

$$variables = \begin{array}{c} \mathbf{Bid} \quad P_H \quad P_G \quad P_P \quad E \quad d \quad T \quad P_{DL} \\ \left[\begin{array}{cccccccc} x0 & x1 & x2 & x3 & x4 & x5 & x6 & x7 \end{array} \right] \end{array}$$

$$\begin{array}{c|cc|c} Bid_i & 0 & & x_0 \\ P_{Hs} & 1 & s & x_1 \\ P_{Gs} & s+1 & 2s & x_2 \\ P_{Ps} & 2s+1 & 3s & x_3 \\ E_s & 3s+1 & 4s & x_4 \\ d_s & 4s+1 & 5s & x_5 \\ T_s & 5s+1 & 6s & x_6 \\ P_{DLs} & 6s+1 & 7s & x_7 \end{array}$$

Equations:

Eq. 0: Regulation costs for period $i \Rightarrow [0 : 0]$

$$T_i = \sum_{s=1}^S T_{s,i} \cdot \rho_s$$

$$Eq. 0 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} \\ 0 & 0 & 0 & 0 & 1 & -1 \cdot \rho & 0 \end{bmatrix}$$

$$b = [0]$$

Eq. 1: Hydro Pumped Storage (1) $\Rightarrow [1 : s]$

\hookrightarrow 1st hour

$$E_{s,1} = e_begin$$

$$Eq. 1 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$b = [e_begin]$$

Eq. 2: Hydro Pumped Storage (2) $\Rightarrow [s + 1 : s + 1]$

\hookrightarrow 1st hour

$$P_{H,i} = e_begin \cdot \frac{\eta_{hydro}}{t}$$

$$Eq. 2 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$b = [e_begin \cdot \frac{\eta_{hydro}}{t}]$$

Eq. 3: Hydro Pumped Storage (3) $\Rightarrow [s + 2 : 2s + 1]$

\hookrightarrow remaining hours

$$E_{s,i} = E_{s,i-1} + t \cdot \left[\eta_P \cdot P_{Ps,i} - \frac{P_{Hi}}{\eta_H} \right]$$

$$Eq. 3 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} \\ 1_{t-1} \cdot \frac{t}{\eta_H} & -1_{t-1} \cdot \eta_P \cdot t & 1_{t-1} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$b = [0]$$

Eq. 4: Imbalance for every scenario $\Rightarrow [2s + 2 : 3s + 1]$

$$d_{s,i} = P_{WGi} - P_{Ws,i} + P_{Ps,i} + P_{Sps,i}$$

$$Eq. 4 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} & P_{Spi} & P_{Sps,i} \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} P_{Ws,i} \end{bmatrix}$$

Eq. 5: Spill for period i $\Rightarrow [3s + 2 : 3s + 2]$

$$P_{Spi} = \sum_{s=1}^S P_{Sps,i}$$

$$Eq. 5 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} & P_{Spi} & P_{Sps,i} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \end{bmatrix}$$

Eq. 6: Pump Power for period i $\Rightarrow [3s + 3 : 3s + 3]$

$$P_{Pi} = \sum_{s=1}^S P_{Ps,i}$$

$$Eq. 6 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} & P_{Spi} & P_{Sps,i} & P_{Pi} \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \end{bmatrix}$$

Eq. 7: Pump Power for period 24 $\Rightarrow [3s + 4 : 4s + 3]$

$$P_{Ps,i_{24}} = 0$$

$$Eq. 6 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} & P_{Spi} & P_{Sps,i} & P_{Pi} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \end{bmatrix}$$

Eq. 8: Energy for period 24 $\Rightarrow [4s + 4 : 5s + 3]$

$$E_{s,i_{24}} - \frac{P_{H_{24}}}{\eta_H} = 0$$

$$Eq. 6 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} & P_{Spi} & P_{Sps,i} & P_{Pi} \\ -\frac{1}{\eta_H} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \end{bmatrix}$$

Inequalities:

Ine. 0: Hydro Generation $\Rightarrow [0 : s - 1]$

$$P_{Hi} \leq \eta_H \cdot \left[\frac{E_{s,i}}{t} + \eta_P \cdot P_{Ps,i} \right]$$

$$Ad. 0 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} & P_{Spi} & P_{Sps,i} & P_{Pi} \\ 1 & -t \cdot \eta_H \cdot \eta_P & -\frac{\eta_H}{t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$bd = [0]$$

Ine. 1: Epigraph Form (1) $\Rightarrow [s : 2s - 1]$

$$d_{s,i} \cdot p_i^+ \leq T_{s,i}$$

$$Ad. 1 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} & P_{Spi} & P_{Sps,i} & P_{Pi} \\ 0 & 0 & 0 & 0 & 0 & -1 & p_plus & 0 & 0 & 0 \end{bmatrix}$$

$$bd = [0]$$

Ine. 2: Epigraph Form (2) $\Rightarrow [2s : 3s - 1]$

$$-d_{s,i} \cdot p_i^- \leq T_{s,i}$$

$$Ad. 2 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} & P_{Spi} & P_{Sps,i} & P_{Pi} \\ 0 & 0 & 0 & 0 & 0 & -1 & -p_minus & 0 & 0 & 0 \end{bmatrix}$$

$$bd = [0]$$

Ine. 3: Hydro + Pump Constraint $\Rightarrow [3s : 4s - 1]$

$$P_H + P_{Ps,i} \leq P_H^M$$

$$Ad. 3 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} & P_{Spi} & P_{Sps,i} & P_{Pi} \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$bd = [P_H^M]$$

Ine. 4: Wind Power Constraint $\Rightarrow [4s : 5s - 1]$

$$P_{Ws,i} \leq P_{WGi} + P_{Ps,i} + P_{Sps,i}$$

$$Ad. 4 = \begin{bmatrix} P_{Hi} & P_{Ps,i} & E_{s,i} & P_{Gi} & T_i & T_{s,i} & d_{s,i} & P_{Spi} & P_{Sps,i} & P_{Pi} \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$$bd = [-P_{Ws,i}]$$

Ine. 5: Pump + Spill Constraint $\Rightarrow [5s : 6s - 1]$

$$P_{Ps,i} + P_{Sps,i} \leq P_{Ws,i}$$

$$Ad. 4 = \begin{array}{c} P_{Hi} \quad P_{Ps,i} \quad E_{s,i} \quad P_{Gi} \quad T_i \quad T_{s,i} \quad d_{s,i} \quad P_{Spi} \quad P_{Sps,i} \quad P_{Pi} \\ \left[\begin{array}{cccccccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \end{array}$$

$$bd = \left[P_{Ws,i} \right]$$