

## Phase 11 a : Function Types

Total functions (every element in domain has a mapping):

$$\mid f : X \rightarrow Y \quad g : \mathbb{N} \rightarrow \mathbb{N}$$

Partial functions (some elements may not have a mapping):

$$\mid f : X \rightarrowtail Y$$

Injections (one-to-one mappings):

$$\mid f_{total} : X \rightarrowtail Y \quad f_{partial} : X \rightarrowtail Y$$

Surjections (onto mappings):

$$\mid f_{total} : X \twoheadrightarrow Y \quad f_{partial} : X \twoheadrightarrow Y$$

Bijections (one-to-one and onto):

$$\mid f : X \xrightarrow{\sim} Y$$

Complex function types (nested and higher-order):

$$\mid f : X \rightarrow Y \rightarrow \mathbb{Z} \quad g : X \rightarrow (Y \rightarrow \mathbb{Z}) \quad h : \mathbb{N} \rightarrow \mathbb{N} \rightarrow (\mathbb{N} \rightarrow \mathbb{N})$$

Mixed operators (relations and function types work together):

$$\left| \begin{array}{l} relation : X \leftrightarrow Y \\ function : X \rightarrow Y \end{array} \right| \quad function \subseteq relation$$