Free types and induction

$[N]$

$Tree ::= stalk \mid leaf \langle\!\langle N \rangle\!\rangle \mid branch \langle\!\langle Tree \times Tree \rangle\!\rangle$

Solution 44

The two cases of the proof are established by equational reasoning: the first by

reverse $(\langle\rangle\ ^{t}) = reverset[cat.1a] = (reverset)\langle\rangle$ [cat.1b]

where cat.1a is $\langle\rangle$ s = s$andcat.1biss\langle\rangle$ = s

and the second by

reverse $((\langle x\rangle\ ^{u})^{t}) = reverse(\langle x\rangle\ ^{(}u^{t}))[cat.2]$

= reverse (u $^{t})\langle x\rangle$ [reverse.2]

= (reverse t $^{r}everseu)\langle x\rangle$ [anti-distributive]

= reverse t $^{(}reverseu\langle x\rangle)$ [cat.2]

= reverse t $^{r}everse(\langle x\rangle\ ^{u})[reverse.2]$

Solution 45

The base case:

reverse (reverse $\langle\rangle$) = reverse $\langle\rangle$ [reverse.1] = $\langle\rangle$ [reverse.1]

The inductive step:

reverse (reverse ($\langle x\rangle$ ^{t}))

= reverse ((reverse t) $\langle x\rangle$) [reverse.2]

= reverse ($\langle x\rangle$) $^r everse(reverset)[anti - distributive]$

= reverse ($\langle x\rangle$ $\langle\rangle$) $^r everse(reverset)[cat.1]$

= ((reverse $\langle\rangle$) $\langle x\rangle$) $^r everse(reverset)[reverse.2]$

= ($\langle\rangle$ $\langle x\rangle$) $^r everse(reverset)[reverse.1]$

= $\langle x\rangle$ $^r everse(reverset)[cat.1]$

= $\langle x\rangle$ $^t[reverse(reverset) = t]$

Solution 46

(a)

count : $Tree \to N$

count $stalk = 0$

$\forall\, n \colon N \bullet \mathit{count}(\text{leaf n}) = 1$

$\forall\, t1, t2 \colon \mathit{Tree} \bullet \mathit{count}(\mathit{branch}(t1, t2)) = \mathit{count}\text{t1} + \text{count t2}$

(Blocked by: recursive free types and pattern matching)

(b)

flatten : $\mathit{Tree} \to \mathit{seq}$ N

flatten stalk = $\langle\rangle$

$\forall\, n \colon N \bullet \mathit{flatten}(\text{leaf n}) = \langle n \rangle$

$\forall\, t1, t2 \colon \mathit{Tree} \bullet \mathit{flatten}(\mathit{branch}(t1, t2)) = \mathit{flatten}\text{t1}\ ^{f}\mathit{lattent}2$

(Blocked by: recursive free types and pattern matching)

Solution 47

First, exhibit the induction principle for the free type:

P stalk and ($\forall\, n \colon N \bullet P$(leaf n)) and ($\forall\, t1, t2 \colon \mathit{Tree} \bullet \mathbb{P}\ t1 \land \mathbb{P}\ t2 \Rightarrow \mathbb{P}\ \mathit{branch}(t1, t2)$)

implies $\forall\, t \colon \mathit{Tree} \bullet \mathbb{P}\ t$

This gives three cases for the proof:

(flatten stalk) = $\langle\rangle$ [flatten] = 0 [] = count stalk [count]

(Remaining cases omitted - require equational reasoning with recursive functions)