu with Precondition

The  $\exists_1$  quantifier provides a precondition to ensure mu is well-defined:

Since the proposition

$$\exists_1 n : \mathbb{N} \bullet \forall m : \mathbb{N} \bullet n \leq m$$

is equivalent to true, we can be certain that

$$(\mu n : \mathbb{N} \mid \forall m : \mathbb{N} \bullet n \leq m)$$

will return a result (which is 0).

### Example 3 : Mu elem Expressions

Using mu to select unique values:

$$(\mu a : \mathbb{N} \mid a = a) = 0$$
$$(\mu z : \mathbb{Z} \mid z = 10) = 10$$

### Example 4 : Undefined Mu Expressions

When no unique value  $\exists$ , mu is undefined:

$$(\mu b : \mathbb{N} \mid b = b)$$

This is undefined because the property holds for all natural numbers, not just one.

$$(\mu c : \mathbb{N} \mid c > c)$$

This is also undefined because no natural number satisfies this property.