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1 # Groundwater Modeling Coding Assignment #2
2 # Jim Finnegan
3 # 1D Transport Equation
4 # Finite Element Method
5
6 # SETUP
7 import numpy as np
8 from scipy.sparse import diags
9 from matplotlib import pyplot as plt
10
11 # user inputs
12 D = float(input('Enter D (m^2/d): '))
13 R = float(input('Enter R: '))
14
15 # other parameters
16 v, L, dx, t, dt = 0.1, 200, 2, 400, 10
17 # matrix dimensions
18 rows = int(t / dt) + 1
19 n_el = int(L / dx)
20 cols = int(L / dx) + 1
21
22 # initial conditions
23 C0 = np.zeros(cols)
24 C = np.zeros((rows, cols))
25 C[:, 0] = 1 # boundary condition: C/C0 = 1 at x=0
26
27 # CONSTRUCT STIFFNESS AND STORAGE MATRICES
28 # element matrices
29 alpha = (R * dx) / 6
30 lam_1 = D / dx
31 lam_2 = v / 2
32 Ae = [[lam_1 - lam_2, -lam_1 + lam_2], [-lam_1 - lam_2, lam_1 + lam_2]] # element stiffness matrix
33 Be = [[2 * alpha, alpha], [alpha, 2 * alpha]] # element storage matrix
34 # global matrices
35 A = np.zeros((cols, cols))
36 B = np.zeros((cols, cols))
37 for i in range(cols):
38     A[i, i] += Ae[1][1] # assemble Ae elements
39     A[i, i - 1] += Ae[1][0]
40     A[i - 1, i] += Ae[0][1]
41     A[i - 1, i - 1] += Ae[0][0]
42     B[i, i] += Be[1][1] # assemble Be elements
43     B[i, i - 1] += Be[1][0]
44     B[i - 1, i] += Be[0][1]
45     B[i - 1, i - 1] += Be[0][0]
46
47 # finite element equation matrices
48 A_f = (A/2 + B/dt)
49
50 # TIME STEPPING
51 for k in range(1, rows):
52     b_f = np.dot((-A/2 + B/dt), C[k-1, :]) # solve LHS
53     C[k, :] = np.linalg.solve(A_f, b_f) # solve RHS

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54     C[k][0] = 1                                # boundary condition at
        source
55     C[k][-1] = 0                                # boundary condition at
        end
56
57 # PLOT
58 x = np.linspace(0, L, num=cols)
59 plt.plot(x, C[0, :])
60 plt.plot(x, C[10, :])
61 plt.plot(x, C[20, :])
62 plt.plot(x, C[30, :])
63 plt.plot(x, C[40, :])
64
65 plt.title('Finite Element Solution')
66 plt.xlabel('distance (m)')
67 plt.ylabel('C/C0')
68 plt.legend(['0 days', '100 days', '200 days', '300 days', '400 days'
        ])
69 plt.show()
70
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