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CEE 279 Project 2

June 8th 2011

Objective

Solve the two-dimensional shallow water equations using the second order accurate MacCormack Scheme.

Governing shallow water flow equations

$$U_t + E_x + F_y + S = 0$$

$$U = \begin{bmatrix} h \\ uh \\ vh \end{bmatrix}$$

$$E = \begin{bmatrix} uh \\ u^2h + \frac{gh^2}{2} \\ uvh \end{bmatrix}$$

$$F = \begin{bmatrix} vh \\ uvh \\ u^2h + \frac{gh^2}{2} \end{bmatrix}$$

$$S = \begin{bmatrix} 0 \\ -gh(S_{0_x} - S_{f_x)} \\ -gh(S_{0_y} - S_{f_y)} \end{bmatrix}$$

The MacCormack Scheme (MacCormack 1969) used average of a prediction step and a correcting step to estimate U^{t+1}

Predicting step

$$U_{i,j}^{*} = U_{i,j}^{k} - \frac{\Delta t}{\Delta x} \bigtriangledown_x E_{i,j}^{k} - \frac{\Delta t}{\Delta y} \bigtriangledown_y F_{i,j}^{k} - \Delta t S_{i,j}^{k}$$

Correcting step

$$U_{i,j}^{**} = U_{i,j}^{*} - \frac{\Delta t}{\Delta x} \Delta_x E_{i,j}^{*} - \frac{\Delta t}{\Delta y} \Delta_y F_{i,j}^{*} - \Delta t S_{i,j}^{*}$$

Where $\bigtriangledown$ refers to a backward step and Δ refers to a backward step

Problem 1

Boundary conditions: a square 20 x 50 container, with no leakage.

Initial conditions: with respect to the center of the containing boundaries

$$h_{x,y} = h_0 + h_1 e^{-\frac{x^2+y^2}{4}}$$

$$u_{x,y} = 0$$

Initial conditions

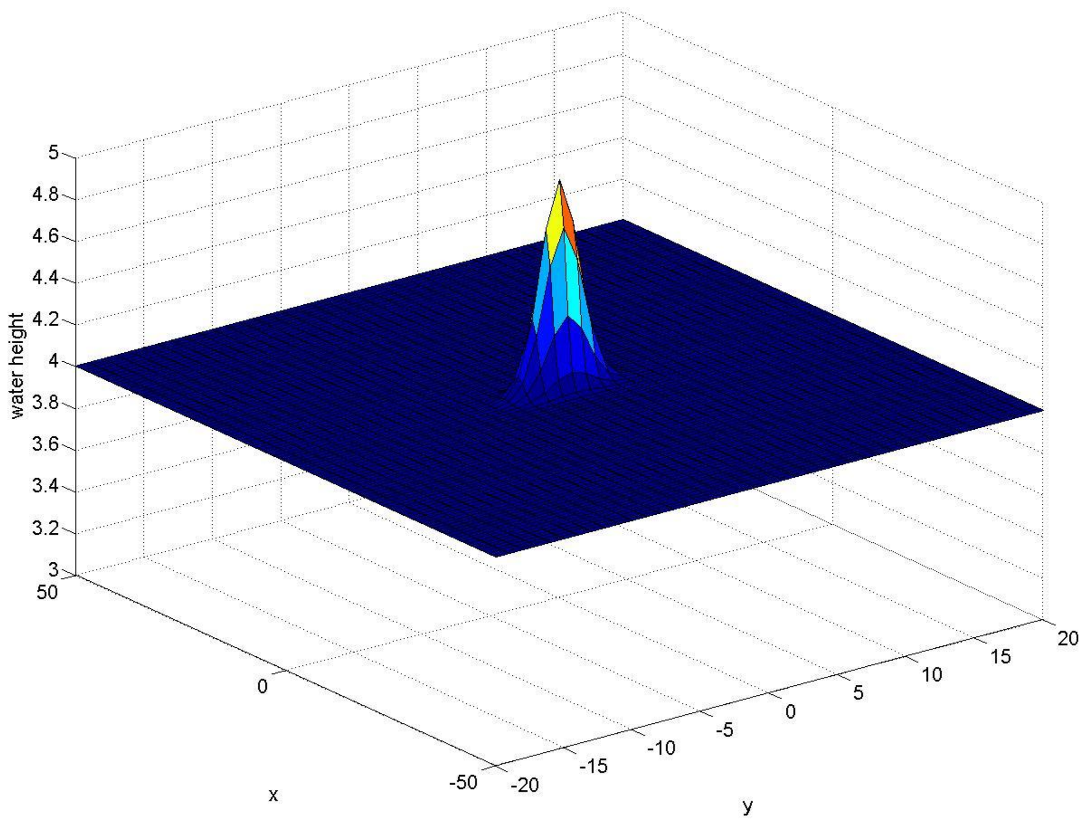


Figure 1 shows the initial condition of the water. The velocity is zero throughout. There is no gradient steep enough to require the use of artificial viscosity.

Initial wave

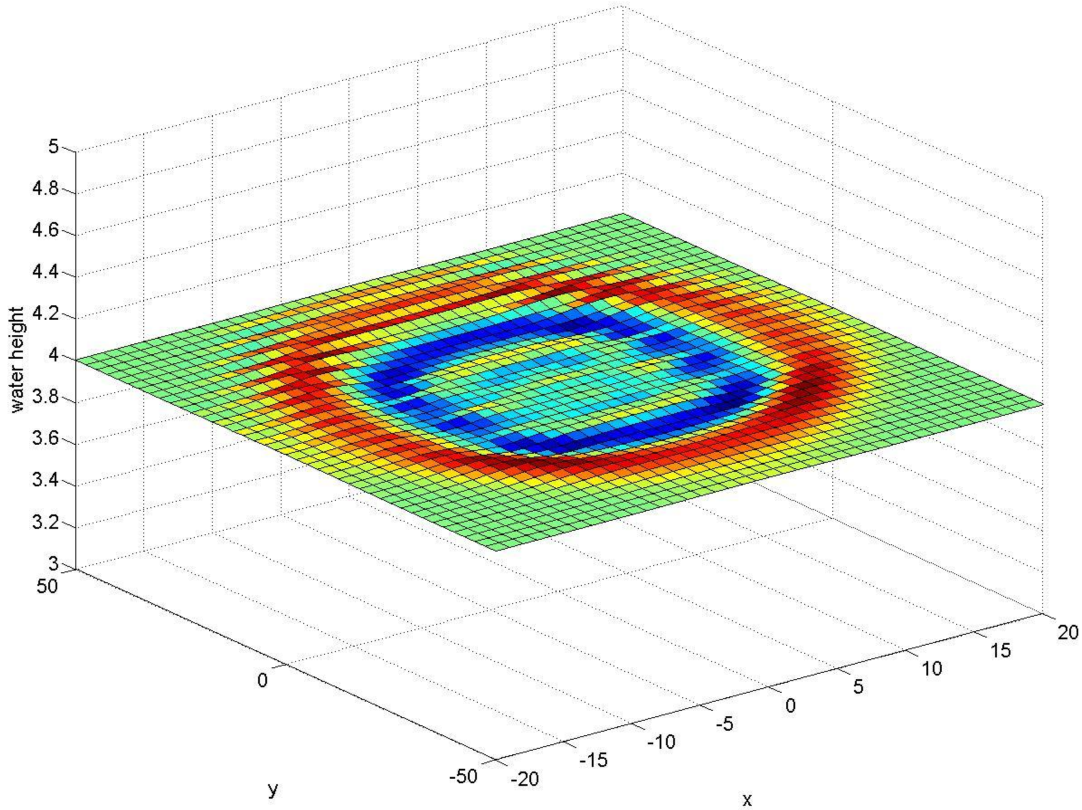


Figure 2 shows the initial wave resulting from the initial condition propagates outward.

Boundary conditions

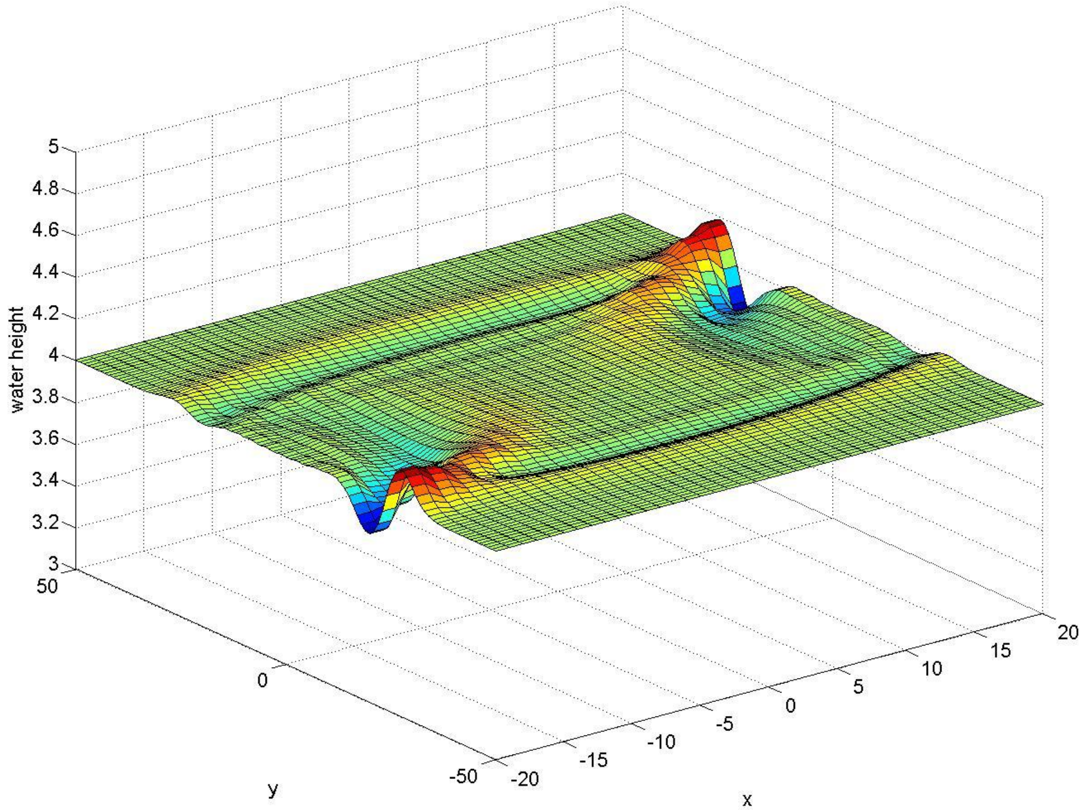
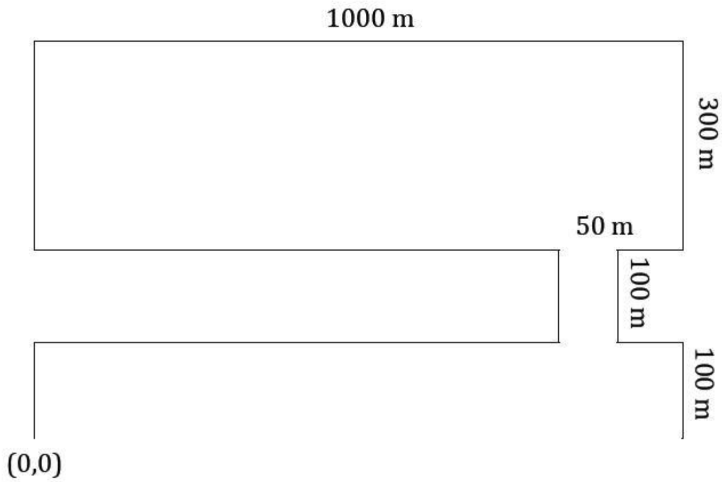


Figure 3. When coming in contact with the no flow boundary, the wave is reflected and refracted back into the system with the same momentum as going into the wall.

Problem 2

Simulating an idealized harbor as a wave is blocked by a barrier



initial conditions: $h(x,y) = 5m; u(x,y) = 0$

boundary condition: $h = \sin\left(\frac{t}{\pi}\right)$ at y=0

To implement the barrier as boundary conditions, a binary grid was created with 1s representing bounds, and 0s representing free surface.

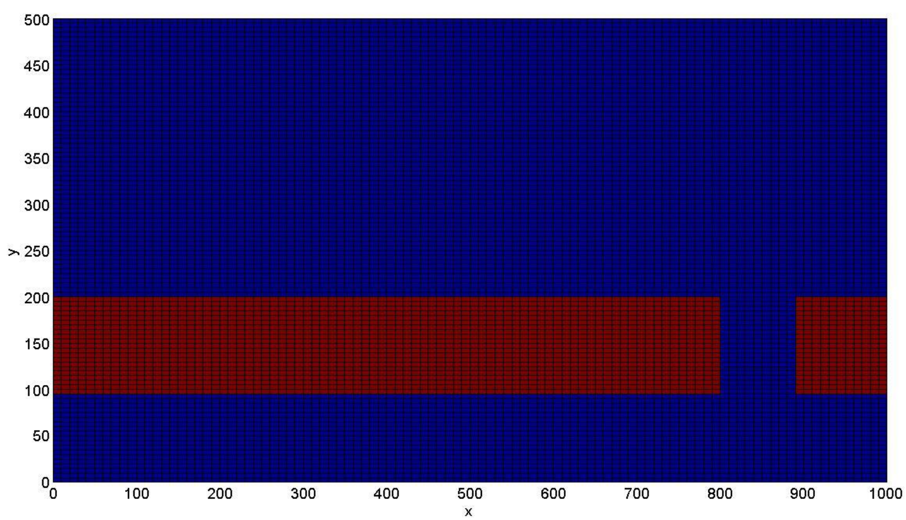


Figure 4. A binary grid representing the boundary of the ideal harbor.

Using conditions in the loop creating the h, uh and vh values, the harbor binary grid was used to block waves directly propagating through the entire system space. The initial condition was test run without the harbor boundary condition for comparison

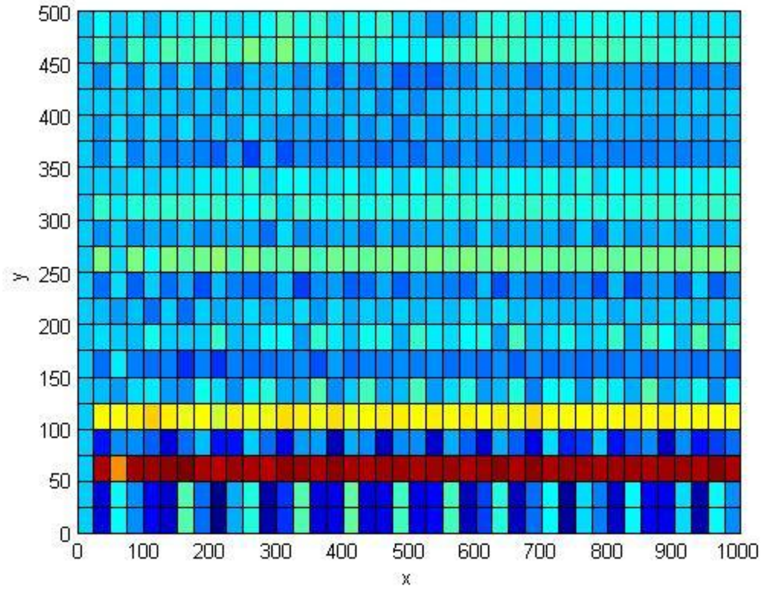


Figure 5. The propagation of waves through a 1000 x 500 boundary at t = 200 with no harbor boundary.

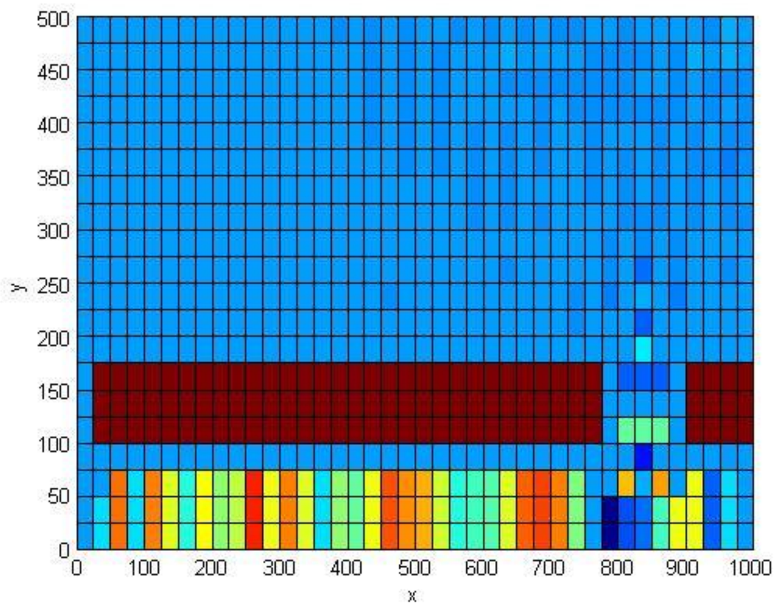


Figure 6. The propagation of waves through a 1000 x 500 boundary at t = 200 with a harbor boundary. It can be seen that the wave activity outside the harbor is far greater than inside.

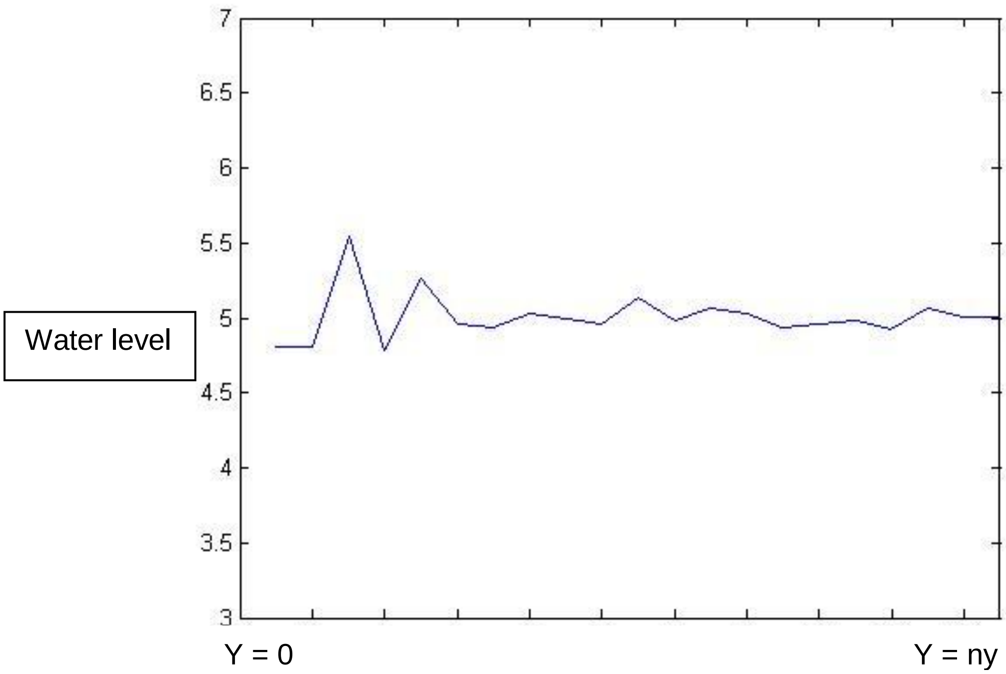


Figure 7. the longitudinal profile of the water surface at time = 200 with no harbor boundary. At x = 900, through the harbor opening.

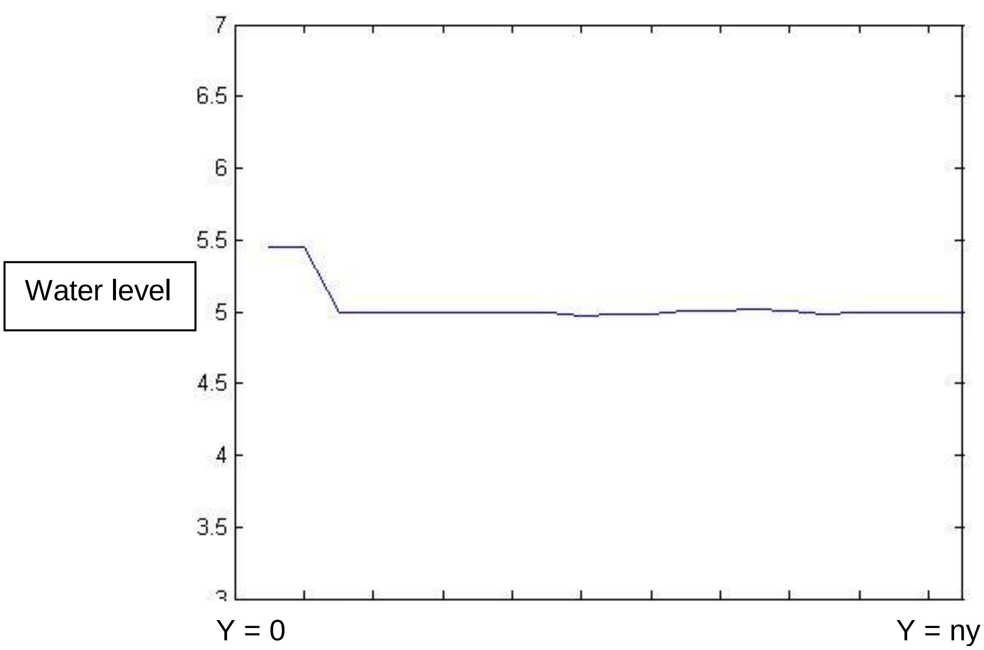


Figure 8. the longitudinal profile of the water surface at time = 200 with a harbor boundary. As compared to figure 6, there is very small amplitude along the latter half of the profile. At x = 900, through the harbor opening.

Discussion

Although the initial condition of the simulation did not have gradients with a great enough magnitude to require the use of artificial viscosity, it was necessary to include artificial viscosity when dealing with boundaries. It can be seen in figure 3 that the reflection of waves off of the no flow boundaries create high gradient sections. It was also possible to dampen these sections by increasing manning's n coefficient to a value higher than a realistic system would have naturally.

There was no benchmarking was done with analytical solutions.